Subexponential parameterized complexity of completion problems
Survey of the upper bounds

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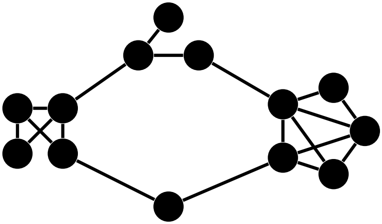
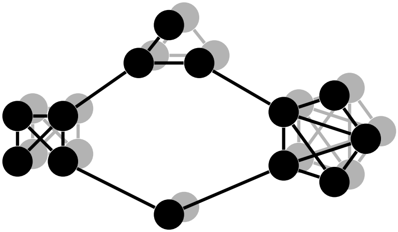
4th Nov 2015

Graph modification problems



- An input graph: a result of an experiment.

Graph modification problems



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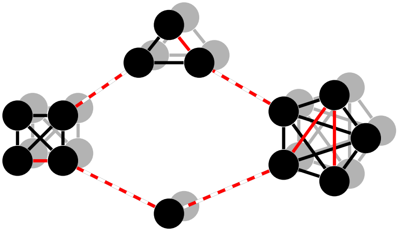

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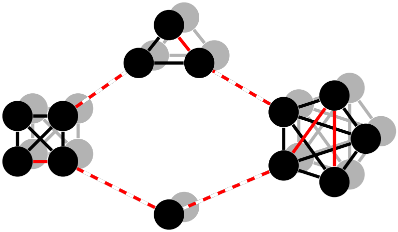
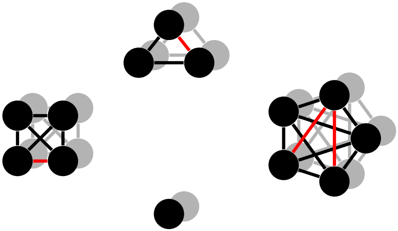


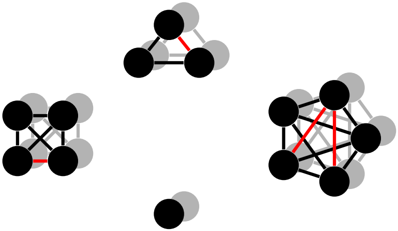
- An input graph: a result of an experiment.
- Theory: result should have some property.
- Modification: error in our measurments.
- A few errors ⇒ #modifications as a parameter.

Completions



completion = only edge insertions are allowed

Completions



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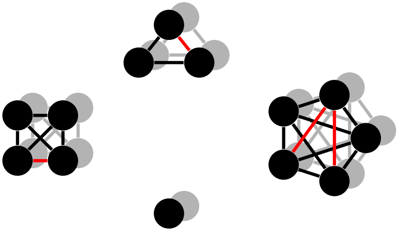
$\mathcal{G}$ Completion
obtain a graph $\in \mathcal{G}$

Completions



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$\mathcal{F}$-COMPLETION
kill all induced subgraphs $\in \mathcal{F}$

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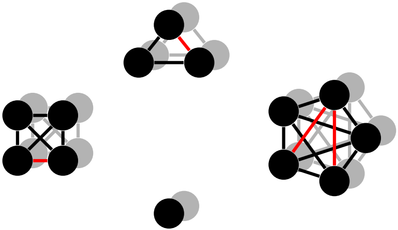

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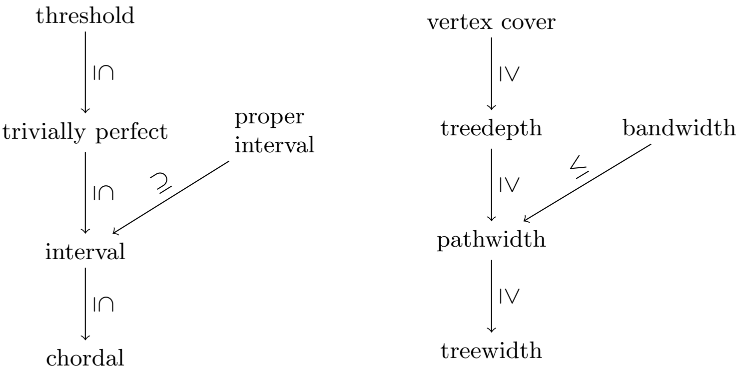
obtain a graph $\in \mathcal{G}$

$\mathcal{F}$-Completion
kill all induced subgraphs $\in \mathcal{F}$

Side note: no known P vs NP dichotomy for completion problems.

