

APPLICATIONS OF (INDISTINGUISHABILITY) OBFUSCATION

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Definition of iO [B⁺01]

- An indistinguishability obfuscator is a PPT algorithm iO that takes a program P as input and is:
 - ▣ **Efficient**: Description/runtime of $iO(P)$ are poly-related to P .
 - ▣ **Functionality-Preserving**: The string $iO(P)$ describes a program with the same input-output behavior as P .
 - ▣ **Pseudo-Canonicalizing**: For any PPT adversary A and any programs P_1 and P_2 of equal complexity and functionality:

$$\left| \Pr [A(iO(P_1))=1] - \Pr [A(iO(P_2))=1] \right| \text{ is negligible.}$$

- ▣ In English: If two programs have same input-output behavior, the adversary cannot distinguish which was obfuscated.
- Circuits: Usually our model of computation.

Plant of the Talk

Amazing Crypto Applications

VBBeanstalk

[BGI+01]

Does not exist.

diO

Subexp

iO

FHE

iO

LWE

IBE

ABE

Short Sigs

PKE

MPC

ZK

Signatures

Warning: Not to scale!

Slide stolen from Elette's and Amit's garden.

Plant of the Talk



Amazing Crypto Applications

VBBeanstalk
[BGI+01]
Does not exist.



diO



Subexp
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iO



FHE

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Shor
Sigs



PKE



MPC

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Slide stolen from Elette's and Amit's garden.



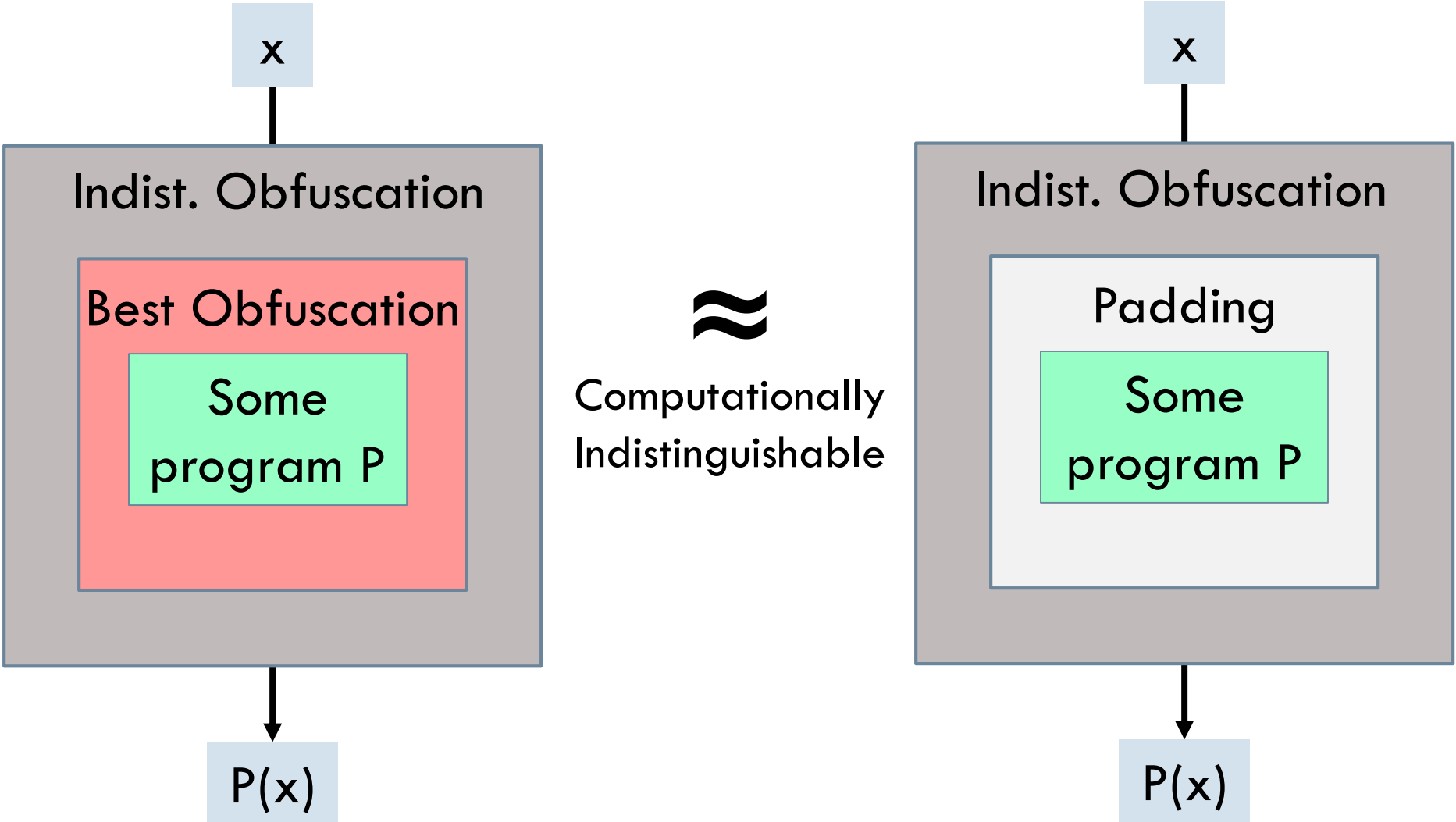
Simple Applications of iO

Best-Possible Obfuscation



An indistinguishability obfuscator is “as good”
as any other obfuscator that exists. [GR07]

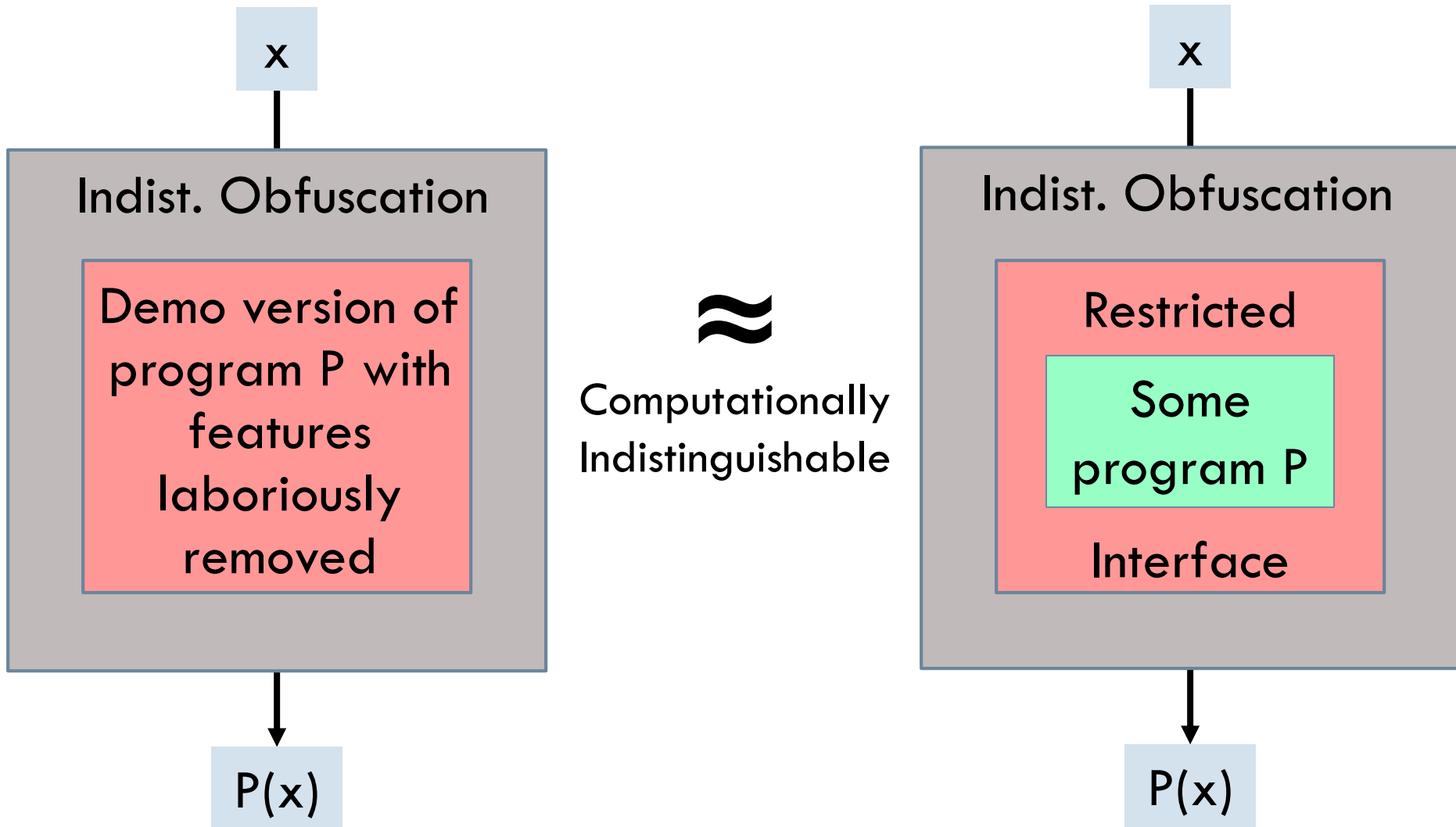
Best-Possible Obfuscation



Restricted-Use Software

- **Setting:** Software developer wants to:
 - ▣ Publish demo version with features removed
 - ▣ Construct multiple tiers of product at different prices
 - ▣ Give an untrusted partner a “dumbed-down” version that only works for relevant tasks
- **The problem:** Removing features is difficult.
 - ▣ Laborious, introduces bugs
 - ▣ End product may still reveal more than intended

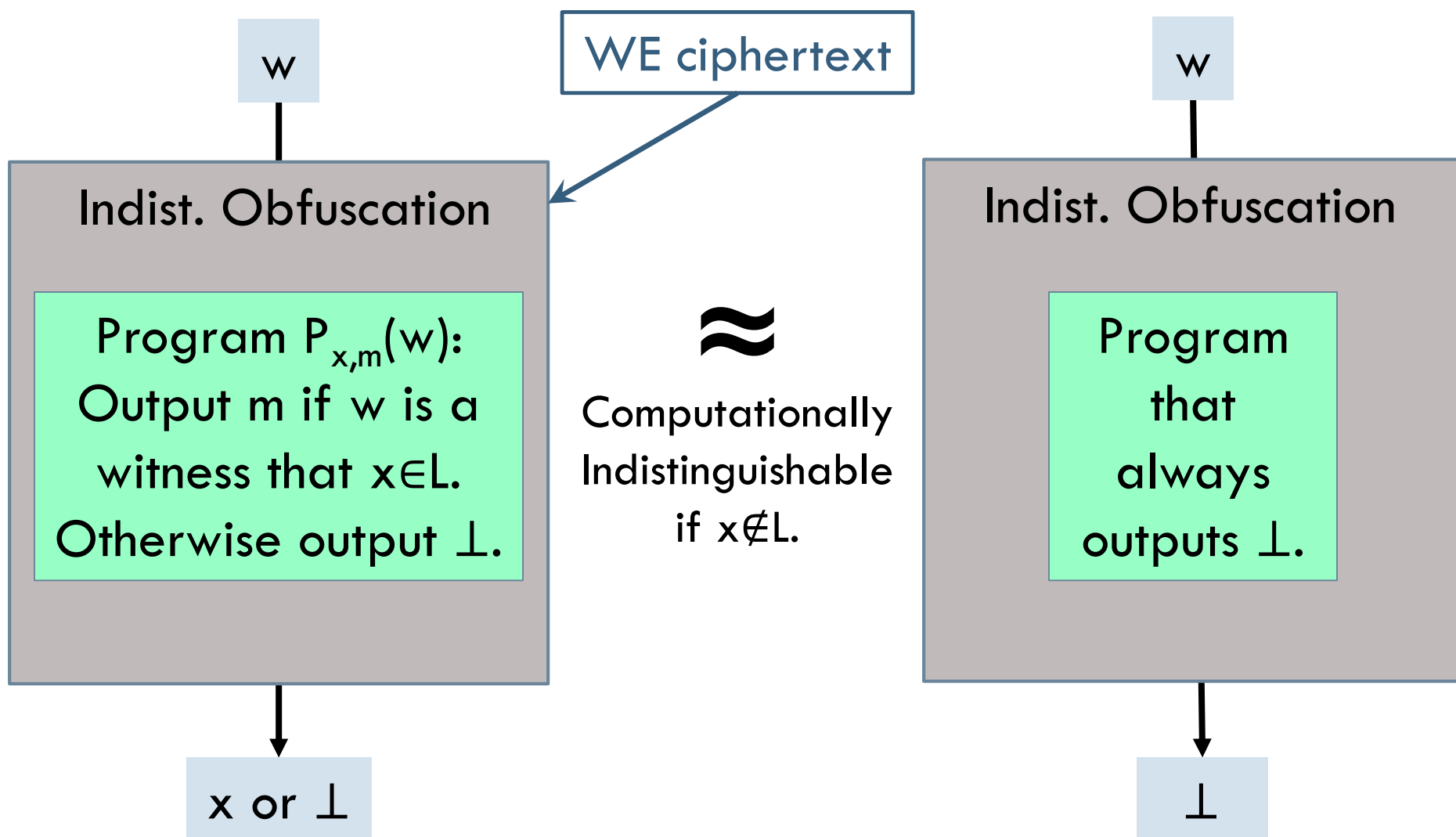
Restricted-Use Software from iO



Witness Encryption [R89,GK05,GGSW13]

- **Goal:** Encrypt m so only someone with proof of Riemann Hypothesis can decrypt.
- **Procedures:**
 - ▣ Encryption: $c \leftarrow \text{WEnc}(x;m)$ encrypts m relative to statement x .
 - ▣ Decryption: $\{m,\perp\} \leftarrow \text{WDec}(w;c)$ works if w is a witness for $x \in L$.
- **Secret key?:** No “secret key” per se.
- **Security:** $\text{WEnc}(x;m_0) \approx \text{WEnc}(x;m_1)$ when $x \notin L$.

Witness Encryption from iO [GGHRSW13]



Relative vs. Absolute Guarantees

- Apps above have weak “relative” security guarantees:
 - ▣ BPO: Obfuscation is as good as best-possible obfuscation
 - ▣ Restricted-use software: As good as restricted interface.
 - ▣ WE: No guarantees when $x \in L$.
- How to get absolute guarantees?
 - ▣ Make an absolute assumption – e.g., existence of OWFs.
 - ▣ But surely iO already implies OWFs...?

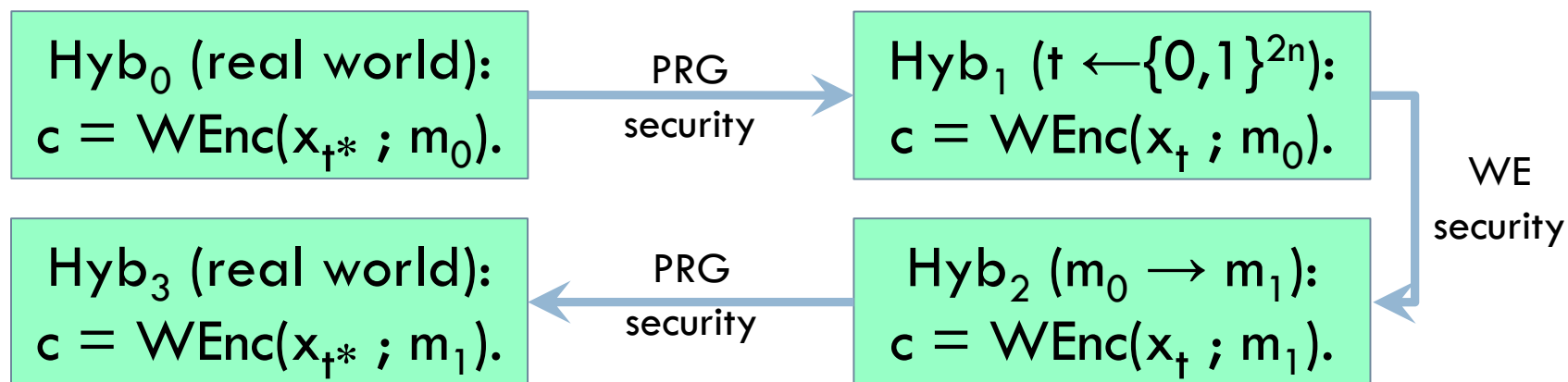
iO and OWFs

- iO \rightarrow OWFs?
 - ▣ No, if $P = NP$.
 - ▣ iO exists if $P = NP$: Obfuscate program by outputting lexicographically first program with same functionality.
- iO \rightarrow OWFs if $NP \not\subseteq BPP$ [KMNPY14].
 - ▣ Candidate OWF: $f(x) = iO(Z;x)$ where Z is unsatisfiable.
 - ▣ Replace challenge with $y_1 = iO(C_1;x_1)$ for **unsatisfiable** C_1 .
By iO, adversary cannot distinguish, and will still invert.
 - ▣ Replace challenge with $y_2 = iO(C_2;x_2)$ for **satisfiable** C_2 .
Adversary cannot invert, since $\nexists x$ such that $f(x) = iO(C_2;x_2)$.
 - ▣ Adversary's success/failure tells us whether C is satisfiable.

