Graph Spectrum
and
Small Set Expansion

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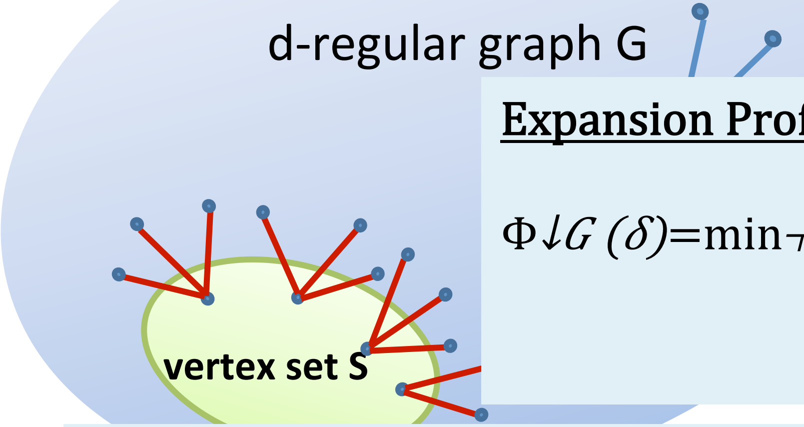
Based on joint works:

Many sparse cuts via higher eigenvalues.
Anand Louis, Prasad Tetali, Santosh Vempala

Making the long code shorter
Boaz Barak, Parikshit Gopalan, Johan Hastad, Raghu Meka, David Steurer

The Small Set Expansion Problem

$vol(S) = |S|/|V|$



$expansion(S) = \frac{\#\ edges\ leaving\ S}{d|S|}$

Expansion Profile : $\Phi_G : (0,1/2] \to [0,1]$

$\Phi_G(\delta) = \min_{S \subset V,\ vol(S) \leq \delta} expansion(S)$

n vertex in S
xpansion(S)

Small Set Expansion Problem:
Given graph G and a constant $\delta \in (0, 1/2]$, approximate $\Phi_G(\delta)$

$SSE(\delta, \epsilon)$
Given a graph G decide if,

- $\Phi_G(\delta) \leq \epsilon$ — G has a non-expanding small set

  OR

- $\Phi_G(\delta) > 1/10$ — G is a ``δ-small set expander"

Expansion Profile : $\Phi_G : (0, 1/2] \rightarrow [0,1]$

$$\Phi_G(\delta) = \min_{S \subset V,\ vol(S) \le \delta} expansion(S)$$

The Small Set Expansion

Small Set Expansion Problem:

Given graph G and a constant $\delta \in (0, 1/2]$, approximate $\Phi_G(\delta)$

Approximation Algorithms for $\Phi_G(1/2)$:

- Cheeger's Inequality [Alon][Alon-Milman]

  Given a graph G, if the normalized adjacency matrix has second eigen value λ_2 then,

$(1-\lambda_2)$ ... ϵ

$SSE(\delta, \epsilon)$ (STILL OPEN)

Given a graph G decide if,

- $\Phi_G(\delta) \le \epsilon$ — G has a non-expanding small set

  OR

- $\Phi_G(\delta) > 1/10$ — G is a ``δ-small set expander"

Any improvement over ``$\Phi_G(\delta) \le \epsilon$ *OR* $\Phi_G(\delta) > \sqrt{\epsilon \log 1/\delta}$ " would solve SSE(δ,ϵ) [R-Steurer-Tulsiani 2012]

Connection to Unique Games Conjecture

BASIC SDP is optimal for …

Constraint Satisfaction Problems [R`08]
MAX CUT [KKMO`06], MAX 2SAT [Au`07]

Metric Labeling Problems [MNRS`08]
MULTIWAY CUT, 0-EXTENSION

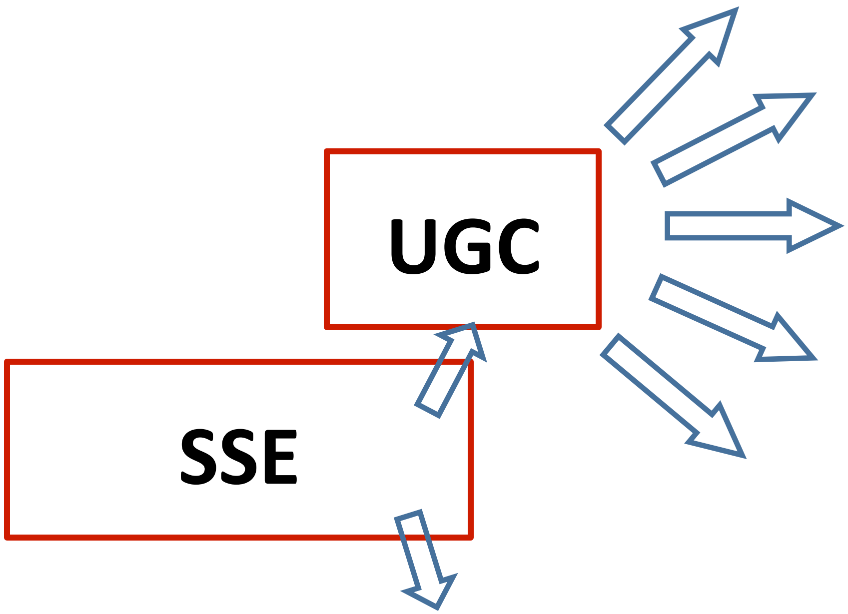
Ordering CSPs [GMR`08]
MAX ACYCLIC SUBGRAPH, BETWEENESS

Strict Monotone CSPs [KMTV`10]
VERTEX COVER, HYPERGRAPH VERTEX COVER

Kernel Clustering Problems [KN`08,10]

Grothendieck Problems [KNS`08, RS`09]

…



Unifor
Minimum L

SSE problem is a bottleneck for existing algorithmic techniques for all these problems

Notation

Let A be the normalized adjacency matrix of the d-regular graph G,

(all entries are 0 $or 1/d$)

$L_G = I - A$ is the Laplacian.

L_G has n eigenvalues $\lambda_1 = 0 \leq \lambda_2 \leq \lambda_3 \leq \ldots \leq \lambda_N \leq 2$

and corresponding eigenvectors e_1 , …, e_n

``top eigen-vectors"~ eigenvectors with small eigenvalues.

Def (Small Set Expander)**:**

A regular graph G is a δ-small set expander

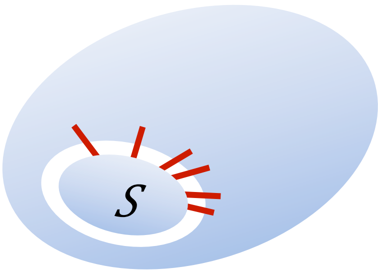
if for every set $S \subset V$,

$|S| \leq \delta N \Rightarrow expansion(S) \geq 1/2$

Non-expanding sets and spectrum

Given: regular graph G with vertex set V, parameter $\delta>0$

Suppose $f=\mathbb{1}_S$ indicator function of a small non-expanding set.



