Deterministic Edge Connectivity in Near-Linear Time

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Edge connectivity and global min-cut

- Simple graph $G = (V, E)$ (no parallel edges).
- Edge connectivity is smallest number of edges whose removal disconnects G.
- Cut defined by $U \subseteq V$, $\emptyset \neq U \neq V$.
  Two sides U and $T = V \setminus U$,
  cut edges $E(U, T) = \partial U = \partial T$ between sides.
- Result Find edge connectivity including minimum cut deterministically in near linear time.

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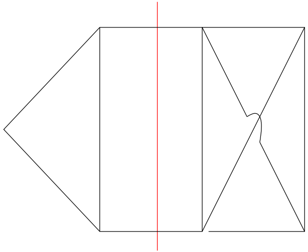

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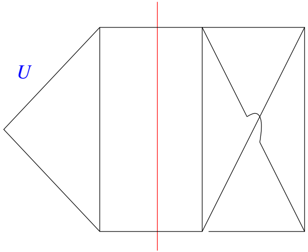

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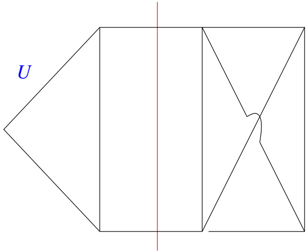

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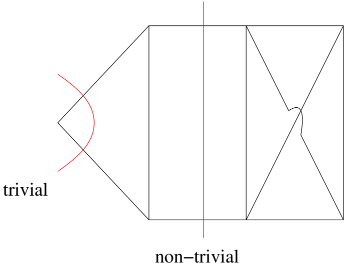
Underlying result

- A cut is trivial if one side is a single vertex.
- For simple graph with min-degree δ, in near-linear time, contract edges producing graph $\overline{G}$ with $\overline{m} = \widetilde{O}(m/\delta)$ edges, preserving all non-trivial min-cuts of G.



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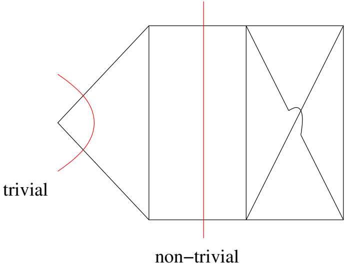

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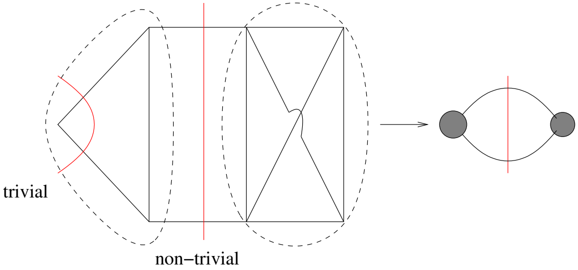

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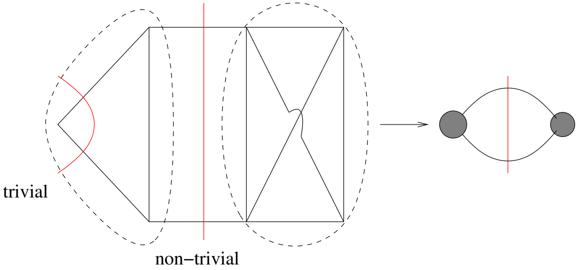

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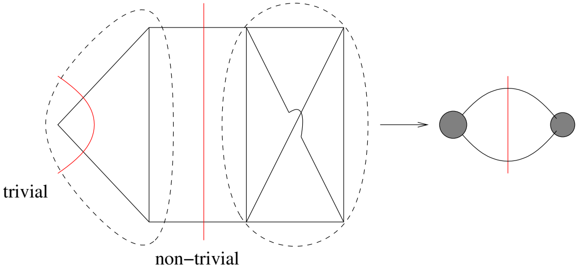

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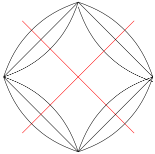

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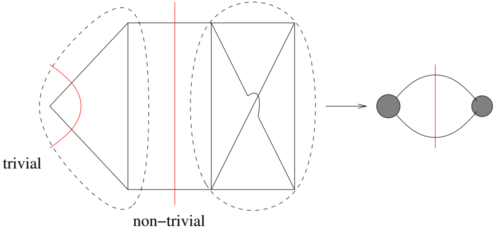

- Run Gabow's min-cut (or cactus) algorithm on $\overline{G}$ in $\widetilde{O}(\lambda\overline{m}) = \widetilde{O}(m)$ time.
- Check against δ to see if trivial min-cuts from G should be included.
- Gives min-cut (or cactus) for original G in $\widetilde{O}(m)$ total time.

