# ADVERSE SELECTION AND AUCTION DESIGN FOR INTERNET DISPLAY ADVERTISING

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# Old Advertisers & New

**"Half the money I spend on advertising is wasted; the trouble is, I don't know which half."**

**- John Wanamaker, Advertising pioneer**

# Old-Fashioned “Brand” Ads



Customer Service Digital Newspaper el Nuevo Herald Classifieds Jobs Cars Real Estate Find & Save Deals Public Notices Place an Ad

Miami Herald

MIAMI HERALD HOME DELIVERY JUST 53¢ A DAY!

FREE SHIPPING DEALS Shop with us today and enjoy great deals everyday Store Locations BrandsMart U.S.A.

Home News Sports Business Lifestyle Entertainment Opinion Obituaries Subscriptions 83°

HIGHLIGHTS

DEADLINE MIAMI

The link to news happening now

- 2 teens in crash, good Samaritan killed after another car plows into them
- Man accused of kicking feces-covered dog in face
- Police-involved shooting at nightclub near airport

Miami Heat

Wade returns to lineup, but Miami Heat loses to Hawks

- Le Batard: Family plan is a huge success for Pat Riley, Heat

Columns

Grimm: Zombie Apocalypse bill

- Hiaasen: First, do no harm — to your bank account
- Pitts: About that missing plane: Give it a rest!

PASSOVER

Jews come together to retell the story of Passover

As Jews retell an ancient story of freedom and share an traditional meal on Monday and Tuesday nights, many have taken to heart a growing Passover tradition in South Florida: Everyone should have a place to go. - 8:56 AM ET

- Davie citrus family preserves Passover traditions
- Cooking for Passover gets creative at local synagogues

THE EVERGLADES

Fishing heals war vets' unseen wounds

A veteran and an angler's fishing trips serve as a form of therapy from the stresses of military combat and coming home. - 8:56 AM ET

- Fishing as healing

BREAKING NEWS

- Stabbing suspect at nightclub shot by Miami-Dade officer - Updated 6 minutes ago
- Bus hits truck on Mexico highway; 36 reported dead - Updated 8 minutes ago
- 9/11 defendant's competency is focus of Guantánamo hearings - Updated 28 minutes ago
- Summer teasers, zany honors at MTV Movie Awards - Updated 28 minutes ago
- 2 teens in crash, good Samaritan killed when car hits them - 11:11 AM ET

» More

COMMUNITY NEWS

Select your community

KEY BISCAYNE

Key Biscayne to spend $80,000 on traffic study under no-bid contract

PINECREST

Pinecrest may limit ‘McMansions’

HIALEAH

Homeowners in Hialeah neighborhoods live on crumbling streets

SCHOOLS

In Miami, an ‘F’ school strives to make the grade

South MOTORS OIL & WASH starting from only $24.95*

*Conventional oil. Factory filter. Outside wash and vacuum. Pricing varies by model.

