Spatial Coupling vs. Block Coding: A Comparison

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Coding: From Theory to Practice
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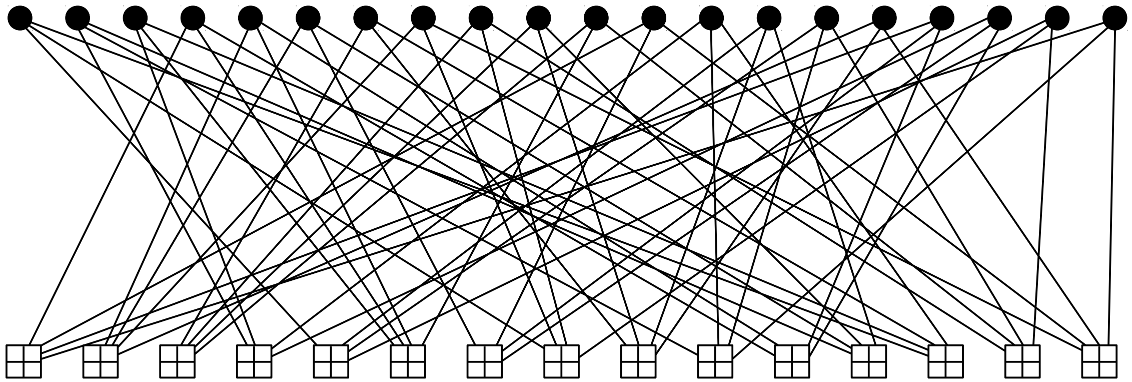
Research Collaborators: David Mitchell, Michael Lentmaier, and Ali Pusane

LDPC Block Codes

- LDPC codes are defined on a sparse bipartite graph



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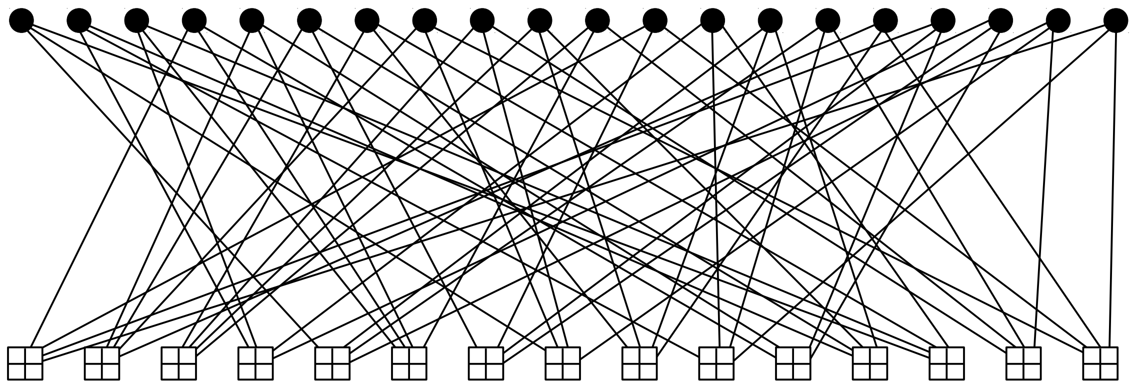
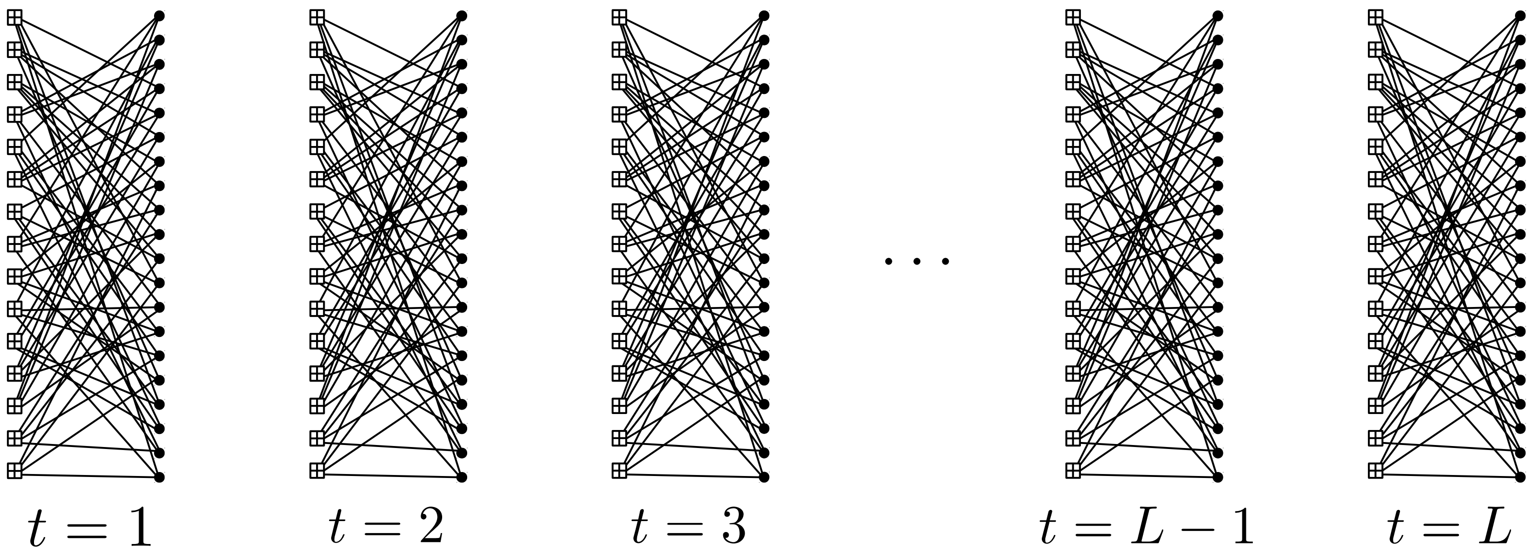


