Google's AdWords Market: How Theory Influenced Practice

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Two major achievements of Google

1). Search: That is robust to spam

2). Advertising: Business model

Google Search: vioxx

http://www.google.com/search?q=vioxx&sourceid=mozilla-search

Getting Started | Latest Headlines | Computational Co... | The New York Tim... | Slate Magazine | Harvard EconCS Group

Office Depot Online - Search Res... | Google Search: vioxx

Google

Web | Images | Groups | News | Froogle | Local New! | more »

vioxx [Search] Advanced Search | Preferences

Web Results **1** - **10** of about **9,990,000** for **vioxx**. (**0.38** seconds)

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Vioxx-Center.com Expected to reach $18,000,000,000! For **Vioxx** Damages - Get More Info.

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www.VioxxSettlements.com Receive compensation for injuries sustained due to taking **Vioxx**

[vioxx.com](#)
MERCK. Print Button. Once Daily **VIOXX** ... Merck Announces Voluntary Worldwide Withdrawal of **VIOXX** ®. WHITEHOUSE STATION, NJ, Sept. 30, 2004—Merck & Co., Inc. ...
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This page will be redirected to the **Vioxx** Information Page in 10 seconds. Or you may click on this link http://www.fda.gov/cder/drug/infopage/**vioxx**/default.htm.
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[Vioxx Side Effects, and Drug Interactions - Rofecoxib - RxList ...](#)
... Approximately 3600 patients with osteoarthritis were treated with **VIOXX**; approximately 1400 patients received **VIOXX** for 6 months or longer and approximately ...
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Vioxx Heart Attack or Stroke?
Protect Your Legal Rights Here.
Vioxx.AlexanderLaw.com
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Revolution in advertising

- Matching merchants with customers
  - How can a merchant pitch ads in a targeted way to customers who are interested in his goods.
- Difficulty: How to find out needs/desires of customers in a super-efficient manner?
- Insight: A user's search queries reveals to Google a succinct window into her mind/needs

- Solution: Auction off queried keywords to advertisers!
- This converts a giant undefined matching problem into a gigantic auction!
- Google is world’s biggest auction house: billions daily!

How to allocate keywords to advertisers?

- Advertisers bid for specific keywords
- Maximize Google's revenue – efficient solution!

- GREEDY: Display the ad of highest bidder
  - Assume: only 1 ad is shown

- Charge bid, or

  second highest bid (Vickery auction)

This soln. was being used!

Google

Web Images Groups News Froogle Local New! more »

vioxx Search Advanced Search Preferences

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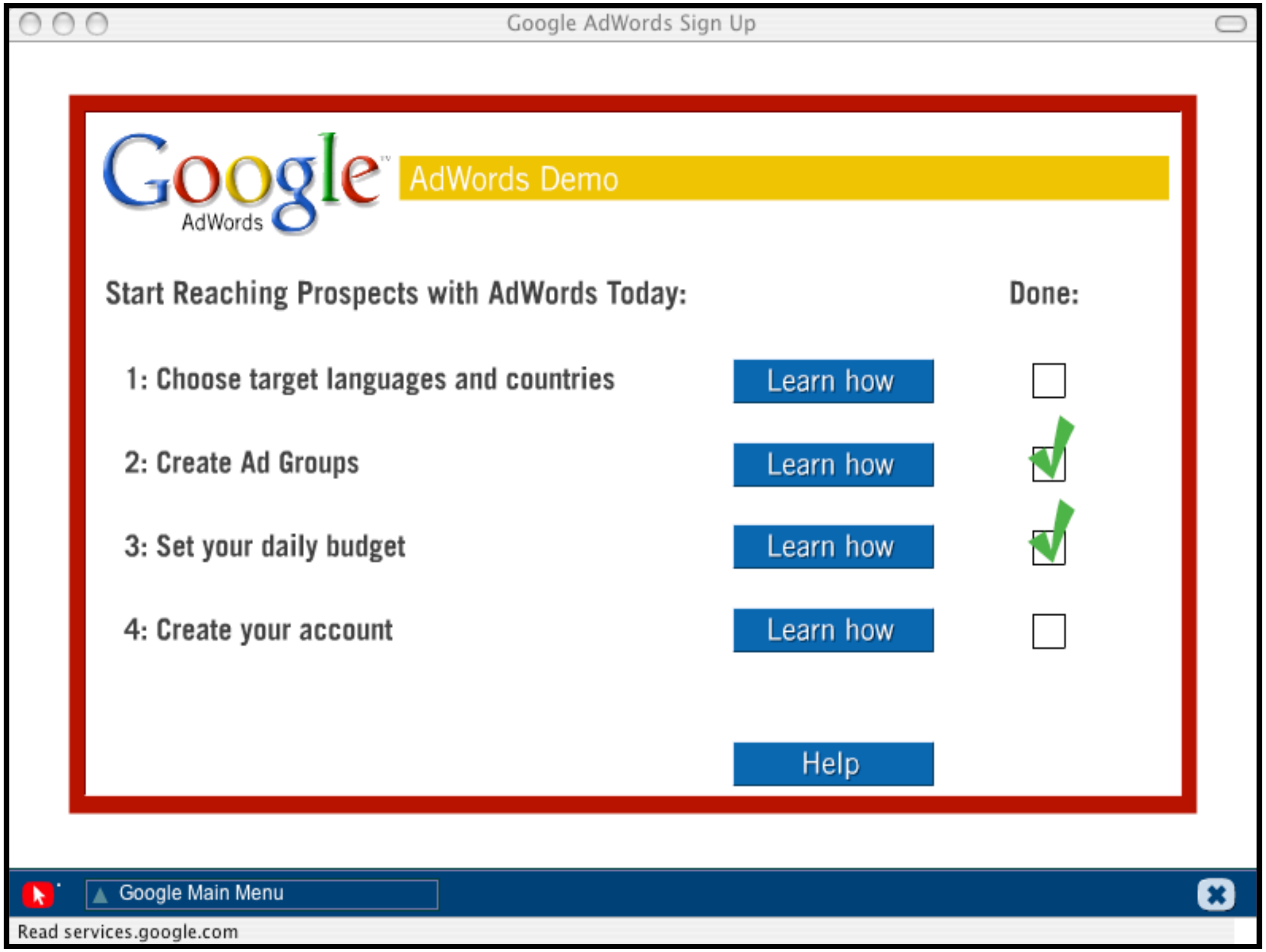
Problem …

