2015-05-04 Overlapping qubits

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OVERLAPPING QUBITS

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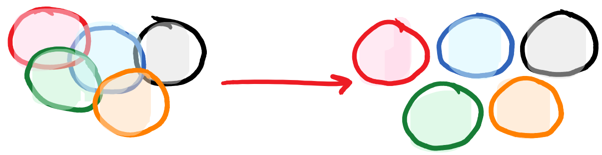
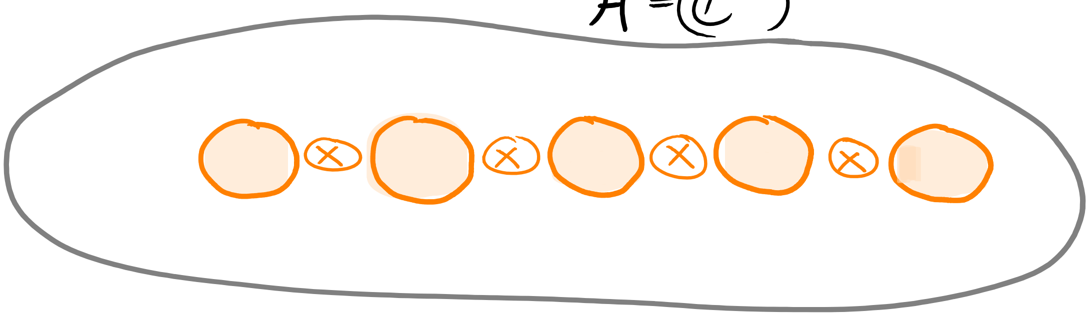


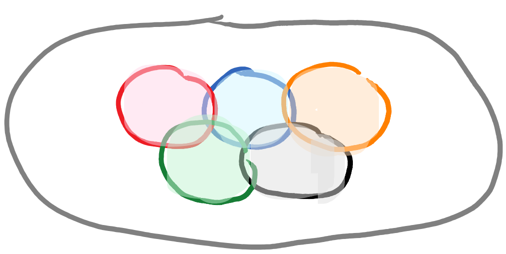
Quantum Hamiltonian Complexity Reunion Workshop
Berkeley, CA 5/4/2015

Quantum information theory with overlapping qubits

$\mathcal{H} = (\mathbb{C}^2)^{\otimes n}$

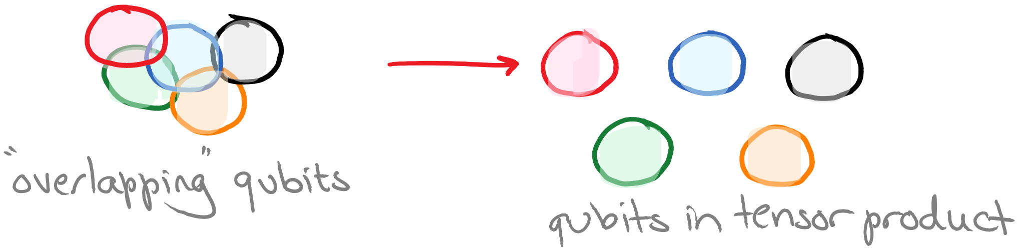


$\mathcal{H}$



Problem:

How to "pull apart" qubits that are almost in tensor product...?

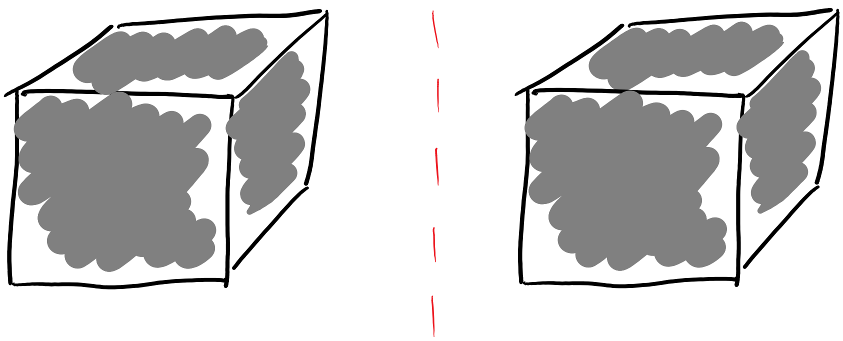


qubits: X, Y, Z $\quad X, Y, Z$ $\quad X, Y, Z$

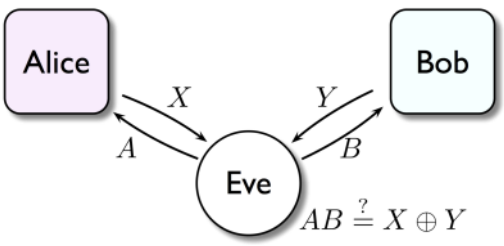
X, Y, Z $\quad X, Y, Z$

Motivation:

Goal: Establish substantial entanglement (with structure and control) between two untrusted devices—by giving them black-box tests.



