Structural Properties of Erasure Codes for Streaming Communication

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Joint work with:
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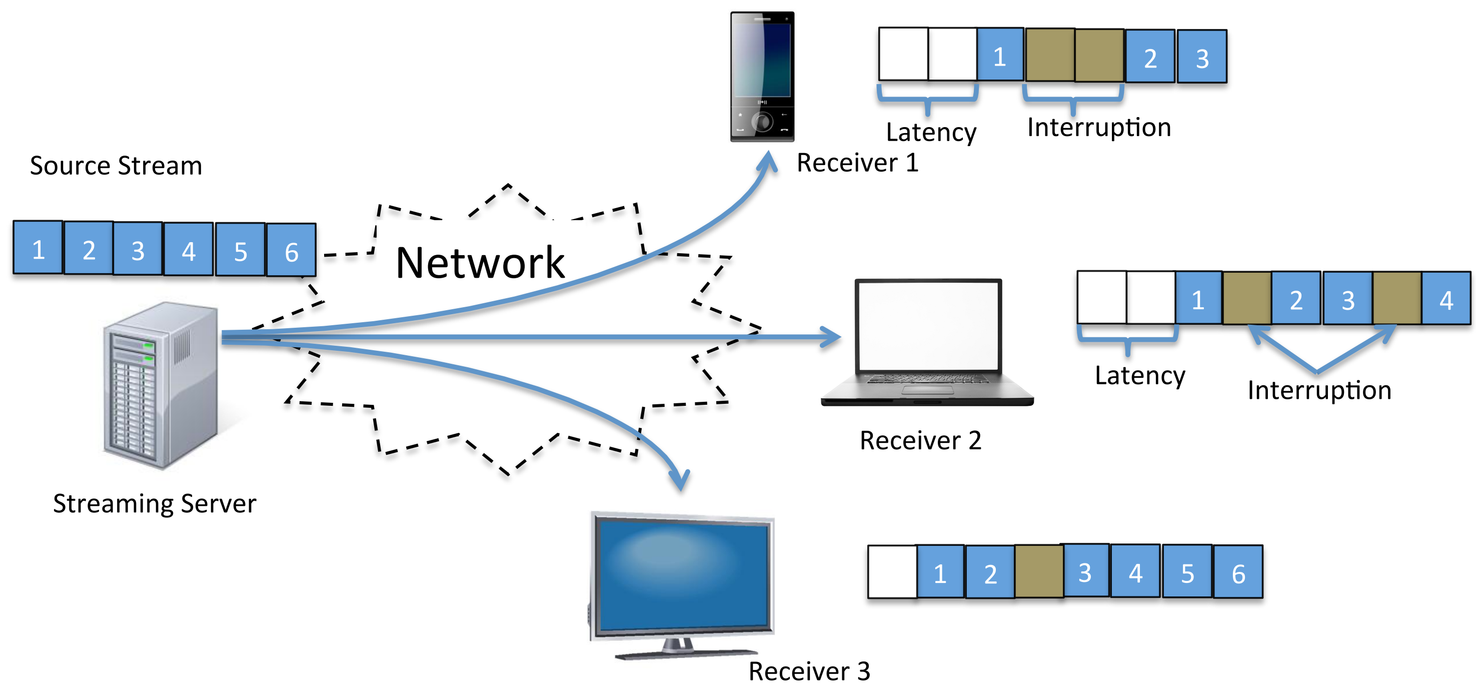
Multimedia Streaming



Applications

Application	Bit-Rate (Mbps)	MSDU (B)	Delay (ms)	PLR
Video Conf.	2 Mbps	1500	100 ms	10^{-4}
Interactive Gaming	1 Mbps	512	50 ms	10^{-4}
Video Streaming	4 Mbps	1500	500 ms	10^{-6}

Real-Time Communication System

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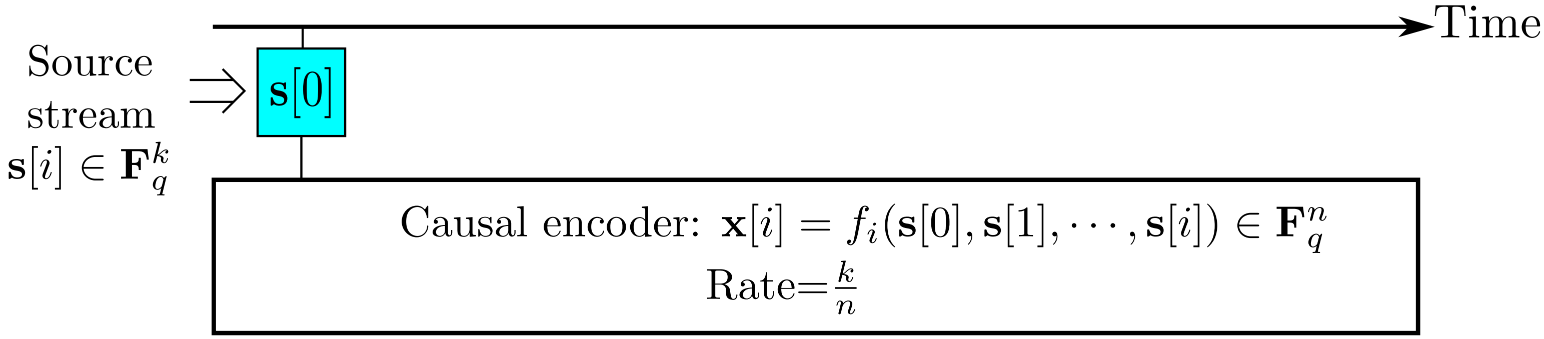