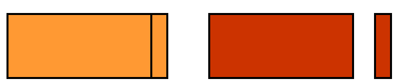

Google AdWords Sign Up
Google
AdWords
AdWords Demo
Start Reaching Prospects with AdWords Today:
Done:
1: Choose target languages and countries
Learn how
2: Create Ad Groups
Learn how
3: Set your daily budget
Learn how
4: Create your account
Learn how
Help
Google Main Menu
Read services.google.com

Example:

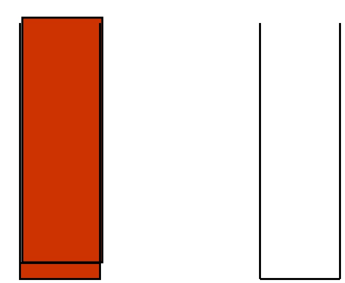
	Bidder1	Bidder 2
Book	\$1	\$0.99
CD	\$1	\$0

B1 = B2 = \$100

Queries: 100 Books then 100 CDs



LOST



Bidder 1 Bidder 2

Revenue
\$100

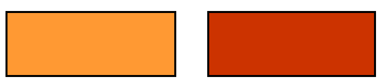
Algorithm Greedy

Example:

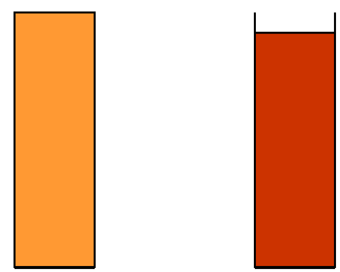
	Bidder1	Bidder 2
Book	\$1	\$0.99
CD	\$1	\$0

B1 = B2 = \$100

Queries: 100 Books then 100 CDs



Optimal Allocation



Bidder 1 Bidder 2

Revenue
\$199

Must incorporate budgets in solution

- Large fraction of advertisers are budget-constrained
- Budget distribution is heavy-tailed

  Must handle small budget advertisers adequately
- Solution must be real time and particularly simple!

Adwords Problem

Mehta, Saberi, Vazirani & V, 2005:

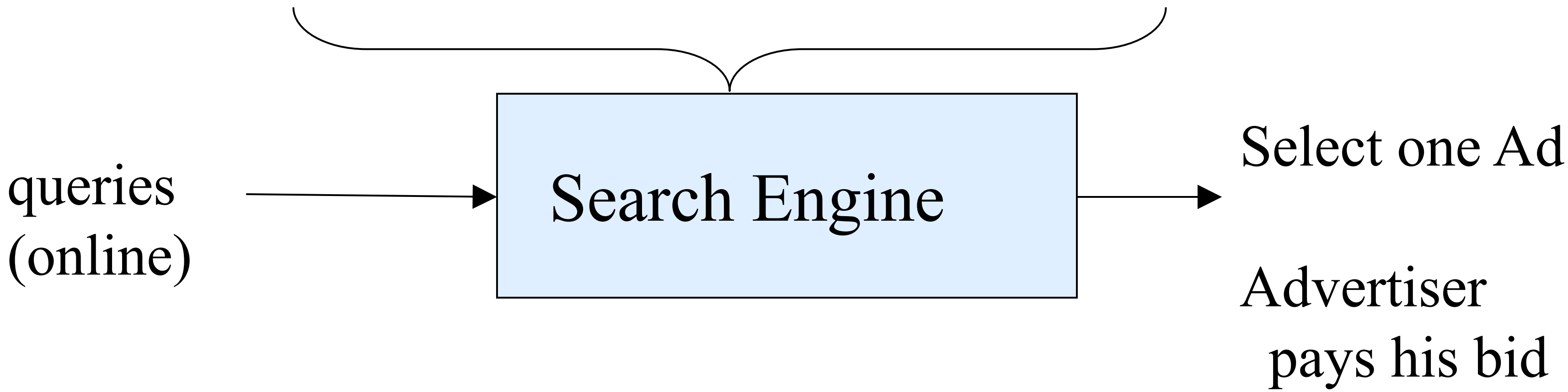
Simple framework that captures essential features.

Will add bells and whistles later!

The Adwords Problem

N advertisers.

- Daily Budgets $B_1, B_2, \ldots, B_N = \$1$
- Each advertiser provides bids for keywords he is interested in.



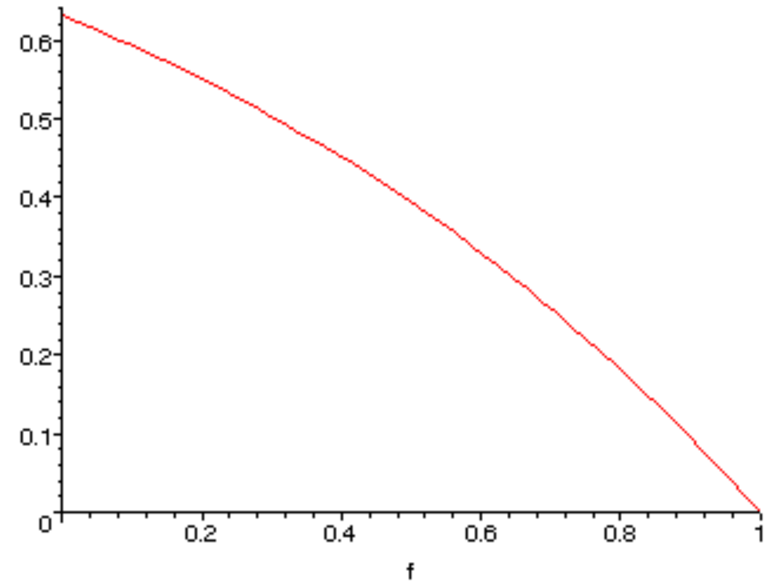
Online competitive analysis

- Compare revenue of online algorithm with that of best offline allocation.
- E.g., GREEDY has competitive ratio of ½.