Approach: Repeated CHSH games



- If devices pass the tests (w.h.p.), then they must share lots of entanglement, which they measure in a very particular way

MAIN THEOREM: [R., Unger, Vazirani '13]

$$\underbrace{\underbrace{\times\ \times\ \times\ \times}_{n\text{ games}} \quad \underbrace{\times\ \times\ \times\ \times}_{n\text{ games}} \quad \underbrace{\times\ \times\ \times\ \times}_{n\text{ games}} \quad \cdots \quad \underbrace{\times\ \times\ \times\ \times}_{n\text{ games}}}_{\substack{N \text{ blocks of } n \\ \text{sequential CHSH games}}}$$

- If: $\mathbb{P}[\text{win} \geq \omega^* - \varepsilon \text{ of games}] \geq 1 - \varepsilon$
- Then: At the beginning of a random block of n games,

  Alice & Bob's strategy for those games $\approx$ Ideal strategy on $(|00\rangle + |11\rangle)^{\otimes n}$

  pair j for game j

Proof idea:

Start with arbitrary shared state, arbitrary strategy.

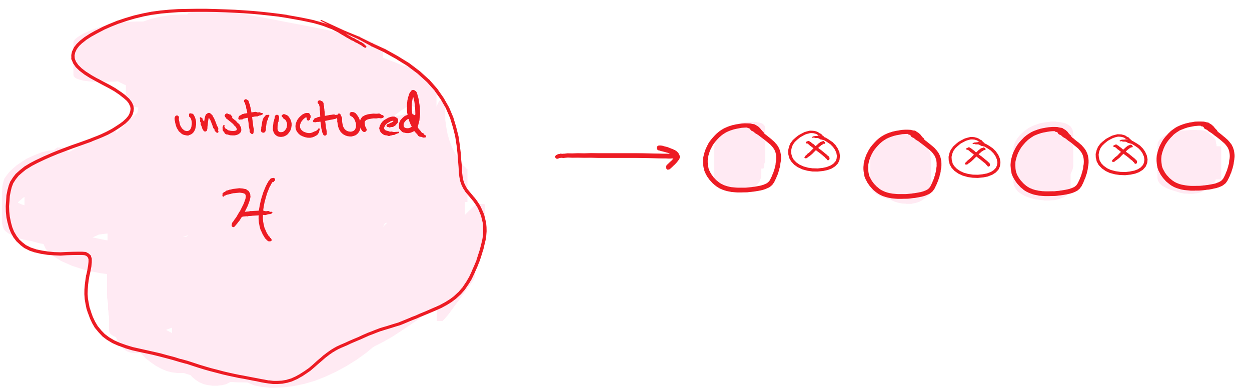
① Find subsequence of sequential games so

$\forall j$, $\mathbb{P}[\text{win game } j] \approx \omega^*$
regardless of games 1 to $j-1$

2. Find near-EPR pairs,
move them into tensor product,
glue together history dependence,
replace with perfect EPR pairs,
...

Big picture:

(operational, approximate)
Independence assumption $\longrightarrow$ EPR pairs in tensor product



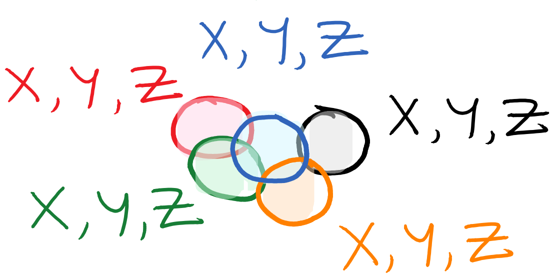
This gives lots of entanglement, in a very nice form
but inefficiently: $N = \mathrm{poly}(n)$,
final error $= \varepsilon^{1/c}$.