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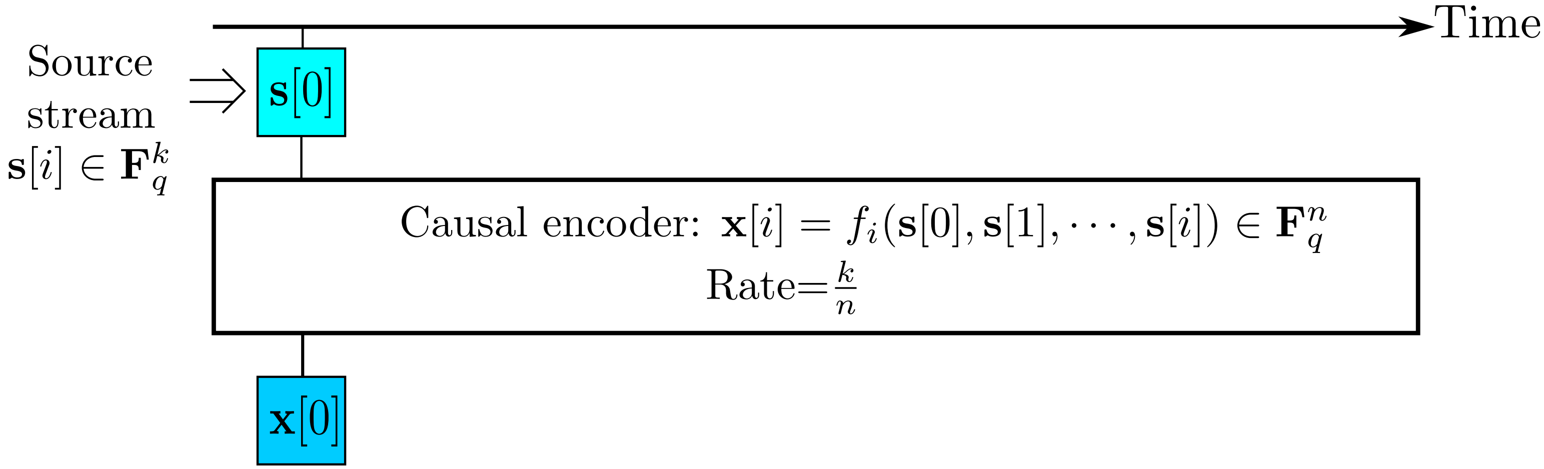

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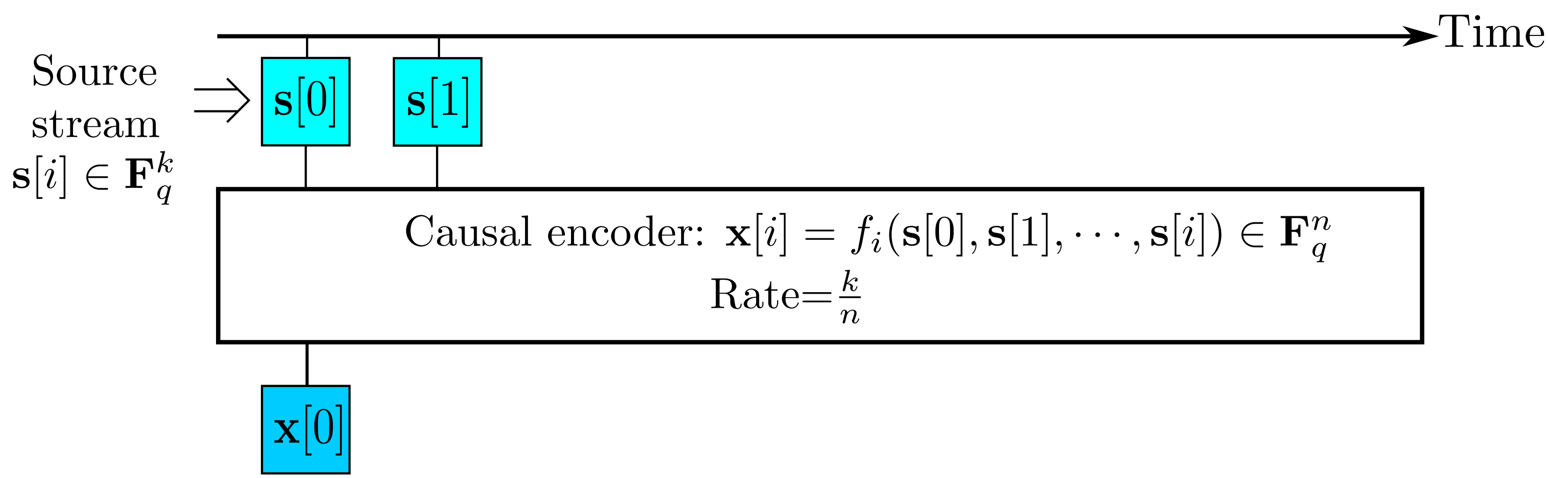