Algorithm

- Will use agents'
  - Bid
  - Fraction of budget spent
- "Correct" tradeoff between them given by a special function f

Algorithm



$$f(\alpha) = 1 - e^{-(1-\alpha)}$$

Award next query to bidder with
max bid $\times f$ (fraction of budget spent)

$1 - 1/e$ competitive ratio, assuming bid << budget

Simple, minimalistic solution!

Optimal!

Impact

- Provided a general framework for thinking about budget constrained auctions for many Ad products

- Bid scaling used widely in industry

  for ad auctions and for display ads:
  - Very fast: one operation per bid
  - Low on memory: one number per bid
  - No extra communication

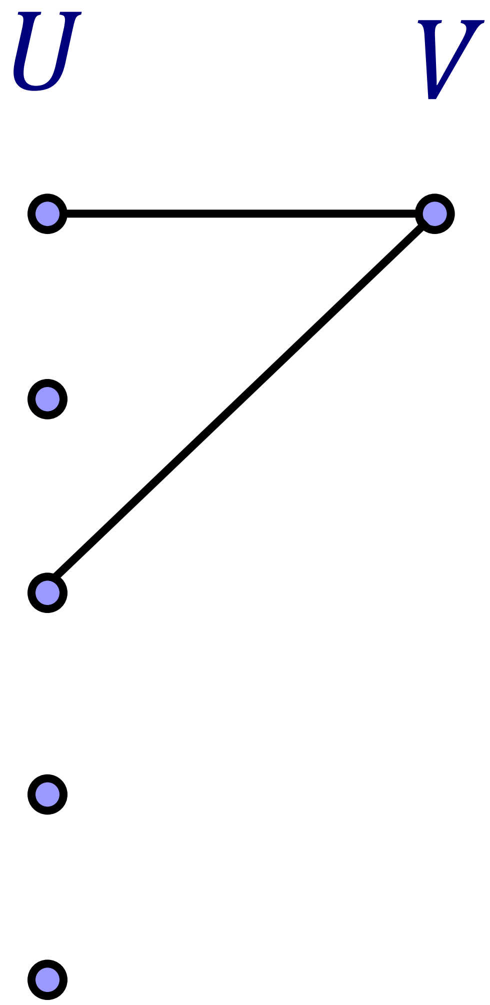
New algorithmic work

- Numerous models:
  - Queries arrive in random order
  - Queries are picked from a distribution
  - Submodular constraints on allocations
  - Use historical data about query arrival
  - . . .

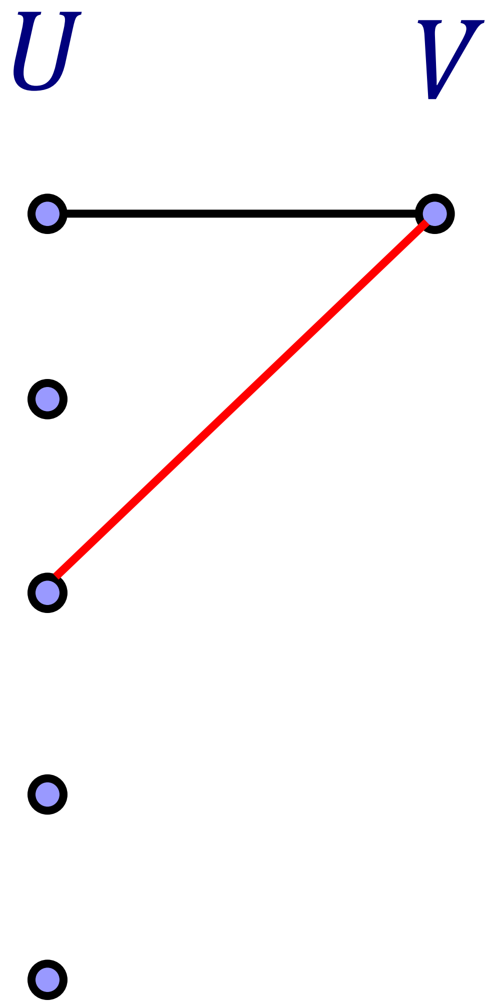
Solution has roots in "pure" theory, in particular, Matching Theory!

