Tractable Learning in Structured Probability Spaces

Adnan Darwiche
Computer Science Department
UCLA

Representation learning workshop
Simons Institute, 3/29/2017

References

Probabilistic Sentential Decision Diagrams
Doga Kisa, Guy Van den Broeck, Arthur Choi and Adnan Darwiche
KR, 2014

Learning with Massive Logical Constraints
Doga Kisa, Guy Van den Broeck, Arthur Choi and Adnan Darwiche
ICML 2014 workshop

Tractable Learning for Structured Probability Spaces
Arthur Choi, Guy Van den Broeck and Adnan Darwiche
IJCAI, 2015

Tractable Learning for Complex Probability Queries
Jessa Bekker, Jesse Davis, Arthur Choi, Adnan Darwiche, Guy Van den Broeck.
NIPS, 2015

Structured Features in Naive Bayes Classifiers
Arthur Choi, Nazgol Tavabi and Adnan Darwiche
AAAI, 2016

Tractable Operations on Arithmetic Circuits
Jason Shen, Arthur Choi and Adnan Darwiche
NIPS, 2016

Structured probability spaces?

Running Example

Courses:

- Logic (L)
- Knowledge Representation (K)
- Probability (P)
- Artificial Intelligence (A)

Data

L	K	P	A	Students
0	0	1	0	6
0	0	1	1	54
0	1	1	1	10
1	0	0	0	5
1	0	1	0	1
1	0	1	1	0
1	1	0	0	17
1	1	1	0	4
1	1	1	1	3

Prior Knowledge

- Must take at least one of Probability or Logic.
- Probability is a prerequisite for AI.
- The prerequisites for KR is either AI or Logic.

Probability Space

unstructured

L	K	P	A
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

Structured Probability Space

unstructured

L	K	P	A
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

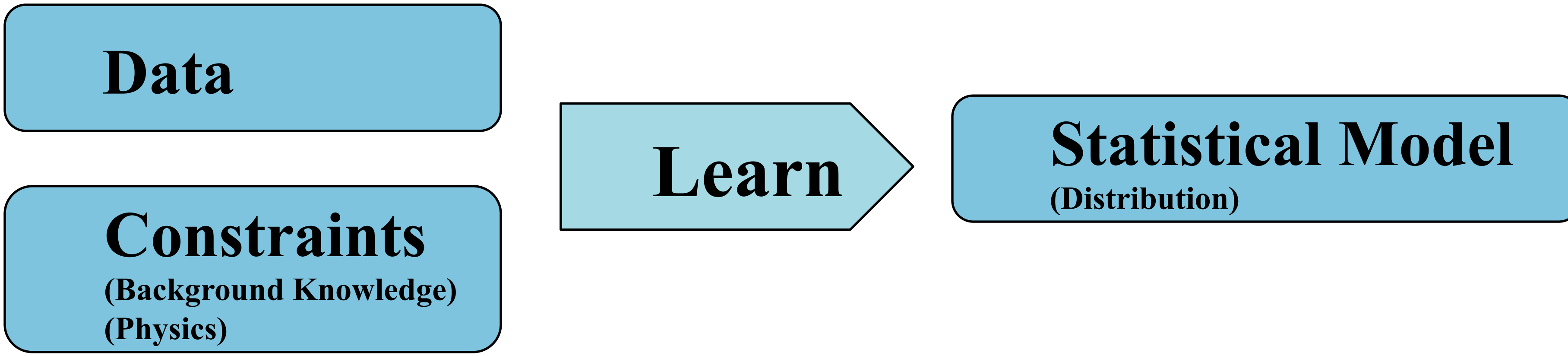
- Must take at least one of Probability or Logic.
- Probability is a prerequisite for AI.
- The prerequisites for KR is either AI or Logic.

7 out of 16 instantiations are impossible

structured

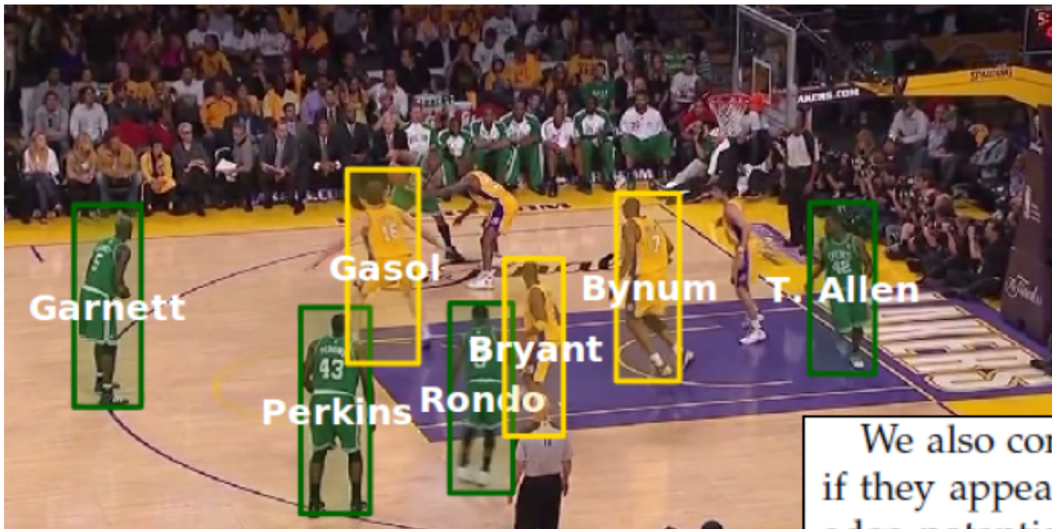
L	K	P	A
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

Learning with Constraints



Learn a statistical model that assigns
zero probability
to instantiations that violate the constraints.

Example: Video



We also connect all pairs of identity nodes $y_{t,i}$ and $y_{t,j}$ if they appear in the same time t. We then introduce an edge potential that enforces mutual exclusion:

$$\psi_{\text{mutex}}(y_{t,i}, y_{t,j}) = \begin{cases} 1 & \text{if } y_{t,i} \neq y_{t,j} \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

This potential specifies the constraint that a player can be appear only *once* in a frame. For example, if the i-th detection $y_{t,i}$ has been assign to Bryant, $y_{t,j}$ cannot have the same identity because Bryant is impossible to appear twice in a frame.

[Lu, W. L., Ting, J. A., Little, J. J., & Murphy, K. P. (2013). Learning to track and identify players from broadcast sports videos.]

Example: Language

- Non-local dependencies:

  At least one verb in each sentence

- Sentence compression

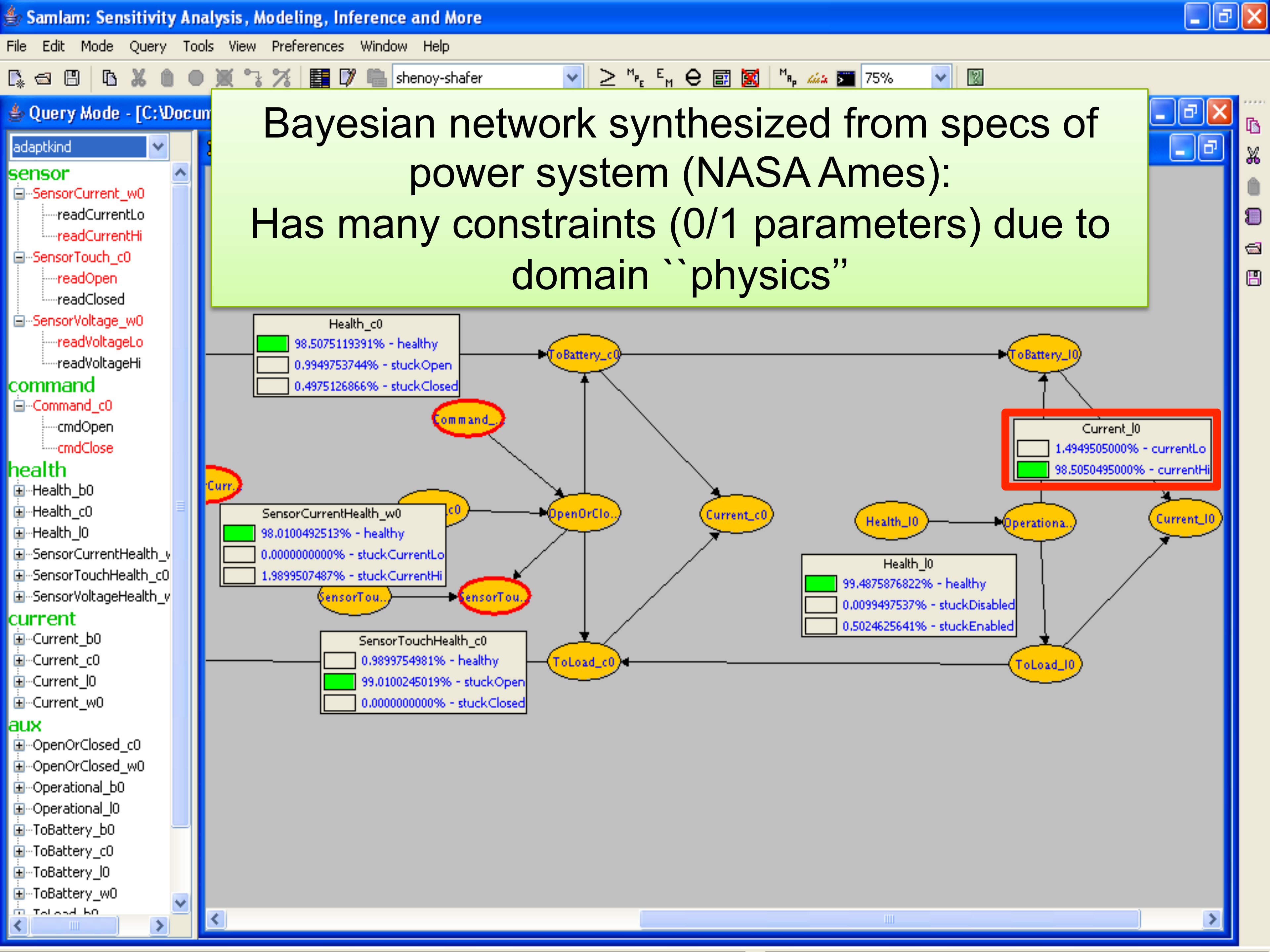
  If a modifier is kept, its subject is also kept

- Information extraction
- Semantic role labeling
- … and many more!

