The Whole is Greater than the Sum of its Parts: Linear Garbling and Applications

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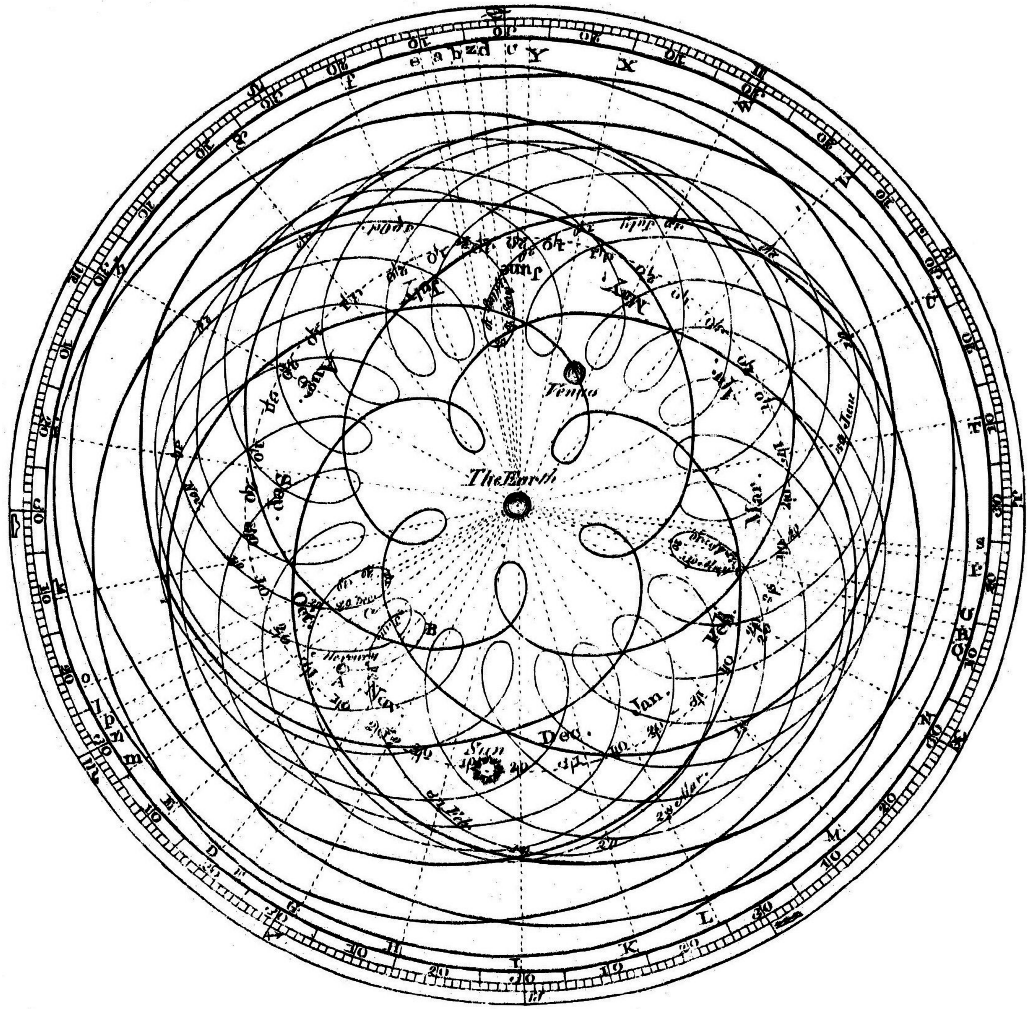
June 10, 2015

Some complex system...

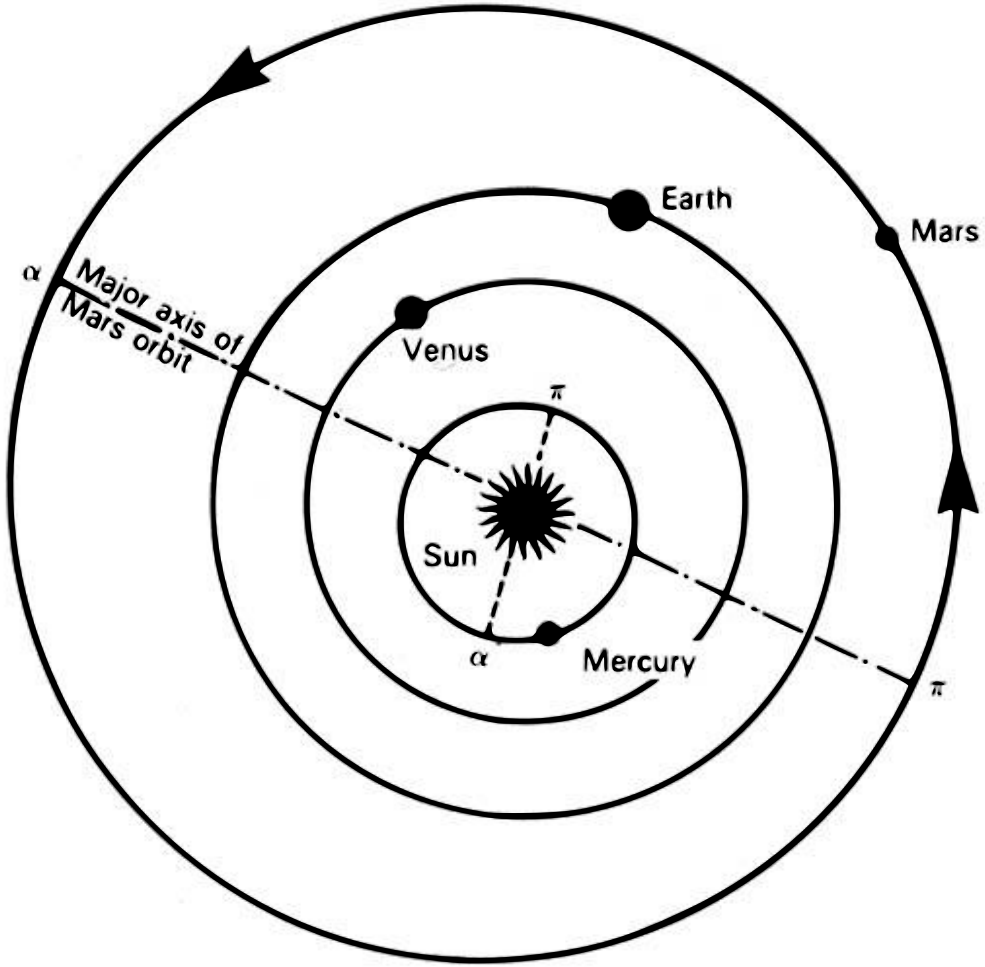
The solar system: Geocentric Model – 1400 AD



Credit: `http://en.wikipedia.org/wiki/Deferent_and_epicycle`

...can made simple, by changing perspective.