Online bipartite matching

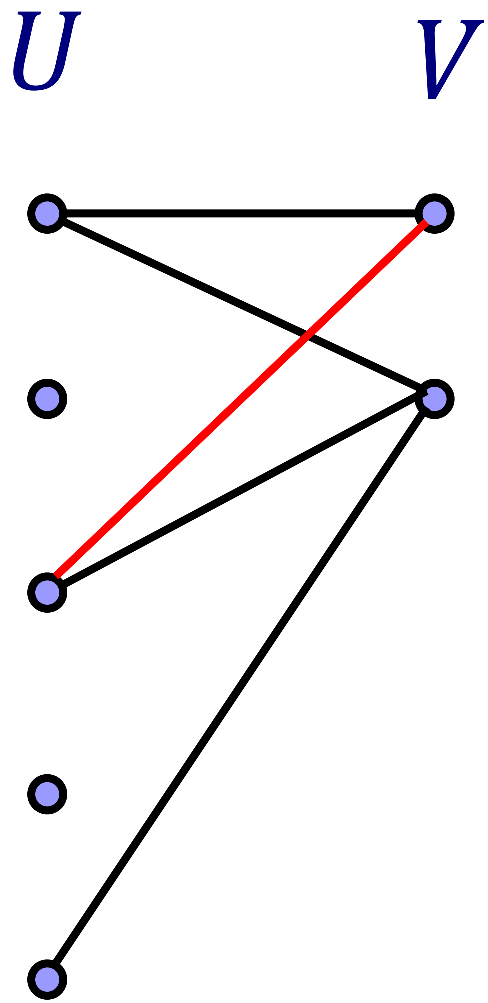
Online bipartite matching



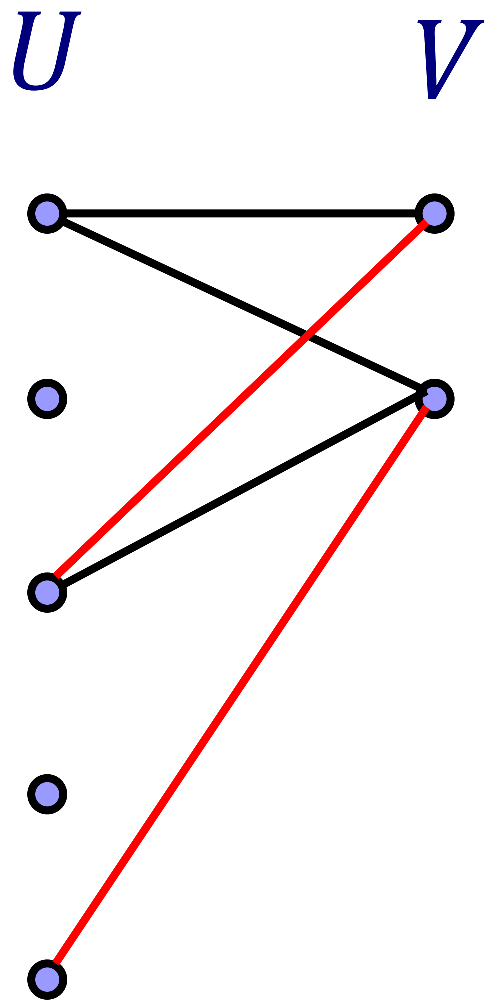
Online bipartite matching



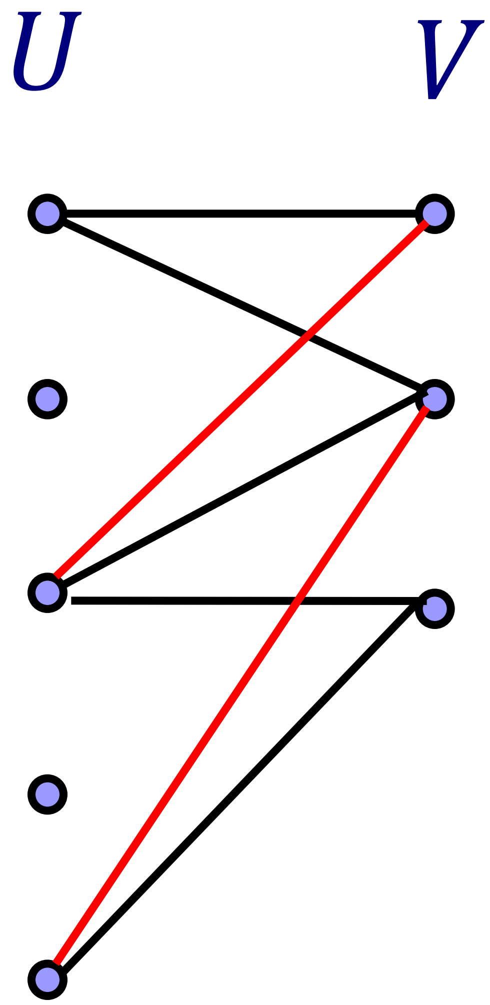
Online bipartite matching



Online bipartite matching



Online bipartite matching



- Any deterministic algorithm has competitive ratio of 1/2.

- Karp, Vazirani & V., 1990: $1 - 1/e$ factor randomized algorithm

Optimal!

- Same as simple case of Adwords problem: All budgets \$1, all bids \$0/1.

Online *b*-matching

- Kalyanasundaram & Pruhs, 1996:

  $1 - 1/e$ factor algorithm for *b*-matching:

  Daily budgets $\$b$, bids $\$0/1$, $b >> 1$

BALANCE: Assign query to interested bidder who has spent least so far.

Optimal!

