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How Fair is Your Protocol?

A Utility-based Approach to Protocol Optimality

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[PODC 2015]

Two Coin-Toss Protocols

Protocol A

1. Each party commits to a bit.
2. Both parties open their commitments.
3. The result is the XOR.

Protocol B

Two Coin-Toss Protocols

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Two Coin-Toss Protocols

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Protocol B

1. Each party commits to a bit.
2. They toss a coin $i^* \in \{1,2\}$.

Two Coin-Toss Protocols

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2. They toss a coin $i^* \in \{1,2\}$.
3. $p_{(3-i^*)}$ opens its commitment to p_{i^*}

Two Coin-Toss Protocols

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4. p_{i^*} opens its commitment to $p_{(3-i^*)}$

Two Coin-Toss Protocols

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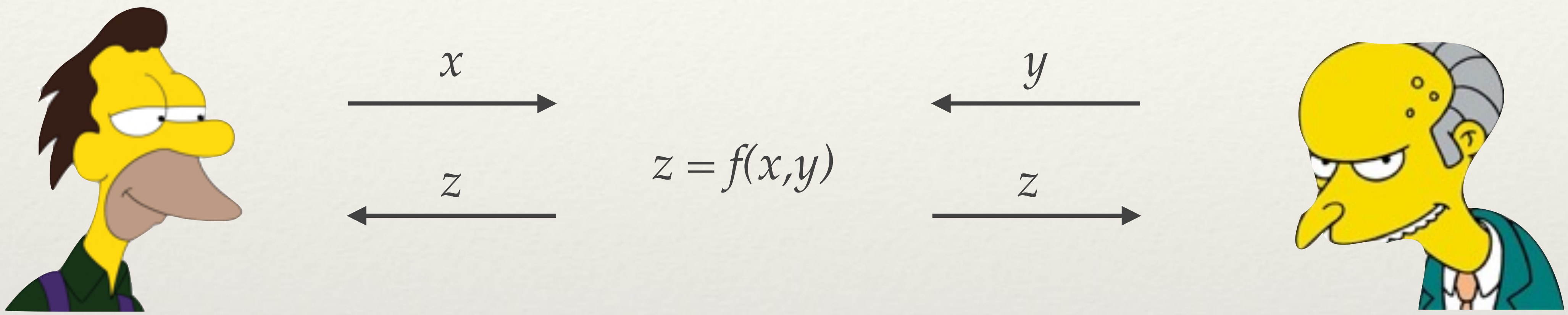
3. $p_{(3-i^*)}$ opens its commitment to p_{i^*}
4. p_{i^*} opens its commitment to $p_{(3-i^*)}$
5. The result is the XOR.

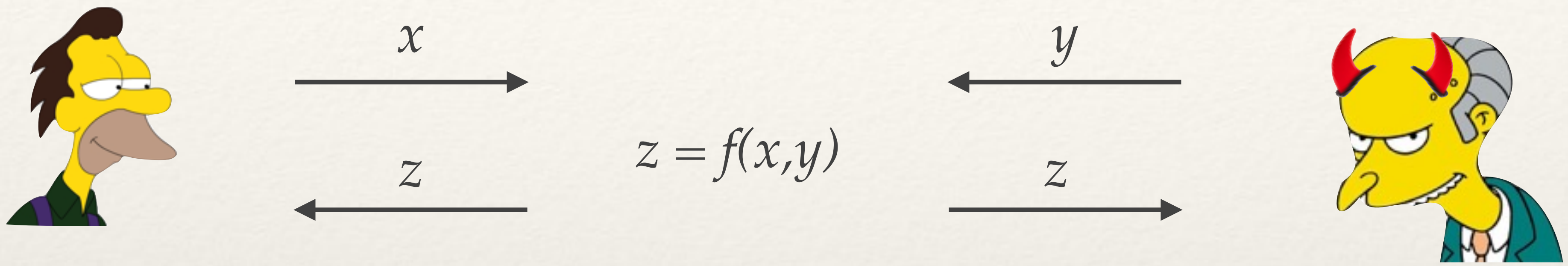
Fairness in SFE



Fairness:

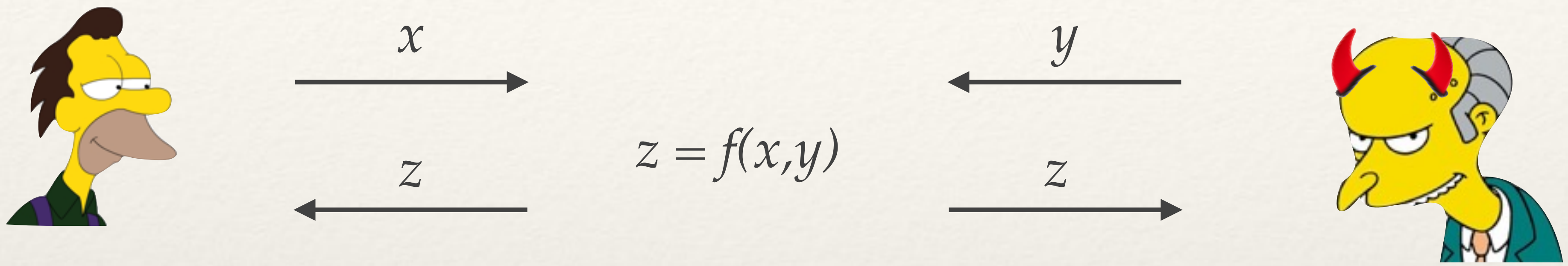
- "if one party learns the output, the other party also learns it,"
- generally impossible in 2PC [Cleve, STOC'86].

Fairness in SFE



	does not get z		does not get z
	does not get z		gets z
	gets z		does not get z
	gets z		gets z

Fairness in SFE



Possible outcomes (intuitively):

	does not get z		does not get z
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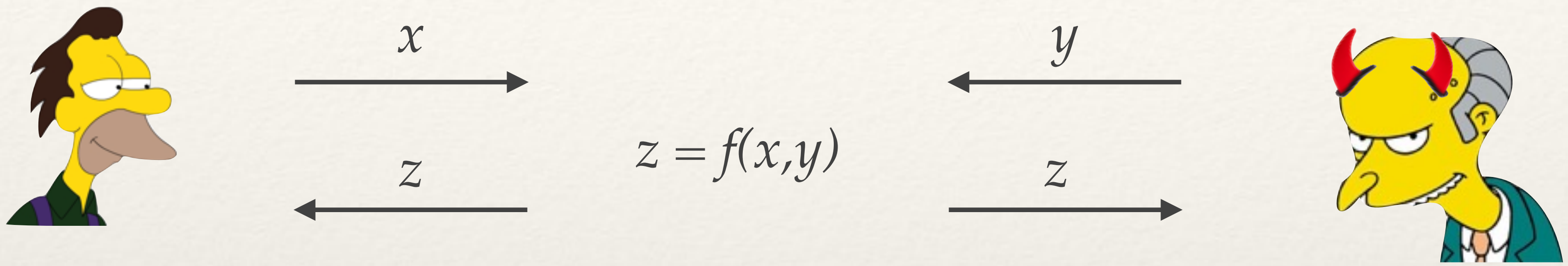
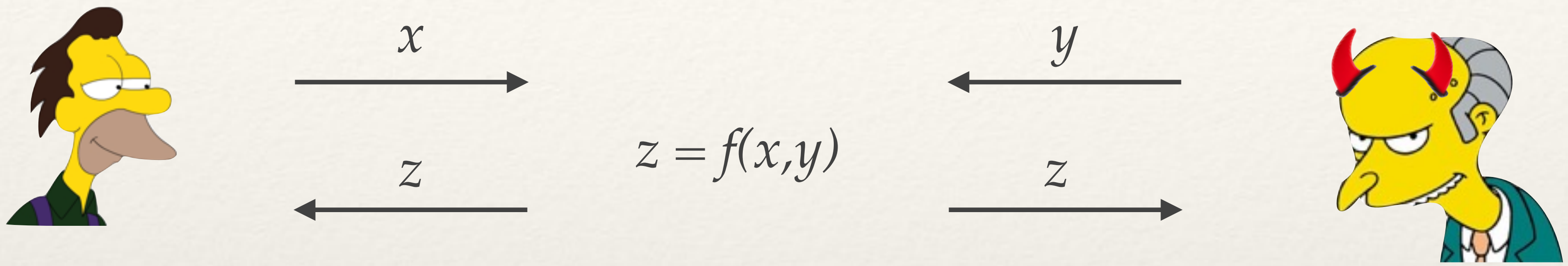
	gets z		does not get z
	gets z		gets z

Fairness in SFE



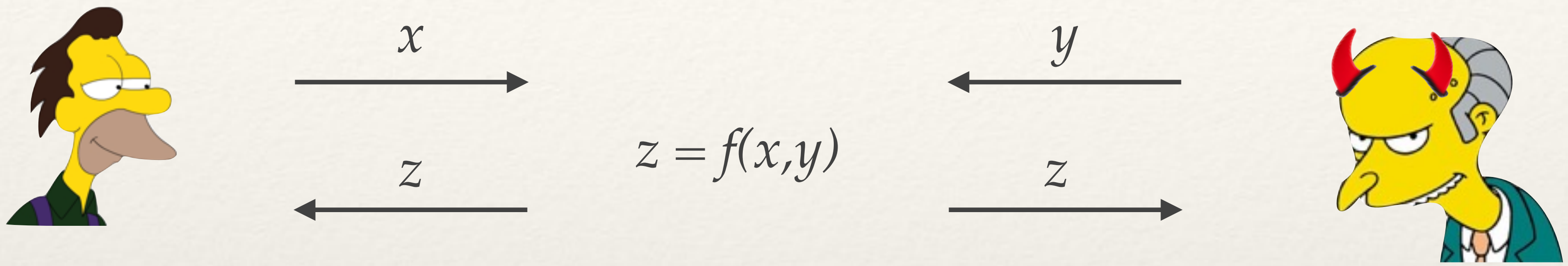
Possible outcomes (intuitively):				"Fairness"
	does not get z		does not get z	good
	does not get z		gets z	bad
	gets z		does not get z	good
	gets z		gets z	good

Fairness in SFE



Possible outcomes (intuitively):				"Fairness"	Utility
	does not get z		does not get z	good	γ_{00}
	does not get z		gets z	bad	γ_{10}
	gets z		does not get z	good	γ_{01}
	gets z		gets z	good	γ_{11}

