

Separation distance for k shuffles of 52 cards

| k | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|-----------|-------|-------|-------|-------|-------|-------|--------|--------|--------|--------|--------|--------|
| BD-92 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 0.995 | 0.928 | 0.729 | 0.478 | 0.278 |
| blackjack | 1.00 | 1.00 | 1.00 | 1.00 | 0.999 | 0.970 | 0.834* | 0.596* | 0.366* | 0.204* | 0.108* | 0.056* |
| ♠♦♥♣ | 1.00 | 0.997 | 0.997 | 0.976 | 0.884 | 0.683 | 0.447 | 0.260 | 0.140 | 0.073 | 0.037* | 0.019* |
| A♠ | 1.00 | 1.00 | 0.993 | 0.875 | 0.605 | 0.353 | 0.190 | 0.098 | 0.050 | 0.025 | 0.013 | 0.006 |
| redblack | 0.890 | 0.890 | 0.849 | 0.708 | 0.508 | 0.317 | 0.179 | 0.095 | 0.049 | 0.025 | 0.013 | 0.006 |
| ☐♥♠♣ | 1.00 | 1.00 | 0.993 | 0.943 | 0.778 | 0.536 | 0.321 | 0.177 | 0.093* | 0.048* | 0.024* | 0.012* |

| Deck | Method | $a = 16$ | 32 | 64 | 128 | 256 | 512 | 1024 |
|--|-----------------|----------|--------|--------|--------|--------|--------|--------|
| 52 Distinct | Exact | 1.0000 | 0.9237 | 0.6135 | 0.3341 | 0.1672 | 0.0854 | 0.0429 |
| 123... (52) | $44.0571a^{-1}$ | 2.7536 | 1.3768 | 0.6884 | 0.3442 | 0.1721 | 0.0860 | 0.0430 |
| Ordered Bridge $N^{13}E^{13}S^{13}W^{13}$ | Monte Carlo | 0.9902 | 0.7477 | 0.4230 | 0.2183 | 0.1104 | 0.0550 | 0.0274 |
| | $27.9095a^{-1}$ | 1.7443 | 0.8722 | 0.4361 | 0.2180 | 0.1090 | 0.0545 | 0.0273 |
| Cyclic Bridge (NESW) ¹³ | Monte Carlo | 0.2349 | 0.0735 | 0.0346 | 0.0169 | 0.0084 | 0.0042 | 0.0021 |
| | $2.1469a^{-1}$ | 0.1342 | 0.0671 | 0.0335 | 0.0168 | 0.0084 | 0.0042 | 0.0021 |
| Back-Forth Bridge (NESWSEN) ⁸ (NESW) | Monte Carlo | 0.3118 | 0.0260 | 0.0073 | 0.0022 | 0.0008 | 0.0003 | 0.0002 |
| | $0.1651a^{-1}$ | 0.0103 | 0.0052 | 0.0026 | 0.0013 | 0.0006 | 0.0003 | 0.0002 |

Table 1: Variation distances from uniform after an a -shuffle for a deck of distinct cards and 3 methods of dealing bridge; $a = 16$ means 4 riffle shuffles, $a = 32$ is 5 riffle shuffles, etc.

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