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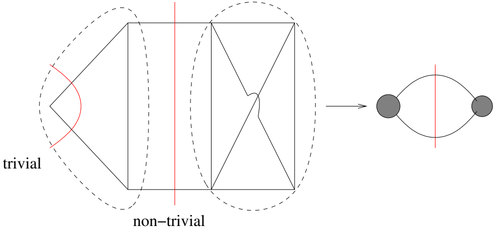

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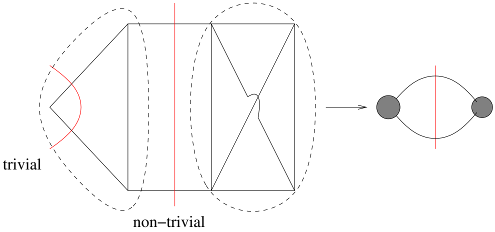

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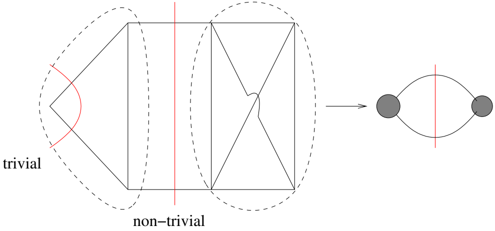

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Involving cut conductance

- The volume of vertex set $U \subseteq V$ is $\#$ edge end-points in U:

$$vol(U) = \sum_{v \in U} d(v).$$

- Recall $\partial U = E(U, V \setminus U)$.
- Conductance of cut around U is

$$\Phi(U) = \frac{|\partial U|}{\min\{\mathrm{vol}(U),\ 2m - \mathrm{vol}(U)\}} = \Phi(V \setminus U)$$

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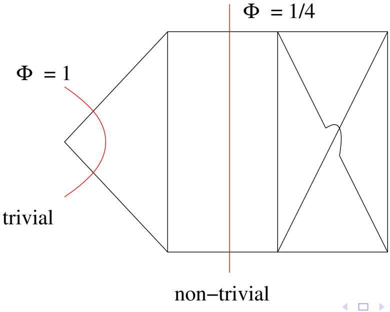

Non-trivial min-cuts have low-conductance

Obs Any non-trivial min-cut S has conductance $\leq 1/\delta$.

- $|\partial S| \geq |S|(\delta - (|S| - 1))$.
- $|\partial S| \leq \delta$ and $|S| > 1 \implies |S| \geq \delta$.
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We assume min-degree $\delta \geq \lg^6 n$; otherwise apply Gabow.

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(i) certify all min-cuts of G are trivial, or

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Both (i) and (ii) alone are difficult deterministically.

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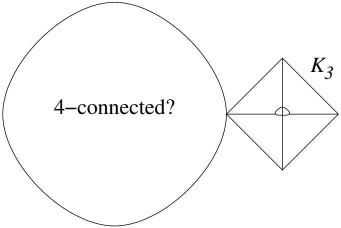

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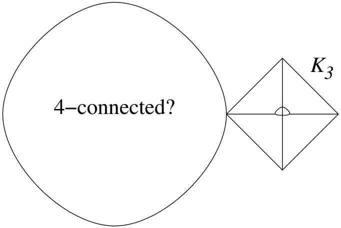

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Certify-or-cut(C, G) C subgraph of G with min-degree $\frac{2}{5}\delta$.

(i) certify no min-cut of G splits more than 2 vertices from C.

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Claim If C has been certified, we can contract a large "core" of C in G preserving all non-trivial cuts of G. No proof in this talk

- Set $H = G$.
- While some component C of H has not been certified.

- Contract cores of components C of H in G.

Lemma Most edges remain in components C of H:

- charge cut edges as $o(1/\log m)$ per small-side edge.
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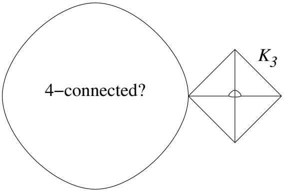

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teleportation constant $\alpha = 1/\lg^5 n$

slack $\varepsilon \in (0, 1)$.

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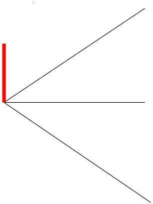

PageRank [Andersen, Chung, Lang FOCS'06]

PageRank($p^\circ, \alpha, \varepsilon$)

initial mass distribution $p^\circ : V \to \mathbb{R}_{\geq 0}$, $p^\circ(V) = \sum_{v \in V} p^\circ(v) = 1$

teleportation constant $\alpha = 1/\lg^5 n$

slack $\varepsilon \in (0, 1)$.

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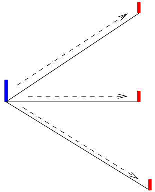

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But when can we promise finding low-conductance cut?

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Thm [ACL'06] If $S \subseteq V$ has $p^*(S) - \mathrm{vol}(S)/(2m) = \Omega(1)$ then PageRank finds T with conductance $\Phi(T) = o(1/\log m)$ with $\mathrm{vol}(T) = \widetilde{O}(\mathrm{vol}(S))$ in $\widetilde{O}(\mathrm{vol}(T))$ time.

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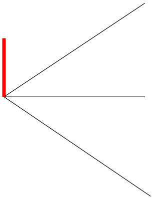

Lemma Starting from p°, with settled mass p, netflow over (u, v) was $(1 - \alpha)(p(u)/d(u) - p(v)/d(v))/\alpha$.