Interesting classes for completion



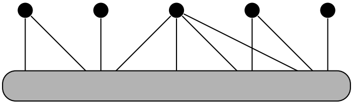
$$\mathrm{measure}(G) \simeq \min\{\omega(H) : H \in \mathcal{G} \text{ and } H \text{ is a completion of } G\}.$$

Disclaimer

In this talk we mostly focus on $f(k)$
in the FPT running time $f(k)n^{\mathcal{O}(1)}$.

We denote $\mathcal{O}^*(f(k)) = f(k)n^{\mathcal{O}(1)}$.

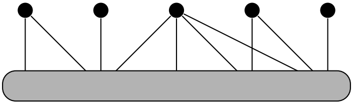
Finite set $\mathcal{F}$



Split Completion

$\{2K_2, C_4, C_5\}$-Completion

Finite set $\mathcal{F}$



SPLIT COMPLETION

$\{2K_2, C_4, C_5\}$-COMPLETION

Theorem (Cai, IPL'96)

A simple branching strategy $\Rightarrow \mathcal{O}^*(c^k)$ *FPT algorithm*

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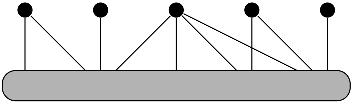

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Works also for:
CO-CLUSTER, COGRAPH, THRESHOLD, PSEUDOSPLIT, ...

Chordal completion

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$\{C_4, C_5, C_6, \ldots\}$-COMPLETION

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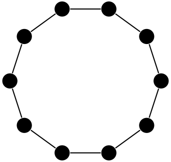

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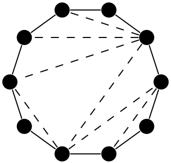
CHORDAL COMPLETION $\{C_4, C_5, C_6, \ldots\}$-COMPLETION



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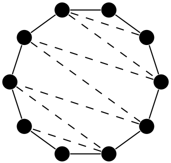

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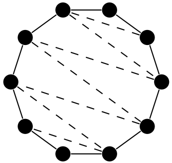

Theorem (Kaplan, Shamir, Tarjan, SICOMP'99)

Large hole
$\Rightarrow$ *many options but big cost (Catalan number $C_{\ell-2}$ for $\ell - 3$ edges)*
$\Rightarrow$ $\mathcal{O}^*(4^k)$ *branching.*

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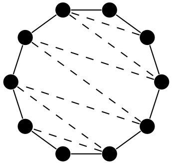

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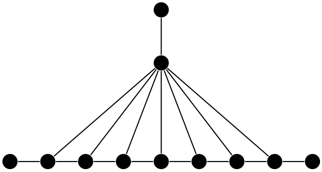
Gives also $\mathcal{O}^*(c^k)$ FPT algorithm for Proper Interval Completion, as Chordal $\longrightarrow$ Proper Interval means killing $\{S_3, \mathrm{claw}, \mathrm{net}\}$.

Interval completion

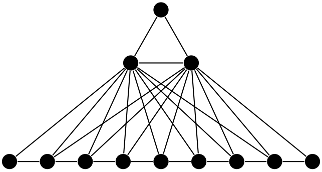
INTERVAL COMPLETION $\{\text{holes}, \text{ATs}\}$-COMPLETION

Interval completion

Interval Completion



{holes, ATs}-Completion



Interval completion

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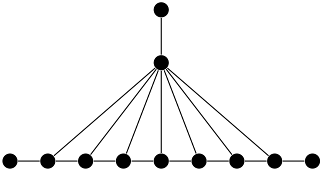

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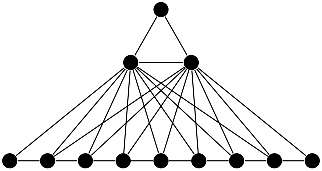

Long-standing open problem for more than a decade.

