Low-Error Two-Source extractors from efficient non-malleable extractors

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Joint work with
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Today's talk

* Two-source extractors.
* Non-malleable extractors.
* Current constructions of two-source extractors via non-malleable extractors and where they fail in achieving small error.
* Constructing low-error two-source extractors given "good" non-malleable extractors.

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Two-source extractors

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- We say that a source X over $\{0,1\}^n$ has k min-entropy if for every x, $\Pr[X=x] \leq 2^{-k}$. This is how we model *weak* sources.
- Alternatively, we can think of a weak source X as uniformly distributed over a subset of size 2^k.

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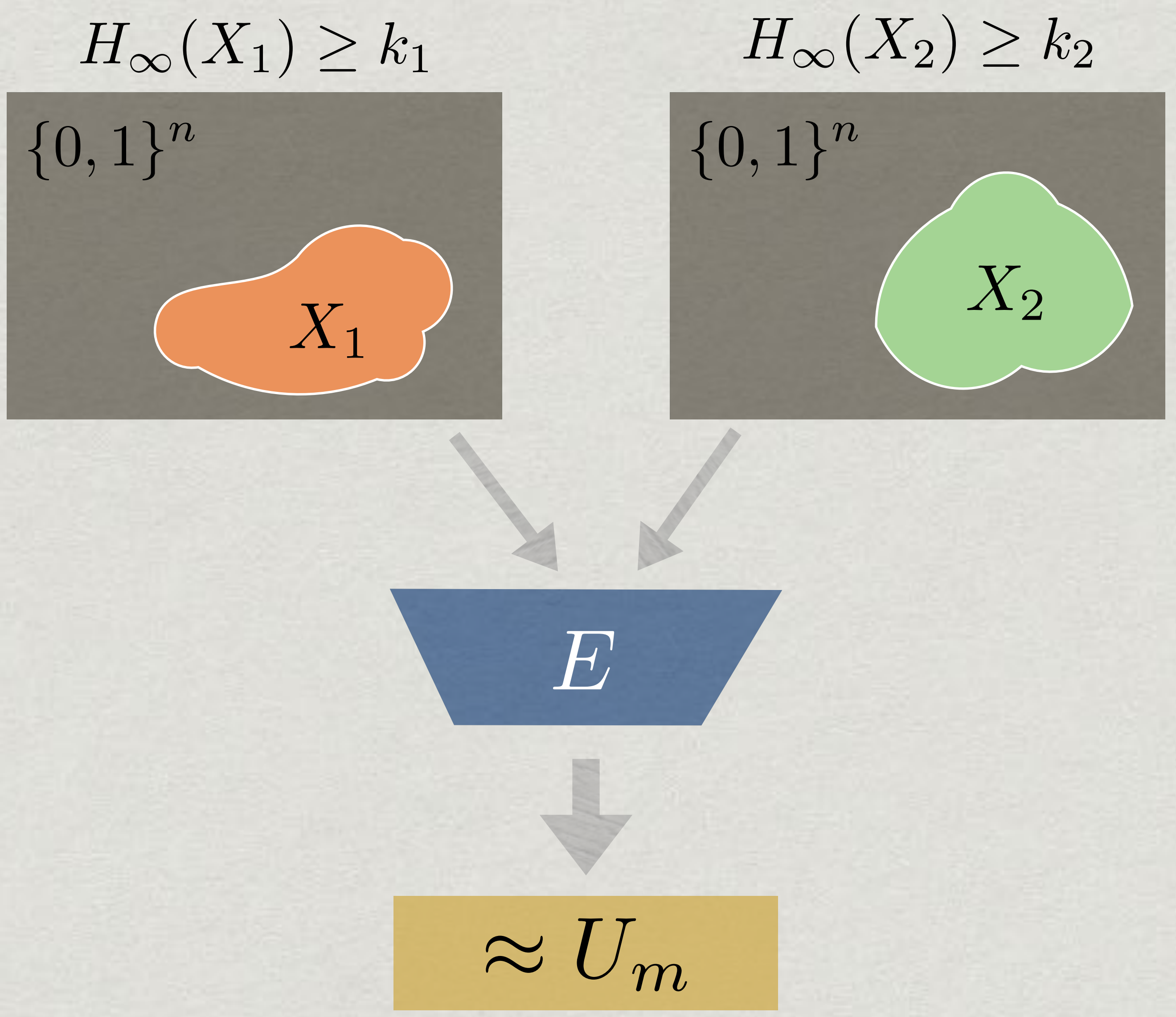
- Alternatively, we can think of a weak source X as uniformly distributed over a subset of size 2^k.
- Given two **independent** weak source X_1 and X_2, we want to extract almost-uniform bits (potentially, almost all the entropy).

Two-source extractors



$$E(X_1, X_2) \approx_\varepsilon U_m$$

Two-source extractors

- Known results for **constant** error.
- Omitted here: many constructions of multi-source extractors.

	min-entropy
Non-explicit	$\log n + O(1)$
[CG88]	$(½+\delta)n$
[Raz05]	$(½+\delta)n, O(\log n)$
[Bourgain05]	$0.499n$
[CZ15]	$\mathrm{polylog}(n)$
[BDT16]	$\log^{1+o(1)} n$
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A closer look at the error

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- Viewing it differently: We want the construction to run in time polylog($1/\varepsilon$) instead of poly($1/\varepsilon$).

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Our goal: Low-error two-source extractors, even for δn min-entropy.

(Preferably outputting many bits as well, but it often goes together…)

Bipartite Ramsey graphs

- The very-high error case is also interesting…
- In every $N \times N$ bipartite graph there is a $\frac{1}{2}\log N \times \frac{1}{2}\log N$ monochromatic subgraph (a bipartite clique or an independent set).

Bipartite Ramsey graphs

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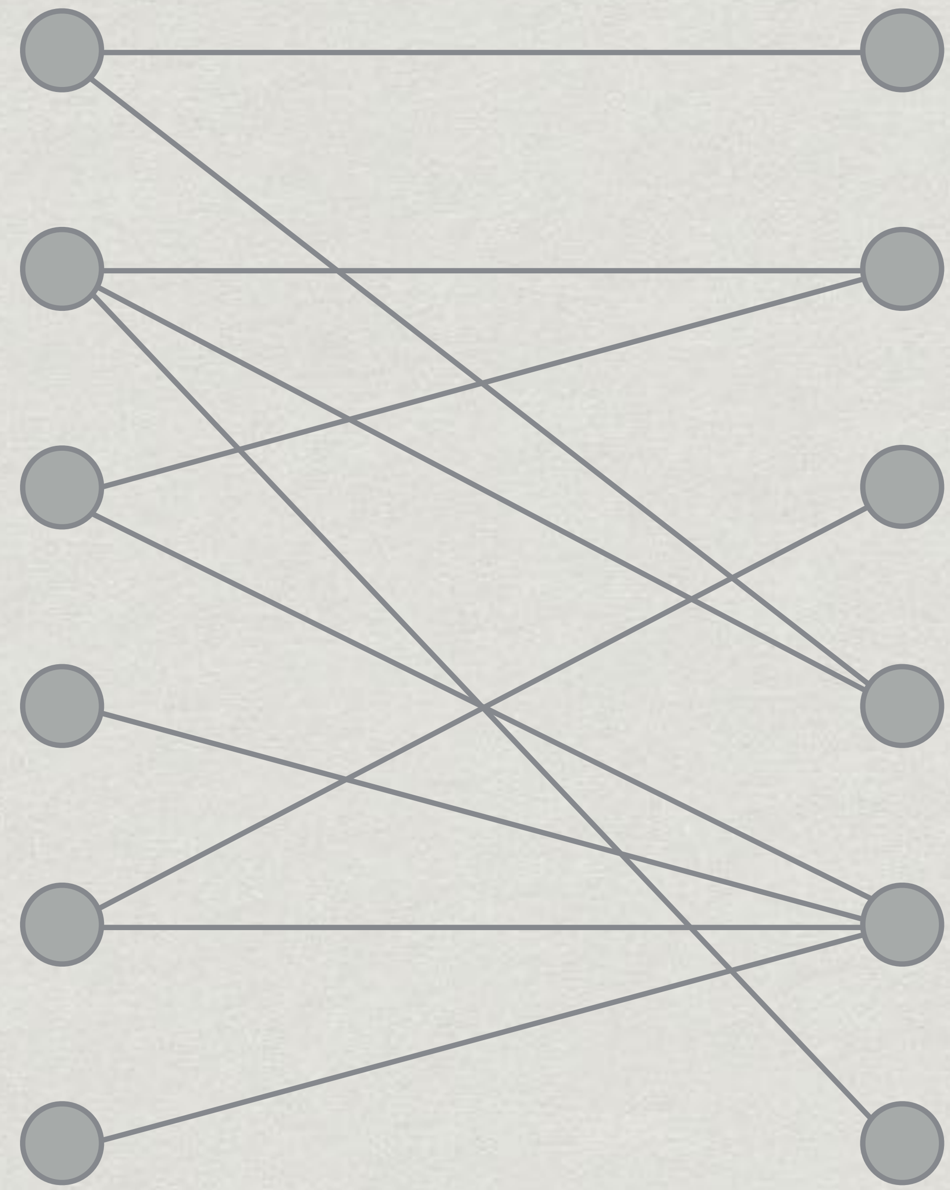

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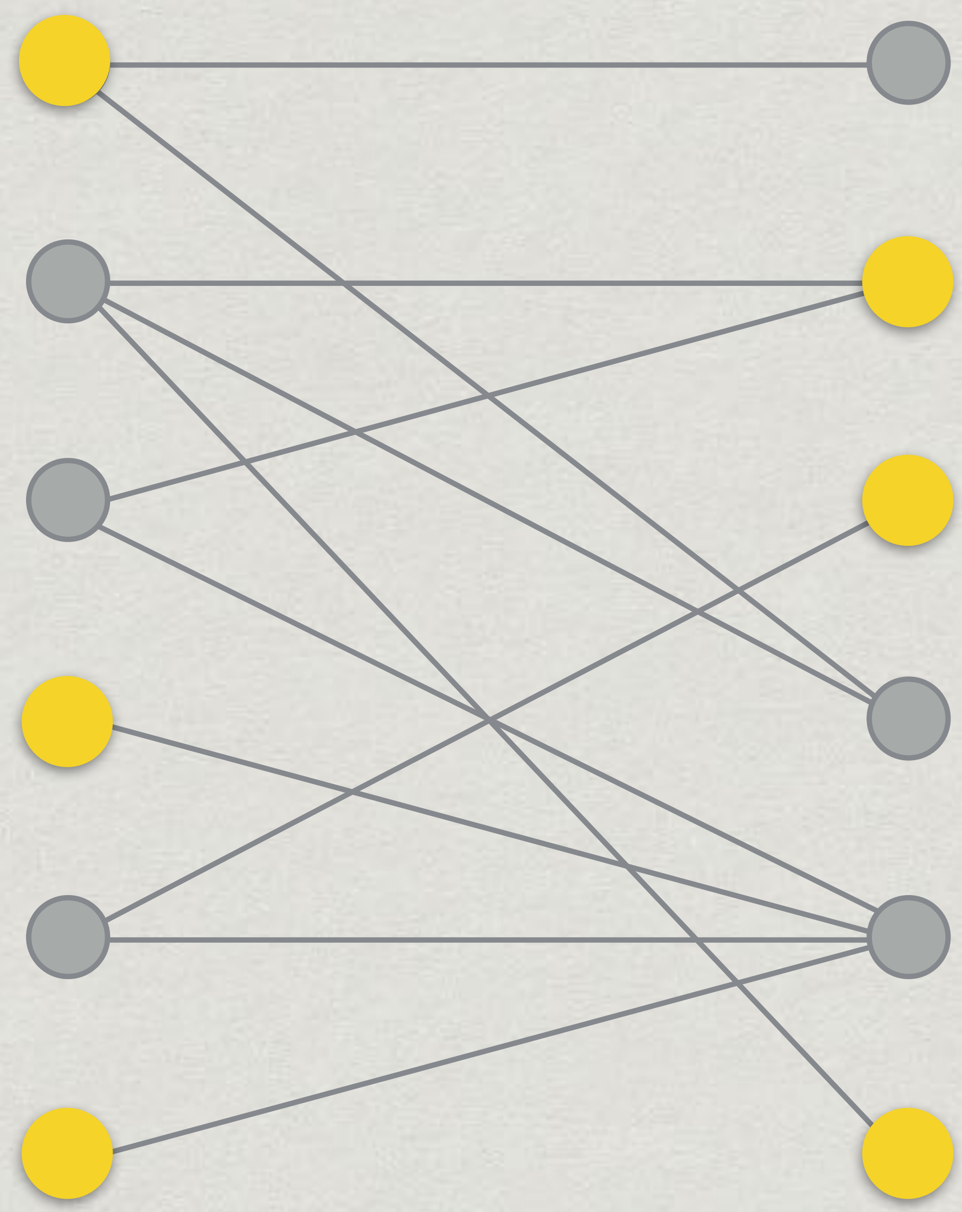
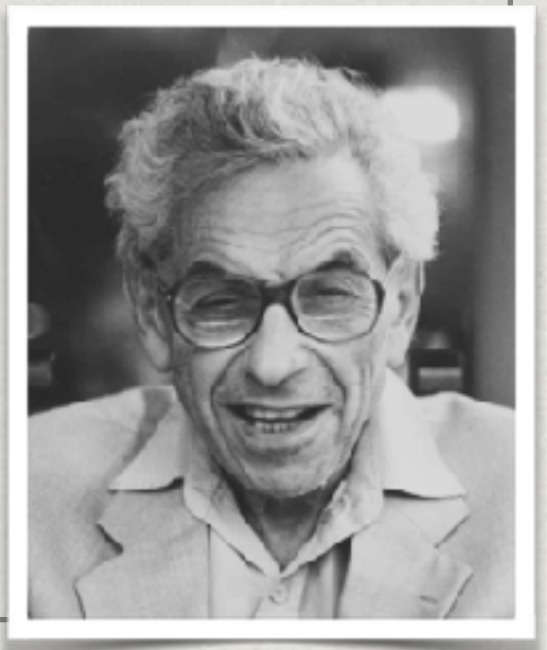


Bipartite Ramsey graphs

- Erdős (1947) — there exists an $N \times N$ bipartite graph with **no** $K \times K$ monochromatic subgraphs, for $K = 2\log N$.
- A random graph has this property.
- The Erdős \$100 challenge — find such an explicit graph, even for $K = O(\log N)$.
- Still open…



Bipartite Ramsey graph

- We can view every bipartite graph naturally as a function E:[*N*]×[*N*]→{0,1}.
- The bipartite Ramsey problem: construct explicit matrices with no *K*×*K* **constant** sub-matrices.

N

N

0	1	0	1	0	1	1	1	0
0	0	0	1	1	0	0	1	0
0	0	1	0	1	1	1	0	1
0	1	0	0	0	0	1	0	0
1	1	0	1	0	1	0	1	0
0	0	1	0	0	0	1	0	1
0	1	1	1	0	1	0	0	1
0	1	1	0	0	1	1	0	0
1	0	0	1	1	0	0	1	1

Bipartite Ramsey graph

- We can view every bipartite graph naturally as a function E:[N]×[N]→{0,1}.
- The bipartite Ramsey problem: construct explicit matrices with no K×K **constant** sub-matrices.
- The low-error two-source extractors problem: Insist on **unbiased** sub-matrices, with a **very small bias**.

