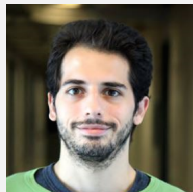


# ridesharing

Sid Banerjee

School of ORIE, Cornell University



based on work with – D. Freund, T. Lykouris (Cornell),  
– C. Riquelme & R. Johari (Stanford),

special thanks to the data science team at Lyft

# ridesharing platforms

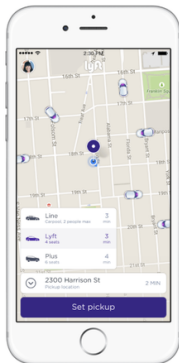


- critical components of modern urban transit
- crucible for Real-Time Decision Making/Ops Management/EconCS

## How Lyft Works

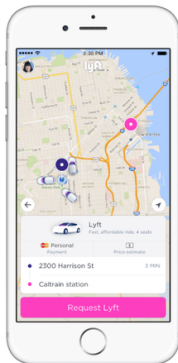
### 1. Request

Whether you're riding solo or with friends, you've got options. Tap to request Lyft, Lyft Line, or Lyft Plus.



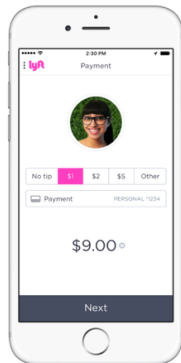
### 2. Ride

Get picked up by the best. Our reliable drivers will get you where you need to go.



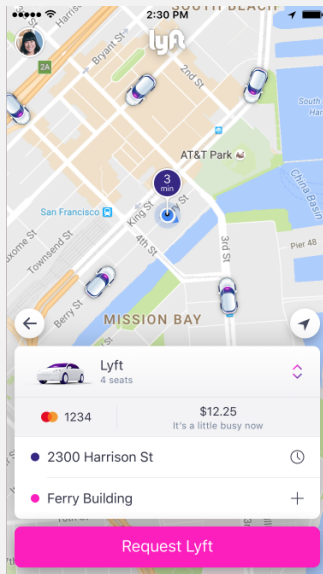
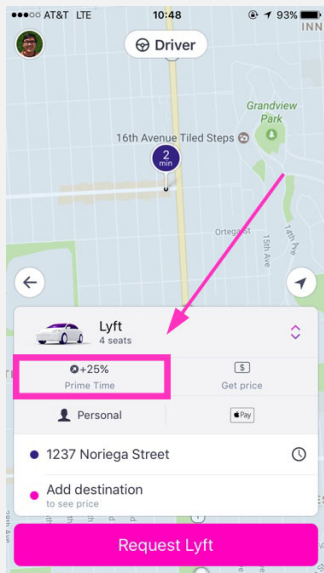
### 3. Pay

When the ride ends, just pay and rate your driver through your phone.



credit: lyft.com


# ridesharing: pricing



credit: lyft.com

# rideshare platforms: pricing

**Thanks for riding with [redacted]!**  
Ride ending January 31 at 12:35 AM



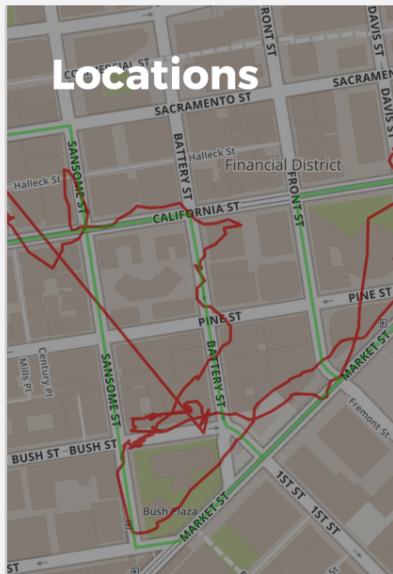
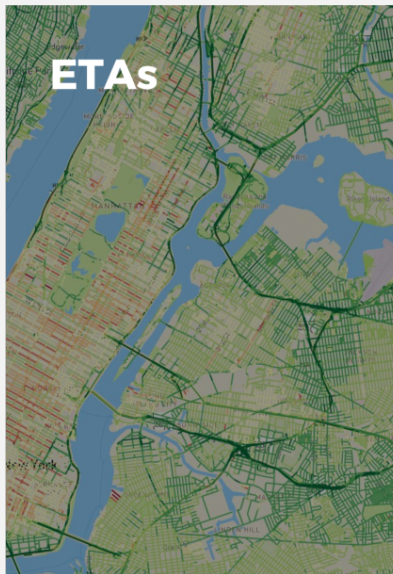
Pickup: [redacted]  
Dropoff: [redacted]

Ride 2.5 mi & 10 min:	\$8.28
Prime Time*:	\$2.07
Trust & Safety Fee:	\$1.50
<b>Total charged to [redacted]</b>	<b>\$11.85</b>

\*25% Prime Time was included in your total. Prime Time encourages more people to drive when Lyft gets really busy.  
[Learn More](#)

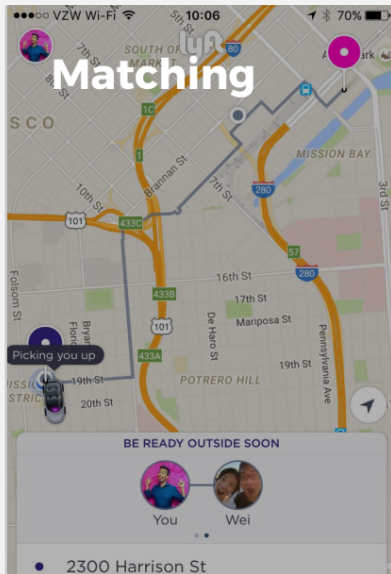
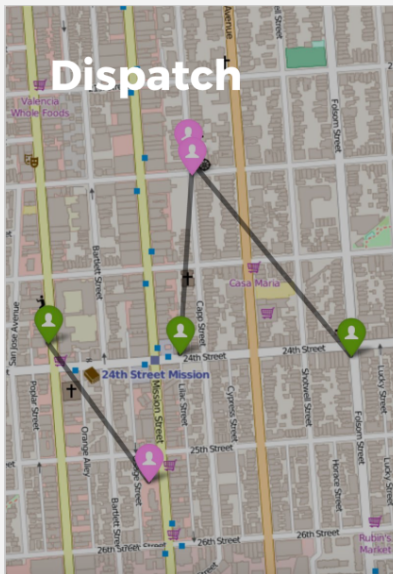
credit: lyft.com