MIAMI'S NEW CENTER Brickell Heights

SOULCYCLE EQUINOX RELATED

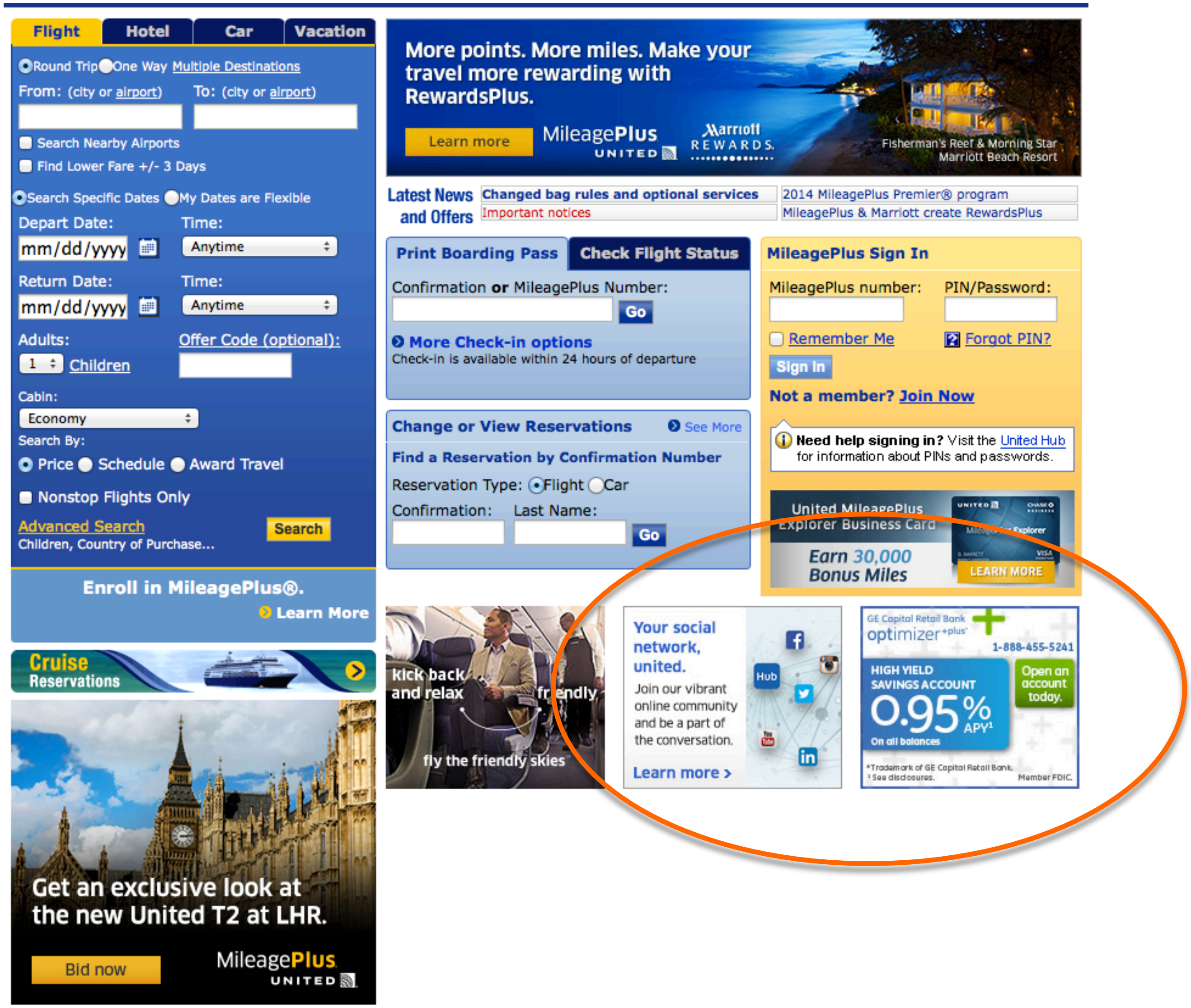
BRICKELLHEIGHTS.COM 786.623.5719

Get the Deal!

Boat rental (seats up to 6) --

# New-Fashioned "Performance" Ads





# Display Advertisement Types



## Brand Ads

- Goal: awareness and image
  - Reach and repetition.
- Common Characteristics
  - Targeted to a large group
  - Large number of Impressions
  - Guaranteed delivery
- Sample Advertisers
  - Ford (weekend auto sale)
  - Disney (movie openings)
  - Shopping Center (location)

## Performance Ads

- Goal: measurable action now
  - Click, fill form, or buy.
- Common Characteristics
  - Targeted to an individual (based on cookies)
  - Smaller number of impressions
  - Bought one by one
- Sample Advertisers
  - Hertz (car rental)
  - Amazon (re-targeting)
  - Quicken mortgage (refinance)

# Danger of Adverse Selection



| Brand Advertisers | Performance Advertisers |
| --- | --- |
| May select impressions en masse ("road block" ads) | Mostly use private cookies to select impressions |
| Receive deferred, aggregated data about performance of the whole ad campaign | Receive immediate, detailed data about the performance of individual ads |
| Cannot easily distinguish low-performing ads and publishers | Can quickly identify low-performing ads and publishers |

*If the value of ad impressions is positively correlated for both types of advertisers, then brand advertisers may suffer adverse selection.*