N (columns) × N (rows)

0	1	0	1	0	1	1	1	0
0	0	0	1	1	0	0	1	0
0	0	1	0	1	1	1	0	1
0	1	0	0	0	0	1	0	0
1	1	0	1	0	1	0	1	0
0	0	1	0	0	0	1	0	1
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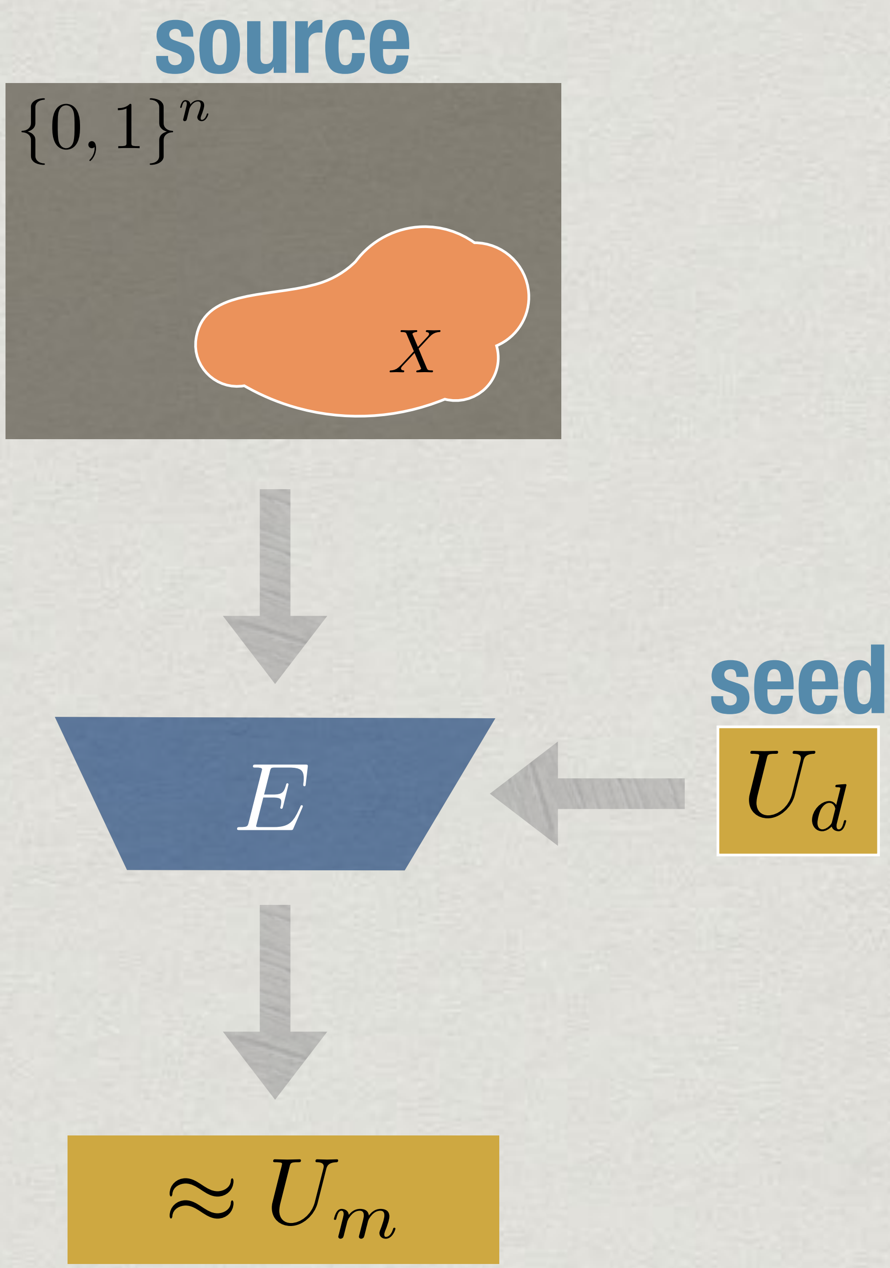
Seeded extractors

- A special case of two-source extractors is when one source is completely uniform, the *seed*.
- The seed length can be as small as $2\log(n/\varepsilon)$.



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- We say a seeded extractor is ***strong*** if the output is uniform even given the seed: $(E(X,Y),Y) \approx_\varepsilon (U,Y)$.

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- Equivalently, for every source X with entropy at least k there exists a set of good seeds of density at least $1-\varepsilon$ such that for every good seed $y \in \{0,1\}^d$, $E(X,y) \approx_\varepsilon U$.
- We have good strong seeded extractors [LRVW03,GUV07,…].

Non-malleable extractors [DW09]

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- A generalization of strong seeded-extractors.
- An adversary cannot distinguish between the output nmE(X,Y) and a uniform string, even given the seed Y and the output of nmE on t **correlated** seeds.

Non-malleable extractors [DW09]

- A generalization of strong seeded-extractors.
- An adversary cannot distinguish between the output nmE(X,Y) and a uniform string, even given the seed Y and the output of nmE on t **correlated** seeds.
- (nmE(X,Y),nmE(X,f_1(Y)),…,nmE(X,f_t(Y)),Y) is ε-close to (U,nmE(X,f_1(Y)),…,nmE(X,f_t(Y)),Y).

Non-malleable extractors

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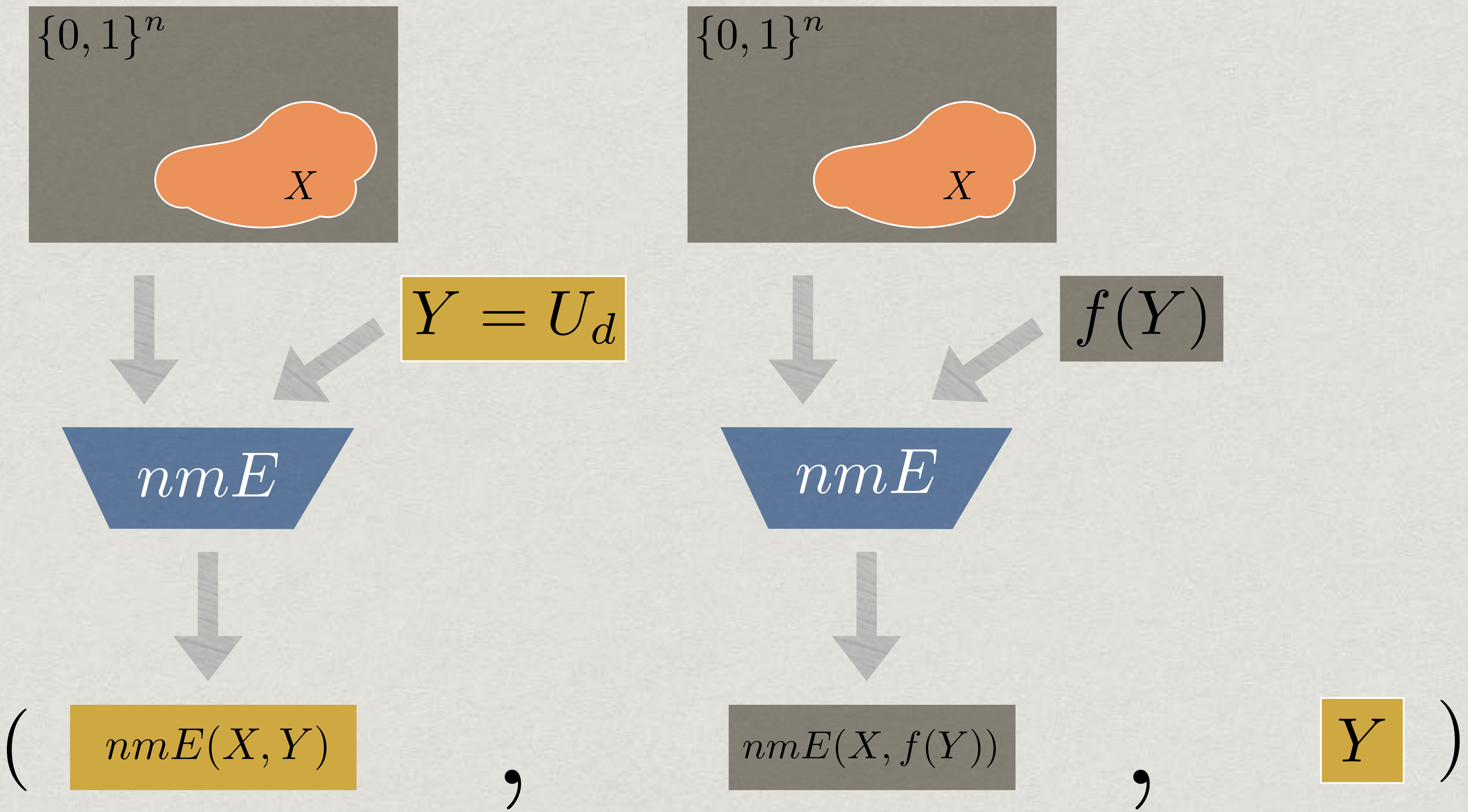
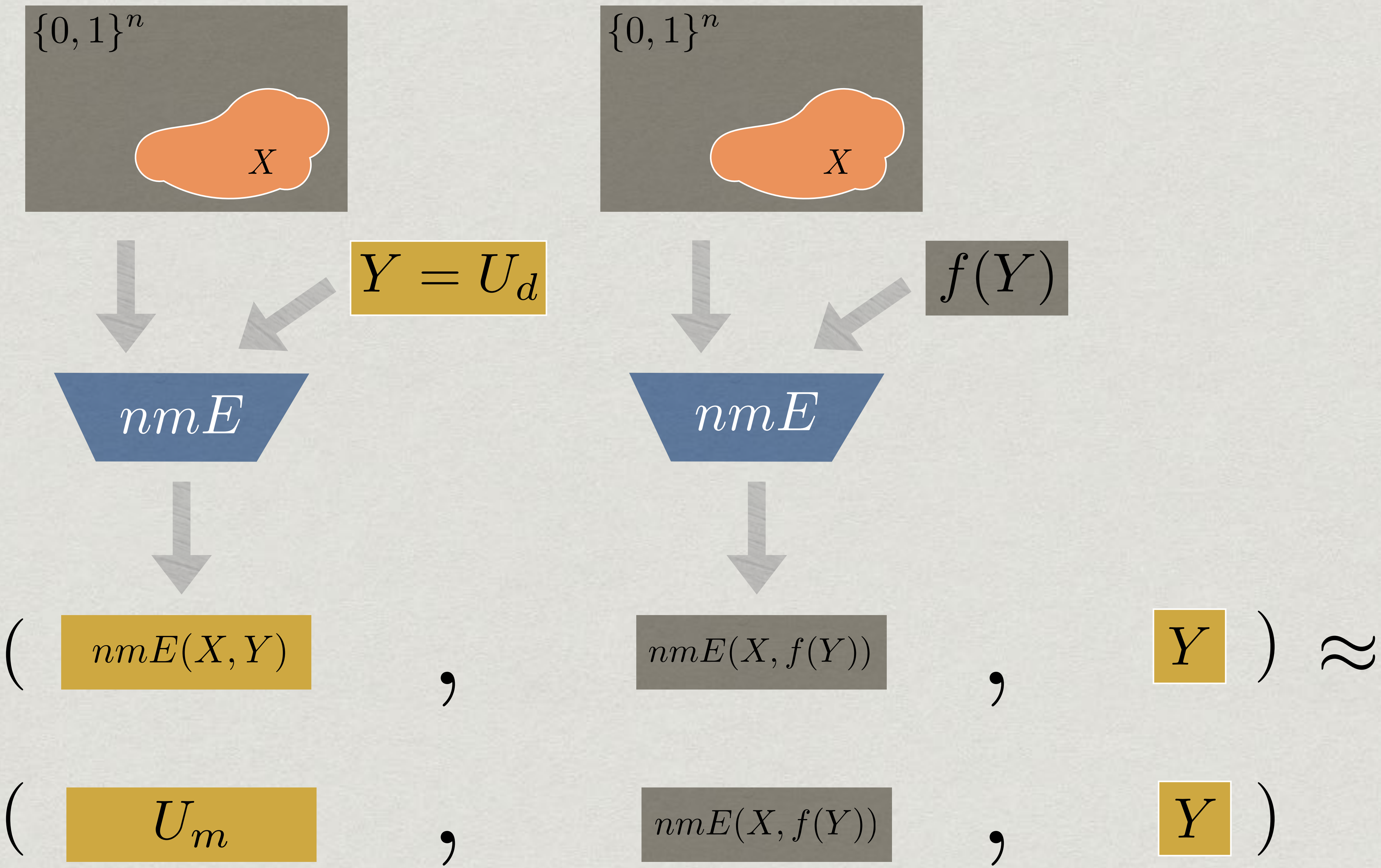


Non-malleable extractors



Non-malleable extractors

- Known explicit constructions for ***t*=1** (a partial list). A reduction by [Cohen16] allows us to go to an arbitrary t by roughly paying a factor of t in the entropy and t^2 in the seed-length.

	seed length	min-entropy
[CRS12,DLWZ11]	$\log(n/\varepsilon)$	$(\frac{1}{2}+\delta)n$
[Li12]	$\log(n/\varepsilon)$	$0.499n$
[CGL15]	$\log^2(n/\varepsilon)$	$\Omega(d)$
[Cohen16]	$\log(n/\varepsilon)\log(\log(n)/\varepsilon)$	$\Omega(d)$
[CL16]	$\log^{1+o(1)}(n/\varepsilon)$	$\Omega(d)$
[Cohen16]	$\log(n)+\log(1/\varepsilon)\mathrm{poly}(\log\log(1/\varepsilon))$	$\Omega(d)$
[Li16]	$\log(n)+\log(1/\varepsilon)\log\log(1/\varepsilon)$	$\Omega(d)$

Non-malleable extractors

Non-malleable extractors

- We will use an equivalent definition (up to some loss in the error) [CZ15,Cohen16].
- nmE is a n.m. extractor if every source induces a set of good seeds of high density such that the output of the extractor on a good seed is close to uniform even conditioned on its output on t other distinct seeds.

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- nmE is a n.m. extractor if every source induces a set of good seeds of high density such that the output of the extractor on a good seed is close to uniform even conditioned on its output on t other distinct seeds.
- For every X there exists a set of G of density at least $1\text{-}\varepsilon$ such that for every $y \in G$ and any $y_1,\ldots,y_t \in \{0,1\}^d \setminus \{y\}$ it holds that $(\mathrm{nmE}(X,y),\mathrm{nmE}(X,y_1),\ldots,\mathrm{nmE}(X,y_t))$ is ε-close to $(U,\mathrm{nmE}(X,y_1),\ldots,\mathrm{nmE}(X,y_t))$.