Simple App: WE+OWF \rightarrow PKE [GGSW13]

Super-fast
KeyGen!

- KeyGen: $\text{PRG} : \{0,1\}^n \rightarrow \{0,1\}^{2n}$.
 - ▣ Secret key: $s^* \in \{0,1\}^n$. Public key: $t^* = \text{PRG}(s^*)$.
- Encryption:
 - ▣ Let x_{t^*} be the statement “ $\exists s$ such that $t^* = \text{PRG}(s)$ ”.
 - ▣ $c \leftarrow \text{WEnc}(x_{t^*}; m)$.
- Decryption: $m \leftarrow \text{WDec}(s^*; c)$.



Other Apps of WE (+ Simple Primitives)

- Identity-based encryption
- Attribute-based encryption for circuits
- Secret sharing for monotone NP access structures [KNY14]
- ...

Hiding Secrets in Software with iO

Two main techniques:

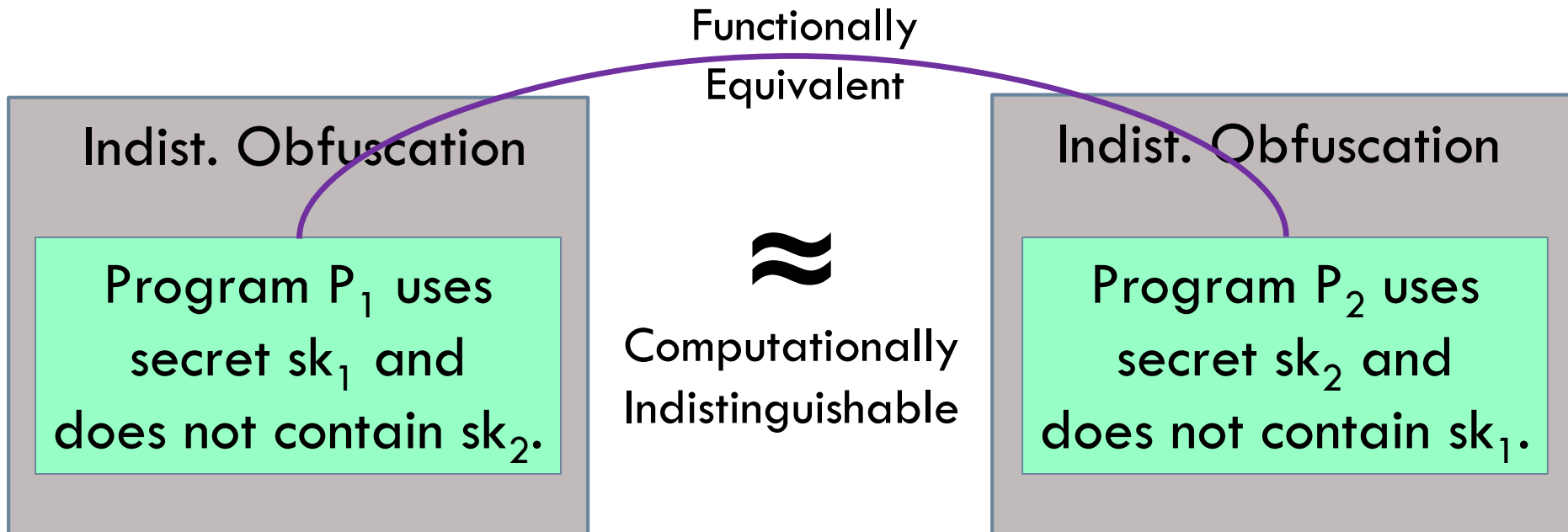
1. Shell games with secrets
2. Punctured programs



1st Technique: Shell Games with Secrets

Shell Games with Secrets

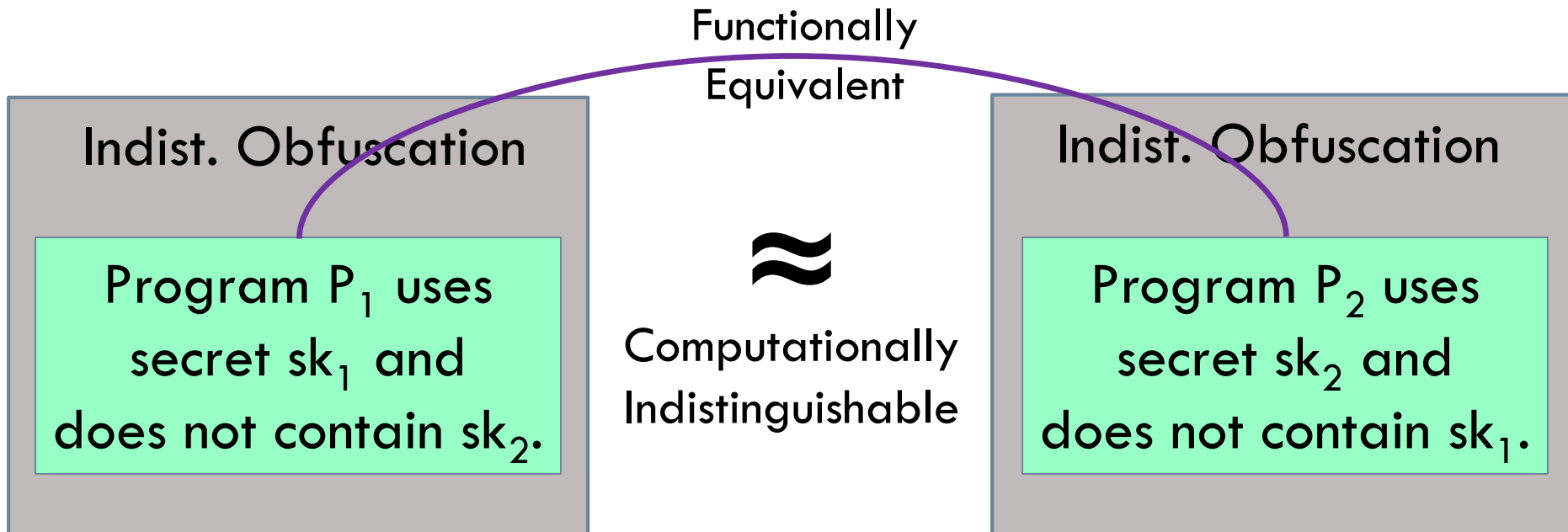
Thought Experiment



Does either obfuscation reveal sk_1 or sk_2 ?

Shell Games with Secrets

Thought Experiment

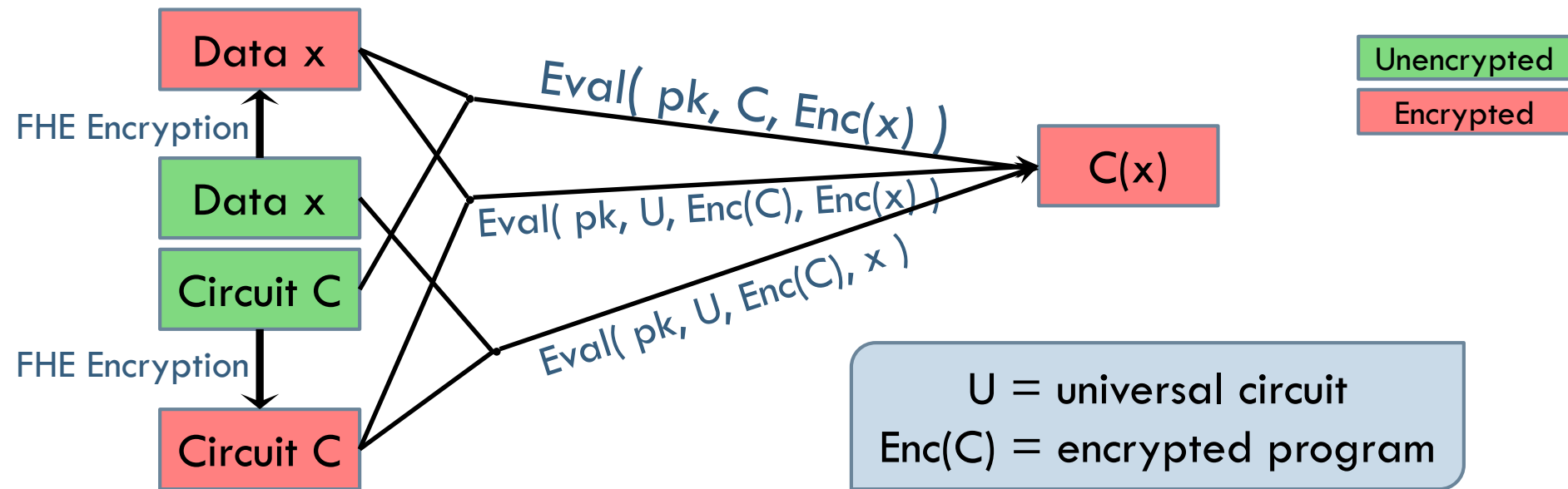


- ▣ P_1 hides sk_2 and P_2 hides sk_1 .
- ▣ But $iO(P_1) \approx iO(P_2)$.
- ▣ So, $iO(P_1)$ hides sk_1 and $iO(P_2)$ hides sk_2 .
- ▣ “Two-key technique” used many times before ([NY90], ...).

Shell Game Application: iO for Circuits

from (iO for NC^1) + (FHE with decryption in NC^1)

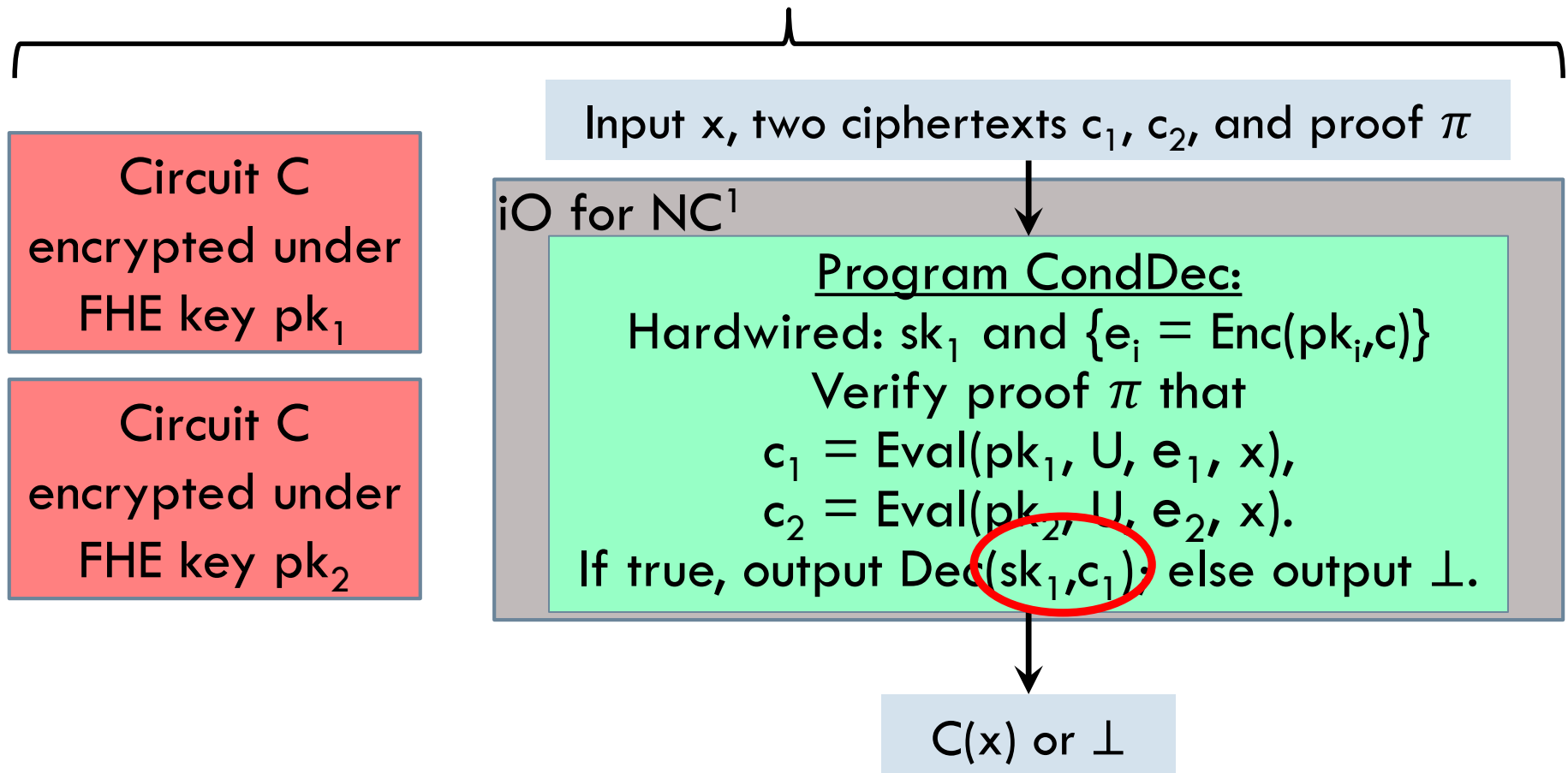
Reminder about FHE [RAD78, Gen09, ...]



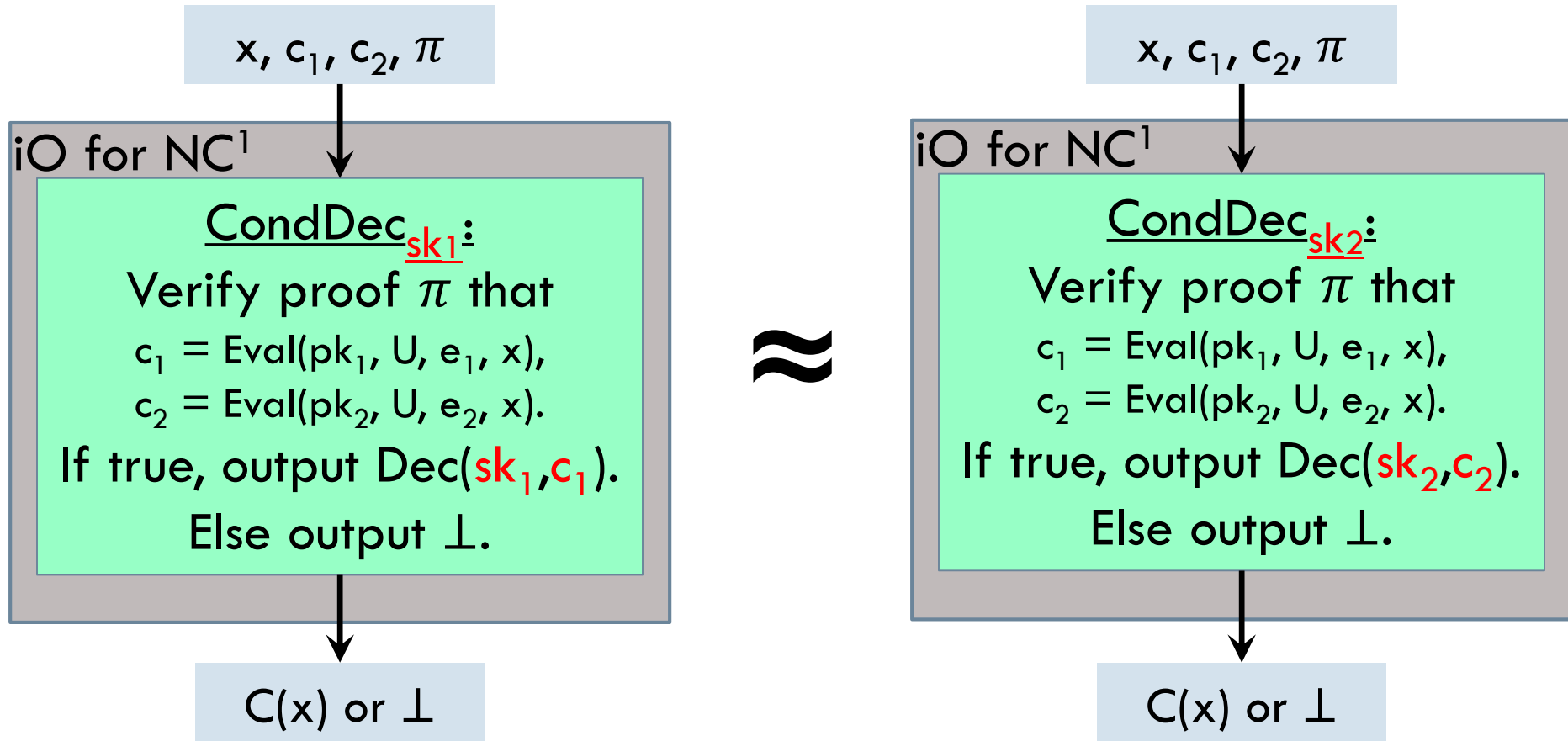
Current FHE schemes have decryption procedures that can be computed by shallow (NC^1) circuits.

