Informational Substitutes and Complements for Prediction

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Joint work with Bo Waggoner

Roadmap

- Information, prediction and prediction markets
- Substitutes and complements of signals
- Equilibria of prediction markets
- Future directions

Information (Bayesian View)

- Event of interest: E
- Signals: $A, B, C \ldots$
- Prior distribution: $\mathrm{P}(e, a, b, c \ldots)$

Replication outcome of a behavioral experiment

Outcome of some related event

Information and Decision

- Event of interest: E
- Signals: $A, B, C \ldots$
- Prior distribution: $\mathrm{P}(e, a, b, c \ldots)$
- A decision problem
  - Decision: $d \in \mathcal{D}$
  - Utility: $u(d, e)$
- Value of information [Börgers et al. `13]

$$\mathcal{V}(A) = \mathbb{E}_a \left[\max_d \mathbb{E}_e [u(d, e) | A = a] \right]$$

Replication outcome of a behavioral experiment

Outcome of some related event

Follow the recommendation of the original result

Information and Prediction

- Event of interest: E

  Replication outcome of a behavioral experiment

- Signals: $A, B, C \ldots$

  Outcome of some related event

- Prior distribution: $\mathrm{P}(e, a, b, c \ldots)$

- A prediction problem
  - Report: $r \in \Delta$

    Prob. distribution of *E*
  - Proper scoring rule: $S(r, e)$

    $S(r, e) = \log r_e$
- Value of information

$$V(A) = \mathbb{E}_a \mathbb{E}_{e \sim p_a} S(p_a, e) = \mathbb{E}_a G(p_a)$$

Prediction Markets

$1	if the study is replicated
$0	otherwise

Replicated or not r^0 r^1 r^2 r^3 r^4 . . .

- Market Scoring rules (MSR) [Hanson`03, `07]
  - Participant at time t receives $S(r^t, e) - S(r^{t-1}, e)$ when the event outcome is e.
  - Current report measures the population's collective belief
  - Implemented as a market maker offering contracts in practice
  - Many applications: Political events, economic events, entertainment, and business forecasts

Prediction markets correctly predicted the outcome of 71% replications of 41 psychology studies. [Drebera et al. `15]

Information Aggregation in Prediction Markets

- With strategic participants, how is information revealed and aggregated in prediction markets?

- Modeled as a Bayesian extensive-form game
  - Event of interest: E
  - Signals: $A, B, C \ldots$
  - Common prior distribution: $\mathrm{P}(e, a, b, c \ldots)$
  - Fixed order of participation
  - Either finite or infinite rounds

Information Aggregation in Prediction Markets

- [Ostrovsky `12] characterizes a condition under which information is fully aggregated in the limit (as time approaches infinity) in any PBE of any MSR.
- [Iyer, Johari, & Moallemi `14] extends the setting to risk-averse participants.
- [Chen et al. `07]: With conditionally independent signals, LMSR only has all-rush equilibria
- [Dimitrov and Sami `08]: With independent signals, LMSR can not have an all-rush equilibrium
- [Gao, Zhang, and Chen `13]: With independent signals, LMSR only has all-delay equilibria in a finite-round game

Substitutes and Complements

Substitutes

Complements

Strong connection between substitutes and existence of equilibria in markets for goods or matching markets

Can we define informational substitutes and complements that have similar impacts?

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Prior Definition [Börgers et al. `13]

$$\mathcal{V}(A) = \mathbb{E}_a \left[\max_d \mathbb{E}_e [u(d, e) | A = a] \right]$$

Two signals A and B are substitutes if for every decision problem

$$\mathcal{V}(A) + \mathcal{V}(B) \geq \mathcal{V}(A \vee B) + \mathcal{V}(\bot)$$

$E_1 = A_1 = B_1$ Substitutes

$E_2 = A_2 \oplus B_2$ Complements

- Doesn't depend on the decision problem
- Doesn't depend on the internal structure of the signals

$$E = (E_1, E_2), \quad A = (A_1, A_2), \quad B = (B_1, B_2)$$

Place a structure on signals

- Blackwell informativeness criterion [Blackwell`53]

$A' \preceq A$: A' is less informative than A if A' is a ``garbling" of A

Any randomized strategy given A is a ``garbling'' of A.

Substitutes of signals

$$\mathcal{V}(A) = \mathbb{E}_a \left[\max_d \mathbb{E}_e [u(d, e) | A = a] \right]$$

Signals A and B are substitutes in the context of a prior p and decision problem u if for all $A' \preceq A$

$$\mathcal{V}^{u,p}(A' \vee B) - \mathcal{V}^{u,p}(A') \geq \mathcal{V}^{u,p}(A \vee B) - \mathcal{V}^{u,p}(A)$$

and analogously for all $B' \preceq B$

"Diminishing returns" for substitutes.

Substitute of a set of signals

- A set of signals $A_1, \ldots, A_n$ are substitutes if the signals $A_i \vee C$ and $A_j \vee C$ are pairwise substitutes for any A_i, A_j and C, where C is the join of a subset of signals.