The solar system – today



Credit: http://history.nasa.gov/SP-4212/p427.html

More Context:

Our system: linear garbling

New perspective: linear garbling seen as linear secret sharing
simple properties $\Rightarrow$ simulation-based security

Why? simpler model $\Rightarrow$ more advanced schemes

What is garbling? [BHR12]

C

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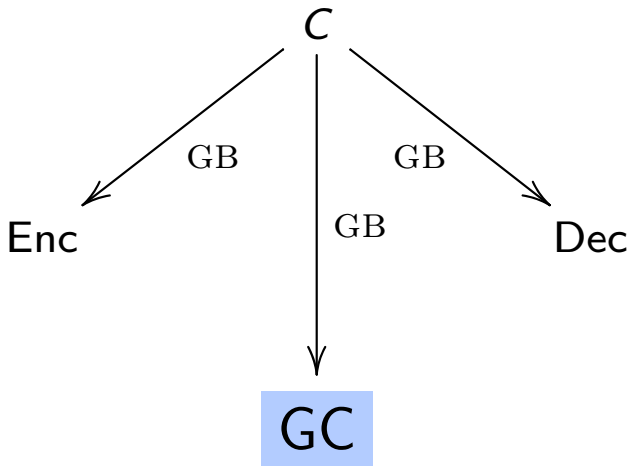

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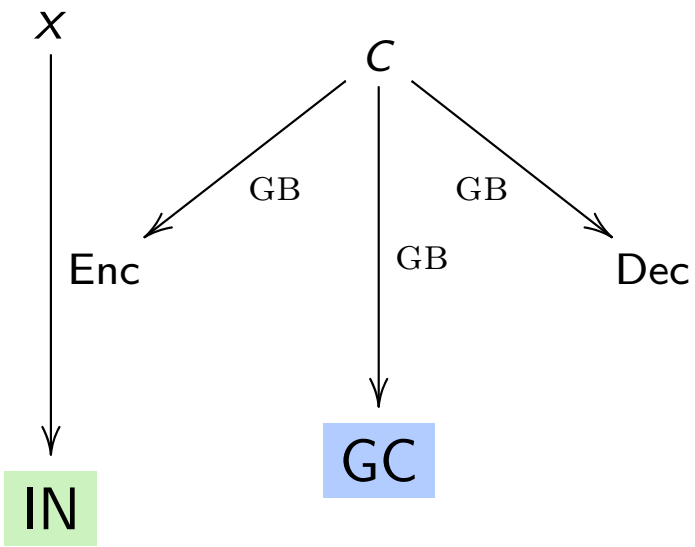

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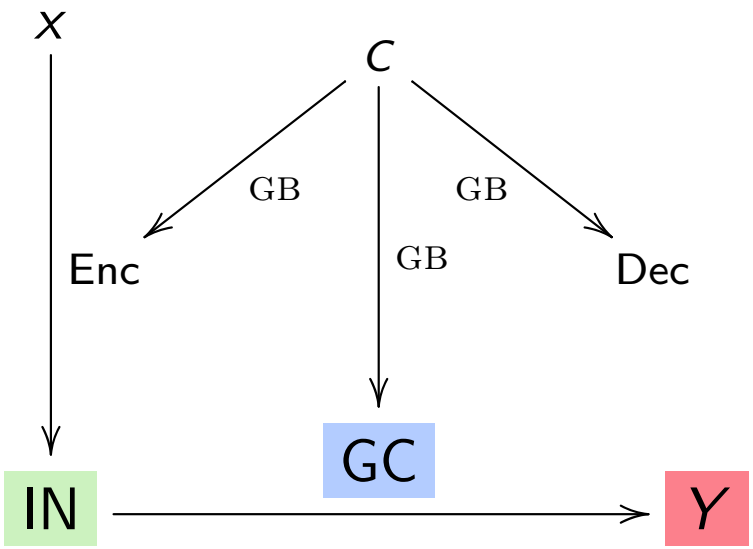

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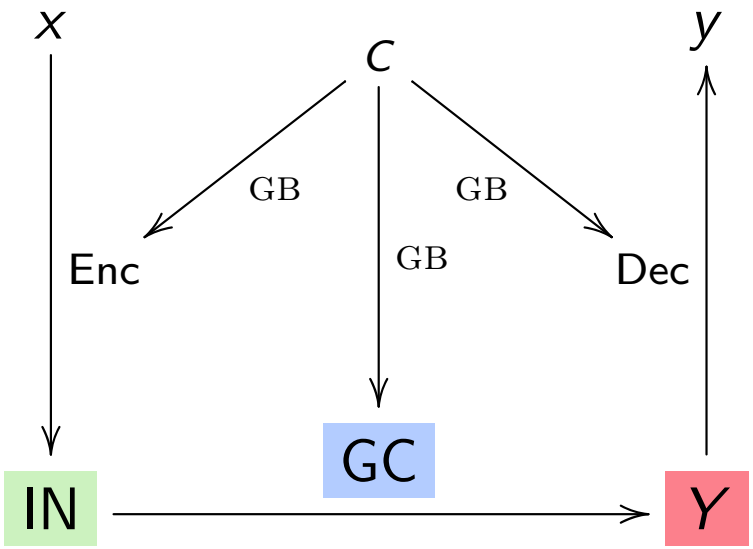

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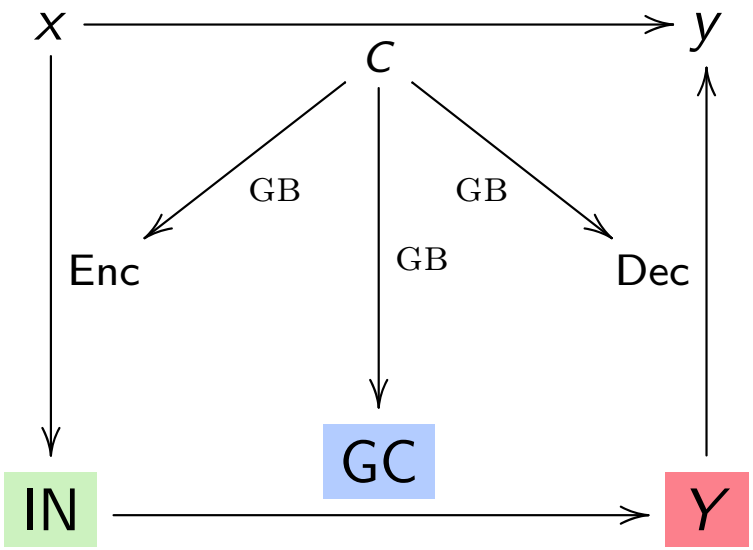

