Consumer behavior and algorithm design

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Google™

Warm up

A small demonstration on using our understanding of behavior

Connectivity Server

- In-memory support for queries on the web graph
  - Which URLs point to a given URL?
  - Which URLs does a given URL point to?
- Applications
  - Crawl control
  - Web graph analysis
  - Link analysis

Adjacency lists

- Each URL represented by an integer
  - E.g., for a 4 billion page web, need 32 bits per node
- Naively, need 64 bits to represent each edge
- Will show scheme achieving 3 bits/edge
  - Further optimizations get to 2 bits/edge

Adjaceny list compression

- Properties exploited in compression:
  - Similarity (between lists)
  - Locality (many links from a page go to "nearby" pages)
  - Use gap encodings in sorted lists
  - Distribution of gap values

Main ideas of Boldi/Vigna

- Consider lexicographically ordered list of all URLs, e.g.,
  - …
  - www.stanford.edu/alchemy
  - www.stanford.edu/biology
  - www.stanford.edu/biology/plant
  - www.stanford.edu/biology/plant/copyright
  - www.stanford.edu/biology/plant/people
  - …

Basic idea

- Each of these URLs has an adjacency list
- Due to templates, the adjacency list of a node is similar to one of the 7 preceding URLs in the lexicographic ordering
- Express adjacency list in terms of one of these
- E.g., consider these adjacency lists
  - 1, 2, 4, 8, 16, 32, 64
  - 1, 4, 9, 16, 25, 36, 49, 64
  - 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144
  - 1, 4, 8, 16, 25, 36, 49, 64

Encode as (-2), remove 9, add 8

Summary

- Depends on similarity/locality in canonical ordering
  - Lexicographic ordering works well for the web, thanks to the behavior of users and page creators
- Adjacency queries can be answered efficiently
  - To find neighbors, trace back the chain of prototypes
  - This chain is typically short in practice (since similarity is mostly intra-host)
- Easy to implement one-pass algorithm

Three themes

How consumer behavior is changing
what we compute, and how

Three themes

- Subjective functions
- Riding the fat tails
- PRAM lost, parallelism regained

Subjective functions

- Consumers drive consumption of cycles, bandwidth, storage
- Lots of cycles still expended in SORT and SELECT but – a large and growing chunk in "making users happy"
- 7 billion "objective" functions
  - Don't bother trying to write them down
  - Find and optimize proxies

Example subjective function

- What's a good search ranking?

  Proxy: Make each document a vector in the space of all words; do the same for a query

  Score = dot product of query and document vectors

  Algorithmically: Select 100 documents of highest dot product to query, efficiently.

(Salton, circa 1975.)

A few other subjective functions

- Find me beautiful pictures of sunsets
  - *What's a sunset?*
  - *What's beautiful?*
- Given feeds from 6 cameras at a stadium, find the best camera view at each point in time
- Show me songs I'll like
- [Spelling correction]

Subjective function characteristics

- How do we assess our algorithm?
  - Ask users! (What does this mean?)
- Users have some tolerance for junk – no notion of absolute correctness
- "Infinite" computing doesn't help
  - Resources typically not the primary constraint in algorithm design
  - Even with unbounded computing, don't know how to do better

The first 25 years of search ranking

- Focus on tweaking the weights in the basic vector space
  - Presumption of expert "information scientists" seeking information
- Starting 1995, sea changes
  - Ordinary consumers with extraordinary expectations
  - Adversarial content

Better search ranking

- Say your student comes to you and says – I have a new score for documents, called Pagerank
  - How do you combine dot product score and Pagerank?
- Explore combinations of scores that regress well to user judgments
  - Explicit surveys of trained raters
  - Crowd ratings
  - Clicks

More generally

- The world is full of signals for good search ranking (or beautiful pictures, likeable songs, memorable slogans)
- Combining them left to Machine Learning
  - At core, large math programs
- Algorithmic improvement entails
  - Inventing new signals ("*features*")
  - Volume of rating data to train ML

Upshot

- We learn functions we can't cleanly describe
- Approximate, and iteratively improve (proxy for) user happiness
  - Standard methodology in research/industry
  - Data volume is paramount
- Abdicate algorithm design to machine learning kernels