Shell Game Application: iO for Circuits from (iO for NC^1) + (FHE with decryption in NC^1)

Obfuscation of General Circuit C [GGHRSW13]



Shell Game Application: iO for Circuits from (iO for NC^1) + (FHE with decryption in NC^1)

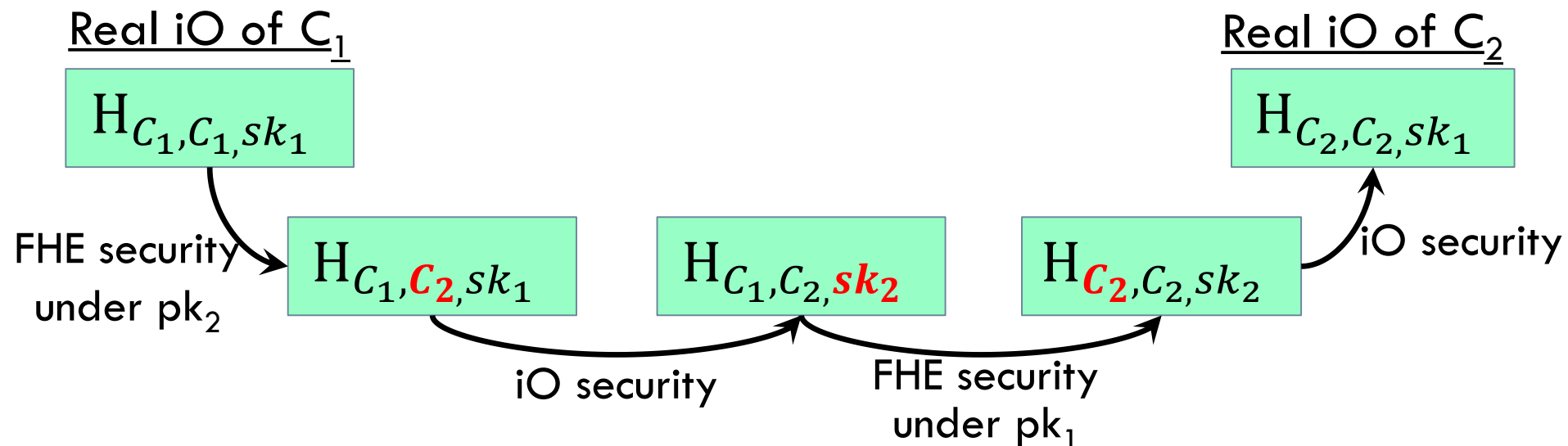


- CondDec_{sk1} and CondDec_{sk2} have same input-output behavior.
- So, their obfuscations are indistinguishable, and hide sk_1 and sk_2 .

Shell Game Application: iO for Circuits from (iO for NC^1) + (FHE with decryption in NC^1)

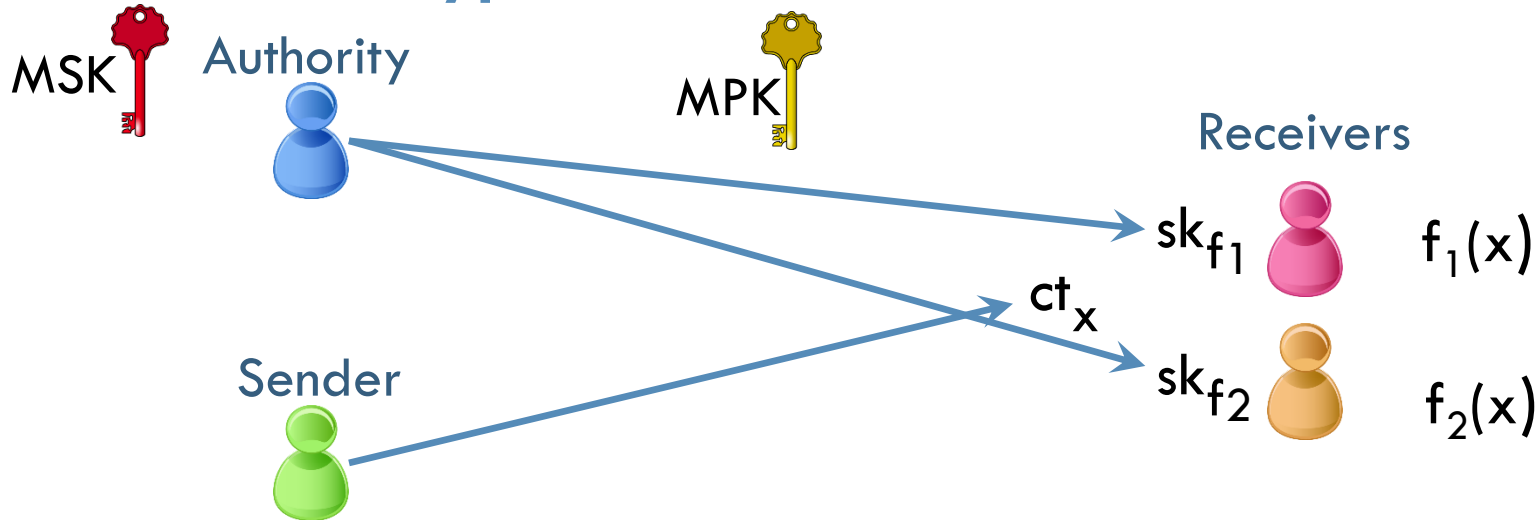
Security Proof for iO Scheme:

- Suppose circuits C_1, C_2 have same functionality.
- Hybrids: In $H_{C_{b_1}, C_{b_2}, sk_{b_3}}$ the obfuscation consists of:
 $e_1 = \text{Enc}(pk_1, C_{b_1}), e_2 = \text{Enc}(pk_2, C_{b_2}), \text{iO}(\text{CondDec}_{sk_{b_3}})$



Shell Game Application: Functional Encryption for Circuits

Functional Encryption [S84, SW05, BSW11, ...]

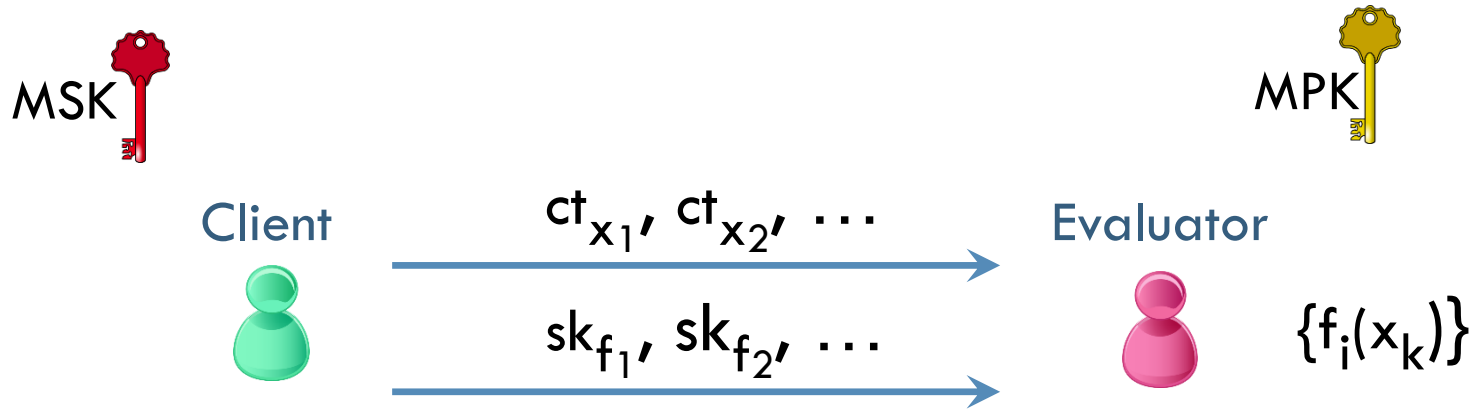


Syntax

- $(MPK, MSK) \leftarrow FE.Setup(1^\lambda)$
- $sk_f \leftarrow FE.KeyGen(MSK, f)$
- $ct_x \leftarrow FE.Enc(MPK, x)$
- $f(x) \leftarrow FE.Dec(sk_f, ct_x)$

Shell Game Application: Functional Encryption for Circuits

Functional Encryption [S84, SW05, BSW11, ...]



Syntax

- ▣ $(MPK, MSK) \leftarrow FE.Setup(1^\lambda)$
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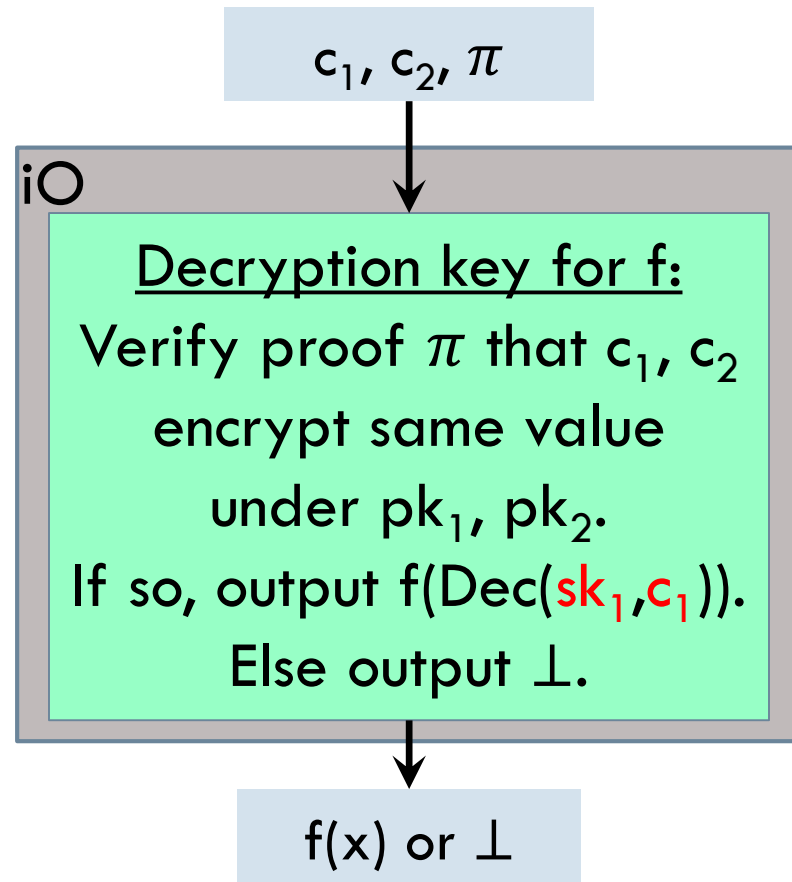
(Selective) Security Game

- ▣ Adversary selects x_1, x_2 .
- ▣ Challenger sends MPK, challenge ct .
- ▣ Key queries: for f_i such that $f_i(x_1) = f_i(x_2)$.
- ▣ Adversary guesses.

Shell Game Application: Functional Encryption for Circuits

Functional Encryption from IO [GGHRSW13]

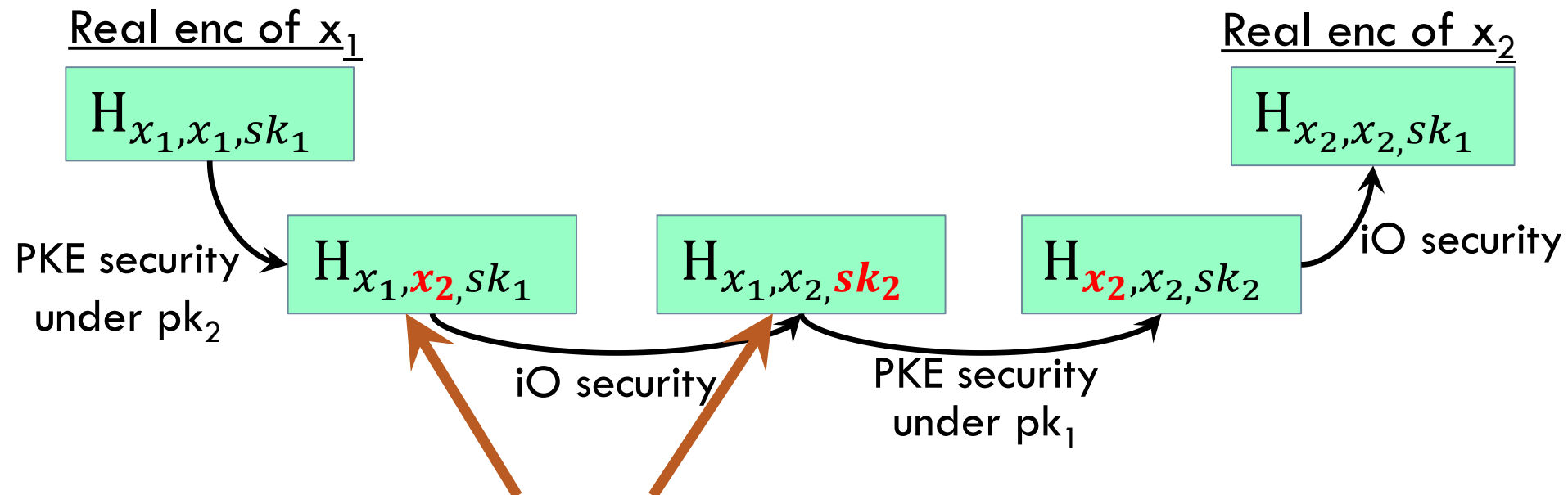
- FE.Setup: Generate:
 - ▣ PKE key-pairs $(pk_1, sk_1), (pk_2, sk_2)$.
 - ▣ CRS for stat. sim. sound NIZK proof.
- FE.Enc(MPK,x): Generate:
 - ▣ $c_1 \leftarrow \text{Enc}(pk_1, x; r_1)$,
 - ▣ $c_2 \leftarrow \text{Enc}(pk_2, x; r_2)$,
 - ▣ NIZK proof π that c_1, c_2 encrypt same value: that $\exists r_1, r_2$ s.t. $c_1 = \text{Enc}(pk_1, x; r_1), c_2 = \text{Enc}(pk_2, x; r_2)$.



Shell Game Application: Functional Encryption for Circuits

Security Proof for FE Scheme:

- Hybrids: In $H_{x_{b_1}, x_{b_2}, sk_{b_3}}$ the challenge ciphertext is c_1^*, c_2^*, π^* , where c_1^* encrypts x_{b_1} , c_2^* encrypts x_{b_2} , and user keys decrypt under sk_{b_3} .