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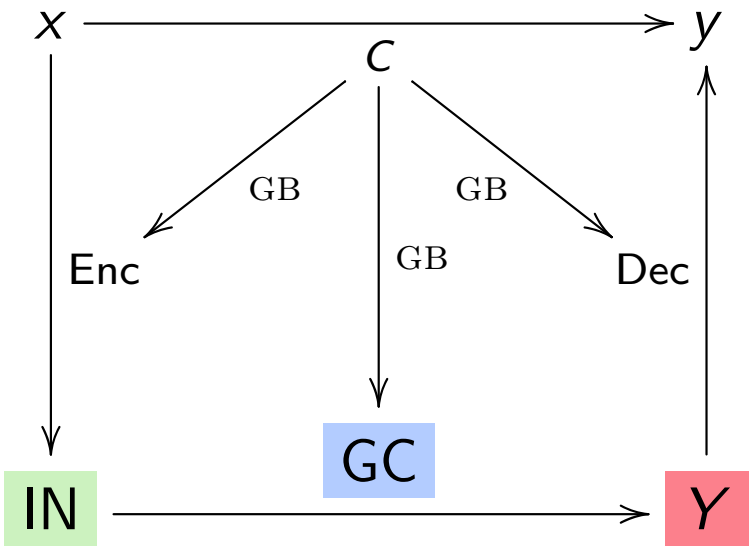

Security:

$$\left\{\left(\boxed{\text{GC}}, \text{Enc}, \text{Dec}\right) \leftarrow \text{GB}(1^\lambda, C), \boxed{\text{IN}} \leftarrow \text{Enc}(x) : \left(\boxed{\text{GC}}, \boxed{\text{IN}}, \text{Dec}\right)\right\}_\lambda \approx_c \left\{S(1^\lambda, C, C(x))\right\}_\lambda$$

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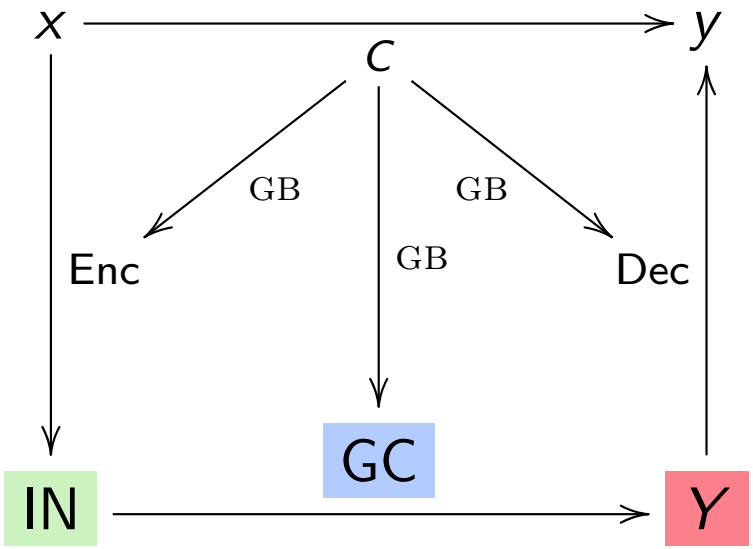

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Focus on: boolean circuits, communication complexity (size of $\boxed{\text{GC}}$)

Can we do better?

	$\times\lambda$ bits	
Scheme	XOR	AND
Yao [Yao82]	4	4
GRR2 [PSSW09]	2	2
Free-XOR + GRR3 [KS08, NPS99]	0	3
FleXOR [KMR14]	2/1/0	2
Half-gates [ZRE15]	0	2

Table : Per-gate communication complexity.

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[ZRE15]: any linear, gate-by-gate scheme		≥ 2

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Approaching "**not** gate-by-gate" garbling:

- slice circuit in small "units"
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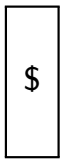
Large units $\Rightarrow$ hard proofs $\Rightarrow$ need for easier framework

Linear garbling [ZRE15]

Intuition: garbler and evaluator: **RO calls** and **linear functions** only



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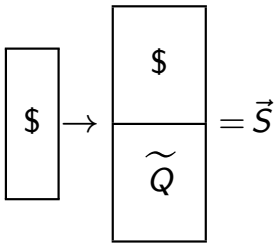

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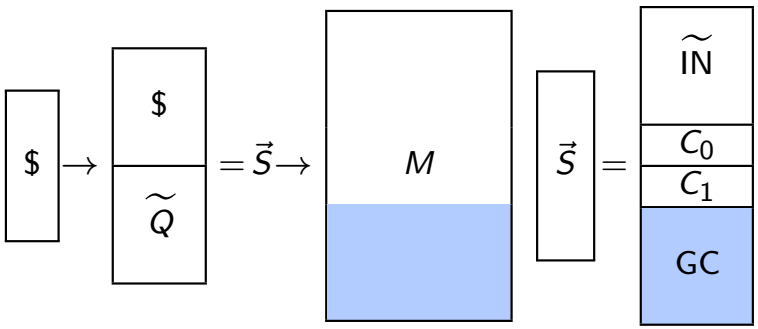

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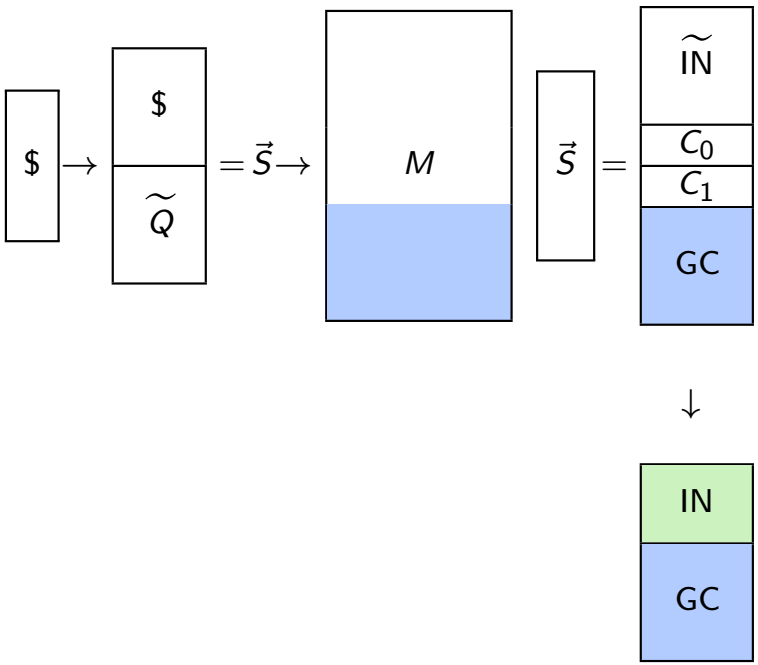