# Matching with Adverse Selection

…and "Not-Quite-Optimal" Market Design

# Model



- There are $N + 1$ advertisers, with $N \geq 2$
- The value of an impression to advertiser *i* is $X_i = CM_i$
- $C$ is the (random) ***common value factor*** and
  - $M_i$ is the (random) ***match value factor*** for bidder *i*

- **Key Assumptions**
  1. Advertiser 0 (the "brand advertiser') does not observe $X_0$
  2. Performance advertisers $n = 1, \dots, N$ observe their values $X_n$ Define $X = (X_1, \dots, X_n)$.
  3. The common value factor $C$ is statistically independent of $M \stackrel{\text{def}}{=} (M_0, \dots, M_N)$

# A Market Design Challenge



- Compare the restricted-worst-case performance of different mechanisms on efficiency grounds
- The mechanisms considered are:

1. A benchmark: "Omniscient" mechanism with C observed
2. "Optimal" (expected-efficiency maximizing) mechanisms
3. Second-price auction
4. "Modified second-bid auction"
   in which the highest performance bidder wins if the ratio of the highest to second-highest performance bid exceeds a threshold.

# The Omniscient Benchmark

OMN, in which the auctioneer observes both the bids and C

# OMN Benchmark



- If the auctioneer could separately gather *perfect information* about the common factor C and decide the allocation accordingly (no incentive constraints), it could achieve this value:

$$V(OMN) = E[\max(C \cdot E[M_0], X_1, \ldots, X_n)\,]$$

- Performance of other mechanisms will be compared to $V(OMN)$.

# Bayesian Optimal Mechanism

OPT …and its drawbacks

# Optimal Mechanism Formulation



- $z_i(X)$ is probability that $i$ wins, given $X$
- $p_i(X)$ is $i$'s expected payment, given $X$

- Efficiency Objective
  - Goal is to maximize $E[\sum_{i=0}^{n} X_i z_i(X)]$
    - subject to dominant-strategy incentive constraints and participation constraints
  - Let OPT be the mechanism that does that

# OPT in an Example



- Assume that $M_1, \ldots, M_n$ are IID and that…

$$P\{C = 1\} = P\{C = 2\} = \frac{1}{2}$$

$$P\{M_n = 1\} = P\{M_n = 2\} = P\{M_n = 4\} = \frac{1}{3}$$

$$3 < E[M_0] < 4$$

- So, optimally, only a performance advertiser $n$ with $M_n = 4$ ought to be assigned this impression.

# Example Solved



- The expected-efficiency-maximizing assignment with $N = 2$ is:
  - If $X_{(1)} \in \{1,2\}$, then $M_{(1)} \leq 2 < E[M_0] \Rightarrow$ brand advertiser
  - If $X_{(1)} = 8$, then $M_{(1)} = 4 > E[M_0] \Rightarrow$ top performance advertiser
  - If $X_{(1)} = 4$, assignment hinges on whether $E[M_{(1)}|X_{(1)}, X_{(2)}] \gtrless E[M_0]$.
    - If $X_{(2)} = 1$, then $M_{(1)} = 4 \Rightarrow$ top performance advertiser
    - If $X_{(2)} = 2$, then $\mathrm{E}\left[\mathrm{M}_{(1)} \middle| X_{(1)}, X_{(2)}\right] = 3 < E[M_0] \Rightarrow$ brand advertiser
      - In this case, $\Pr\left\{C = 1, M_{(1)} = 4, M_{(2)} = 2 \middle| X_{(1)}, X_{(2)}\right\} = \Pr\left\{C = 2, M_{(1)} = 2, M_{(2)} = 1 \middle| X_{(1)}, X_{(2)}\right\} = \frac{1}{2}$.
    - If $X_{(2)} = 4$, then $\mathrm{E}\left[\mathrm{M}_{(1)} \middle| X_{(1)}, X_{(2)}\right] = 3 < E[M_0], \Rightarrow$ brand advertiser
      - In this case, $\Pr\left\{C = 1, M_{(1)} = M_{(2)} = 4 \middle| X_{(1)}, X_{(2)}\right\} = \Pr\{C = 2, M_{(1)} = M_{(2)} = 2 | X_{(1)}, X_{(2)}\} = \frac{1}{2}$.