S has δ-fraction of vertices ➔ $\|f\|_2^2 = E[f^2]=\delta$

Fraction of edges leaving S = $E_{\text{random edge } (x,y)} (f(x)-f(y))^2$

$$=\langle f, L_G f\rangle$$

If $expansion(S)\leq 0.001$ then

So,

$$\langle f, L_G f\rangle \leq 0.001 \|f\|_2^2$$

(f is close to the span of eigenvectors of G with eigenvalue ≤ 0.01)

Conclusion:

Indicator function of a small non-expanding set f = $\mathbb{1}_S$ is a

- sparse vector
- close to the span of the top eigenvectors of G

Subspace Search Algorithm

- Let

$$V_{top} = Span\{e_i \mid \lambda_i \leq 10^{-6}\}$$

- Pick an ϵ-net for unit ball in V_{top}
- For each $v \in \epsilon$-net

test if $S_v = \{i \mid \sqrt{\delta n}\, v_i \geq 1/2\}$ is non-expanding.

Running Time = $O(n^{O(\dim(V_{top}))})$

Small Set Expansion is *easy* on graphs with $\dim(V_{top})$ is small.

Overview

- Higher Order Cheeger Inequalities
- Small Set Expanders with many large eigenvalues.

Higher Order Cheeger Inequalities

Fix a graph $G=(V,E)$

let

$$\lambda_1 = 0 \leq \lambda_2 \leq \lambda_3 \leq \ldots \lambda_k \leq \ldots \leq \lambda_n$$

are eigenvalues of the normalized Laplacian of G.

Cheeger's Inequality (Easy Direction)

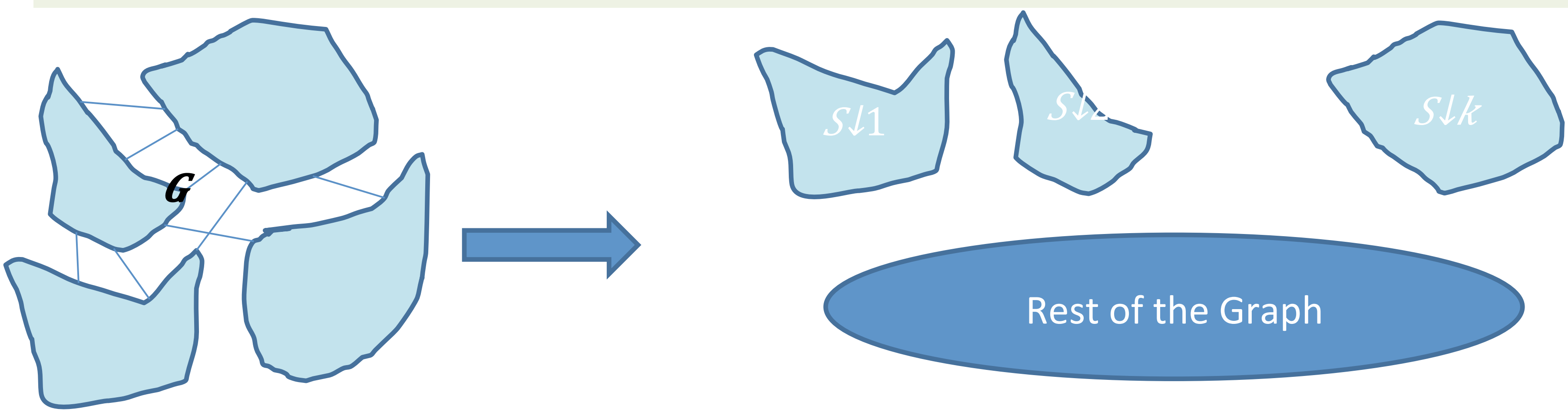
for every subset $S \subset V, |S| \leq |V|/2$,

$\text{expansion}(S) \geq \lambda_2 / 2$

Higher Order Cheeger (Easy Direction)

For every k disjoint subsets $S_1, S_2, \ldots, S_k$, $|S_i| \leq |V|/2$

$\max_i \text{ expansion}(S_i) \geq \lambda_k / 2$



Cheeger's Inequality (Difficult Direction)

There exists a subset $S \subset V, |S| \leq |V|/2$,

$\text{expansion}(S) \leq \sqrt{2\lambda_2}$

Higher Order Cheeger (Difficult Direction)

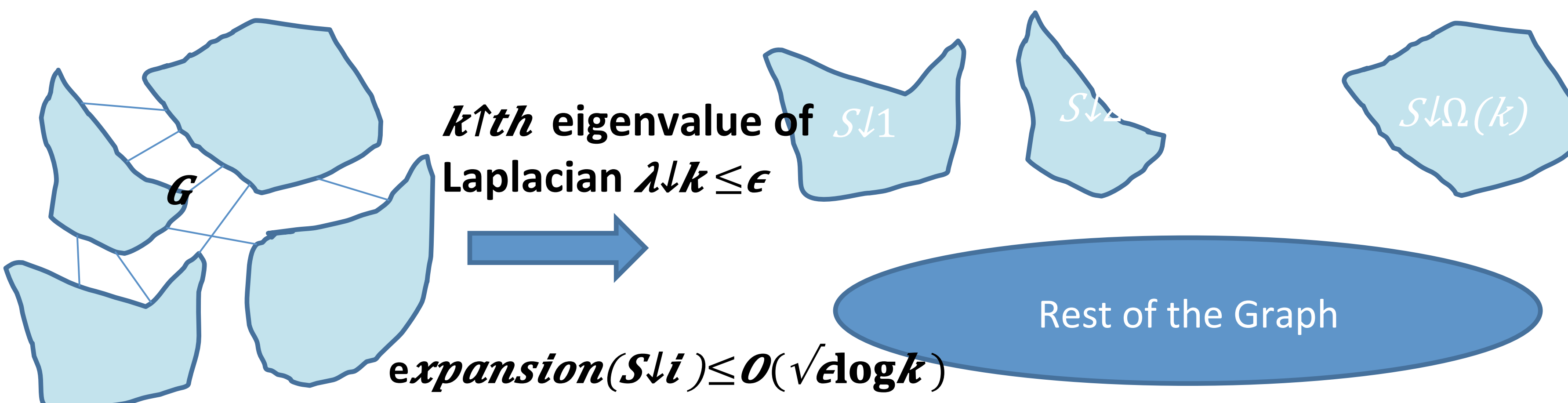
There exists $\Omega(k)$ disjoint subsets $S_1, S_2, \ldots, S_{\Omega(k)}$, $|S_i| \leq |V|/2$

$$\max_i \ \text{expansion}(S_i) \leq O(\sqrt{\lambda_k \log k})$$

[Lee-Oveis Gharan-Trevisan] [Louis-R-Tetali-Vempala]

Exactly k sets with $\max_i \ expansion(S_i) \leq k^2 \sqrt{\lambda_k}$

[Lee-Oveis Gharan-Trevisan]



Fix a graph $G=(V,E)$

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Cheeger's Inequality (Difficult Direction)

There exists a subset $S \subset V, |S| \leq |V|/2$,

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Can Efficiently find the sets

Higher Order Cheeger (Difficult Direction)

There exists $\Omega(k)$ disjoint subsets $S_1, S_2, \ldots, S_{\Omega(k)}$, $|S_i| \leq |V|/2$

$\max_i \text{expansion}(S_i) \leq O(\sqrt{\lambda_k \log k})$

OPEN: Exactly sets?

loss necessary.