- Graph-based codes can be decoded with low complexity using iterative **belief propagation** decoding
- Desirable properties of LDPC codes:
  - ➡ Low error floors (typical of regular LDPC codes)
  - ➡ Waterfall performance close to capacity (typical of optimized irregular LDPC codes)

What are Spatially Coupled Codes?

- Consider the transmission of consecutive LDPC block code codewords



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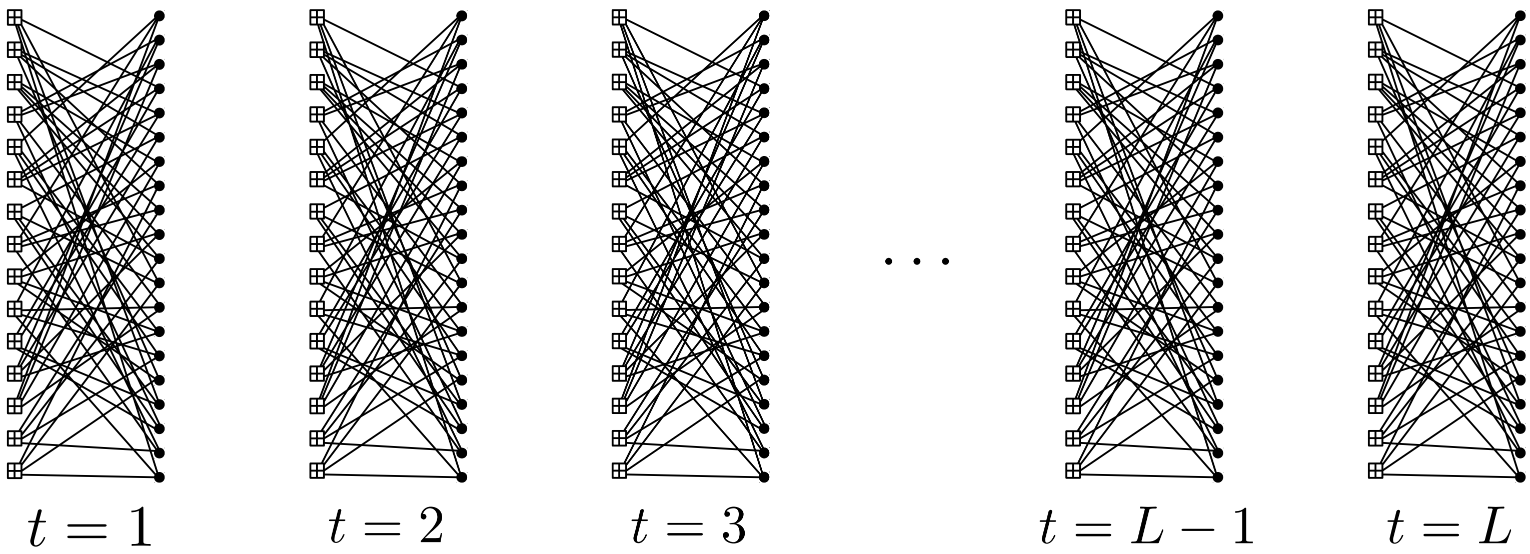