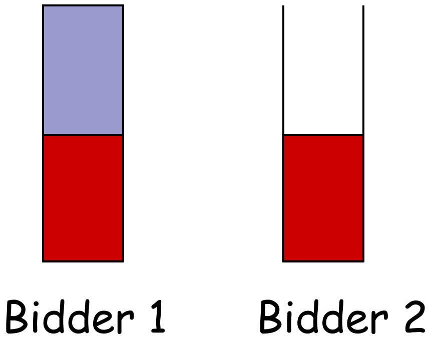
Example: Balance

	Bidder1	Bidder 2
Book	$1	$1
CD	$1	$0

$B_1 = B_2 = \$100$

Queries: 100 Books then 100 CDs

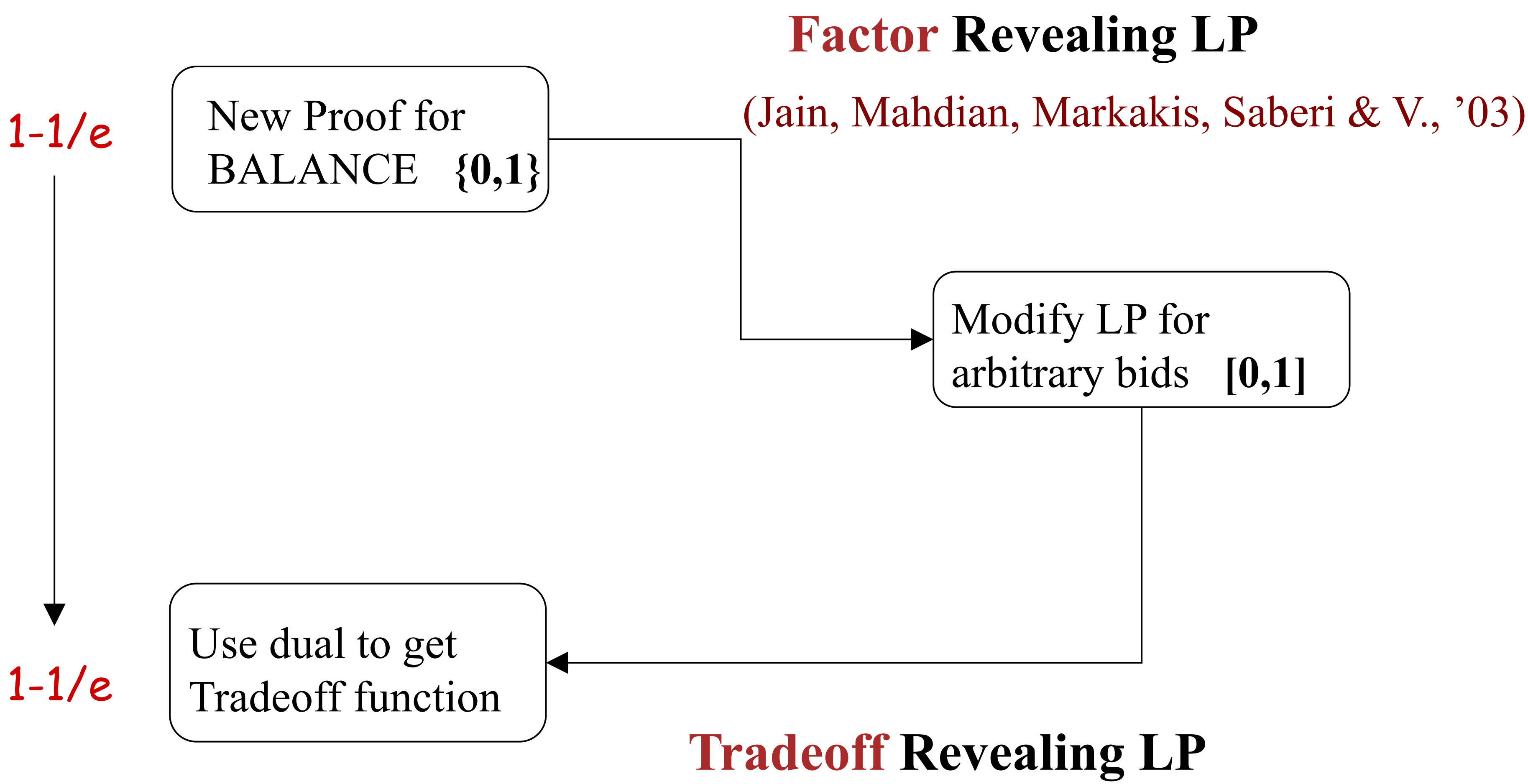
Balance Algorithm



Revenue
$1.50

$\frac{3}{4}$-competitive

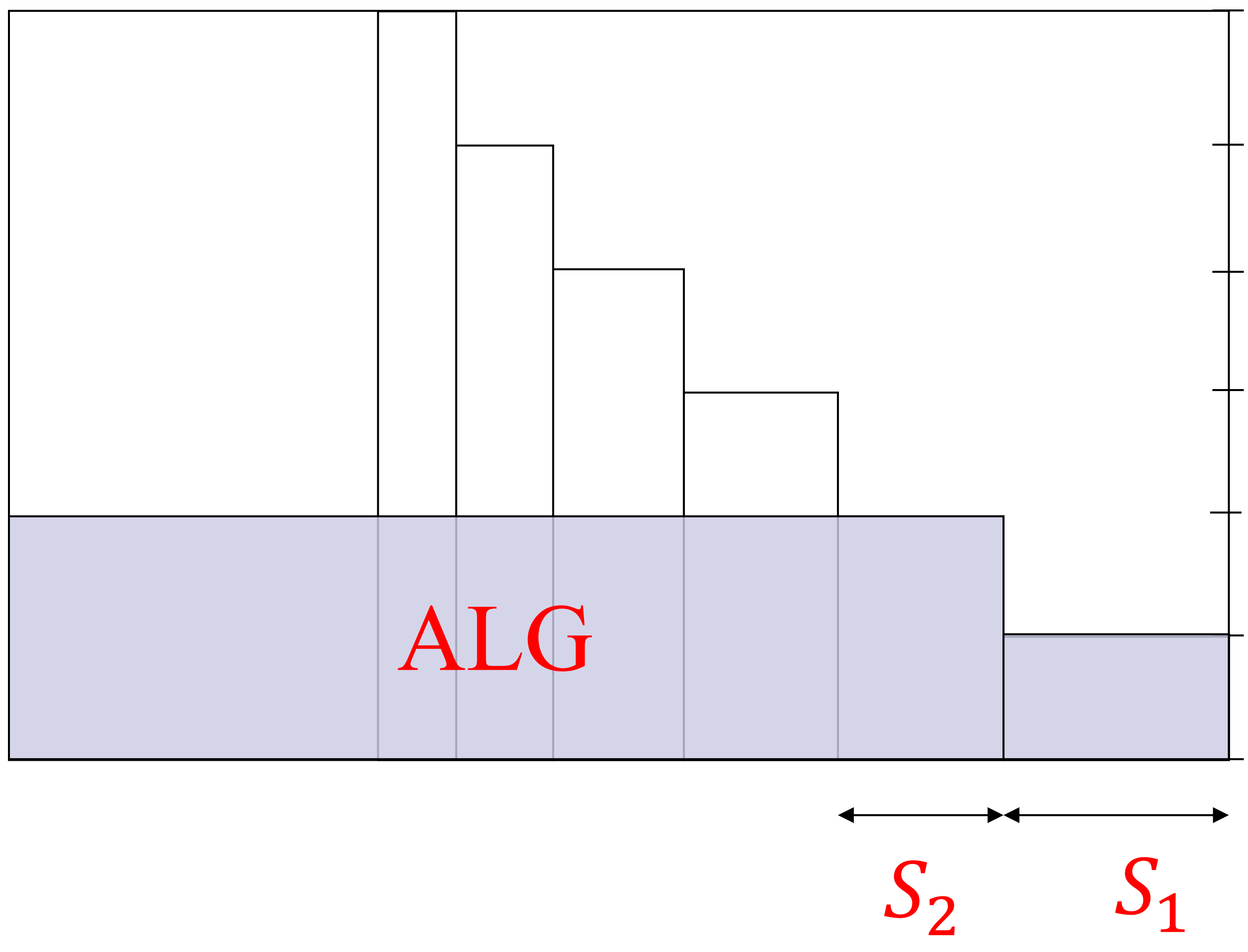
Where did this tradeoff function come from?