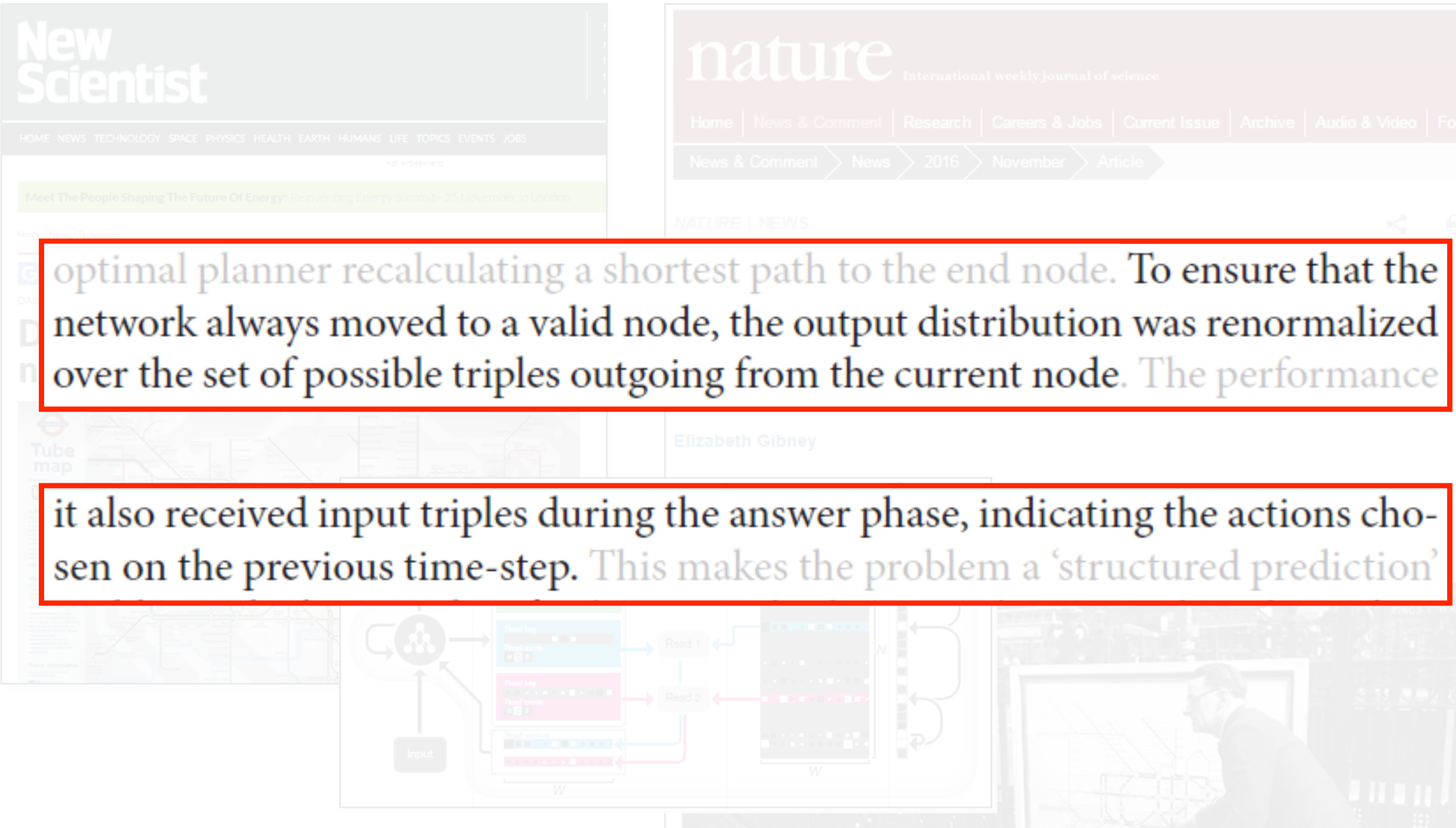
Citations	
Start	The citation must start with author or editor.
AppearsOnce	Each field must be a consecutive list of words, and can appear at most once in a citation.
Punctuation	State transitions must occur on punctuation marks.
BookJournal	The words *proc*, *journal*, *proceedings*, *ACM* are *JOURNAL* or *BOOKTITLE*.
...	...
TechReport	The words *tech*, *technical* are *TECH_REPORT*.
Title	Quotations can appear only in titles.
Location	The words *CA*, *Australia*, *NY* are *LOCATION*.

[Chang, M., Ratinov, L., & Roth, D. (2008). Constraints as prior knowledge],…, [Chang, M. W., Ratinov, L., & Roth, D. (2012). Structured learning with constrained conditional models.], [https://en.wikipedia.org/wiki/Constrained_conditional_model]

Bayesian network synthesized from specs of power system (NASA Ames):
Has many constraints (0/1 parameters) due to domain ``physics”



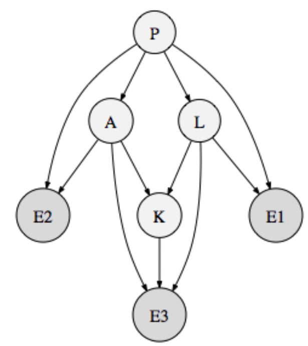
Example: Deep Learning



[Graves, A., Wayne, G., Reynolds, M., Harley, T., Danihelka, I., Grabska-Barwińska, A., et al.. (2016). Hybrid computing using a neural network with dynamic external memory. *Nature*, *538*(7626), 471-476.]

What are people doing now?

- Ignore constraints
- Handcraft into models
- Use specialized distributions
- Find non-structured encoding
- Try to learn constraints
- Hack your way around



Accuracy ?
Specialized skill ?
Intractable inference ?
Intractable learning ?
Waste parameters ?
Risk predicting out of space ?

you are on your own

Structured Probability Spaces

- Everywhere in ML!
  - Configuration problems, inventory, video, text, deep learning
  - Planning and diagnosis (physics)
  - Causal models: cooking scenarios (interpreting videos)
  - Combinatorial objects: parse trees, rankings, directed acyclic graphs, trees, simple paths, game traces, etc.

No statistical ML boxes out there that take constraints as input!

Goal: Constraints as important as data! General purpose!

Specification Language: Logic

Structured Probability Space

unstructured

L	K	P	A
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

- Must take at least one of Probability or Logic.
- Probability is a prerequisite for AI.
- The prerequisites for KR is either AI or Logic.

7 out of 16 instantiations are impossible

structured

L	K	P	A
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

Boolean Constraints

unstructured

L	K	P	A
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

$$P \vee L$$
$$A \Rightarrow P$$
$$K \Rightarrow (P \vee L)$$

7 out of 16 instantiations are impossible

structured

L	K	P	A
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

Combinatorial Objects: Rankings

rank	sushi
1	fatty tuna
2	sea urchin
3	salmon roe
4	shrimp
5	tuna
6	squid
7	tuna roll
8	see eel
9	egg
10	cucumber roll

rank	sushi
1	shrimp
2	sea urchin
3	salmon roe
4	fatty tuna
5	tuna
6	squid
7	tuna roll
8	see eel
9	egg
10	cucumber roll

10 items:
3,628,800
rankings

20 items:
2,432,902,008,176,640,000
rankings

Combinatorial Objects: Rankings

rank	sushi
1	fatty tuna
2	sea urchin
3	salmon roe
4	shrimp
5	tuna
6	squid
7	tuna roll
8	see eel
9	egg
10	cucumber roll

rank	sushi
1	shrimp
2	sea urchin
3	salmon roe
4	fatty tuna
5	tuna
6	squid
7	tuna roll
8	see eel
9	egg
10	cucumber roll

A_{ij} item *i* at position *j*
(*n* items require n^2 Boolean variables)

An item may be assigned to more than one position

A position may contain more than one item

Encoding Rankings in Logic

A_{ij} : item i at position j

	pos 1	pos 2	pos 3	pos 4
item 1	A_{11}	A_{12}	A_{13}	A_{14}
item 2	A_{21}	A_{22}	A_{23}	A_{24}
item 3	A_{31}	A_{32}	A_{33}	A_{34}
item 4	A_{41}	A_{42}	A_{43}	A_{44}

constraint: each item i assigned to a unique position (n constraints)

$$\bigvee_j A_{ij} \wedge \left(\bigwedge_{k \neq j} \neg A_{ik} \right)$$

constraint: each position j assigned a unique item (n constraints)

$$\bigvee_i A_{ij} \wedge \left(\bigwedge_{k \neq i} \neg A_{kj} \right)$$

total constraints	$2n$
unstructured space	2^{n^2}
structured space	$n!$

Structured Space for Paths


nature

Home | News & Comment | Research | Careers & Jobs | Current Issue | Archive | Audio & Video

News & Comment > News > 2016 > November > Article

NATURE | NEWS

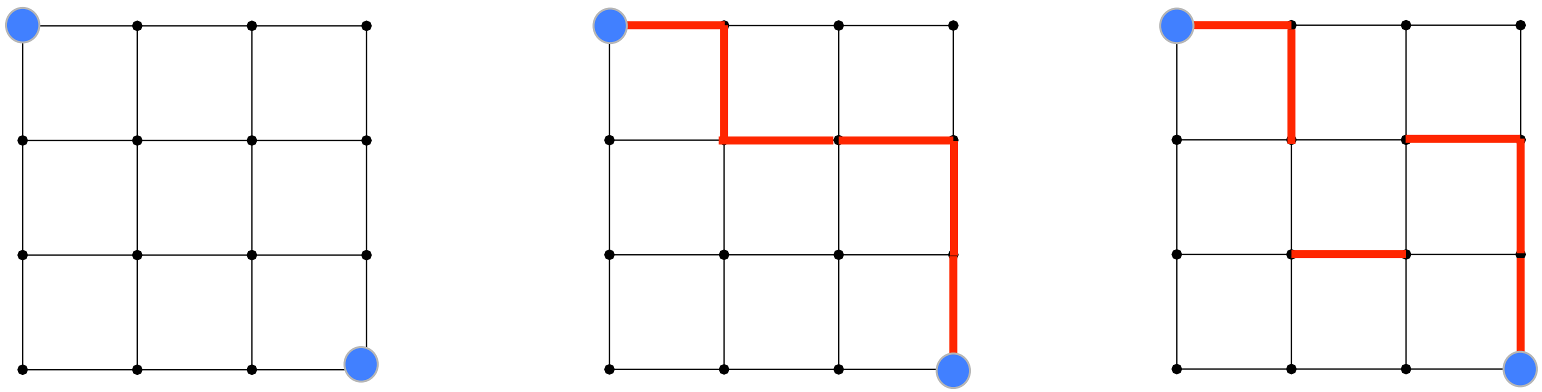
Google's AI reasons its way around the London Underground

DeepMind's latest technique uses external memory to solve tasks that require logic and reasoning — a step toward more human-like AI.

Elizabeth Gibney

13 October 2016

Rights & Permissions



Good variable assignment (represents route)