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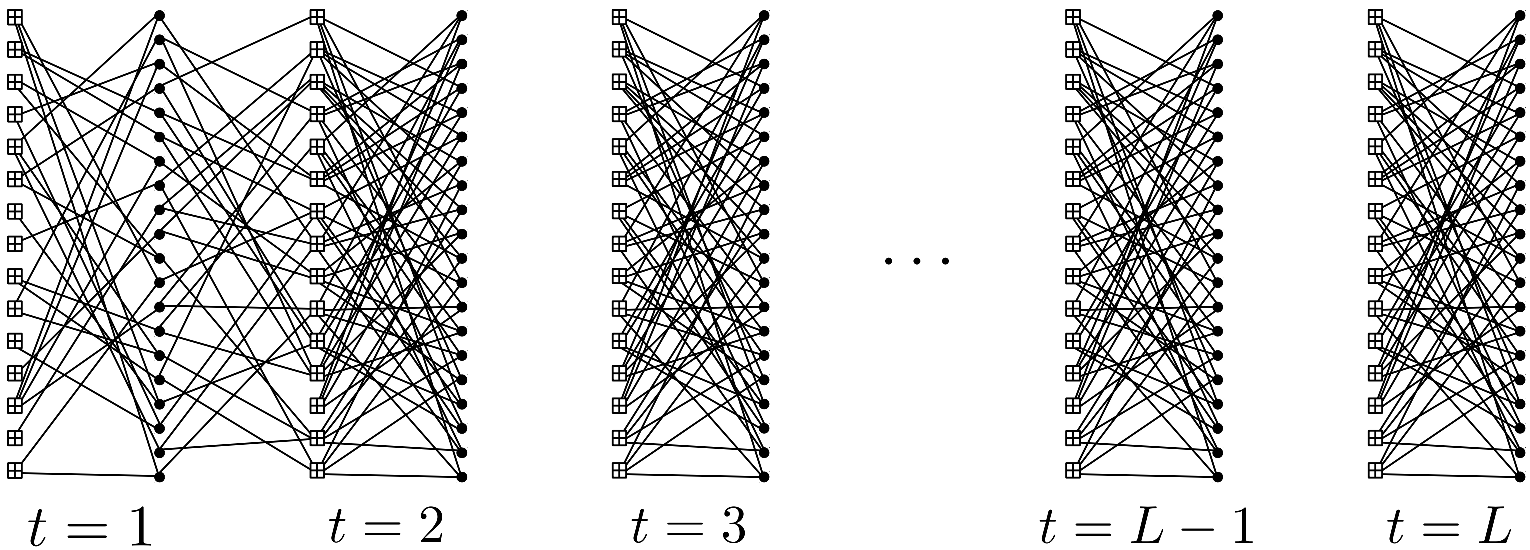

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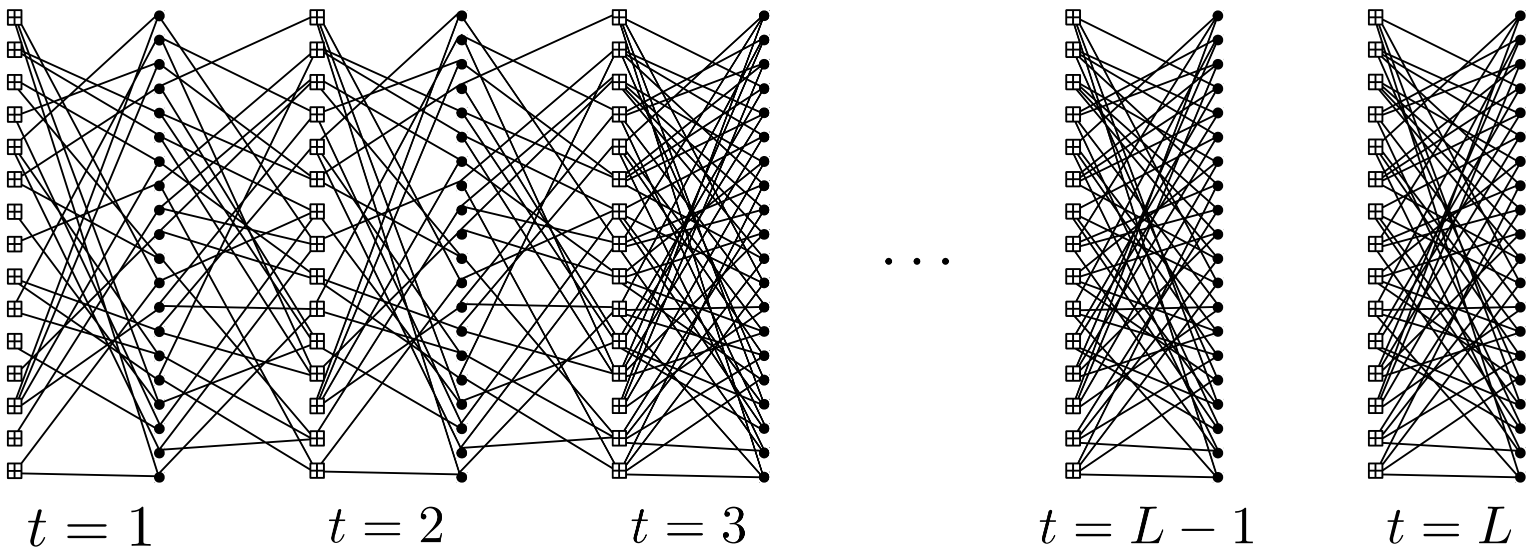