New proof of BALANCE

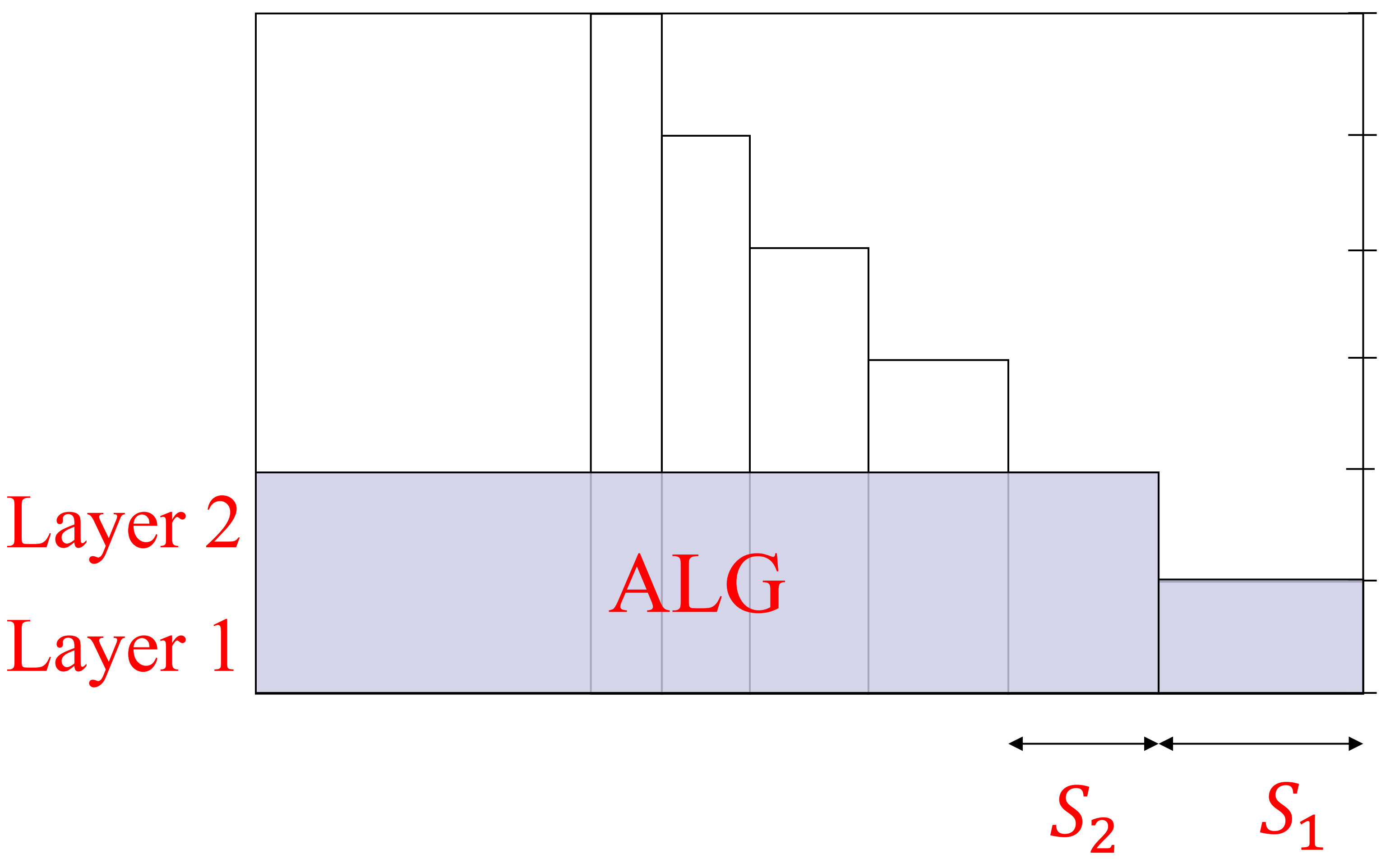
- N bidders, \$1 each. Each bid is $\$\epsilon$.
- OPT: optimal offline, ALG: BALANCE
- OPT = N
- Will show: ALG $\geq N\left(1 - \frac{1}{e}\right)$

- Idea: Upper bound no. of bidders who spent less!
- Assume k large
- S_i: Bidders who spent $\left[\frac{i-1}{k}, \frac{i}{k}\right)$
- Let $|S_i| = x_i$. Will constrain $x_1,\ x_1 + x_2, \ldots$


ALG
S_2
S_1

- Partition revenue of ALG

- $\$\varepsilon \in layer\ i$ if bidder had $\left[\frac{i-1}{k}, \frac{i}{k}\right)$ when this money was spent.