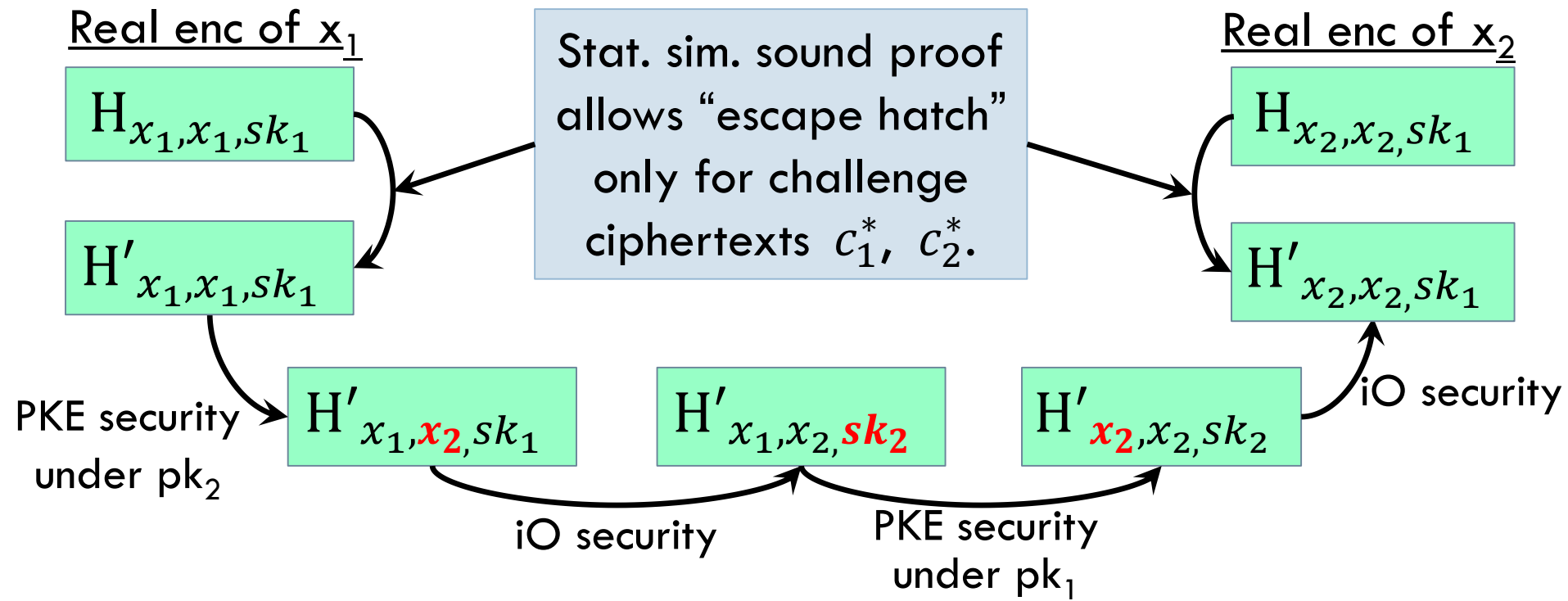


Oops!! NIZK proof cannot work in these hybrids!!!

Shell Game Application: Functional Encryption for Circuits

Security Proof for FE Scheme:

- Hybrids: In $H_{x_{b_1}, x_{b_2}, sk_{b_3}}$ the challenge ciphertext is c_1^*, c_2^*, π^* , where c_1^* encrypts x_{b_1} , c_2^* encrypts x_{b_2} , and user keys decrypt under sk_{b_3} .

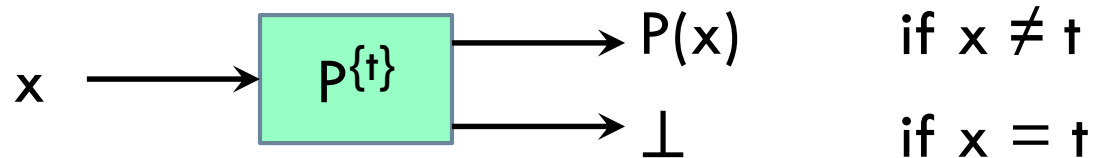




2nd Technique: Punctured Programs

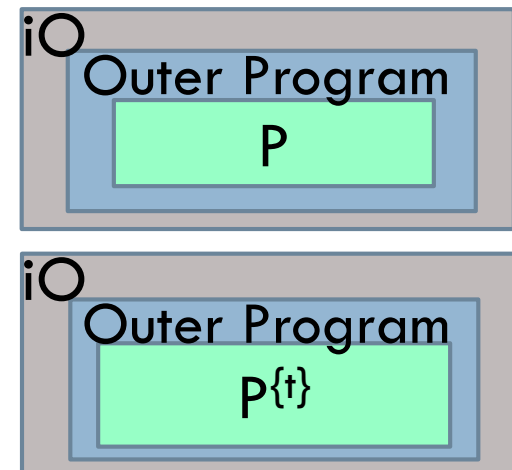
Punctured Programming [SW13]

Definition: $P^{\{t\}}$ is program P punctured at input t .



Punctured Programming Strategy:

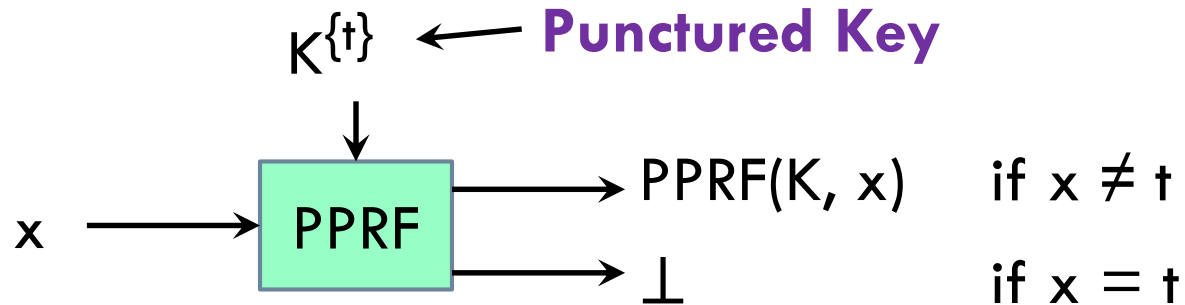
- Show iOs of two programs are indistinguishable.
- Show adversary needs $P(t)$ to win game.
- Show that $P^{\{t\}}$ keeps $P(t)$ secret.



“The idea of the technique is to alter a program (which is to be obfuscated) by surgically removing a key element of the program, without which the adversary cannot win the security game it must play, but in a way that does not alter the functionality of the program.”

Punctured PRFs

Definition:



Security: PPRF(K, t) is pseudorandom given $K^{\{t\}}$ and t.

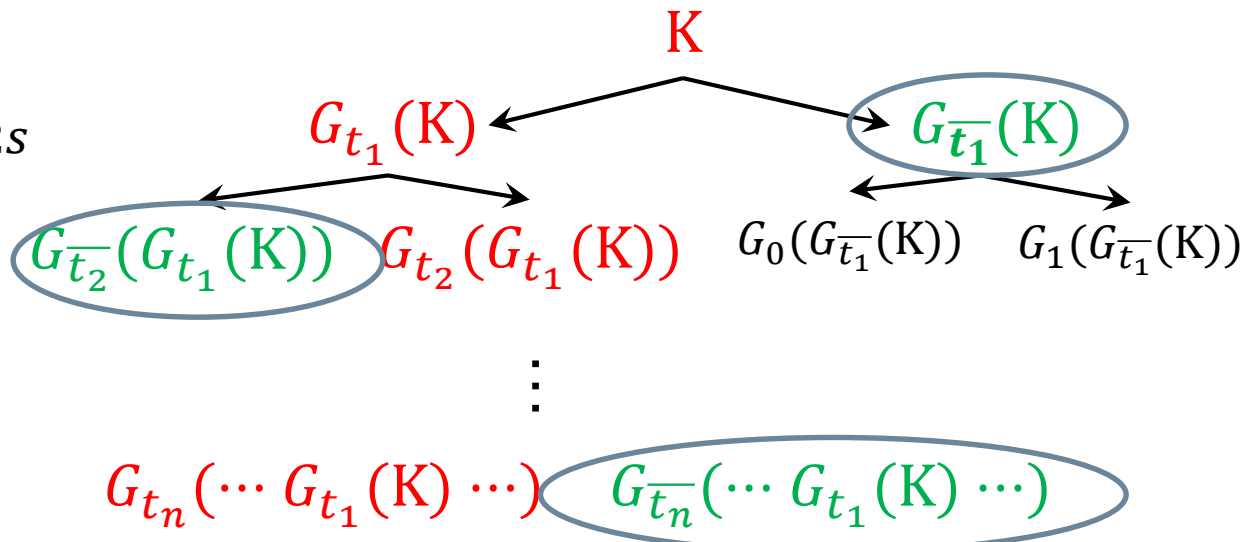
From GGM:

PRG $G : \{0,1\}^s \rightarrow \{0,1\}^{2s}$

$G(r) = G_0(r) \parallel G_1(r)$

PRF(K, x)

$= G_{x_n}(\dots G_{x_1}(K) \dots)$

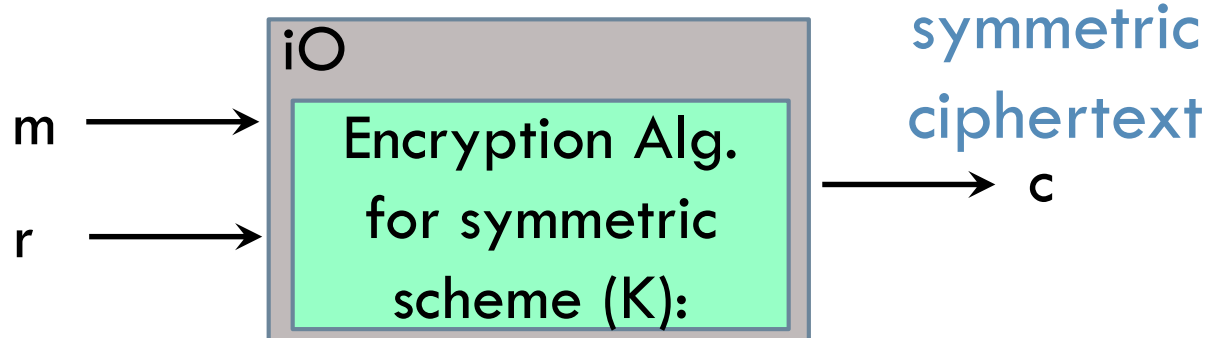


PKE Using Punctured PRFs [SW13]

Secret Key:

Key K for symmetric encryption

Public Key:



Diffie-Hellman '76: Get PKE by obfuscating encryption:

“If the [encryption] program were to be made purposefully confusing through the addition of unneeded variables and statements then determining an inverse algorithm could be made very difficult.”

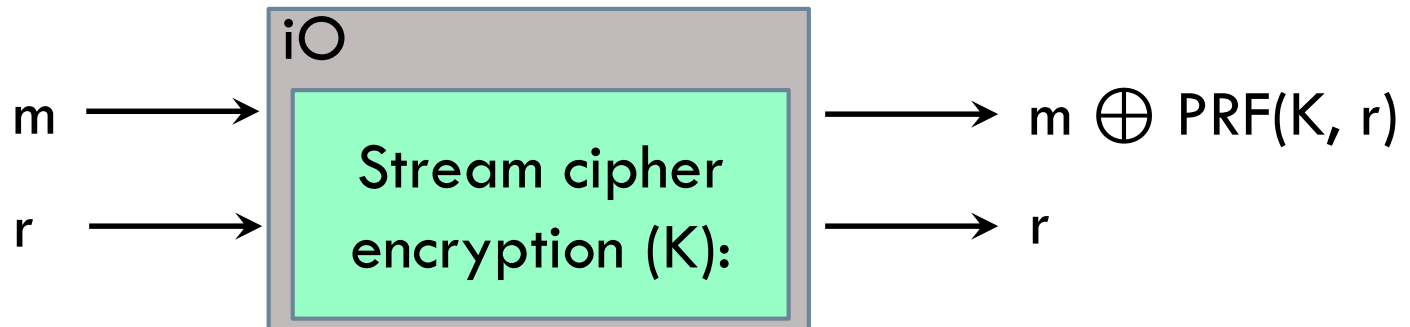
PKE Using Punctured PRFs [SW13]

Naïve Attempt

Secret Key:

Key K for PRF

Public Key:



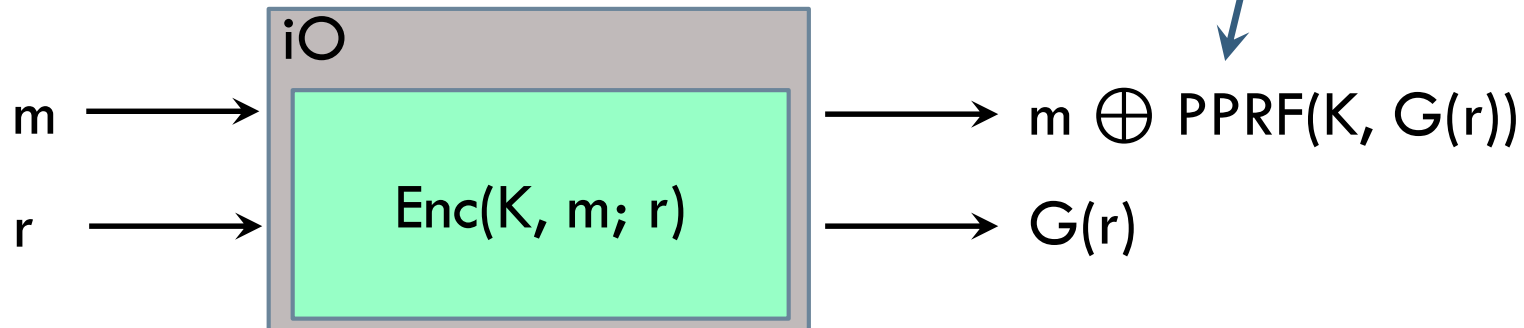
Problem: Stream cipher encryption is its own inverse!

PKE Using Punctured PRFs [SW13]

Secret Key:

Key K for PRF

Public Key: Let $G : \{0,1\}^s \rightarrow \{0,1\}^{2s}$ be a PRG.



Challenge ciphertext: $c^* = (t, m \oplus \text{PPRF}(K, t)), t = \text{PRG}(r)$

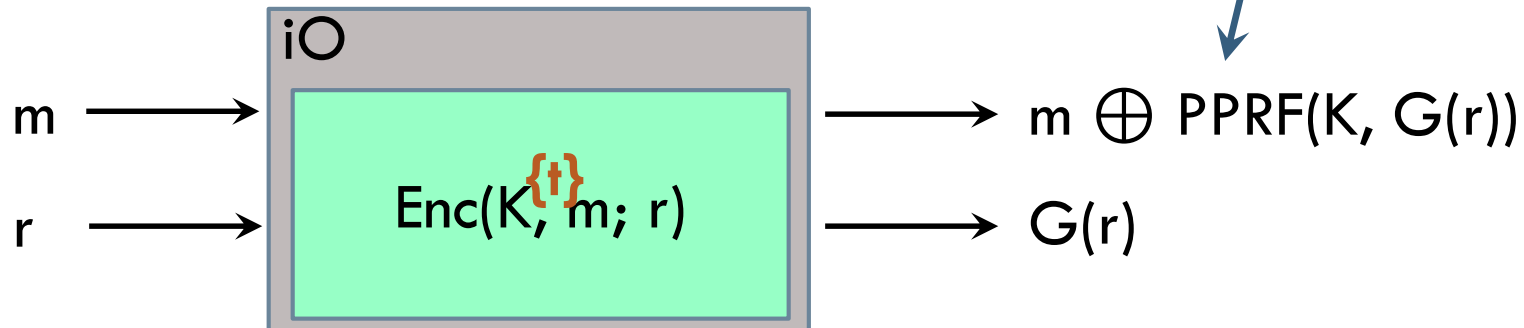
Make t uniform in $\{0,1\}^{2s}$. (PRG security)

PKE Using Punctured PRFs [SW13]

Secret Key:

Key K for PRF

Public Key: Let $G : \{0,1\}^s \rightarrow \{0,1\}^{2s}$ be a PRG.



Challenge ciphertext: $c^* = (t, m \oplus \text{PPRF}(K, t))$, t uniform in $\{0,1\}^{2s}$.

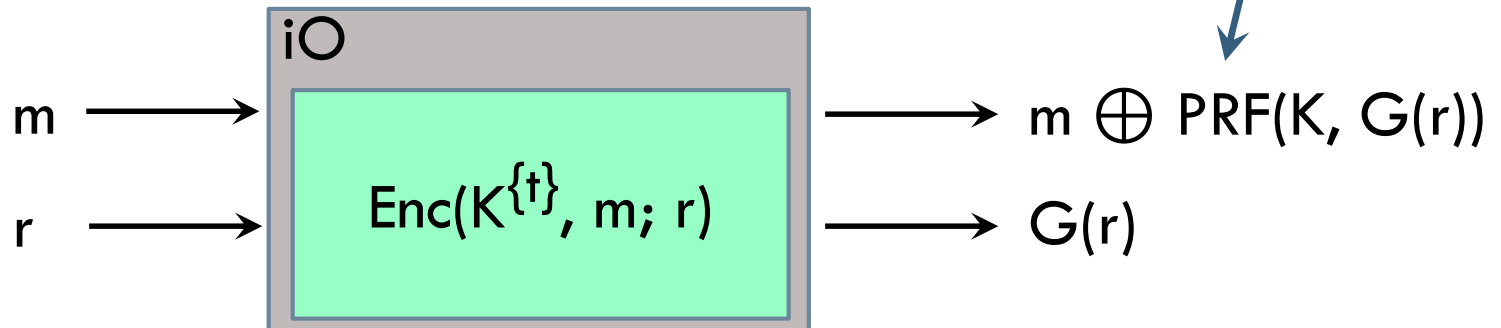
Use $\text{PPRF}(K^{\{t\}}, \cdot)$ instead of $\text{PPRF}(K, \cdot)$ inside Enc.
(iO security, since t is almost certainly not in range of G .)