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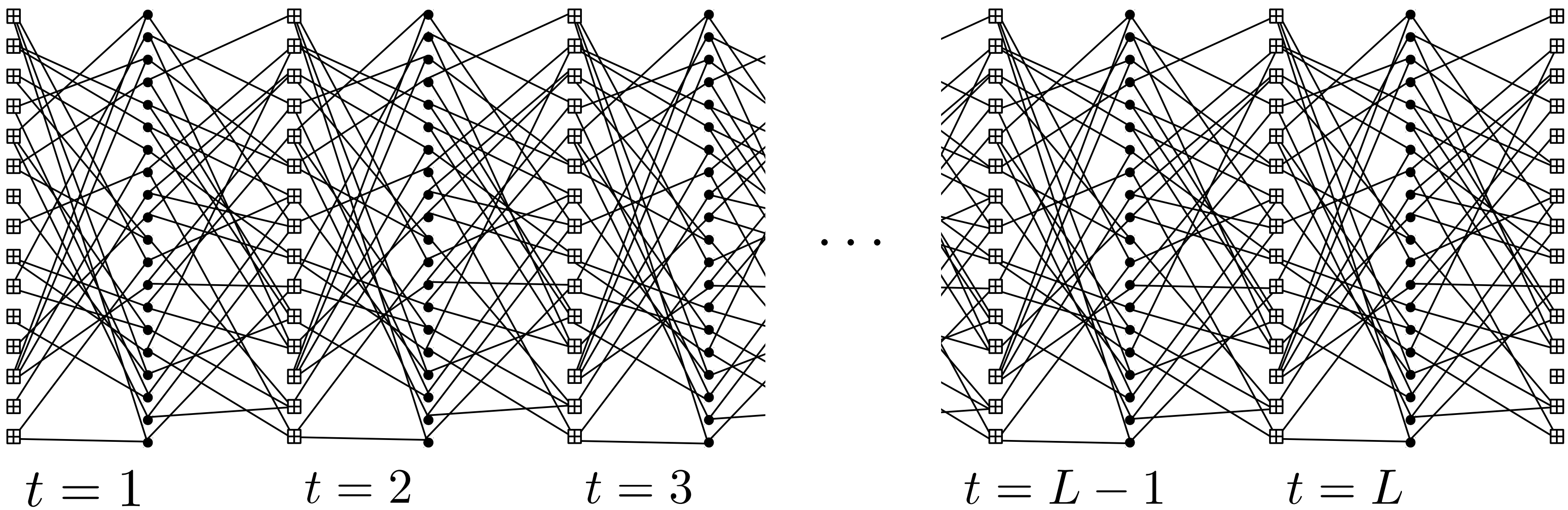

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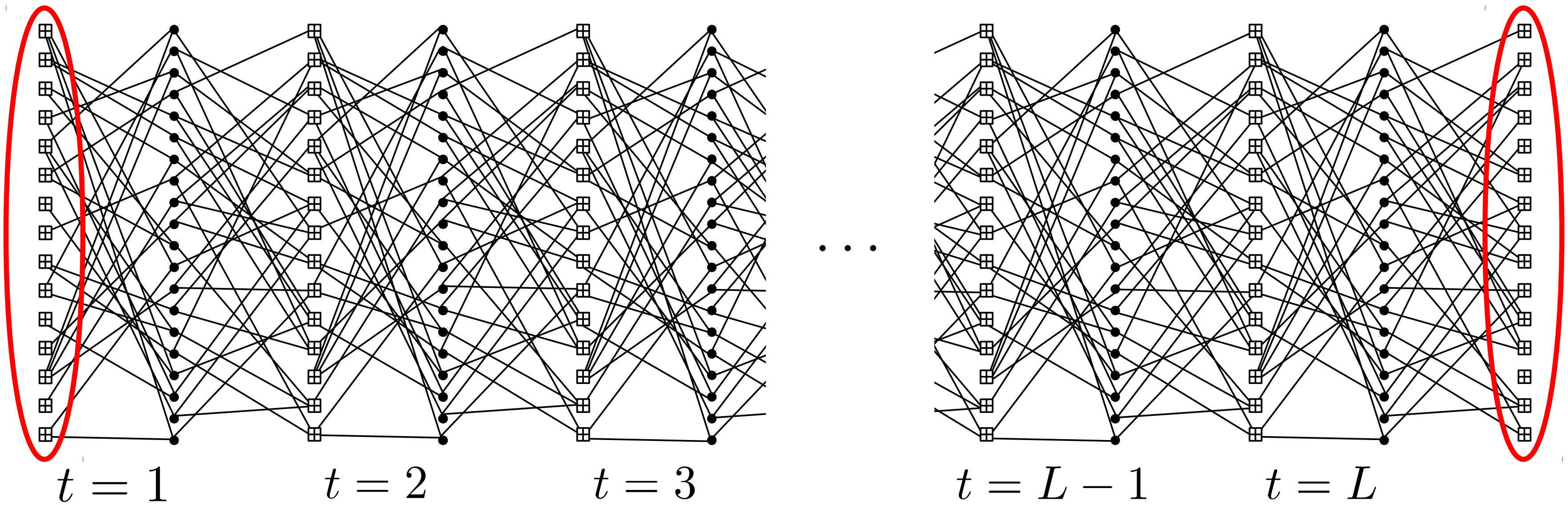