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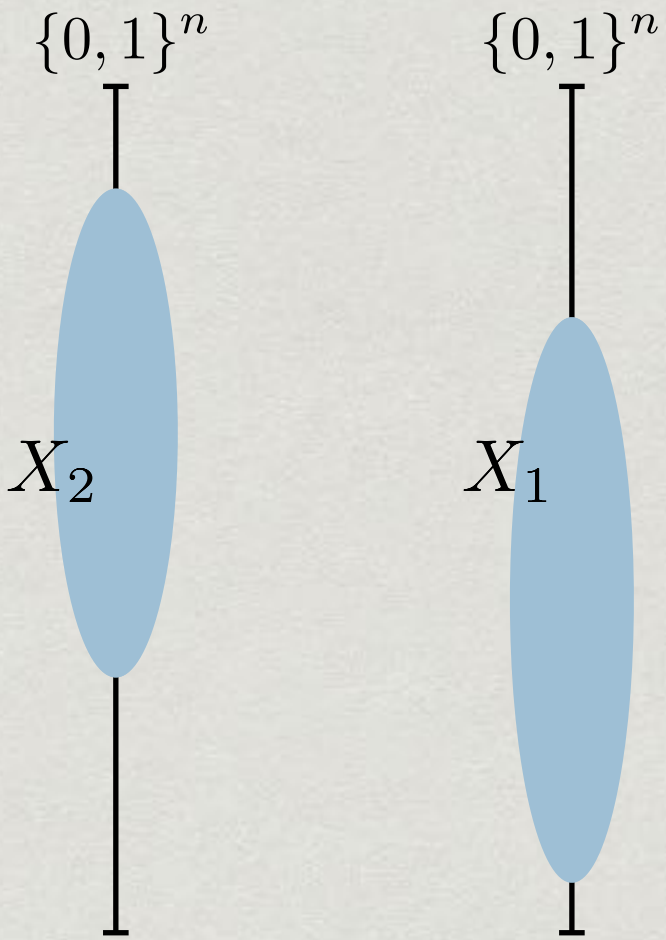
- A bird's-eye view of these constructions: Given two inputs x_1 and x_2,
  - Generate a table of nmE(x_1,i) for all seeds $i \in \{0,1\}^d$.
  - Using x_2, sample a subset of the rows.
  - Apply a *resilient* function on the reduced table.

Current constructions of two-source extractors



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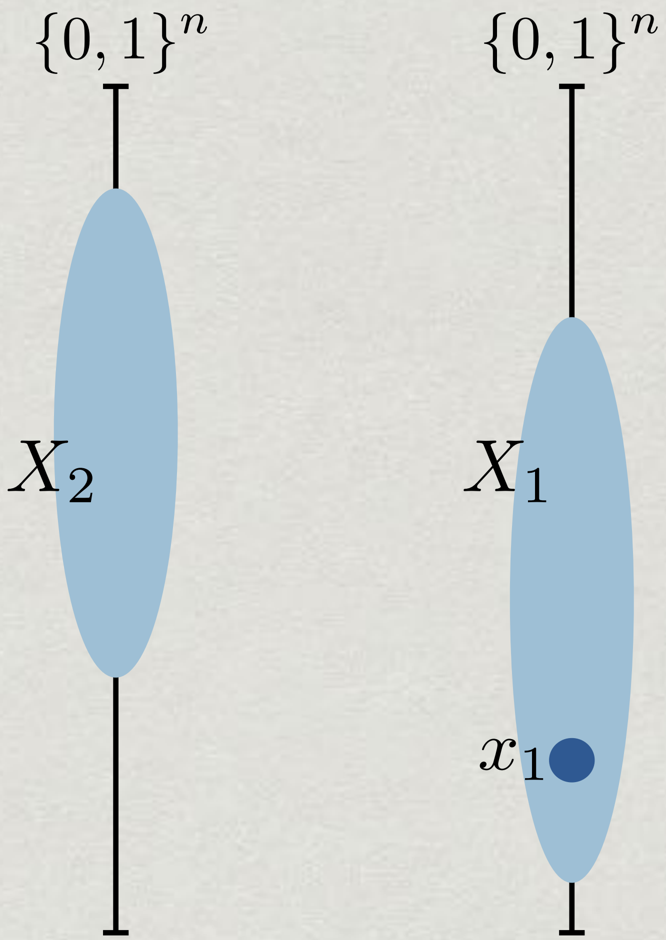

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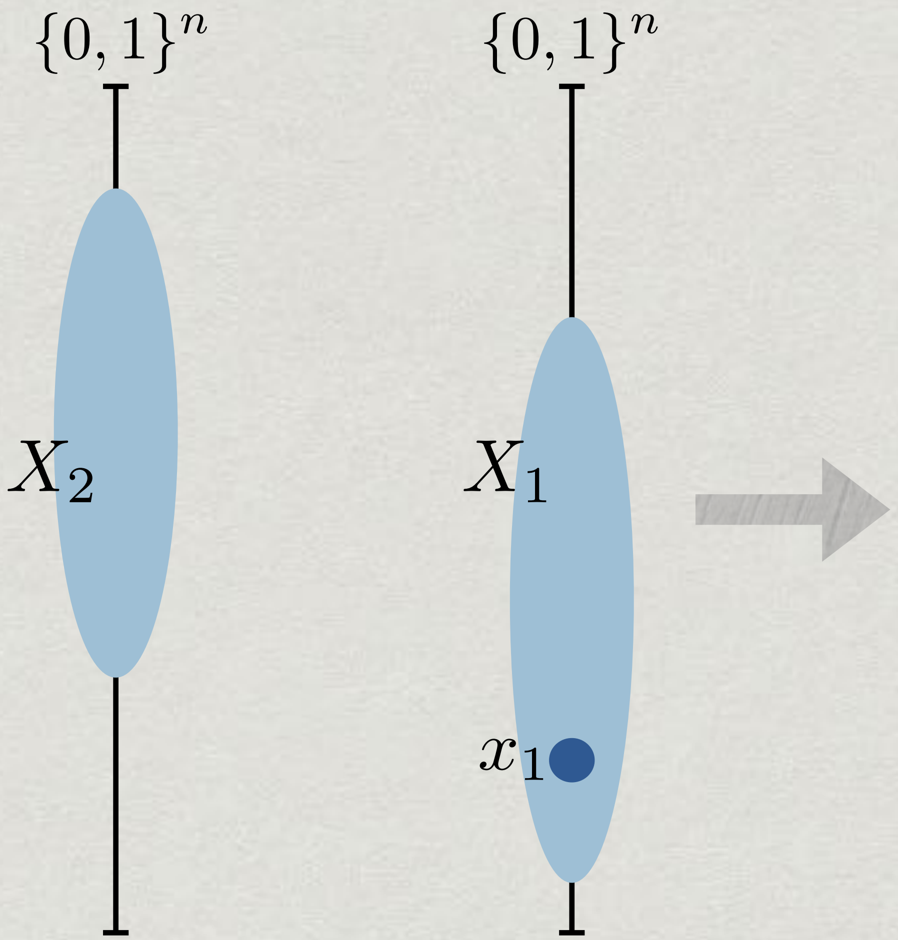

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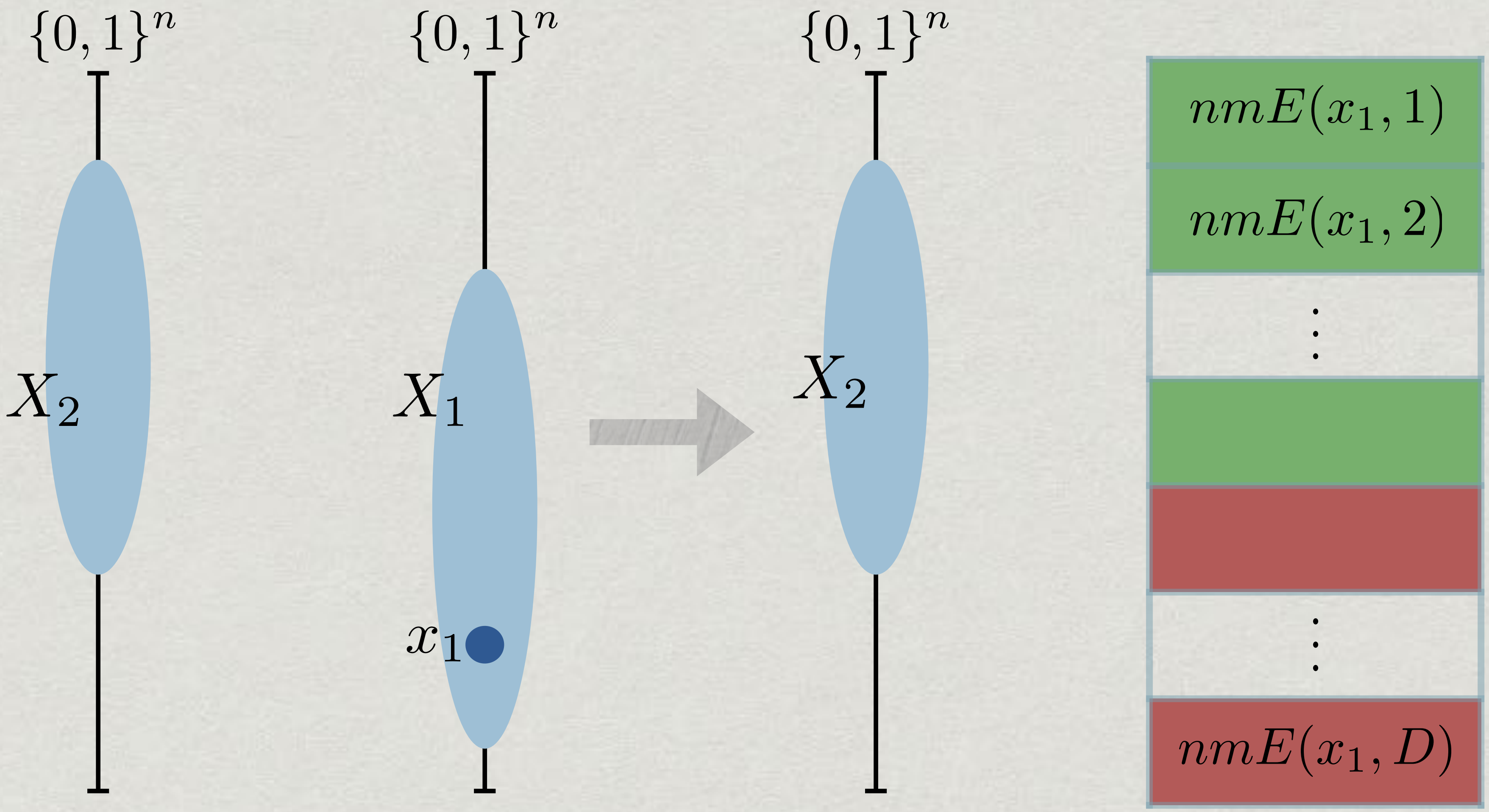

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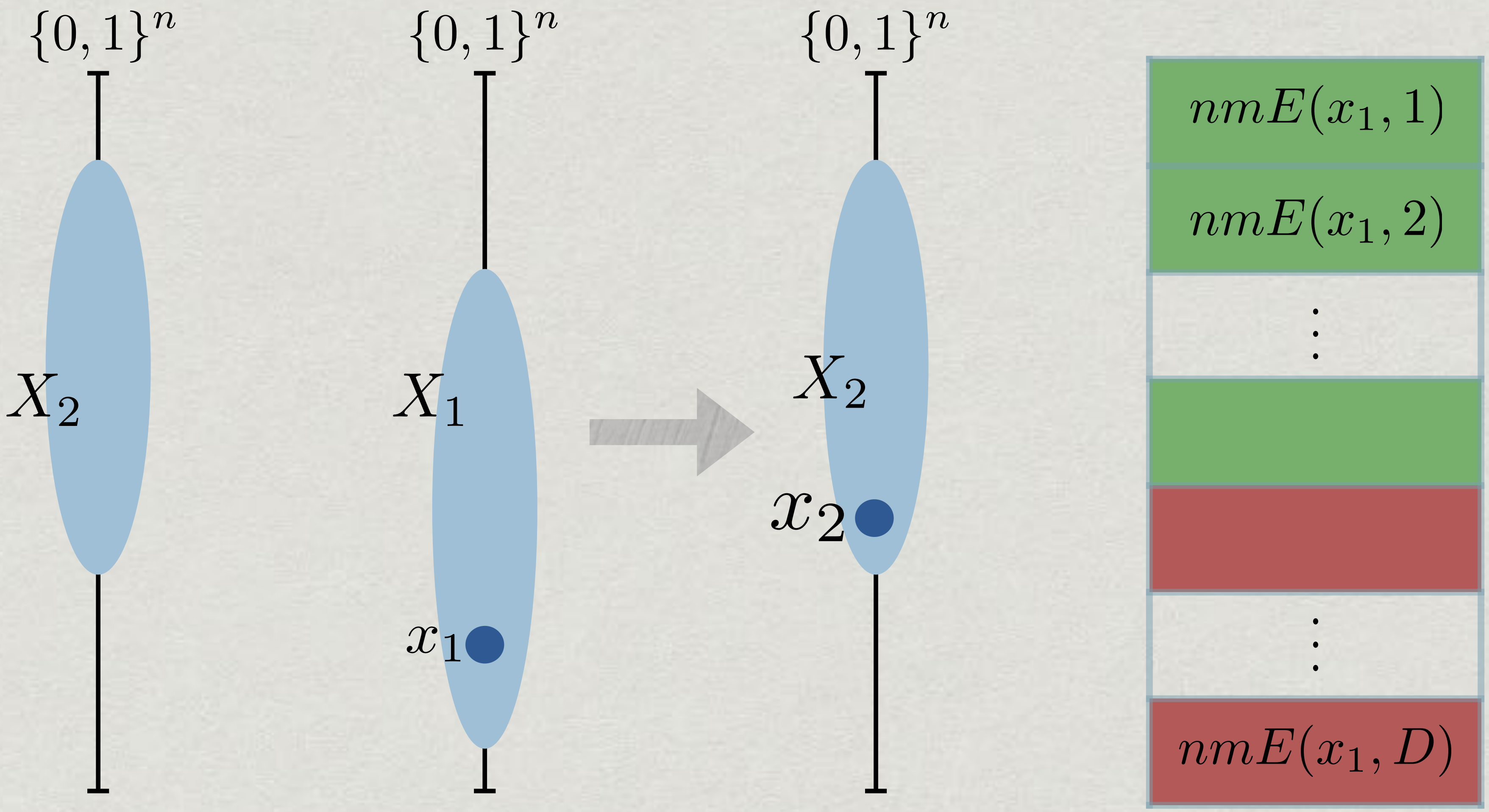