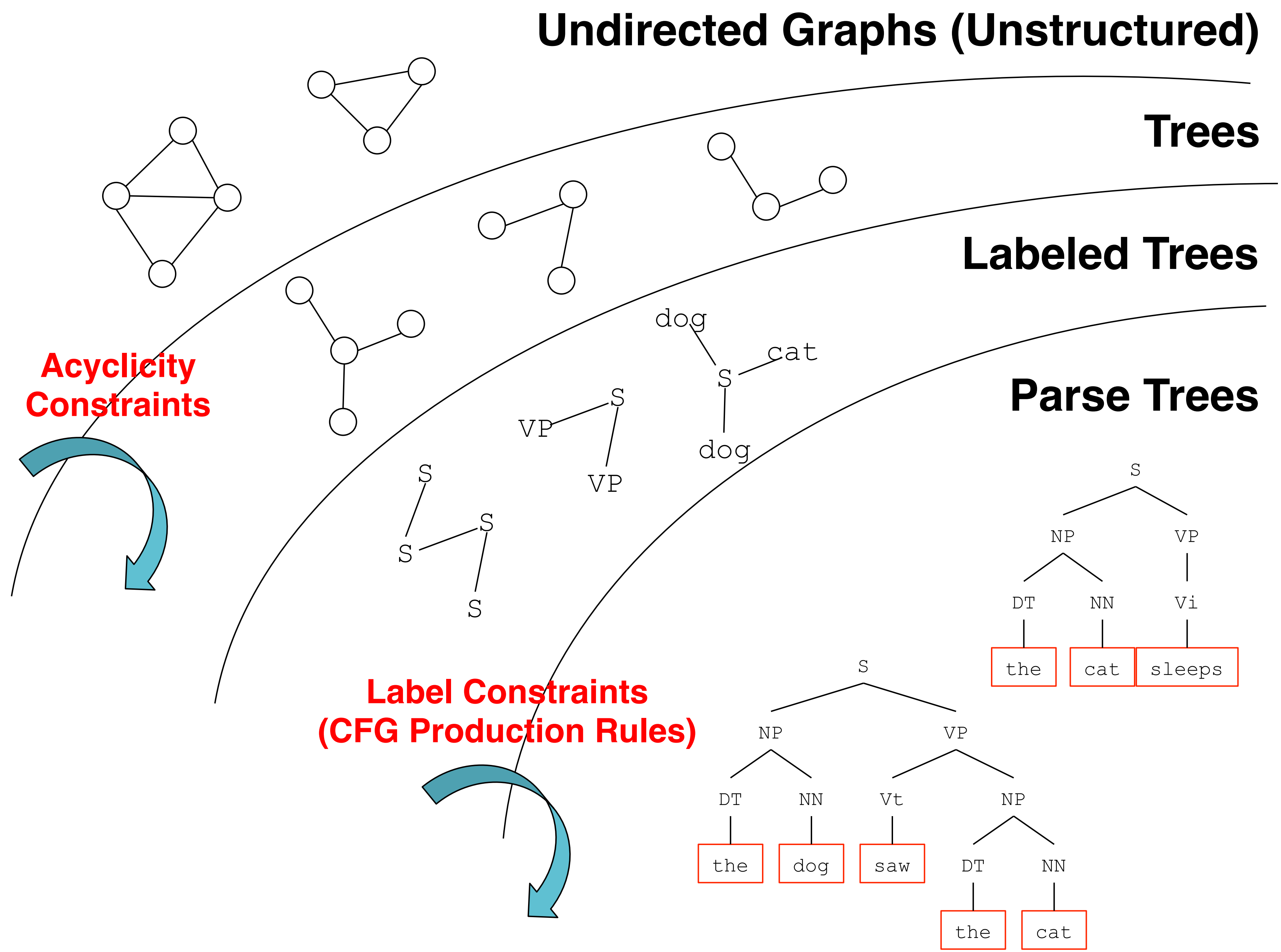
184

Bad variable assignment (does not represent route)

16,777,032

Space easily encoded in logical constraints

Unstructured probability space: 184+16,777,032 = 2^{24}

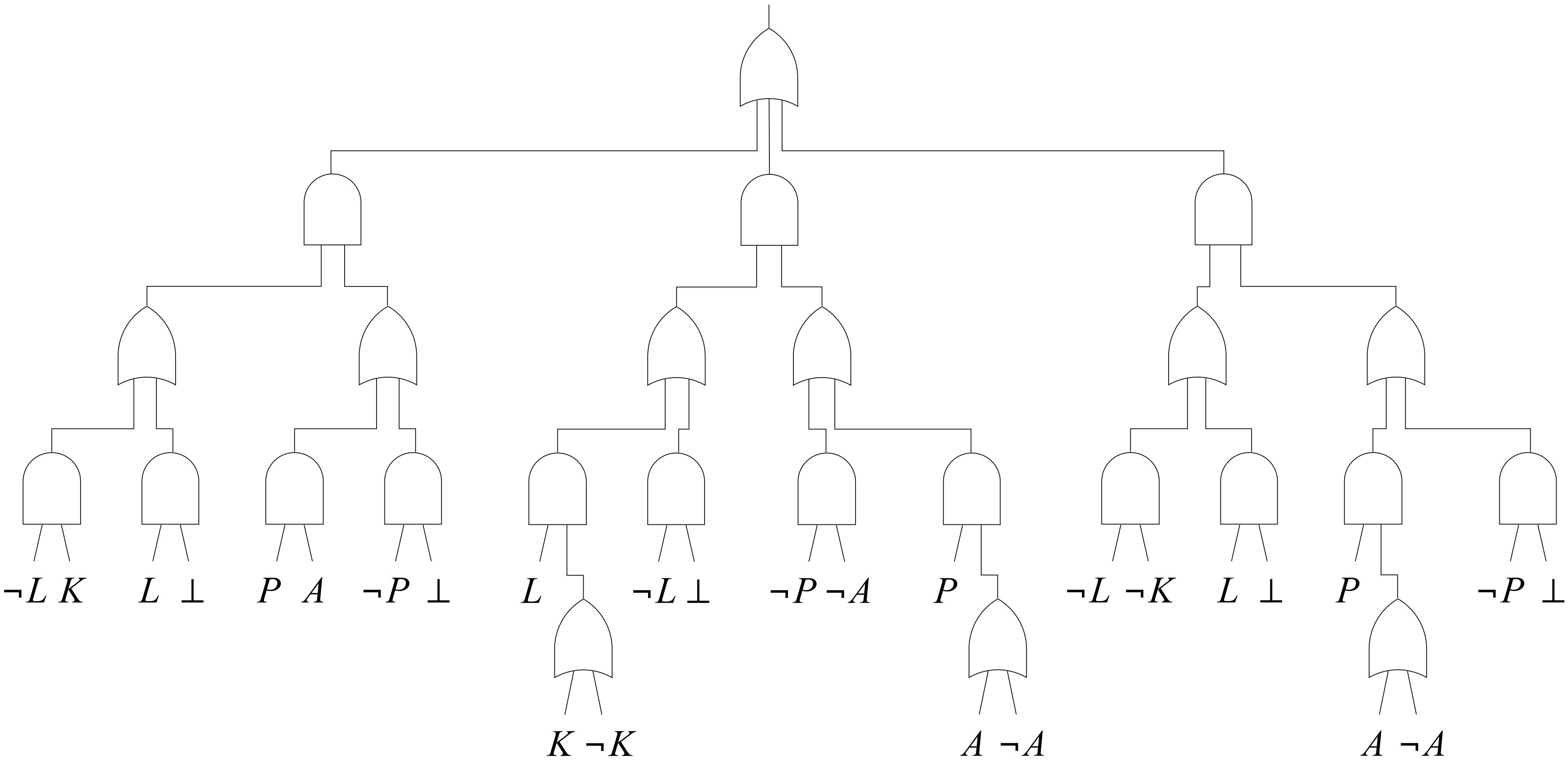

Undirected Graphs (Unstructured)
Trees
Labeled Trees
Parse Trees
Acyclicity Constraints
Label Constraints (CFG Production Rules)
dog
cat
S
dog
VP
S
VP
S
S
S
S
S
NP
VP
DT
NN
Vi
the
cat
sleeps
S
NP
VP
DT
NN
Vt
NP
the
dog
saw
DT
NN
the
cat

“Deep Representation”