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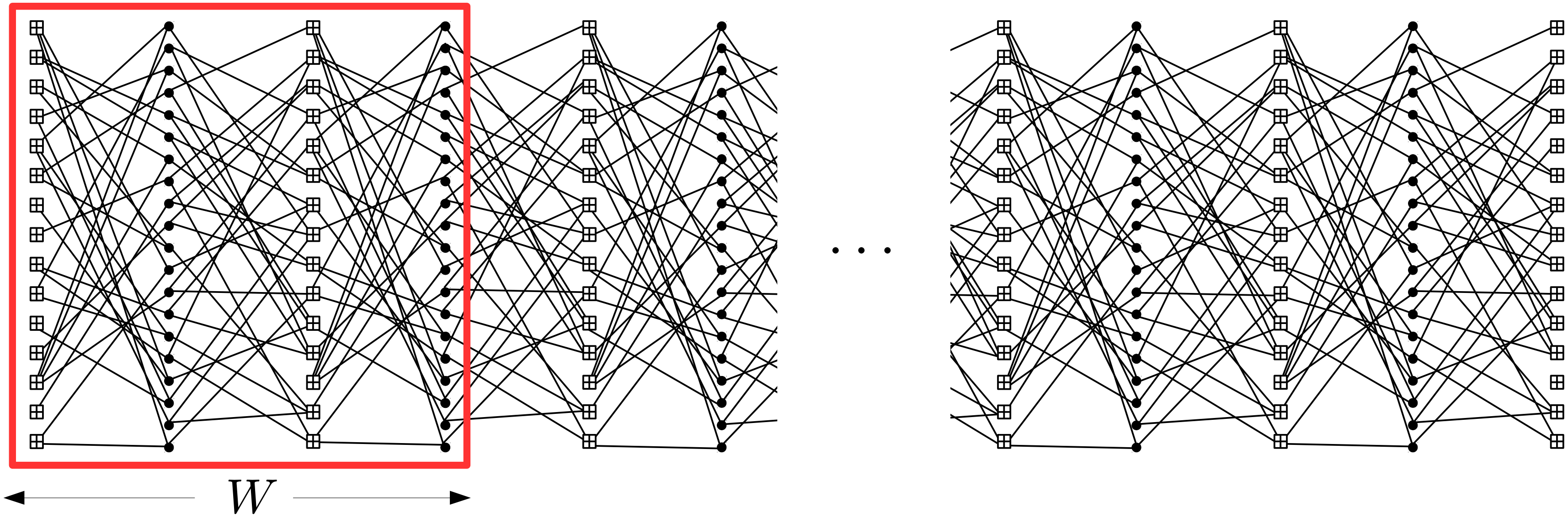

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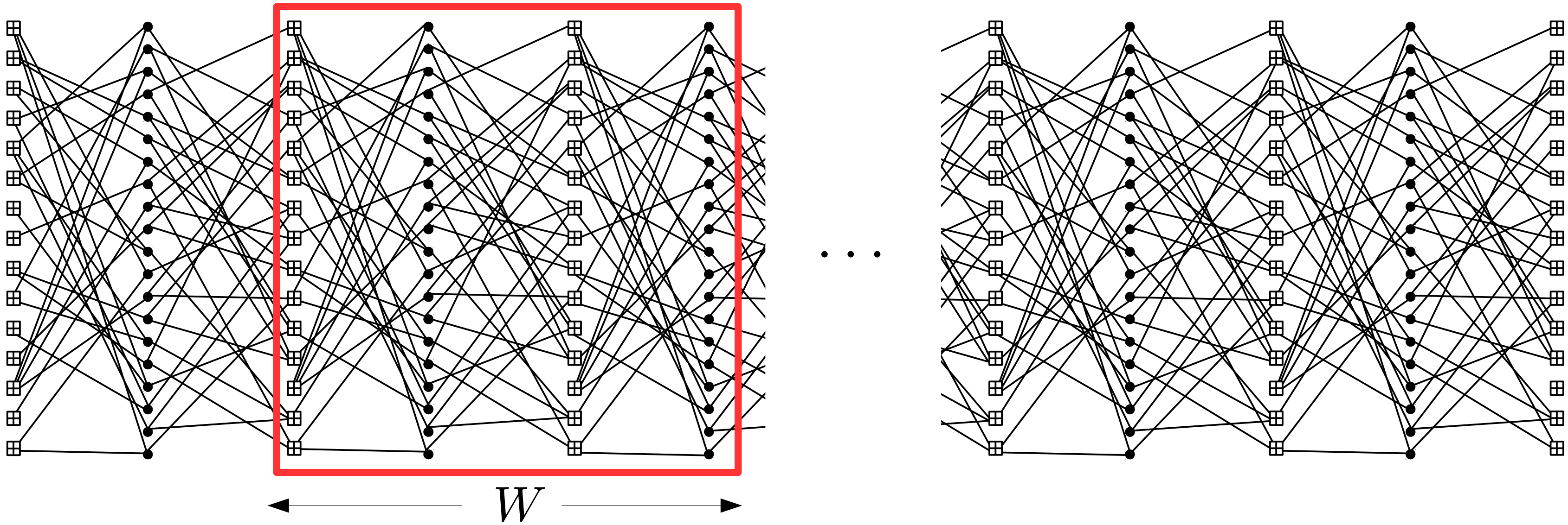

