Packing degenerate graphs using pseudorandomness

Julia Böttcher

London School of Economics

Simons Institute, "Proving and Using Pseudorandomness"

March 2017

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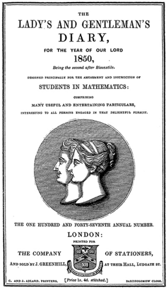

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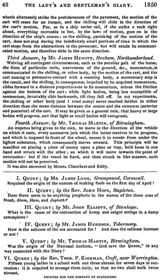

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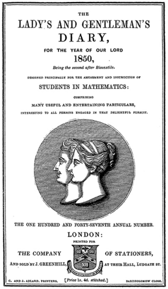

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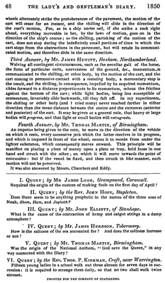

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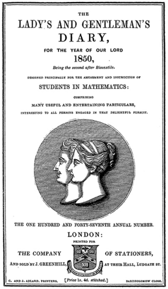

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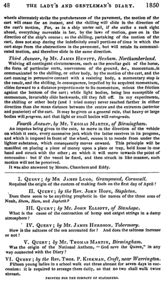

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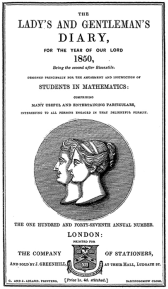

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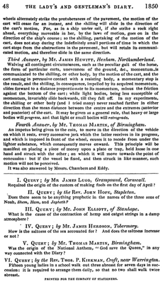

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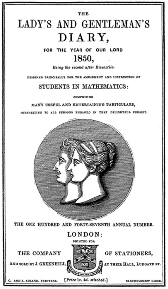
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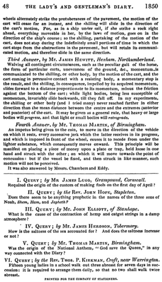
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- but we need more: parallel classes which are themselves partitions of the vertices into disjoint blocks (resolvable)

The graph case

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Ray-Chaudhuri and Wilson show an analogous result for $S(2, k, n)$ for all k.

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Rödl nibble:

- in a first round choose few $K_k^{(t)}$-copies randomly and of these select only those without overlaps
- delete the edges in the selected copies
- continue with a second round in the remainder, and so on

Help from modern extremal combinatorics

Breakthrough:

Theorem KEEVASH

For large n, if the obvious divisibility conditions are satisfied, then a Steiner system $S(t, k, n)$ exists.

Divisibility conditions: $\binom{k-i}{t-i}$ should divide $\binom{n-i}{t-i}$ for $1 \leq i \leq t-1$

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Recently: alternative proof and more GLOCK, KÜHN, LO, OSTHUS

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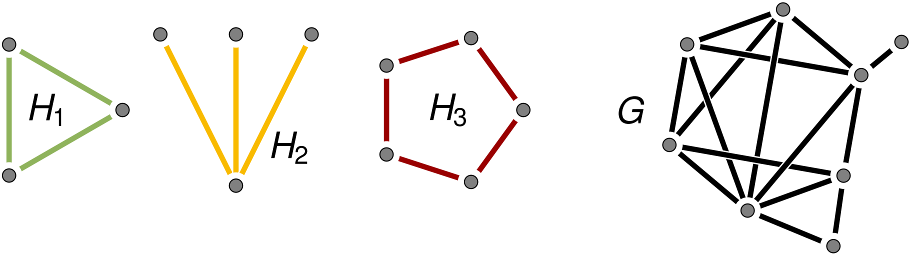