Fairness in SFE



Possible outcomes (intuitively):				"Fairness"	Utility
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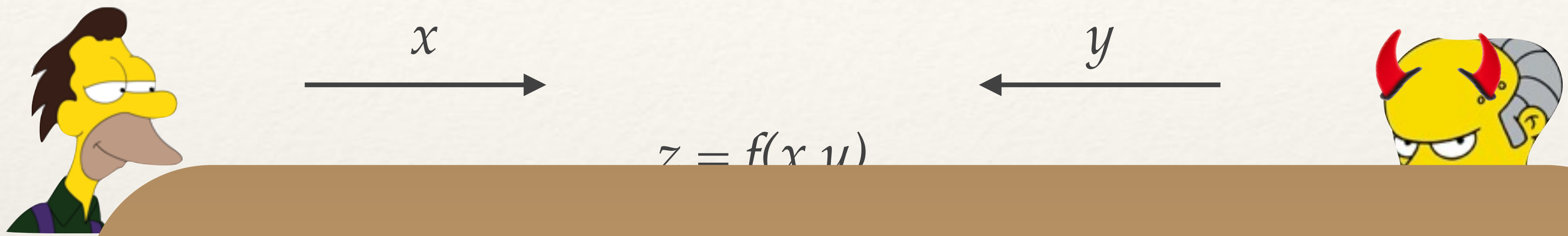
	does not get z		gets z	bad	γ_{10}
	gets z		does not get z	good	γ_{01}
	gets z		gets z	good	γ_{11}

Natural conditions: $\gamma_{01} < \gamma_{00}, \gamma_{11}$ and $\gamma_{00}, \gamma_{11} < \gamma_{10}$

Fairness in SFE



Protocol comparison and optimality:

- the utilities for the individual outcomes define an expected payoff for each adversarial strategy,
- a protocol is *better* (fairer) if the expected payoff of the *best* adversarial strategy is smaller.

	Utility
	γ_{00}
	γ_{10}
	γ_{01}
	γ_{11}

Natural conditions: $\gamma_{01} < \gamma_{00}, \gamma_{11}$ and $\gamma_{00}, \gamma_{11} < \gamma_{10}$

Other Relaxed Notions of Fairness

- "Gradual Release"-type approaches [Goldwasser-Levin, 1990; Garay-MacKenzie-Prabhakaran-Yang, 2005; …]
- Rational fairness [Asharov-Canetti-Hazay, 2011]
- *1/p*-Security [Gordon-Katz, 2010; …]

Rational Protocol Design

Protocol π →

← Adversary strategy for π

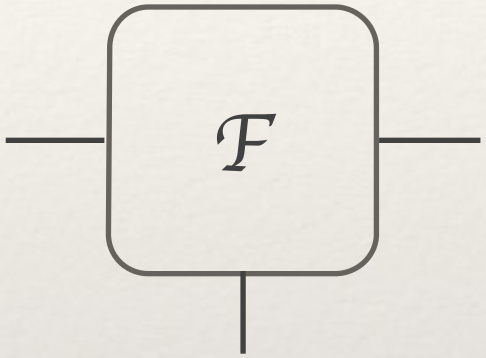
Protocol Designer

Attacker

- Two-move "meta" game,
- zero-sum: $u_D = -u_A$,
- ε-subgame-perfect equilibrium.

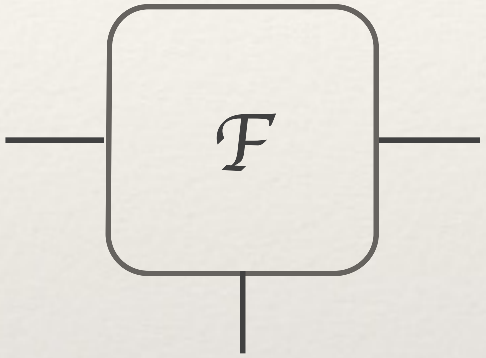
[Garay-Katz-Maurer-T-Zikas, FOCS 2013]

Rational Protocol Design



Rational Protocol Design

Step 1: Relax functionality



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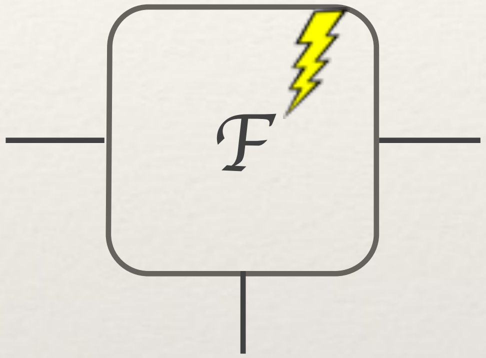
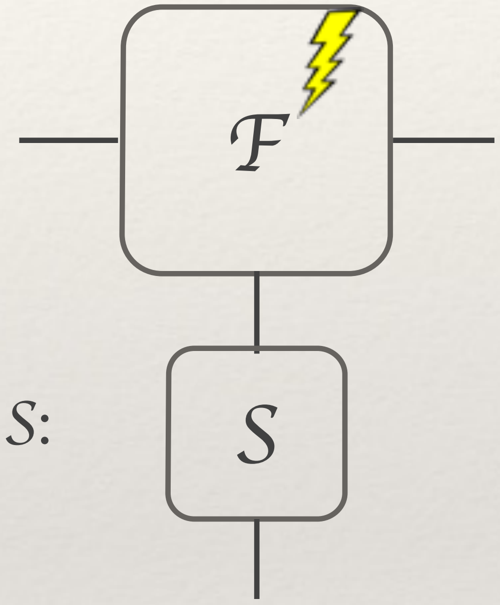


Rational Protocol Design

Step 1: Relax functionality

s.t. ∃ $\mathcal{S}$:

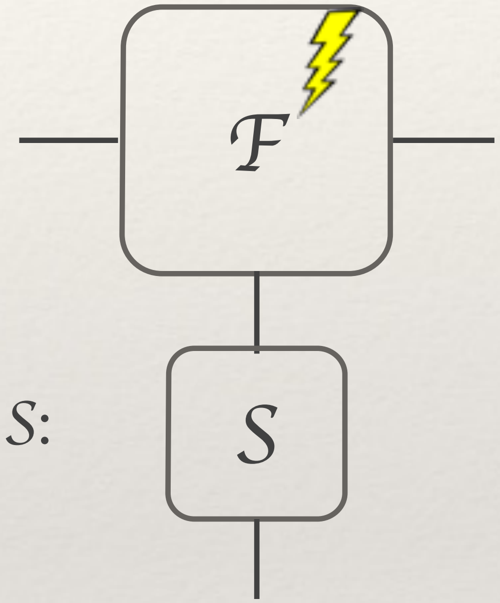


Rational Protocol Design

Step 1: Relax functionality

Step 2: Define events in *ideal*

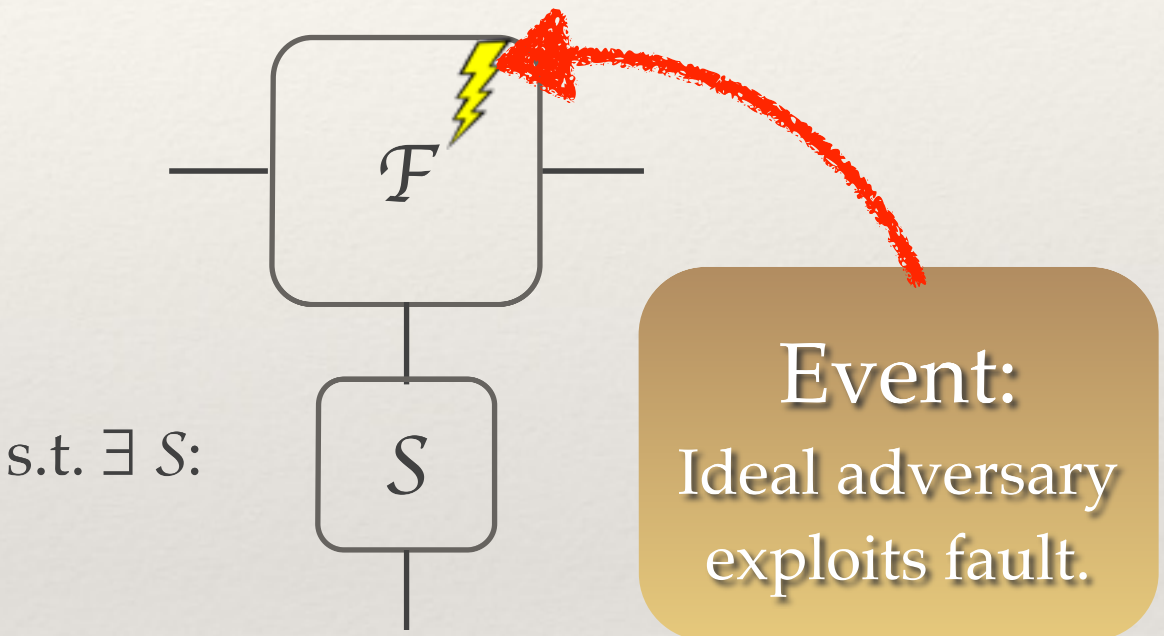
s.t. $\exists\ \mathcal{S}$:



Rational Protocol Design

Step 1: Relax functionality

Step 2: Define events in *ideal*

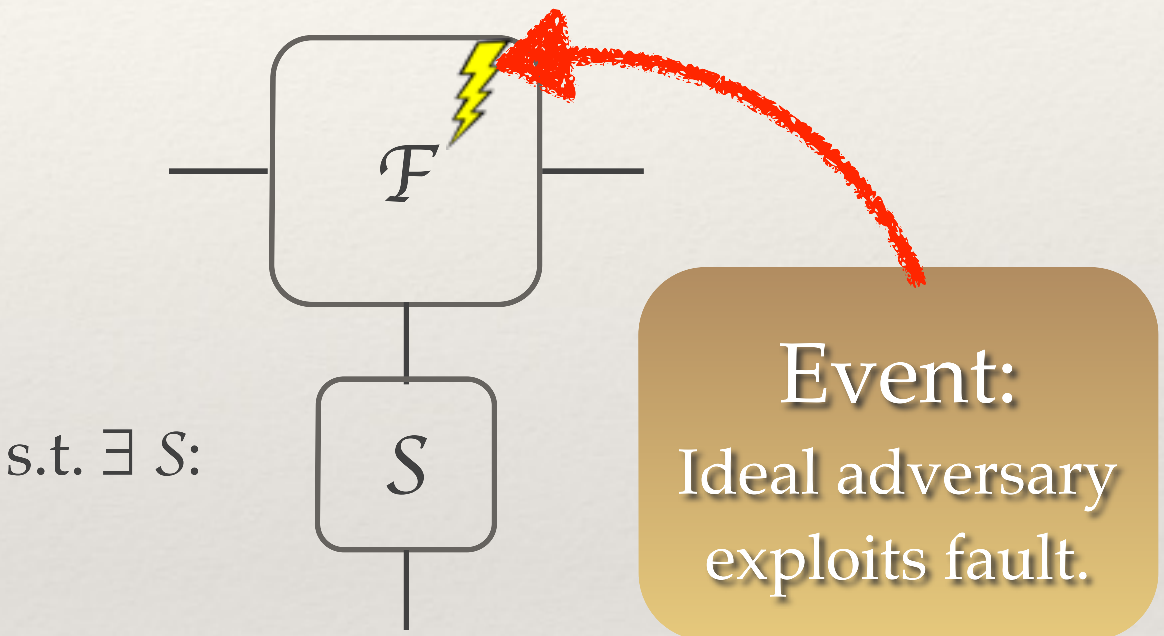


Rational Protocol Design

Step 1: Relax functionality

Step 2: Define events in *ideal*

Step 3: Define payoff



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Step 1: Relax functionality

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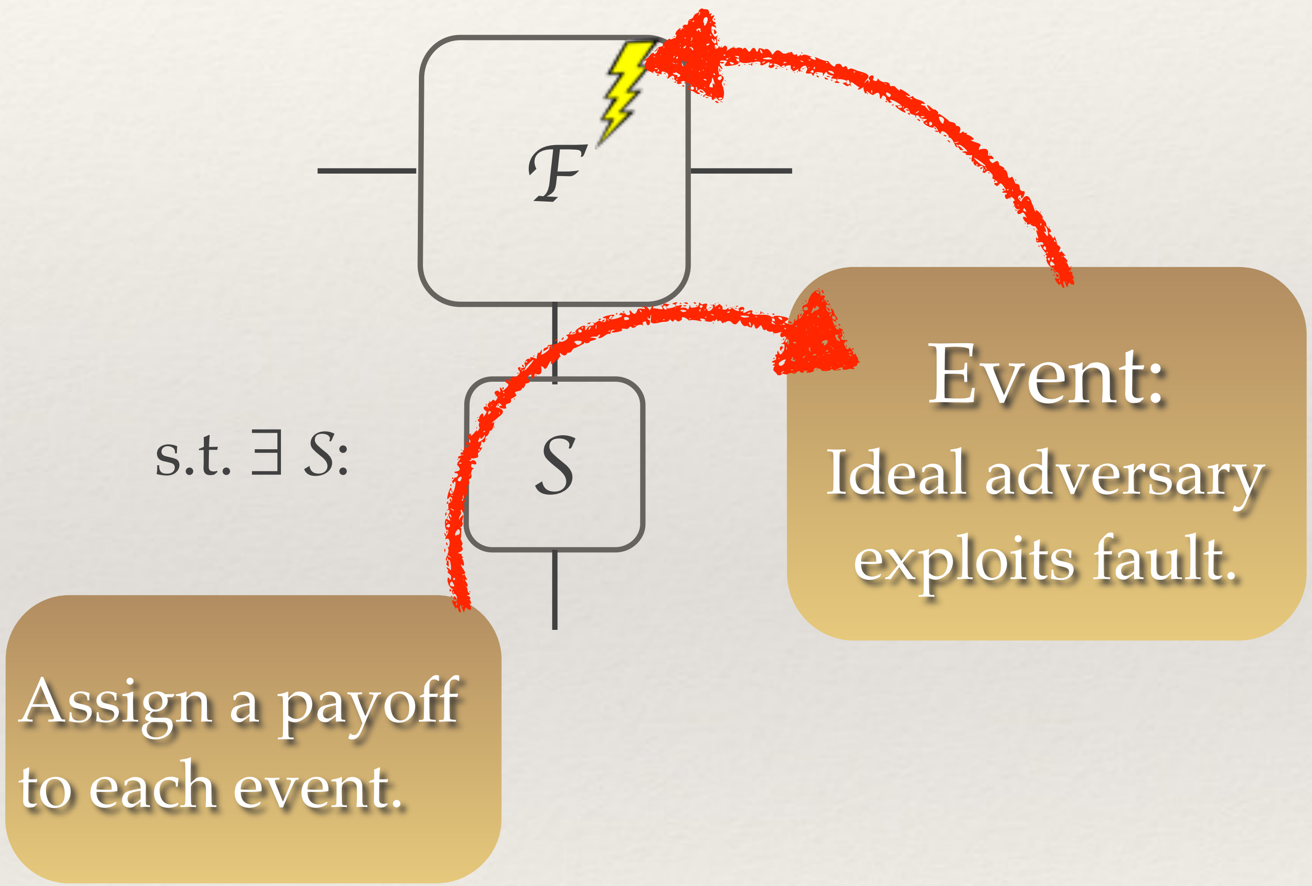
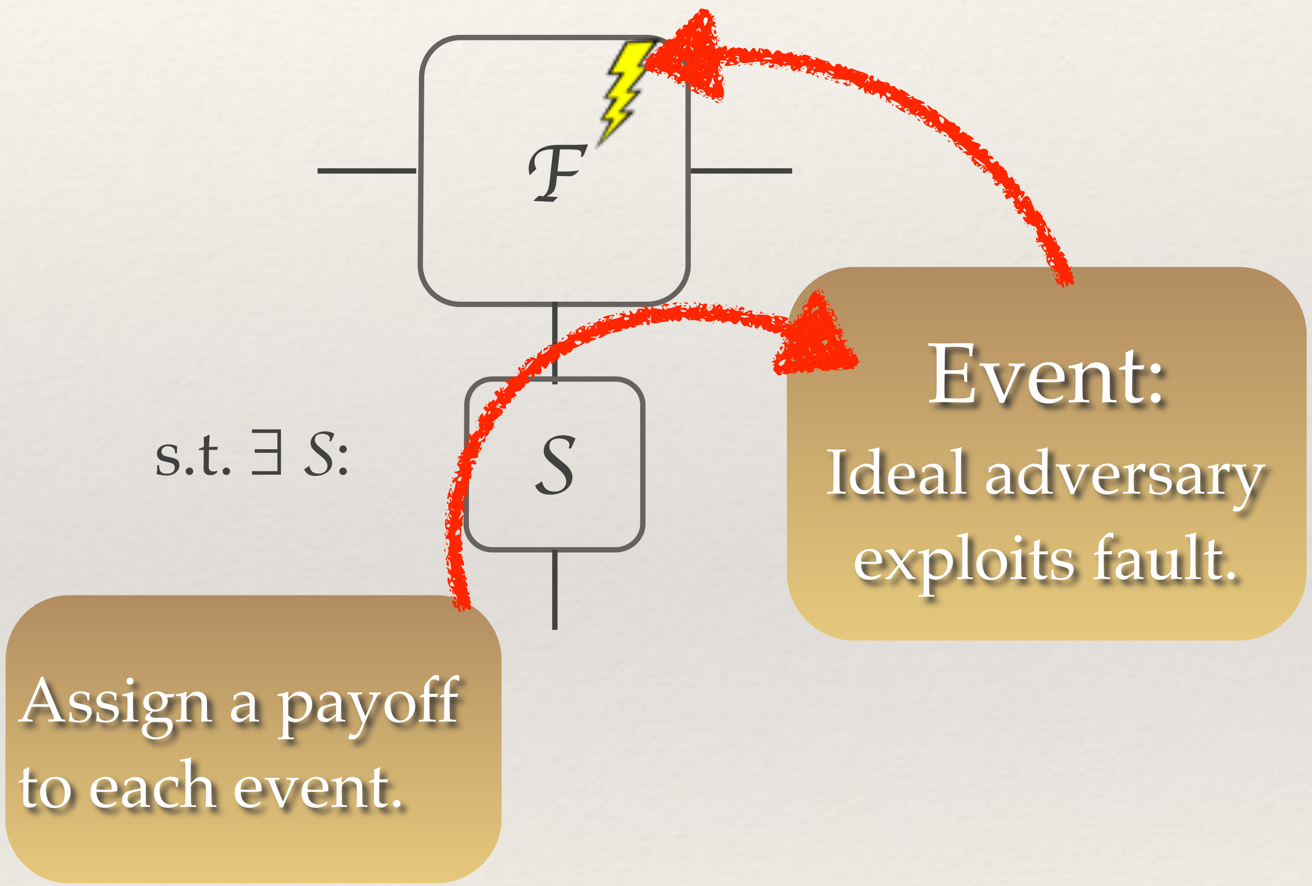


Rational Protocol Design

Step 1: Relax functionality

Step 2: Define events in *ideal*

Step 3: Define payoff



$$\text{Payoff}(\mathcal{A}) = \min_{\text{"good" } \mathcal{S}} \text{payoff}(\mathcal{S})$$