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(3,6)	0.46 dB	1.11 dB
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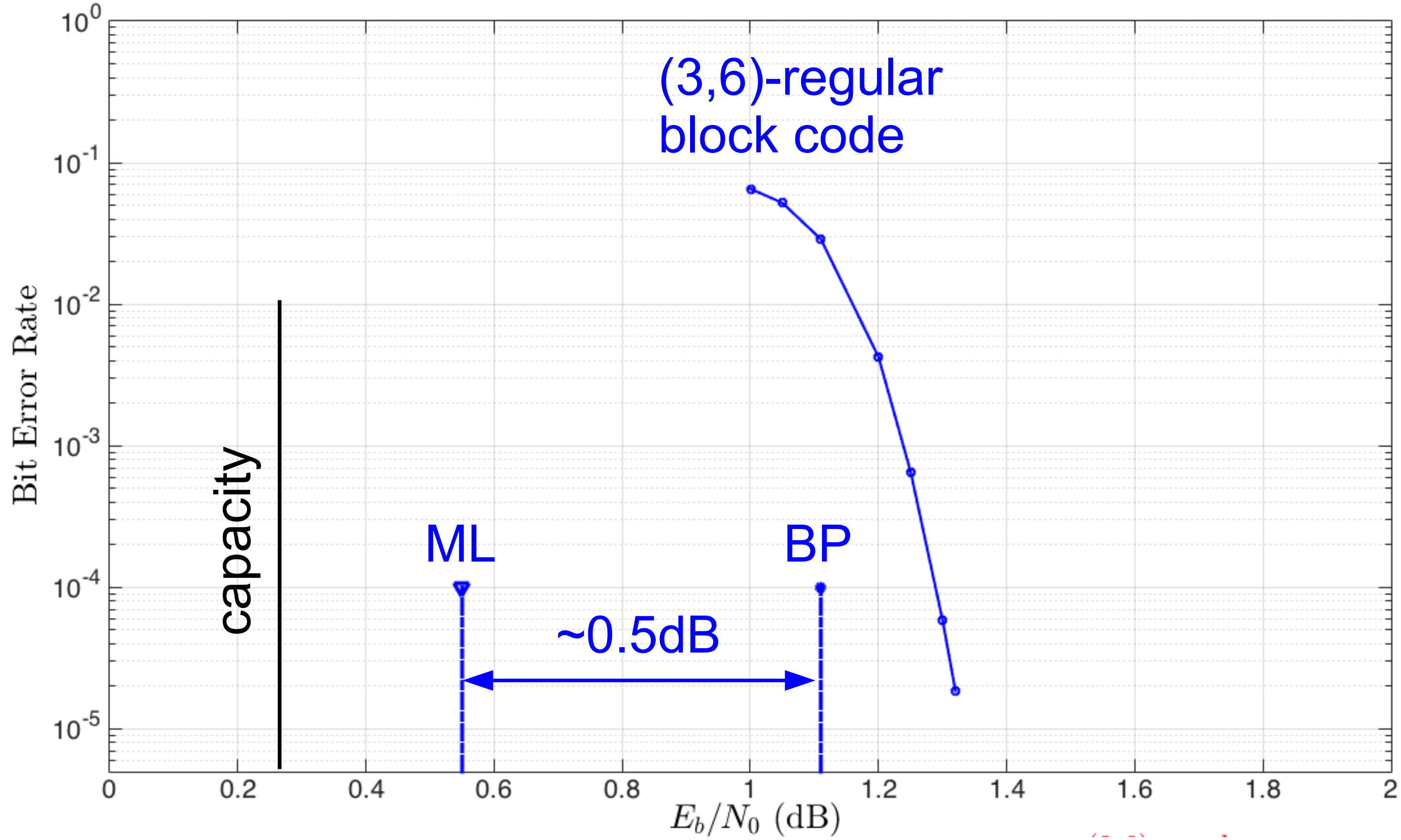
- In contrast to LDPC block codes, the thresholds of spatially coupled codes **improve** as the graph density increases!

Threshold Saturation (AWGNC)

BP = iterative (suboptimal) decoding threshold
ML = maximum likelihood (optimal) decoding threshold