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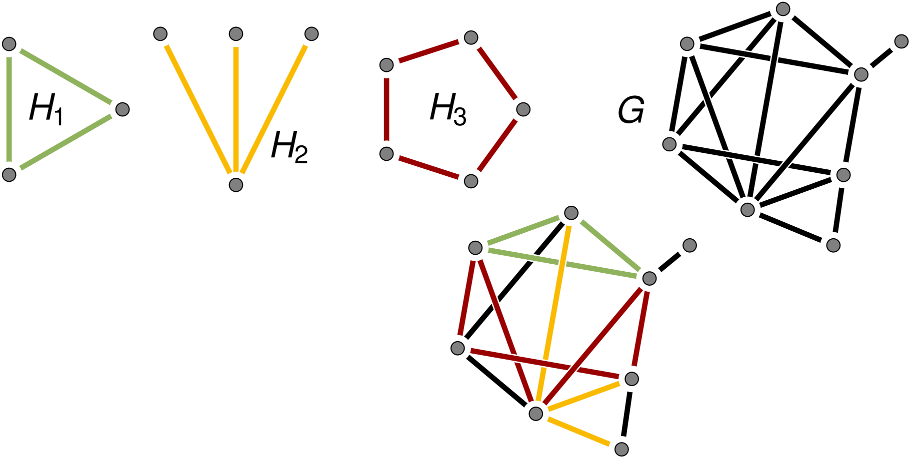

The Tree Packing Conjecture

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Any family $T_1, T_2, \ldots, T_n$ of trees with $v(T_i) = i$ packs into K_n.

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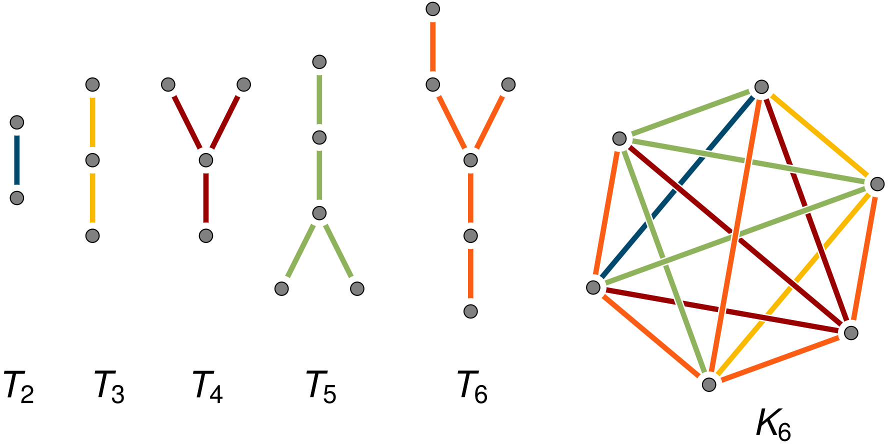

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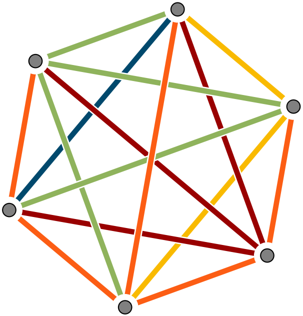
Magic:

1. perfect packing: $\sum_{i=1}^{n} e(T_i) = \binom{n}{2}$



K_6

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- also gives a perfect packing
- bipartite versions of these packing conjectures exist

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Definition GRACEFUL LABELLING

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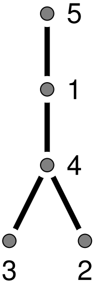

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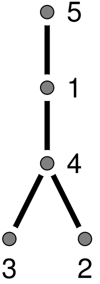

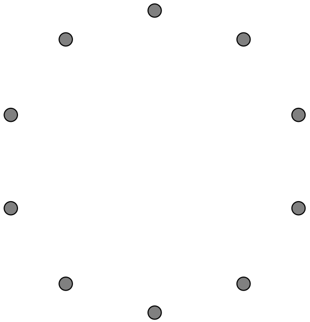

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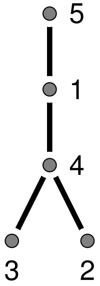