# rtdm in ridesharing: mapping



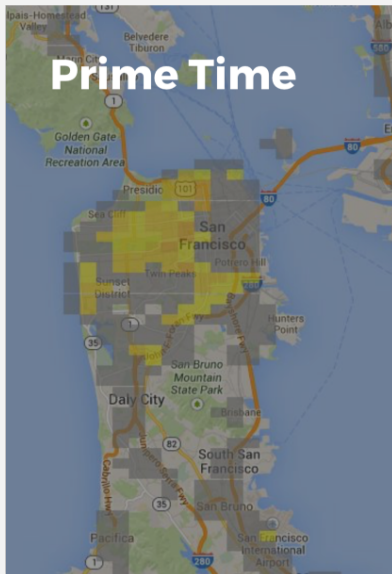
credit: lyft data science team

# rdtm in ridesharing: logistics



credit: lyft data science team

# rdtm in ridesharing: market design



A screenshot of the Lyft mobile app interface. At the top, it shows 'pages.lyftmail.com', the time '9:54 AM', and a battery level of 31%. The main heading is 'Supply Levels' written vertically on the left. To the right, there are icons for a dollar bill, a large dollar sign, and a car. Below this is the text 'Easier. More Money. The Power Driver Bonus Upgrade.' followed by a paragraph explaining the new Power Driver Bonus (PDB) features. At the bottom, there is a table with two columns: 'DRIVE' and 'GET'. The table lists various ride goals and the corresponding bonuses.

	DRIVE	GET
NEW	<b>30 Total Rides</b> 10 PEAK HOUR RIDES	<b>\$50 Bonus</b>
NEW	<b>50 Total Rides</b> 20 PEAK HOUR RIDES	<b>\$100 Bonus</b>
	<b>80 Total Rides</b> 30 PEAK HOUR RIDES	<b>10% Back + \$150 Bonus</b>
	<b>100 Total Rides</b> 40 PEAK HOUR RIDES	<b>20% Back + \$150 Bonus</b>
NEW	<b>120 Total Rides</b> 48 PEAK HOUR RIDES	<b>20% Back + \$200 Bonus</b>

Plus, we added 19 more eligible peak hours that count toward your bonus.

credit: lyft data science team



## the bigger picture: on-demand transportation



- fast operational timescales; complex network externalities
- new **control-levers**: dynamic pricing/dispatch, incentives, pooling
- new(er) challenges: competition, effect on public transit, urban planning

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## this talk

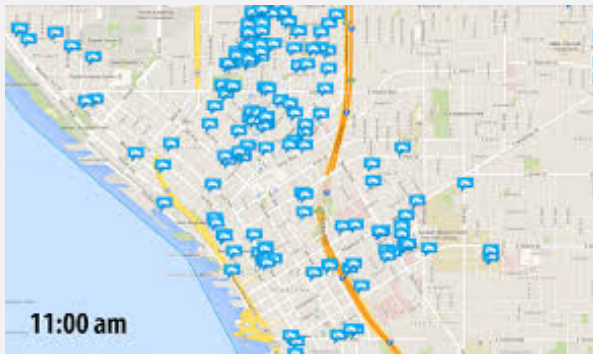
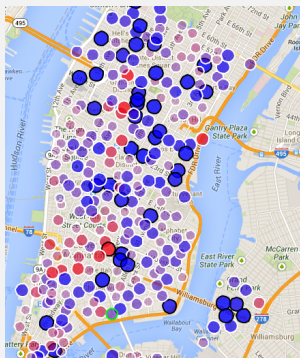
- 'where do we come from?'
  - ▶ simple framework for ridesharing: data, state, controls
- 'where are we?'
  - ▶ **approximate optimal control** for ridesharing **logistics**
  - ▶ **market mechanisms** as a tool for algorithmic self-calibration
- 'where are we going?'

# main challenge: rebalancing

demand heterogeneity  $\Rightarrow$  non-uniform supply across space and time

logistical 'solution': rebalance the vehicle fleet

economic 'solution': incentives for passengers and drivers



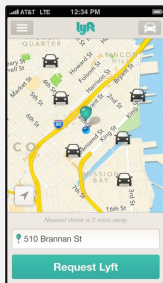
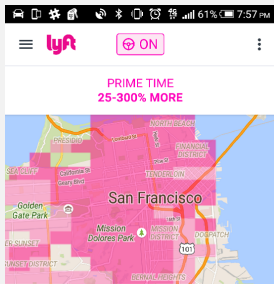
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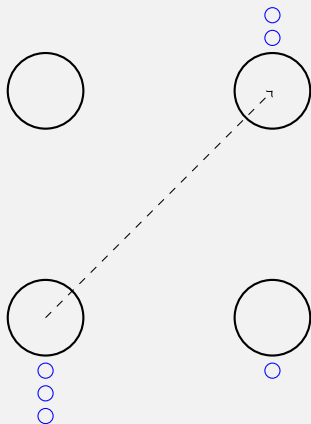
logistical 'solution': rebalance the vehicle fleet