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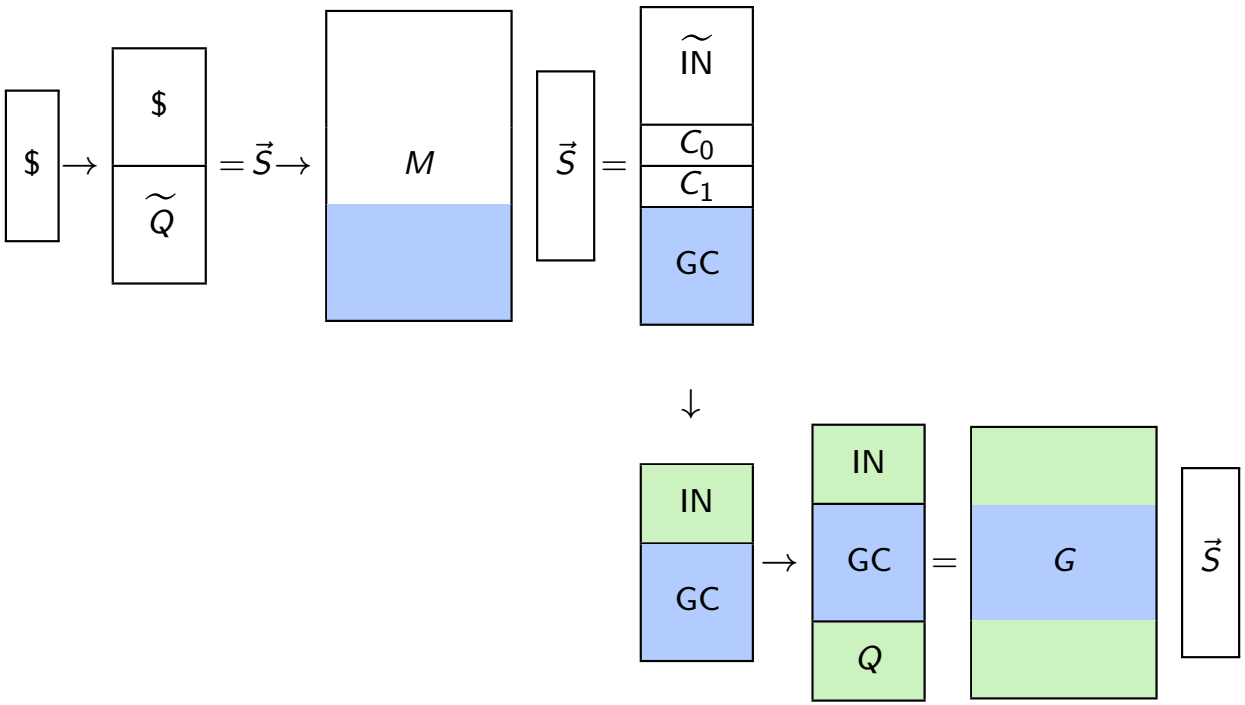

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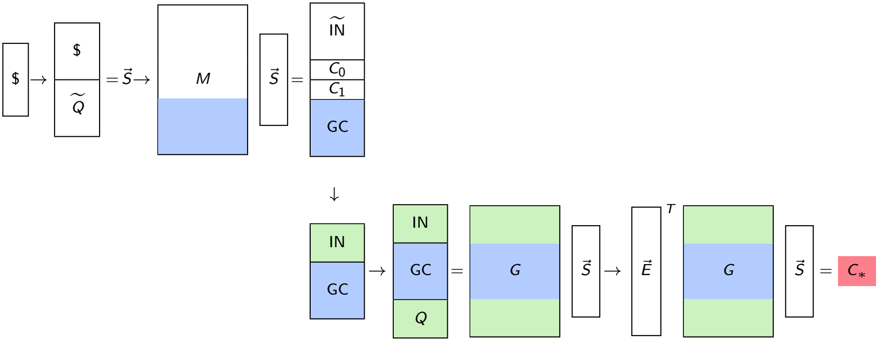

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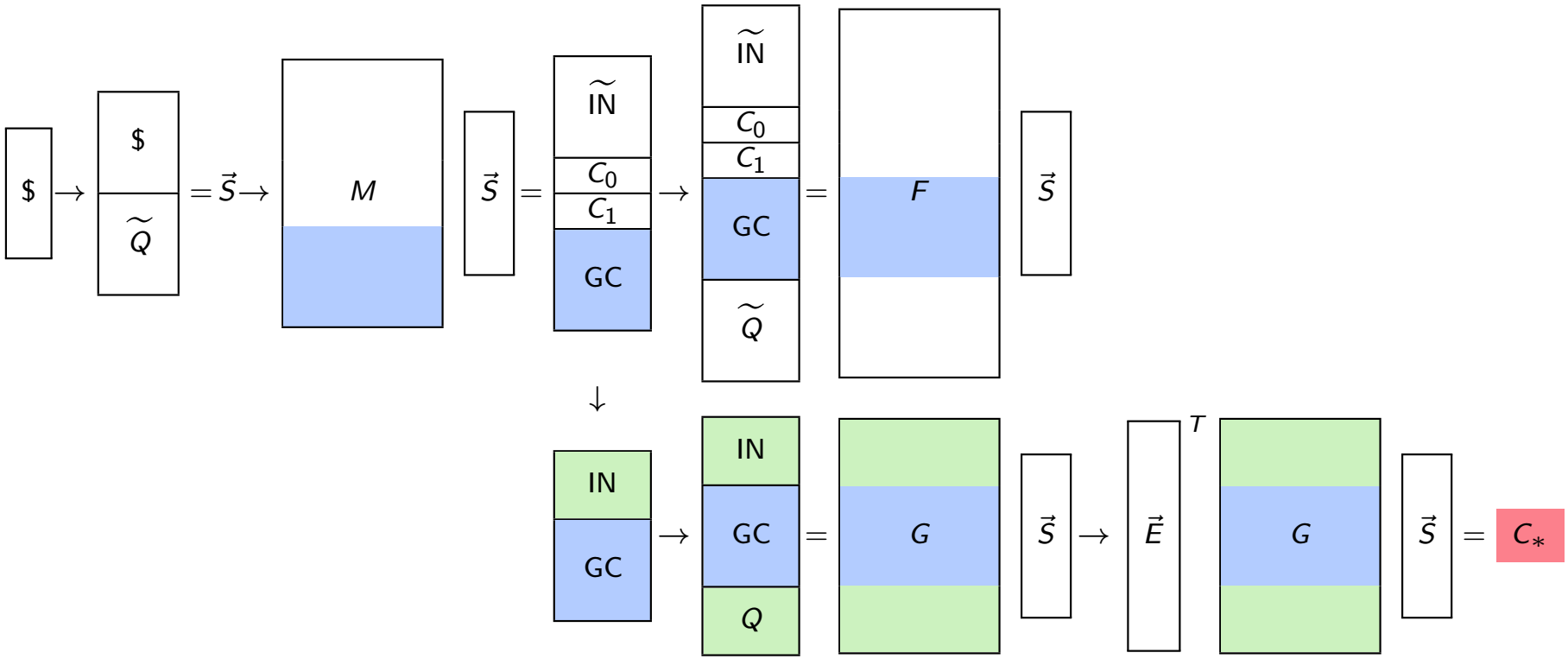