Open question:

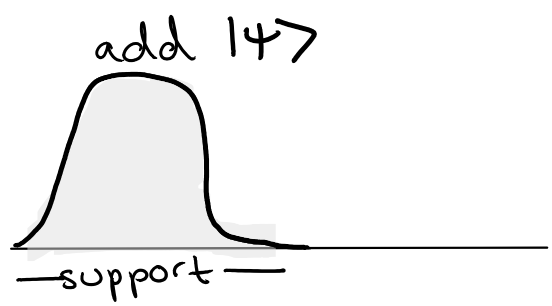
- Make the test more efficient:
  Can we determine a tensor-product structure even starting with a constant noise rate?

Outline: State-independent separation
State-dependent separation

State independent:



State dependent:



(State-independent)

Separation of overlapping qubit operators

Given: Pauli operators

$$\sigma_x^1, \sigma_z^1 \quad , \quad \dots \quad , \sigma_x^n, \sigma_z^n$$

almost commuting:

$$\|[\sigma_\alpha^i, \sigma_\beta^j]\| < \varepsilon$$

Goal: Find nearby operators (possibly on an extended space) so

$$[\tilde{\sigma}_\alpha^i, \tilde{\sigma}_\beta^j] = 0,$$

i.e., operators for qubits in tensor product.

Claim: $\Theta(n\varepsilon)$ movement is sufficient & necessary.

$$\boxed{\text{Nearby} = \Theta(n \cdot \varepsilon)}$$

Remark: **Related work** (more sophisticated)

Lin '95: almost-commuting Hermitian pairs close to commuting Hermitians

if $\|[A, B]\| < \varepsilon$, $\exists$ $A' \approx_\delta A$, $B' \approx_\delta B$

A, B Hermitian; A', B' Hermitian, $[A', B'] = 0$

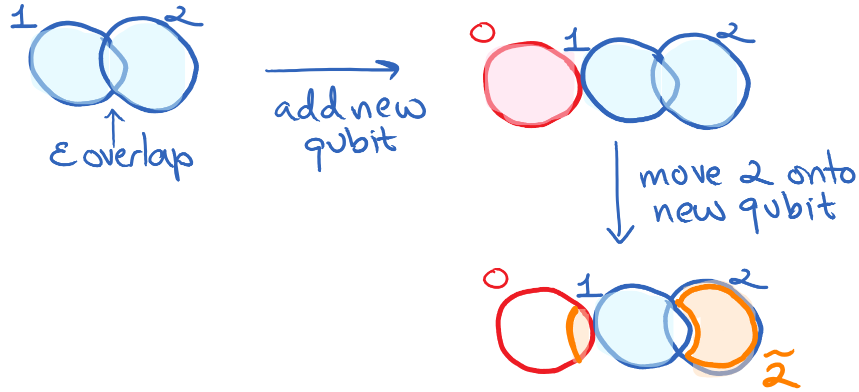
Hastings '10: $\delta = \tilde{O}(\varepsilon^{1/5})$. dimension-independent!

Hastings '10: $\delta = O(\varepsilon^{\cdot\cdot})$. dimension-independent!

- false for almost-commuting unitary pairs
- false for almost-commuting Hermitian triples
  (no dimension-independent bound)
- but Paulis have much more structure
  Hermitian & unitary $\Rightarrow$ e-values ± 1

Upper bound: $O(n\varepsilon)$ is enough.

Proof: • 2 qubits



$$\tilde{\sigma}_2^2 = SWAP_{0,1}\, \sigma_2^2\, SWAP_{0,1} \approx \sigma_2^2$$

• n qubits
- $SWAP_{0,1}$ to fix qubits 2 to n
- $SWAP_{0',2}$ to fix qubits 3 to n
- $SWAP_{0'',3}$ for qubits 4 to n

⋮

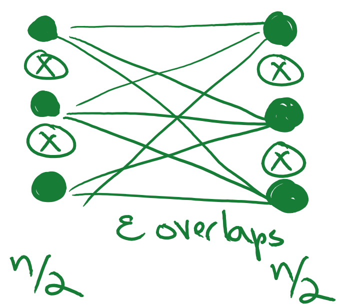
$$\tilde{\sigma}_2^2 = SWAP_{0,1}\, \sigma_2^2\, SWAP_{0,1}$$ same σ_2

$\tilde{\sigma}^3_\alpha = SWAP_{0,1}\, \sigma^3_\alpha\, SWAP_{0,1}$ same commutator $\to \sigma_3$

$\Rightarrow \sigma^n$ moves $n\varepsilon$ total ✓

Lower bound: $\Omega(n\varepsilon)$ is sometimes required

Idea:



$\Omega(n)$ qubits need to be moved by $\Omega(n\varepsilon)$?