economic 'solution': incentives for passengers and drivers

control-levers: pricing/incentives, dispatch, empty-car rebalancing

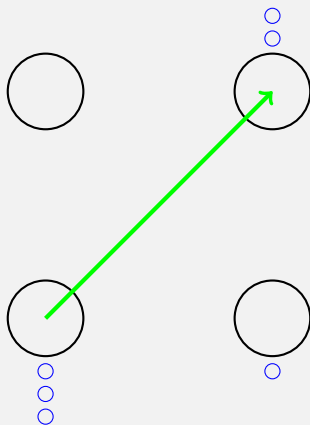


## (stochastic-network) model for ridesharing



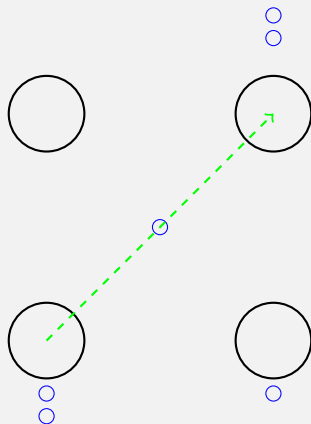
- $m$  units (cars) across  $n$  stations (here, we have  $m = 6, n = 4$ )
- system state  $\in \mathcal{S}_{n,m} = \{(x_i)_{i \in [n]} \mid \sum_{i=1}^n x_i = m\}$
- $i \rightarrow j$  passengers arrive via Poisson process with rate  $\phi_{ij}$

## (stochastic-network) model for ridesharing



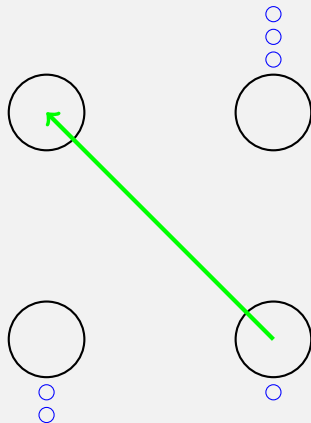
- platform sets **state-dependent** prices  $p_{ij}(\mathbf{X})$
- **quantile**  $q_{ij}(\mathbf{X}) = 1 - F_{ij}(p_{ij}(\mathbf{X}))$ : fraction willing to pay  $p_{ij}(\mathbf{X})$

## (stochastic-network) model for ridesharing



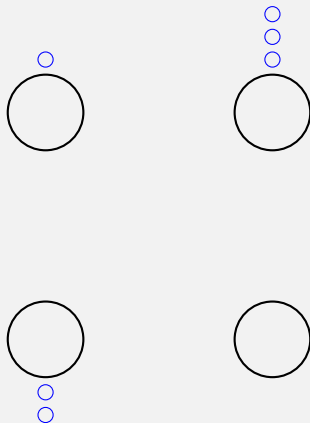
- car travels with passenger to destination
- (this talk: assume travel-times are zero)

# (stochastic-network) model for ridesharing

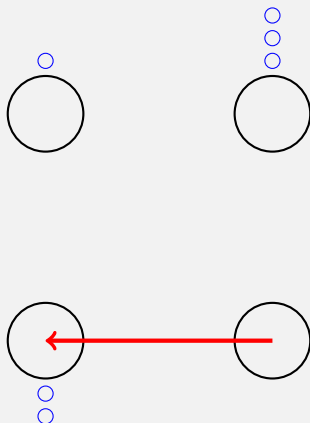




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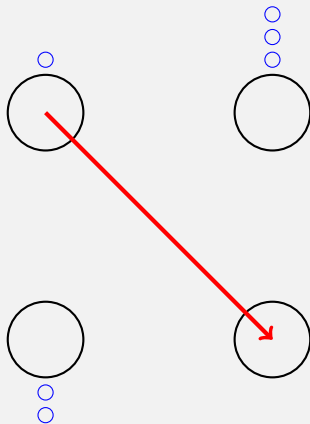


## (stochastic-network) model for ridesharing



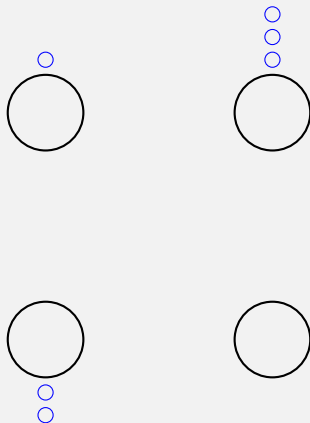
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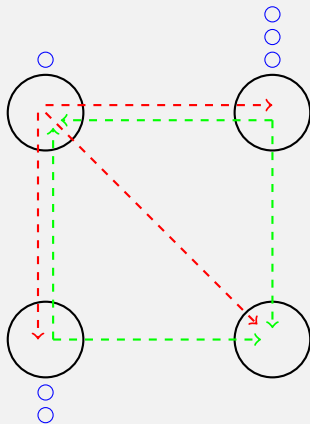
- myopic customers: abandon system if
  - vehicle unavailable or
  - price too high

## (stochastic-network) model for ridesharing



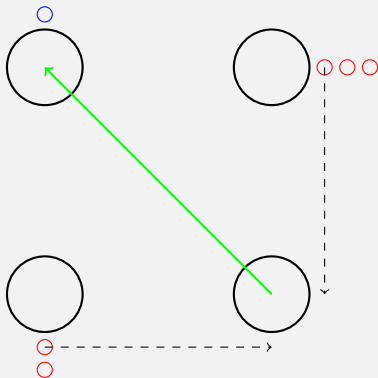
- objective:
  - optimize chosen **long-run average** system objective
  - objectives: revenue, welfare, customer engagement, etc.

# control levers for ridesharing



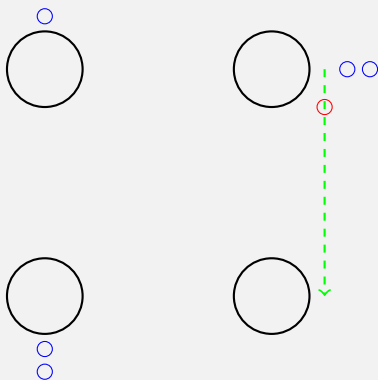
- pricing
  - modulates demand between locations
  - dynamic, state-dependent

## control levers for ridesharing



- **dispatch**: choose 'nearby' car to serve demand  
– can use any car within 'ETA target'

## control levers for ridesharing



- **rebalancing**: re-direct free car to empty location
  - incur a cost for moving the car
  - driver ‘nudges’ (heat-maps), autonomous vehicles

## intermezzo: why model?

### scales and economics

- need controls that work in real-time, at large-scales
- complex controls need more resources; non-commensurate (?) impact



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### known(?) unknowns

- errors in estimation and forecasting
- difficulties in learning demand/supply curves

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## unknown unknowns

**UBER** Uber NYC   
@Uber\_NYC Following

Surge pricing has been turned off at #JFK Airport. This may result in longer wait times. Please be patient.