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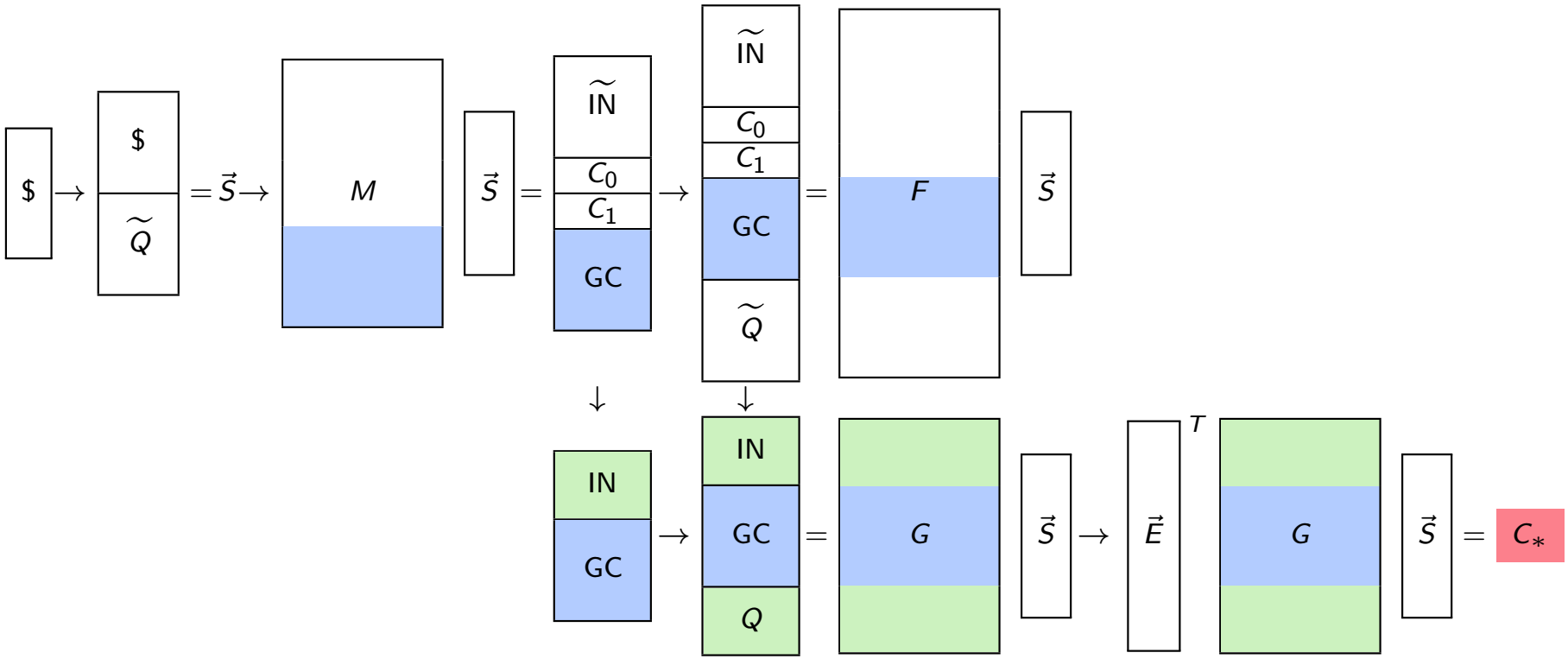

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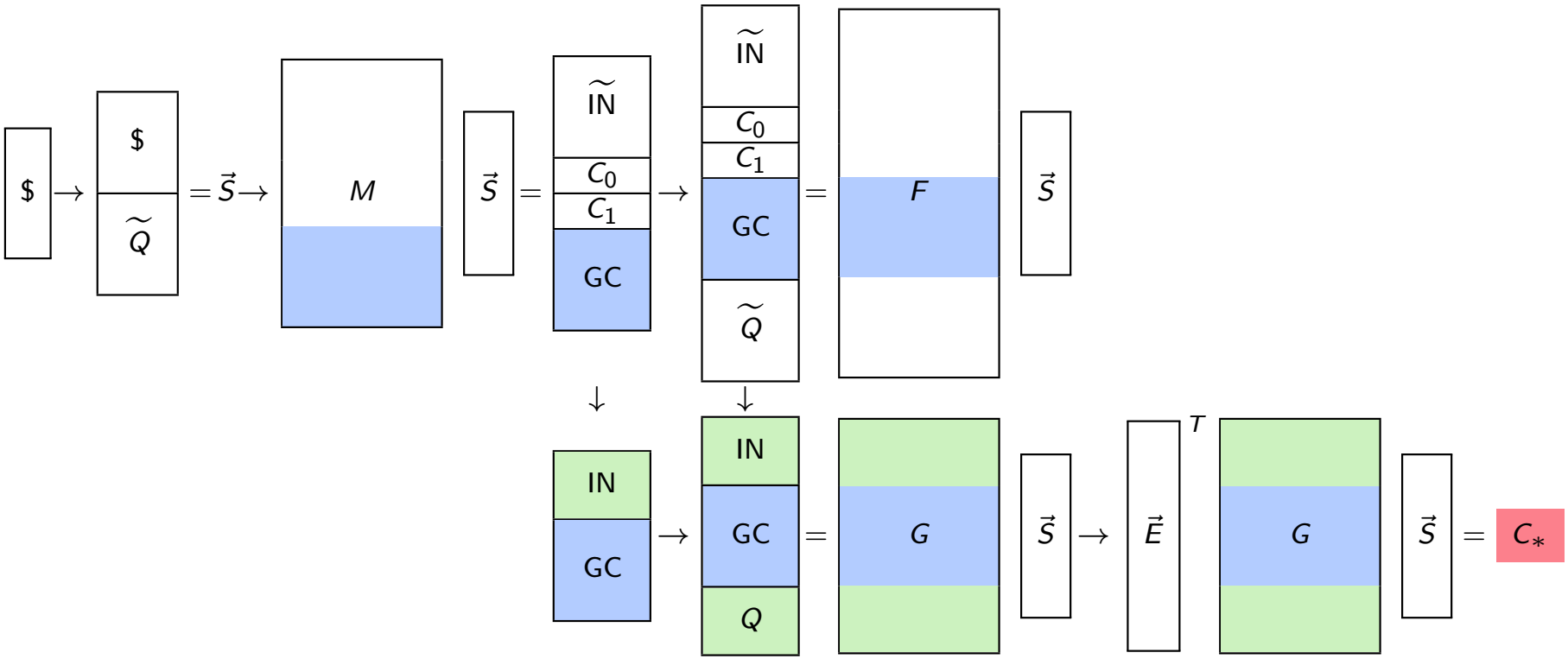
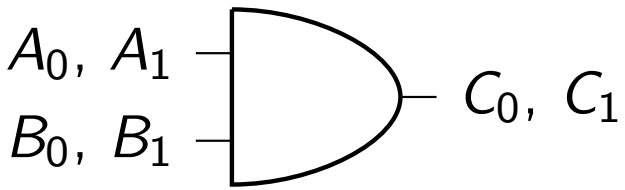


Possible interpretation:

- F: secret sharing scheme for both C_0, C_1
- G: rows corresponding to shares given to evaluator

Yao Garbling – GB (M matrix)



$$
\begin{aligned}
G_{0,0} &= H(A_0 \| B_0) \oplus C_0 = \mathrm{Enc}_{A_0,B_0}(C_0) \\
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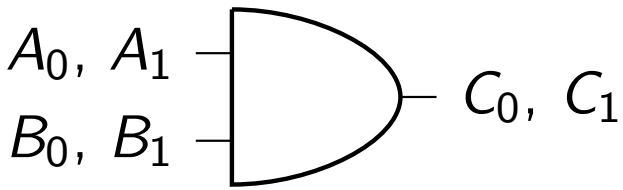