Ex: noisy hypercube.

Connection to Small-Set Expansion

Higher Order Cheeger (Difficult Direction)

There exists $\Omega(k)$ disjoint subsets $S_1, S_2, \ldots, S_{\Omega(k)}$, $|S_i| \leq |V|/2$

$\max_i \ \text{expansion}(S_i) \leq O(\sqrt{\lambda_k \log k})$

If there are k eigenvalues $\leq \epsilon$, then one can efficiently find a set S such that

$$vol(S) \leq 1/k \quad and \quad expansion(S) \leq O(\sqrt{\epsilon \log k})$$

- SSE(ϵ,δ) is easy for $\delta \geq 2^{-1/\epsilon}$
- Any improvement over ``$\Phi_G(\delta) \leq \epsilon$ *OR* $\Phi_G(\delta) > \sqrt{\epsilon \log 1/\delta}$'' would solve SSE($\delta$,$\epsilon$)

[R-Steurer-Tulsiani 2012]

Cheeger's Inequality **(Difficult Direction)**

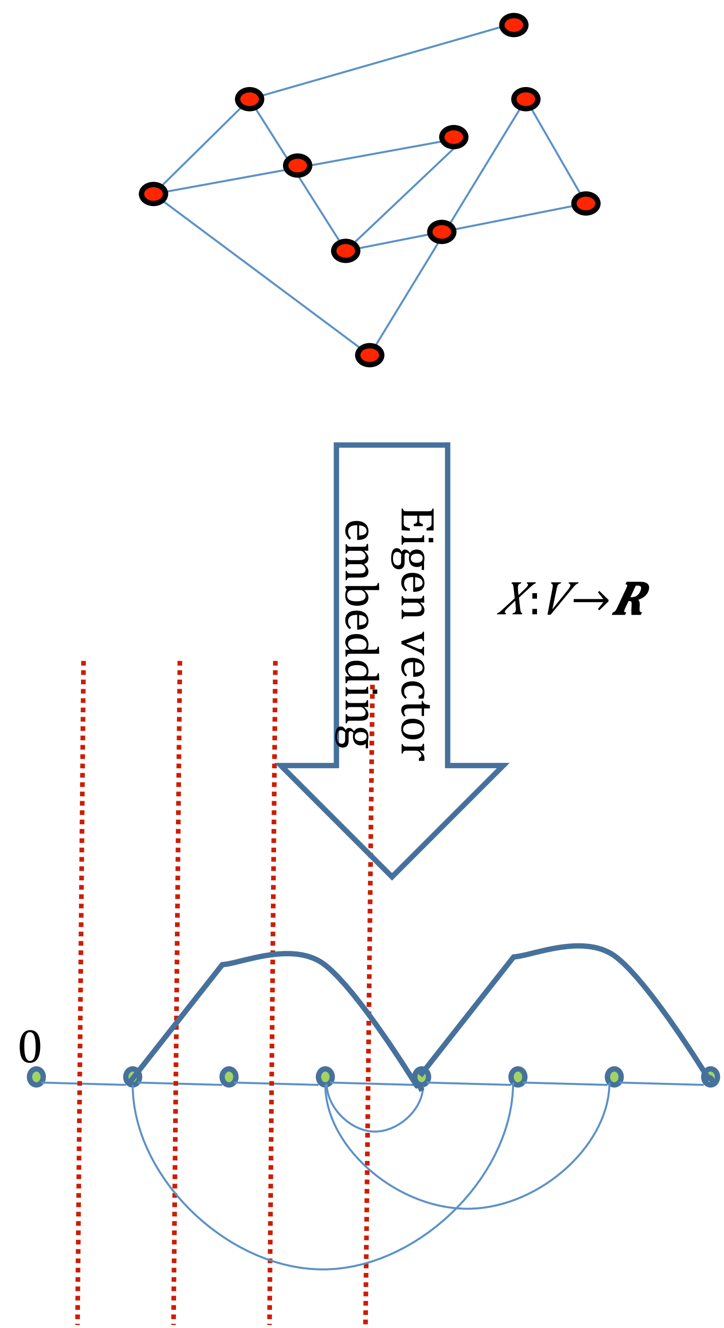
There exists a subset $S \subset V, |S| \le |V|/2$,

$$\text{expansion}(S) \le \sqrt{2\lambda_2}$$

Second eigenvector of the $Laplacian(G)$

$$X = (x_1, x_2, \ldots, x_n) \in \boldsymbol{R}^n$$

$$\sum_{(i,j)\in E} (x_i - x_j)^2 \Big/ \sum_i x_i^2 \le \lambda_2$$



Algorithm

- Scan the embedding from left to right
- Output the level set S_i that minimizes expansion.

Higher Order Cheeger (Difficult Direction)

There exists $\Omega(k)$ disjoint subsets $S_1, S_2, \ldots, S_{\Omega(k)}$, $|S_i| \leq |V|/2$

$\max_i \text{ expansion}(S_i) \leq O(\sqrt{\lambda_k \log k})$

k- eigenvectors of the $Laplacian(G)$

$$X_1 = x_{11} \; x_{12} \; x_{13} \; x_{14} \ldots\ldots x_{1n}$$

$$X_2 = x_{21} \; x_{22} \; x_{23} \; x_{24} \ldots\ldots x_{2n}$$

...

....

$$X_k = x_{k1} \; x_{k2} \; x_{k3} \; x_{k4} \ldots\ldots x_{kn}$$

$$X_i \perp 1$$

$$X_i \perp X_j \text{ for } i \neq j$$

$$X_i^T L X_i / X_i^T X_i \leq \lambda_k$$

Our Result **(Difficult Direction)**

There exists $\Omega(k)$ disjoint subsets $S_1, S_2, \ldots, S_{\Omega(k)}$, ... $\leq |V|/2$

$\max_i$ exp... $(S_i) \leq O(\sqrt{\lambda_k}$ log...

V_1 V_2 V_n

k- eigenve... ...he $Laplacian($...