PKE Using Punctured PRFs [SW13]

Secret Key:

Key K for PRF

Public Key: Let $G : \{0,1\}^s \rightarrow \{0,1\}^{2s}$ be a PRG.



Challenge ciphertext: $c^* = (t, m \oplus \text{PPRF}(K, t))$, t uniform in $\{0,1\}^{2s}$.

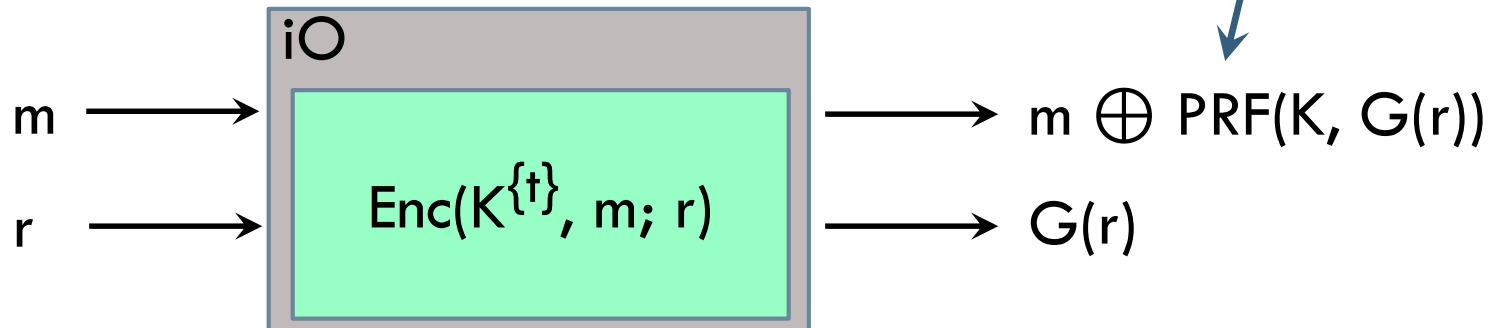
Replace $\text{PPRF}(K, t)$ with random value u . (Punctured PRF security)

PKE Using Punctured PRFs [SW13]

Secret Key:

Key K for PRF

Public Key: Let $G : \{0,1\}^s \rightarrow \{0,1\}^{2s}$ be a PRG.



Challenge ciphertext: $c^* = (t, m \oplus u)$, t uniform, u uniform.

Message is perfectly hidden.

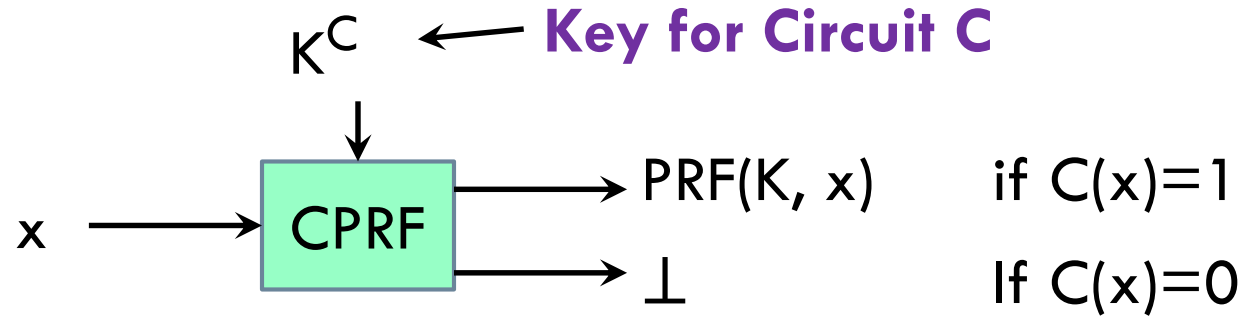
Another Useful Trick: Complexity Leveraging

- Construct “selectively secure” scheme.
 - Adversary forced to pre-commit to input $t \in \{0,1\}^k$ to “attack”.
 - Successful attack on t breaks iO or PPRF (or whatever).
 - $\epsilon_{\text{selective}}(\lambda) \leq \epsilon_{\text{iO}}(\lambda) + \epsilon_{\text{PPRF}}(\lambda)$.
- Go from selective security to adaptive security
 - Challenger randomly guesses t that adversary will target.
 - Probability that adaptive adversary wins and happens to pick t is $\epsilon_{\text{adaptive}}(\lambda)/2^k \leq \epsilon_{\text{selective}}(\lambda) \leq \epsilon_{\text{iO}}(\lambda) + \epsilon_{\text{PPRF}}(\lambda)$.
 - So $\epsilon_{\text{adaptive}}(\lambda) \leq 2^k (\epsilon_{\text{iO}}(\lambda) + \epsilon_{\text{PPRF}}(\lambda))$.
 - Choose $\lambda = \text{poly}(k)$ so that $\epsilon_{\text{adaptive}}(\lambda)$ is negligible.

Security at one input boosted to security at many inputs.

Constrained PRF

Definition:



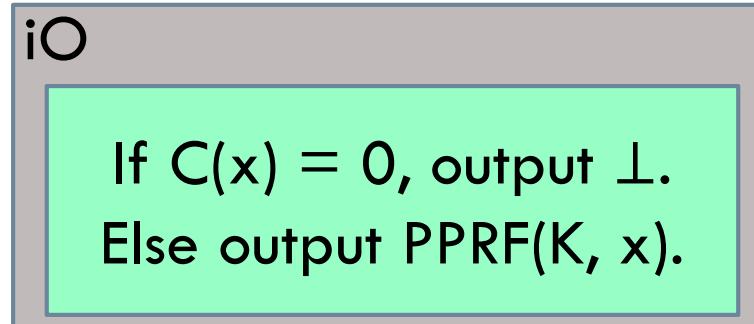
Security: PRF(K, x) is pseudorandom for all unsatisfying x.

Problem: How can we puncture the key at an exponential number of points?

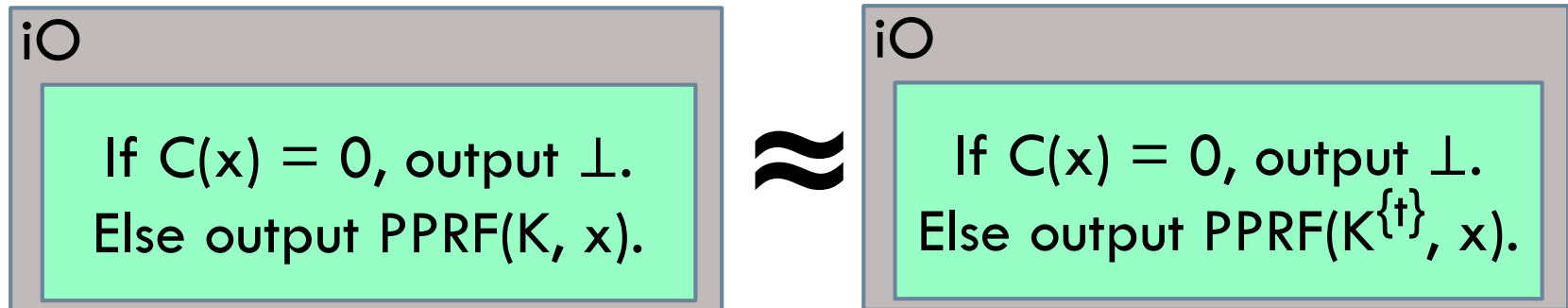
Constrained PRF

Construction:

PPRF $\{0,1\}^n \rightarrow \{0,1\}^m$



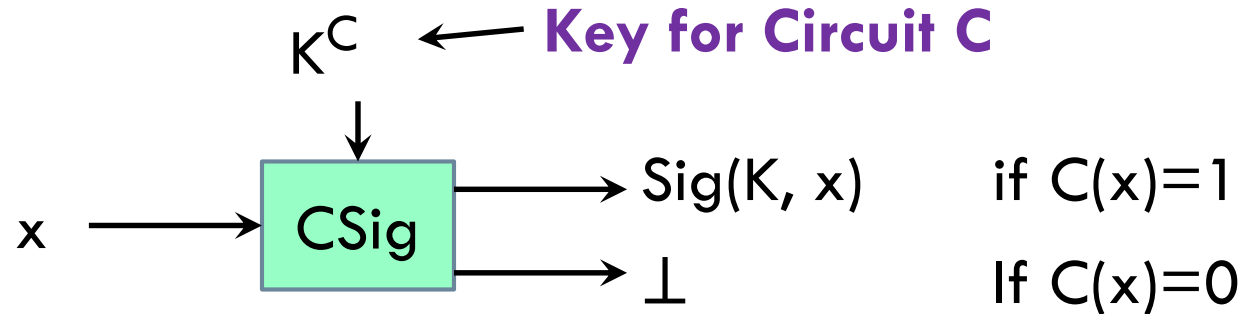
Security: Sample random t such that $C(t)=0$.



- Adversary outputs PPRF(t) with probability $> \varepsilon_{CPRF}/2^n - \varepsilon_{iO}$.
- **Complexity leveraging:** iO and PPRF need sub-exp security.

Constrained Signature Scheme

Definition:



(Plus a verification algorithm Ver.)

For unsatisfying x , we get usual signature scheme security.

Applications:

- Mobile agents: Agent's signature looks just like Principal's (on messages that it is permitted to sign).
- Delegation: Signature on x is an argument that $C(x)=1$.

Constrained Signature Scheme

Construction:

PPRF $\{0,1\}^n \rightarrow \{0,1\}^m$. Let f be a public OWF. $|x| = k$.

Constrained signing

iO
If $C(x) = 0$, output \perp .
Else output PPRF(K, x).

Verification

iO
Output $f(\text{PPRF}(K, x))$.

Security: Pick random t such that $C(t)=0$. Hybrids:

- Use $K^{\{t\}}$ and hardwire $f(\text{PPRF}(K,t))$ into Ver (iO security).
- Change $f(\text{PPRF}(K,t))$ to $f(v)$ in Ver for random v (PPRF security).
- Adv forges $v = \text{Sig}(t)$ with prob $> \varepsilon_{\text{CSig}}/2^k - 2\varepsilon_{\text{IO}} - \varepsilon_{\text{PPRF}}$
 - ▣ OWF needs $> 2^k$ security $\rightarrow m > k \rightarrow$ CSigs longer than messages.

Non-Deterministic Constrained PRFs/Sigs

Witness PRFs: [Zhandry '14] Get PRF on x if input satisfies $R(x,w)=1$

Applications:

- Multiparty NIKE without trust setup
 - ▣ Similar to Boneh-Zhandry protocol, but without iO
- Reusable WE and ABE with short cts (independent of relation size)
- Reusable secret sharing for NP with shorter shares
- Fully distributed broadcast encryption
- Maybe not enough for to achieve some things achievable via iO , like Boneh-Zhandry's traitor tracing protocol.

NIZK args for NP: [SW13] Get sig on x if input satisfies $R(x,w)=1$

A Few Constrained PRF/Sig papers

- Boneh, Waters: “Constrained Pseudorandom Functions and Their Applications”. Asiacrypt 2013.
- Boyle, Goldwasser, Ivan: “Functional Signatures and Pseudorandom Functions”. PKC 2014.
- Kiayias, Papadopoulos, Triandopoulos: “Delegatable Pseudorandom Functions and Applications”. CCS 2013.
- Bellare, Fuchsbauer: “Policy-Based Signatures”. PKC 2014.
- Backes, Meiser, Schroder: “Delegatable Functional Signatures”. ePrint 2013.

- Chen, Zhang: “**Publicly Evaluable** Pseudorandom Functions and Their Applications”. SCN 2014
- Georg Fuchsbauer, “Constrained **Verifiable** Random Functions”. SCN 2014.
- Chandran, Raghuraman, Vinayagamurthy: “Constrained Pseudorandom Functions: **Verifiable** and Delegatable”. ePrint 2014.
- Fuchsbauer, Konstantinov, Pietrzak, Rao: “**Adaptive Security** of Constrained PRFs”. Asiacrypt 2014.
- Hofheinz, Kamath, Koppula, Waters: “**Adaptively Secure** Constrained Pseudorandom Functions”. ePrint, 2014.
- Hohenberger, Koppula, Waters: **Adaptively Secure** Puncturable Pseudorandom Functions in the Standard Model”. ePrint 2014.
- Abusalah, Fuchsbauer, Pietrzak: “Constrained PRFs for **Unbounded Inputs**”. ePrint 2014.
- Cohen, Goldwasser, Vaikuntanathan: “**Aggregate** Pseudorandom Functions and Connections to Learning”. TCC 2015.

Another Useful Trick: Extraction

- Show $iO(P) \approx iO(P^{\{t\}})$ even though $P(t) \neq \perp$.
 - ▣ Show distinguisher can extract “differing input” t .
 - ▣ Show t is hard to extract assuming OWFs.
- Show that $P^{\{t\}}$ lacks some property necessary for the adversary’s attack.

iO and Differing Inputs Obfuscation (diO)

- Differing inputs obfuscation (diO):
 - ▣ Security definition: For every diO distinguisher, there is an extractor that gives a differing input.
- $\text{diO} \rightarrow \text{iO}$
- $\text{iO} \rightarrow \text{diO}$ if # of differing inputs is *very small* [Boyle, Chung, Pass TCC 2014]
 - ▣ Apply iO scheme to programs P, P' that differ at one input t .
 - ▣ If $\text{iO}(P) \approx \text{iO}(P')$, iO implies we can **extract** t .
 - If $P' = P^{\{t\}}$, we can extract t .
 - In general, Extractor's work scales with # of differing inputs.

iO \rightarrow diO for One Differing Input

- Suppose $P(x) = P'(x)$ for all x except t .
- Program P_k : If $x \geq k$ output $P(x)$, else output $P'(x)$.



- Assuming iO, if $iO(P) \neq iO(P')$, we can find t by binary search.

Random Puncture is Undetectable

Let's Give $P^{\{t\}}$ Some Code:

Suppose t 's domain supports an injective OWF f .

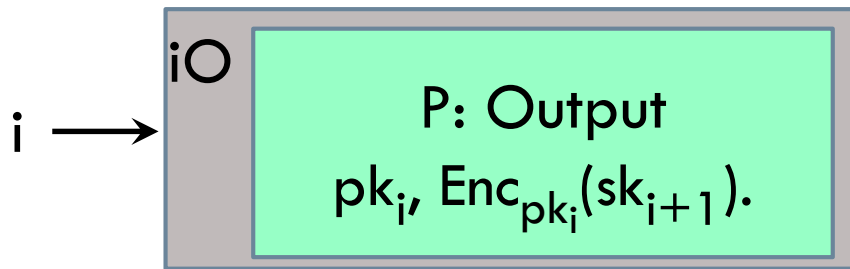
Let $y = f(t)$.

If $f(x) = y$, output \perp .
Else, output $P(x)$.

- Assuming iO , if $iO(P) \not\approx iO(P^{\{t\}})$, we can break the OWF.
- Assuming iO and OWF, $iO(P) \approx iO(P^{\{t\}})$.