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=
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1
\end{pmatrix}
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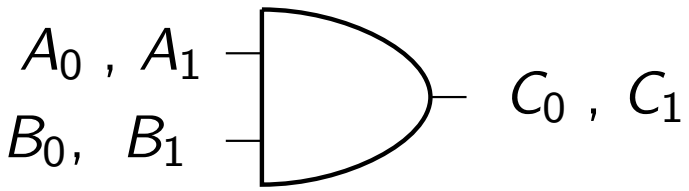

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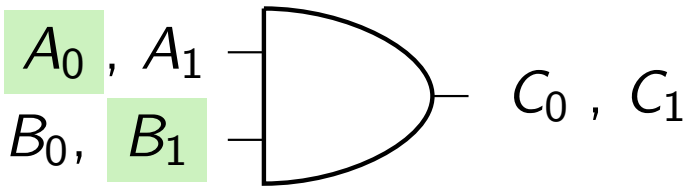

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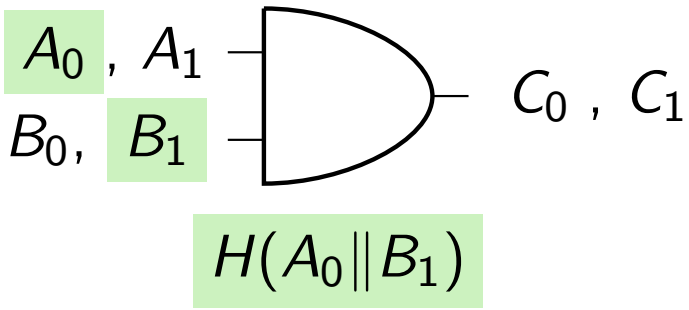

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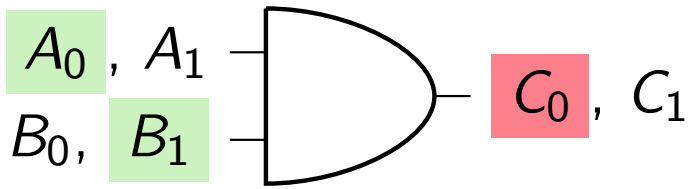

$$C_0 \leftarrow H(A_0 \| B_1) \oplus G_{0,1}$$

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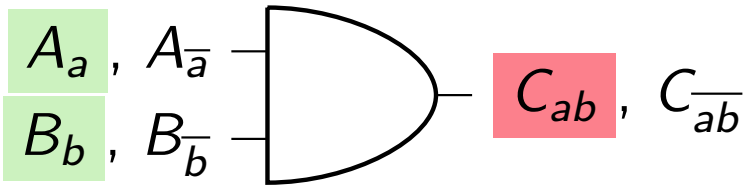
In general:



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$$\begin{pmatrix} A_a \\ B_b \\ C_{ab} \\ C_{\overline{ab}} \\ G_{0,0} \\ G_{0,1} \\ G_{1,0} \\ G_{1,1} \\ H(A_a \| B_b) \end{pmatrix} = \begin{pmatrix} \overline{a} & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{b} & b & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \overline{ab} & ab & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & ab & \overline{ab} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & \overline{ab} & \overline{a}b & a\overline{b} & ab \end{pmatrix} \begin{pmatrix} A_0 \\ A_1 \\ B_0 \\ B_1 \\ C_0 \\ C_1 \\ H(A_0 \| B_0) \\ H(A_0 \| B_1) \\ H(A_1 \| B_0) \\ H(A_1 \| B_1) \end{pmatrix}$$

A Different Interpretation of Correctness/Security

$$\begin{pmatrix} A_a \\ B_b \\ C_{ab} \\ C_{\overline{ab}} \\ G_{0,0} \\ G_{0,1} \\ G_{1,0} \\ G_{1,1} \\ H(A_a \| B_b) \end{pmatrix} = \begin{pmatrix} \overline{a} & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{b} & b & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \overline{ab} & ab & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & ab & \overline{ab} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & \overline{a}\overline{b} & \overline{a}b & a\overline{b} & ab \end{pmatrix} \begin{pmatrix} A_0 \\ A_1 \\ B_0 \\ B_1 \\ C_0 \\ C_1 \\ H(A_0 \| B_0) \\ H(A_0 \| B_1) \\ H(A_1 \| B_0) \\ H(A_1 \| B_1) \end{pmatrix}$$

■ $\in$ Span(■ $\cup$ ■): linear reconstruction

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$\in$ Span($\cup$): linear reconstruction

$\notin$ Span($\cup$): linear privacy

A Different Interpretation of Correctness/Security

$$\begin{pmatrix} A_a \\ B_b \\ C_{ab} \\ C_{\overline{ab}} \\ G_{0,0} \\ G_{0,1} \\ G_{1,0} \\ G_{1,1} \\ H(A_a \| B_b) \end{pmatrix} = \begin{pmatrix} \overline{a} & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{b} & b & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \overline{ab} & ab & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & ab & \overline{ab} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & \overline{a}\overline{b} & \overline{a}b & a\overline{b} & ab \end{pmatrix} \begin{pmatrix} A_0 \\ A_1 \\ B_0 \\ B_1 \\ C_0 \\ C_1 \\ H(A_0 \| B_0) \\ H(A_0 \| B_1) \\ H(A_1 \| B_0) \\ H(A_1 \| B_1) \end{pmatrix}$$

$\in$ Span($\cup$): linear reconstruction

$\notin$ Span($\cup$): linear privacy

Theorem

Linear reconstruction & linear privacy $\Rightarrow$ simulation-based security

Warm up

Half-gate technique [ZRE15]:

$$\underbrace{v_A}_{\substack{\text{color bit,}\\ \text{known by evaluator}}} = \overbrace{a}^{\text{input}} + \underbrace{p_A}_{\substack{\text{permutation bit,}\\ \text{known by garbler}}}$$

Warm up

Half-gate technique [ZRE15]: $\underbrace{v_A}_{\substack{\text{color bit,}\\ \text{known by evaluator}}} = \overbrace{a}^{\text{input}} + \underbrace{p_A}_{\substack{\text{permutation bit,}\\ \text{known by garbler}}}$

$$\left(\begin{array}{ccccccc}
1 & 0 & v_A & 0 & 0 & 0 & 0 \\
0 & 1 & v_B & 0 & 0 & 0 & 0 \\
0 & 0 & ab + p_A p_B & 1 & 1 & 0 & 0 \\
0 & 0 & 1 + ab + p_A p_B & 1 & 1 & 0 & 0 \\
0 & 0 & p_B & 1 & 0 & 1 & 0 \\
1 & 0 & p_A & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 + v_A & 0 & v_A & 0 \\
0 & 0 & 0 & 0 & 1 + v_B & 0 & v_B
\end{array}\right)$$