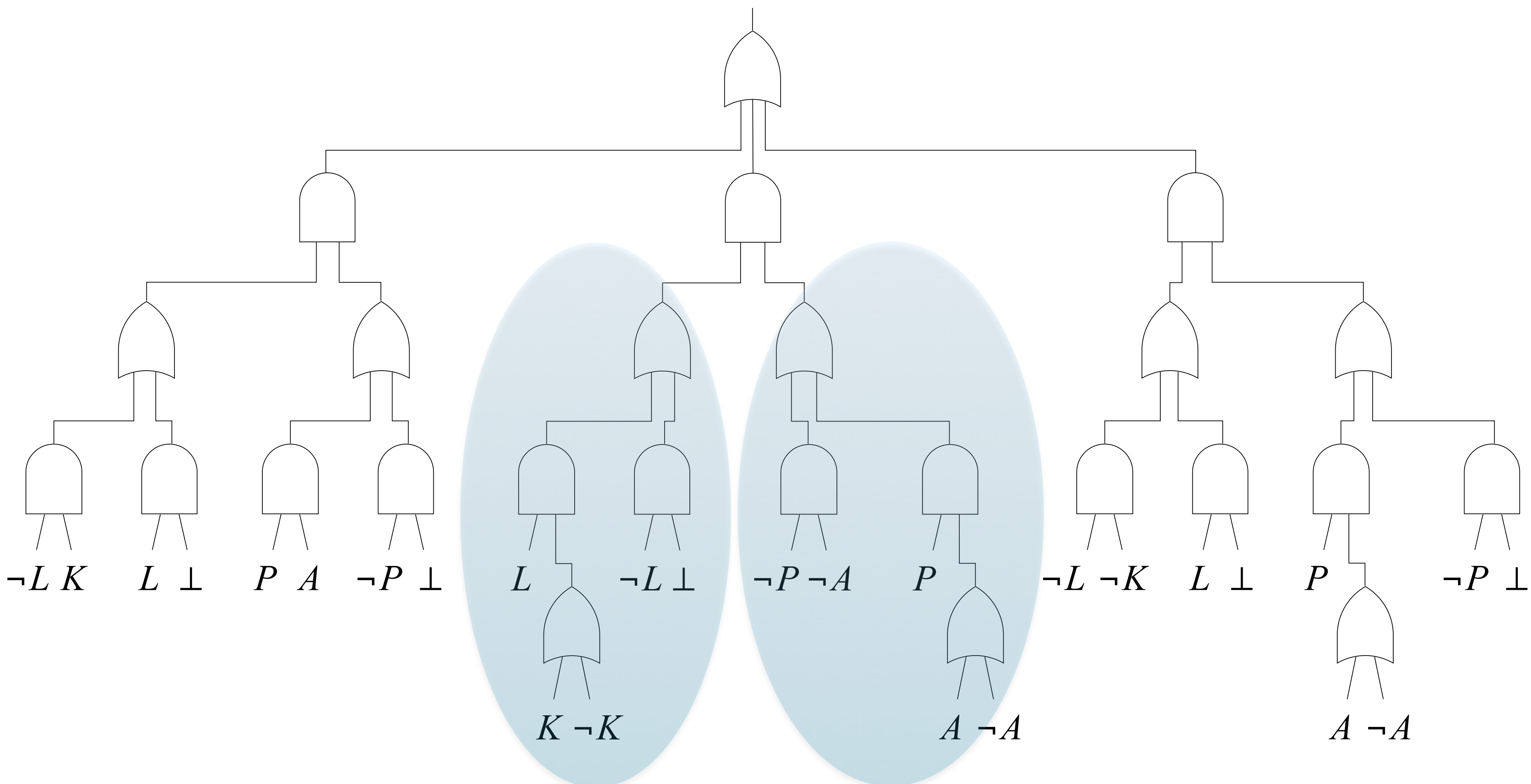
Logic + Probability

Logical Circuits

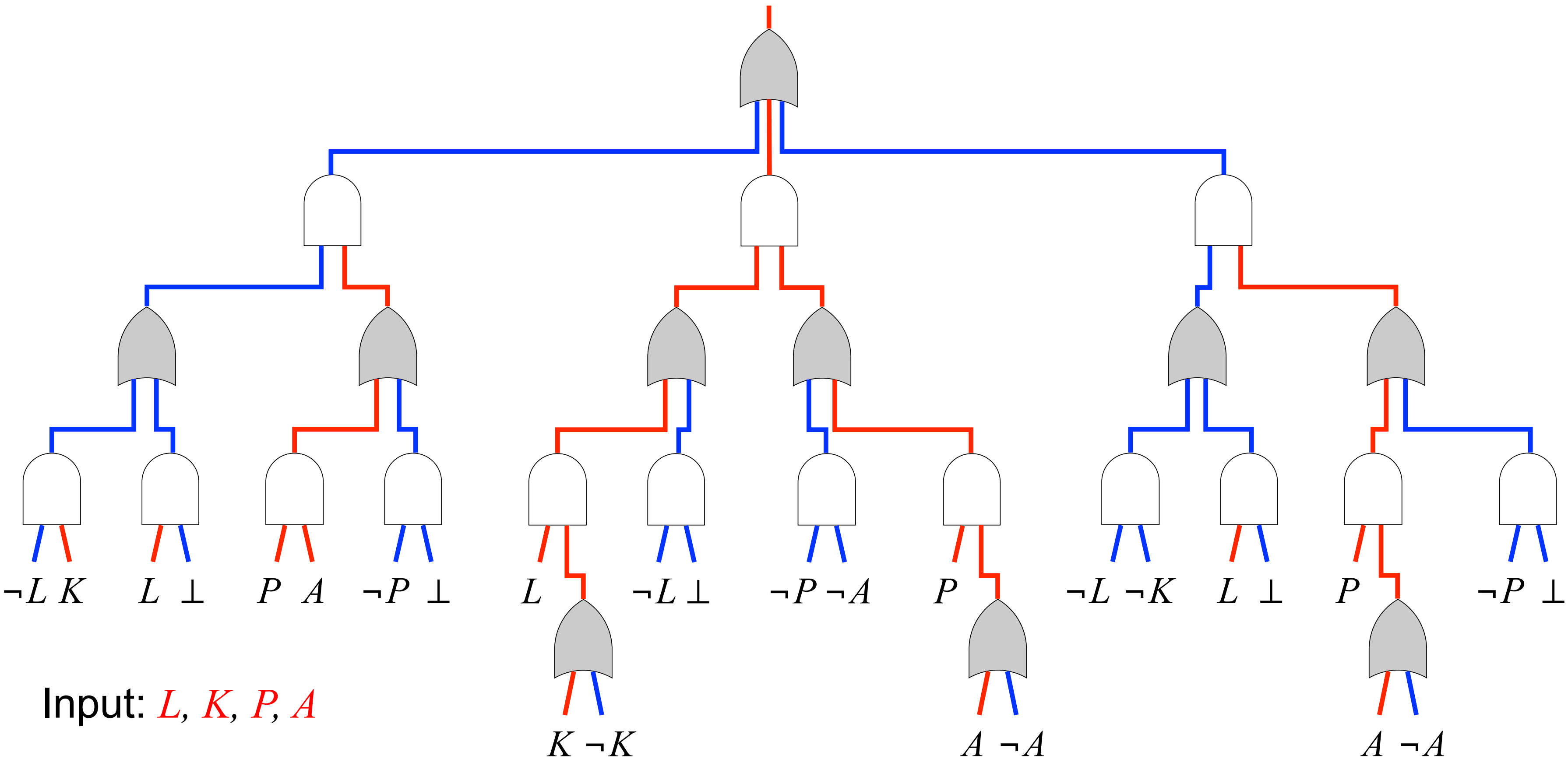
$P \lor L$
$A \Rightarrow P$
$K \Rightarrow (P \lor L)$



Property: Decomposability

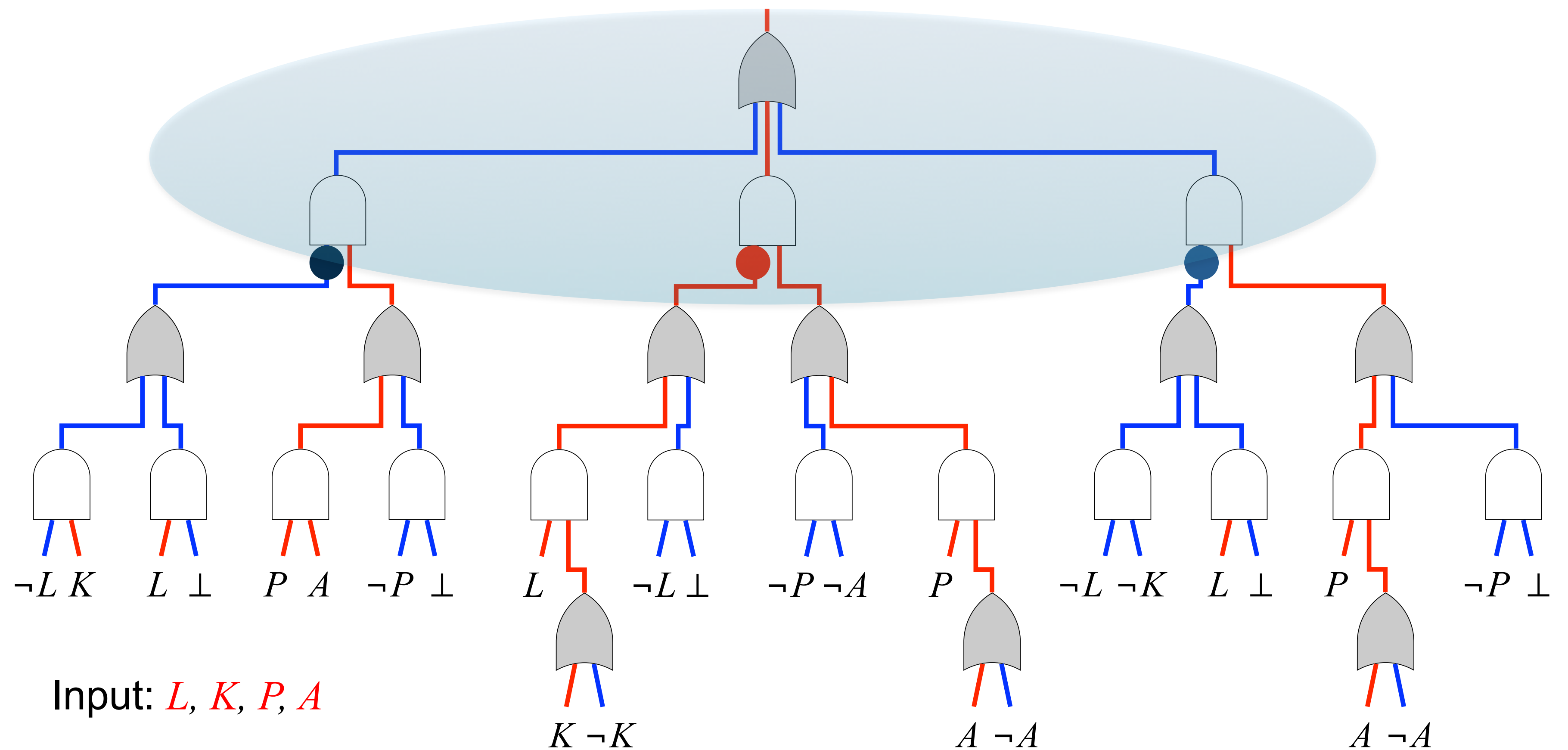


Property: Determinism



Input: L, K, P, A

Sentential Decision Diagram (SDD)



Input: *L*, *K*, *P*, *A*

Tractable for Logical Inference

- Is structured space empty? (SAT)
- Count size of structured space (#SAT)
- Check equivalence of spaces
- Algorithms linear in circuit size
  (pass up, pass down, similar to backprop)

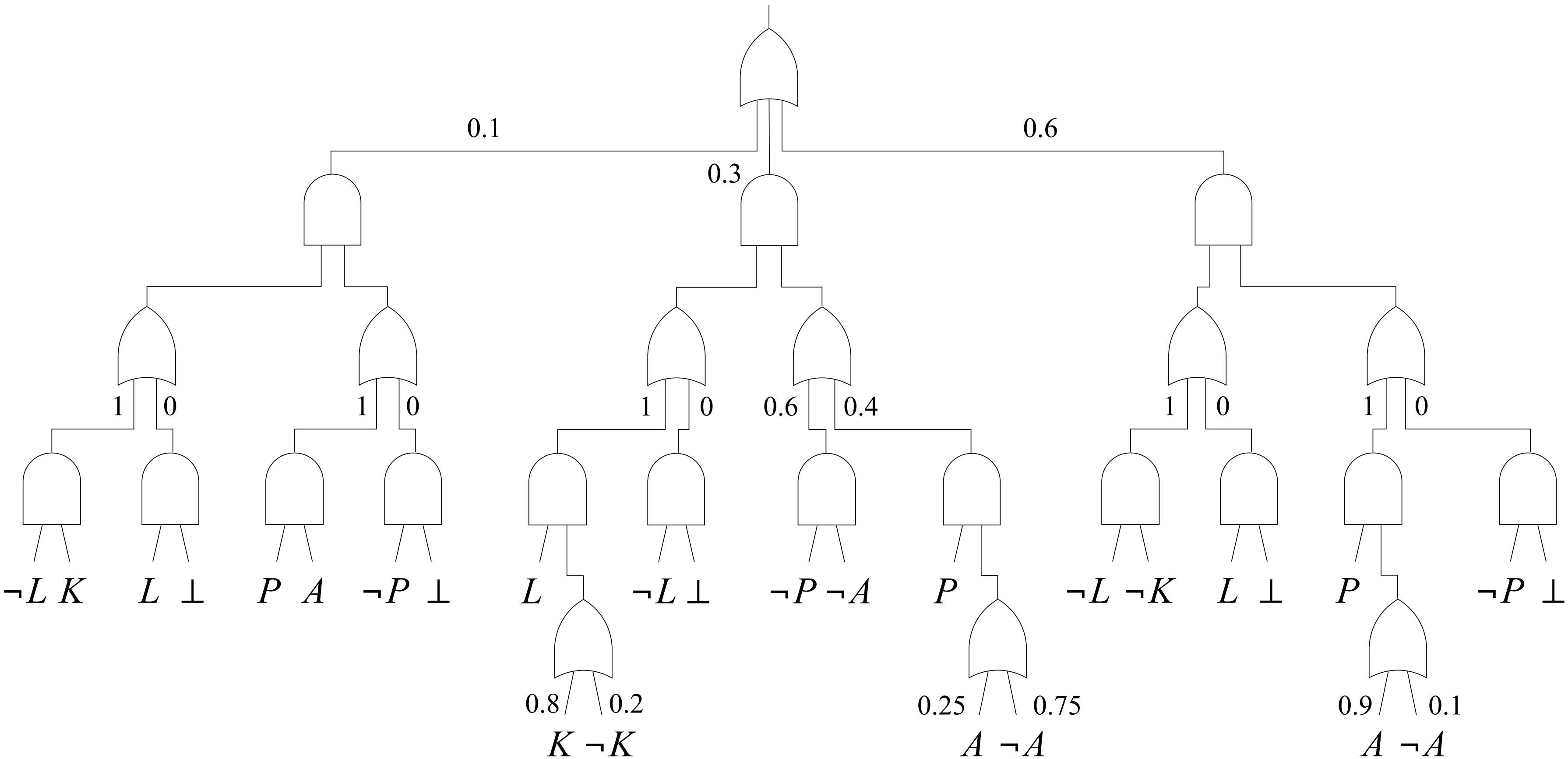
SCIENCE + TECHNOLOGY

Artificial intelligence framework developed by UCLA professor now powers Toyota websites