Interval completion

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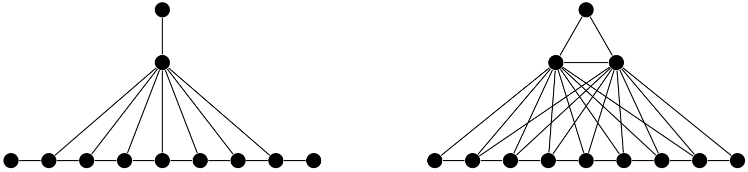

Long-standing open problem for more than a decade.

Theorem (Villanger, Heggernes, Paul, Telle, SICOMP'09)

Branching still doable! An $\mathcal{O}^*(k^{2k})$ FPT algorithm.

Theorem (Cao, SODA'16)

Can be solved in $\mathcal{O}(c^k(n+m))$ time.

Polynomial kernels

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Theorem (Kratsch, Wahlström, IWPEC'09)

There is a finite $\mathcal{F}$ *such that* $\mathcal{F}$-COMPLETION *does not admit a polynomial kernel unless* $\mathrm{NP} \subseteq \mathrm{coNP/poly}$.

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Theorem (Guillemot, Havet, Paul, Perez, Algorithmica'13)

COGRAPH COMPLETION *admits a polynomial kernel.*

Theorem (Bessy, Perez, Inf. Comp.'13)

PROPER INTERVAL COMPLETION *admits a polynomial kernel.*

Look at the bright side!

$\mathcal{G}$ COMPLETION

$\mathcal{F}$-COMPLETION

Look at the bright side!

$\mathcal{G}$ Completion $\mathcal{F}$-Completion

Fomin, Villanger, SODA'12:
think positively!

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Theorem (Fomin, Villanger, SODA'12)

Chordal Completion *can be solved in time* $\mathcal{O}^*(k^{\mathcal{O}(\sqrt{k})})$.

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Build the structure of $(G + \mathrm{completion})$ by dynamic programming.

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Key observation: there are $2^{o(k)}$ reasonable 'partial structures', and hence $2^{o(k)}$ reasonable DP states.

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3. Candidate structure $\longrightarrow$ DP state.

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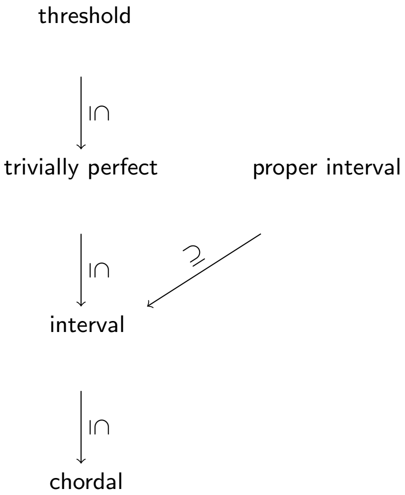


Look at the bright side!

threshold
$\mathcal{O}^*\left(k^{\mathcal{O}(\sqrt{k})}\right)$

$\downarrow \subseteq$

trivially perfect
$\mathcal{O}^*\left(k^{\mathcal{O}(\sqrt{k})}\right)$

proper interval
$\mathcal{O}^*\left(k^{\mathcal{O}(k^{2/3})}\right)$

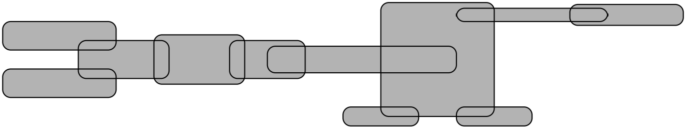
$\downarrow \subseteq$

interval
$\mathcal{O}^*\left(k^{\mathcal{O}(\sqrt{k})}\right)$

$\downarrow \subseteq$

chordal
$\mathcal{O}^*\left(k^{\mathcal{O}(\sqrt{k})}\right)$

Chordal completion



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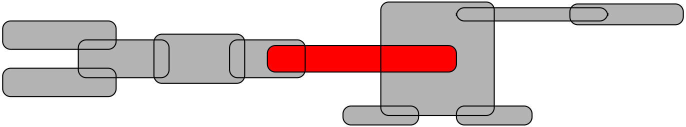
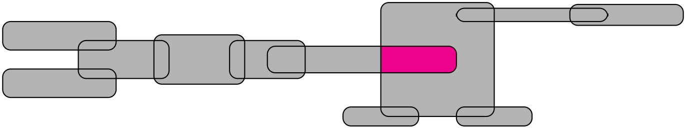


crucial structure = maximal clique

Chordal completion



crucial structure = maximal clique / minimal clique separator

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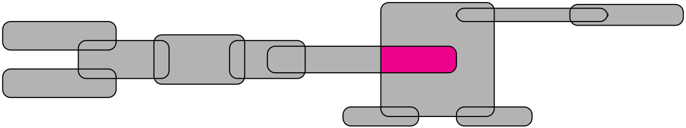