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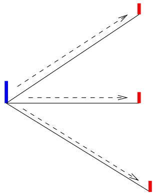

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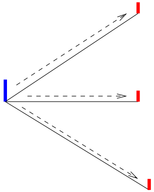

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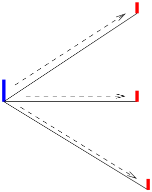

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Starting from any vertex on small side of min-cut

- We have min-degree $\delta \geq \lg^6 n$ and $\alpha = 1/\lg^5 n$.
- Let S with $\text{vol}(S) \leq m/2$ be small side of min-cut.
- For arbitrary $v \in S$, start with $p^\circ(v) = 1$ and push from v
- At least half mass stays in S.
- On every vertex u, residual mass $r(u) \leq 1/d(v) \leq 1/\delta$
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- Thus $1/2 - o(1)$ mass remains in S, so

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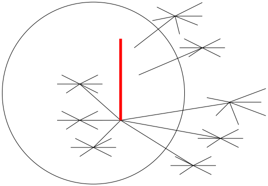

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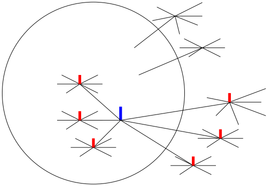

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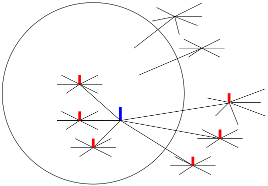

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Starting from any vertex on small side of min-cut

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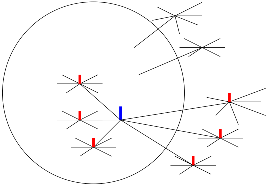

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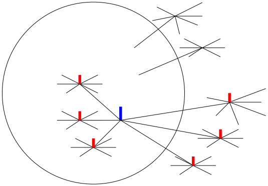

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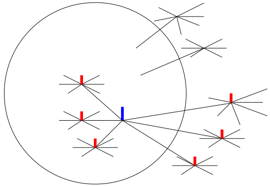

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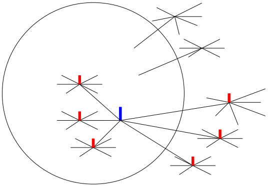

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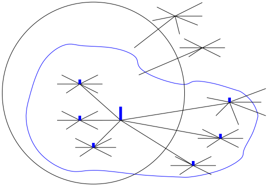

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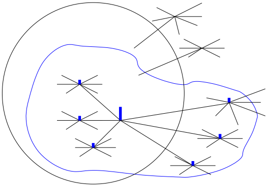

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Balanced min-cut

- Suppose min-cut side S with $m/2 \leq \text{vol}(S) \leq 3m/2$.
- < 16 vertices incident to $\geq \delta/8$ cut edges.
- Trying 16 vertices separately.
- One v has $7/8$ neighbors on same side.
- Pushing to limit from v, we get

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- For some $s \leq m/2$, know $\text{vol}(S) \leq s$ for any min-cut S.
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- Try $8m/(s\alpha)$ different v in $\widetilde{O}(m)$ time. None succeeds.
- Give each of them initial mass $s\alpha/(8m)$
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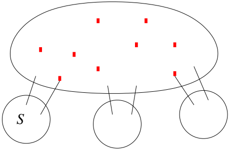

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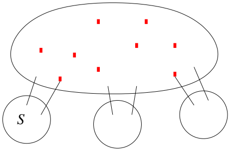

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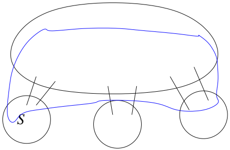

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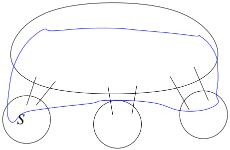

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Cores to contract in full algorithm

- C subgraph of G with min-degree $\frac{2}{5}\delta$.
- Certified: no min-cut of G splits > 2 vertices from C.
- Vertex $v \in C$ loose if $\leq d(v)/2 + 1$ neighbors in C.
- All other vertics of C in core.

Lemma Core of C can be contracted preserving all non-trivial cuts of G.

- Consider non-trivial min-cut (U, T) of G.
- If (U, T) cuts C, at most two verteces, v and w in $U \cap C$.
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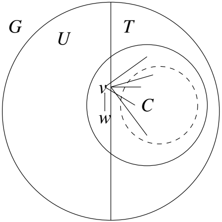

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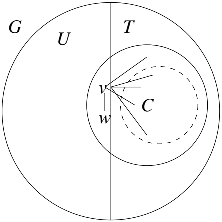

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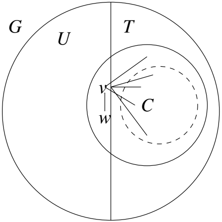

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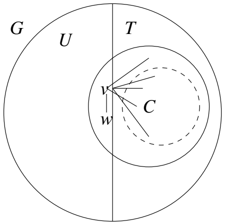

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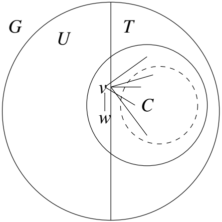

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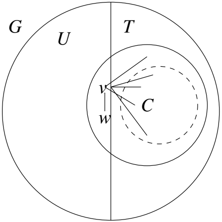

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