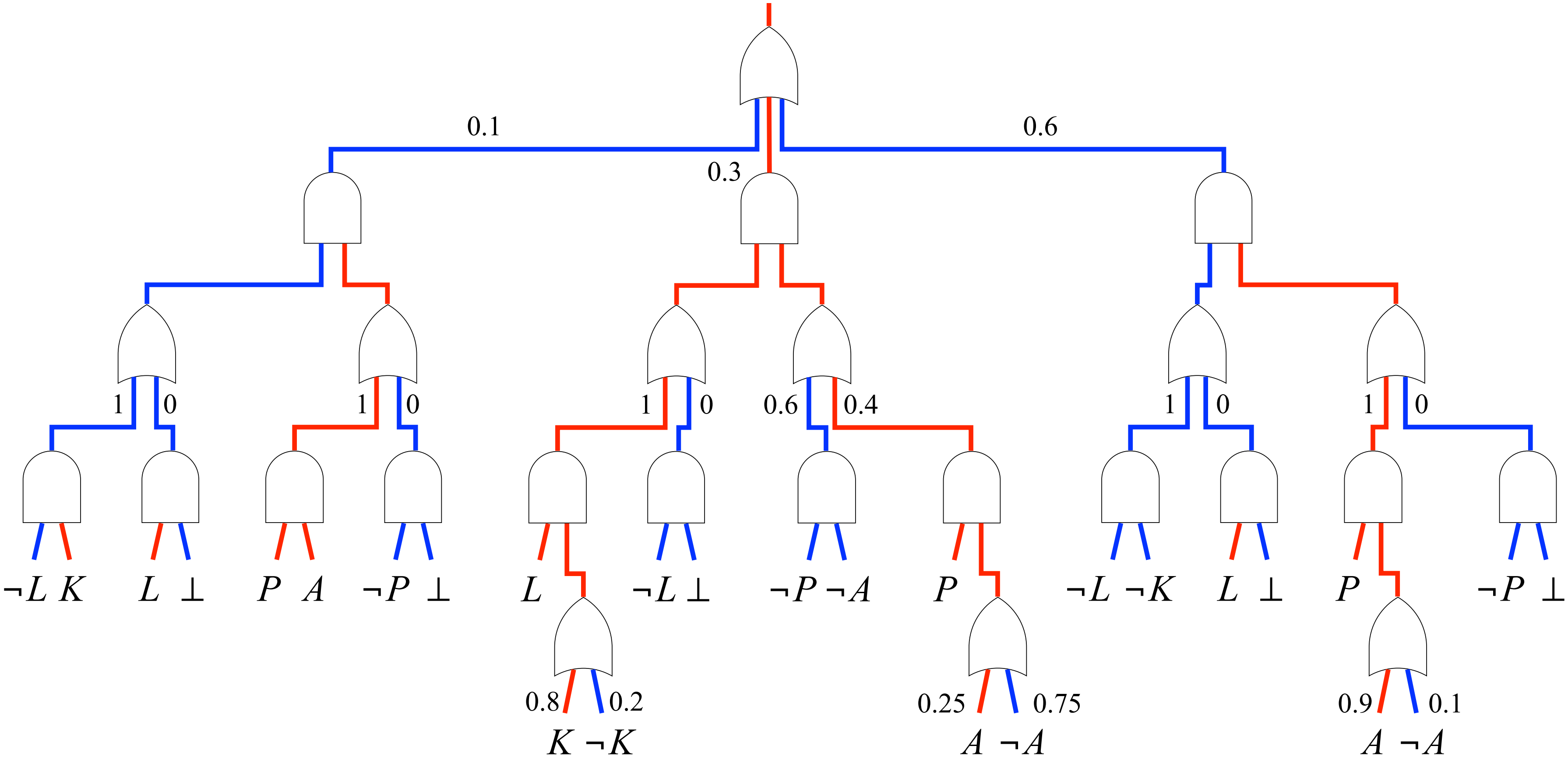
Adnan Darwiche's invention helps consumers customize their vehicles online

Matthew Chin | May 12, 2016

PSDD: Probabilistic SDD

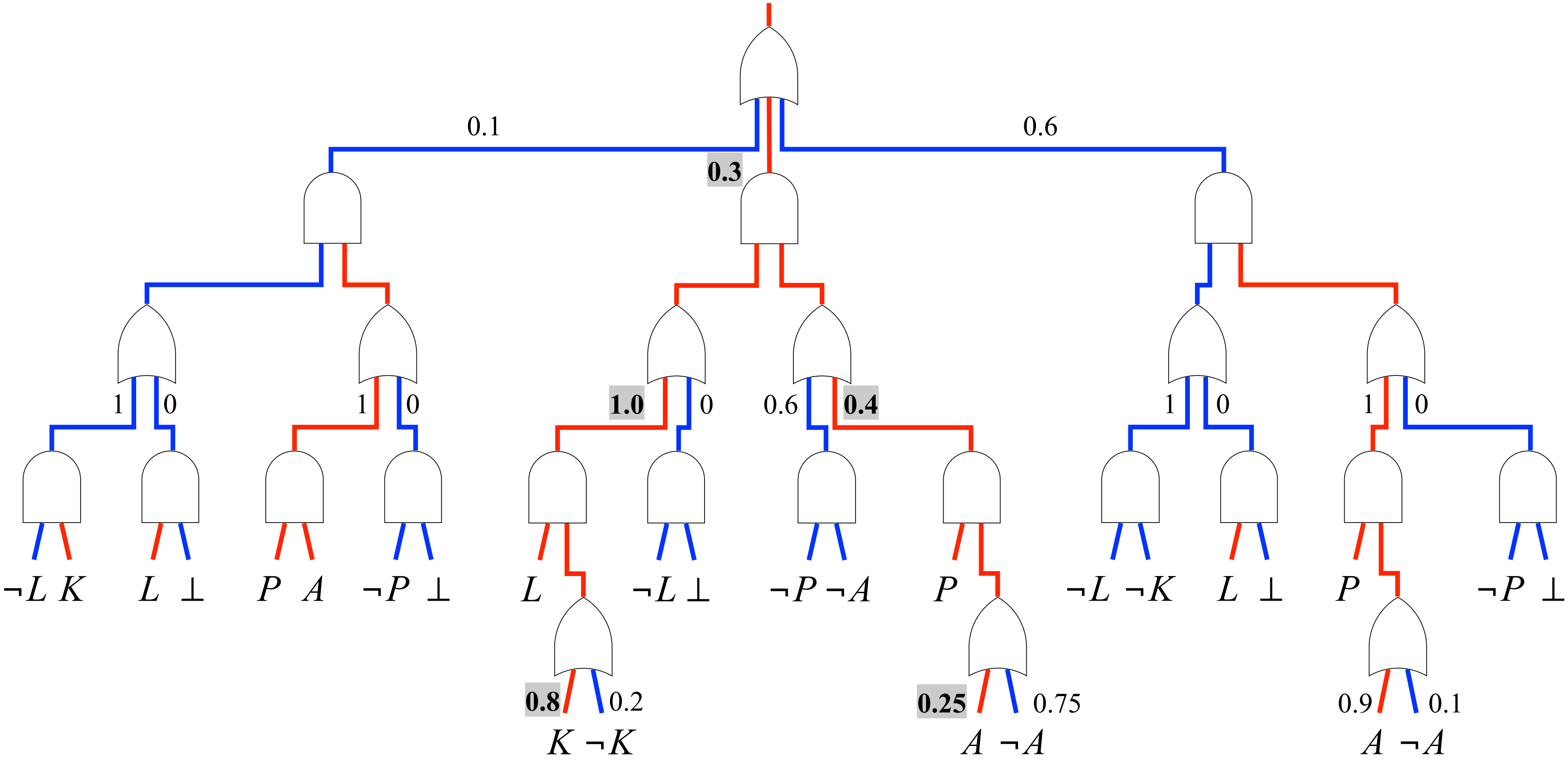


PSDD: Probabilistic SDD



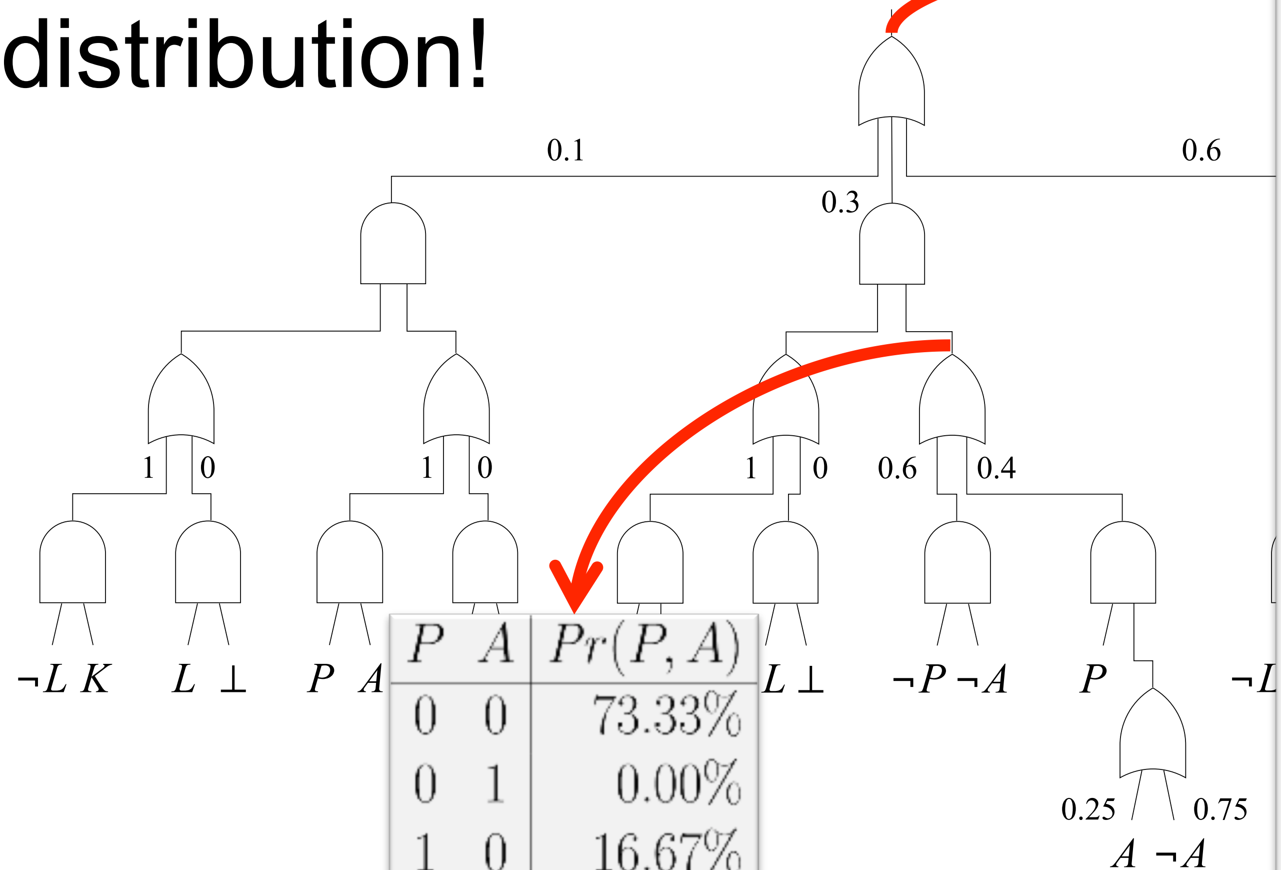
Input: *L*, *K*, *P*, *A*

PSDD: Probabilistic SDD



Input: *L*, *K*, *P*, *A* Pr(*L*,*K*,*P*,*A*) = 0.3 x 1.0 x 0.8 x 0.4 x 0.25 = 0.024

PSDD nodes induce a normalized distribution!