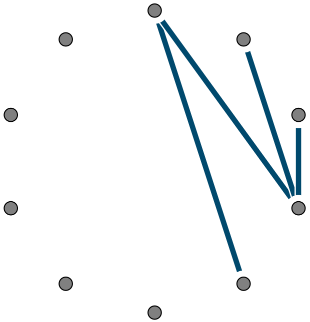

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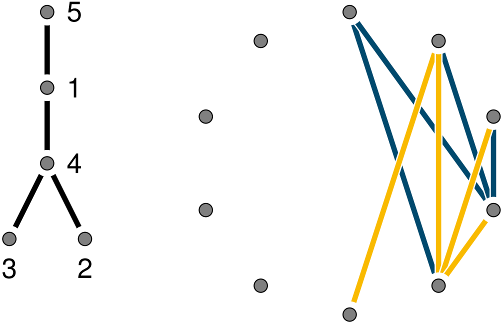

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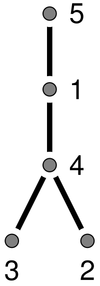

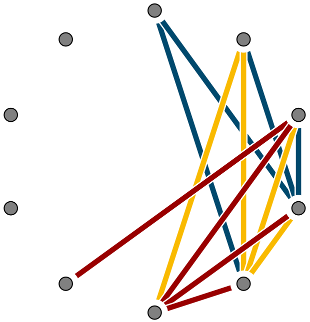

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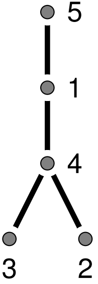

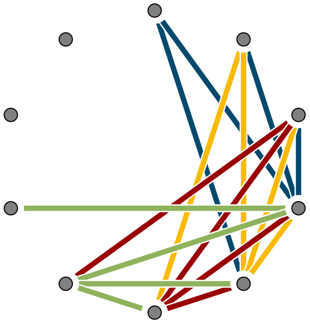

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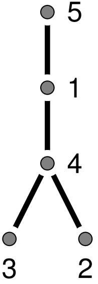

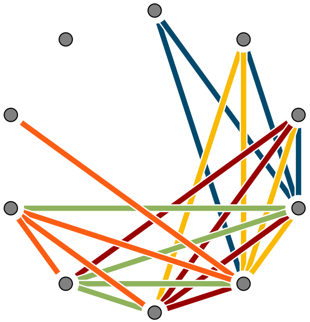

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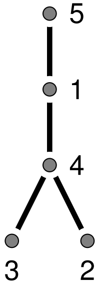

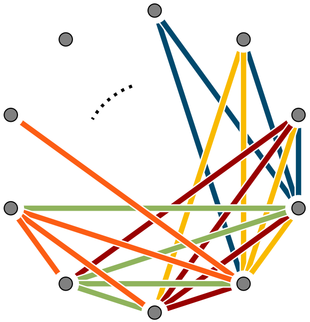

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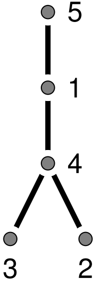
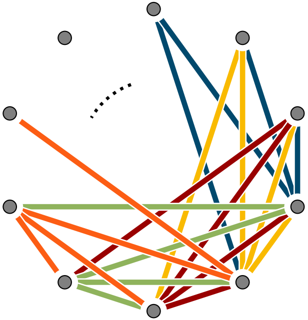
Conjecture

Every tree is graceful.





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Conjecture — ROSA '67

Every tree is graceful.

- paths and caterpillars, firecrackers, banana trees, olive trees, . . .
- trees of diameter at most 7

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Also gives near-perfect version for the conjecture of Ringel:

- $2n + 1$ copies of a tree T with $v(T) = n + 1$ pack into K_{2n+1}.

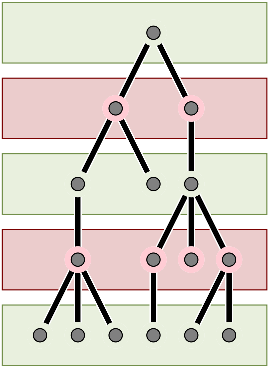
Idea of near-perfect tree packing

Let T be a tree, G a host graph.