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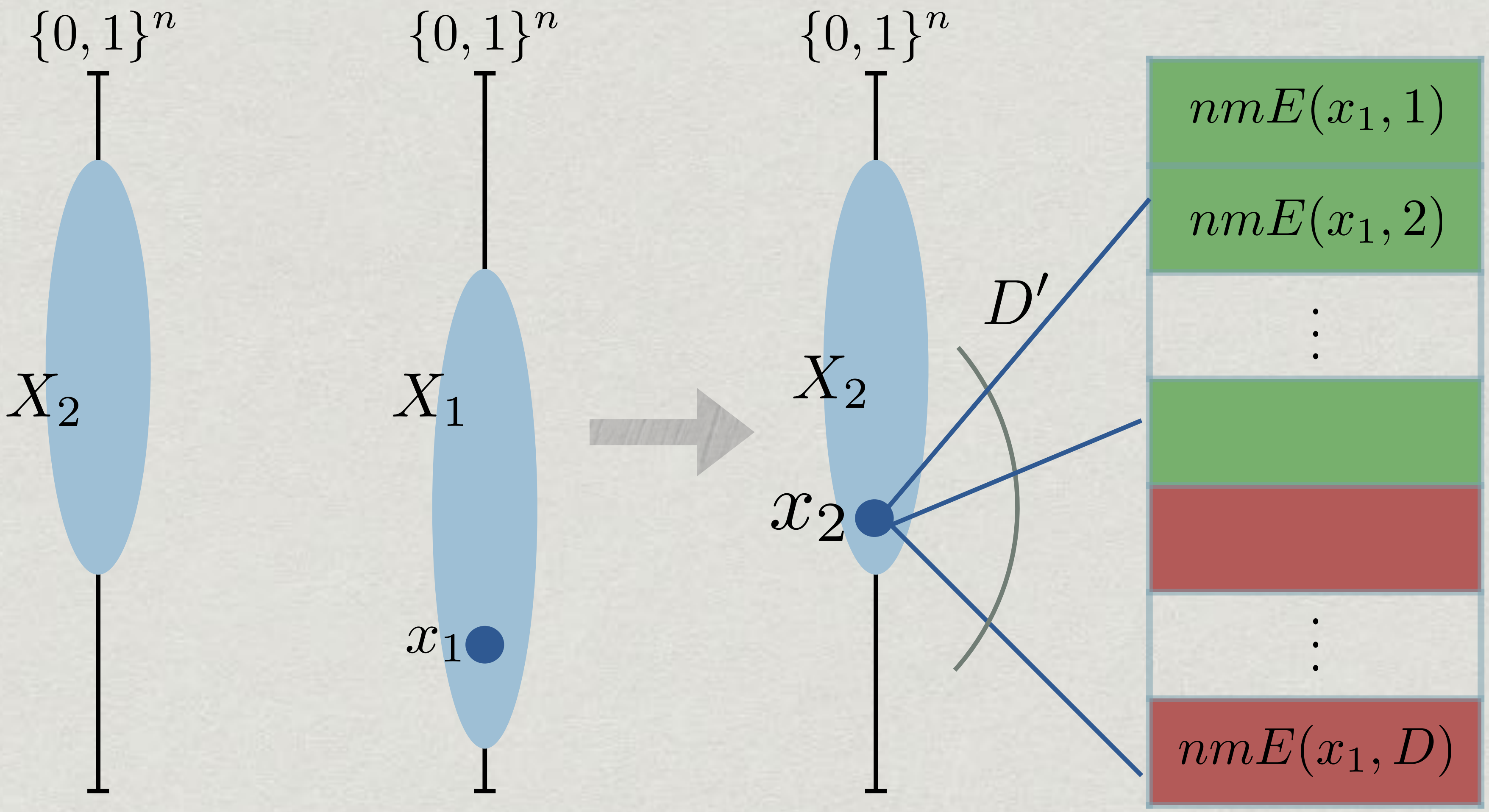

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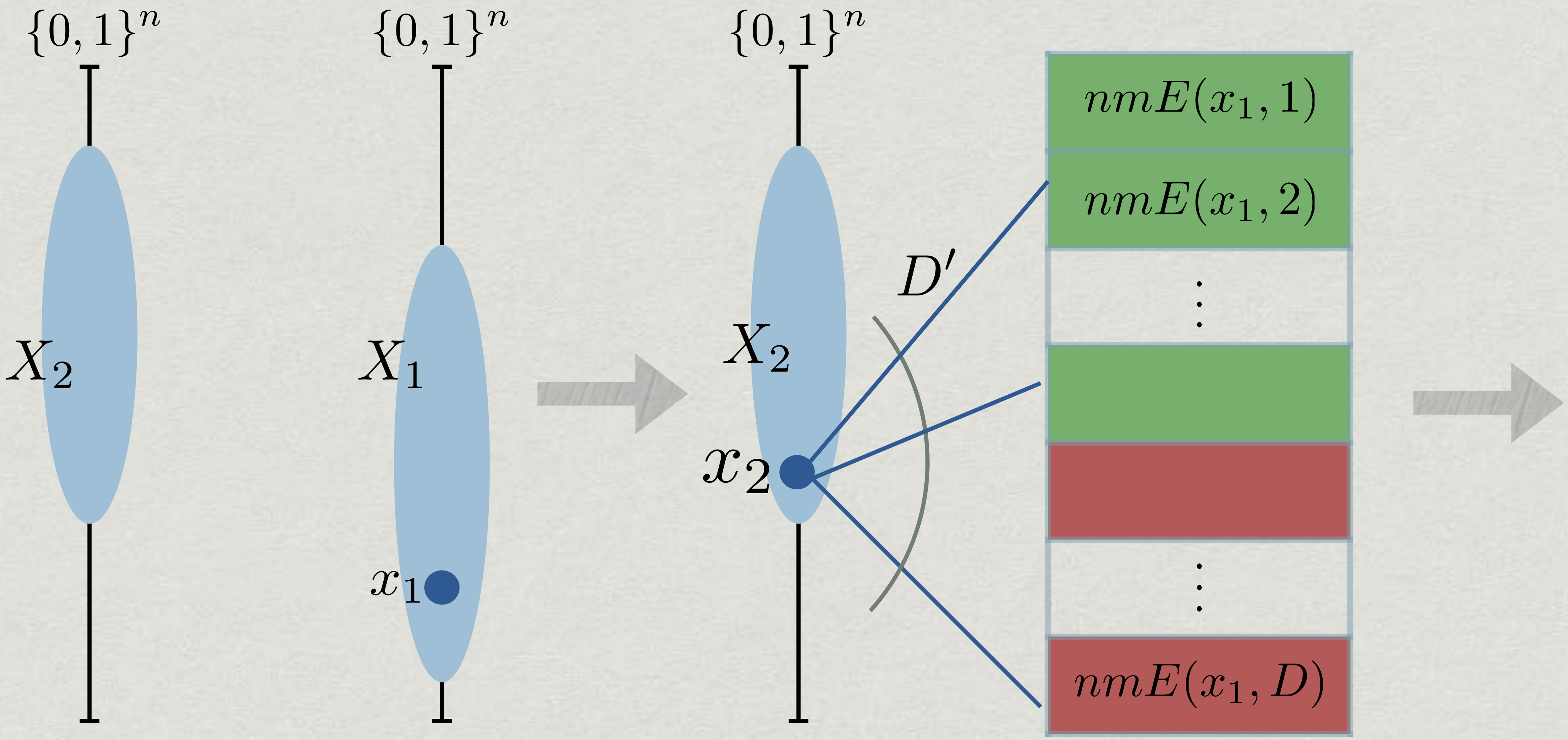

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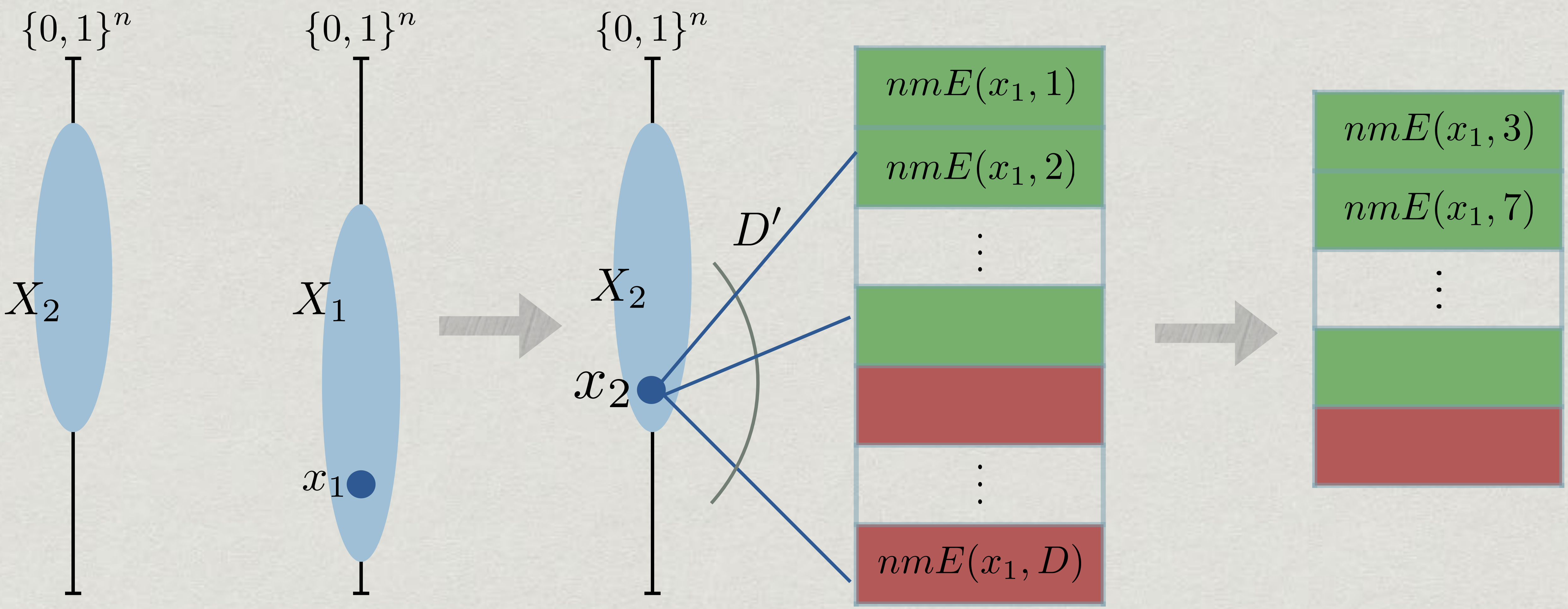

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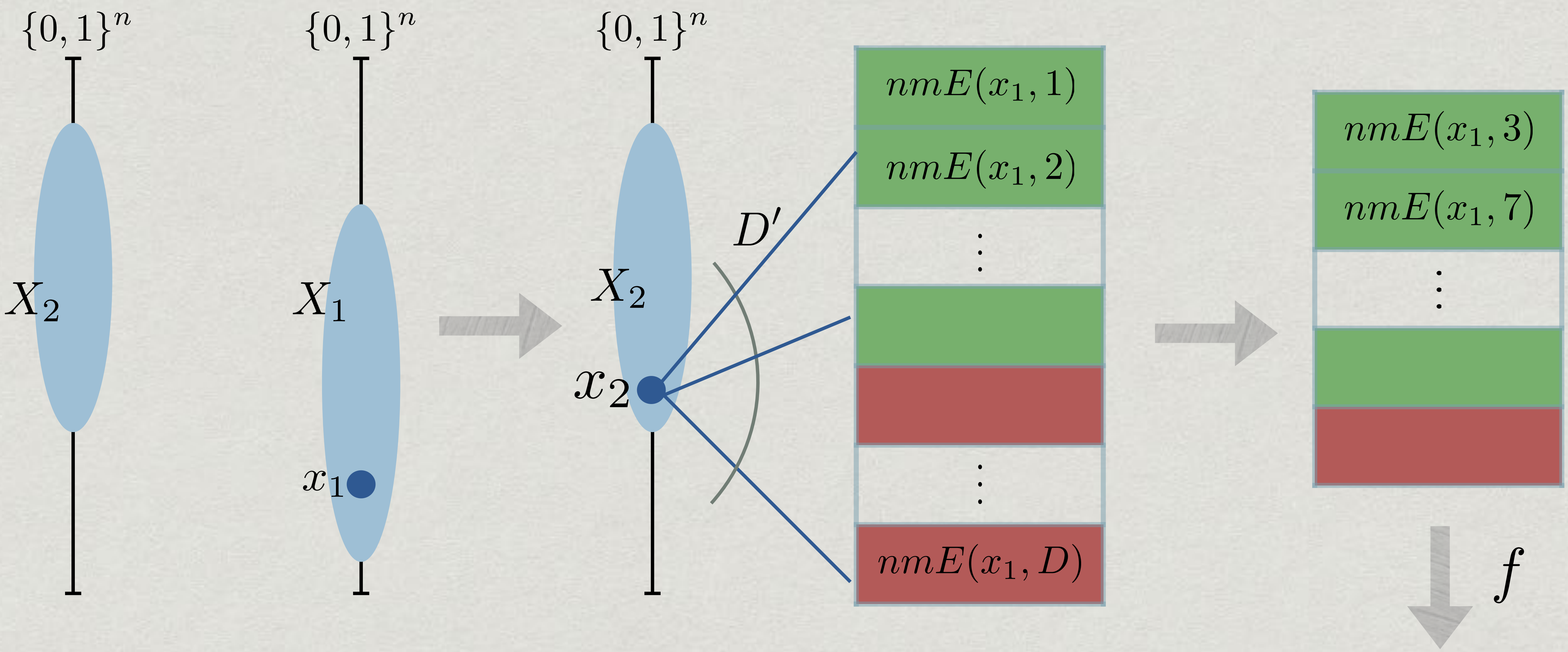

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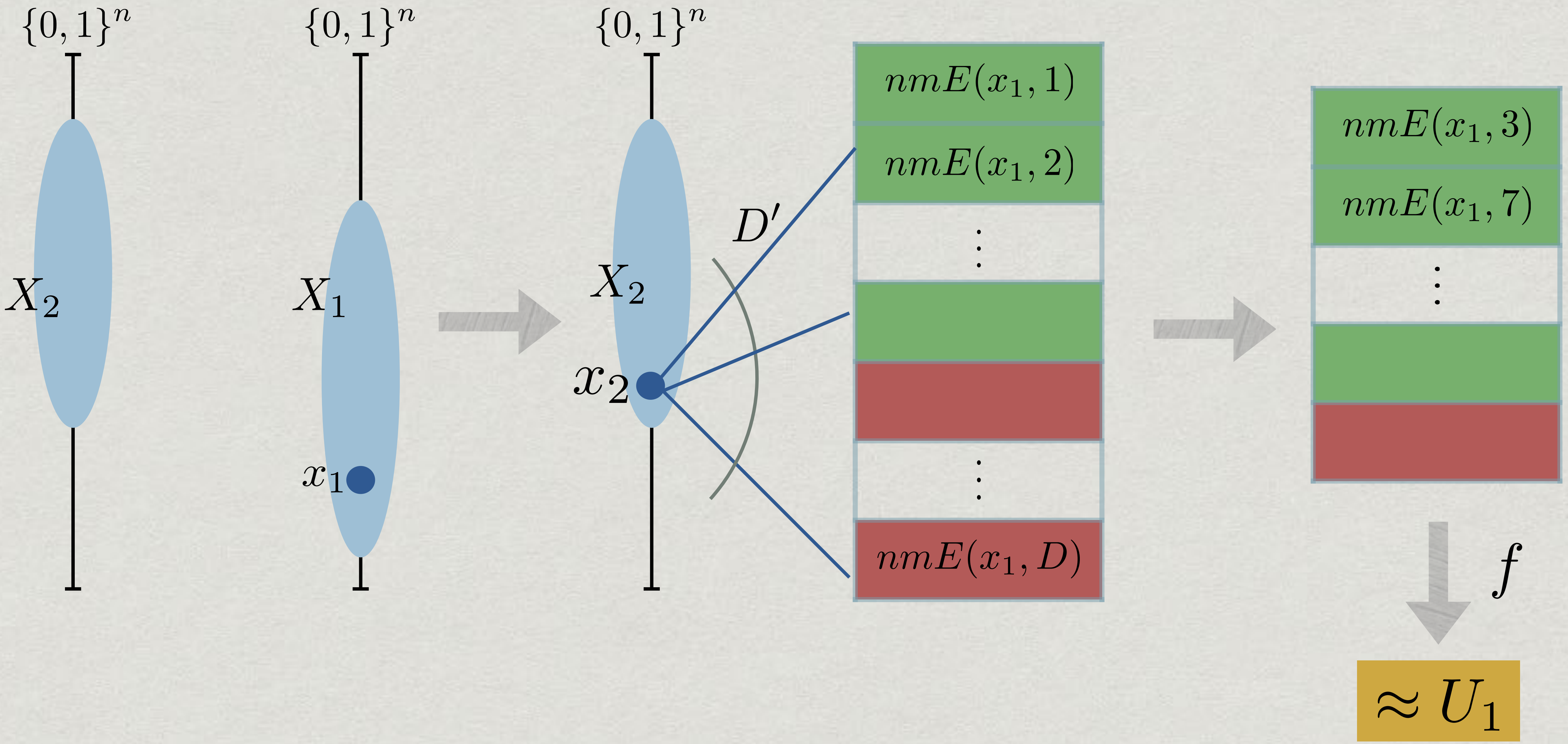

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  - The honest players draw their random bit and later the malicious players draw as they wish.
  - With high probability, the outcome is not biased — the malicious players cannot substantially bias the outcome.