RETWEETS 513 LIKES 1,141

4:36 PM - 28 Jan 2017

1.3K 513 1.1K

## intermezzo: why this model?

### assumption 1: timescales of platform operations

- number of cars, arrival rates, demand elasticities remain constant over time
- time-varying rates (re-solve policies at change-points. . .)
- driver entry/exit behavior
- effect of bursty arrivals?

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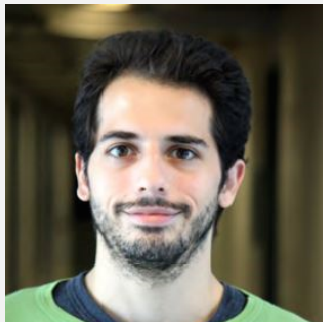
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### assumption 3: availability of data

- platform has perfect knowledge of arrival rates, demand elasticities
- is that really true?
- is that really needed?

# data-driven optimization for vehicle-sharing

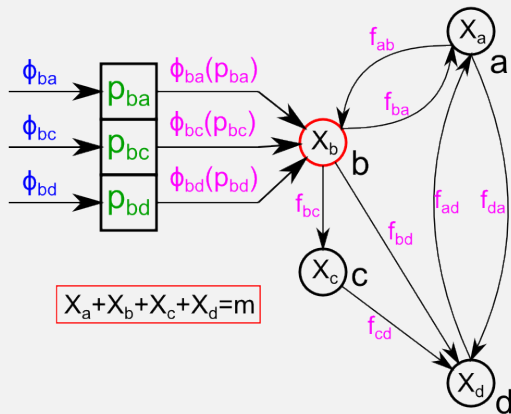


## Pricing and Optimization in Shared Vehicle Systems

Banerjee, Freund & Lykouris (2016)

<https://arxiv.org/abs/1608.06819>

## model recap



- $m$  units spread across  $n$  nodes
- control: state-dependent pricing policy  $\vec{p} = \{p_{ij}(\mathbf{x})\}$  (or quantiles  $\vec{q}$ )
- flows of cars in network: realized via Markov chain dynamics

# technical challenges

## objective

$$\max_{\mathbf{q}=\{q_e(\mathbf{x})\}} \underbrace{\sum_{\mathbf{x}} \pi_{\bar{q}}(\mathbf{x})}_{\text{long-run avg under control } \mathbf{q}} \left( \sum_{e=(i,j)} \mathbb{E}[\text{reward rate from } i \rightarrow j \text{ rides}] \right)$$



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assumption:  $q l_{ij}(q)$  is concave

true for throughput; welfare; revenue under regular  $F_{ij}$

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## objective

$$\max_{\mathbf{q} \in [0,1]^{|E|}} \mathbb{E}_{\pi_{\mathbf{q}}}(\mathbf{X}) \left[ \sum_e \phi_e q_e(\mathbf{X}) l_e(q(\mathbf{X})) \right]$$

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## challenges

- exponential size of policy
- non-convex problem: even with state-independent  $q_{ij}$

# approximately optimal control policies

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- exponential number of states
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## theorem [Banerjee, Freund & Lykouris 2016]

convex relaxation gives state-independent pricing policy with approximation factor of  $1 + \frac{\text{number of stations}}{\text{number of cars}}$

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- extends to dispatch, rebalancing
- large-supply/large-market optimality: factor goes to 1 as system scales

# proof roadmap

relaxation + resource augmentation

**step 1:** elevated flow relaxation: convex program that upper bounds performance, encodes essential conservation laws

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## relaxation + resource augmentation

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**step 2:** show EFR is tight for a class of state-independent pricing policies, in the 'infinite-unit system' (i.e.,  $m \rightarrow \infty$ )



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$$\boxed{\text{OBJ}_m(\mathbf{p}_m(\mathbf{X}))} \leq \boxed{\text{EFR}(\mathbf{p}^*)} = \boxed{\text{OBJ}_\infty(\mathbf{p}_\infty)} \leq \boxed{(1/\alpha_{mn})\text{OBJ}_m(\mathbf{p}_\infty)}$$

## the elevated flow relaxation

objective

$$\max_{\mathbf{q} \in [0,1]^{|E|}} \mathbb{E}_{\pi_{\mathbf{q}}(\mathbf{X})} \left[ \sum_{i,j} \phi_{ij} q(\mathbf{X}) l_{ij}(q(\mathbf{X})) \right]$$

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idea: **strengthen relaxation by adding additional constraints on  $\mathbf{q}$**

- circulation:  $\sum_j \phi_{ij} q_{ij} = \sum_k \phi_{ki} q_{ki} \quad \forall i \in V$
- Little's law:  $\mathbb{E}[\text{units in transit}] \leq m$

in summary

$$\boxed{\text{OBJ}_m(\mathbf{p}_m(\mathbf{X}))} \leq \boxed{\text{EFR}(\mathbf{p}^*)} = \boxed{\text{OBJ}_\infty(\mathbf{p}_\infty)} \leq \boxed{(1/\alpha_{mn})\text{OBJ}_m(\mathbf{p}_\infty)}$$

theorem [Banerjee, Freund & Lykouris 2016]

state-independent prices  $\vec{p}_\infty$  (from EFR) in  $m$ -unit system gives

$$\text{OBJ}_m(\vec{p}_\infty) \geq \alpha_{mn} \text{OPT}_m \quad , \quad \text{where } \alpha_{mn} = \frac{m}{m+n-1}$$



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main takeaway

new technique for optimizing stochastic dynamical system in steady-state

- can extend to more complex settings (?)  
(travel-times, multi-objective, pooling, reservations)

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- can extend to more complex settings (?)  
(travel-times, multi-objective, pooling, reservations)
- but where do we get the demand-rate and price-elasticity estimates?