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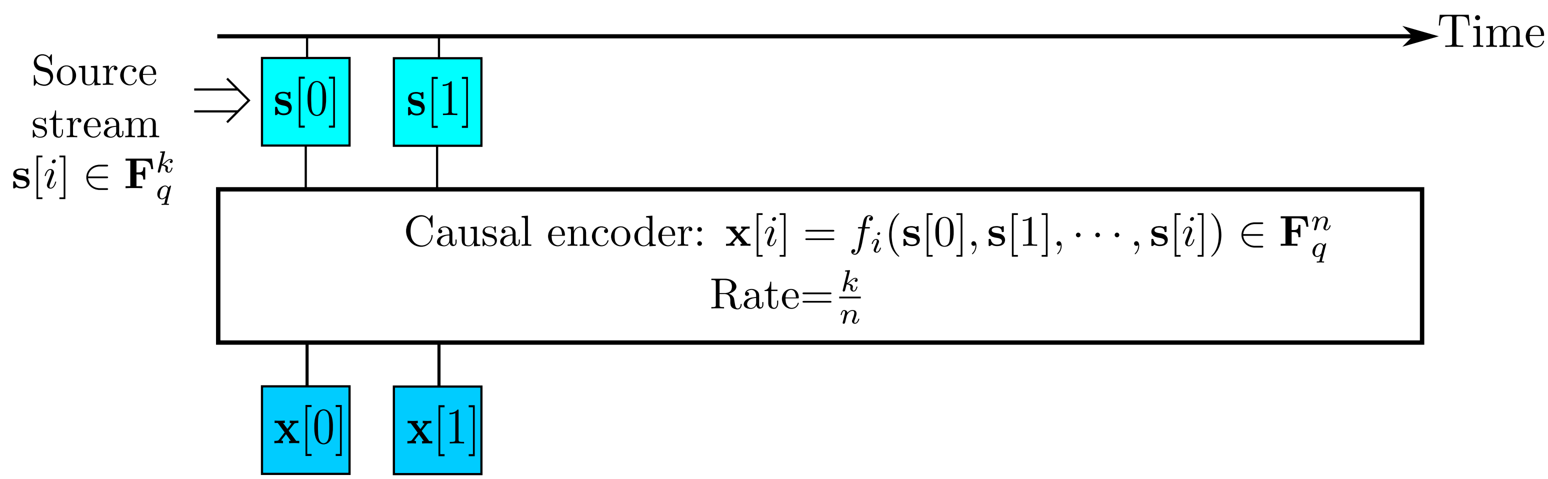

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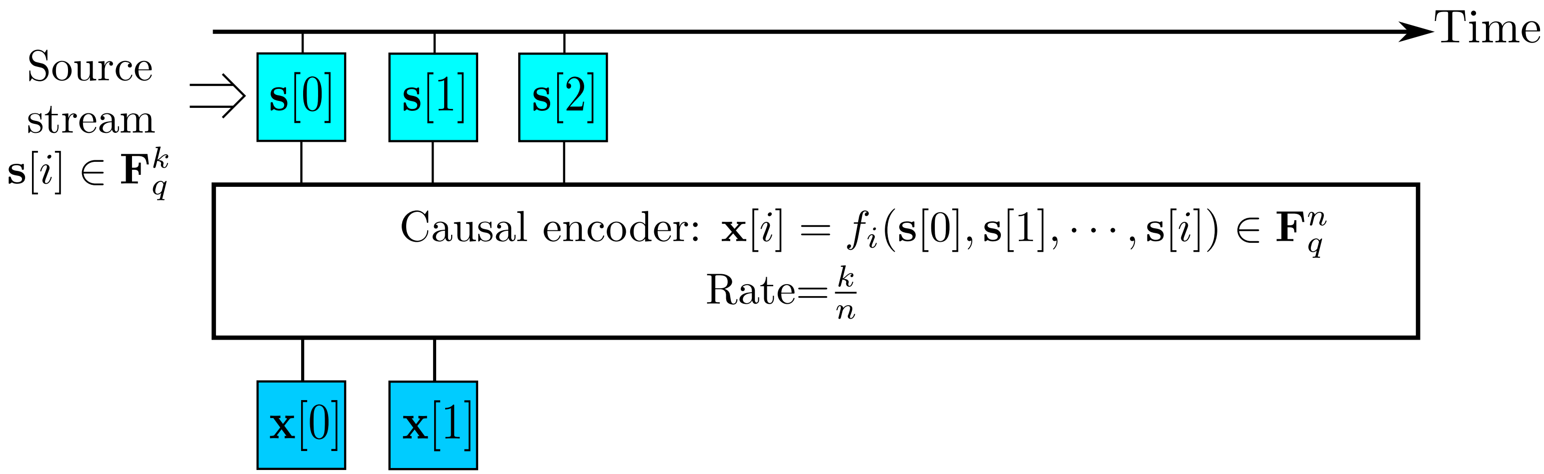