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- A corollary of [KKL88] — even one malicious player can bias the output with probability at least $\log D'/D'$.
- We cannot hope for an error smaller than $1/D'$, and D' is the size of our table.
- Thus, the running time is at least $1/\varepsilon$.

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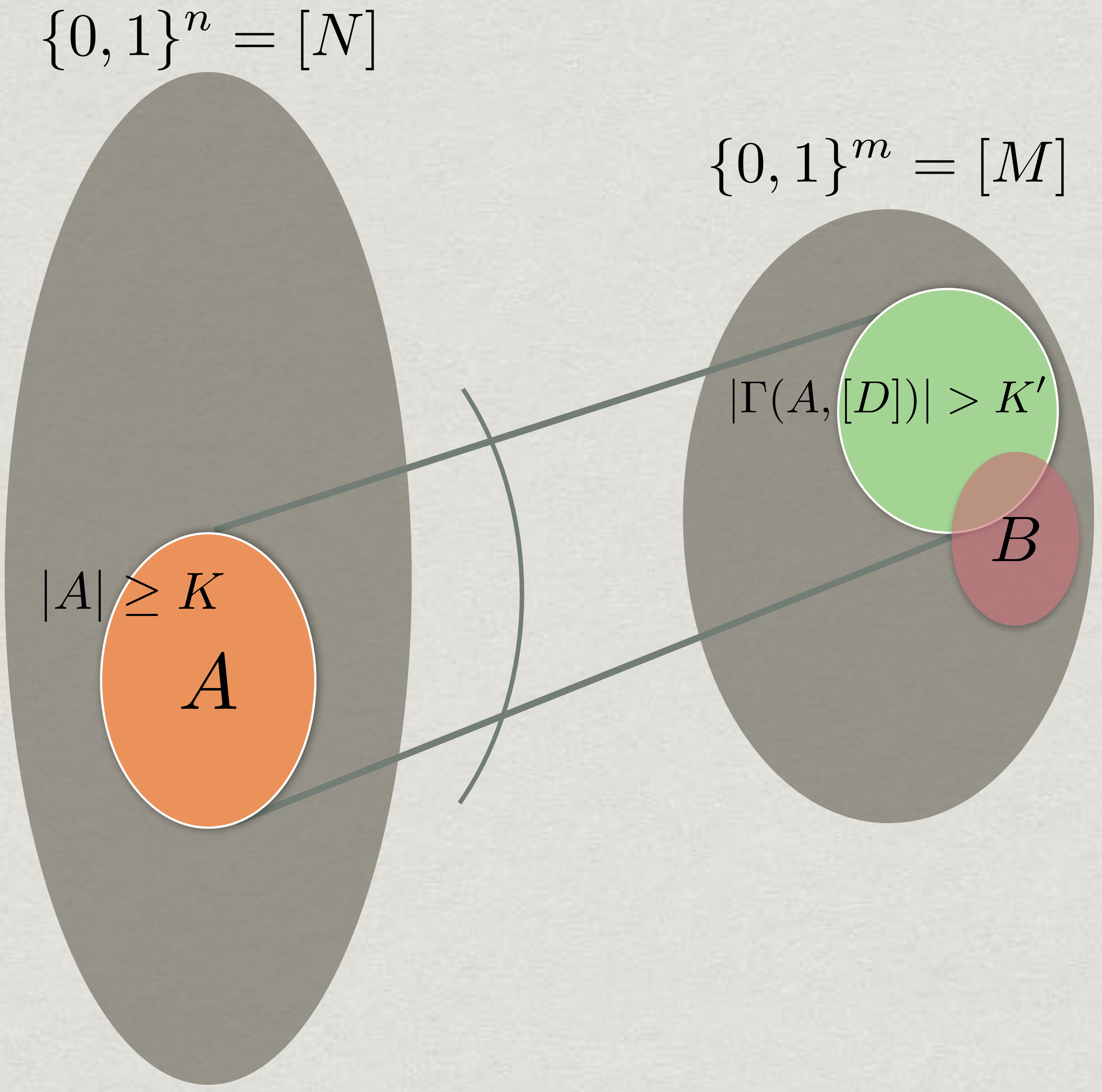
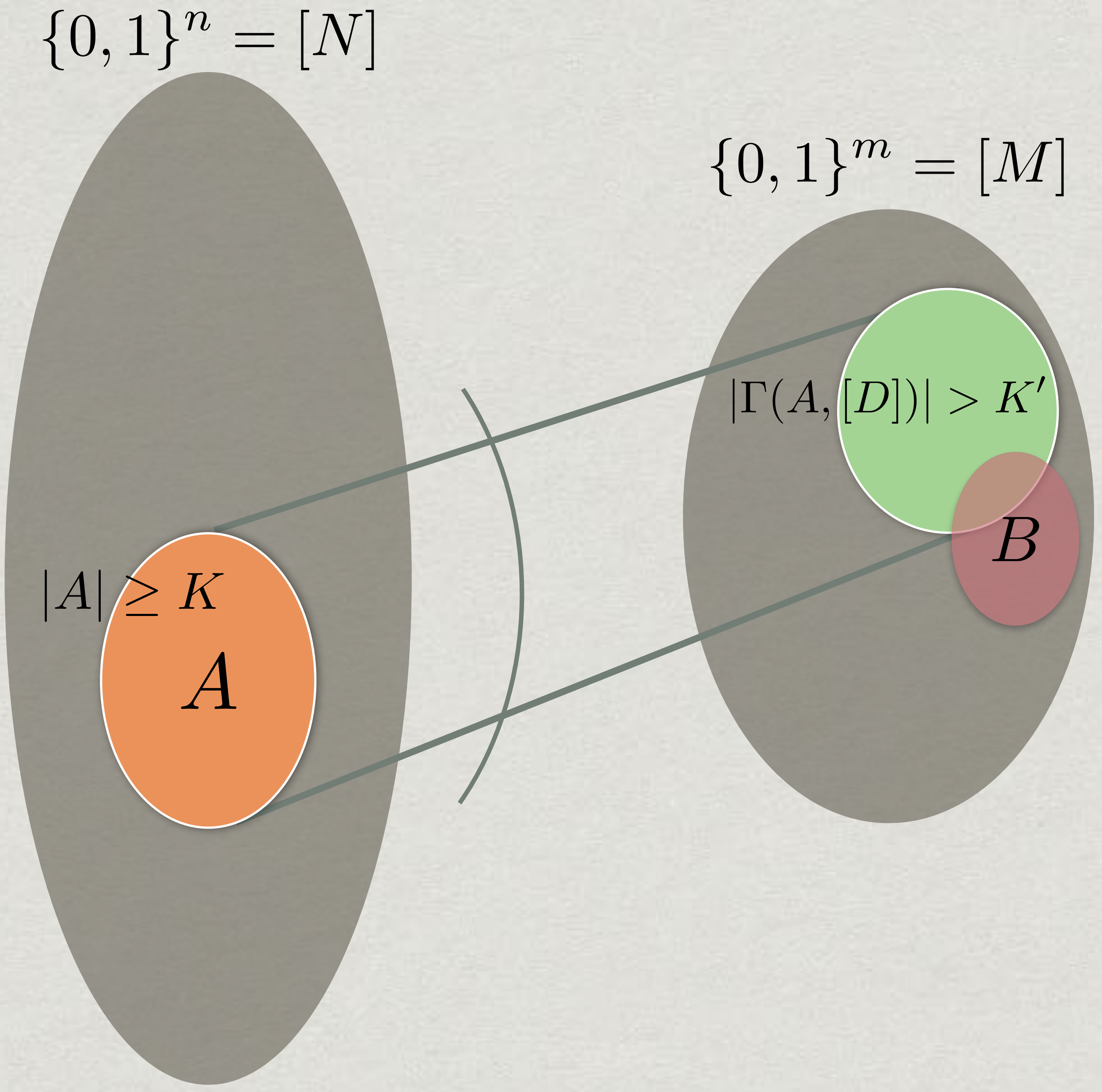
* Instead of trying to sample and then employ t-wise in the good rows, let's just try and **hit** a good row.
* As usual, we hit with a disperser…

Dispersers



Dispersers

- Γ:$\{0,1\}^n \times [D] \to \{0,1\}^m$ is a (K,K')-disperser if for every set A of cardinality at least K, Γ maps A to a set of cardinality greater than K'.



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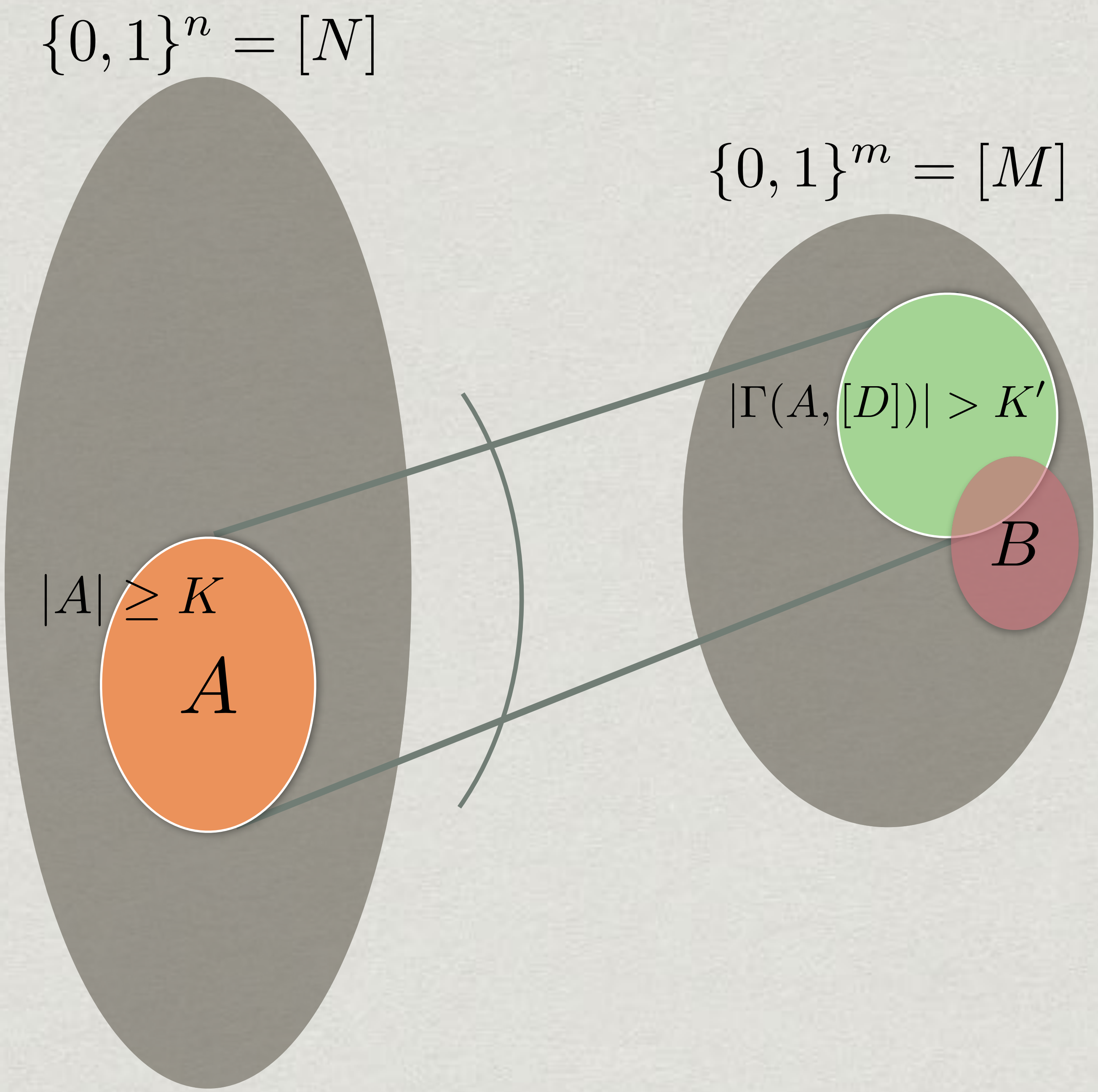
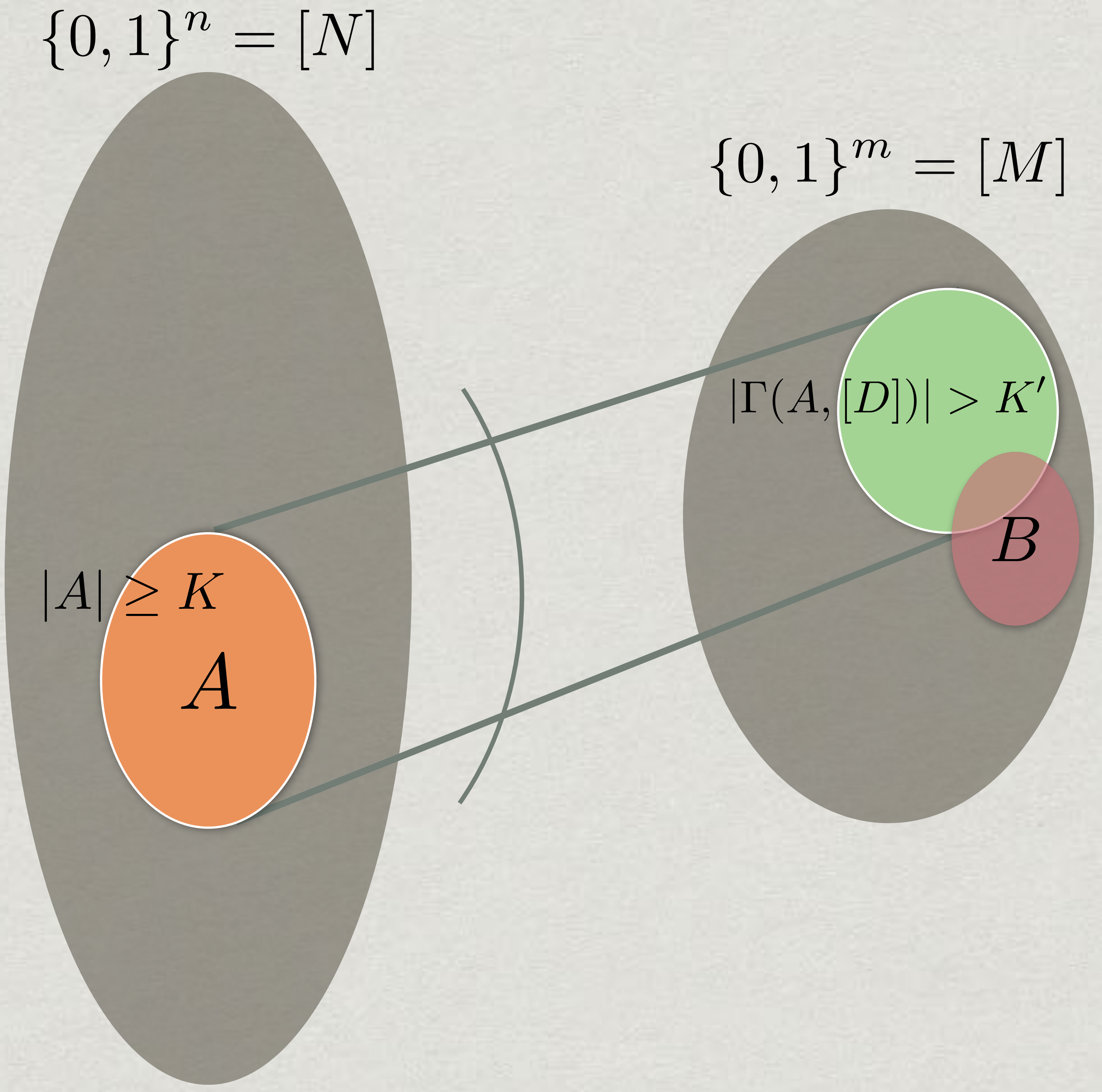
- Γ:$\{0,1\}^n \times [D] \rightarrow \{0,1\}^m$ is a (K,K')-disperser if for every set A of cardinality at least K, Γ maps A to a set of cardinality greater than K'.
- We are interested in the case where K' is small compared to 2^m. That is, we want to avoid **small** bad sets.