Defining Fairness (1)

x →

← z

$\mathcal{F}_{\text{fair-sfe}}$

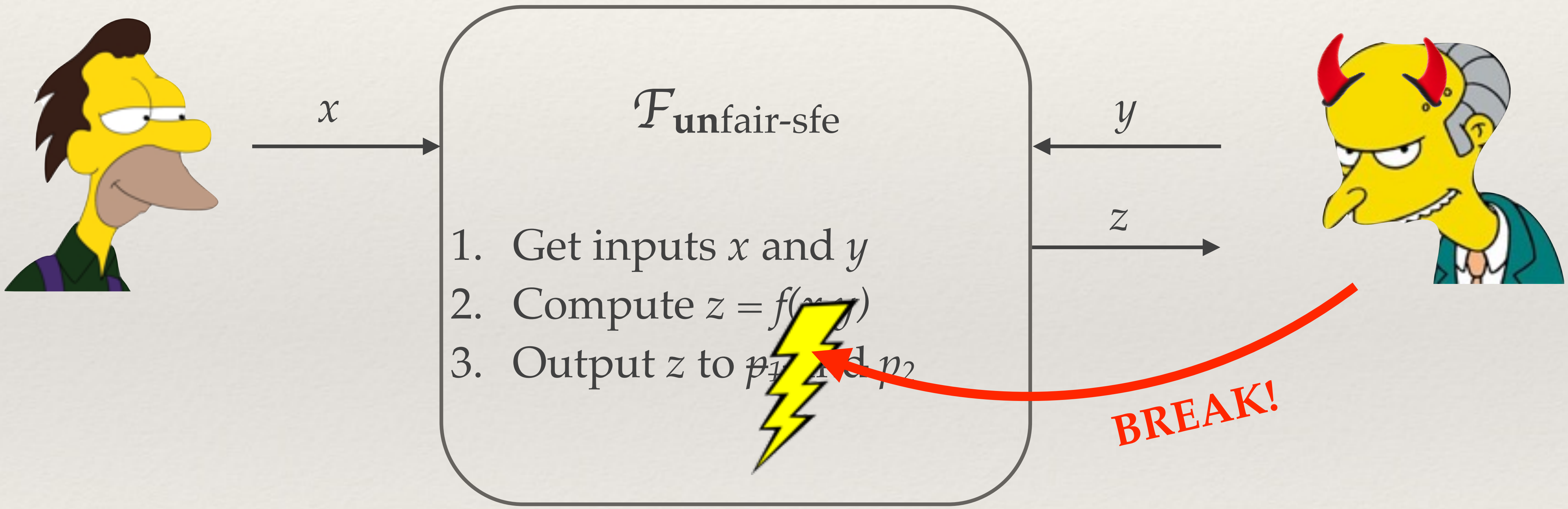
1. Get inputs x and y
2. Compute $z = f(x,y)$
3. Possibly: Output z to p_1 and p_2

← y

z →

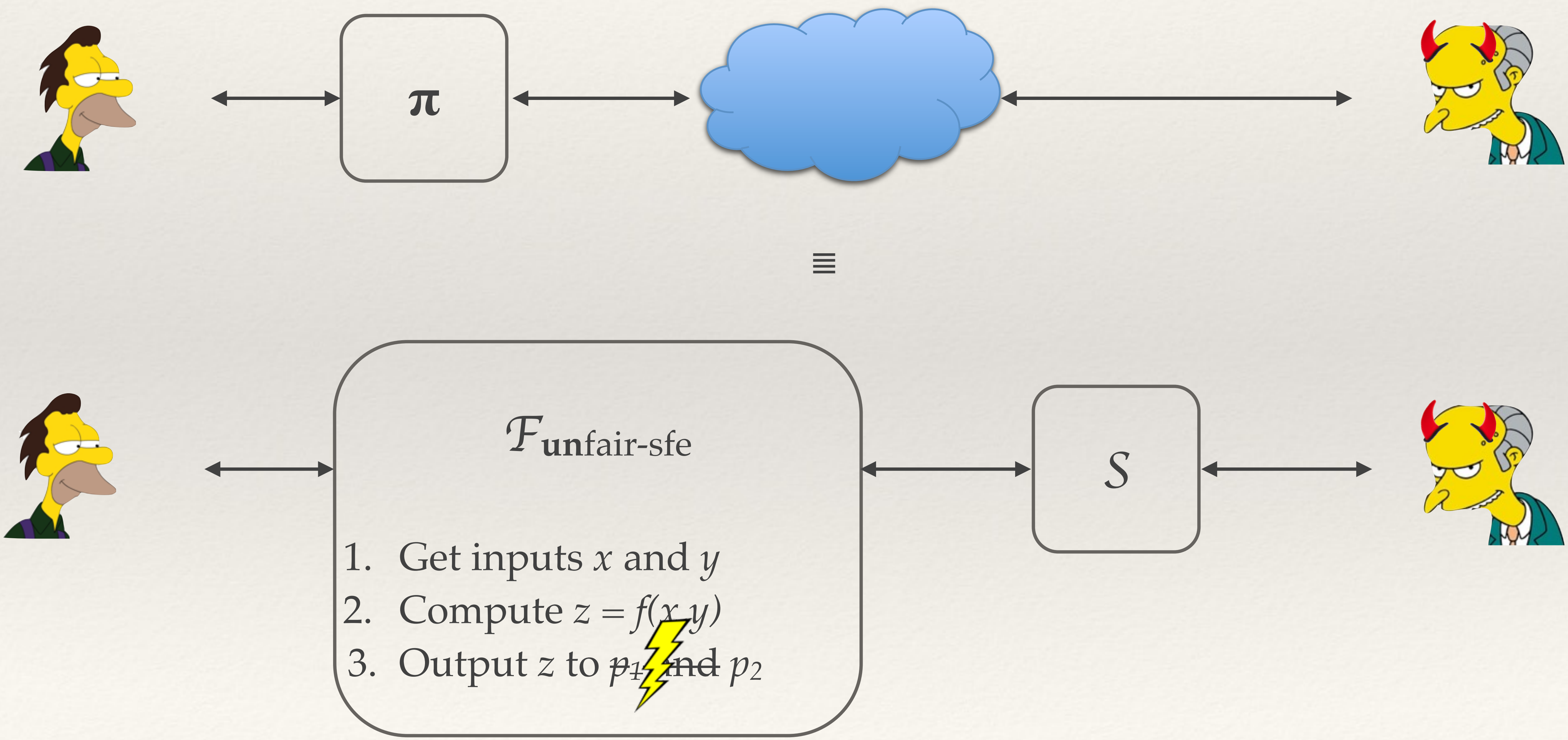
Defining Fairness (2)

Step 1:



Defining Fairness (3)

The protocol $\boldsymbol{\pi}$ realizes $\mathcal{F}_{\textbf{un}\text{fair-sfe}}$, i.e., there is S:



Defining Fairness (4)

Step 2: Define events in the *ideal* execution:

(a) Neither party gets the output: E_{00}, payoff γ_{00}

(b) Only honest party gets the output: E_{01}, payoff γ_{01}

(c) Only corrupted party gets the output: E_{10}, payoff γ_{10}

(d) Both parties get the output: E_{11}, payoff γ_{11}

Natural conditions: $\gamma_{01} < \gamma_{00}, \gamma_{11}$ and $\gamma_{00}, \gamma_{11} < \gamma_{10}$

Defining Fairness (5)

Step 3: Define the expected payoff for each S:

$$\text{payoff}(S) = \sum_{i,j \in \{0,1\}} \Pr(E_{ij}) \cdot \gamma_{ij}$$

The payoff of an adversary is the expected payoff of the *best* simulator:

$$\text{Payoff}(\mathcal{A}) = \min_{\text{"good" } S} \text{payoff}(S)$$

Optimal Protocol for Two Party SFE

- The protocol achieves $\frac{\gamma_{10} + \gamma_{11}}{2}$.
- This is optimal (see next slide).

1. In an *unfair* SFE:
   (a) choose $i^* \in \{1,2\}$
   (b) compute a sharing of the output value
   (c) output i^* and one share to each party
2. in case of abort, restart with default input for other party
3. $p_{(3-i^*)}$ sends its share to p_{i^*}
4. p_{i^*} sends its share to $p_{(3-i^*)}$

Optimal Protocol for Two Party SFE

- The protocol achieves $\frac{\gamma_{10} + \gamma_{11}}{2}$.
- This is optimal (see next slide).

Proof idea:

- secure w/o fairness (based on underlying SFE and repeat before leaking output)
- the simulator chooses i^* uniformly at random

1. In an *unfair* SFE:
   (a) choose $i^* \in \{1,2\}$
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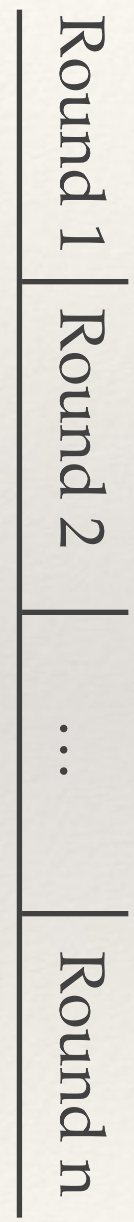
Optimal Protocol for Two Party SFE

There exist functions such that…



Optimal Protocol for Two Party SFE

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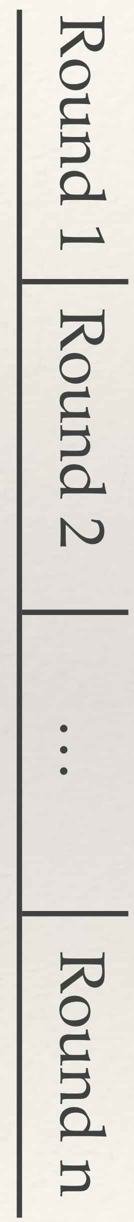