Proposal 1: In $(\mathbb{C}^2)^{\otimes n/2}$,

● ⊗ ● ⊗ ● usual qubits

$\mathcal{U}$ ● ⊗ ● ⊗ ● $\mathcal{U}^\dagger$ same, conjugated by a random unitary

Intuition: Overlap $\|[\sigma^i_\alpha, \sigma^{n/2+j}_\beta]\| \approx 1/(n/2)$??

Wrong! **Fact:** In $(\mathbb{C}^2)^{\otimes n}$, for random unitary $\mathcal{U}$,

$$\|[\sigma^1_\alpha, \mathcal{U}\sigma^1_\beta\mathcal{U}^\dagger]\| \approx 2 \text{ (maximal)}$$

Moral: Qubit overlap is not "monogamous".

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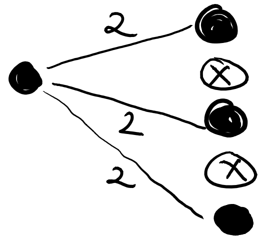


huge overlaps $\Rightarrow$ need more structure

Proposal 2: In $(\mathbb{C}^2)^{\otimes n}$

perturb, e.g., by

$e^{-i\varepsilon H}$, $H = \sum_{j=1}^{n/2} X_j$

$\Rightarrow \tilde{X}_j = X_j$, $\tilde{Z}_j \approx Z_j + \varepsilon Y_j \otimes \sum_{i=1}^{n/2} X_i$

Track total commutator

$$C = \sum_{\substack{i \neq j \\ \alpha, \beta}} \| [\sigma^i_\alpha, \sigma^j_\beta] \|_F$$

Frobenius norm

$\approx \left(\frac{n}{2}\right)^2 \times \varepsilon$ initially

Claim: Move qubit by δ (spectral norm)

$\Rightarrow C$ changes by $O(\delta)$.

$\Longrightarrow \Omega(n^2 \varepsilon)$ total change needed to make $C = 0$. ✓

Proof:

$$\sum_{i=1}^{n/2} \left\| \left[Z_k + \varepsilon Y_k \otimes \sum_{j=1}^{n/2} X_j + \Delta, Z_i \right] \right\|_F$$

$$= \sum_{i \in [n/2]} \| [\varepsilon Y_k \otimes X_i, Z_i] + [\Delta, Z_i] \|_F$$

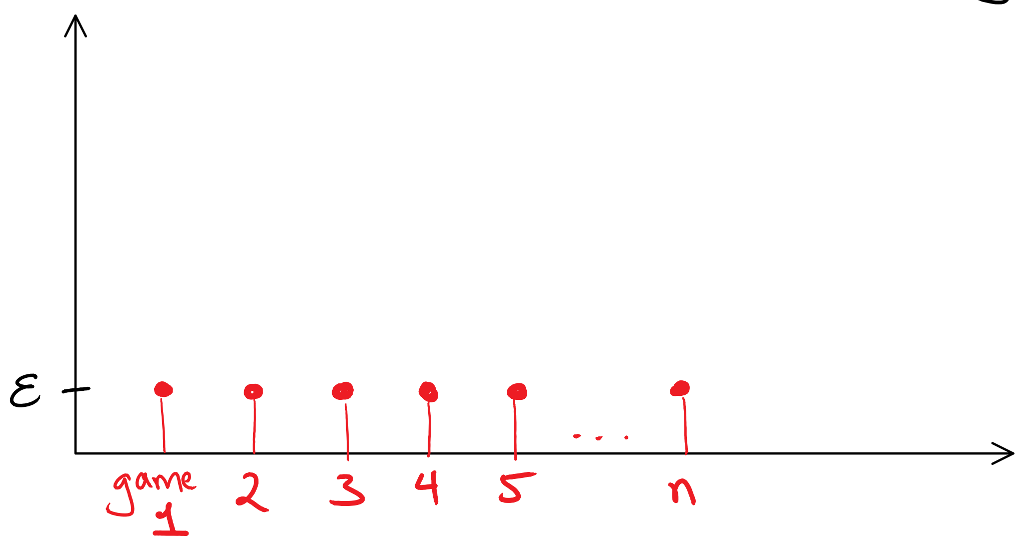
$\rightarrow$ best choice $\Delta = -\frac{f}{n} Y_k \otimes \sum_{i \le n/2} X_i$