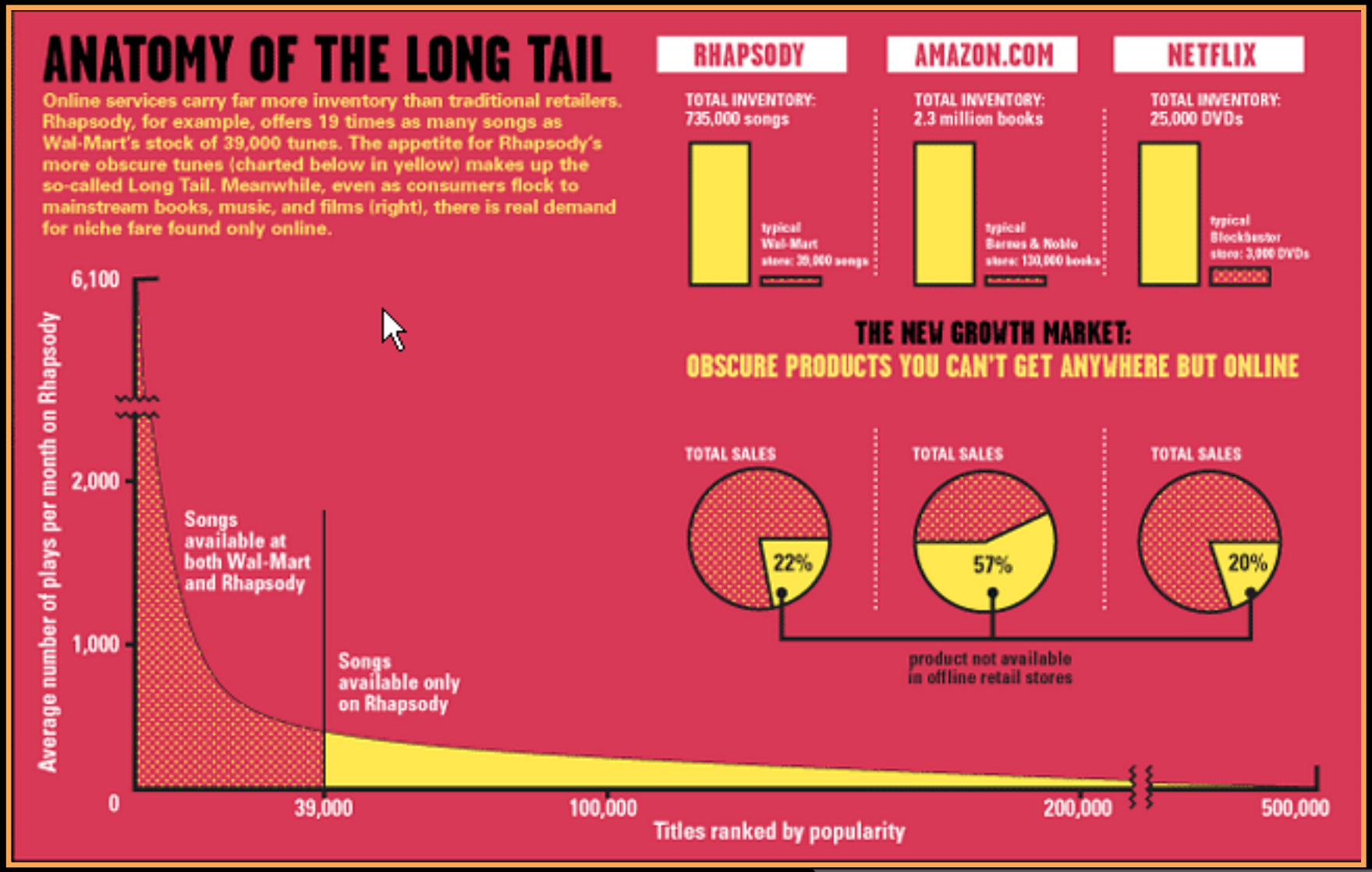
Computational aesthetics

- Consider a question such as: *Which of these web pages is more appealing?*
- An assessment of aesthetics
- We can do a decent job on such problems using the general approach above
  - No pretense of understanding aesthetics
  - Nor of synthesizing creativity

Riding the fat tails

Graphs with small average degree, but some nodes have massive degrees

“The Long tail”