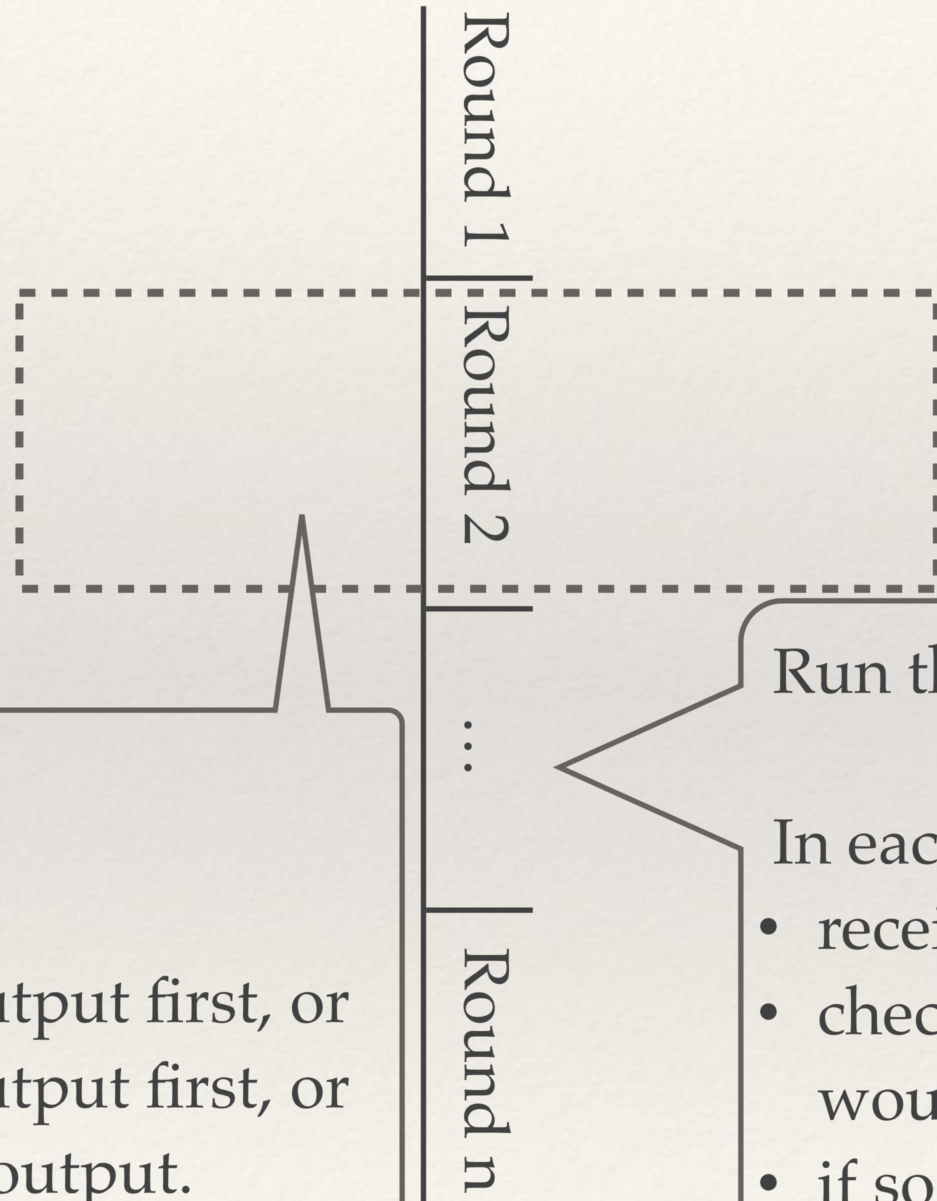
Run the honest protocol as follows.

In each round:

- receive the honest party's message,
- check whether the honest protocol would generate output,
- if so, then abort,
- otherwise, send the honestly computed message for this round

Optimal Protocol for Two Party SFE

There exist functions such that…



In each round:

- p_1 receives the output first, or
- p_2 receives the output first, or
- both receive the output.

Run the honest protocol as follows.

In each round:

- receive the honest party's message,
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Optimal Protocol for Two Party SFE

There exist functions such that…

Round 1

Proof idea:

- a protocol can be improved by never outputting to both in the same round
- at least one party is *first* with probability at least 1/2

In each round:

- p_1 receives the output first, or
- p_2 receives the output first, or
- both receive the output.

ound n

…ocol as follows.

… party's message,

- check whether the honest protocol would generate output,
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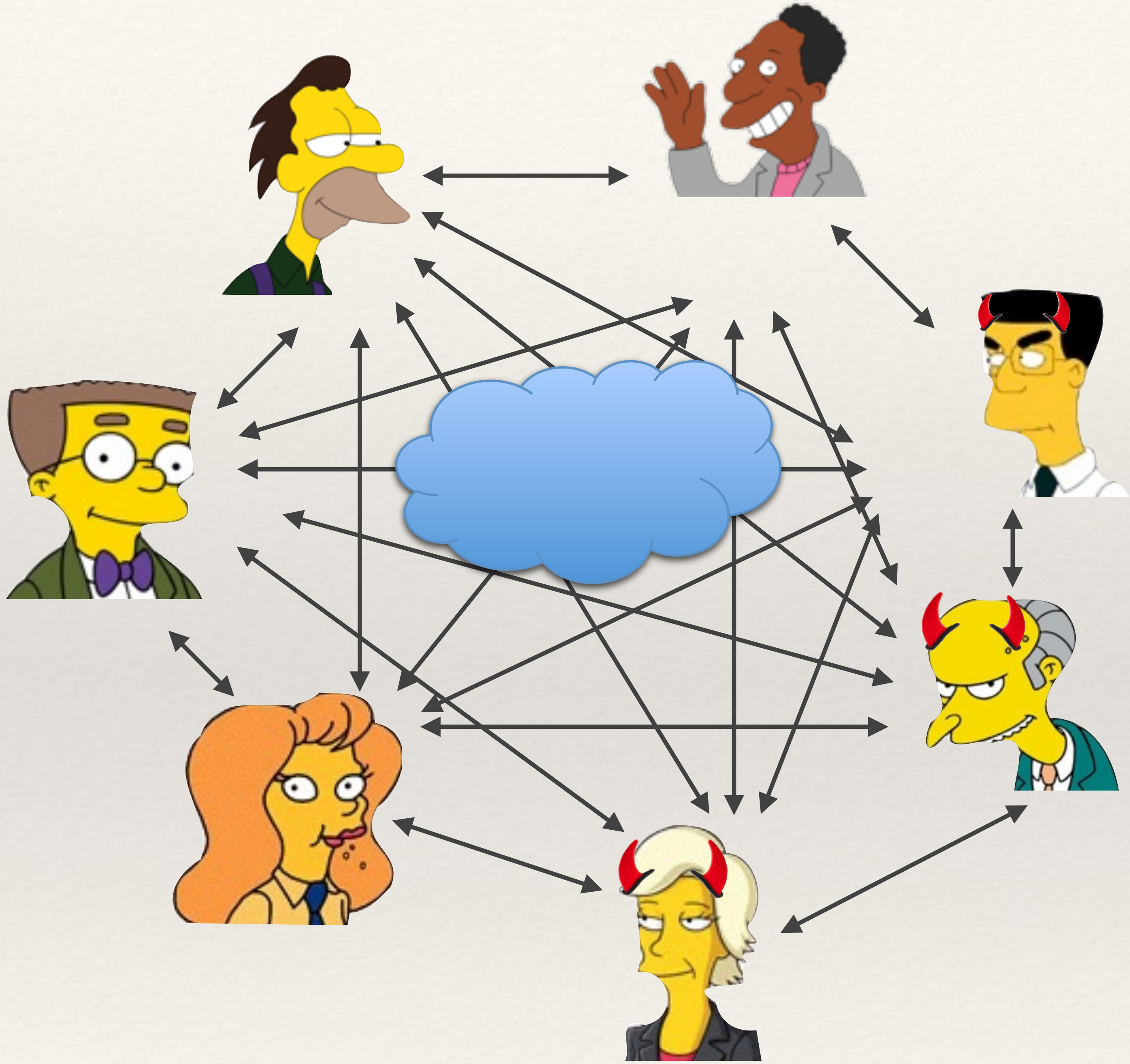
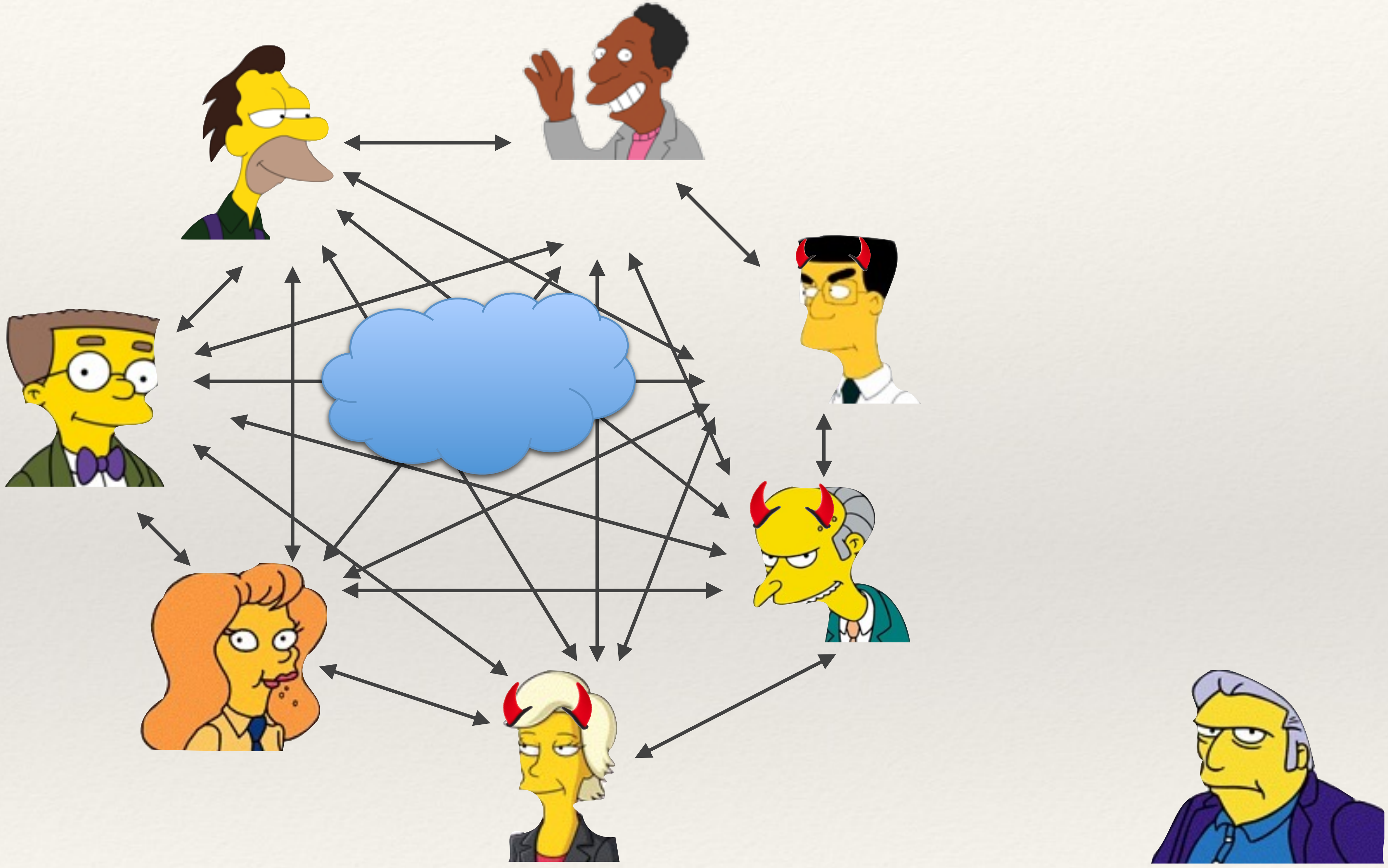
The Multi-Party Case