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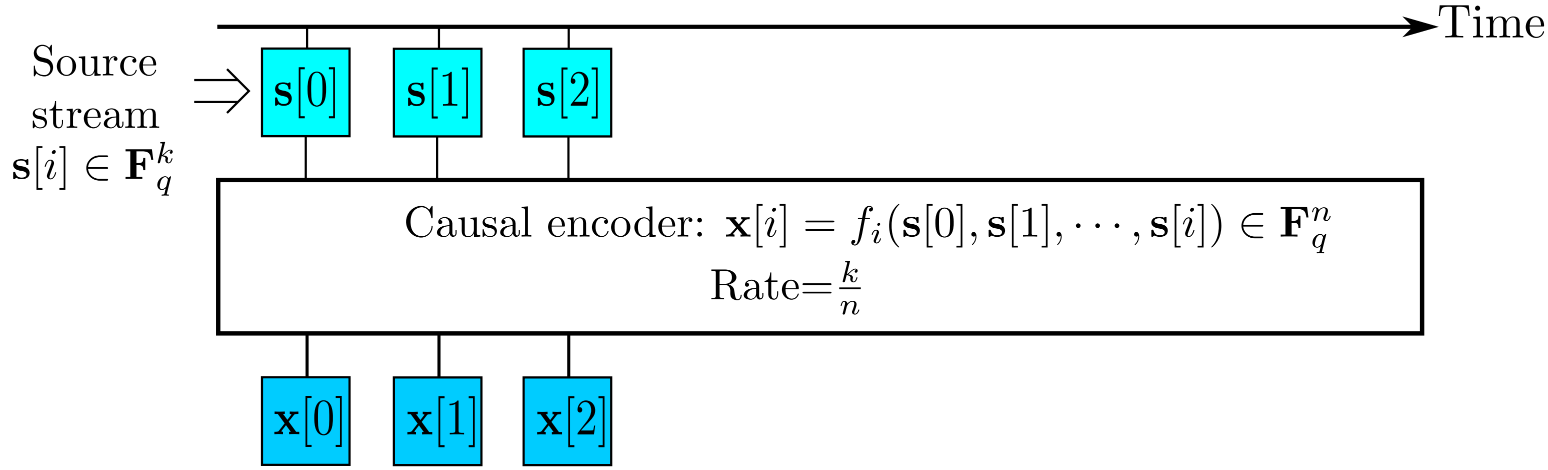

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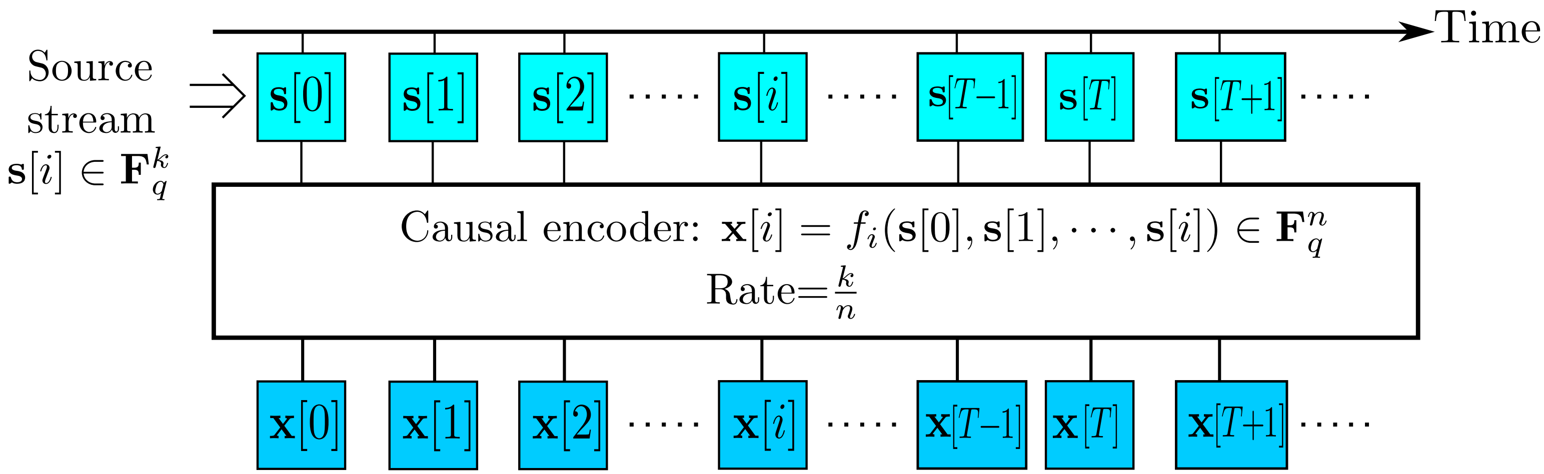