$\approx n\left(\varepsilon - \frac{f}{n}\right) = n\varepsilon - f$ ✓

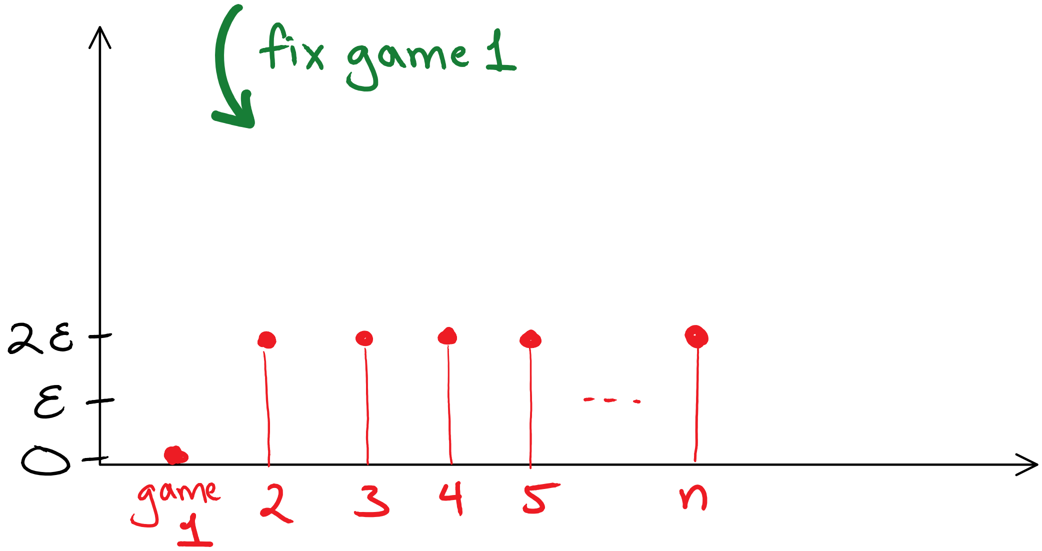
(State-dependent)

Separation of overlapping qubits

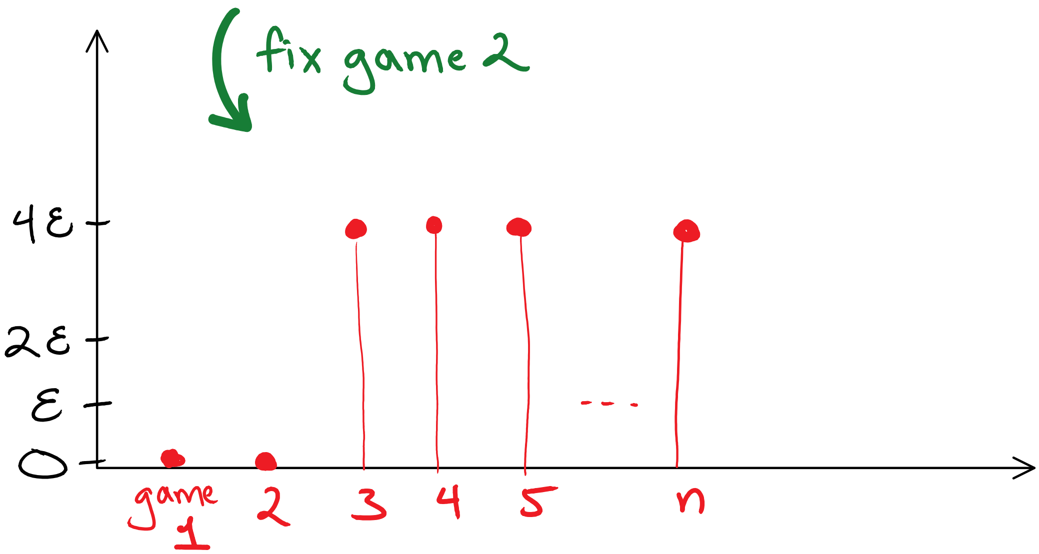
Goal: Understand error accumulation in analysis



fix game 1



fix game 2

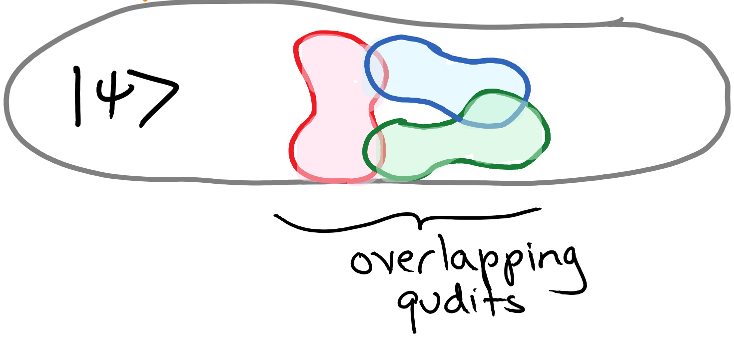


Possible theorem statements

Must we lose a factor of n?