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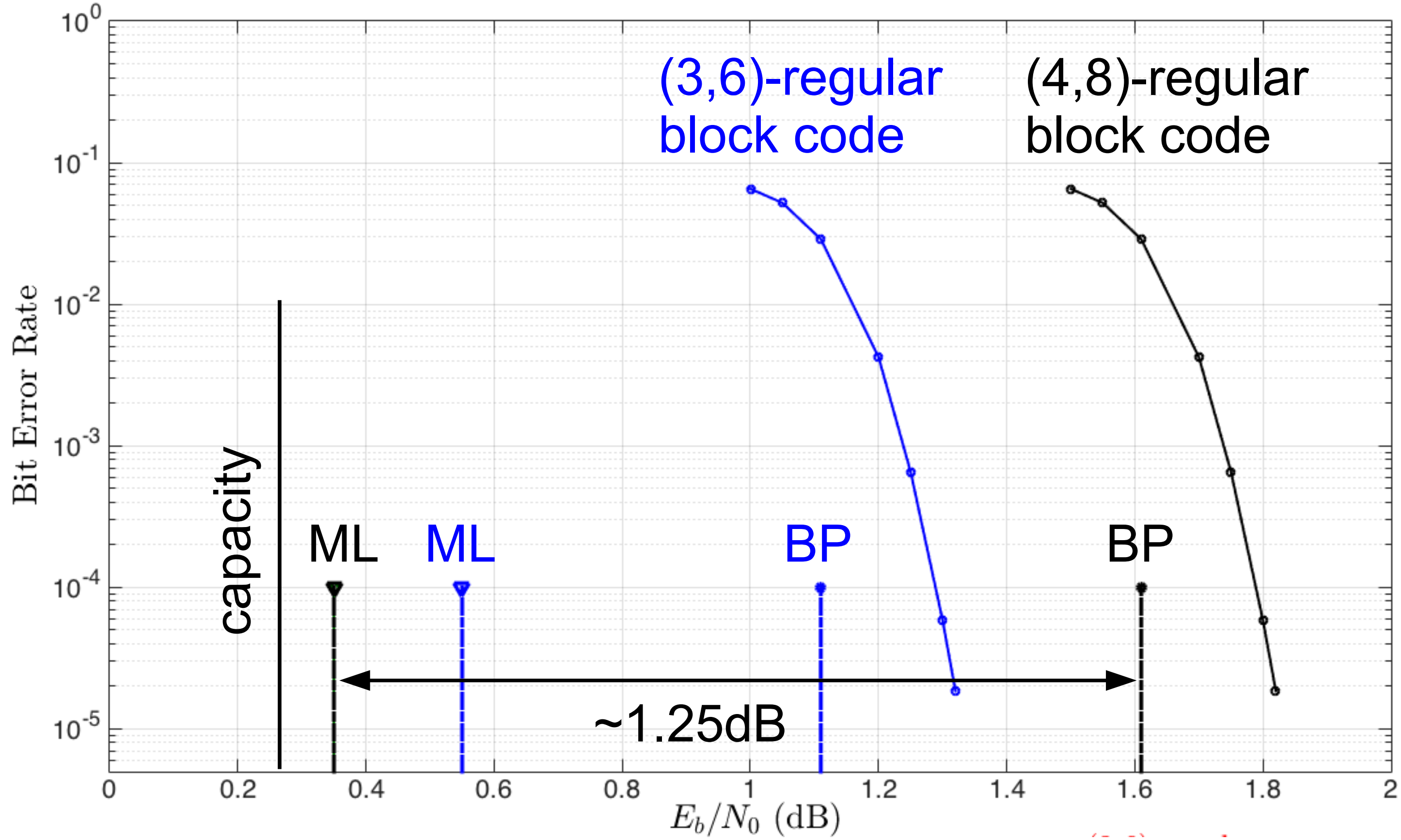

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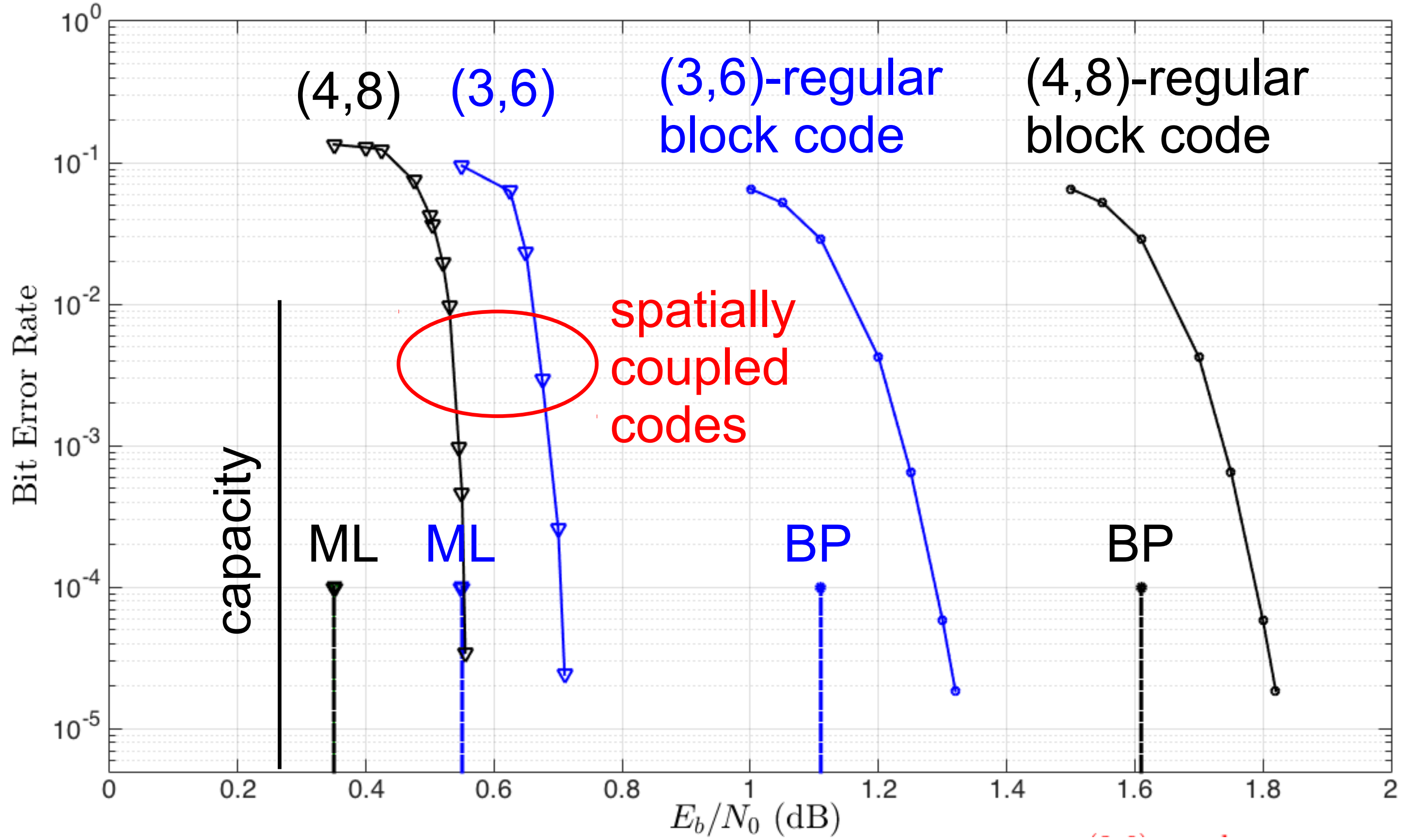