- even layers of T: primary vertices
- odd layers of T: secondary vertices

Random process:

1. map primary vertices randomly to $V(G)$,
2. map secondary vertices randomly into neighbourhoods



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Near-graceful labelings: for trees with maximum degree $O(n/\log n)$

ADAMASZEK, ALLEN, GROSU AND HLADKÝ

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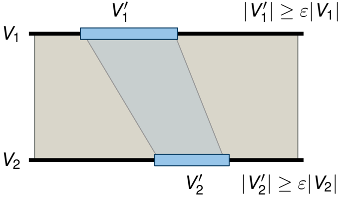

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For Δ fixed, in an (ε, d)-superregular pair (V_1, V_2), we can embed any bipartite H with classes X_1 and X_2 with $|X_i| = |V_i|$ and max. degree $\leq \Delta$.

The idea of perfect tree packing with bounded degrees

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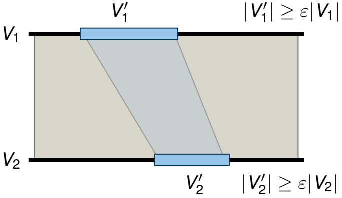

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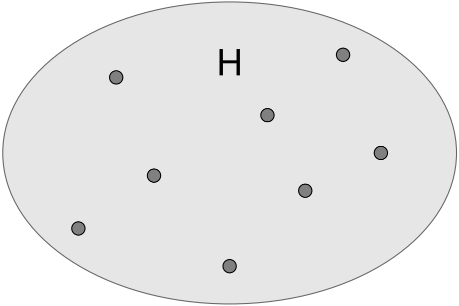
Packing version: analogous result for near-perfect packing of such H

A natural random packing process

For packing $G_1, \ldots, G_t$ in H:



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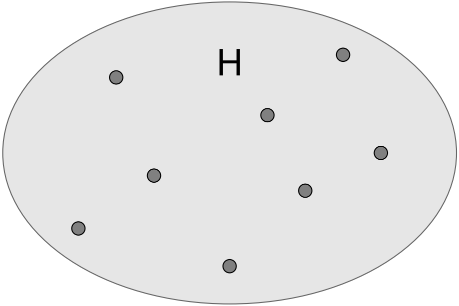

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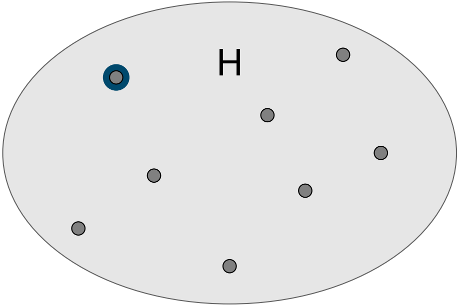

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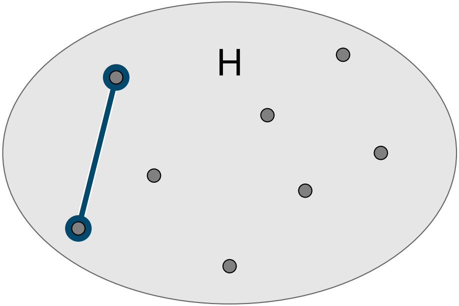

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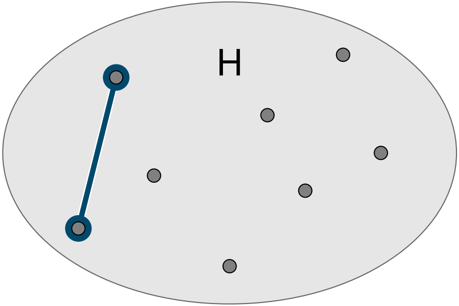

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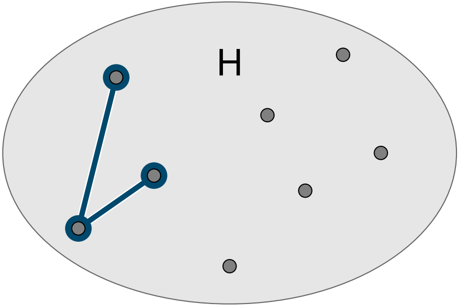