Chris Anderson, Wired, Oct 2004

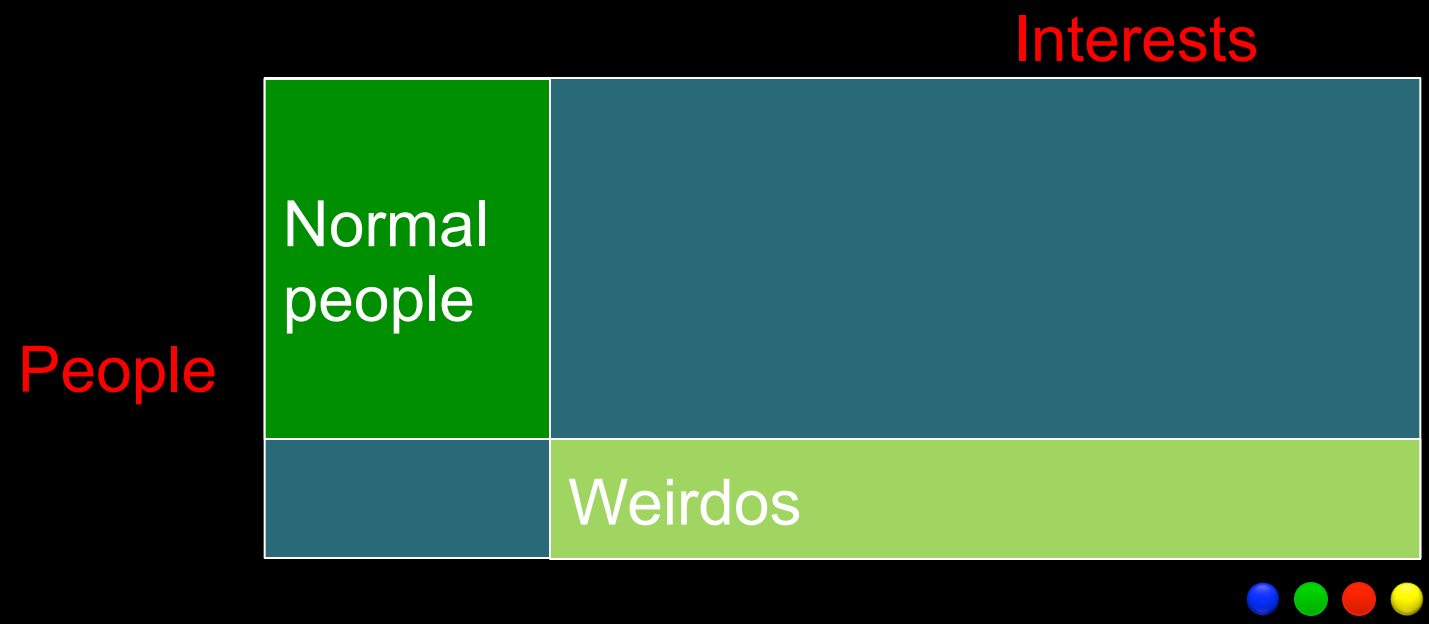
Infinite-inventory stores: two properties

1. The vast majority of products are "misses"
2. These "worst-sellers" in aggregate account for a large fraction of sales

- 25% of Netflix sales and 30% of Amazon sales are reportedly for products not available in traditional stores
- But … How many people care about the tail?
  - Could the world be bimodal?

Could this be two normal distributions?