The Multi-Party Case



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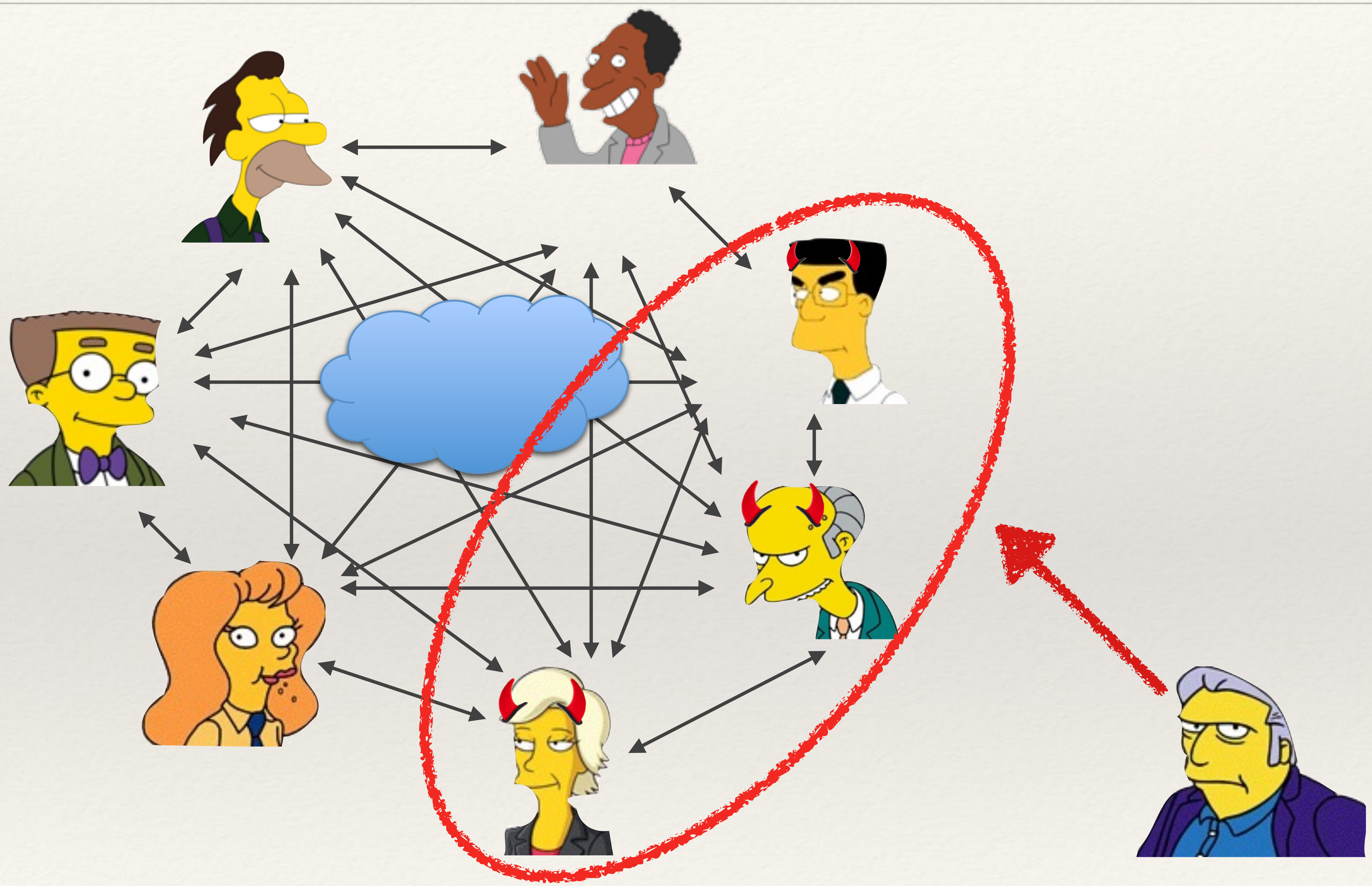


The Multi-Party Case

$\mathcal{F}_{\text{fair-sfe}}$

1. Get inputs $x_1, x_2, \ldots, x_n$
2. Compute $z = f(x_1, x_2, \ldots, x_n)$
3. Possibly: Output z to $p_1, p_2, \ldots, p_n$

The Multi-Party Case

$\mathcal{F}_{\text{fair-sfe}}$

1. Get inputs $x_1, x_2, \ldots, x_n$
2. Compute $z = f(x_1, x_2, \ldots, x_n)$
3. Possibly: Output z to $p_2, \ldots, p_n$

BREAK!

Give output only to me!

Multi-Party Fairness

Step 2: Define events in the *ideal* execution:

A. No party gets the output: E_{00}, payoff γ_{00}

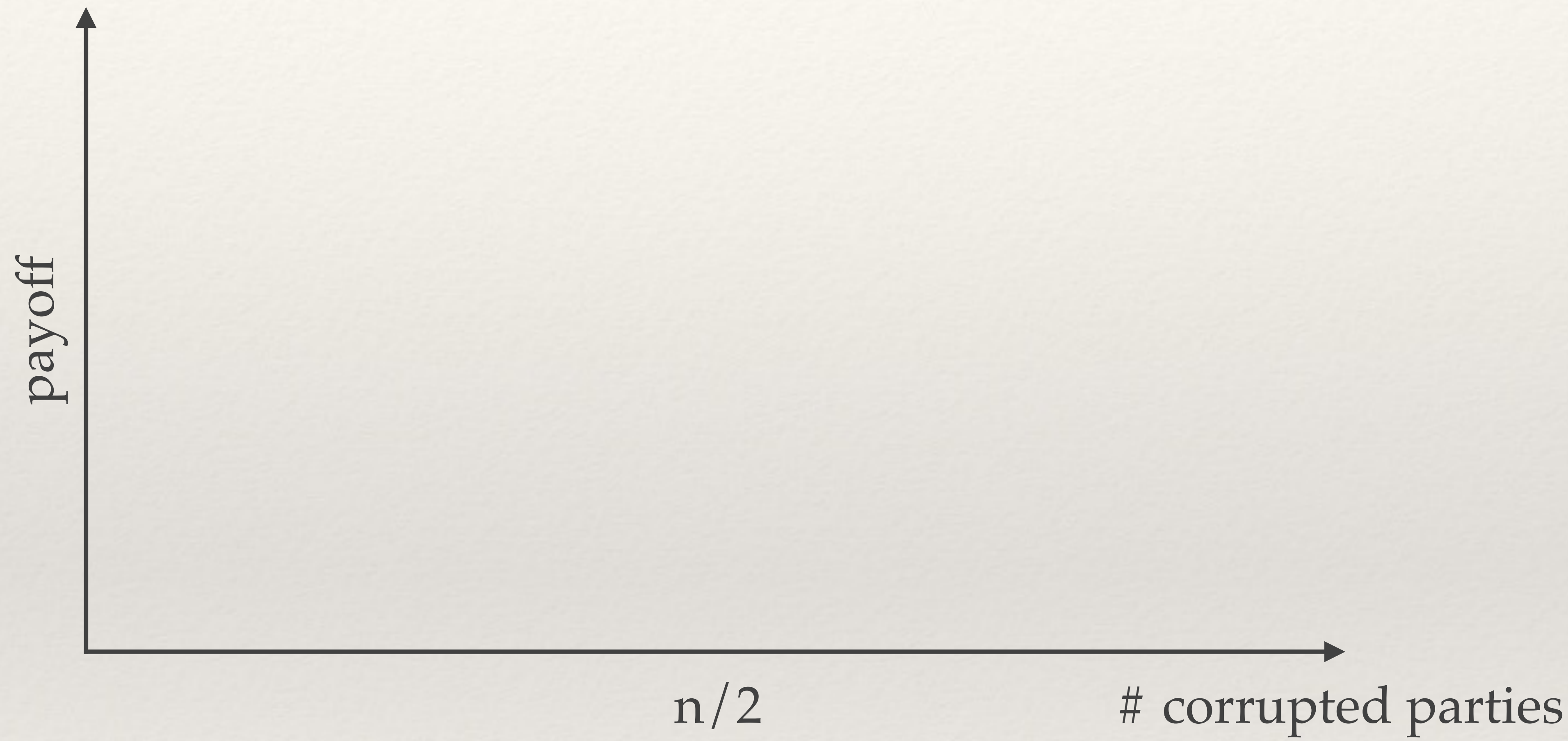
B. Exactly all honest parties get the output: E_{01}, payoff γ_{01}

C. Not all honest parties, but some corrupted party gets the output: E_{10}, payoff γ_{10}

D. All honest parties and some corrupted party get the output: E_{11}, payoff γ_{11}

Here: stronger condition $\gamma_{01} < \gamma_{00} < \gamma_{11} < \gamma_{10}$.