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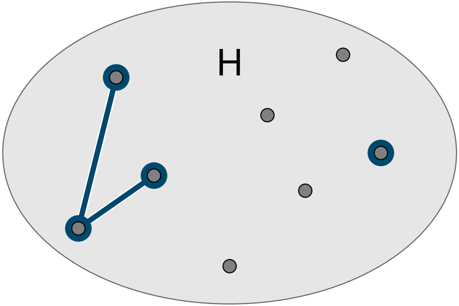

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For packing $G_1, \dots, G_t$ in H:

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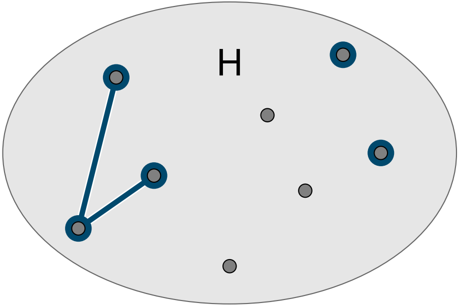

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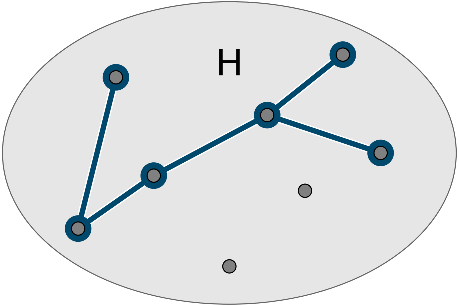

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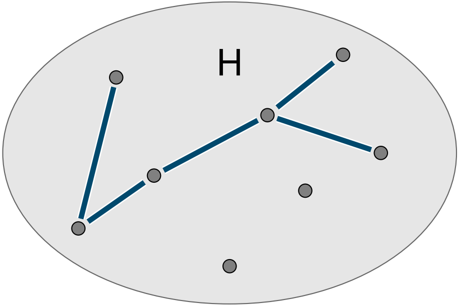

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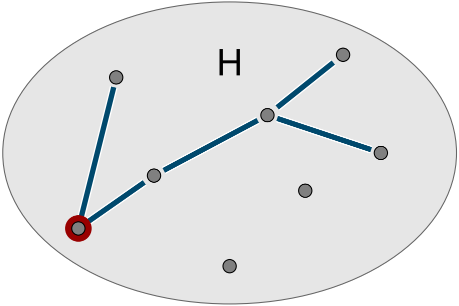

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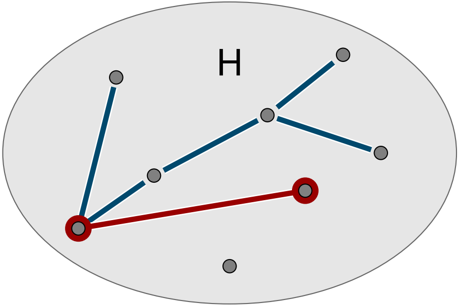

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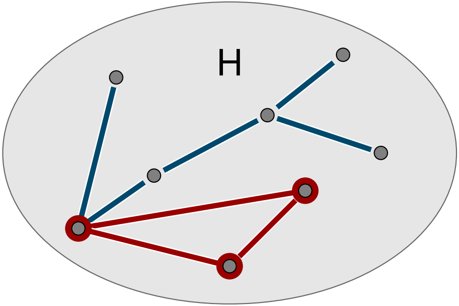

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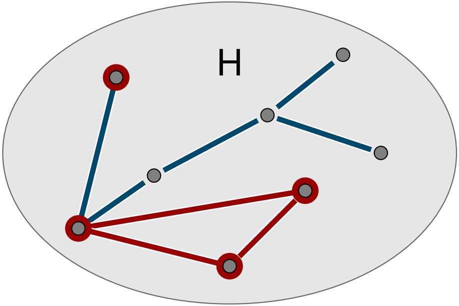