Warm up

Half-gate technique [ZRE15]: $\underbrace{v_A}_{\substack{\text{color bit,}\\ \text{known by evaluator}}} = \overbrace{a}^{\text{input}} + \underbrace{p_A}_{\substack{\text{permutation bit,}\\ \text{known by garbler}}}$

$$\begin{pmatrix} \varnothing \\ \varnothing \end{pmatrix}^T \begin{pmatrix} 1 & 0 & v_A & 0 & 0 & 0 & 0 \\ 0 & 1 & v_B & 0 & 0 & 0 & 0 \\ 0 & 0 & ab + p_A p_B & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 + ab + p_A p_B & 1 & 1 & 0 & 0 \\ 0 & 0 & p_B & 1 & 0 & 1 & 0 \\ 1 & 0 & p_A & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 + v_A & 0 & v_A & 0 \\ 0 & 0 & 0 & 0 & 1 + v_B & 0 & v_B \end{pmatrix} =$$

Warm up

Half-gate technique [ZRE15]: $\underbrace{v_A}_{\substack{\text{color bit,}\\ \text{known by evaluator}}} = \overbrace{a}^{\text{input}} + \underbrace{p_A}_{\substack{\text{permutation bit,}\\ \text{known by garbler}}}$

$$\begin{pmatrix} 0 \\ \varnothing \\ \varnothing \end{pmatrix}^T \begin{pmatrix} 1 & 0 & v_A & 0 & 0 & 0 & 0 \\ 0 & 1 & v_B & 0 & 0 & 0 & 0 \\ 0 & 0 & ab + p_A p_B & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 + ab + p_A p_B & 1 & 1 & 0 & 0 \\ 0 & 0 & p_B & 1 & 0 & 1 & 0 \\ 1 & 0 & p_A & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 + v_A & 0 & v_A & 0 \\ 0 & 0 & 0 & 0 & 1 + v_B & 0 & v_B \end{pmatrix} =$$

$$\begin{matrix} 0 & ? \end{matrix}$$

$? = 0$

Warm up

Half-gate technique [ZRE15]: $\underbrace{v_A}_{\substack{\text{color bit,}\\ \text{known by evaluator}}} = \overbrace{a}^{\text{input}} + \underbrace{p_A}_{\substack{\text{permutation bit,}\\ \text{known by garbler}}}$

$$\begin{pmatrix} \\ 0 \\ \varnothing \\ \varnothing \\ v_A \\ \\ 1 \\ \\ \end{pmatrix}^T \begin{pmatrix} 1 & 0 & v_A & 0 & 0 & 0 & 0 \\ 0 & 1 & v_B & 0 & 0 & 0 & 0 \\ 0 & 0 & ab + p_A p_B & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 + ab + p_A p_B & 1 & 1 & 0 & 0 \\ 0 & 0 & p_B & 1 & 0 & 1 & 0 \\ 1 & 0 & p_A & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 + v_A & 0 & v_A & 0 \\ 0 & 0 & 0 & 0 & 1 + v_B & 0 & v_B \\ \\ & 0 & ? & 1 & & 0 & \end{pmatrix} =$$

$$? = v_A p_B$$

Warm up

Half-gate technique [ZRE15]: $\underbrace{v_A}_{\substack{\text{color bit,}\\ \text{known by evaluator}}} = \overbrace{a}^{\text{input}} + \underbrace{p_A}_{\substack{\text{permutation bit,}\\ \text{known by garbler}}}$

$$\begin{pmatrix} \\ 0 \\ \varnothing \\ \varnothing \\ v_A \\ v_B \\ 1 \\ 1 \end{pmatrix}^T \begin{pmatrix} 1 & 0 & v_A & 0 & 0 & 0 & 0 \\ 0 & 1 & v_B & 0 & 0 & 0 & 0 \\ 0 & 0 & ab + p_A p_B & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 + ab + p_A p_B & 1 & 1 & 0 & 0 \\ 0 & 0 & p_B & 1 & 0 & 1 & 0 \\ 1 & 0 & p_A & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 + v_A & 0 & v_A & 0 \\ 0 & 0 & 0 & 0 & 1 + v_B & 0 & v_B \end{pmatrix} =$$

v_B	0	?	1	1	0	0

$$? = v_A p_B + v_B p_A$$

Warm up

Half-gate technique [ZRE15]: $\underbrace{v_A}_{\substack{\text{color bit,}\\ \text{known by evaluator}}} = \overbrace{a}^{\text{input}} + \underbrace{p_A}_{\substack{\text{permutation bit,}\\ \text{known by garbler}}}$

$$\begin{pmatrix} v_B \\ 0 \\ \varnothing \\ \varnothing \\ v_A \\ v_B \\ 1 \\ 1 \end{pmatrix}^T \begin{pmatrix} 1 & 0 & v_A & 0 & 0 & 0 & 0 \\ 0 & 1 & v_B & 0 & 0 & 0 & 0 \\ 0 & 0 & ab + p_A p_B & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 + ab + p_A p_B & 1 & 1 & 0 & 0 \\ 0 & 0 & p_B & 1 & 0 & 1 & 0 \\ 1 & 0 & p_A & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 + v_A & 0 & v_A & 0 \\ 0 & 0 & 0 & 0 & 1 + v_B & 0 & v_B \end{pmatrix} =$$

$$\begin{array}{ccccccc} 0 & 0 & ? & 1 & 1 & 0 & 0 \end{array}$$

$$\begin{aligned} ? &= v_A p_B + v_B p_A + v_A v_B \\ &= (a + p_A) p_B + (b + p_B) p_A + (a + p_A)(b + p_B) \\ &= (a + p_A) b + (b + p_B) p_A \\ &= ab + p_A p_B \end{aligned}$$

QED

Our Scheme

Observation: [ZRE15] obtains $ab + p_A p_B$ in a clever way:

1. reveal one time pads (additive secret shares) of inputs ($v_A = a + p_A$)
2. reconstruct $ab + p_A p_B$ linearly in p_A, p_B

Very similar to Beaver's technique to compute MULT gates [Bea91].

This can be extended:
one product $\Rightarrow$ any polynomial of degree d in n variables

Example

$$f(a,b,c,d) = ab + ac + ad + bc + bd + cd$$

$$\left(\begin{array}{cccc|c|cccccccc}
1 & 0 & 0 & 0 & v_A & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & v_B & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & v_C & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & v_D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & f(a,b,c,d) + f(p_A,p_B,p_C,p_D) & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 + f(a,b,c,d) + f(p_A,p_B,p_C,p_D) & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & p_B + p_C + p_D & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & p_C + p_D & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & p_D & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & \overline{v_A} & v_A & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \overline{v_B} & v_B & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \overline{v_C} & v_C & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \overline{v_D} & v_D
\end{array}\right)$$

Generalized half-gates

Theorem

Our scheme garbles any quadratic polynomial in n variables using n λ-bits.

Earlier example,

$$f(a, b, c, d) = ab + ac + ad + bc + bd + cd$$

can be garbled using 4 λ-bit strings.

Comparison 1

Trivial circuit C_1 for f:

$$ab + ac + ad + bc + bd + cd$$

6 AND gates $\Rightarrow$ 12 λ-bit strings required by [ZRE15] on C_1

Generalized half-gates

Theorem

Our scheme garbles any quadratic polynomial in n variables using n λ-bits.