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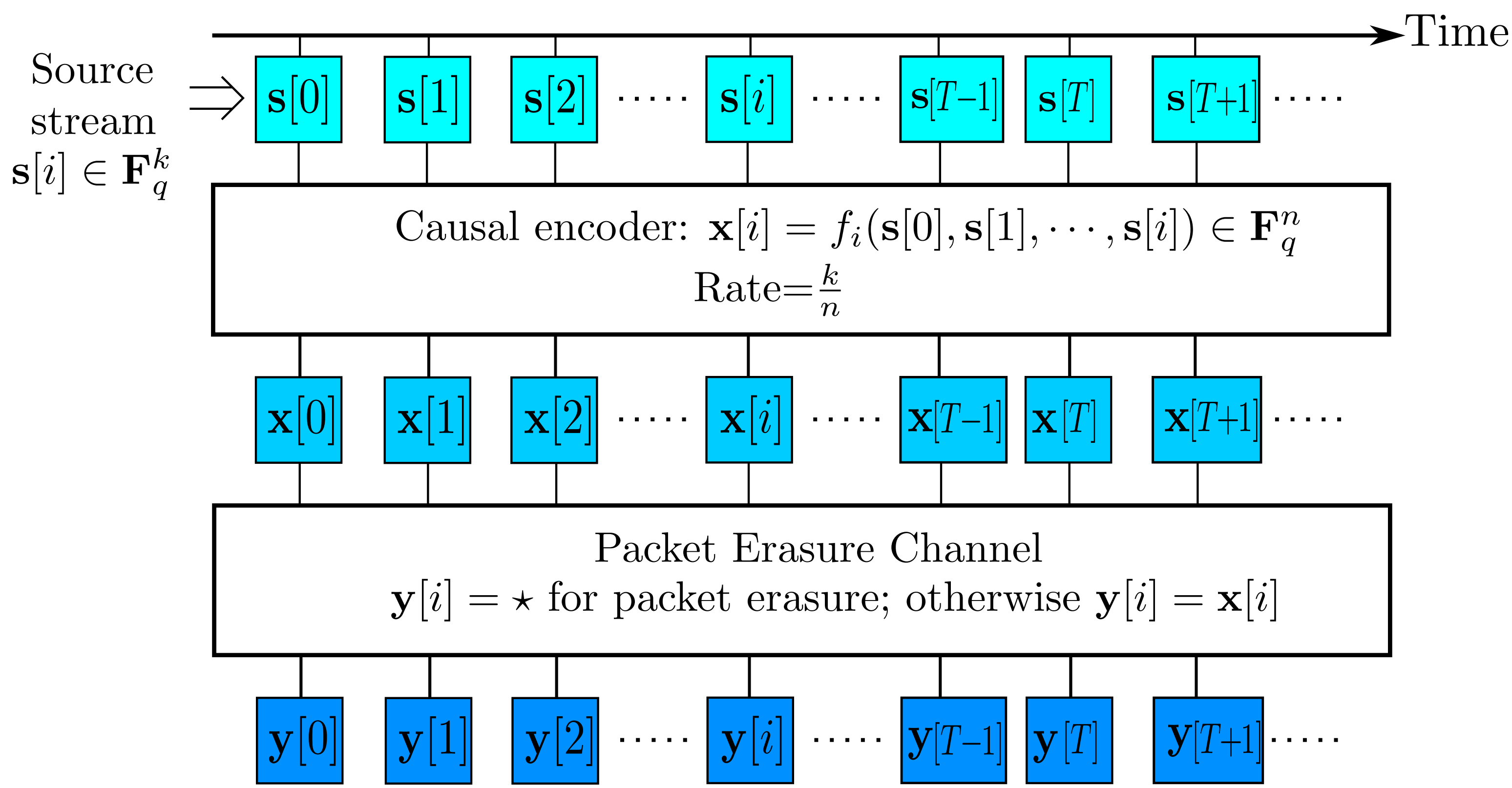