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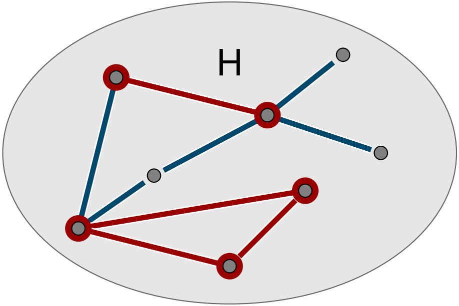

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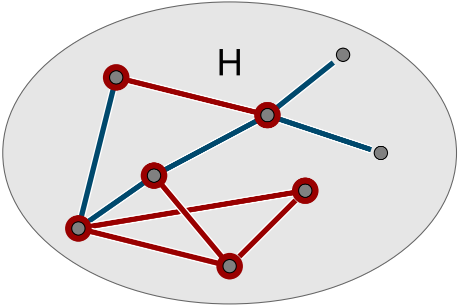

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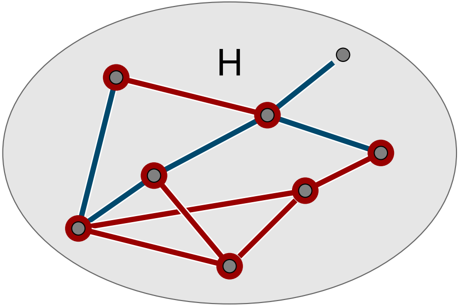

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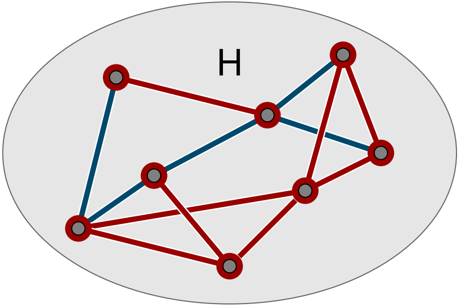
Hope: after embedding some G_i, remainder of H is quasirandom



A near-perfect packing of spanning degenerate graphs

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Theorem — Allen, B, Hladký, Piguet

For all $\varepsilon > 0$, $D \in \mathbb{N}$ there are $c > 0$ and $n_0 \in \mathbb{N}$ such that for $n \geq n_0$:
Let $G_1, \ldots, G_t$ be a family of D-degenerate graphs with

- $v(G_i) \leq n$,
- $\sum_{i=1}^{t} e(G_i) \leq (1-\varepsilon)\binom{n}{2}$,
- $\Delta(G_i) \leq cn/\log n$.

Then $G_1, \ldots, G_t$ pack into K_n.

- covers more general graph class than all previous near-perfect packing results

Some remarks about the proof

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- we need to control sequential dependencies
- for extending each G_i' to a copy of G_i:
  - before starting the process, reserve $\frac{1}{2}\varepsilon\binom{n}{2}$ random edges H^* of K_n
  - choose $G_i' \setminus G_i$ as independent set
  - use a matching argument to show G_i' can be completed in H^*

Sequential dependencies

Lemma

Let

- Ω be a finite probability space,
- $(\mathcal{F}_0, \mathcal{F}_1, \ldots, \mathcal{F}_n)$ be partitions of Ω, with $\mathcal{F}_i$ refining $\mathcal{F}_{i-1}$.
- Y_i be nonnegative random variables, constant on each part of $\mathcal{F}_i$.
- $\mathcal{E}$ be an event.

Suppose that almost surely, either

- $\mathcal{E}$ does not occur, or
- $\sum_{i=1}^n \mathbb{E}\left[Y_i \middle| \mathcal{F}_{i-1}\right] = \mu \pm \nu$, $\sum_{i=1}^n \mathrm{Var}\left[Y_i \middle| \mathcal{F}_{i-1}\right] \leq \sigma^2$, and $0 \leq Y_i \leq R$

Then

$$\mathbb{P}\left[\mathcal{E} \text{ and } \sum_{i=1}^n Y_i \neq \mu \pm (\nu + \varrho)\right] \leq 2 \exp\left(-\frac{\varrho^2}{2\sigma^2 + 2R\varrho}\right).$$

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Many thanks!