# market design in ride-share platforms



## Pricing in Ride-Share Platforms

Banerjee, Johari & Riquelme (2015)

(EC'15: <https://ssrn.com/abstract=2568258>)

## why market design? and why ridesharing?

*Over the next 10 years, the major breakthrough of economics will be in applications of market design, which improves the efficiency of markets using a combination of game theory, economics and algorithm design. We've already seen fruitful application in search and spectrum auctions, kidney exchange and school assignment. (2016 will be the year that) Silicon Valley recognizes that the value of Uber is its marketplace, not the data...*

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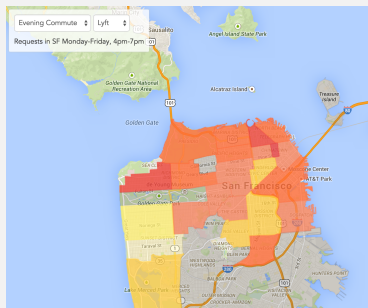
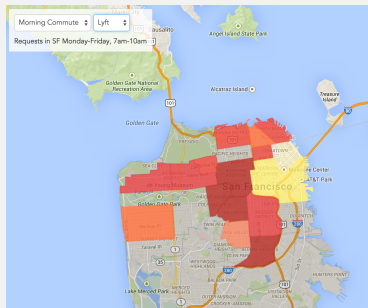
### data-driven optimization vs. market design

- default approach for complex operational problems:  
model – calibrate from data – optimize specific problem instance
- market mechanisms self-calibrate to solve the optimization problem
- ridesharing unique among online marketplaces: *platform sets prices*

## quasi-static vs. dynamic

for a large **block of time** (e.g., few hours), **region** (e.g., city-neighborhood), mean system parameters are constant, predictable.

*why not have hourly location-based prices?*



Source: [whatsthefare.com](http://whatsthefare.com)

## dynamic pricing vs. static pricing

- **dynamic**: price changes instantaneously, in response to system state
- **(quasi) static**: constant over several hours (predictably changing)

# model for studying rideshare pricing

focus on a **single block of time**, and a **single region**.

**system state** = number of available drivers

**assumption 1**: mean system parameters stay constant

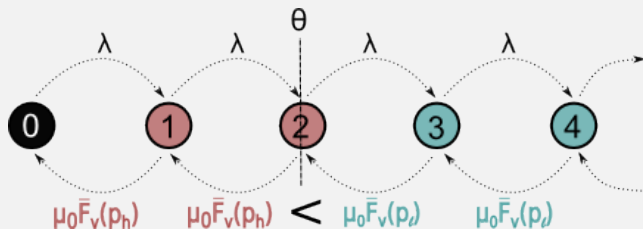
- state-dependent (dynamic) pricing policy:  
if # of available drivers =  $A$ , then price for ride =  $P(A)$
- platform earns a (fixed) fraction  $\gamma$  of every dollar spent

**assumption 2**: the two sides react at different time-scales

- myopic passengers: sensitive to **instantaneous prices**, availability
- drivers are sensitive to **long-term (average) earnings** and ride-volume

# rideshare pricing model: the details

stochastic dynamics + passenger/driver strategic behavior



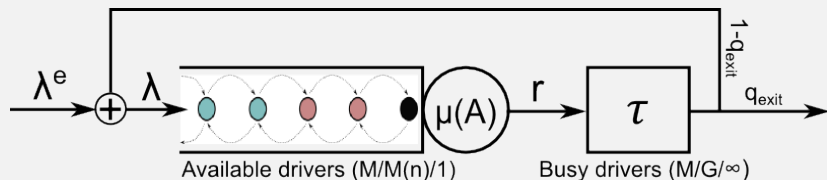
## strategic model for passengers

- a (potential) passenger requests a ride iff:  
reservation value  $V >$  current price, and driver available
  - ▶  $V \sim F_V$ , i.i.d. across ride requests
- $\mu_0$  = exogenous rate of “app opens”,  $\mu$  = actual rate of requests when  $A$  drivers present:  $\mu = \mu_0 \bar{F}_V(P(A))$



# rideshare pricing model: the details

stochastic dynamics + passenger/driver strategic behavior

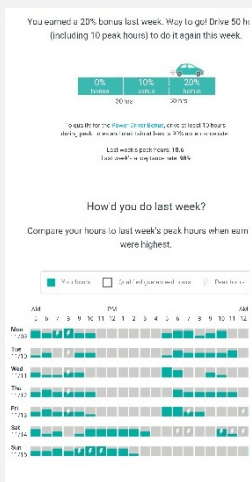


## strategic behavior of drivers

- a driver works on the platform iff:  
reservation rate  $C \times \mathbb{E}[\text{per-ride time spent}] < \mathbb{E}[\text{per-ride earning}]$ 
  - ▶  $C \sim F_C$ , i.i.d. across drivers
- $\Lambda_0$  = “potential” driver-arrival rate,  $\lambda$  = actual driver-arrival rate

$$\lambda = \frac{\Lambda_0}{q_{\text{exit}}} F_C \left( \frac{\mathbb{E}[\text{Per-ride earning}]}{\mathbb{E}[\text{Idle (waiting) time} + \text{Ride time}]} \right)$$

# driver decision aids



### Earnings

Day	Trips	Receipts <small>(Tolls)</small>	Lift Fees	Your Earnings
Nov 9	21	\$391.86 + \$9.00	\$70.37	\$201.19
Nov 10	21	\$366.37 + \$29.00 + <b>\$8.00</b>	\$73.27	\$321.10
Nov 11	14	\$247.24 + \$3.00	\$49.45	\$200.79
Nov 12	20	\$367.37 + \$16.00 + <b>\$4.00</b>	\$73.49	\$315.88
Nov 13	25	\$371.58 + \$13.00 + <b>\$5.00</b>	\$74.32	\$315.26
Nov 14	30	\$336.02 + \$9.00	\$67.23	\$277.79
Nov 15	28	\$157.51 + \$15.00	\$73.52	\$309.09
<b>Totals</b>	<b>160</b>	<b>\$2,408.05 + \$87.00 + \$17.00</b>	<b>\$481.65</b>	<b>\$2,030.40</b>

Drivers keep 100% of all tips.  
Passenger pays additional \$1.55 Tues. & Sat. surfee per ride.