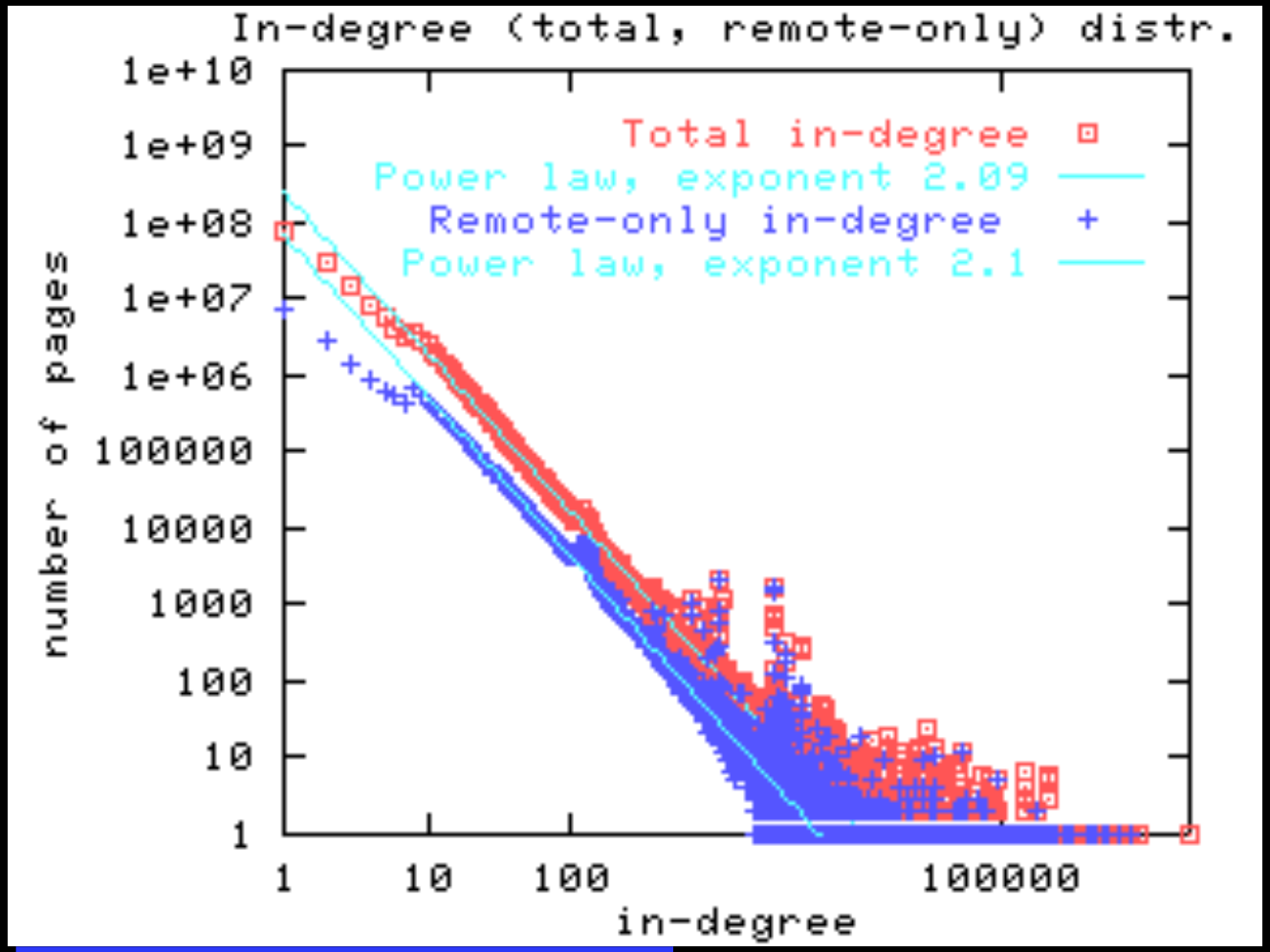
- Two views of human behavior consistent with heavy tails
  1. Bimodal population?



Many datasets show in fact that …

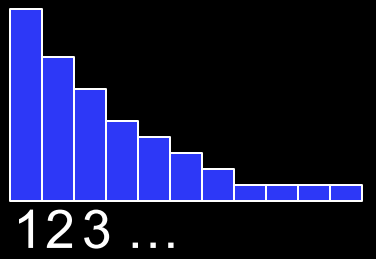
- Two views of human behavior consistent with heavy tails
  2. Or "homogenous" population?
- We are all partially eclectic

How many links into a random web page?



$$\Pr_u[Deg(u) = i] \cong i^{-2.1}$$

Heavy tails



- Decreasing probability distributions over integers in *[1,n]*
  - E.g., search query frequencies in decreasing order
- For any fixed *k* (say, 10000), the probability mass in all buckets *>k* is a "big" constant
- Arise in observed statistics stemming from human behavior
  - Number of friends, popularity of movies …

A typical heavy tailed distribution

- *p[i] ~ 1/ i^{β}* - so-called *power law*
  - log-log plot is a straight line
  - For $\beta > 1$, bounded expectation
- Other distributions – log-normal, Pareto
  - cf Mitzenmacher survey
- Misnomer:

  Long-tailed distributions

Copying model
aka *preferential attachment*

- Each page is a node, each hyperlink is a directed edge
- Edges arrive in sequence
- For $0 \leq \alpha \leq 1$,
  - With probability α, link to a random page
  - With probability $1-\alpha$, copy a random link
- Explore-exploit of news stories on news sites

Informal analysis

- Let $X_j(t)$ be the number of pages of degree j at time t

$$\frac{dX_j}{dt} = \alpha \frac{X_{j-1}}{t} - \alpha \frac{X_j}{t} + (1-\alpha)(j-1)\frac{X_{j-1}}{t} - (1-\alpha)\frac{X_j}{t}$$

- Limiting distribution

$$X_i \approx i^{-(2-\alpha)/(1-\alpha)}$$

- Proof uses bounded difference argument

Consequence for probabilistic analysis

- Consumer phenomena yield heavy tails
- Most probabilistic analysis uses light-tailed distributions (binomials, Poisson ...)
  - Reason: we use simple generative models based on the superposition of independent events
- Work ahead – analysis of data structures and algorithms under heavy-tailed inputs
  - Early work on inverted indexes and graph compression

Work ahead

- Better modeling and analysis
  - Underlying individual and social behavior
- Behavior of consumers correlated
  - To each other
  - To "market forces"
    - Recommendation systems
    - What's on the front page of the NY Times
- What distributions arise from consumers subject to such forces?
  - How real is the "filter bubble"?