P	A	$Pr(P, A)$
0	0	73.33%
0	1	0.00%
1	0	16.67%
1	1	10.00%

L	K	P	A	$Pr(L, K, P, A)$
0	0	0	0	0.00%
0	0	0	1	0.00%
0	0	1	0	6.00%
0	0	1	1	54.00%
0	1	0	0	0.00%
0	1	0	1	0.00%
0	1	1	0	0.00%
0	1	1	1	10.00%
1	0	0	0	4.40%
1	0	0	1	0.00%
1	0	1	0	1.00%
1	0	1	1	0.60%
1	1	0	0	17.6%
1	1	0	1	0.00%
1	1	1	0	4.00%
1	1	1	1	2.40%

Can read probabilistic independences off the circuit structure

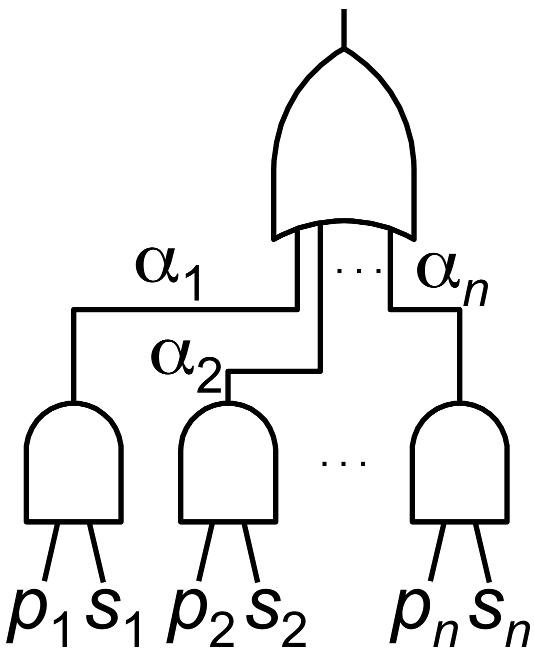
Tractable for Probabilistic Inference

- **MAP inference**: Find most-likely assignment (otherwise NP-complete)
- Computing **conditional probabilities** Pr(x|y) (otherwise PP-complete)
- **Sample** from Pr(x|y)

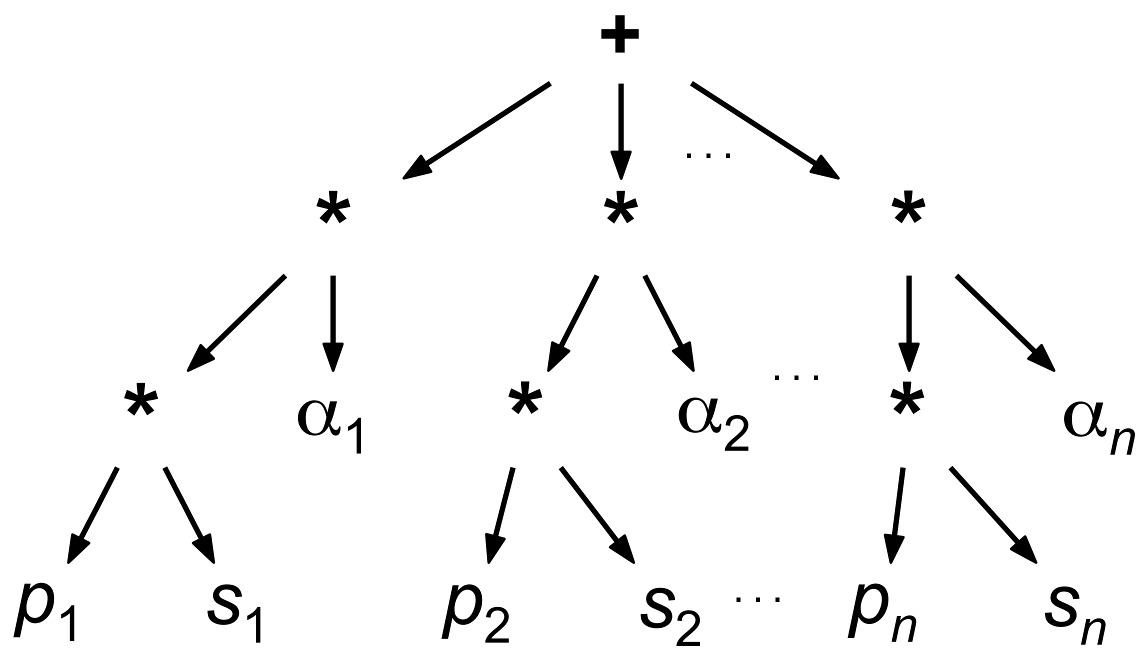
- Algorithms linear in circuit size (pass up, pass down, similar to backprop)

PSDDs are Arithmetic Circuits

[Darwiche, JACM 2003]



PSDD



AC

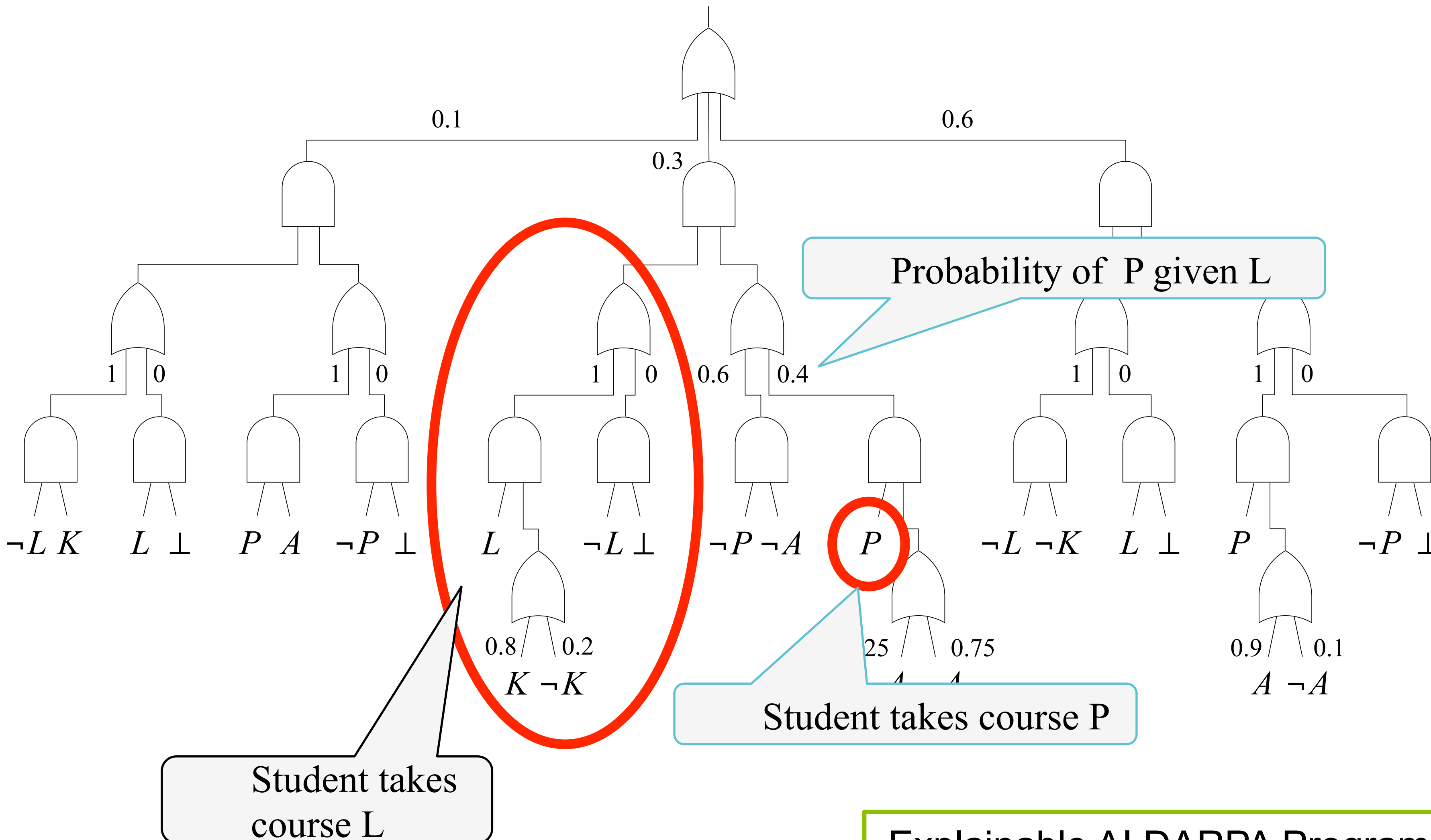
Known in the ML literature as SPNs
UAI 2011, NIPS 2012 best paper awards

[ICML 2014]
(SPNs equivalent to ACs)

Learning PSDDs

Logic + Probability + ML

Parameters are Interpretable



Explainable AI DARPA Program

Learning Algorithms

- Parameter learning:

  Closed form max likelihood from complete data

  One pass over data to estimate parameters

Note a lot to say: very easy!

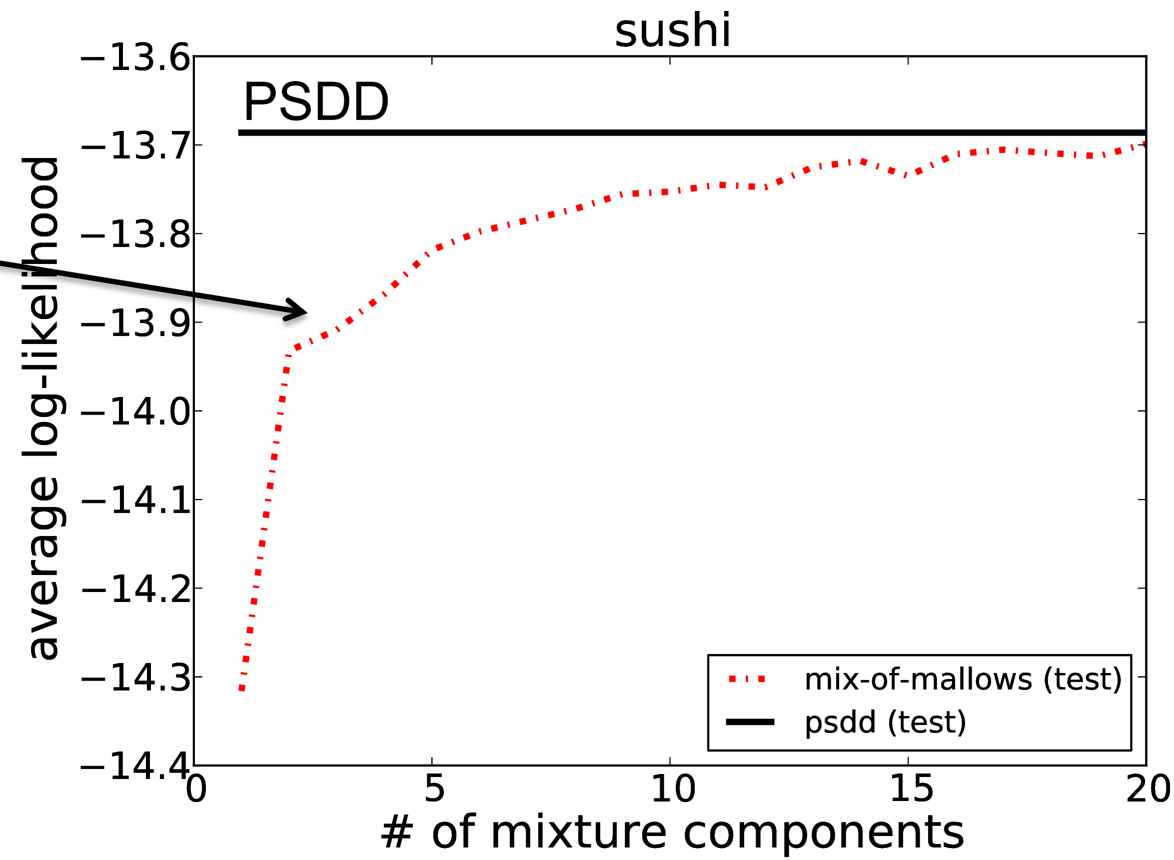
- Structure learning:
  - Compile constraints to SDD (naive)

    Use SAT solver technology (SDD library)
  - Search for structure to fit data (ongoing work)