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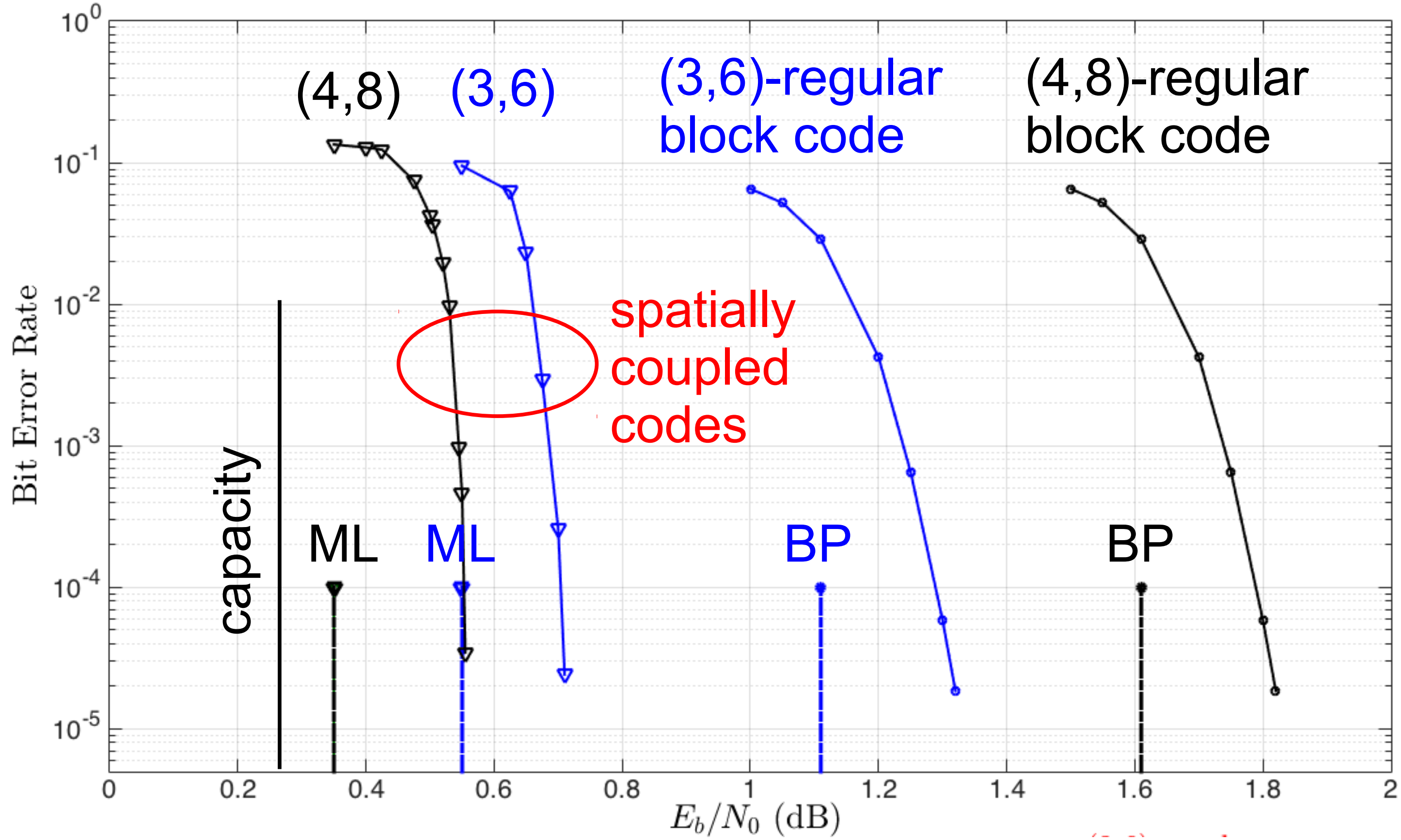

➡ Approach **optimal** performance with **suboptimal** iterative decoding!

A Comparison?

- Asymptotic performance comparison

	LDPC block code	SC-LDPC codes
Linear minimum distance growth	(J,K)-regular ensembles, some irregular ensembles	(J,K)-regular ensembles, some irregular ensembles
Capacity approaching BP thresholds	Optimized irregular ensembles only	(J,K)-regular ensembles, irregular ensembles

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Latency	Block length n	Window size W
Complexity	Graph density, iterations	Graph density, iterations

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Factors: Code design (QC), stopping rules, absorbing sets, scaling, quantization, …, and so on!