Dispersers

- Used to reduce error in one-sided probabilistic algorithms.
- [RT]: When *K'* is not too large, say *K'*=*εM*, the lower bound on the degree is

$$D = \Omega\left(\frac{\log \frac{N}{K}}{\log \frac{1}{\varepsilon}}\right)$$



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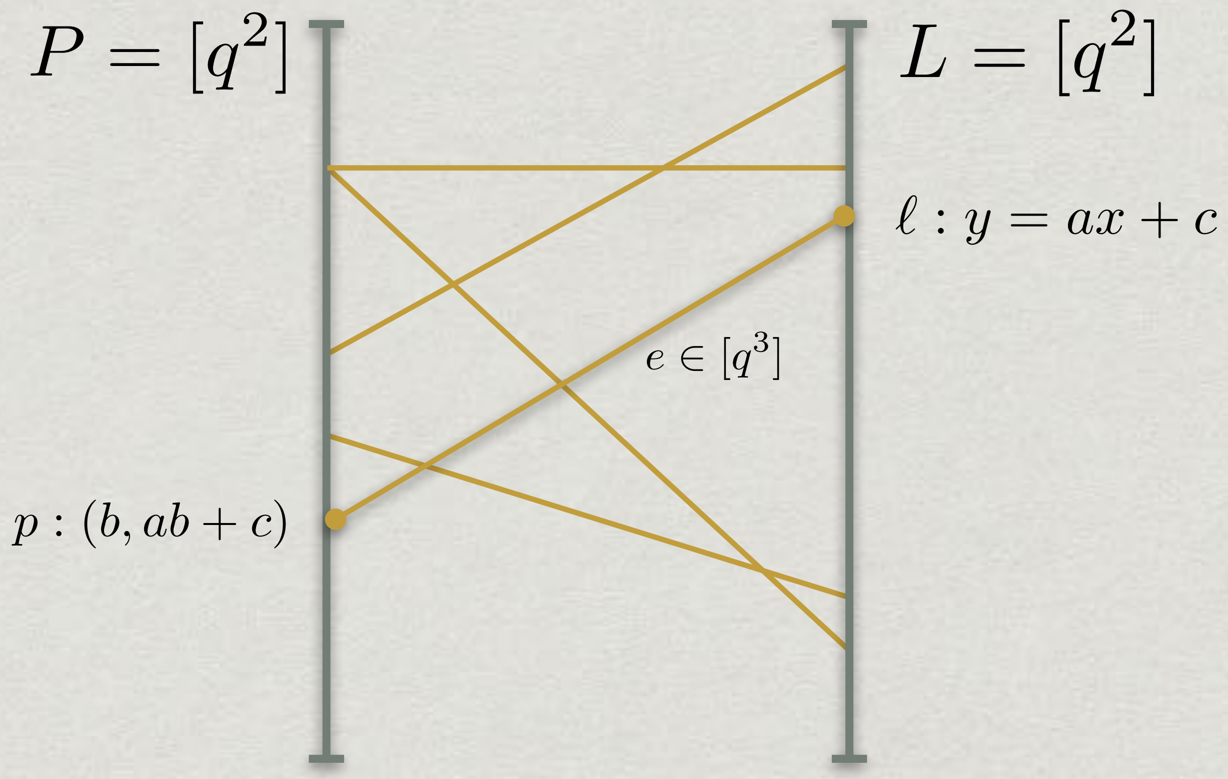
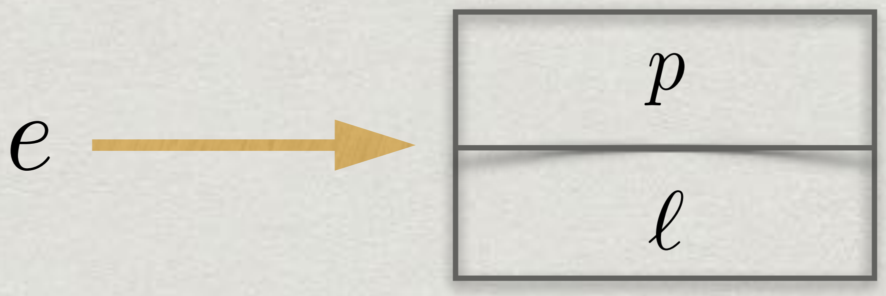
- The key ingredient in Zuckerman's construction: A points-lines incidence graph.

Zuckerman's disperser

The input source is distributed, over $[q]^3$, among the edges of the graph.



$\Gamma : \mathbb{F}_q^3 \times [2] \to \mathbb{F}_q^2$



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- For $K=N^{\delta}$, where δ is arbitrary, the dependence is

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- Also, the output length is determined by the number of recursion steps, and we have $m=\delta^{O(1)}n$.

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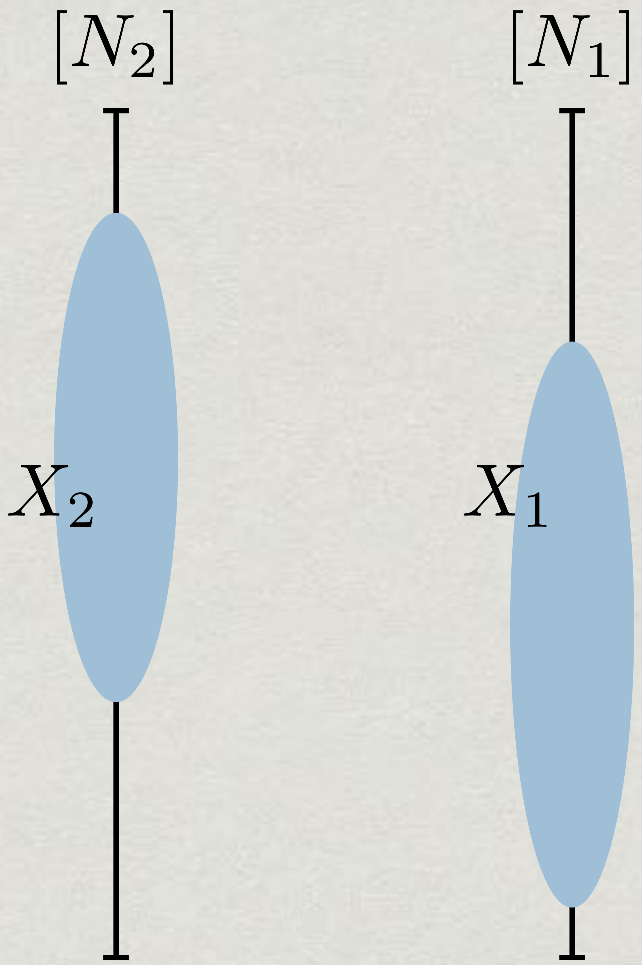
  - Γ: $[N_2]\times[\boldsymbol{t+1}]\rightarrow[D]$, a $(\varepsilon K_2,\varepsilon D)$-disperser.
- On input x_1,x_2, output $\oplus_{i\in[t+1]}\mathrm{nmE}(x_1,\Gamma(x_2,i))$.

Our reduction



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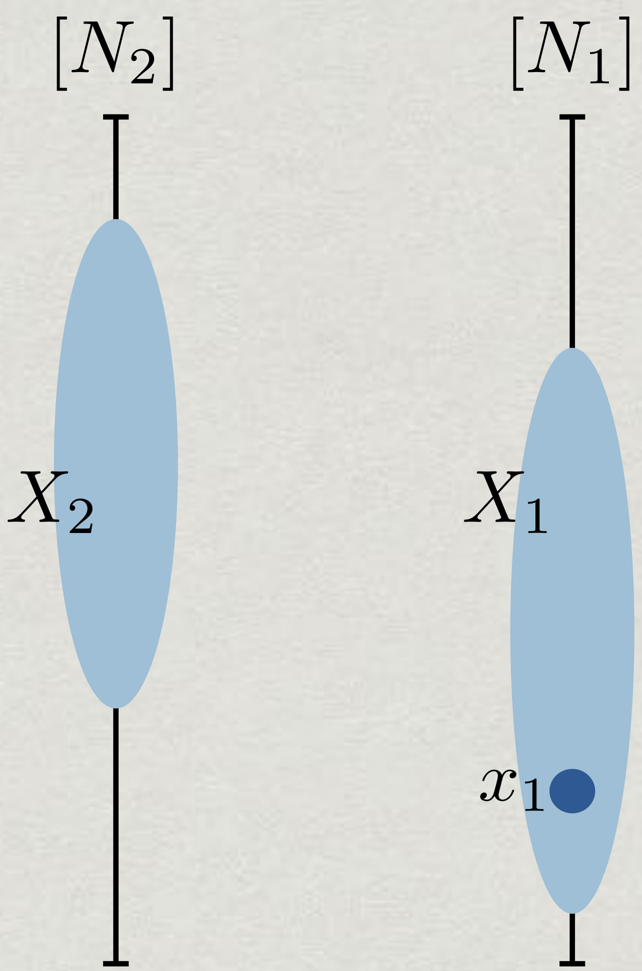
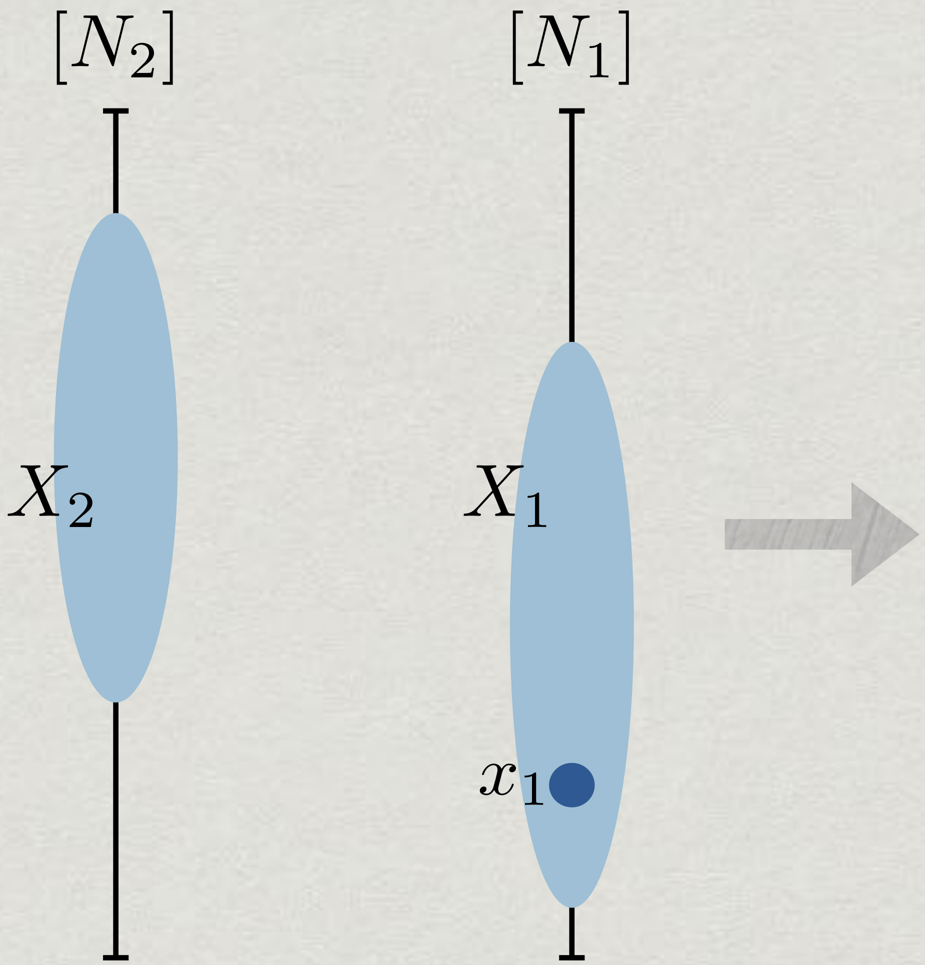


Our reduction



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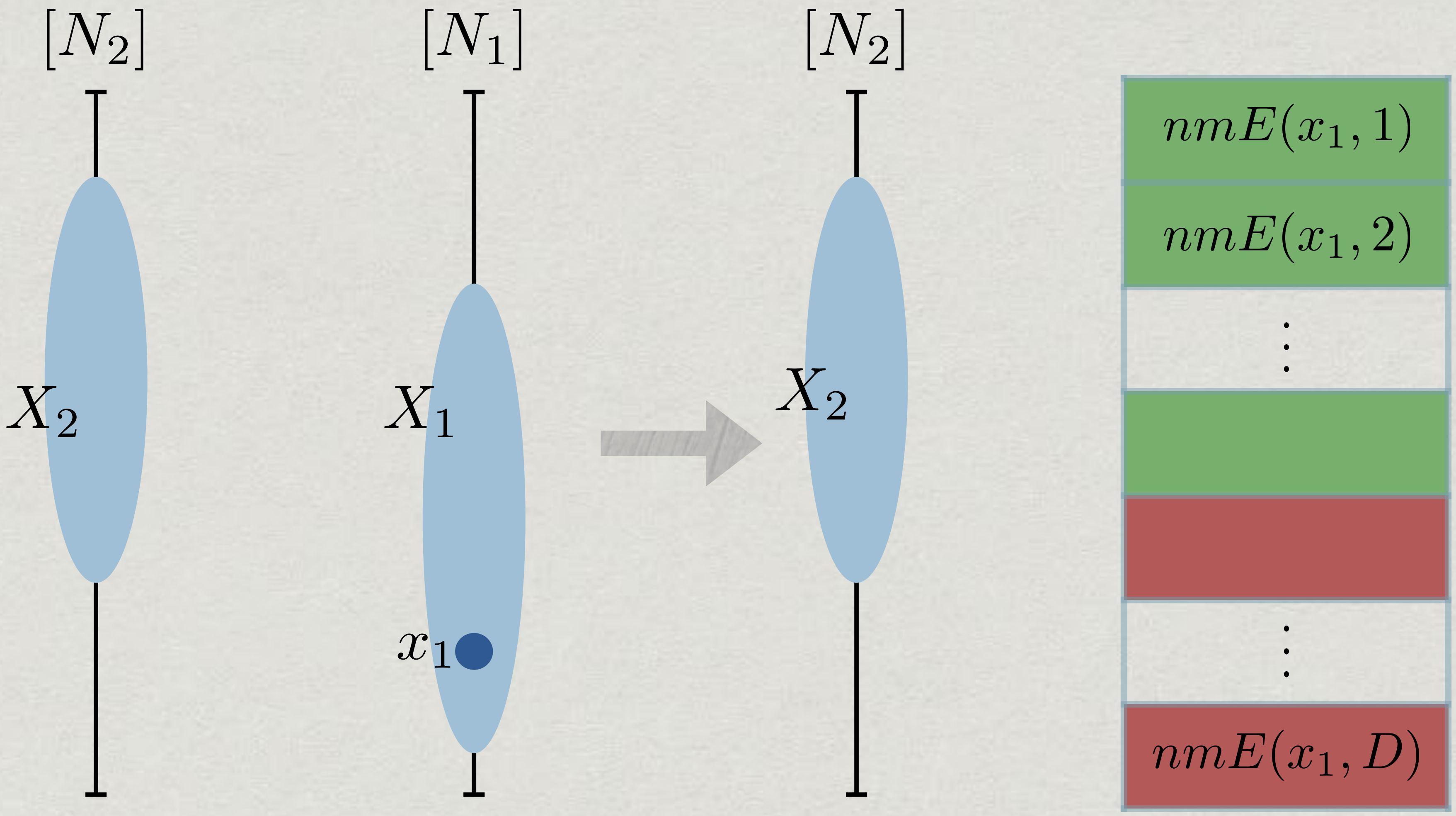