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One can either enumerate $k^{\mathcal{O}(\sqrt{k})}$ candidate maximal cliques and candidate clique separators, or perform a $k^{\mathcal{O}(\sqrt{k})}$ branching.

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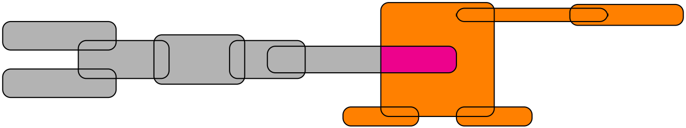

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DP state = clique separator Ω +

one connected component C of $G \setminus \Omega$,

value = minimum completion of $G[C \cup \Omega]$ that cliquifies Ω.

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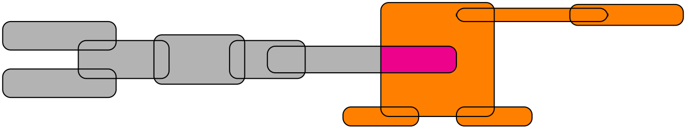

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value = minimum completion of $G[C \cup \Omega]$ that cliquifies Ω.

Corollary: Chain Completion in $\mathcal{O}^*(k^{\mathcal{O}(\sqrt{k})})$ time.

Enumerating potential maximal cliques

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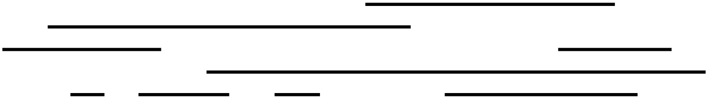
Theorem (Bliznets, Fomin, P., Pilipczuk, 2014)

There are $n^{\mathcal{O}(\sqrt{k})}$ reasonable candidates for maximal cliques in the Interval Completion *case.*

Maximal cliques in Interval Completion



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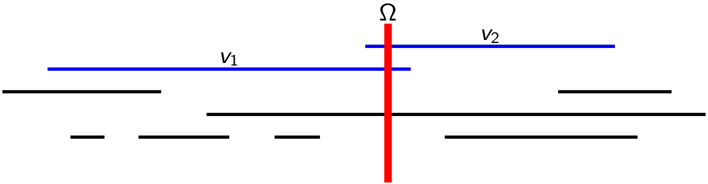

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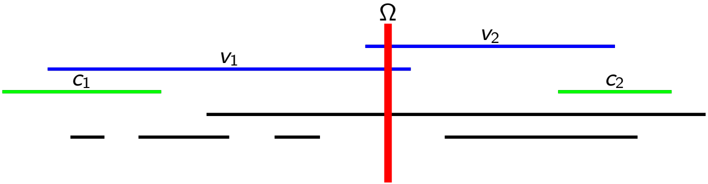
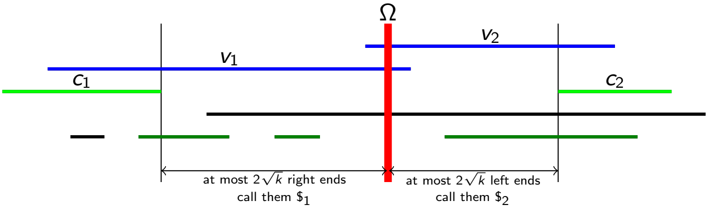


cheap v = at most $\sqrt{k}$ incident solution edges.
c_1 := last ending cheap before Ω
c_2 := first starting cheap after Ω