$$X_1 = x_{11} \;\; x_{12} \;\; x_{13} \;\; x_{14} \;\ldots\ldots x_{1n}$$

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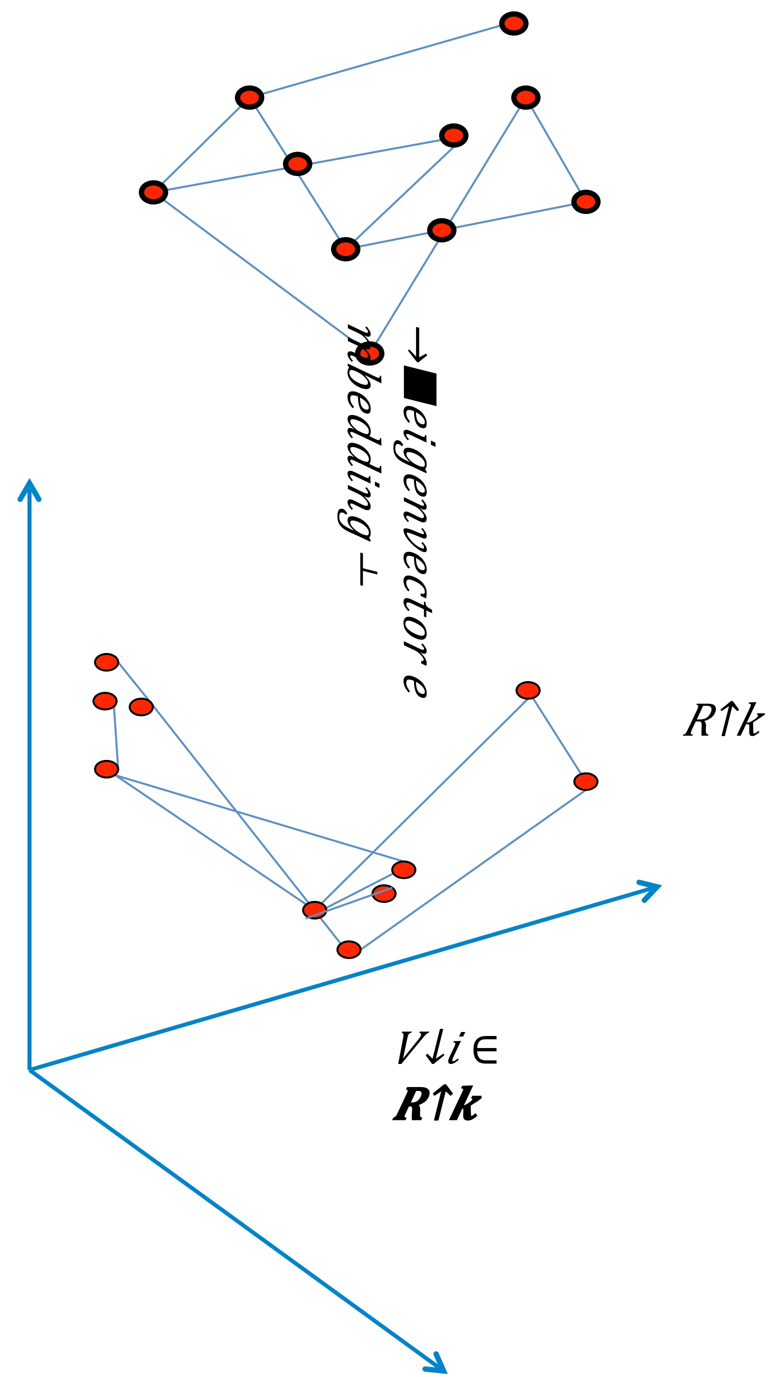
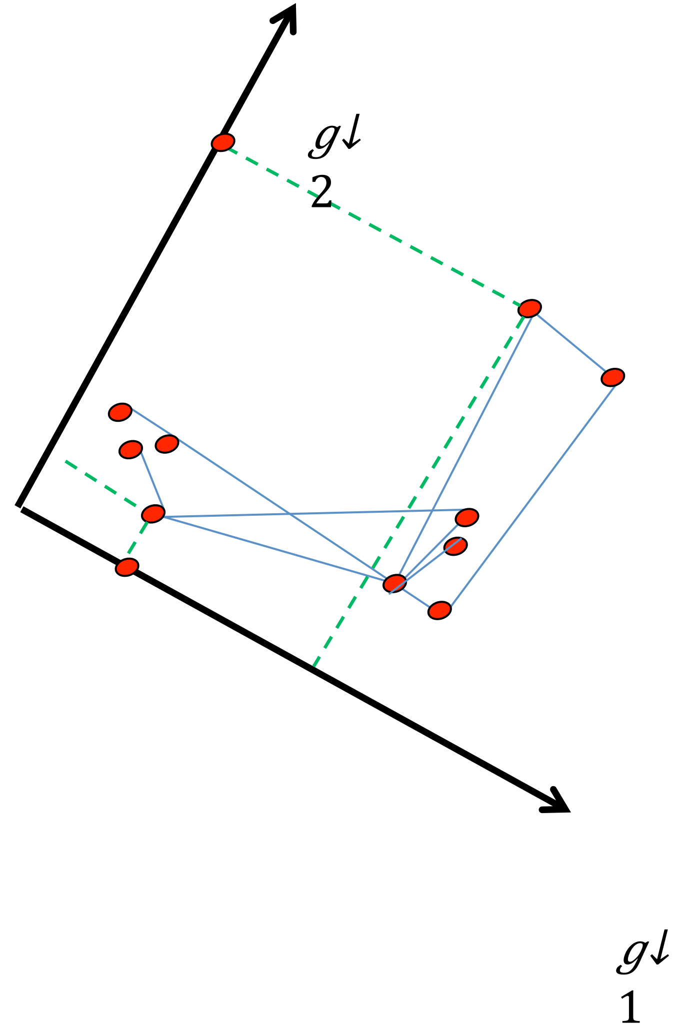


Algorithm

- Pick k random Gaussian vectors $g_1, g_2, \ldots, g_k \in \boldsymbol{R}^{\boldsymbol{k}}$
- For each vertex i,
    1. Project V_i along Gaussian directions $g_1, g_2, \ldots g_k$
    2. Let $j = argmax_p \langle V_i, g_p \rangle$
    3. Move (Project Down) the vector V_i to direction g_j.
- Run Cheeger rounding along each direction g_i, output the cuts of expansion $\leq O(\sqrt{\lambda_k} \log k)$



Algorithm

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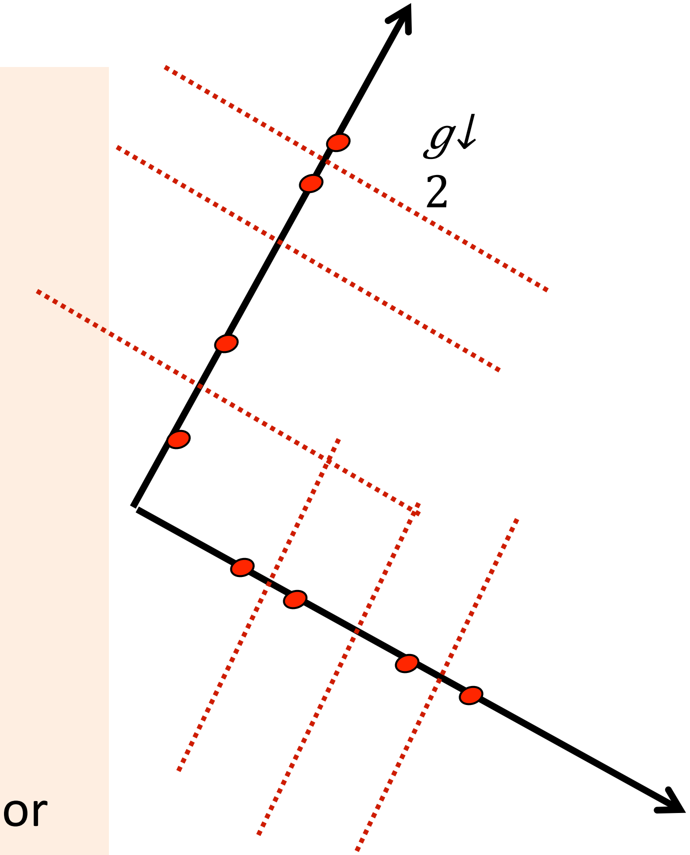
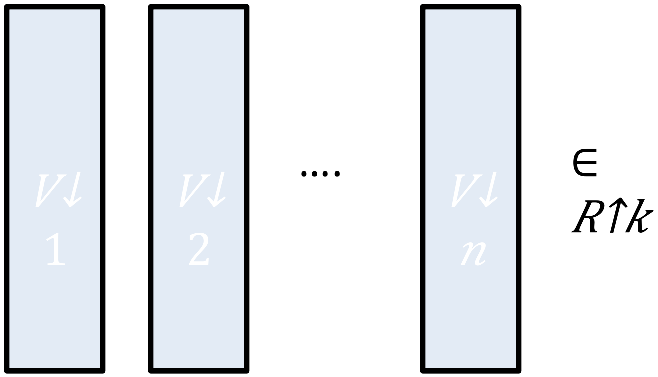