Layer 2
Layer 1
ALG
S_2
S_1

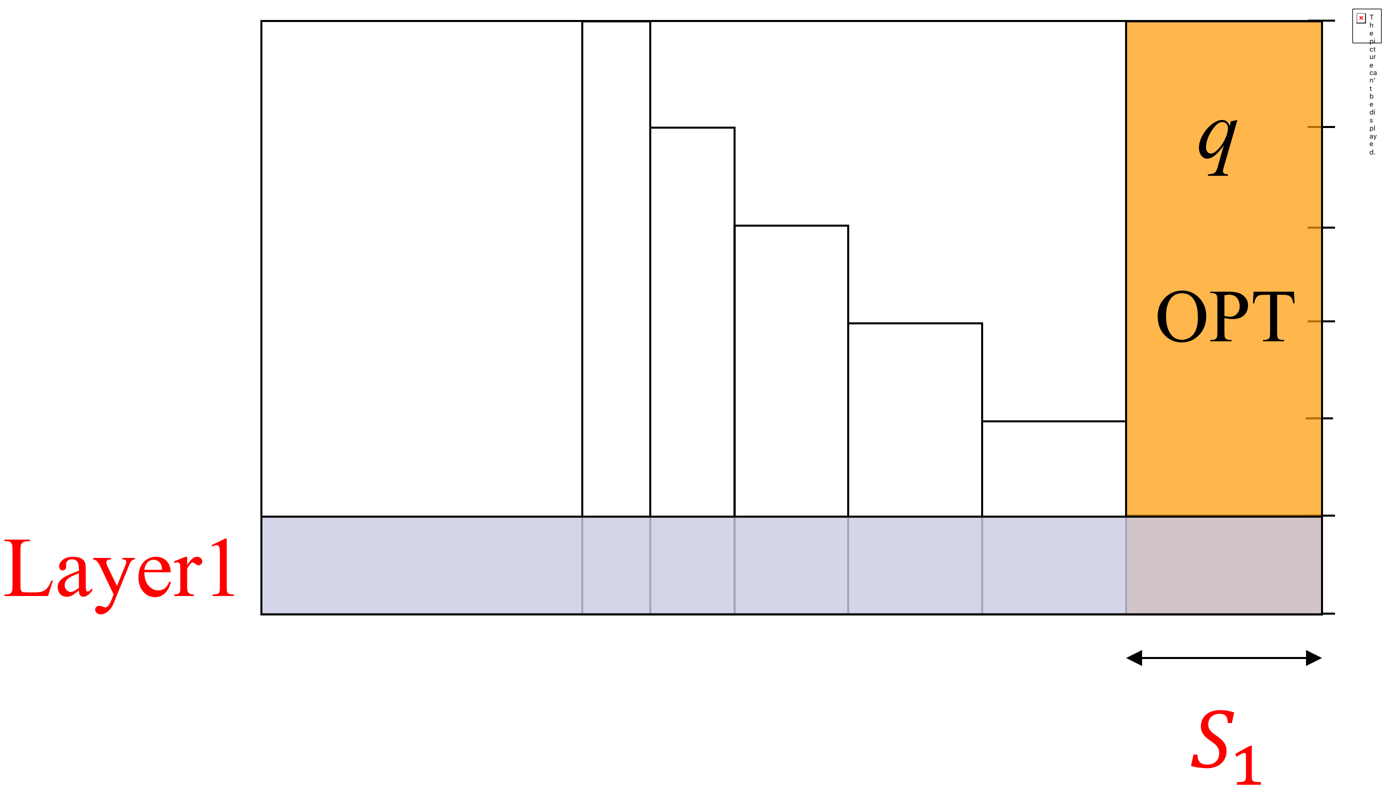
- Partition revenue of ALG

- $\$\varepsilon \in layer\ i$ if bidder had $\left[\frac{i-1}{k}, \frac{i}{k}\right)$ when this money was spent.

- BALANCE assigns next query to interested bidder who has spent least so far.