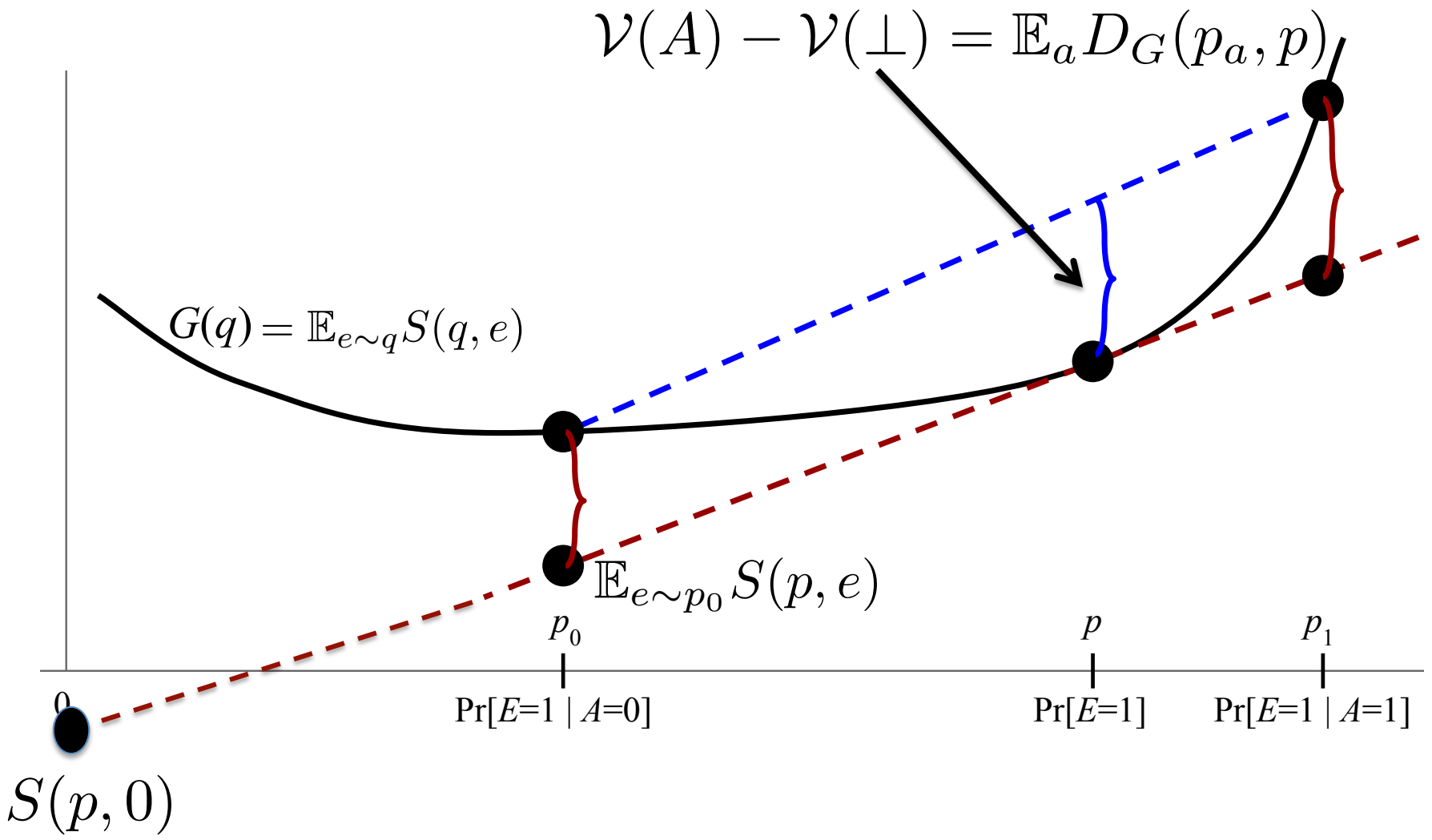
"Revelation Principle"

- For any decision problem u, there exists a proper scoring rule S such that for all prior p and signals A,

$$\mathcal{V}^{S,p}(A) = \mathcal{V}^{u,p}(A)$$

$S(r, e) = u(d_r^*, e)$ is a proper scoring rule

$$V(A) = \mathbb{E}_a \mathbb{E}_{e \sim p_a} S(p_a, e) = \mathbb{E}_a G(p_a)$$


$\mathcal{V}(A) - \mathcal{V}(\perp) = \mathbb{E}_a D_G(p_a, p)$
$G(q) = \mathbb{E}_{e\sim q} S(q, e)$
$\mathbb{E}_{e\sim p_0} S(p, e)$
p_0
p
p_1
0
Pr[E=1 | A=0]
Pr[E=1]
Pr[E=1 | A=1]
$S(p, 0)$

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Equilibria of Prediction Markets

- If signals are strict substitutes, then every BNE is all-rush.

- If signals are not substitutes, then there exists a trading order where some participant initially withholds information.

Equilibria of Prediction Markets

- If signals are strict complements, then every BNE is all-delay (for a finite-round game).

- If signals are not complements, then there exists a trading order where some participant initially reveals information.

Signal Classes

- Independent signals are complements in any decision problem where G has a jointly convex Bregman divergence $D_G(p, q)$.
  - Independent signals are complements for both log and quadratic scoring rules

- Conditionally independent signals are substitutes for the log scoring rule, but not the quadratic scoring rule.

Other Comments

- Substitutes/complements of signals connect to submodular/supermodular set functions (over signals)
  - Algorithmic results for a combinatorial signal selection problem

- Substitutes/complements of signals also have an information theoretic interpretation

Ongoing and Future Directions

- Design market scoring rules to make given signals substitutes
- Characterize signal (and decision problem) classes
- Connection to substitutes/complements of goods