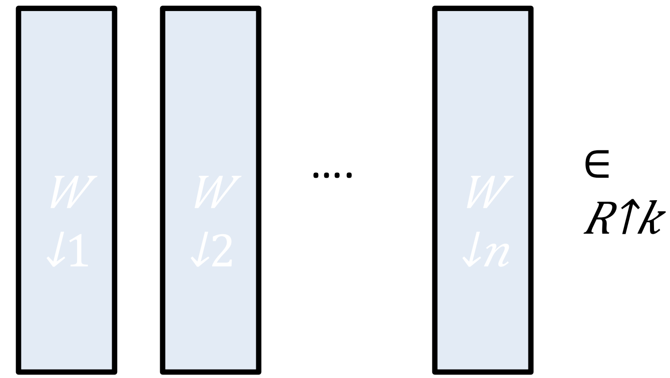
Observation:

Sets produced are disjoint

Algorithm Again



Project along k random directions.

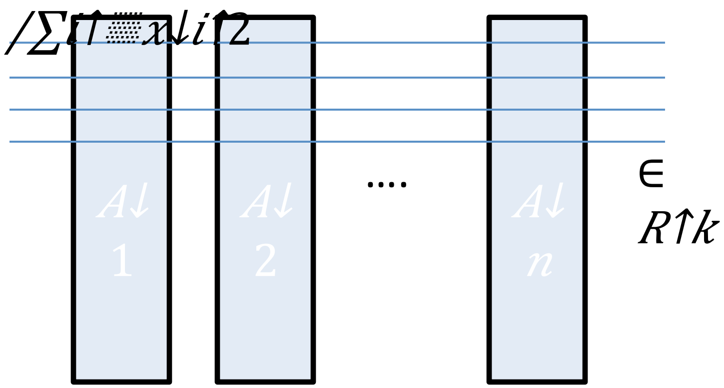


Need to Prove:

For at least a constant fraction of rows of A
The Rayleigh coefficient $\leq O(\lambda_k \log k)$

For each W_i
Zero out all but the largest coordinate.

$$X^T L X / X^T X = \sum_{(i,j)\in E} (x_i - x_j)^2 \Big/ \sum_i x_i^2$$



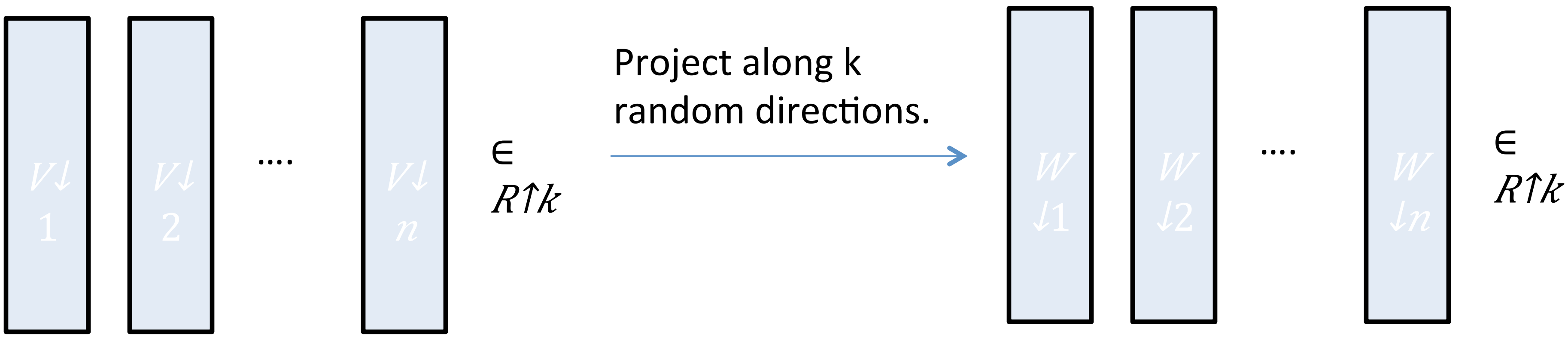
Run Cheeger style rounding on each row.

Algorithm Analysis

Need to Prove:

For at least a constant fraction of rows of A
The Rayleigh coefficient $\leq O(\lambda_k \log k)$

$$X^T LX / X^T X = \sum_{(i,j)\in E} (x$$



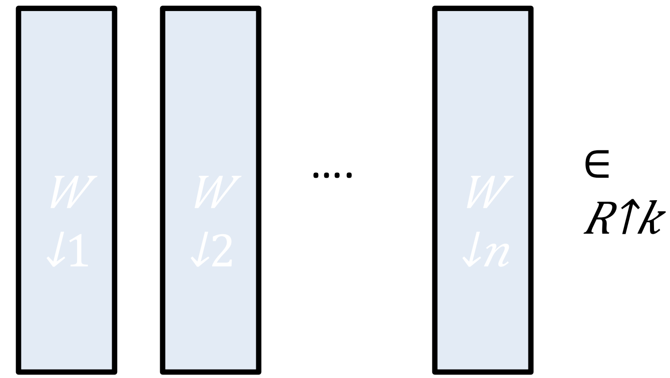
Fact:
Gaussian projection preserves squared distances in expectation.

Rayleigh coefficient of rows of W is roughly the Rayleigh coefficient of rows of V
$\leq \lambda_k$

Need to Prove:

For at least a constant fraction of rows of A
The Rayleigh coefficient $\leq O(\lambda_k \log k)$

Algorithm Analysis



For each W_i
Zero out all but the largest coordinate.

Denominator in Expectation:

$$E\sum_i A_{1i}^2 = O(\log k)/k \; E\sum_i W_{1i}^2$$

$1/k$-fraction of the mass survives , but the topmost survives.