Maximal cliques in Interval Completion



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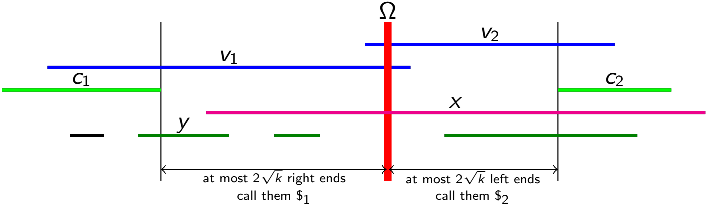

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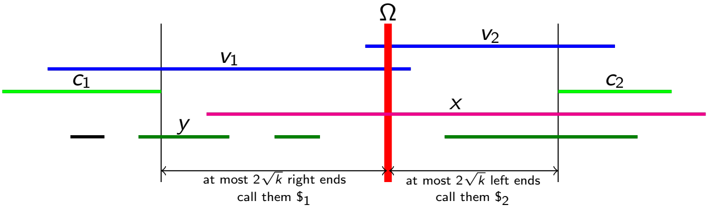
Why $x \in \Omega$?

x has left end before Ω because some $y \in N_G(x)$ has right end before Ω.

Maximal cliques in Interval Completion



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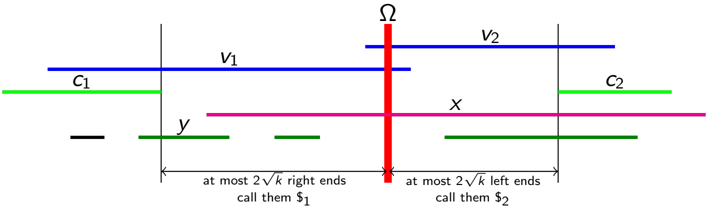
$y \in N_G(x)$ for some y with right end before $\Omega \Rightarrow$
$x \in N_G(v_1) \cup N_G(\$_1) \cup N_{G+F}(c_1)$.

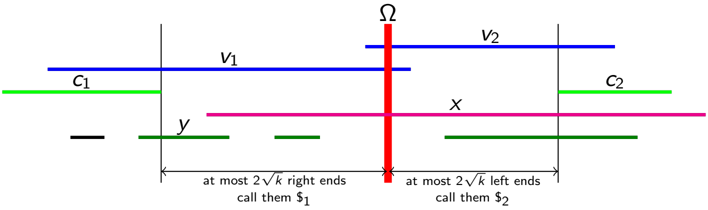
Maximal cliques in Interval Completion



Lemma

$$\Omega = (N_G[v_1] \cup N_G(\$_1) \cup N_{G+F}(c_1)) \cap (N_G[v_2] \cup N_G(\$_2) \cup N_{G+F}(c_2)).$$

Maximal cliques in Interval Completion



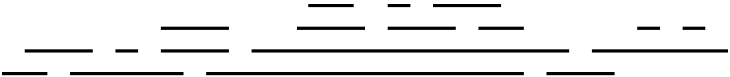
Lemma

$$\Omega = (N_G[v_1] \cup N_G(\$_1) \cup N_{G+F}(c_1)) \cap (N_G[v_2] \cup N_G(\$_2) \cup N_{G+F}(c_2)).$$

There are $n^{\mathcal{O}(\sqrt{k})}$ choices for:

- v_1, v_2, c_1, c_2;
- $\$_1$ and $\$_1$, as there are of size $\mathcal{O}(\sqrt{k})$;
- solution edges incident to c_1 and c_2, as both c_1 and c_2 are cheap.

Interval Completion



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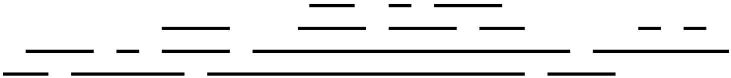

Problem 1: no known polynomial kernel!

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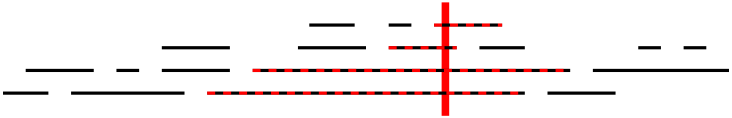

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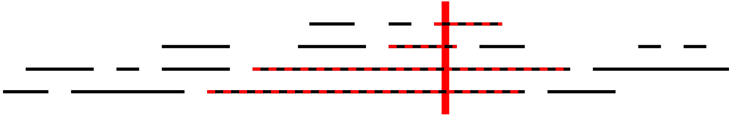