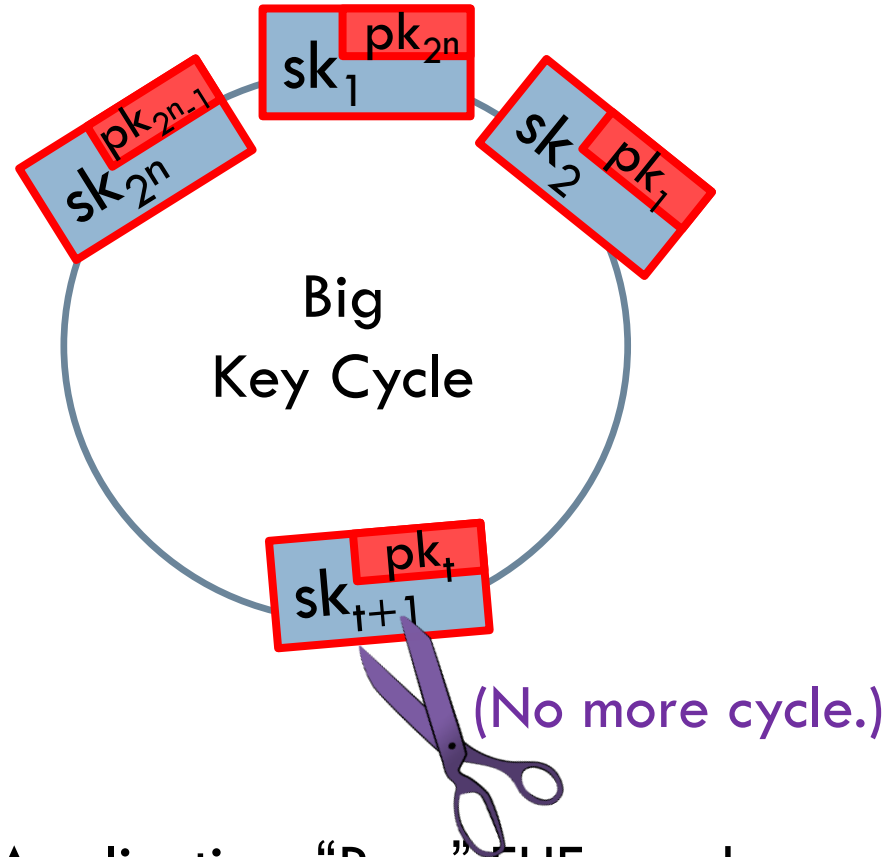
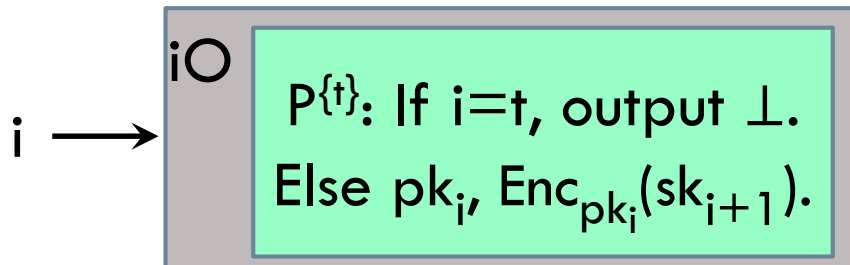
Circular Security from iO

Public Key: pk_1 and $iO(P)$.



If domain
large enough
to support
OWFs

For random t :



Application: “Pure” FHE uses key cycle to handle unbounded depth.

Circular Security from iO: Rest of Proof

Additional Requirements: Encryption scheme is not only IND-CPA, but pairs $(pk_i, c_i = \text{Enc}(pk_i, m))$ look pseudorandom.

$P^{\{t\}}$: If $i = t$, output \perp . Else output $P(i)$.

$Q^{\{t\}}$: If $i = t$, output \perp . Else output $Q(i)$.

P:

$(r_i, s_i) \leftarrow \text{PPRF}(K, i)$
 $(r_{i+1}, s_{i+1}) \leftarrow \text{PPRF}(K, i+1)$
 $(sk_i, pk_i) \leftarrow \text{KGen}(r_i)$
 $(sk_{i+1}, pk_{i+1}) \leftarrow \text{KGen}(r_{i+1})$
 $c_i \leftarrow \text{Enc}(pk_i, sk_{i+1}; s_i)$
Output (pk_i, c_i)

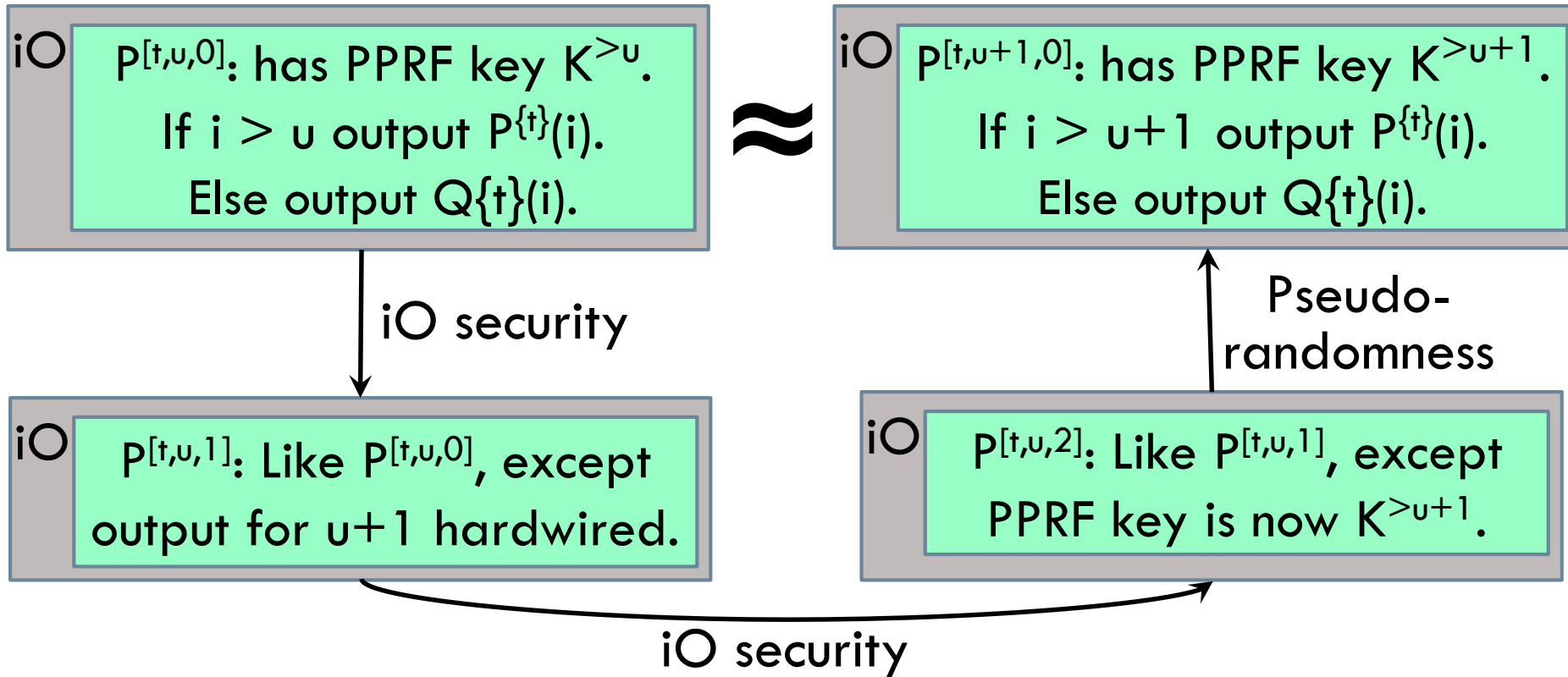
Q:

Set $(pk_i', c_i') \leftarrow \text{PRF}(K', i)$
except $pk_1' = pk_1$.
Output (pk_i', c_i')

Clearly IND-CPA is preserved when $iO(Q^{\{t\}})$ is added to public key.

Circular Security from iO: Rest of Proof

Claim:



Related Work on Circular Security vs. iO

- iO + LFHE \Rightarrow “Almost pure FHE”
 - ▣ Canetti, Lin, Tessaro, Vaikuntanathan: “Obfuscation of Probabilistic Circuits and Applications”. TCC 2015.
 - ▣ Clear, McGoldrick: “Bootstrappable Identity-Based Fully Homomorphic Encryption”. CANS, 2014.
- Negative results on circ. security (for poly-size cycles):
 - ▣ Marcedone, Orlandi:
“iO \Rightarrow (IND-CPA $\not\Rightarrow$ Circular Security)”. SCN 2014.
 - ▣ Koppula, Ramchen, Waters: “Separations in Circular Security for Arbitrary Length Key Cycles”. TCC 2015.

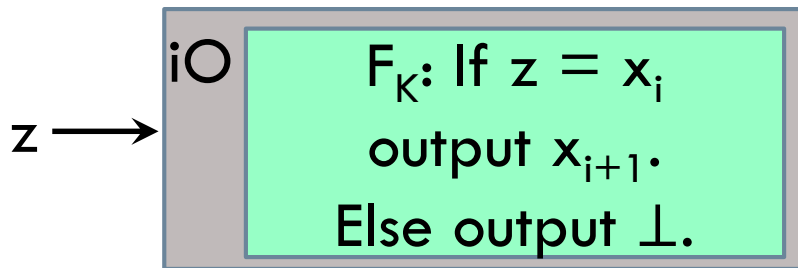
Trapdoor Permutation from iO [BPW15]

Trapdoor:

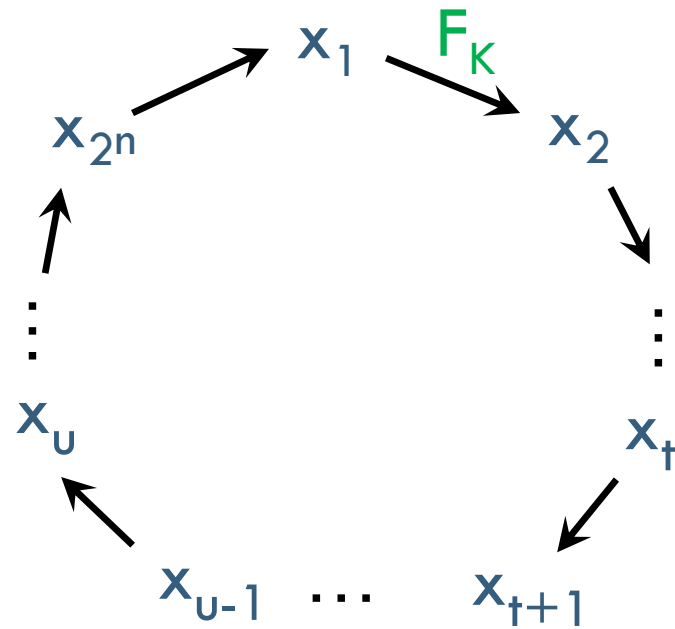
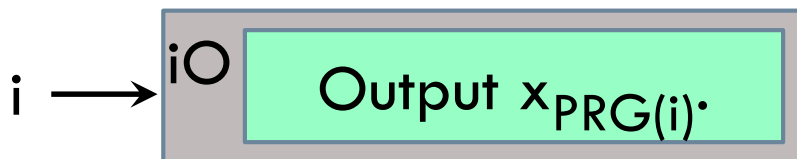
Key K for PRF

Public Key:

Obfuscated program for F_K .



Pseudorandom sampler:



$$x_i = (i, \text{PRF}(K, i))$$

Remark: x_i 's like secret keys in a key cycle.
Replace $iO(F_K)$ with $iO(P)$ that outputs $\text{Enc}(pk_i, sk_{i+1})$.
Get sk_{i+1} from sk_i via decryption.

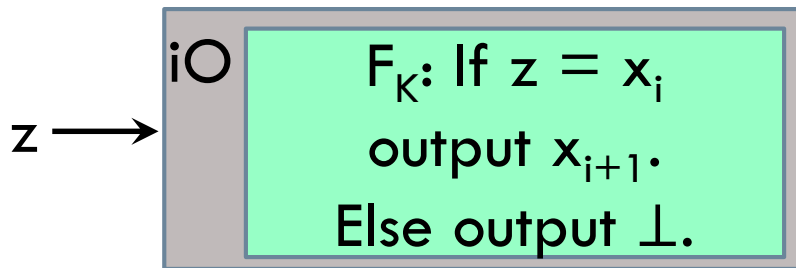
Trapdoor Permutation from iO [BPW15]

Trapdoor:

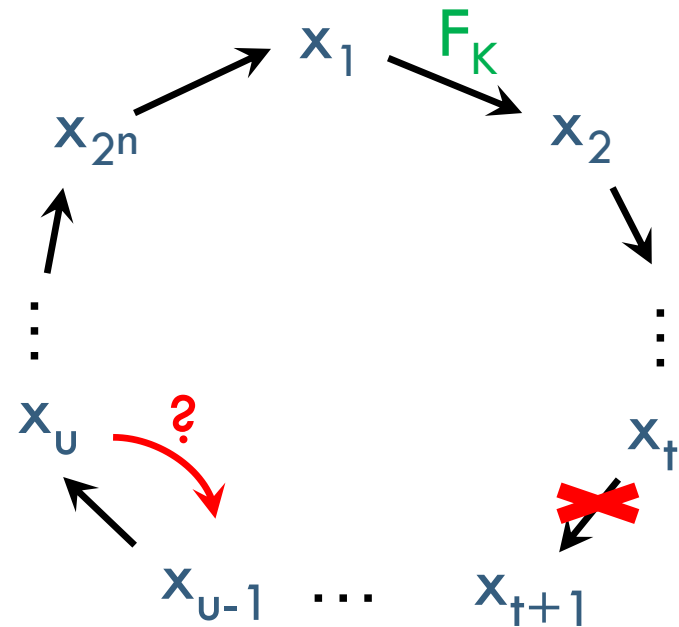
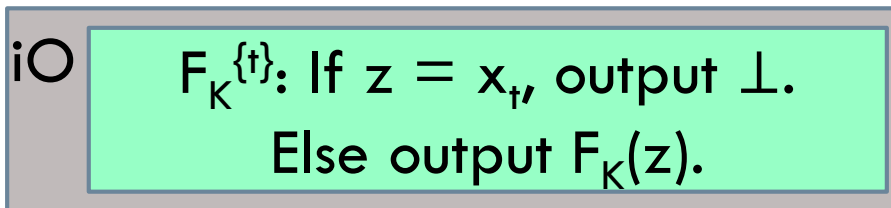
Key K for PRF

Public Key:

Obfuscated program for F_K .



For random t :



$$x_i = (i, \text{PRF}(K, i))$$

Other Work on iO vs. Delicate Graphs

- Bitansky, Paneth, Rosen: “On the Cryptographic Hardness of Finding a Nash Equilibrium”. ePrint 2015.
 - ▣ Prove that finding a Nash equilibrium of a game is hard, assuming the existence of iO and sub-exp OWFs.
 - ▣ Nash is PPAD-complete: [DGP09, CDT09].
 - ▣ END-OF-THE-LINE: Canonical PPAD-complete problem. Given succinct program P_G representing exponential-size directed graph G over $\{0,1\}^n$ with in/out degrees ≤ 1 , and a source node s , find some other source/sink node.
 - ▣ $iO(P_G)$ for long-line-graph indistinguishable from $iO(P'_G)$ where end-of-the-line is inaccessible.

Scope of iO Applications

Roughly grouped by area:

1. FHE
2. Multilinear maps
3. Delegation
4. Secure multiparty computation
5. Garbled circuits
6. RAM computations
7. Differential privacy
8. Odds and Ends

FHE from iO and Re-randomizable PKE

Canetti, Lin, Tessaro, Vaikuntanathan:
“Obfuscation of Probabilistic Circuits and
Applications”. TCC 2015.

Leveled FHE from $iO+OWFs$?