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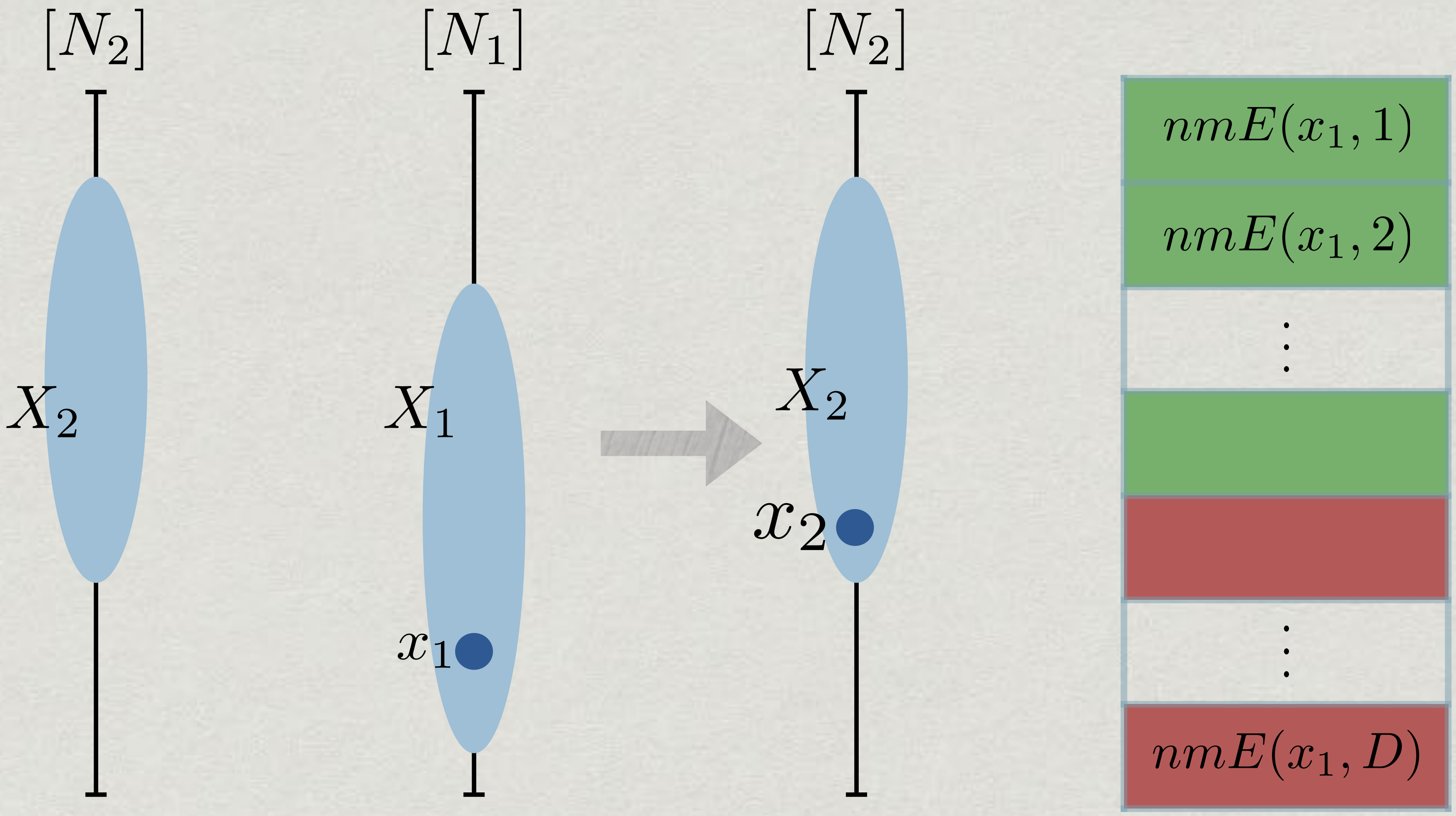

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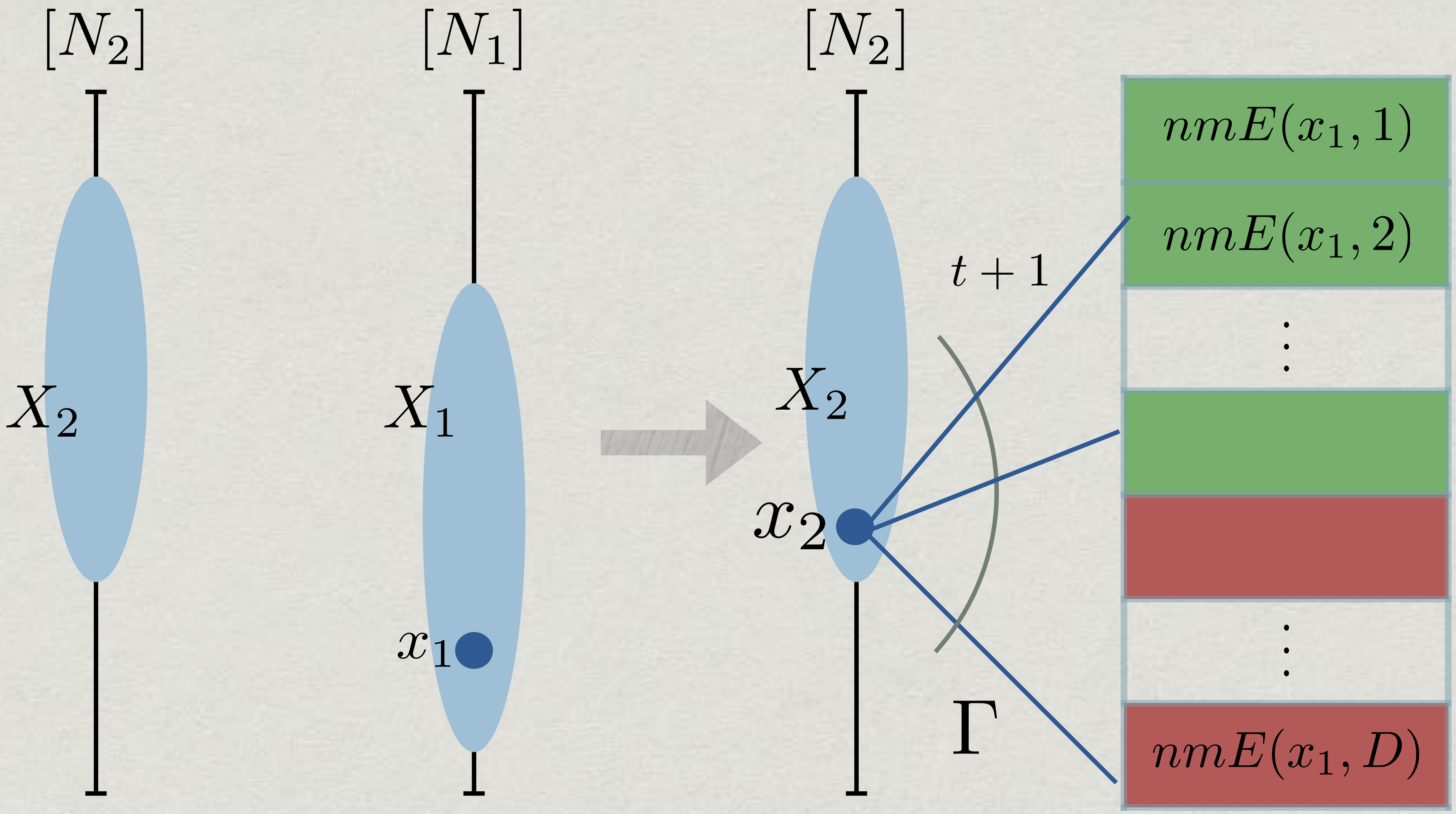

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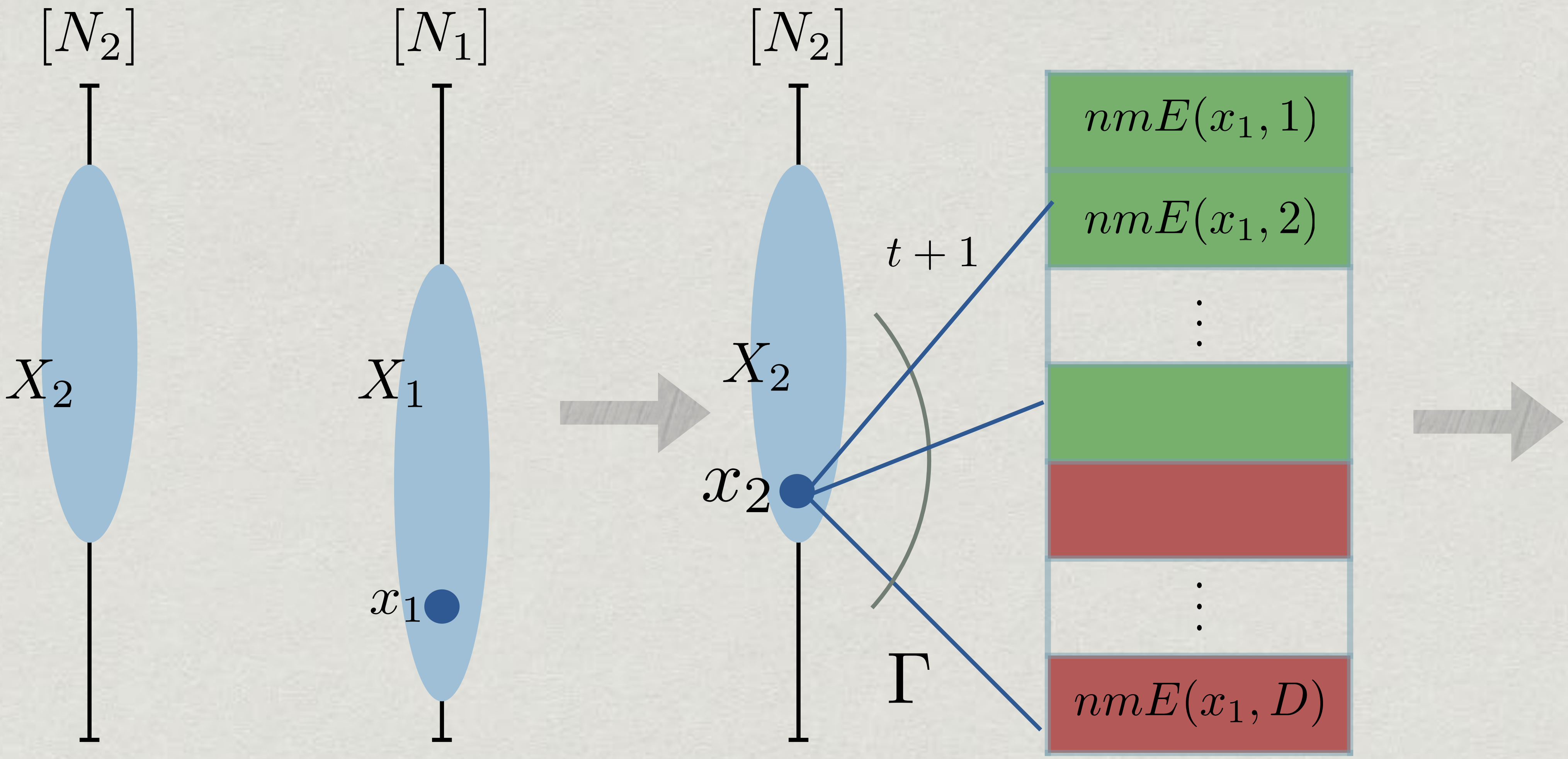

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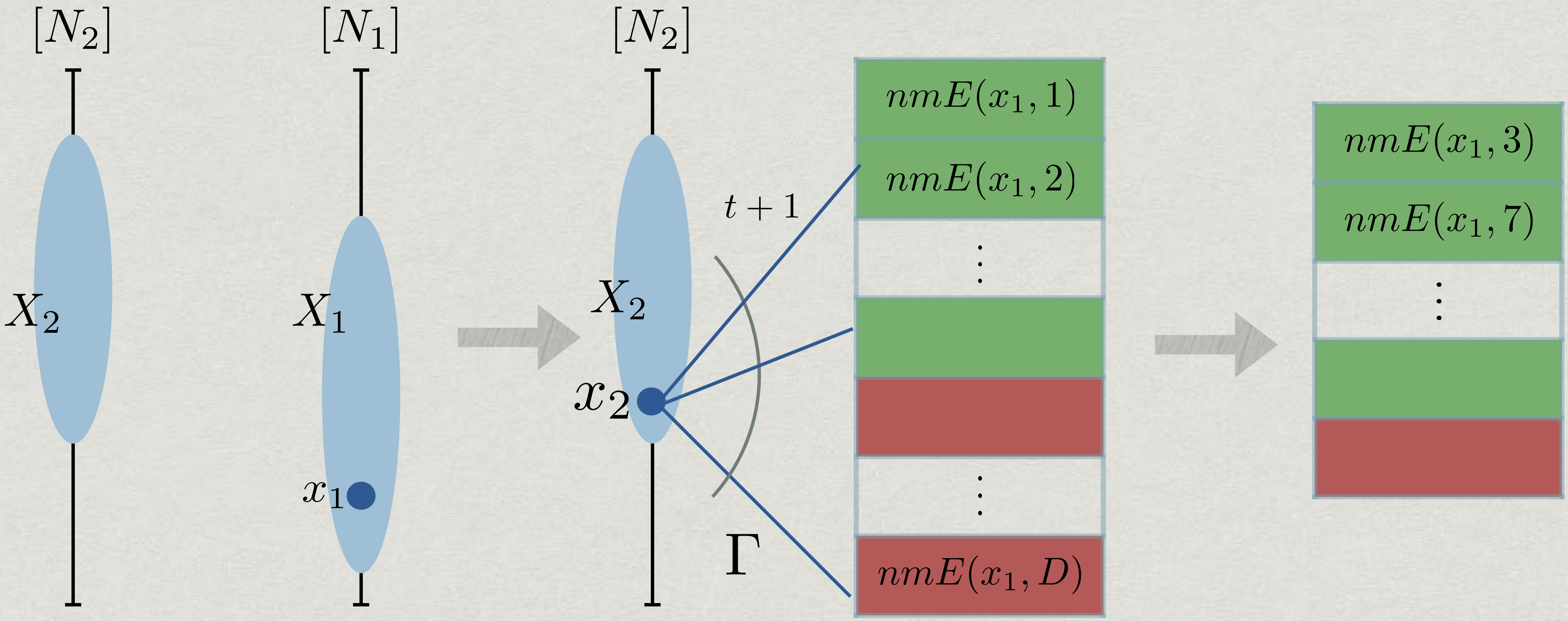