Multi-Party Fairness



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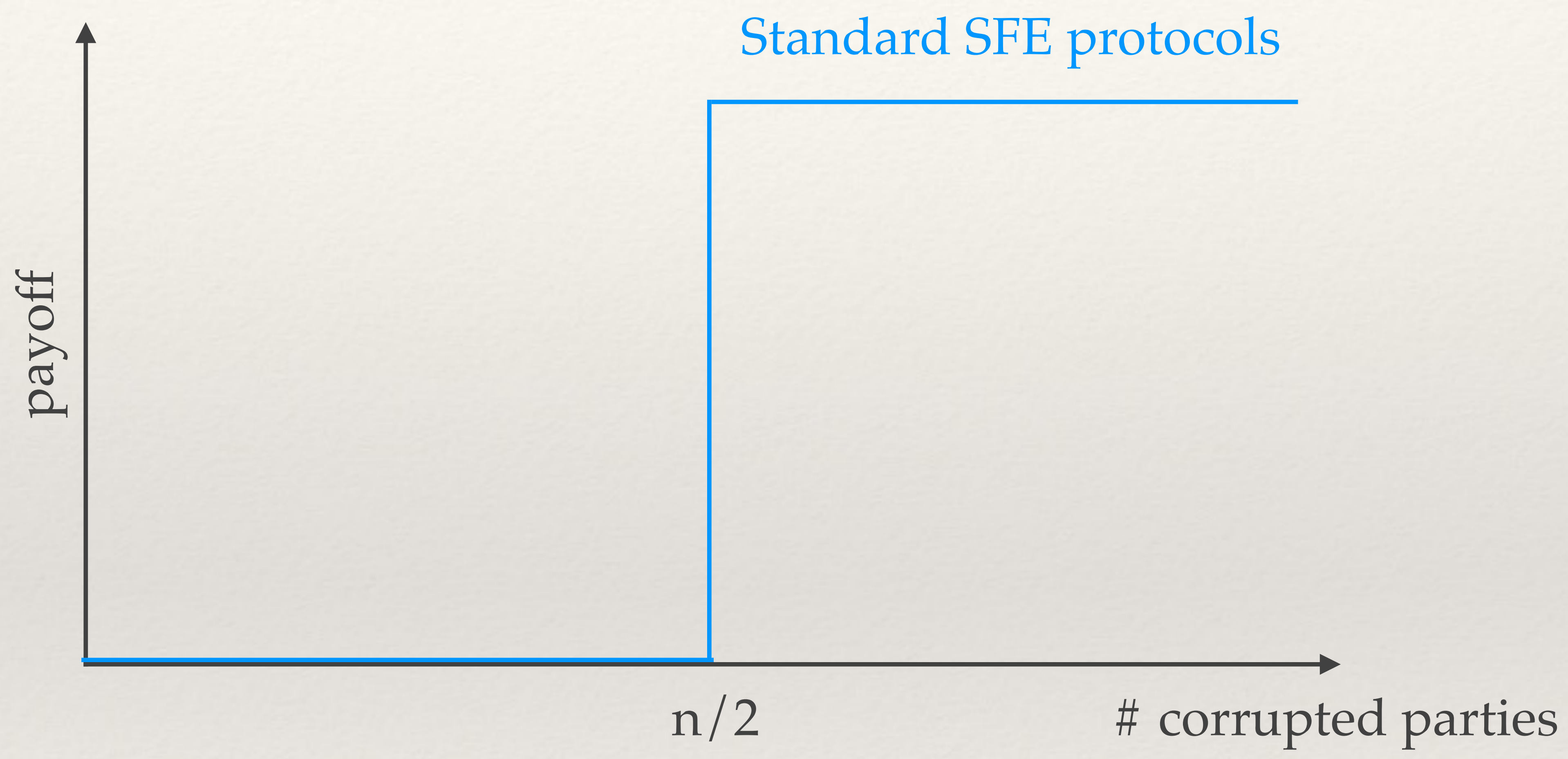

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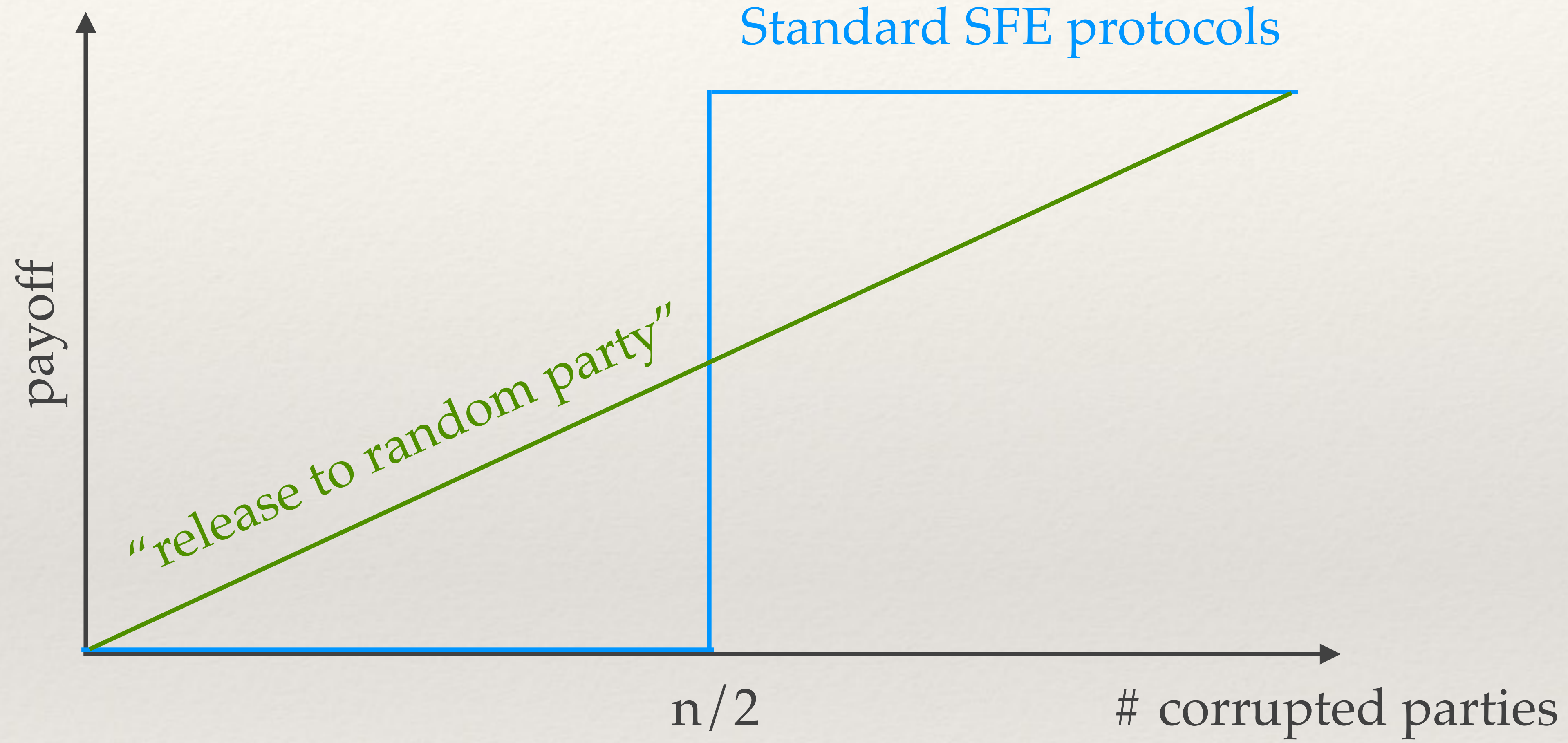

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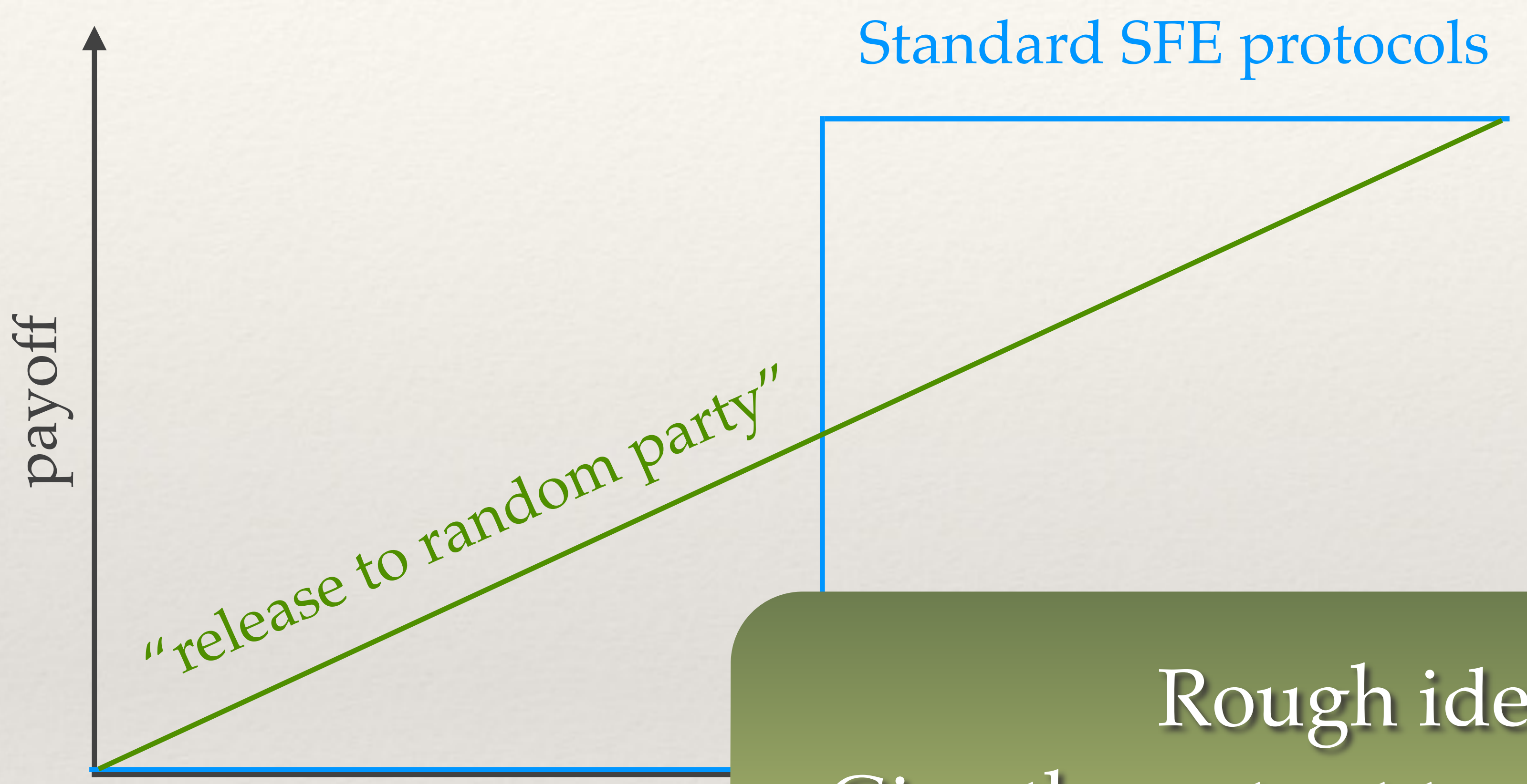


Rough idea:
Give the output to some party,
let him distribute.

Other Relaxed Notions of Fairness

- "Gradual Release"-type approaches [Goldwasser-Levin, 1990; Garay-MacKenzie-Prabhakaran-Yang, 2005, …]
- Rational fairness [Asharov-Canetti-Hazay, 2011]
  - Not closely related, after all…
- *1/p*-Security [Gordon-Katz, 2010]
  - Similar (quantitative) guarantee,
  - protocols for functions with small domain or range,
  - formally more relaxed definition.