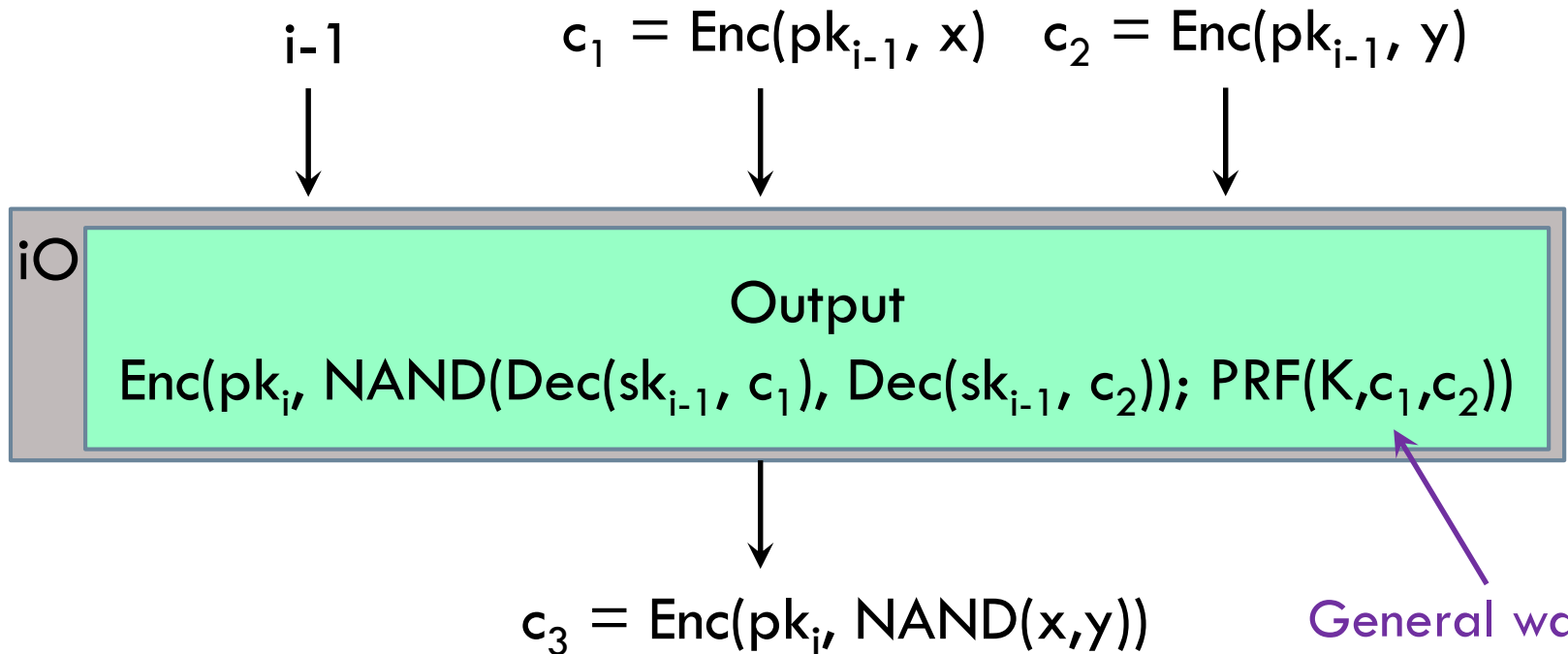
- Compact additive HE \rightarrow CRHFs.
 - Let $c_1 = \text{Enc}(a)$, $c_2 = \text{Enc}(b)$.
 - $H(x_1 \| x_2) = \text{Enc}(ax_1 + bx_2)$ (computed homomorphically).
 - Collision gives linear equation on (a,b) , violating semantic security.
- But don't know how to get CRHFs from $iO+OWFs$!
 - Asharov, Segev: "Limits on the Power of iO and FE" (ePrint 2015): "There is no fully black-box construction with a polynomial security loss of a collision-resistant function family from a general-purpose indistinguishability obfuscator and a... trapdoor permutation."

Leveled FHE from iO +Re-rand PKE

- Canetti, Lin, Tessaro, Vaikuntanathan: “Obfuscation of Probabilistic Circuits and Applications”. TCC 2015.
 - “Assume the existence of a sub-exponentially indistinguishable iO for circuits, and a sub-exponentially secure OWF. Then any perfectly rerandomizable encryption scheme can be transformed into a leveled homomorphic encryption scheme.”

L-Levelled FHE from iO + Re-rand PKE: Natural Approach

KeyGen as in PKE, but with obfuscated NANDs:



General way to handle obfuscation of programs that use internal coins

L-Levelled FHE from iO + Re-rand PKE: Security Proof

- Uses 2 types of PKE scheme:
 - ▣ Normal
 - ▣ Trapdoor/Lossy: $\text{Enc}(\text{tpk}, m; r)$ and $\text{Enc}(\text{tpk}, 0; r)$ have statistically identical distributions.
- Hybrid H_k :
 - ▣ Use normal public keys pk_i for $i \leq k$.
 - ▣ Trapdoor keys tpk_i for $i > k$, and obfuscate program that just outputs encryptions of 0.
 - ▣ H_L : real game.
 - ▣ H_0 : Challenge ct is always an encryption of 0.

L-Levelled FHE from iO + Re-rand PKE: Hybrids

$H_{i-1,0}$ iO Output $\text{Enc}(\text{pk}_i, \text{NAND}(\text{Dec}(\text{sk}_{i-1}, c_1), \text{Dec}(\text{sk}_{i-1}, c_2))); \text{PRF}(K, c_1, c_2))$

Trapdoor key indistinguishability

$H_{i-1,1}$ iO Output $\text{Enc}(\text{tpk}_i, \text{NAND}(\text{Dec}(\text{sk}_{i-1}, c_1), \text{Dec}(\text{sk}_{i-1}, c_2))); \text{PRF}(K, c_1, c_2))$

iO security

$H_{i-1,2,j,1}$ For $j = (c_1^*, c_2^*)$, hardcode response to $(i-1, j)$ and puncture PRF there.

PPRF security

$H_{i-1,2,j,2}$ iO Output $\text{Enc}(\text{tpk}_i, \text{NAND}(\text{Dec}(\text{sk}_{i-1}, c_1^*), \text{Dec}(\text{sk}_{i-1}, c_2^*))); r$

Statistical re-randomizability

$H_{i-1,2,j,3}$ iO Output $\text{Enc}(\text{tpk}_i, 0; r)$

PPRF security and iO security

$H_{i-1,2,j,4}$ iO Output $\text{Enc}(\text{tpk}_i, 0; \text{PRF}(K, j))$

L-Levelled FHE from iO + Re-rand PKE: Hybrids

$H_{i-1,0}$ iO Output $\text{Enc}(pk_i, \text{NAND}(\text{Dec}(sk_{i-1}, c_1), \text{Dec}(sk_{i-1}, c_2)); \text{PRF}(K, c_1, c_2))$

Trapdoor key indistinguishability

$H_{i-1,1}$ iO Output $\text{Enc}(tpk_i, \text{NAND}(\text{Dec}(sk_{i-1}, c_1), \text{Dec}(sk_{i-1}, c_2)); \text{PRF}(K, c_1, c_2))$

iO security

$H_{i-1,2,j,1}$ For $j = (c_1^*, c_2^*)$, hardcode response to $(i-1, j)$ and

PPRF security

$H_{i-1,2,j,2}$ iO Output $\text{Enc}(tpk_i, \text{NAND}(\text{Dec}(sk_{i-1}, c_1^*), \text{Dec}(sk_{i-1}, c_2^*)))$

Statistical re-randomizability

Need $> 2^{2len}$ security where len is then length of a ciphertext.

$H_{i-1,2,j,3}$ iO Output $\text{Enc}(tpk_i, 0; r)$

PPRF security and iO security

Hence need statistical re-randomizability.

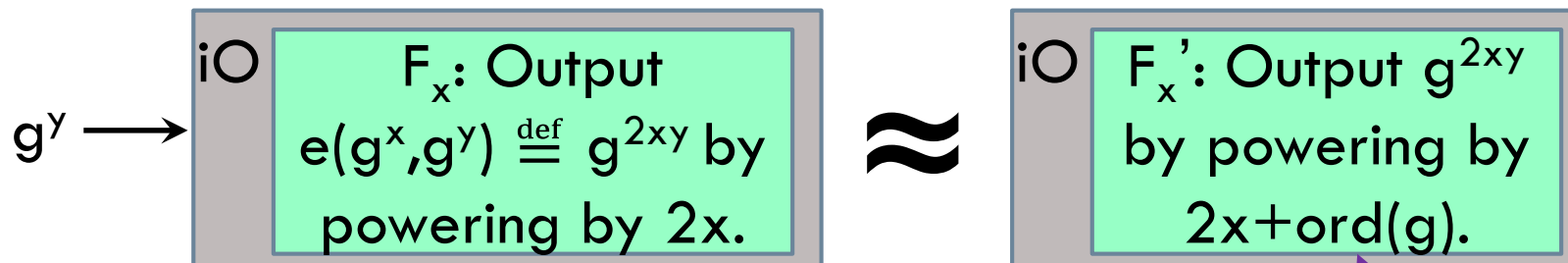
$H_{i-1,2,j,4}$ iO Output $\text{Enc}(tpk_i, 0; \text{PRF}(K, j))$



Toward iO-based Multilinear Maps?

A Positive Result

- Yamakawa, Yamada, Hanaoka, Kunihiro: “Self-bilinear Map on Unknown Order Groups from Indistinguishability Obfuscation and Its Applications”. Crypto 2014.
- Encoding of x consists of the obfuscation:



- Degree of maps is unbounded-polynomial! Odd number
- Applications to non-interactive key agreement and distributed broadcast encryption schemes.

The Problem with Truly Unbounded MMaps

- van Dam, Hallgren, Ip, “Quantum Algorithms for Some Hidden Shift Problems”. SODA 2003.
 - ▣ FHE scheme with equality test can be broken in quantum polynomial time.
 - ▣ Seems difficult to base security of truly unbounded mmap on quantum-resistant assumption like LWE.



Delegation using iO

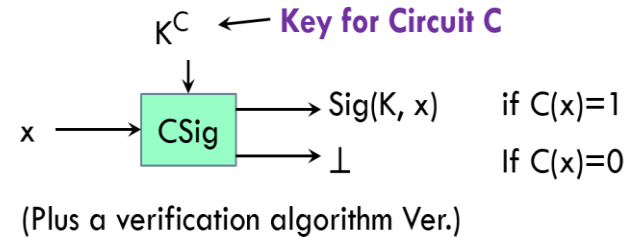
Delegation using iO

□ Constrained sigs via iO

□ But better options?

- Yael's no-signaling proofs
 - Paneth/Rothblum use mmapps to get public verifiability here
- Reusable garbled circuits
- Fully homomorphic sigs
- Quadratic arithmetic programs

Definition:



For unsatisfying x , we get usual signature scheme security.

□ Boneh, Gupta, Mironov, Sahai, “Hosting Services on an Untrusted Cloud”, Eurocrypt 2015.

- iO used to outsource provider's entire multi-client service, with privacy for both the provider and clients against the host.



Improving Secure MPC with iO

Some Applications of iO to MPC

Two-round adaptively secure MPC

- ▣ Canetti, Goldwasser, Poburinnaya: “Adaptively Secure 2PC From iO”. TCC 2015.
- ▣ Dachman-Soled, Katz, Rao: “Adaptively Secure, Universally Composable MPC in Constant Rounds”. TCC 2015.
- ▣ Garg, Polychroniadou: “Two-Round Adaptively Secure MPC from iO”. TCC 2015.
- ▣ Garg, Gentry, Halevi, Raykova: “Two-round [static] secure MPC from iO”. TCC 2014.

SFE with Long Output

- ▣ Hubacek, Wichs, “On the Communication Complexity of SFE with Long Output”. ITCS 2015.
- ▣ Jakobsen, Orlandi, “How to Bootstrap Anonymous Communication”. ePrint 2015.

Two-Round Adaptively-Secure MPC

Useful Ingredient: Deniable encryption, Explainability Compilers

- **Definition [Explainability Compiler]** (following [SW13, DKR15])
A PPT algorithm Comp is an explainability compiler if for every efficient randomized circuit Alg , the following hold:
 - **Polynomial slowdown:** There is a polynomial $p(\cdot)$ such that for any $(\text{Alg}^*, \text{Explain})$ output by $\text{Comp}(1^\lambda, \text{Alg})$ it holds that $|\text{Alg}^*| \leq p(\lambda) |\text{Alg}|$.
 - **Statistical functional equivalence:** Distributions of $\text{Alg}^*(x)$ and $\text{Alg}(x)$ are statistically close for all x .
 - **Explainability:** Adversary A has negl advantage in following game:

$A(1^\lambda)$ outputs x^* of its choice (selects target input).
 $(\text{Alg}^*, \text{Explain}) \leftarrow \text{Comp}(1^\lambda, \text{Alg})$.
Choose uniform coins $r_0 \in \mathcal{R}$ and compute $y^* = \text{Alg}^*(x^*; r_0)$.
Compute $r_1 \leftarrow \text{Explain}(x^*, y^*)$.
Choose random bit b and give (Alg^*, y^*, r_b) to A .
 A tries to guess b .

Explain allows simulator to generate consistent randomness.

Explainability Compiler: Construction

Alg*:

Hardwired keys: K_1, K_2, K_3

Input: x and randomness $u = u[1]||u[2]$

1. Sparse Hidden Trigger: If $(x', y', r') \leftarrow \text{PPRF}(K_3, u[1]) \oplus u[2]$ satisfies $x = x'$ and $u[1] = \text{PPRF}(K_2, (x', y', r'))$, output y' .
2. Normal: Output $\text{Alg}(x; \text{PPRF}(K_1, (x, u)))$.

u is a nonmalleable
“encryption” of (x, y')

Explain:

Hardwired keys K_2, K_3

Input: x, y' and randomness r

1. $u[1] \leftarrow \text{PPRF}(K_2, (x, y', \text{PRG}(r)))$ $u[2] \leftarrow \text{PPRF}(K_3, u[1]) \oplus (x, y', \text{PRG}(r))$.
2. Output $u \leftarrow u[1]||u[2]$.

Outputs an
“encryption” of (x, y')

Communication Complexity of SFE with Long Output [Hubacek,Wichs'15]

- SFE with Long Output Scenario:
 - ▣ Function $f : \{0,1\}^A \times \{0,1\}^B \rightarrow \{0,1\}^t$.
 - ▣ Output is long: $t \gg A$. Maybe also $t \gg B$.
 - ▣ Want SFE protocol for Bob to get $f(x_A, x_B) \mid \in \{0,1\}^t$.
- Examples:
 - ▣ $x_A = \text{PRF key } K, f(x_A, \emptyset) = \text{PRF}(K, 1), \dots, \text{PRF}(K, L)$.
 - ▣ $x_A = \text{partial decryption key}, x_B = \text{encryption of long output of MPC protocol under threshold PKE}, f = \text{partial decryption procedure}$.
- Communication complexity:
 - ▣ Can we get **communication sublinear in t** ?
 - ▣ Possible with insecure protocols.
 - ▣ FHE doesn't do this.
 - ▣ Constrained PRF? No, not simulatable from PRF outputs alone.

Communication Complexity of SFE with Long Output [Hubacek, Wichs'15]

□ Negative Results:

- Output-dependence inherent when Bob is **malicious** or even **honest-but-deterministic**.
- The Problem: **incompressibility**
 - view_{Bob} allows Bob to compute $\text{PRF}(K,1), \dots, \text{PRF}(K,L)$
 - $|\text{view}_{\text{Bob}}| \ll t$ if Bob is deterministic, $|x_B| \ll t$, and $|\text{comm}| \ll t$.
 - view_{Bob} compresses $\text{PRF}(K,1), \dots, \text{PRF}(K,L)$ (impossible).

□ Positive results

- Use iO to remove output-dependence in **semi-honest case**.
- Requires Bob to use long randomness with succinct decommitments via an iO-friendly Merkle tree hash.

Communication Complexity of SFE with Long Output [Hubacek, Wichs'15]

Real World

Bob sends $z = H(r_1, \dots, r_L)$.

Alice sends $iO(C_{K,z})$.

$iO(C_{K,z})$:

iO

Hardwired: key K , hash output z

Inputs: $(i \leq L, r_i, \pi_i)$

1. Verify decommitment π_i that r_i is i -th bit of hashed randomness.
2. If so, output $y_i = \text{PRF}(K, i)$.

Simulation

$z = H(r_1, \dots, r_L)$ for $r_i = y_i \oplus \text{PRF}(K_{\text{sim}}, i)$.

Modified obfuscation.

$iO(C_{K_{\text{sim}}, \{y_i : i \leq L\}, z})$:

iO

Hardwired: key K_{sim} , H output z

Inputs: $(i \leq L, r_i, \pi_i)$

1. Verify decommitment π_i that r_i is i -th bit of hashed randomness.
2. If so, output $y_i = r_i \oplus \text{PRF}(K_{\text{sim}}, i)$.

Not functionally equivalent when $(i, r'_i \neq r_i, \pi'_i)$ is a valid decommitment!

Communication Complexity of SFE with Long Output [Hubacek, Wichs'15]

iO-Friendly Merkle Tree (somewhere statistically binding hash):

- Functionality of a Merkle tree:
 - ▣ Short hash key: $z = H_{hk}(r_1, \dots, r_L)$
 - ▣ Allows short decommitment of r_i .
- iO-friendly security:
 - ▣ Some index i is statistically binding: For that i , there is no opening to $r_i' \neq r_i$.
 - ▣ Hash key hk does not reveal which index i is binding.
- Construction: from FHE.

More iO-Friendly Techniques

- Koppula, Lewko, Waters: “Indistinguishability Obfuscation for Turing Machines with Unbounded Memory”. STOC 2015.
 - **Positional accumulators:** Similar in concept to somewhere statistically binding hashes. Allows short commitment to large storage that is unconditionally sound for some hidden index.