Earlier example,

$$f(a, b, c, d) = ab + ac + ad + bc + bd + cd$$

can be garbled using 4 λ-bit strings.

Comparison 2

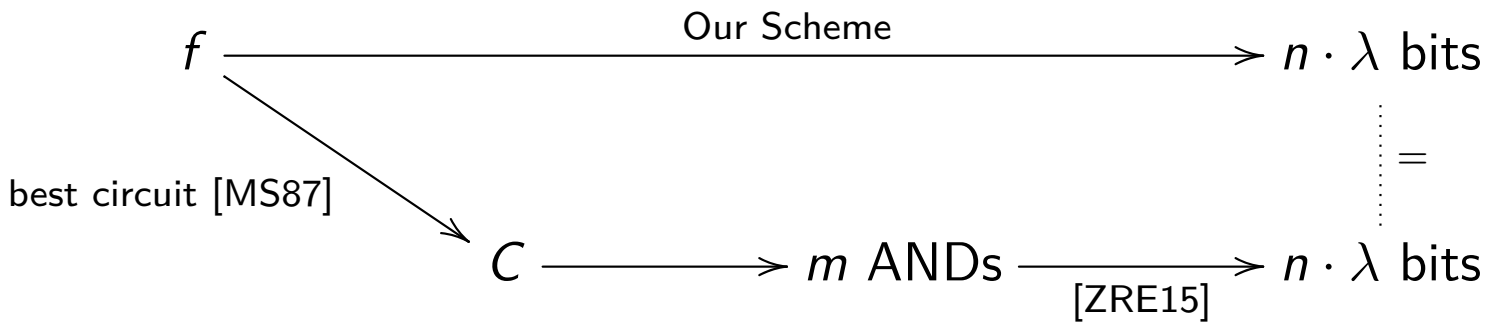
Best circuit C_2 for f:

$$(a + b + c)(a + d) + bc + a$$

2 AND gates $\Rightarrow$ 4 λ-bit strings required by [ZRE15] on C_2

Generalized half-gates

For quadratic polynomial f over $n = 2m$ variables:

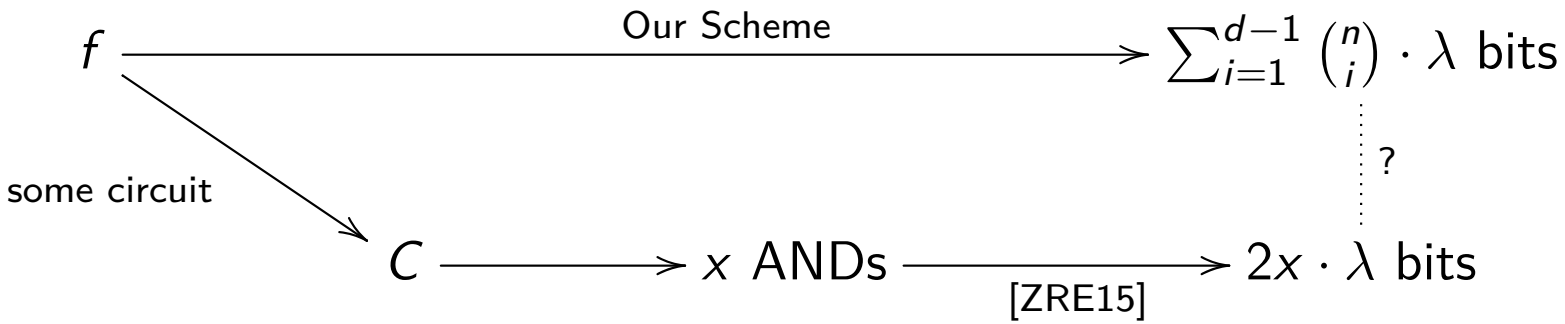


Generalized half-gates

Theorem

We garble any polynomial of degree d in n variables using $\sum_{i=1}^{d-1} \binom{n}{i}$ λ-bits.

In general? (f = degree d polynomial over n variables)



Random constant-degree d polynomial over n variables $\Rightarrow$
$\Rightarrow$ better communication complexity than [ZRE15], but
comparisons depend on C... generally hard to determine AND complexity

Summary, Sneek Peak, and Extras

- New framework: simple span properties $\Rightarrow$ sim-based security
- New boolean garbling scheme (proof in the above framework)
  - not gate-by-gate, garbles polynomials rather than circuits
  - can circumvent comm. complexity lowerbound for linear garbling
  - **calls to RO in each unit performed parallel (1 vs d)**
- New arithmetic garbling scheme (for small finite fields)
- Similar technique to improve Beaver-based MPC
- Non-linear garbling?

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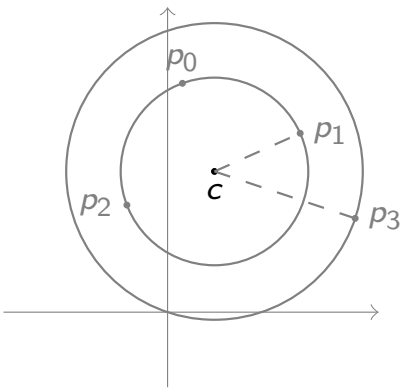

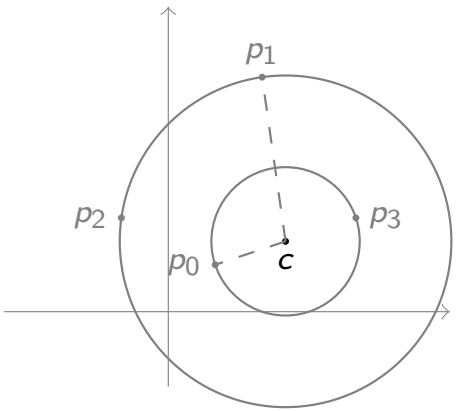

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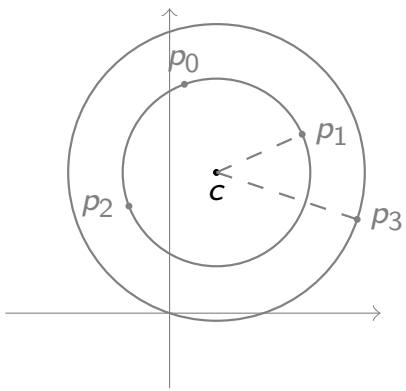
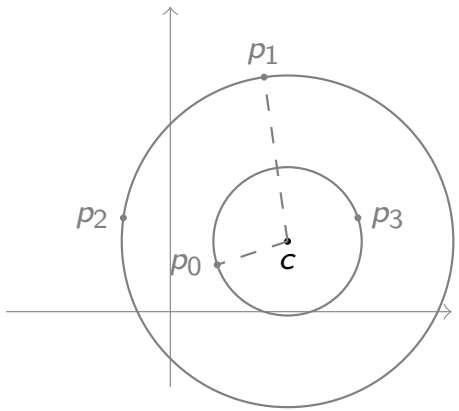


THANKS!



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