Parallelism today

World computer market = 6

(*Howard Aiken;*

mis-attributed to Thomas Watson Sr.)

Rethinking parallelism

- PRAMs – simple abstraction
  - Fine-grained
  - Synchronous
  - Equal access
  - (In principle) straightforward programming
- Modifications
  - Networks of fixed topology (Hypercubes, CCC's)
  - BSP, …

Parallelism – reality

- Fault- and error-prone processors
- Coarse-grained synchrony
- Locality matters
- Software development/maintenance costs dwarf costs of many commodity machines
- Want to solve problems such as
  - "Which search queries co-occur?"
  - "Which friends to recommend?"

Parallel programming is hard

- Threaded programs are difficult to test
  - One successful run is not enough
- Threaded programs are difficult to read
- Threaded programs are difficult to debug
  - Hard to repeat the conditions to find bugs
- More machines means more breakdowns

Special thanks to Sergei Vassilivitskii for some of these slides.

MapReduce: easy parallel programming

- Tracks jobs and restarts if needed
- Takes care of data distribution and synchronization
- But there's no free lunch:
  - Imposes a structure on the data
  - Only allows for certain kinds of parallelism

MapReduce basics

- Data: Represented as <Key, Value> pairs

- Example: A Graph is a list of edges
- Key = (u,v)
- Value = edge weight

MapReduce architecture

- Operations:
- Map: <Key, Value> → List(<Key, Value>)
- Shuffle: Aggregate all pairs with the same key
- Reduce: <Key, List(Value)> → <Key, List(Value)>

MapReduce environments

- 10s to 10,000s processors
- Sublinear Memory: A few Gb of memory/ machine, even for Tb+ datasets
  - Unlike PRAMs: memory is not shared
- Batch Processing

For an input of size *n*

- Cannot store entire data in memory
  - Sublinear memory per machine $O(n^{1-\varepsilon})$
- Machines in a cluster do not share memory
  - Sublinear number of machines: $O(n^{1-\varepsilon})$
- Synchronization
  - Computation proceeds in rounds
  - Aim for O(1) rounds

Counting triangles in a graph

Sequential Version:

```
foreach v in V
    foreach (u,w) in Adjacency(v)
        if (u,w) in E
            Triangles[v]++
```

- Running time: $\Sigma (d_v)^2$
- In practice this is quadratic, as some vertex will have very high degree

Naïve MapReduce version

- Round 1: Generate all possible length 2 paths
- Round 2: Check if the triangle is complete
- Round 3: Sum all the counts

Fat tails strike again

- How much parallelization can we achieve?
  - Generate all the paths to check in parallel
  - The running time becomes $\max_v (d_v)^2$
- Data skew defeats naïve parallelization
- Bottleneck: high degree nodes
  - E.g., 3.2 Million neighbors, must generate 10 Trillion (10^{13}) potential edges to check.
  - Generating 100M edges to check per second, 100K seconds ~ 27 hours.

Pivot on lowest degree

Sequential Version [Schank '07]:

```
foreach v in V
        foreach {u,w} in Adjacency(v)
                if deg(u) > deg(v) && deg(w) > deg(v)
                        if (u,w) in E
                                Triangles[v]++
```

Why does it help?

- Partition nodes into two groups:
  - Low: $\{v : d_v \leq \sqrt{m}\}$
  - High: $\{v : d_v > \sqrt{m}\}$
- There are at most $\sqrt{m}$ High nodes
  - Each produces paths to other High nodes:
    - $O(m)$ paths per node
  - Therefore they generate: $O(m^{3/2})$ paths

Low-degree paths

- Let n_i be the number of nodes of degree i
- The total number of paths generated by Low nodes can also be shown to be $O(m^{3/2})$
- Total work is $O(m^{3/2})$ which is optimal
- But what about (parallel) runtime?

What made this work?

- The algorithm automatically load balances
  - Every node generates O(*m)* paths to check
  - Hence the reducers take about the same time to finish
  - Fat tails get tamed …
- Improvement in per-round runtimes:

  ~2 orders of magnitude on large graphs (10^{9+})

Closing thoughts

- Ordinary users consume a growing share of computation
  - Their expectations shape the computational problems we pursue
  - Their behavior reshapes the data distributions we observe
  - Recasts how we think about and teach algorithms
- Tremendous challenges and benefits to CS
  - Inevitably we transform – and are transformed by – the social sciences