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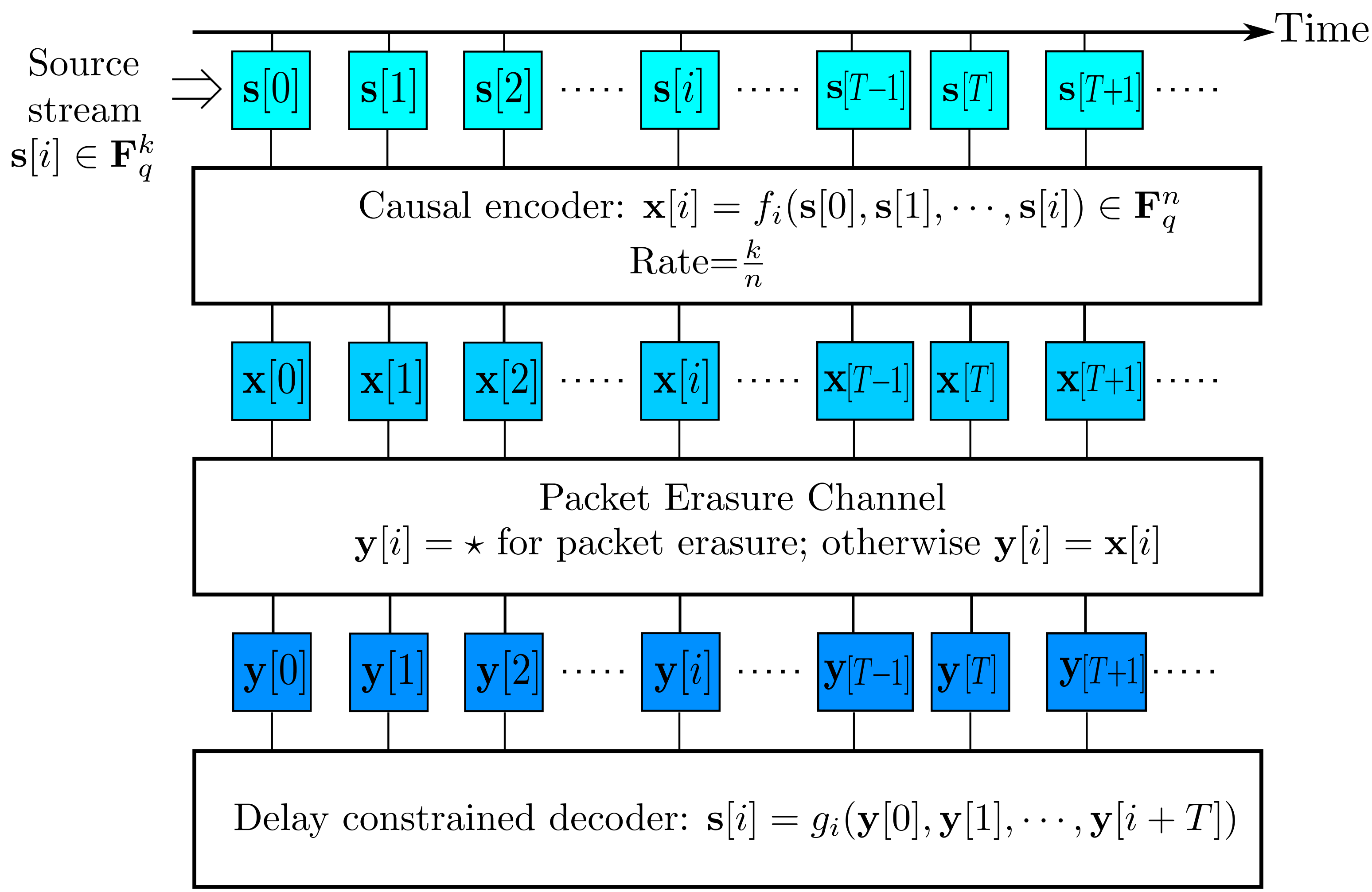

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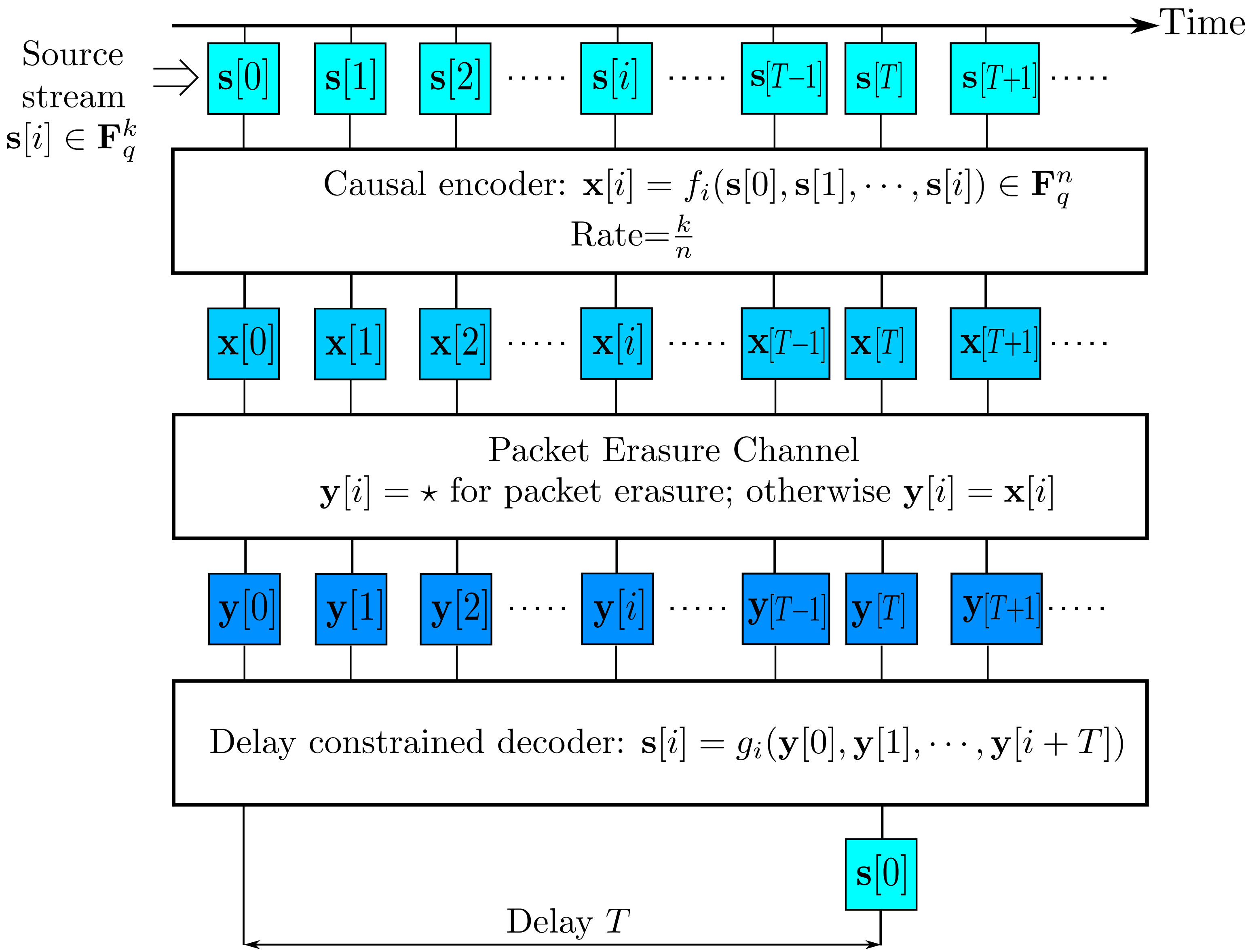