Learning Preference Distributions

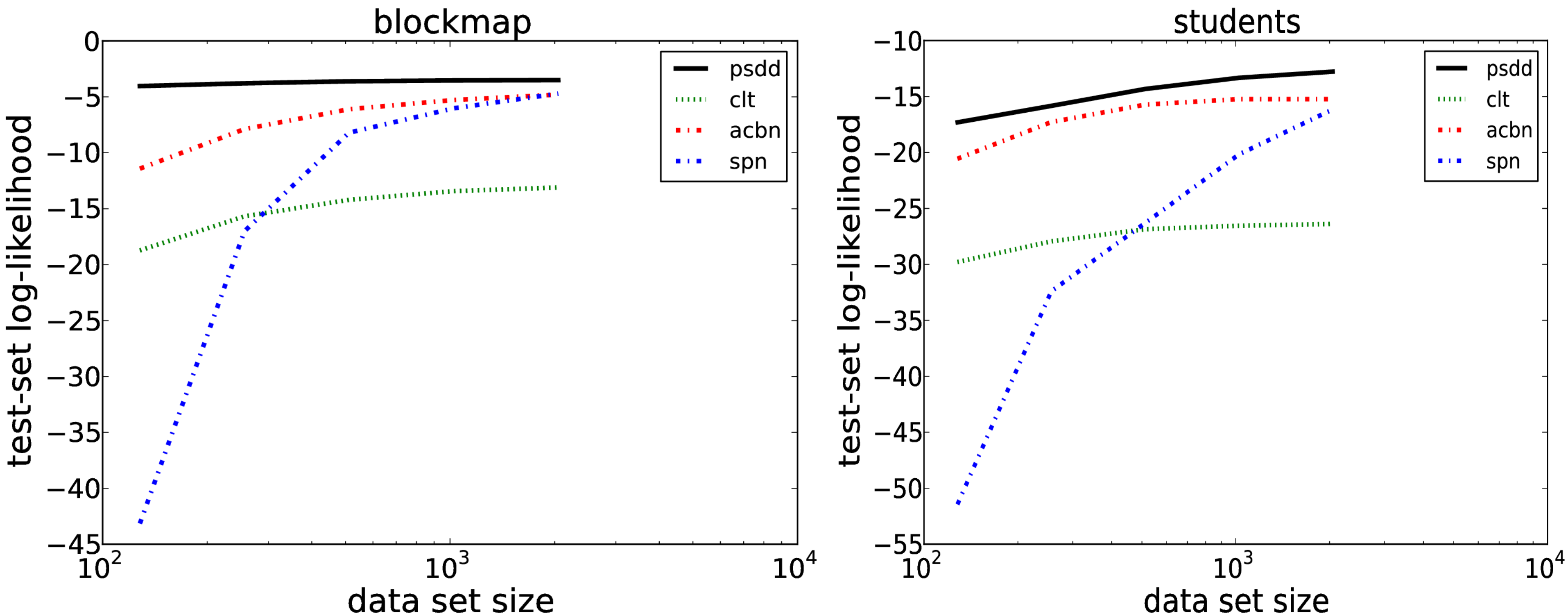
Special-purpose distribution: Mixture-of-Mallows

- # of components from 1 to 20
- EM with 10 random seeds
- implementation of Lu & Boutilier

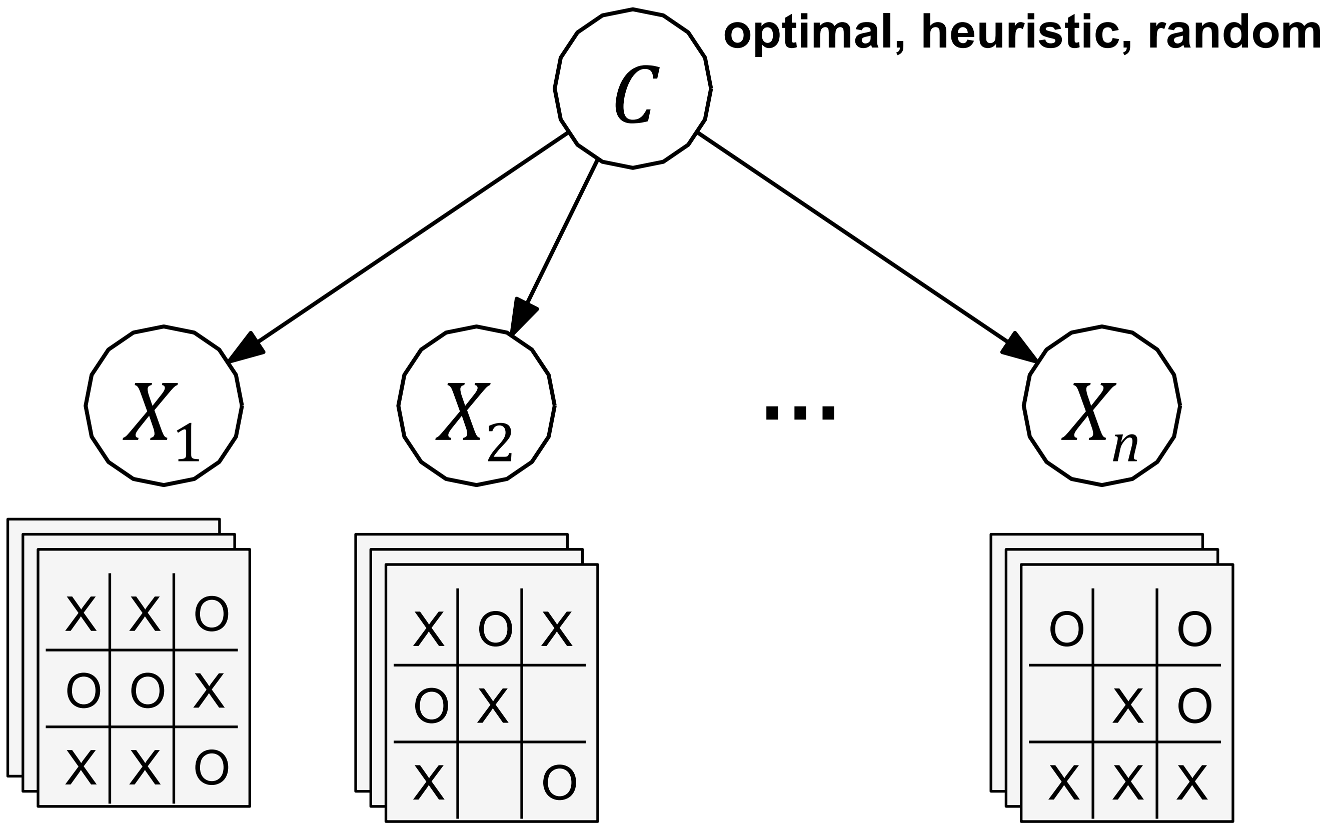


This is the naive approach, without real structure learning!

What happens if you **ignore** constraints?

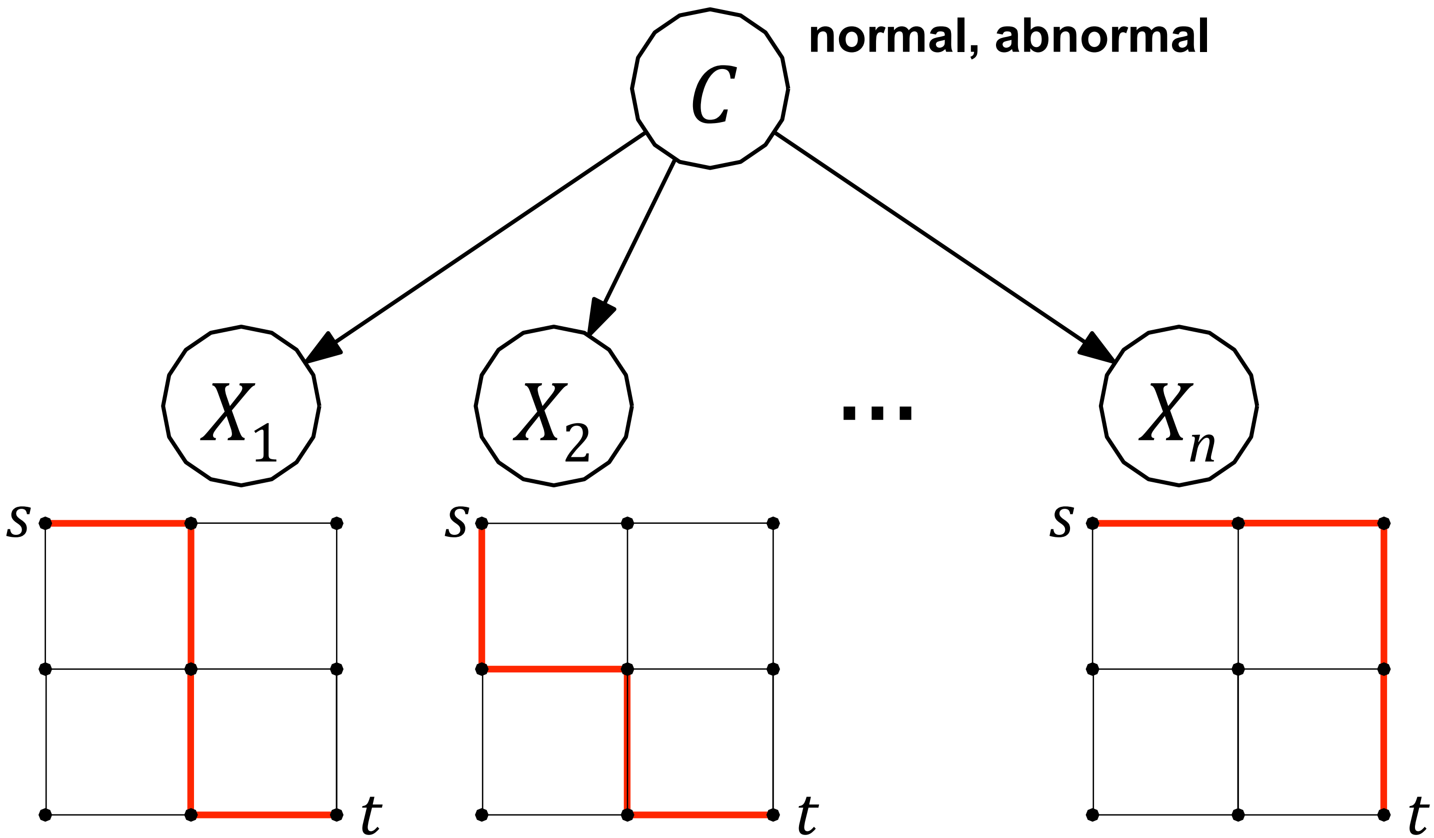


Structured Naïve Bayes Classifier



Attribute with 362,880 values (possible game traces)

Structured Naïve Bayes Classifier



Attribute with 789,360,053,252 values (routes in 8 × 8 grid)
Ongoing work: learn anomalies from Uber data

Structured datasets and queries

Incomplete Data

a classical
complete dataset

id	X	Y	Z
1	x_1	y_2	z_1
2	x_2	y_1	z_2
3	x_2	y_1	z_2
4	x_1	y_1	z_1
5	x_1	y_2	z_2

closed-form
(maximum-likelihood
estimates are unique)

a classical
incomplete dataset

id	X	Y	Z
1	x_1	y_2	?
2	x_2	y_1	?
3	?	?	z_2
4	?	y_1	z_1
5	x_1	y_2	z_2

EM algorithm
(on PSDDs)

a new type of
incomplete dataset

id	X	Y	Z
1	$X \equiv Z$		
2	x_2 and (y_2 or z_2)		
3	$x_2 \Rightarrow y_1$		
4	$X \oplus Y \oplus Z \equiv 1$		
5	x_1 and y_2 and z_2		

Missed in the
ML literature

Structured Datasets

a classical **complete** dataset
(e.g., total rankings)

id	1st sushi	2nd sushi	3rd sushi	…
1	fatty tuna	sea urchin	salmon roe	…
2	fatty tuna	tuna	shrimp	…
3	tuna	tuna roll	sea eel	…
4	fatty tuna	salmon roe	tuna	…
5	egg	squid	shrimp	…

a classical **incomplete** dataset
(e.g., top-*k* rankings)

id	1st sushi	2nd sushi	3rd sushi	…
1	fatty tuna	sea urchin	?	…
2	fatty tuna	?	?	…
3	tuna	tuna roll	?	…
4	fatty tuna	salmon roe	?	…
5	egg	?	?	…