Time in driver mode:	62 hrs, 4 min
Hide payments:	\$2,408.05
Lift fees:	-\$481.65
Tips:	\$87.00
Tolls:	\$17.00
Added tip from Susie's ride on Nov 9	\$0.00
Power driver bonus (20%):	\$481.65
<b>Your earnings:</b>	<b>\$2,517.05</b>

source: [therideshareguy.com](http://therideshareguy.com)

# rideshare pricing model: overview

## putting it together: equilibrium

given pricing policy  $P(\cdot)$ , equilibrium  $(\lambda, \mu, \pi, \eta, \iota)$  such that:

- 1  $\mu$ : passenger-arrival rate, given state  $A$ , satisfies:

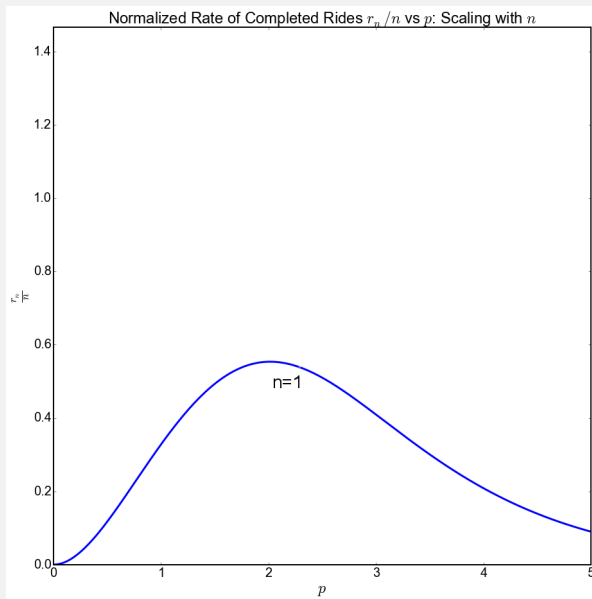
$$\mu = \mu_0 \bar{F}_V(P(A))$$

- 2  $\lambda$ : driver-arrival rate  $\lambda$ , given  $\iota, \eta$ , satisfies:

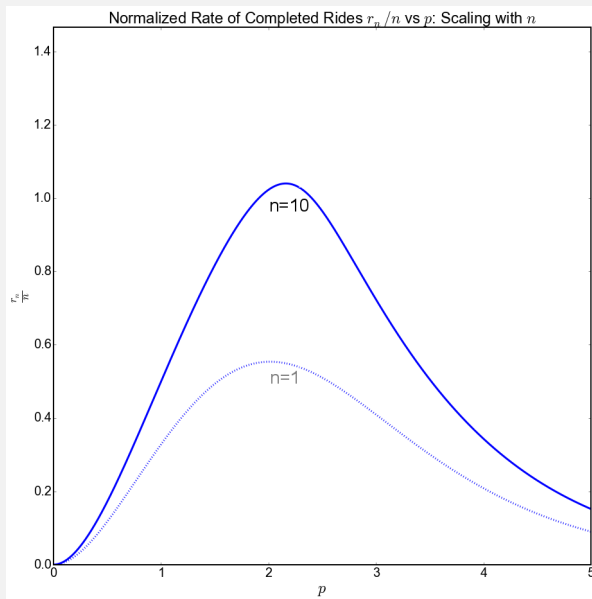
$$\lambda = \Lambda_0 F_C\left(\frac{\eta}{\iota + \tau}\right)$$

- 3  $\pi$ : steady-state distribution of  $A$  given  $\lambda, \mu$
- 4  $\eta$ :  $\mathbb{E}[\text{Earning per ride}]$ , given  $P(\cdot)$  and  $\pi$
- 5  $\iota$ :  $\mathbb{E}[\text{Idle time per ride}]$ , given  $P(\cdot)$  and  $\pi$

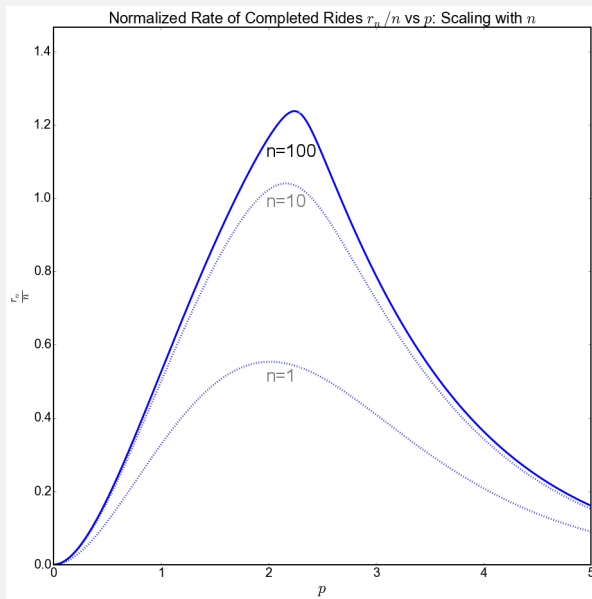
# platform equilibrium under static pricing