More Powerful Protocols via iO

Functionalities:

- Goldwasser, Gordon, Goyal, Jain, Katz, Liu, Sahai, Shi, Zhou: “**Multi-Input Functional Encryption**”. Eurocrypt ‘14.
 - ▣ Given ciphertexts $\text{Enc}(x_1), \dots, \text{Enc}(x_n)$ and the secret key sk_f for n -ary function f , one can compute $f(x_1, \dots, x_n)$, and nothing else about the $\{x_i\}$'s.
- Waters, “A Punctured Programming Approach to **Adaptively Secure FE**”. ePrint 2014.
- Boneh, Zhandry, “Multiparty Key Exchange, Efficient **Traitor Tracing**, and More from Indistinguishability Obfuscation”. Crypto 2014.

More Powerful Protocols via iO

Functionalities from Mmaps (not iO):

- Boneh, Waters, Zhandry, “**Low Overhead BE** from Multilinear Maps”. Crypto 2014.
- Garg, Gentry, Halevi, Zhandry: “**Fully Secure ABE** using Multilinear Maps”. ePrint 2014.
- Garg, Gentry, Halevi, Zhandry: “**Fully Secure FE** without Obfuscation”. ePrint 2014.
- Boneh, Lewi, Raykova, Sahai, Zhandry, Zimmerman: “Semantically Secure **Order-Revealing Encryption**: Multi-Input FE without Obfuscation”. Eurocrypt ‘15.

More Powerful Protocols via iO

ZK, WI, Concurrency:

- Bitansky, Paneth: “ZAPs and NIWI from iO”. TCC 2015.
- Pandey, Prabhakaran, Sahai: “Obfuscation-based Non-black-box Simulation and Four Message Concurrent ZK for NP”. TCC 2015.
- Chung, Lin, Pass: “Constant-Round Concurrent ZK from iO”. ePrint 2014.

More Powerful Protocols via iO

Frameworks:

- Hofheinz, Jager, Khurana, Sahai, Waters, Zhandry: “How to Generate and use **Universal Parameters**”. ePrint 2014.
- Agrawal, Agrawal, Prabhakaran: “Cryptographic Agents: Towards a **Unified Theory of Computing on Encrypted Data**”. Eurocrypt 2015.

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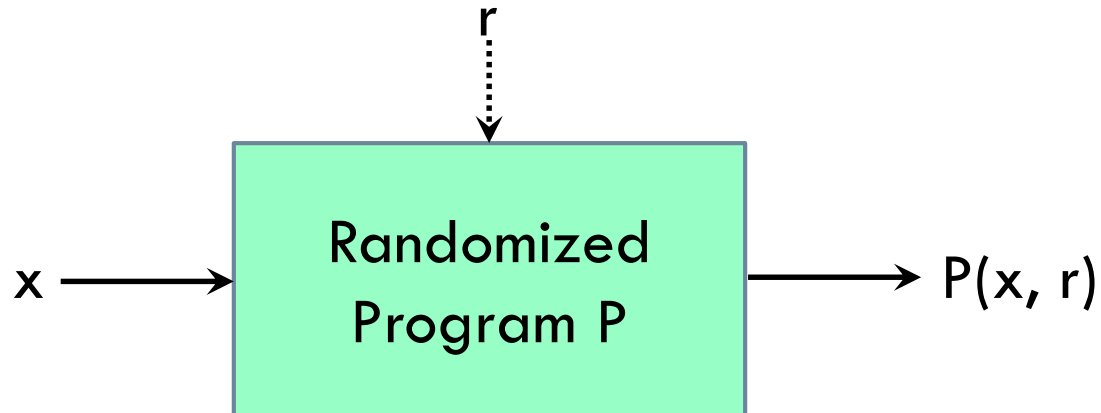
iO of Randomized Functionalities (Like Circuit Garbling)

Papers Considering Randomized Functionalities

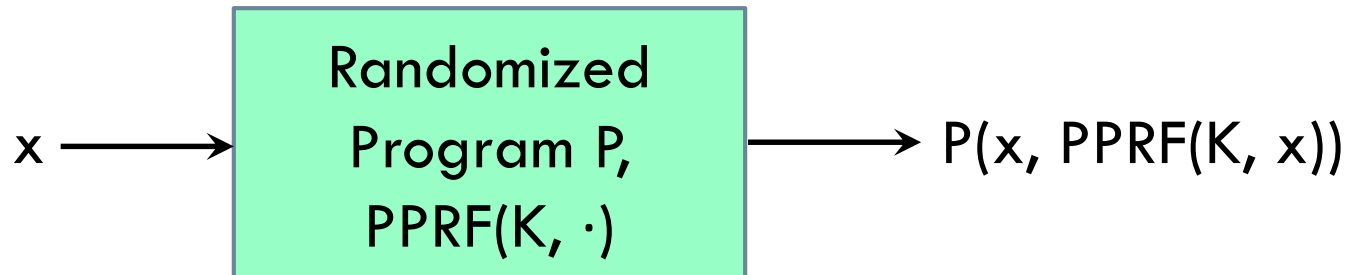
- Alwen, Barbosa, Farshim, Gennaro, Gordon, Tessaro, Wilson: “On the Relationship between Functional Encryption, Obfuscation, and Fully Homomorphic Encryption”. IMACC, 2013.
- Goyal, Jain, Koppula, Sahai: “Functional Encryption for Randomized Functionalities”. TCC 2015.
- Canetti, Lin, Tessaro, Vaikuntanathan: “Obfuscation of Probabilistic Circuits and Applications”. TCC 2015.

iO for Randomized Functionalities

Can we obfuscate a program that flips coins internally?



If we make it pseudorandom?



iO for Randomized Functionalities

The Question:

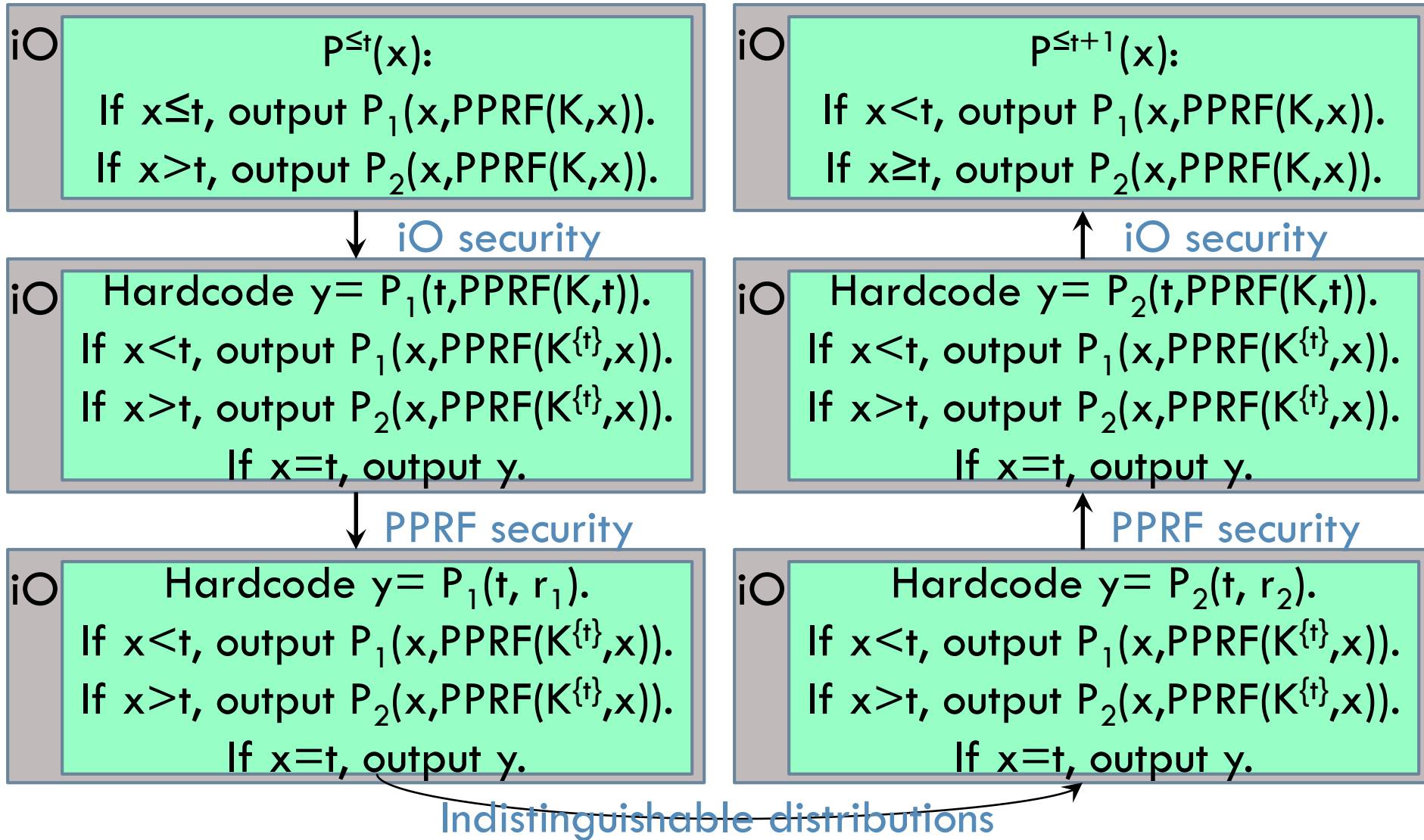
$\{P_1(x, r) : r \in R\} \approx \{P_2(x, r) : r \in R\}$ for every x



$[P_1, P_2, \text{iO}(P_1(x, \text{PPRF}(K, x)))] \approx [P_1, P_2, \text{iO}(P_2(x, \text{PPRF}(K, x)))]$

Certainly not true that $P_1(x, r) = P_2(x, r)$ for all (x, r) !

iO for Randomized Functionalities



Randomized Encodings

Randomized Encoding RE for a circuit family \mathcal{C}

- Given $C \in \mathcal{C}$ input x , $RE(C,x)$ outputs (C',x') such that:
 - ▣ From (C', x') one can efficiently recover $C(x)$
 - ▣ Given $C(x)$, one can efficiently simulate the pair (C',x') , implying $RE(C,x)$ reveals nothing beyond $C(x)$.
 - ▣ Also, RE is typically not only efficient, but has low parallel complexity (e.g., it's in NC^1).

iO and Randomized Encodings



Using Randomized Encodings to Bootstrap iO

- If we have VBB for class WEAK that includes RE and a PRF, we get VBB for general circuits.
 - ▣ $O(C)(x)$ outputs $RE(C,x)$, using $PRF(K,x)$ as randomness.
- [CLTV15] iO suffices for this purpose.
 - ▣ If C_1, C_2 are functionally equiv., $\{RE(C_1, x; r)\} \approx \{RE(C_2, x; r)\}$.
 - ▣ Therefore $iO(C_1(x, PRF(K,x))) \approx iO(C_2(x, PRF(K,x)))$
 - ▣ Don't need FHE to bootstrap iO.



Obfuscating RAM Computations

Obfuscating RAM Computations

- Gentry, Halevi, Raykova, Wichs: “Outsourcing Private RAM Computation”. FOCS 2014.
- Bitansky, Garg, Lin, Pass, Telang: “Succinct Randomized Encodings and their Applications”. STOC 2015.
- Canetti, Holmgren, Jain, Vaikuntanathan: “Succinct Garbling and Indistinguishability Obfuscation for RAM Programs”. STOC 2015.
- Canetti, Holmgren: “Fully Succinct Garbled RAM”. ePrint 2015.
- Chen, Chow, Chung, Lai, Lin, Zhou: “Computation-Trace Indistinguishability Obfuscation”. ePrint 2015.

Garbling RAM Computations

- Lu, Ostrovsky: “How to Garble RAM Programs?” Eurocrypt 2013.
 - ▣ Seminal work on garbling programs without going through circuits.
- Gentry, Halevi, Lu, Ostrovsky, Raykova, Wichs: “Garbled RAM revisited.” Eurocrypt 2014.
- Garg, Lu, Ostrovsky, Scafuro: “Garbled RAM from OWFs”. STOC’15.
 - ▣ Based on OWFs
 - ▣ Runtime proportional to runtime of plaintext RAM program
 - ▣ But garbled program size also proportional to runtime.

Fully Succinct Garbled RAM [CH15]

- Fully succinct garbling scheme for RAM programs assuming iO for circuits and OWFs.
 - ▣ Fully succinct: the size, space requirements, and runtime of the garbled program are the same as those of the input program, up to poly-logarithmic factors and a polynomial in the security parameter.
- Constructs iO for RAMs assuming iO for circuits and sub-exp OWFs.
- Combines iO, garbling, ORAM, and other techniques.



Differential Privacy

iO + Differential Privacy: A Positive App

- Scenario:
 - ▣ Hospitals generate patient records
 - ▣ Medical researchers want access to patient records to test hypotheses
- Differential privacy:
 - ▣ Publish differentially-private “noisy” “sanitized” DB
 - ▣ Hospitals don’t need to interact in each research analysis
 - ▣ Researchers don’t need to share hypothesis or algorithm
 - ▣ Issue: Allowing diverse research analytics → high accuracy loss [DNR⁺09].

iO + Differential Privacy: A Positive App

- Randomized FE/iO can help. Randomized FE approach:
 - ▣ Government is authority of system.
 - ▣ Government issues key to researcher if function is differentially private.
 - ▣ Randomized FE decryption adds noise to exact response, to “sanitize” it. Decryption process adds low-level of noise to exact response.
 - ▣ Pro: Better accuracy, since noise can be added fresh for each function.
 - ▣ Con: Researcher reveals hypothesis to government.
 - Maybe automate government’s role using iO.

iO vs. Differential Privacy

- iO gives very efficient traitor tracing schemes [BZ14].
- Traitor tracing is opposite of differential privacy [Dwork, Naor, Reingold, Rothblum, Vadhan, STOC '09]
 - ▣ Differential privacy: Summarize data in meaningful way while hiding individual information.
 - ▣ Traitor tracing: Prevent keys from being summarized in a way that hides individual information.

More on iO and Complexity Theory

- Bitansky, Paneth, Rosen: “On the Cryptographic Hardness of Finding a Nash Equilibrium”. ePrint 2015.
 - ▣ Finding Nash Eq hard assuming iO and sub-exp OWFs.
- Bun, Zhandry: “Order-Revealing Encryption and the Hardness of Private Learning”. ePrint 2015.
 - ▣ Separates efficient PAC learning from efficient differentially-private PAC learning assuming iO and simple primitives.
- Cohen, Goldwasser, Vaikuntanathan: “Aggregate PRFs and Connections to Learning”. TCC 2015.
 - ▣ [Val84]: PRF in a complexity class C implies the existence of concept classes in C unlearnable by membership queries.
 - ▣ Explores implications of constrained PRFs etc.



Odds and Ends

Software Watermarking

- Nishimaki, Wichs: “Watermarking Cryptographic Programs Against Arbitrary Removal Strategies”. ePrint 2015.
- Cohen, Holmgren, Vaikuntanathan: “Publicly Verifiable Software Watermarking”. ePrint 2015.
- Result: For certain types of programs, like PRFs, they create a marked program $C^\#$ such that:
 - ▣ Evaluates C correctly on overwhelming fraction of inputs
 - ▣ Adversary cannot come up with any program with mark removed that evaluates correctly on even a small fraction of inputs.

Hashing Using iO

- Bellare, Stepanovs, Tessaro: “**Poly-Many Hardcore Bits** for Any One-Way Function and a Framework for Differing-Inputs Obfuscation”. Asiacrypt 2014.
- Brzuska, Mittelbach: “Using Indistinguishability Obfuscation via **UCEs**”. Asiacrypt 2014.
- Canetti, Chen, Reyzin: “On the **Correlation Intractability** of Obfuscated Pseudorandom Functions”. ePrint 2015.
- Hohenberger, Sahai, Waters: “Replacing a Random Oracle: **Full Domain Hash** From Indistinguishability Obfuscation”. Eurocrypt 2014.
- Bernstein, Lange, van Vredendaal et al. ePrint 2015: “**Bad Directions** in Cryptographic Hash Functions”?

Impossibilities Implied by iO

- Bitansky, Canetti, Cohn, Goldwasser, Kalai, Paneth, Rosen: “The Impossibility of Obfuscation with **Auxiliary Input or a Universal Simulator**”. Crypto 2014.
- Brzuska, Farshim, Mittelbach: “**Random-Oracle** Uninstantiability from iO”. TCC 2015.
- Brzuska, Farshim, Mittelbach: “iO and **UCEs**: The Case of Computationally Unpredictable Sources”. Crypto 2014.
- Brzuska, Mittelbach: “iO versus **Multi-Bit Point Obfuscation with Auxiliary Input**”. Asiacrypt 2014.
- Green, Katz, Malozemoff, Zhou, “A Unified Approach to **Idealized Model Separations** via iO”. ePrint 2014.

Thank You! Questions?