Structured Datasets

a classical **complete** dataset
(e.g., total rankings)

id	1st sushi	2nd sushi	3rd sushi	…
1	fatty tuna	sea urchin	salmon roe	…
2	fatty tuna	tuna	shrimp	…
3	tuna	tuna roll	sea eel	…
4	fatty tuna	salmon roe	tuna	…
5	egg	squid	shrimp	…

a new type of **incomplete** dataset
(e.g., **partial** rankings)

id	1st sushi	2nd sushi	3rd sushi	…
1	(fatty tuna > sea urchin) and (tuna > sea eel)			…
2	(fatty tuna is 1st) and (salmon roe > egg)			…
3	tuna > squid			…
4	egg is last			…
5	egg > squid > shrimp			…

(represents constraints on possible *total rankings*)

Learning from Incomplete Data

- Movielens Dataset:
  - 3,900 movies, 6,040 users, 1m ratings
  - take ratings from 64 most rated movies
  - ratings 1-5 converted to pairwise prefs.
- PSDD for **partial** rankings
  - 4 tiers
  - 18,711 parameters

movies by expected tier

rank	movie
1	The Godfather
2	The Usual Suspects
3	Casablanca
4	The Shawshank Redemption
5	Schindler's List
6	One Flew Over the Cuckoo's Nest
7	The Godfather: Part II
8	Monty Python and the Holy Grail
9	Raiders of the Lost Ark
10	Star Wars IV: A New Hope

PSDD Sizes

items	tier size	Size		
n	k	SDD	Structured Space	Unstructured Space
8	2	443	840	$1.84 \cdot 10^{19}$
27	3	4,114	$1.18 \cdot 10^{9}$	$2.82 \cdot 10^{219}$
64	4	23,497	$3.56 \cdot 10^{18}$	$1.04 \cdot 10^{1233}$
125	5	94,616	$3.45 \cdot 10^{31}$	$3.92 \cdot 10^{4703}$
216	6	297,295	$1.57 \cdot 10^{48}$	$7.16 \cdot 10^{14044}$
343	7	781,918	$4.57 \cdot 10^{68}$	$7.55 \cdot 10^{35415}$

Structured Queries

rank	movie
1	Star Wars V: The Empire Strikes Back
2	Star Wars IV: A New Hope
3	The Godfather
4	The Shawshank Redemption
5	The Usual Suspects

- no other Star Wars movie in top-5
- at least one comedy in top-5

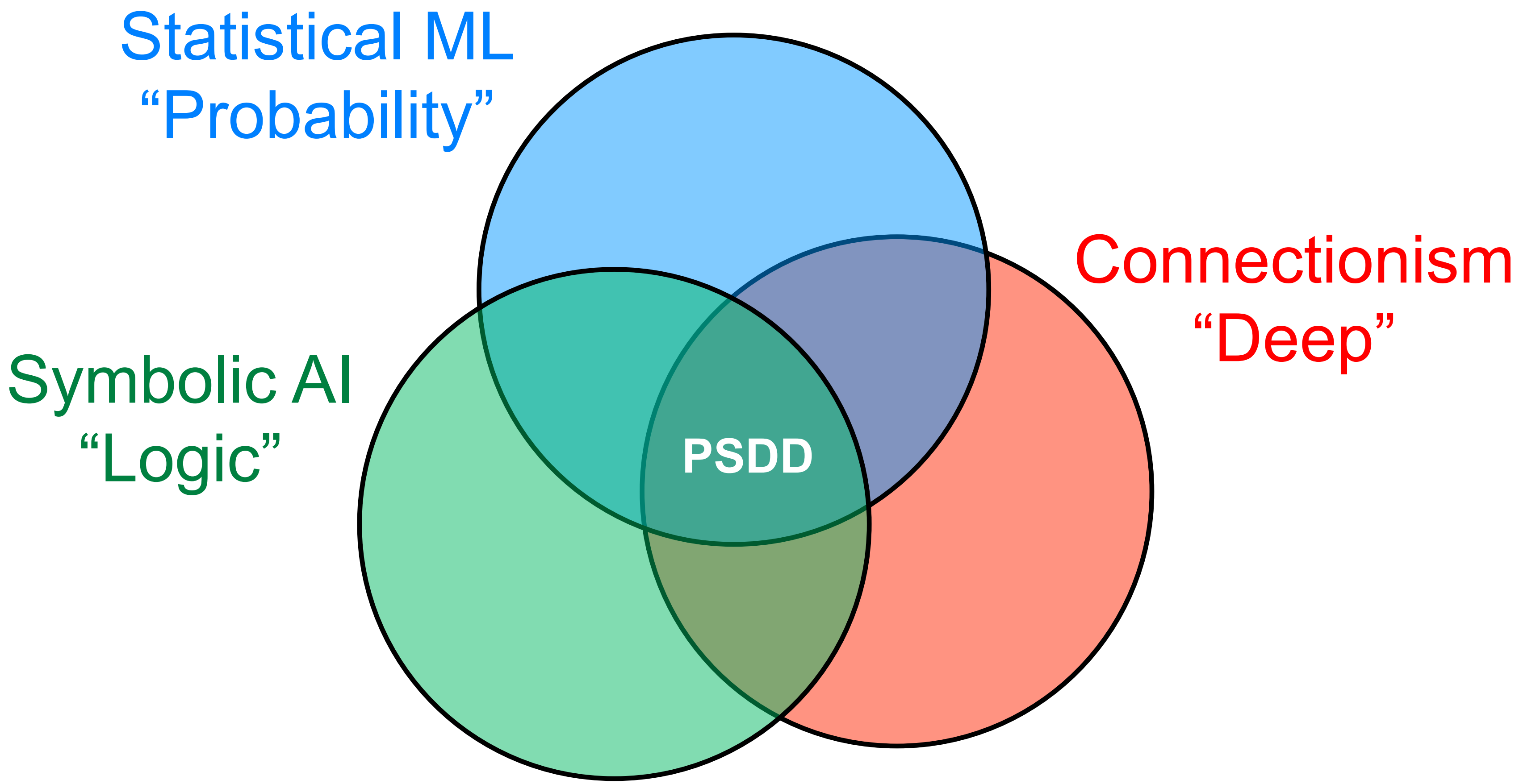
rank	movie
1	Star Wars V: The Empire Strikes Back
2	American Beauty
3	The Godfather
4	The Usual Suspects
5	The Shawshank Redemption

diversified recommendations via
logical constraints

Conclusions

- Structured spaces are everywhere
- Roles of Boolean constraints in ML
  - Domain constraints and combinatorial objects (**structured probability space**)
  - Incomplete examples (**structured datasets**)
  - Questions and evidence (**structured queries**)
- Learn distributions over combinatorial objects
- Strong properties for inference and learning: *Probabilistic sentential decision diagram* (PSDD)

Conclusions



References

Probabilistic Sentential Decision Diagrams
Doga Kisa, Guy Van den Broeck, Arthur Choi and Adnan Darwiche
KR, 2014

Learning with Massive Logical Constraints
Doga Kisa, Guy Van den Broeck, Arthur Choi and Adnan Darwiche
ICML 2014 workshop

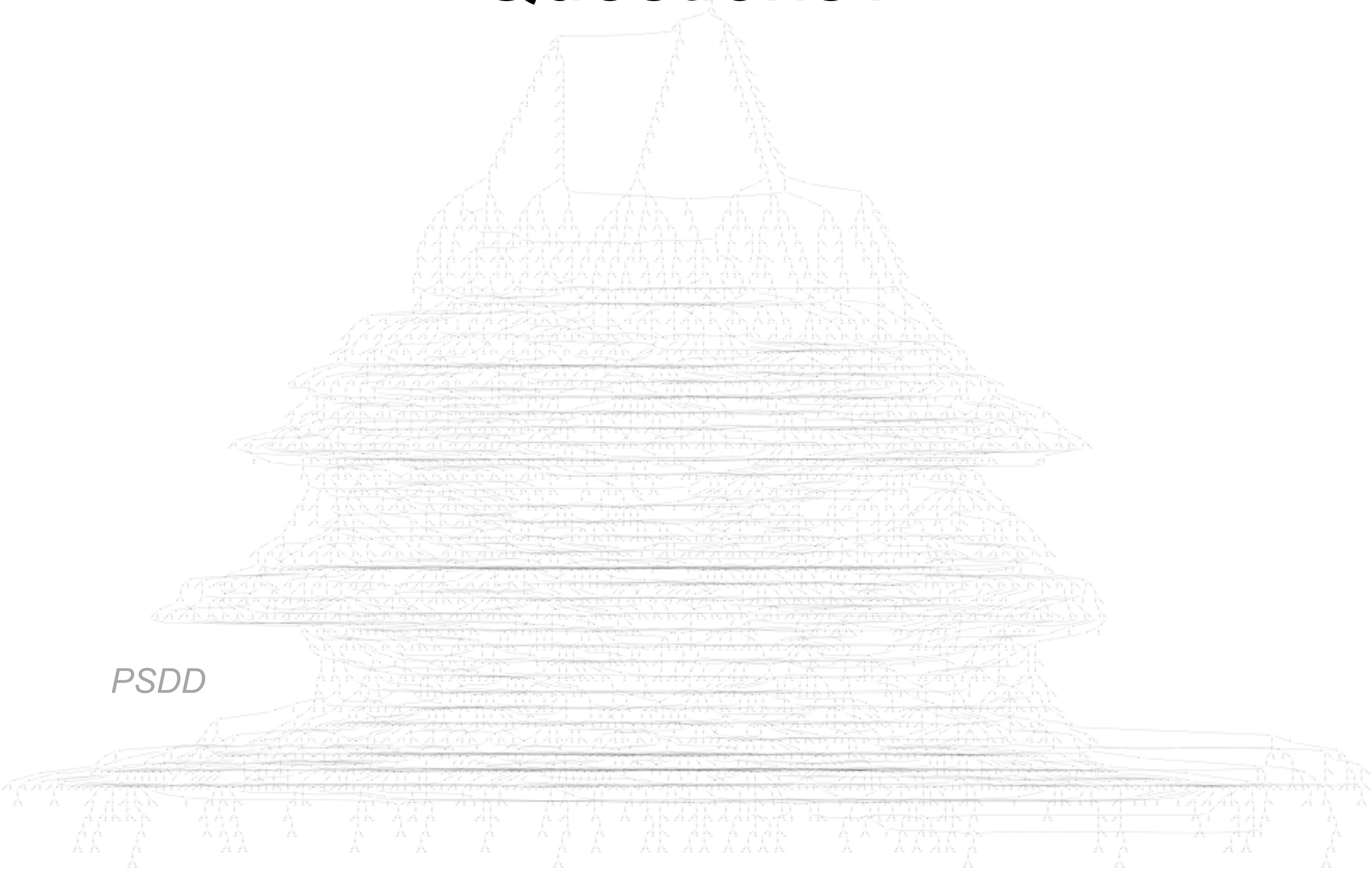
Tractable Learning for Structured Probability Spaces
Arthur Choi, Guy Van den Broeck and Adnan Darwiche
IJCAI, 2015

Tractable Learning for Complex Probability Queries
Jessa Bekker, Jesse Davis, Arthur Choi, Adnan Darwiche, Guy Van den Broeck.
NIPS, 2015

Structured Features in Naive Bayes Classifiers
Arthur Choi, Nazgol Tavabi and Adnan Darwiche
AAAI, 2016

Tractable Operations on Arithmetic Circuits
Jason Shen, Arthur Choi and Adnan Darwiche
NIPS, 2016

Questions?



PSDD