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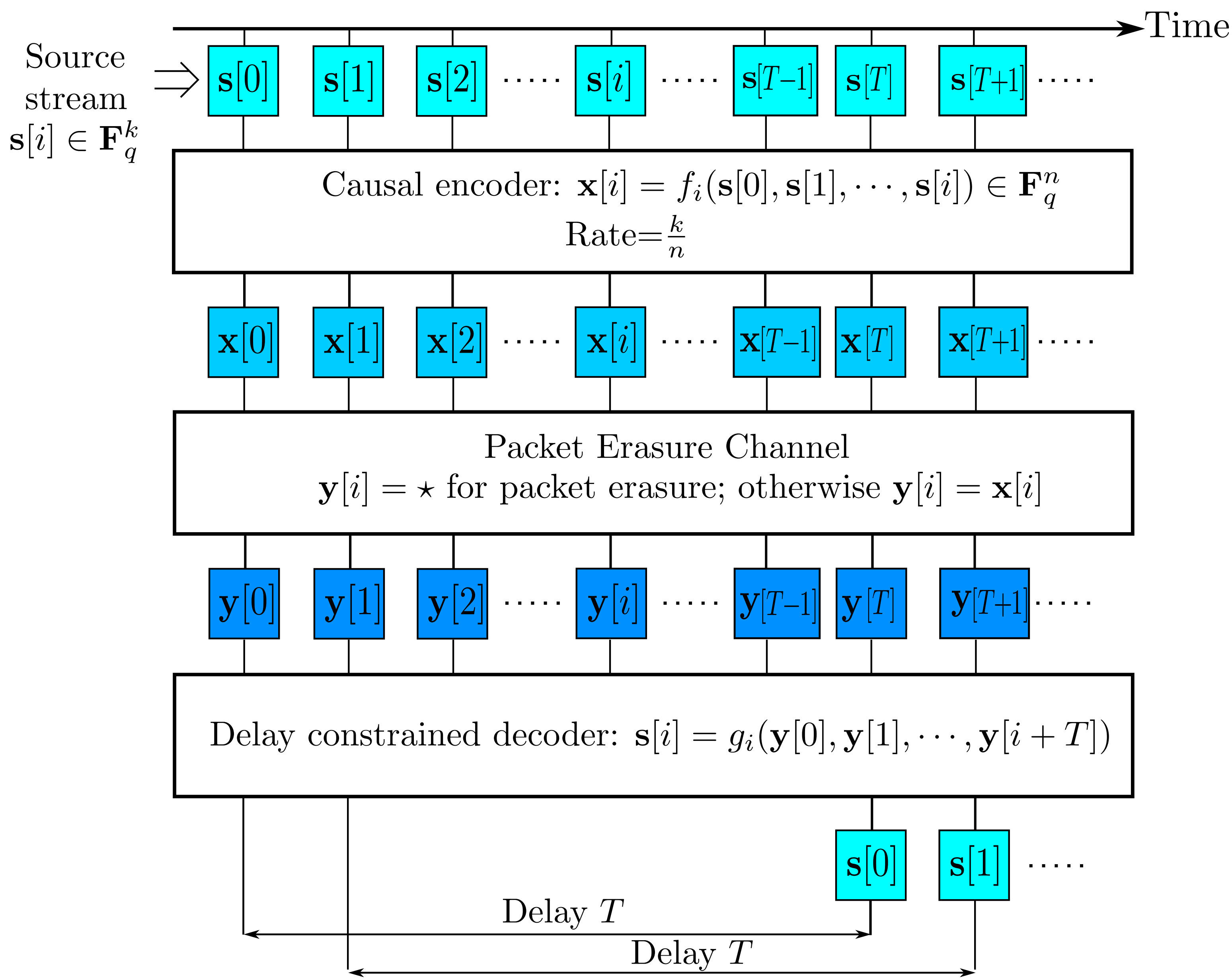
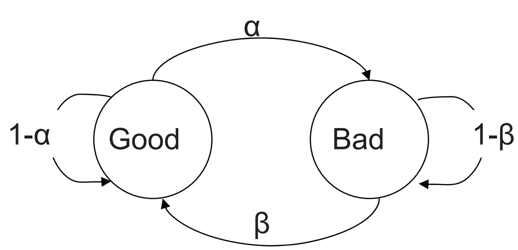
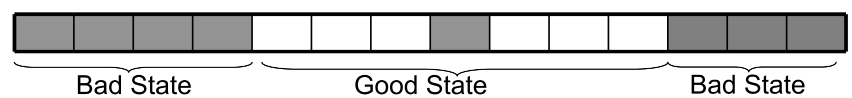