# Main Concerns about OPT



- The example highlights three concerns about OPT
  1. *Sensitivity:* OPT is sensitive to detailed distributional assumptions.
  2. *False-name bidding*: Performance advertiser $n$ with value $X_n = 4$ can only benefit by submitting a *additional*, false-name bid of $X_{\hat{n}} = 1$ (because that leads the auctioneer to infer that $M_n = 4$.)
  3. *Adverse selection*: The brand advertiser wins $4/9$ of high-value impressions, but $7/9$ of low-value ones.
     - Most problematic if the brand advertiser feels uninformed about the impressions and who else may be bidding.

# MSB Characterization

Modified Second Bid auction characterized by its properties

# Properties for Characterization



- A mechanism is
  - *anonymous among performance advertisers* if...
  - *strategy-proof* if…
  - *fully strategy-proof* if, in addition, it is both
    - *bidder false-name proof*: no bidder can benefit by submitting multiple bids, and
    - *publisher false-name proof*: the seller cannot benefit by submitting "low" bids (below all performance bids)
  - *adverse-selection free* if for every joint distribution on $(C, M)$ such that $C$ and $M$ are independent, $z_0(X)$ is statistically independent of $C$.

# Characterization Theorem



- **Definition.** A direct mechanism is a *modified second bid auction* if for some $\alpha \geq 1$,
  - If $\frac{X_{(1)}}{X_{(2)}} > \alpha$, then the highest performance advertiser wins & pays $\alpha X_{(2)}$.
  - If $\frac{X_{(1)}}{X_{(2)}} \leq \alpha$, then the brand advertiser wins (and pays its contract price).

- **Theorem.** A deterministic mechanism $(z, p)$ is anonymous, fully strategy-proof, and adverse selection free *if and only if* it MSB.

# Comparing $MSB_{\alpha}$ and $SP_{r}$ to OMN

$MSB_{\alpha}$ : modified second-bid auction

$SP_{r}$ : second-price auction with reserve

# Assumptions for Comparison



- Evaluate MSB$_\alpha$ and SP$_r$ mechanisms in worst case over a limited family of environments, in which…
  - $M_1, \ldots, M_N$ are IID from a distribution $F$.
  - $C$ is drawn from distribution $G$.
  - $N \geq 2$ and $E[M_0] \geq 0$ are arbitrary.

# Efficiency Performance



- **Theorem.** (Comparing $SP_r$ and $MSB_\alpha$ to $OMN$)

1. Assuming Nash equilibrium bidding by the brand advertiser, both MSB and SP have similar worst case performance:

$$\inf_{F,G,N\geq 2,E[M_0]\geq 0} \max_{\alpha} \frac{V(MSB_\alpha)}{V(OMN)} = \frac{1}{2}$$

$$\inf_{F,G,N\geq 2,E[M_0]\geq 0} \max_{r} \frac{V(SP_r)}{V(OMN)} = \frac{1}{2}$$

2. Further restricting $F$ and/or $G$ to be drawn from power law distributions $\mathcal{P}$,

$$\inf_{F\in\mathcal{P},G\in\mathcal{P},N\geq 2,E[M_0]\geq 0} \max_{r} \frac{V(SP_r)}{V(OMN)} = \frac{1}{2}$$

$$\inf_{F\in\mathcal{P},G,N\geq 2,E[M_0]\geq 0} \max_{\alpha} \frac{V(MSB_\alpha)}{V(OMN)} \approx 0.948$$

# Revenue Performance



- **Theorem.** Fix a number of bidders *N* and assume that the publisher shares in the rents from brand advertising in any fixed proportions, say $(\delta, 1-\delta)$.
- If $F$ is a power law distribution, then there is some $\alpha$ such that $MSB_\alpha$ achieves at least 94.8% of the expected revenue from the corresponding expected-revenue-maximizing strategy-proof auction $REVMAX$.

# Conclusion



- Adverse selection can be neutralized, even without encouraging false-name bidding, provided that $X_n = CM_n$ and $C$ and $M$ are independent.

- The cost of doing that, even without observing the common value factor $C$, is low provided that the tails of the distribution are fat (power law).

- For real applications, we need to evaluate…
  - Is adverse selection important?
  - Are values independent?
  - Are match-value distributions fat-tailed?

# End