Numerator in Expectation:

[Charikar-Makarychev-Makarychev]
Let $X_1, X_2, .. X_k$ and $Y_1, Y_2, .. Y_k$ be Gaussian random variables such that

X_i, Y_i is $1-\epsilon$ correlated
X_i, X_j are uncorrelated,

$$\Pr[\operatorname{argmax}_i X_i \neq \operatorname{argmax}_i Y_i] \leq O(\sqrt{\epsilon \log k})$$

Need to Prove:

For at least a constant fraction of rows of A
The Rayleigh coefficient $\leq O(\lambda_k \log k)$

Algorithm Analysis

Bounding Variance of Denominator

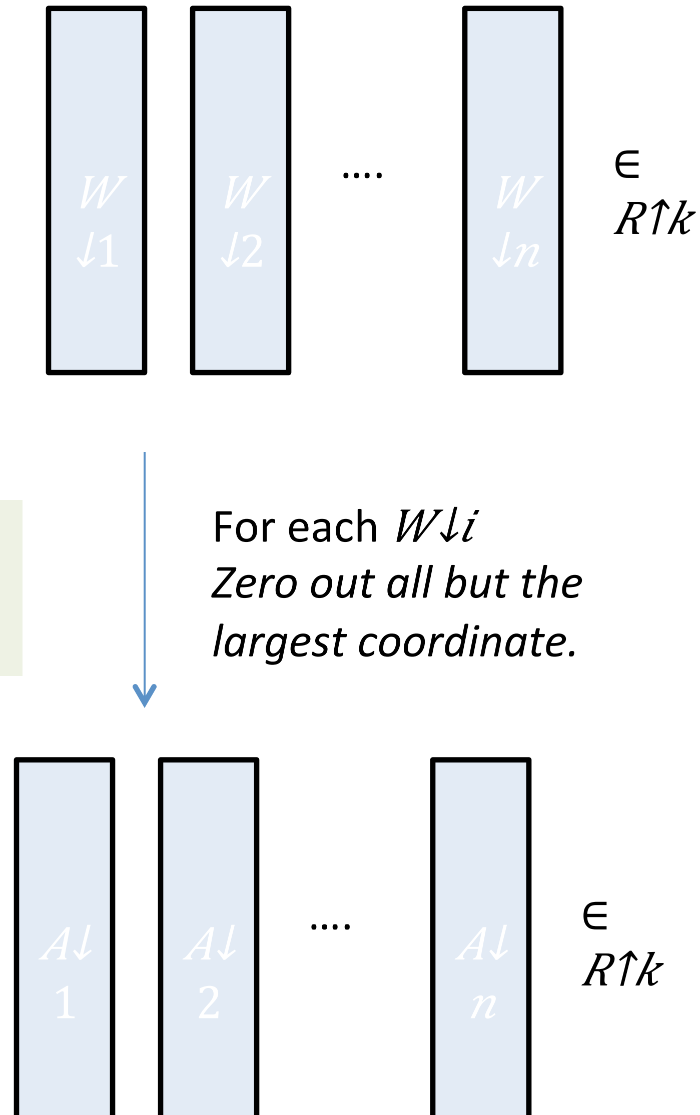
Use the fact that the rows (eigenvectors) are orthogonal!!

→ Average inner product between the columns is low

Lemma: Average squared length of W_i = 1,
The average correlation between random pairs $\leq 1/k$

+ basic Hermite polynomials.

= Bound on the variance.



Small Set Expanders with Many Large Eigenvalues

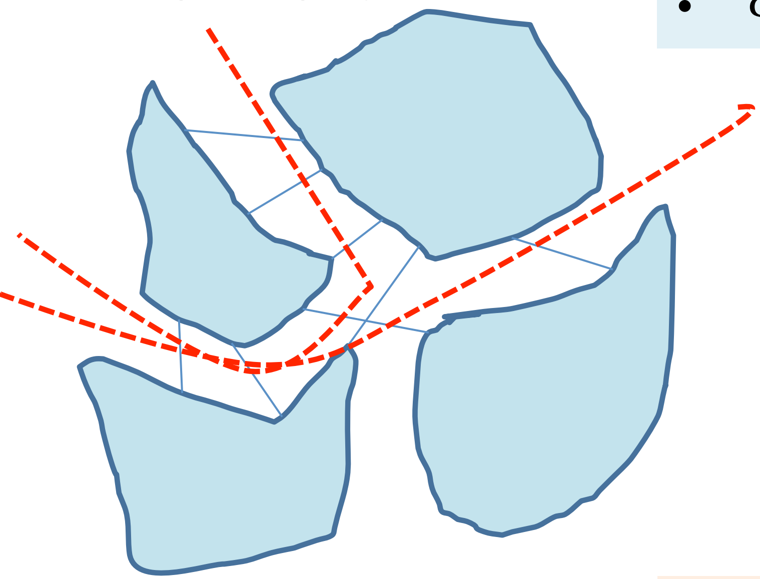
$SSE(\delta,\epsilon)$ (STILL OPEN)

Given a graph G decide if,

- $\Phi_G(\delta) \leq \epsilon$ — G has a non-expanding small set

  OR
- $\Phi_G(\delta) > 1/10$ — G is a ``δ-small set expander"

d-regular graph G



$threshold\ rank_\epsilon(G) \stackrel{\text{def}}{=}$ # of eigenvalues of graph G that are $\leq \epsilon$

Subspace search runs in time $n^{O(\text{threshold rank}_\epsilon(G))}$

Question:
If G is a δ-small set expander,
How many eigenvalues $\leq \epsilon$ can it have?

[Arora-Barak-Steurer 2010]
For a δ–small set expander G $thresholdrank_\epsilon \leq N^{\sqrt{\epsilon}/\delta}$

A subexponential-time algorithm for $SSE(\delta,\epsilon)$

Short Code Graph

[Barak-Gopalan-Hastad-Meka-Raghavendra-Steurer]

For all small constant δ,

There exists a graph (the Short Code Graph) that is a δ–small set expander with $\exp(\log^{\beta} n)$ eigenvalues $\geq 1-\epsilon$., *i.e.*,

$\mathit{threshold\ rank}_{1-\epsilon}(G) \geq \exp(\log^{\beta} N)$
for some β depending on ϵ.

[BGHMRS 11] $\exp(\log^{\beta} N) \leq \mathit{threshold\ rank}_{\epsilon}(G) \leq N^{\epsilon}$
[ABS]

- Led to improved gadget constructions and integrality gaps for semidefinite programming relaxations.

- An n-point L_2^2 -metric that requires distortion $2^{\Omega(\sqrt{\log\log n})}$ to embed in to L_1
  [Kane-Meka]