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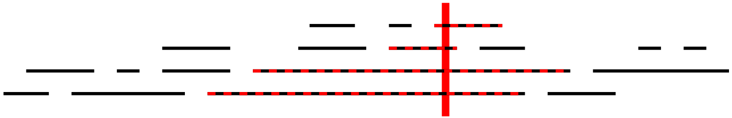

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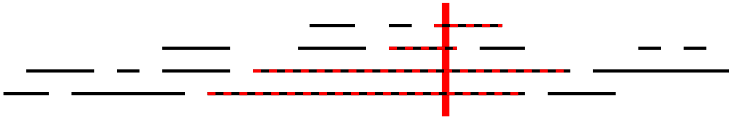

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Theorem (Bliznets, Fomin, P., Pilipczuk, 2014)

There are $k^{\mathcal{O}(\sqrt{k})}n^8$ reasonable candidates for maximal cliques*.

* or we can reduce something.

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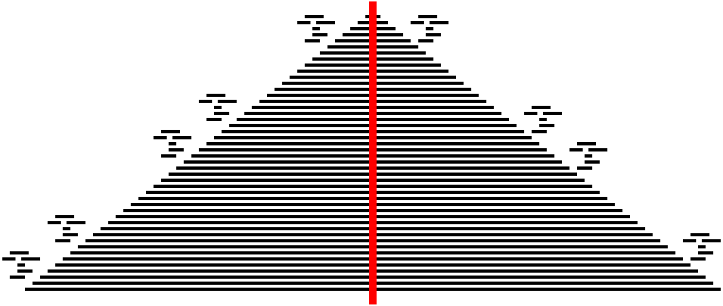

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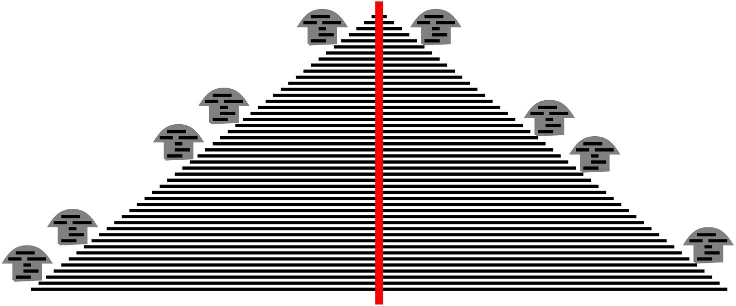

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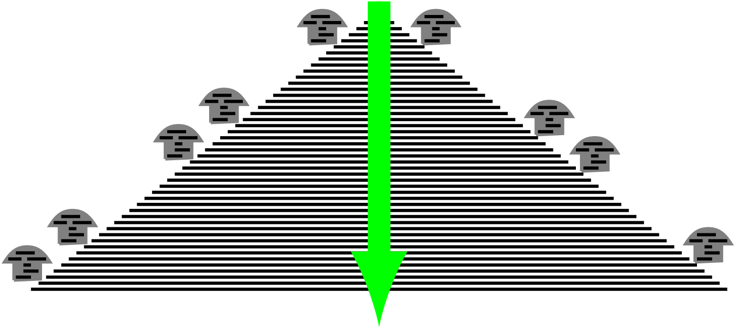

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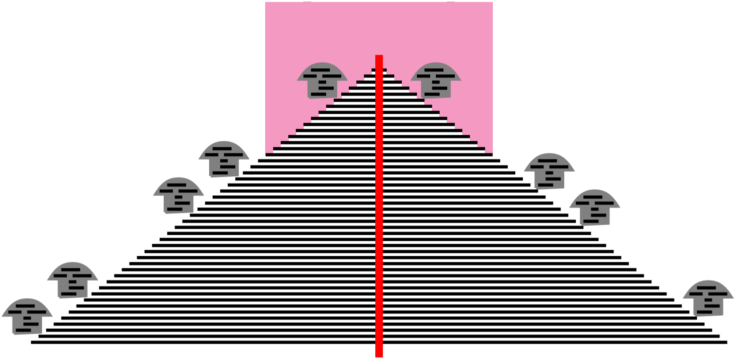