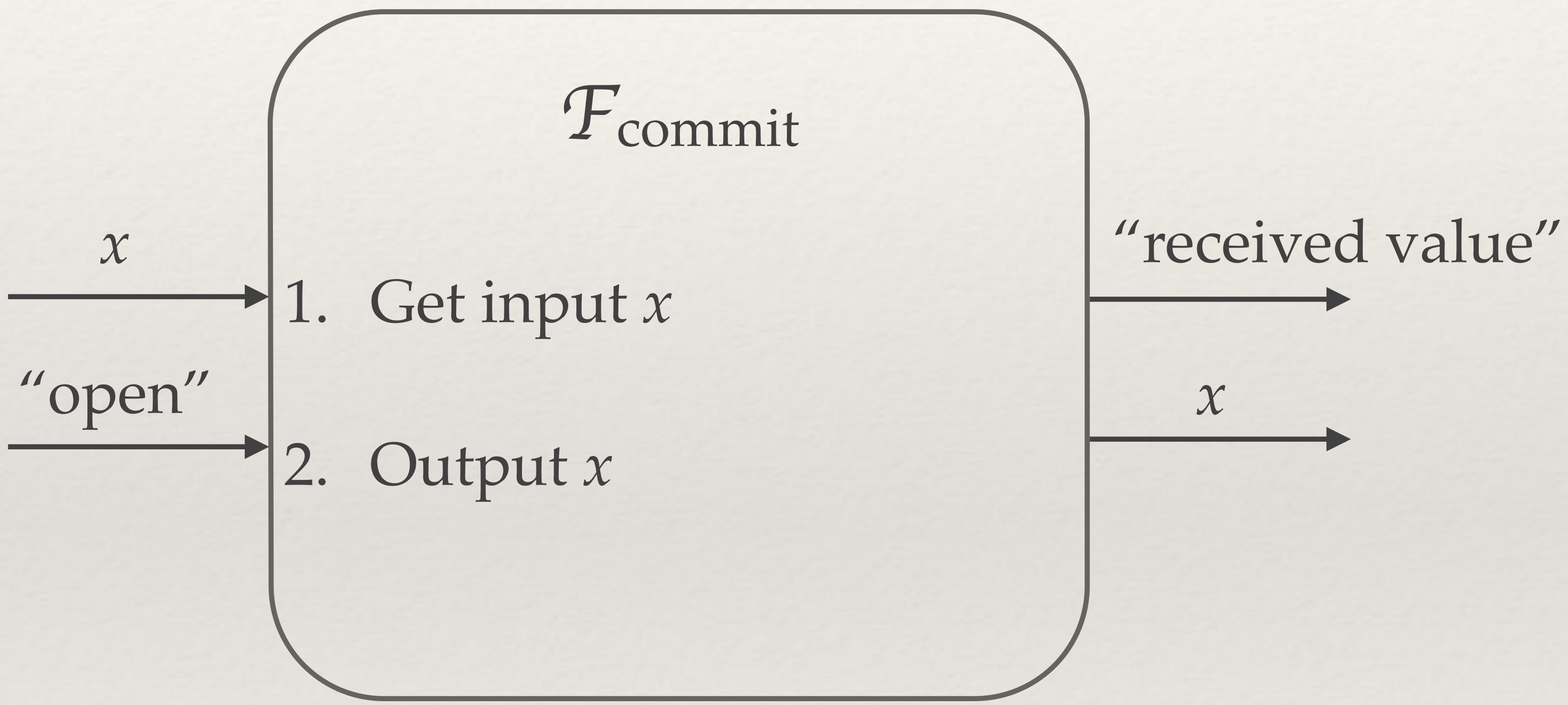
Rational Protocol Design

- General framework (beyond fairness),
- supports composition (via the underlying framework),
- generalizes to reactive functionalities (follow-up).

cf. Ranjit's talk

Rational Protocol Design

"Rational" commitment*:



* as mentioned by Rosario on Monday.

Rational Protocol Design

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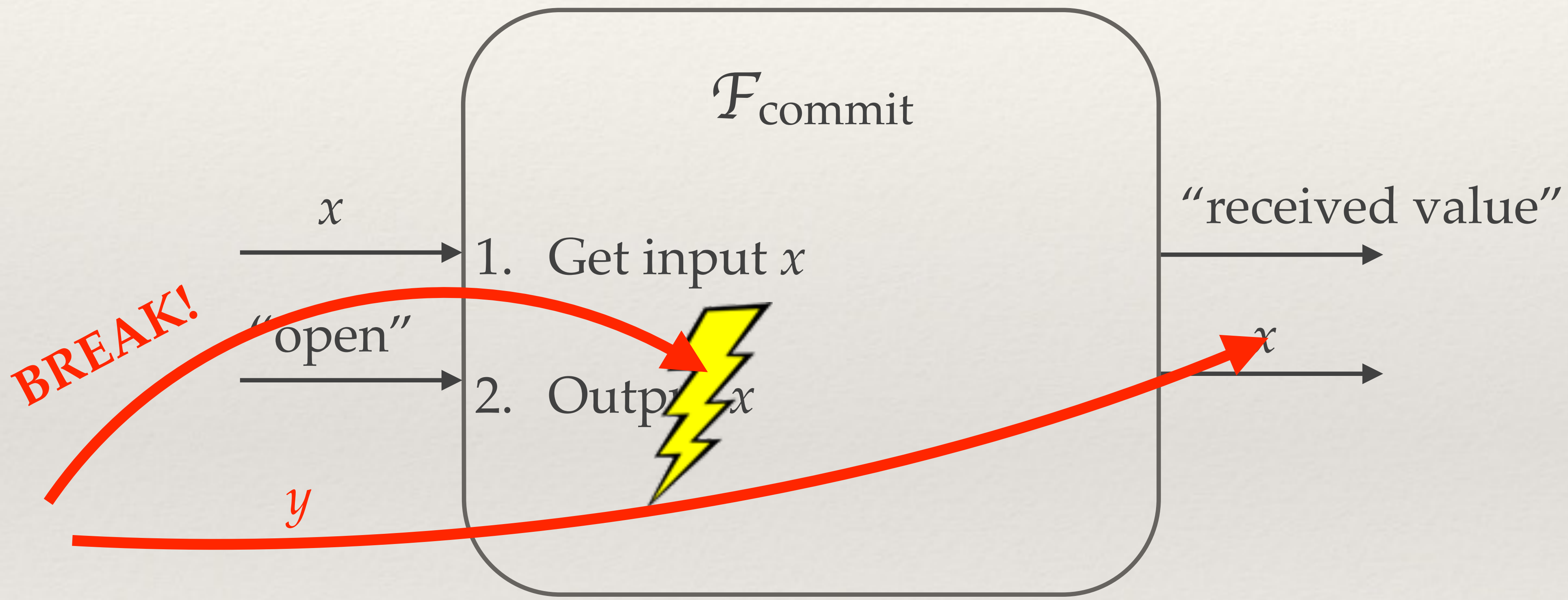

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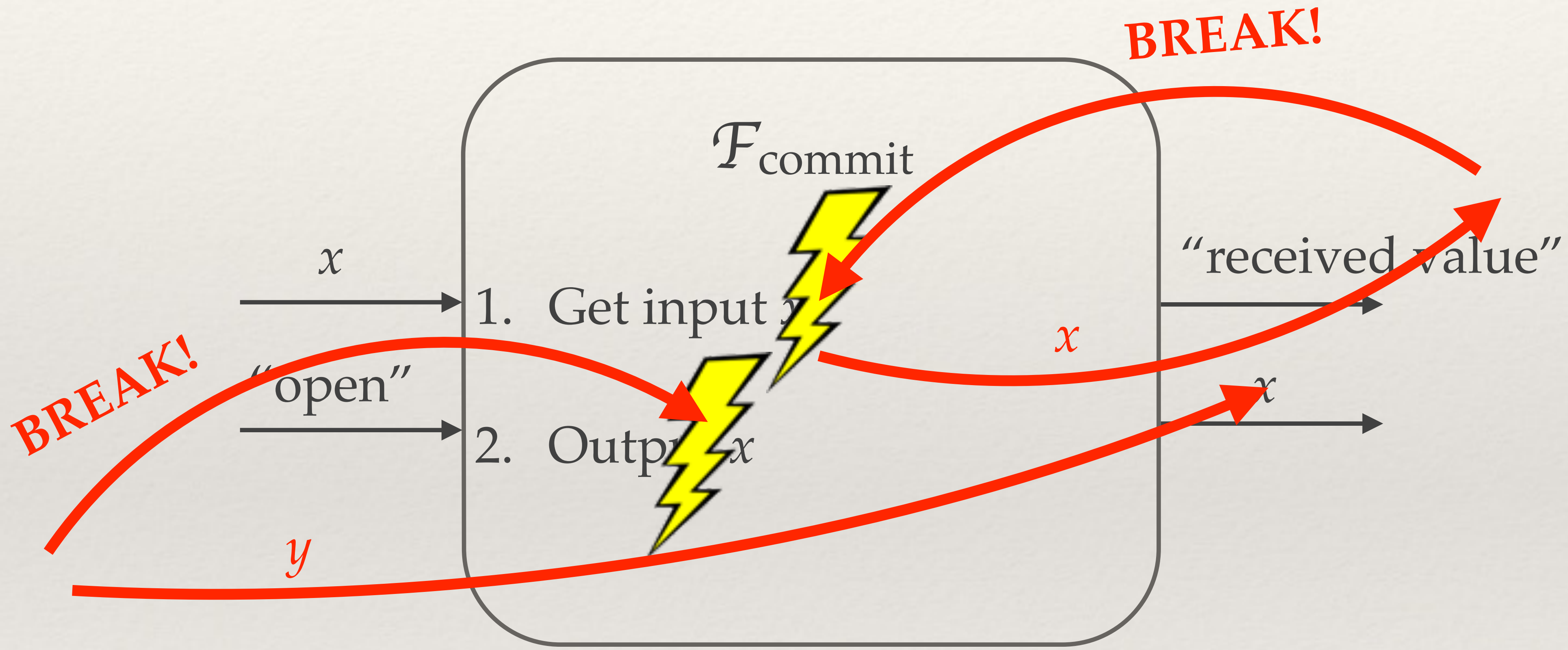

* as mentioned by Rosario on Monday.

Summary

- RPD is a general framework capturing incentives,
- idea: build the best protocol w.r.t. the incentives,
- we showed optimal protocols for fairness in SFE.
- Follow-up: Reactive functionalities.