Overview

Long Code/Noisy Hypercube Graph

- Eigenvectors
- Small Set Expansion

Short Code Graph

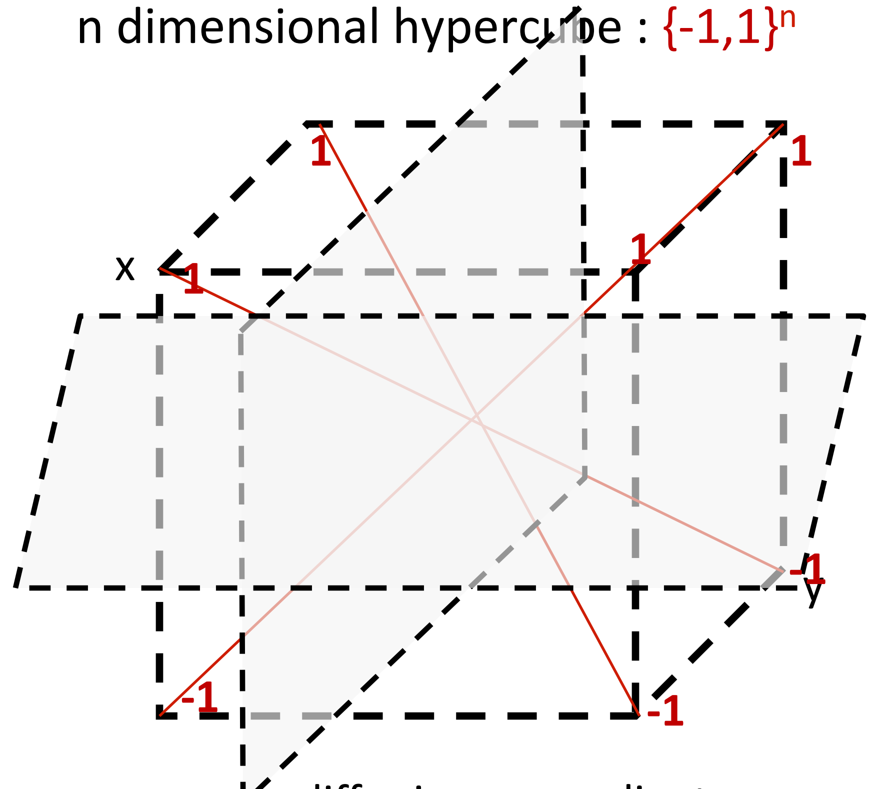
The Long Code Graph aka Noisy Hypercube

Noise Graph/Frankl-Rodl graphs: H_ϵ

Vertices: $\{-1,1\}^n$

Edges: Connect every pair of points in hypercube separated by a Hamming distance of ϵn

n dimensional hypercube : {-1,1}^{n}

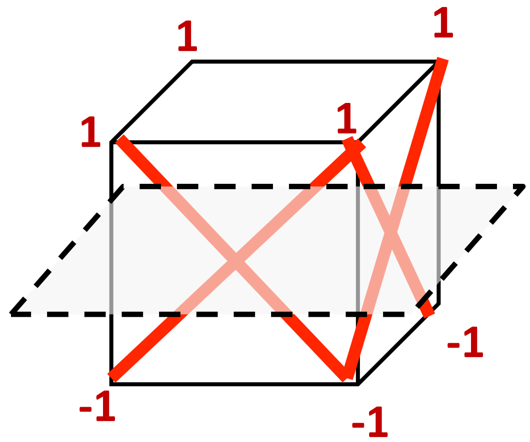


x and y differ in ϵn coordinates.

Eigenvectors are functions on $\{-1,1\}^n$

Dictator cuts: Cuts parallel to the Axis
(given by $F(x) = x_i$)

The dictator cuts yield n sparse cuts in graph H_ϵ



n dimensional hypercube

Sparsity of Dictator Cuts

Connect every pair of vertices in hypercube separated by Hamming distance of ϵn

---

$Fraction\ of\ edges\ cut\ by\ first\ dictator$
$= \Pr_{random\ edge\ (x,y)} [(x,y)\ is\ cut]$
$= \Pr_{random\ edge\ (x,y)} [x_1 \neq y_1]$
$= \epsilon$

Dictator cuts: n-eigenvectors with eigenvalues ϵ for graph H_ϵ
(Number of vertices N = 2^n, so #of eigenvalues = $\log N$)

Eigenfunctions for Noisy Hypercube Graph

Eigenfunctions for the Noisy hypercube graph are multilinear polynomials of fixed degree.

(Noisy hypercube is a Cayley graph on Z_2^n, therefore its eigen functions are characters of the group)

Eigenfunction **Eigenvalue**

$F_1(x)=x_1$, $F_2(x)=x_2$, …, $F_n(x)=x_n$ ϵ

$F_{12}(x)=x_1 x_2$, $F_{23}(x)=x_2 x_3$, …$F_{n-1n}(x)=x_{n-1} x_n$ $\approx 1-(1-\epsilon)^2$

………………………

Degree d multilinear polynomials $\approx 1-(1-\epsilon)^d$

Top Eigenspace = Low degree polynomials over the hypercube.

Hypercontractivity

Definition: (Hypercontractivity)

A subspace $S \in R^N$ is *hypercontractive* if for all $w \in S$

$$\|w\|_4 \le C\|w\|_2$$

Projector P_S in to the subspace S, also called hypercontractive.

(No-Sparse-Vectors)

Roughly, No sparse vectors in a hypercontractive subspace S because,

w is δ-sparse "⇔" $\|w\|_4 / \|w\|_2 > 1/\delta^{1/4}$

Hypercontractivity implies Small-Set Expansion

$P_{1-\epsilon} =$ projector into span of eigenvectors of G with eigenvalue $\ge 1-\epsilon$

$P_{1-\epsilon}$ is hypercontractive

→

No sparse vector in span of top eigenvectors of G

→

No small non-expanding set in G. (G is a small set expander)

Hypercontractivity for Noisy Hypercube

Top eigenfunctions of noisy hypercube are low degree polynomials.

(Hypercontractivity of Low Degree Polynomials)
For a degree d multilinear polynomial f on $\{-1,1\}^n$,

$$\|f\|_4 \leq 9^d \|f\|_2$$

(Noisy Hypercube is a Small-Set Expander)
For constant ϵ, the noisy hypercube is a small-set expander.
Moreover, the noisy hypercube has $N = 2^n$ vertices and n eigenvalues larger than $1-\epsilon$.

Short Code Graph

[Barak-Gopalan-Hastad-Meka-Raghavendra-Steurer 2009]

For all small constant δ,

There exists a graph (the Short Code Graph) that is a δ–small set expander with $\exp(\log^{\beta} n)$ eigenvalues $\geq 1-\epsilon$., *i.e.*,

$$\mathit{threshold\ rank}_{1-\epsilon}(G) \geq \exp(\log^{\beta} N)$$

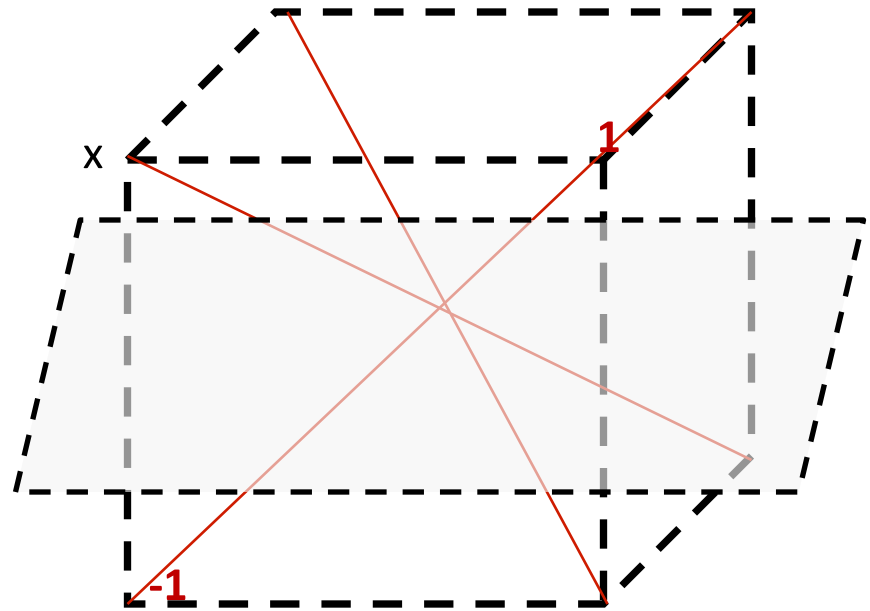
for some β depending on ϵ.