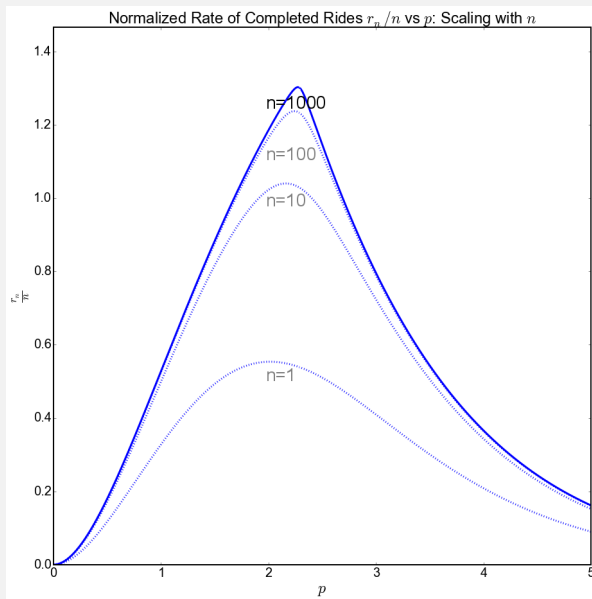
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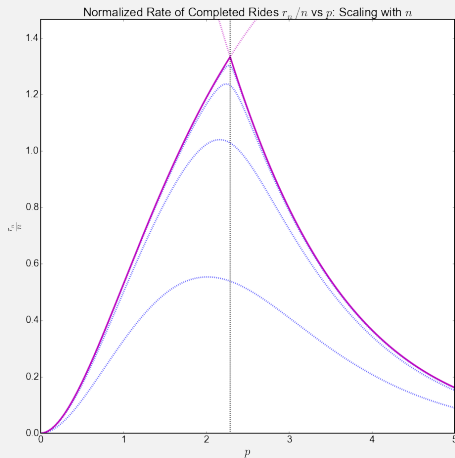
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# platform equilibrium under static pricing



theorem: static pricing in large-market limit  $\Rightarrow$  demand-supply curve  
rate of rides in large-market limit =  $\min\{\text{available supply, available demand}\}$



# Platform Equilibrium under Static Pricing

## Theorem: Static pricing in large-market limit

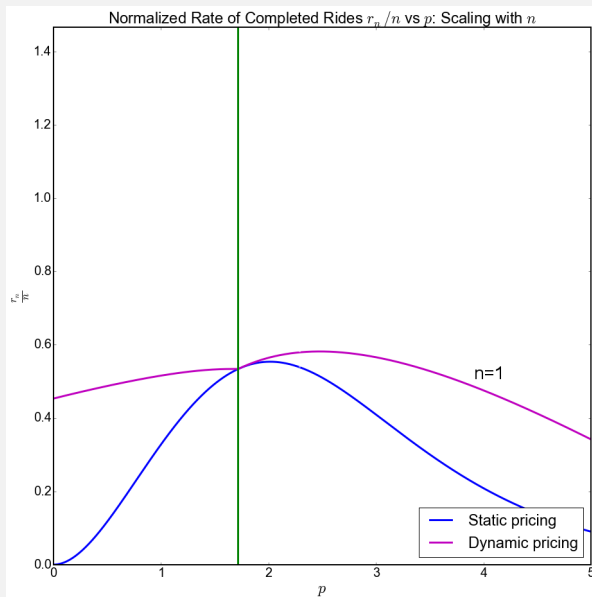
Under static pricing (i.e.,  $P(A) = p \forall A$ ), let  $r_n(p)$  denote the equilibrium rate of completed rides in the  $n^{\text{th}}$  system. Then:

$$r_n(p) \rightarrow \hat{r}(p) \triangleq \min \left\{ \frac{\Lambda_0}{q_{\text{exit}}} F_C \left( \frac{\gamma p}{\tau} \right), \mu_0 (1 - F_V(p)) \right\}$$

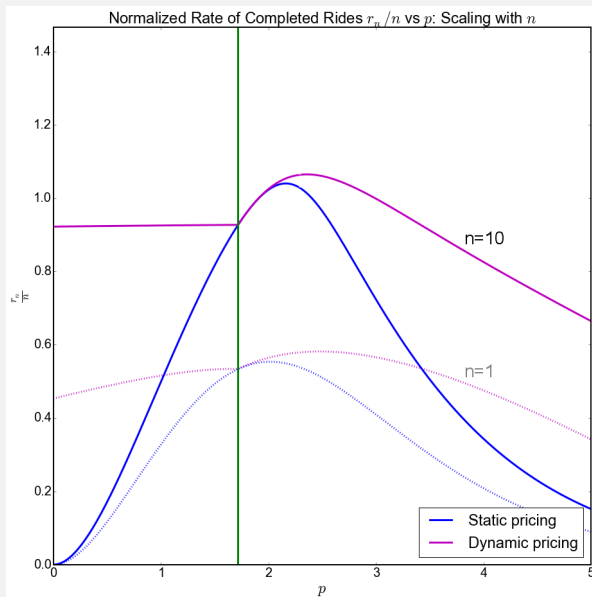
Some intuition:

- At any price, queueing system is always stable (else idle times blow up)
- If supply < demand: Drivers become fully saturated
- If supply > demand: Drivers forecast high idle times and don't enter

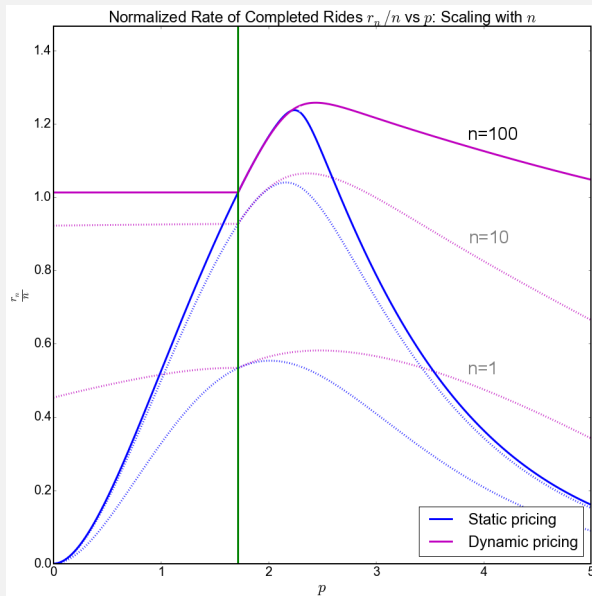
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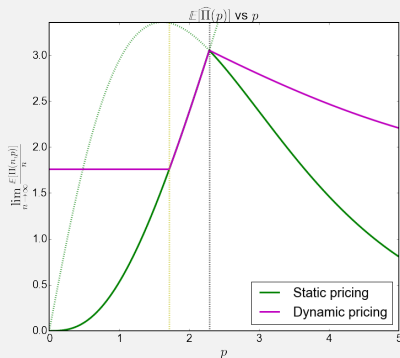
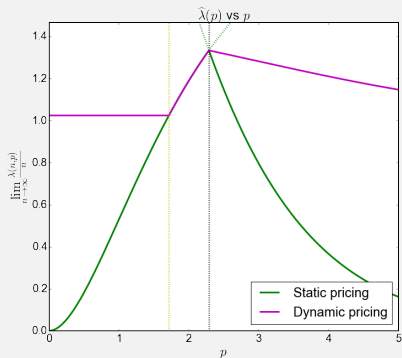
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# platform equilibrium under dynamic pricing