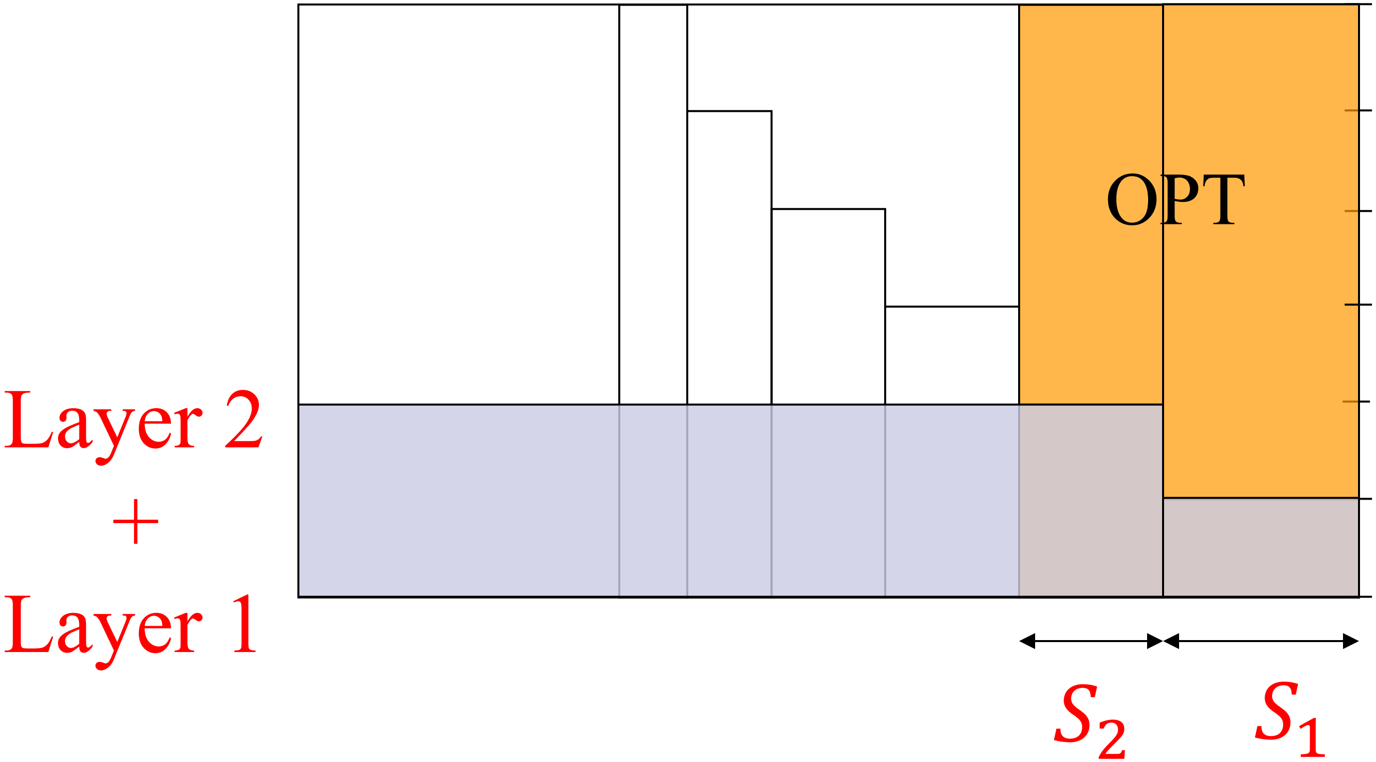
$$\text{Layer } 1 \leq \frac{N}{k}$$

- First Constraint: $|S_1| = x_1 \leq layer\ 1 \leq \frac{N}{k}$



$$\text{Layer 2} \leq \frac{N - x_1}{k}$$

Second Constraint: $|S_1 \cup S_2| = x_1 + x_2 \leq \frac{N}{k} + \frac{N - x_1}{k}$



Lower bound on ALG

LP(N): $\min \sum_i \frac{i}{k} x_i$

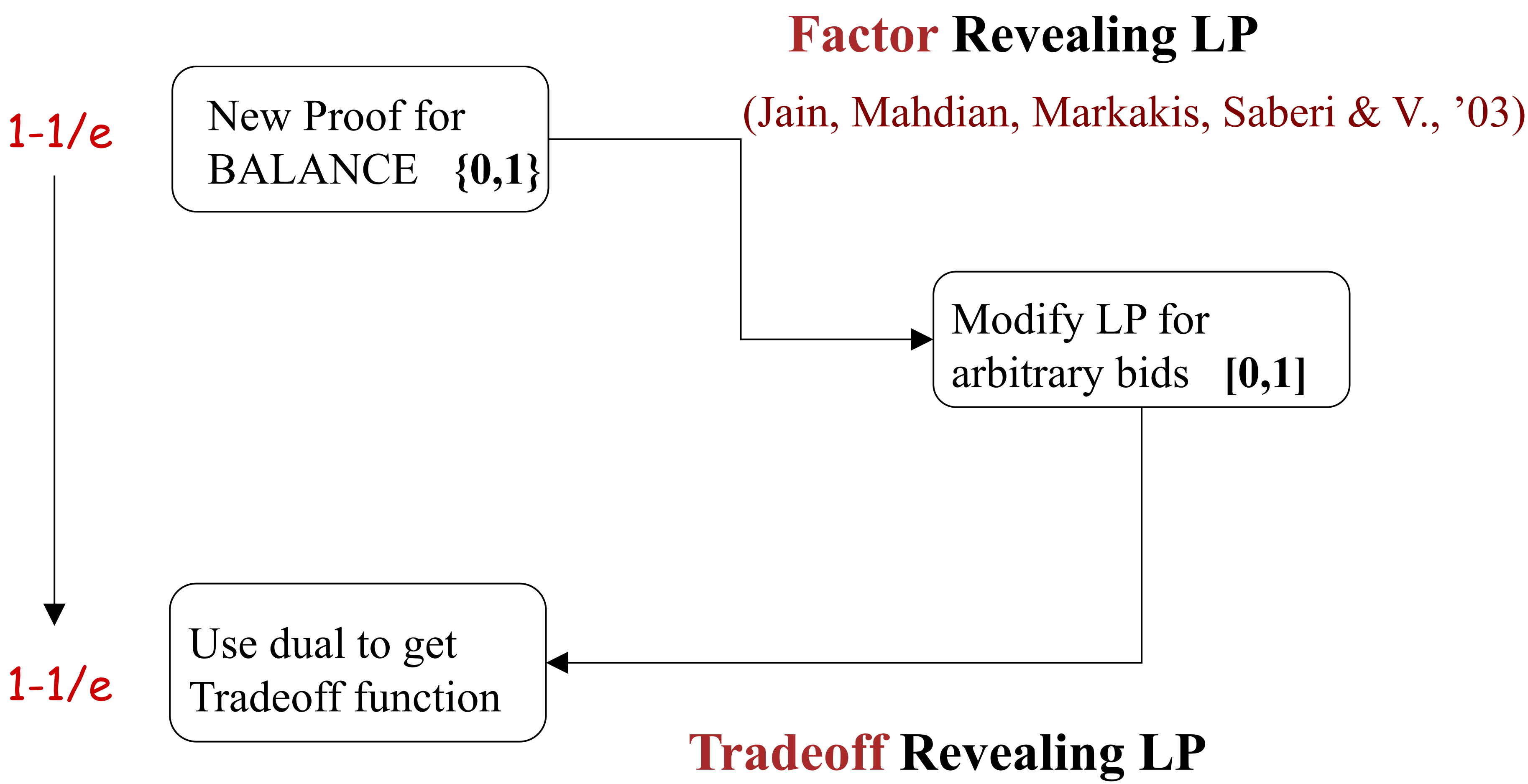
s.t. constraints on $x_1, x_2, \ldots x_k$

$$\text{ALG} \geq N\left(1-\left(1-\frac{1}{k}\right)^k\right) \geq N\left(1-\frac{1}{e}\right)$$

Factor revealing LP

- Jain, Mahdian, Markakis, Saberi & V., '03
- Family of LPs: LP(N) encodes problem of finding lower bound for instance of size N.
- Infimum of optimal solutions gives approximation factor

Where did this tradeoff function come from?



Larger Theme

AdWords, Amazon, eBay,
Yahoo!, Alibaba, Uber,
Apple (iTunes), Airbnb,
Cloud Computing …

Markets on the Internet

- Numerous new algorithmic and game-theoretic issues raised.

- Much scope for creative work that can have a huge impact!