Short Code Graph

Noise Graph: H_{ϵ}

Vertices: $\{-1,1\}^n$

Edges: Connect every pair of points in hypercube separated by a Hamming distance of ϵn



Has n sparse cuts, but $N=2^n$ vertices -- too many vertices!

Idea:
Pick a subset of vertices of the long code graph, and their induced subgraph.

1. The dictator cuts still yield n-sparse cuts
2. The subgraph is a small-set expander!

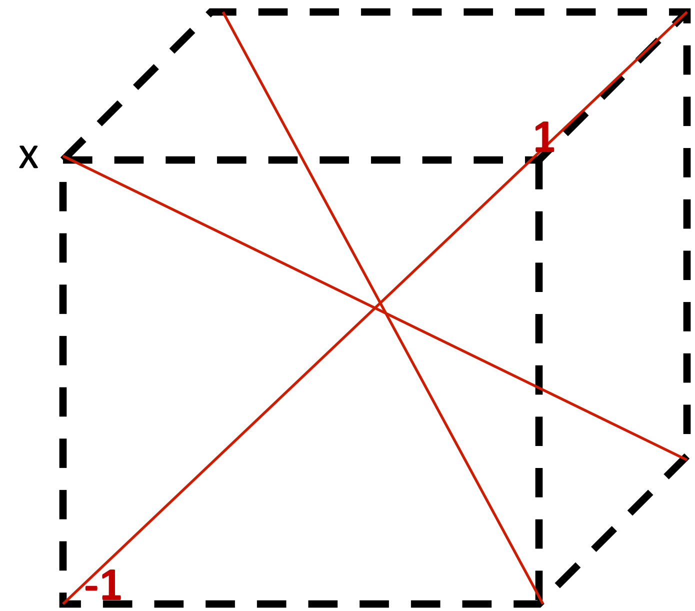
If \ **Choice:** Reed Muller Codewords of large constant degree.
$1/100$

Short Code Graph

Noise Graph: H_ϵ

Vertices: All boolean functions on k bits

Edges: Connect every pair of points in hypercube separated by a Hamming distance of ϵn



Fix $n = 2^k$ and $\epsilon = 2^{-d}$

Short Code Graph

Vertices: All degree d polynomials over $F_2 = \{0,1\}$ in k bits.

Edges: Connect every pair of points separated by a Hamming distance of ϵn

Equivalently,
Connect every pair of polynomials (p,q)

With $p(x) = q(x) + L_1(x) L_2(x) \ldots L_d(x)$

where $L_i(x)$ are linear functions.

Short Code Graph

Short Code Graph

Vertices: All degree d polynomials over $F_2 = \{0,1\}$ in k bits.

Edges: Connect every pair of points separated by a Hamming distance of ϵn

Equivalently,
Connect every pair of polynomials (p,q) x
1
With $p(x) = q(x) + L_1(x) L_2(x) \ldots L_d(x)$

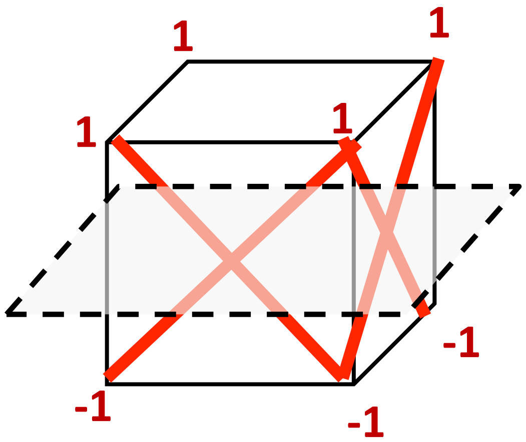
where $L_i(x)$ are linear functions.

Assuming,

1. The dictator cuts still yield n-sparse cuts
2. The subgraph is a small-set expander!

$Threshold\_rank_{\epsilon} = n$ while number of vertices $N = 2^{O(n^d)}$

$= 2^{\log^{1/d} N}$



n dimensional hypercube

Sparsity of Dictator Cuts

Connect every pair of vertices in hypercube separated by Hamming distance of ϵn

Sparsity of dictator cuts holds for every induced subgraph.

Because,

Each edge is cut by exactly ϵn dictators, so at least n/2 dictators cut less than 2ϵ fraction of edges.

Preserving Small Set Expansion

Top eigenfunctions of noisy hypercube are low degree polynomials.

+

(Hypercontractivity of Low Degree Polynomials)
For a degree d multilinear polynomial f on $\{-1,1\}^n$,

$$\|f\|_4 \leq 9^d \|f\|_2$$

(Noisy Hypercube is a Small-Set Expander)
For constant ϵ, the noisy hypercube is a small-set expander.

Preserving hypercontractivity

(Hypercontractivity of Low Degree Polynomials)
For a degree d multilinear polynomial f on $\{-1,1\}^n$,

$$\|f\|_4 \leq 9^d \|f\|_2$$

For a degree d polynomial f,

By hypercontractivity over hypercube,

$$E_{x\in\{-1,1\}^n} [f(x)^4] \leq 9^{4d} (E_{x\in\{-1,1\}^n} [f(x)])^2$$

We picked a subset $S \subset \{-1,1\}^n$ and so we want,

$$E_{x\in S} [f(x)^4] \leq 9^{4d} (E_{x\in S} [f(x)])^2$$

f is degree d, so f^4 and f^2 are degree $\leq 4d$.

If S is a 4d-wise independent set then,

$$E_{x\in S} [f(x)^4] = E_{x\in\{-1,1\}^n} [f(x)^4] \leq 9^{4d} (E_{x\in\{-1,1\}^n} [f(x)])^2$$
$$= 9^{4d} (E_{x\in S} [f(x)])^2$$

Preserving Eigenspaces

Top eigenfunctions of noisy hypercube are low degree polynomials.

We Want:

Only top eigenfunctions on the short code graph are also low degree polynomials.

<u>Bounding the Eigenvalues</u>

Fix $y_1, y_2, \ldots, y_k \in \{0,1\}^k$,

Over choice of random affine forms $L_1(x), L_2(x), \ldots L_d(x)$ over F_2

Show that $\sum_i \prod_j L_j(y_i)$ has bias less than $O(k/2^d)$

Connected to **local-testability** of the dual of the underlying code!

We appeal to local testability result of Reed-Muller codes

[Bhattacharya-Kopparty-Schoenebeck-Sudan-Zuckermann]

Mimicking the hypercube

[Barak-Gopalan-Hastad-Meka-Raghavendra-Steurer 2009]

`Majority is Stablest' theorem also holds for the short code.

Open Questions

Question:
If G is a δ-small set expander,
How many eigenvalues $\leq \epsilon$ can it have?

[BGHMRS 11] $\exp(\log^{\beta} N) \leq \text{threshold rank}_{\epsilon}(G) \leq N^{\epsilon}$
[ABS]

Efficient Certificate for Vertex Expansion:
Given a graph G, find an algorithm to distinguish between,

a) G has a set with vertex expansion $\leq \epsilon$

b) G is a vertex expander (expansion > 0.1)

(No restriction on set size)

Harder than small set expansion
[Louis-R-Vempala 13]

Thank You