# static vs. dynamic pricing: optimality

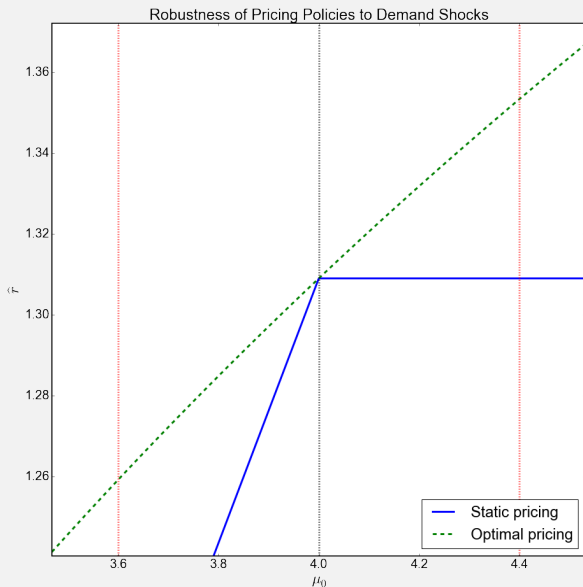


theorem [Banerjee, Johari & Riquelme 2015]

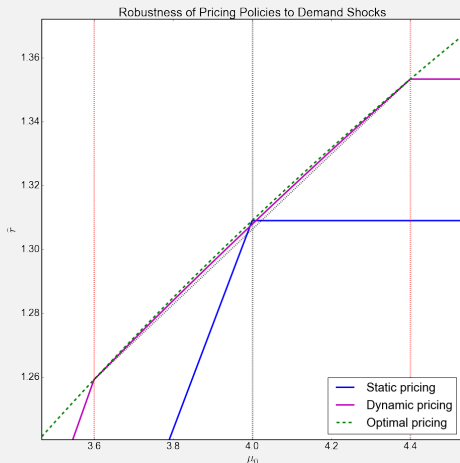
if  $F_V$  has increasing hazard rate: then

rate of rides for any dynamic policy  $\leq$  rate of rides under optimal static pricing.

# static vs. dynamic pricing: sensitivity to parameters



# static vs. dynamic pricing: sensitivity to parameters



theorem [Banerjee, Johari & Riquelme 2015]

dynamic pricing  $\geq$  'linear approximation' of optimal static-pricing throughput

## summary, and the road ahead

### main takeaway

ridesharing platforms: crucible for real-time decision making

- well modeled by steady-state stochastic models
- approximate control via new convex relaxation techniques
- algorithm self-calibration via market mechanisms



# the road ahead

## some short term targets

- the value of state-dependent controls
  - for general controls, objectives: no improvement possible
  - for dispatch: can achieve exponential decay in  $m!$(joint work with Pengyu Qian and Yash Kanoria (Columbia))

# the road ahead

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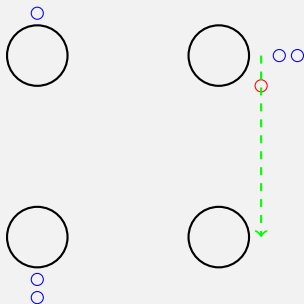
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## going further beyond

- impact of platform competition

## price of fragmentation in ridesharing markets

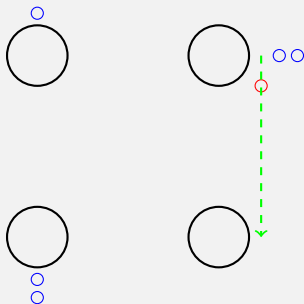
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- what is the ‘societal cost’ of decentralized optimization?
  - multiple platforms with (random) exogenously partitioned demands
  - individual platforms do optimal empty-vehicle rebalancing

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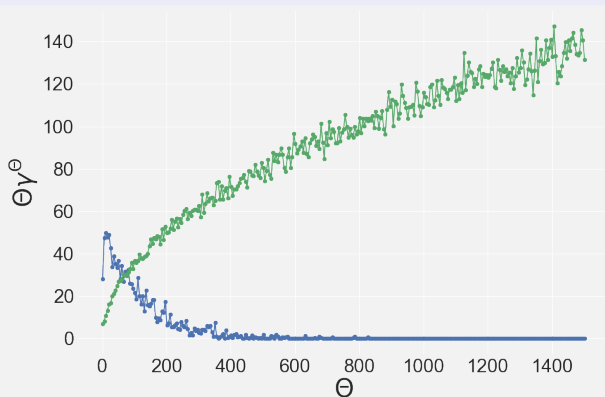
### price of fragmentation

increase in rebalancing costs of multiple platforms (with exogenous demand splits) vs. single platform (under large-market scaling)

# price of fragmentation in vehicle-sharing markets

## result (in brief)

as demand scales, the price of fragmentation undergoes a **phase transition** based on structure of underlying demand flows  
– both regimes observed in NYC taxi-data ( $\approx 10\%$  fragmentation-affected)





# the road ahead

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## going further beyond

- impact of platform competition
- the value of information: forecasting vs. self-calibration
- ridesharing + public transit
- appropriate mix of employees, freelancers and autonomous cars