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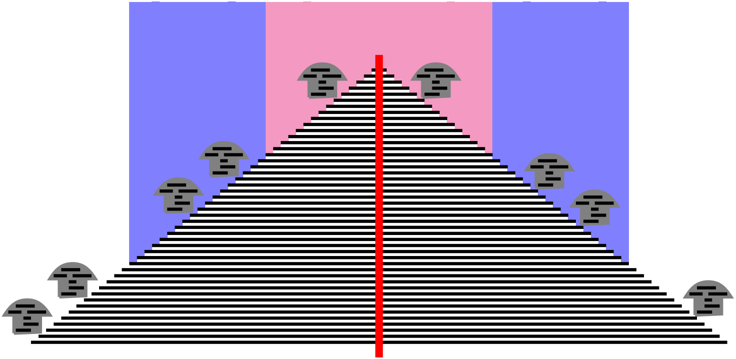

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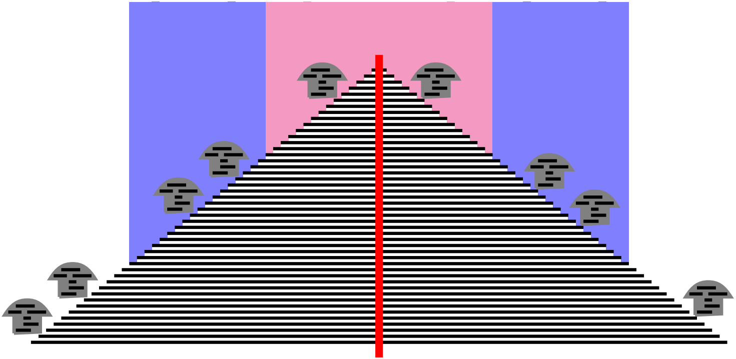

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Solution: make much more complicated DP states.

More completions

Theorem (Ghosh, Kolay, Kumar, Misra, Panolan, Rai, Ramanujan, SWAT'12)

An $\mathcal{O}^*(k^{\mathcal{O}(\sqrt{k})})$ *algorithm for* Split Completion *via chromatic coding.*

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Theorem (Drange, Fomin, Pilipczuk, Villanger, STACS'14)

$\mathcal{O}^*(k^{\mathcal{O}(\sqrt{k})})$ *algorithms for:*

1. TRIVIALLY PERFECT COMPLETION,
2. THRESHOLD COMPLETION
   via chromatic coding + reduction to CHAIN COMPLETION,
3. PSEUDOSPLIT COMPLETION
   similarly as SPLIT COMPLETION.

Summary

CHORDAL	$\mathcal{O}^*(k^{\mathcal{O}(\sqrt{k})})$	[FV, SODA'12]
CHAIN	$\mathcal{O}^*(k^{\mathcal{O}(\sqrt{k})})$	[FV, SODA'12]
SPLIT	$\mathcal{O}^*(k^{\mathcal{O}(\sqrt{k})})$	[GKKMPRR, SWAT'12]
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CO-CLUSTER	ETH-hard	[KU, DAM'12]
COGRAPH	ETH-hard	[DMPV, STACS'14]
CO-TRIVIALLY PERFECT	ETH-hard	[DMPV, STACS'14]

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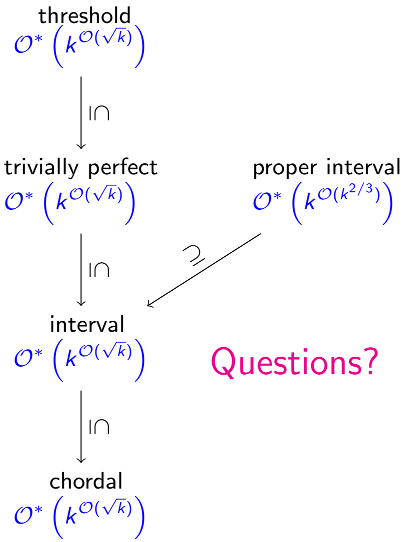
  - An $\mathcal{O}^*(2^{o(\sqrt{k})})$-time algorithm seems hard, as it gives $2^{o(n)}$ bound.
- Is there any meta-explanation why there are subexponential algorithms for this family of problems?

Summary diagram



Questions?