Does fixing one qubit help/hurt the others?

General problem:

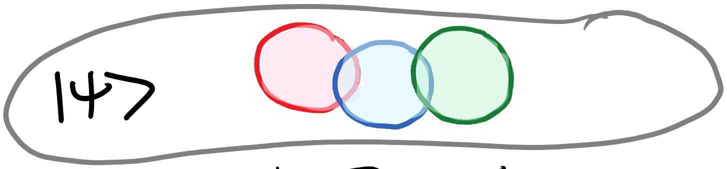


Given:
$$\mathrm{Tr}_{-A}|\psi\rangle\langle\psi| \approx \rho^*$$
$$\mathrm{Tr}_{-B}|\psi\rangle\langle\psi| \approx \rho^*$$
$$\mathrm{Tr}_{-C}|\psi\rangle\langle\psi| \approx \rho^*$$
ideal states ρ^*

Goal: Understand effect of "fixing" $|\psi\rangle$ so
$$\mathrm{Tr}_{-A}|\psi\rangle\langle\psi| = \rho^*$$
(changing state, not qubits)

Example: $\rho^* =$ EPR pair $|00\rangle + |11\rangle \in \mathbb{C}^2 \otimes \mathbb{C}^2$

Toy problem: Depolarizing qubits



$\underbrace{\hphantom{xxxxxxxx}}_{\text{overlapping qubits}}$

$\rho^* = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ maximally mixed state

"fixing" a qubit in $|\psi\rangle$ $\longleftrightarrow$ depolarize it

Question:

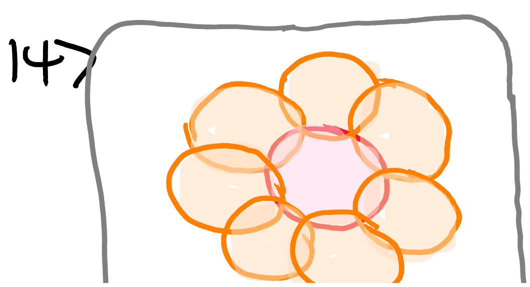
Does depolarizing one qubit help or hurt the others?

help → make more mixed/noisy

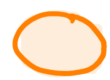
hurt → make less noisy (purify)

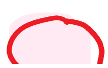
Intuition?: Adding noise on one qubit can only make overlapping qubits more noisy.

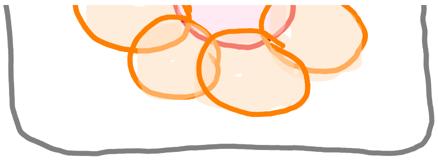
Claim:



 $= \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ initially

$\Big\downarrow$ depolarize  qubits

 $= |0\rangle$ finally



$\circ = |0\rangle$ finally (arbitrarily close to) pure

Moral: Adding noise to some qubits can purify others.

Proof:

$|\psi\rangle = |00\rangle + |11\rangle$, $\circ$ = first qubit initially $\frac{1}{2}\begin{pmatrix}1&0\\0&1\end{pmatrix}$

1) Depolarize $\circ$ $\quad X_L = XX$, $Z_L = 1Z$

$\longrightarrow \frac{1}{2} : |00\rangle + |11\rangle$

$\frac{1}{2} : |00\rangle - |11\rangle$

eigenvalues $\begin{pmatrix}1/2 & & 0\\ & 1/2 & 0 \\ 0 & & 0\end{pmatrix}$

$= \frac{1}{2}\begin{pmatrix}1&0\\0&1\end{pmatrix} \otimes |0\rangle\langle 0|$ in **some** basis

2) (Generalized) Zeno effect:

Any state $\frac{1}{2}, \frac{1}{2}, 0, 0$ $\xrightarrow{\text{slowly rotating depolarizations}}$ Any other $\frac{1}{2}, \frac{1}{2}, 0, 0$

Analogous to Zeno: pure $\xrightarrow[\text{measurements/dephasing}]{}$ pure

✓ □

More problems:

- Extend analysis to more states
- Connect to operational assumptions, eg., parallel CHSH games with constant noise