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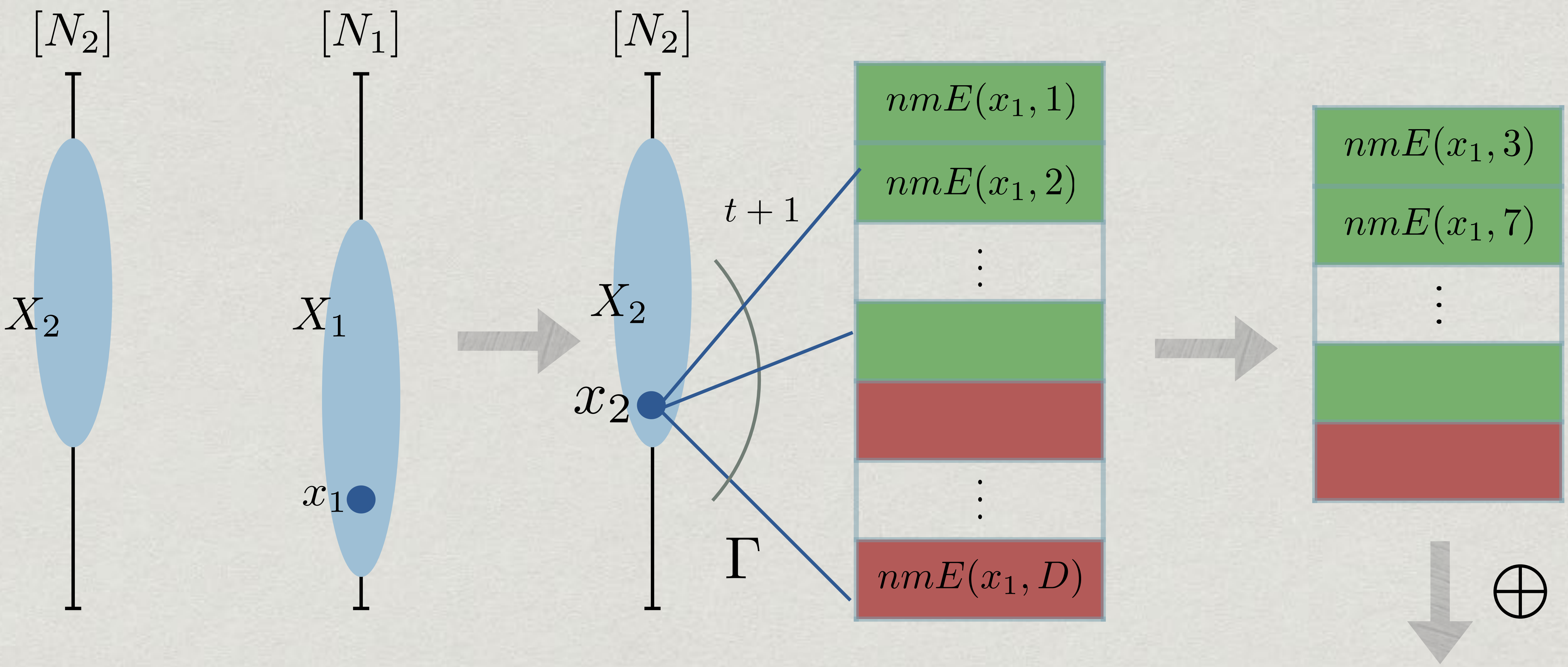

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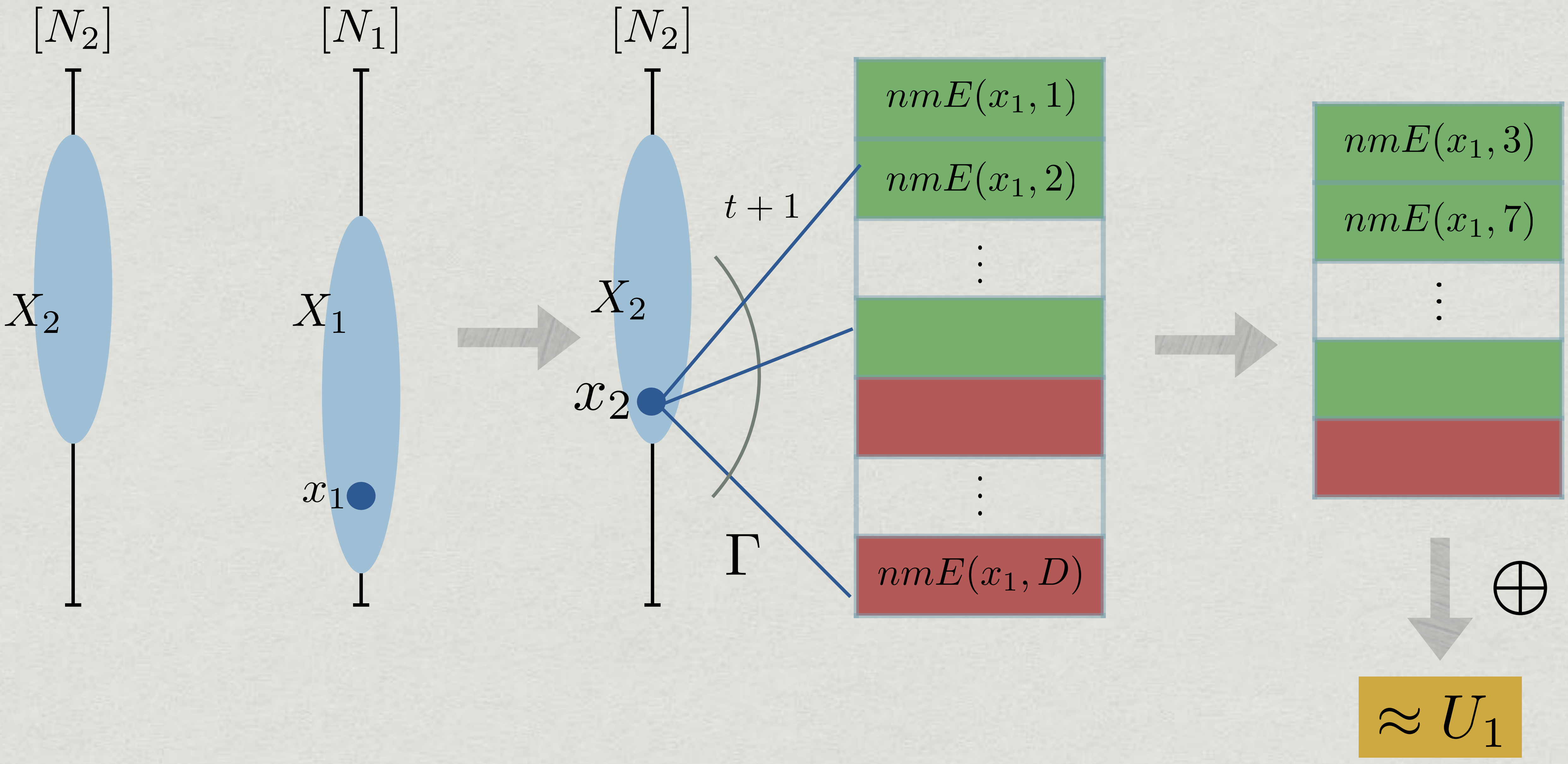

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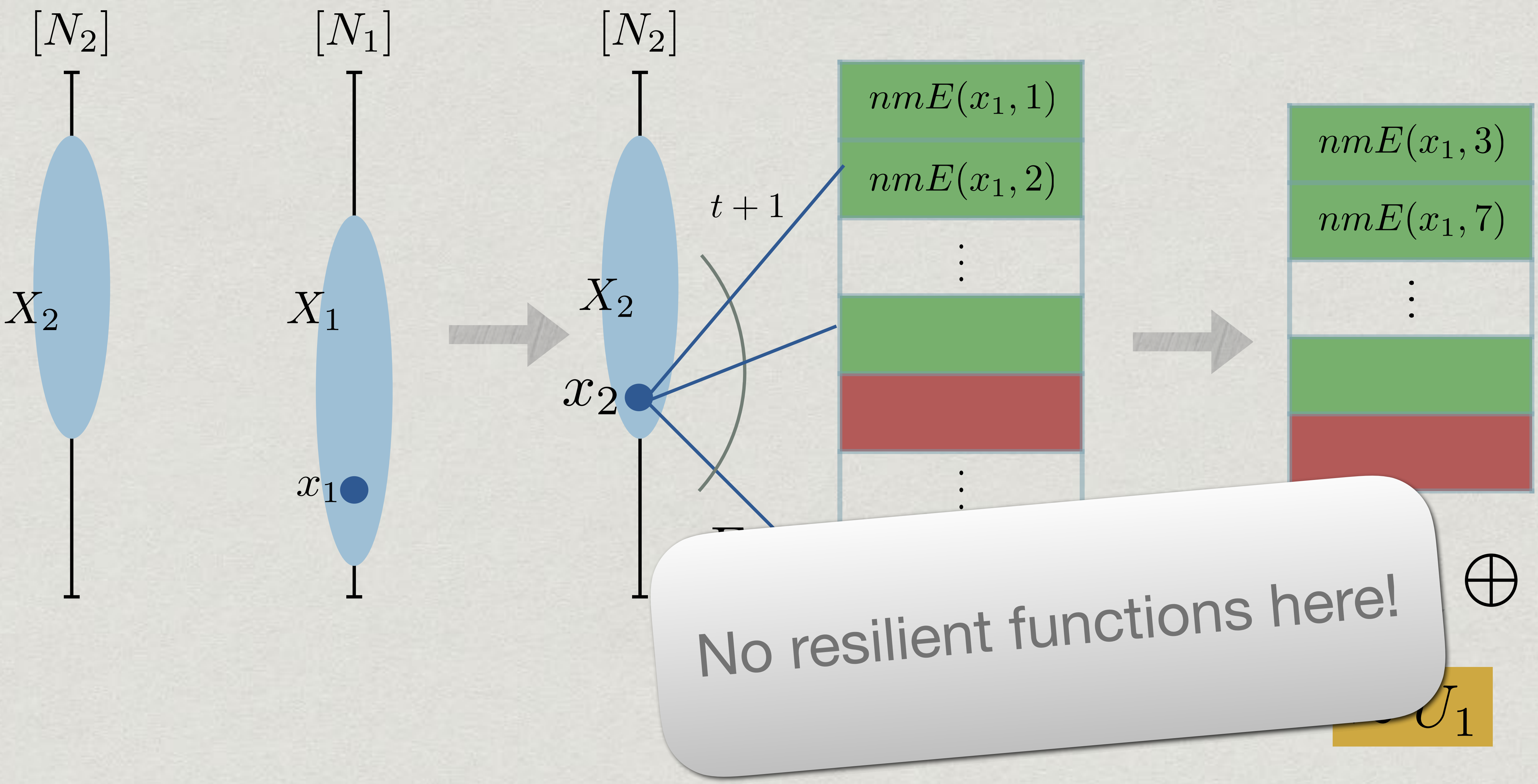

Proof outline

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- The source X_1 defines a set of good and bad seeds. Let G be the set of good seeds, of density at least $1-\varepsilon$.
- By the properties of Γ, the number of elements x_2 for which $\Gamma(x_2,[t+1])$ contains only bad seeds is at most εK_2.
- Thus, with probability at least $1-\varepsilon K_2/K_2=1-\varepsilon$, the input x_2 samples $t+1$ seeds of nmE, one of which, ***y***, is good.

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- For every $y \in G$ and any $y_1,\ldots,y_t \in \{0,1\}^d \setminus \{y\}$ it holds that $(\mathrm{nmE}(X,y),\mathrm{nmE}(X,y_1),\ldots,\mathrm{nmE}(X,y_t))$ is ε-close to $(U,\mathrm{nmE}(X,y_1),\ldots,\mathrm{nmE}(X,y_t))$.

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- Hence, the parity of these random variables is also close to uniform, and the overall error is 2ε.

Our reduction

- So, if the n.m. extractor can support small error (and existing constructions can), we get a construction with a small error.

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- The parity is not resilient… What happened here?
    - Instead of sampling (with a good sampler) *D'* rows from the table and applying a resilient function, we pick a drastically smaller sample set — of size t+1.
    - Instead of requiring that the number of malicious players is small, we have the weaker requirement that not *all* of the players in our sample set are malicious.

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- Or, *when* does it work? We have no option but to look closer into the parameters.
- First, note that the disperser dictates n_2, the length of the second source, and typically it is smaller than n_1.
- A potential circular hazard — the degree of Γ should be at least t+1, but the degree of Γ also depends on the seed length of the n.m. extractor, which in turn depends on t…

But does it work?

* Let’s check this circularity on the board…

Our result

- We see that the seed length of the n.m. extractor plays a crucial role. Say there exists an explicit n.m. extractor with seed length d and supports entropy k_1. Our results:
  - If $d=c\boldsymbol{t}\log(n_1/\varepsilon)$ for a small enough constant c, there exists an explicit two-source extractor with small error for entropies k_1 and $k_2=\alpha n_2$ (for every constant α).

Our result

- We see that the seed length of the n.m. extractor plays a crucial role. Say there exists an explicit n.m. extractor with seed length d and supports entropy k_1. Our results:
  - If $d=\boldsymbol{t}^{\gamma}\log(n_1/\varepsilon)$ for a small enough constant ɣ, there exists an explicit two-source extractor with small error for entropies k_1 and $k_2=n_2^{\beta}$ for some constant β.

Our result

- We see that the seed length of the n.m. extractor plays a crucial role. Say there exists an explicit n.m. extractor with seed length d and supports entropy k_1. Our results:
  - If $d=\log(n_1/\varepsilon)+O(\boldsymbol{t})$, there exists an explicit two-source extractor with small error for entropies k_1 and $k_2=n_2^{\beta}$ for every constant β.

Good n.m. extractors

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- Non-explicitly, our constraints on *d* are easily satisfied. The seed length of a probabilistic construction is $\boldsymbol{d=2\log(n/\varepsilon)+O(\log t)}$.

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- Non-explicitly, our constraints on d are easily satisfied. The seed length of a probabilistic construction is $\boldsymbol{d=2\log(n/\varepsilon)+O(\log t)}$.
- Taking a closer look on recent constructions of non-malleable extractors, we see that $d=\Omega(k)$ and $k=\tilde{O}(t^2\log(n/\varepsilon))$.
- Very roughly, this coupling between d and k is inherent when you do alternating extraction.

Good n.m. extractors

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- To summarize…

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* Due to [CZ15,BDT16] we know that n.m. extractors with short seed length supporting small entropies give rise to good two-source extractors with constant error.

Good n.m. extractors

- To summarize…
- Due to [CZ15,BDT16] we know that n.m. extractors with short seed length supporting small entropies give rise to good two-source extractors with constant error.
- This work: N.m. extractors also give rise to two-source extractors with small error, as long as the seed-length's dependency on *t* is good.

Good n.m. extractors

- The moral: Keep constructing non-malleable extractors, with techniques that go beyond alternating extraction.

Thanks for listening.