Streaming Code: Causal Encoder + Delay Constrained Decoder

Proposed Channel Model



Gilbert-Elliott Model



Loss Pattern

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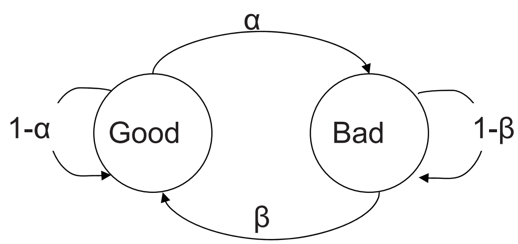

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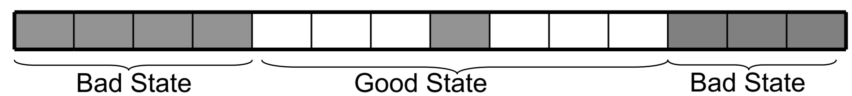

Loss Pattern

Sliding Window Erasure Channel: $\mathcal{C}(N, B, W)$

In any sliding window of length W, the channel can introduce only one of the following:

- An erasure burst of maximum length B
- Upto N erasures in arbitrary positions

Problem Setup - Sliding Window Erasure Channel Model

- Source Model : i.i.d. sequence $\mathsf{s}[t] \sim p_{\mathsf{s}}(\cdot) = \mathrm{Unif}\{(\mathbb{F}_q)^k\}$
- Streaming Encoder: $\mathsf{x}[t] = f_t\,(\mathsf{s}[0], \ldots, \mathsf{s}[t])$, $\mathsf{x}[t] \in (\mathbb{F}_q)^n$
- Erasure Channel: (Sliding Window Model)
- Delay-Constrained Decoder: $\hat{\mathsf{s}}[t] = g_t(\mathsf{y}[0], \ldots, \mathsf{y}[t+T])$
- Rate $R = \frac{k}{n}$

Streaming Capacity

A rate R is achievable over the $\mathcal{C}(N, B, W)$ channel, if there is a sequence of encoding and decoding functions, $f_t(\cdot)$ and $g_t(\cdot)$ respectively over a sufficiently large field $\mathbb{F}_q$, with delay T and rate $R = \frac{k}{n}$. The supremum of achievable rates is the streaming capacity.

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- Worst Case Definition
- Arbitrarily large field size

Main Result

Badr-Patil-Khisti-Tan-Apostolopoulos, IEEE Trans. on Inform. Theory (To Appear 2015)

Theorem

Consider the $\mathcal{C}(N, B, W)$ *channel, with* $W \geq B + 1$, *and let the delay be* T.

Upper-Bound *For any rate* R *code, we have:*

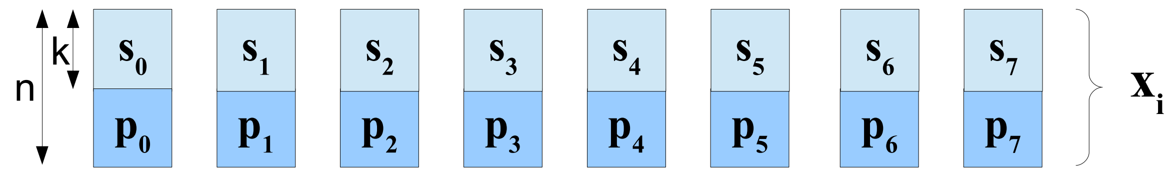
$$\left(\frac{R}{1-R}\right) B + N \leq \min(W, T+1)$$

Lower-Bound: *There exists a rate* R *code that satisfies:*

$$\left(\frac{R}{1-R}\right) B + N \geq \min(W, T+1) - 1.$$

The gap between the upper and lower bound is 1 *unit of delay.*

Baseline Codes - Full Rank Condition



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Erasure Codes:

- Random Linear Codes
- Strongly-MDS Codes (Gabidulin'88, Gluesing-Luerssen'06)

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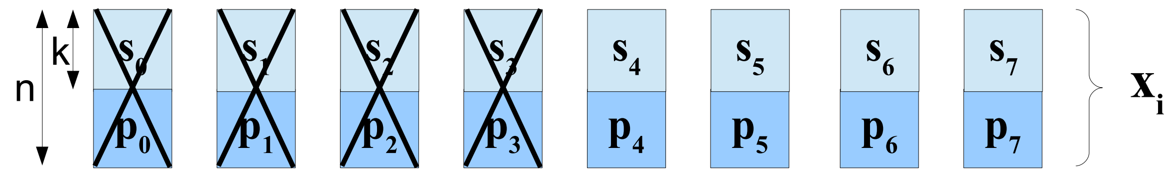

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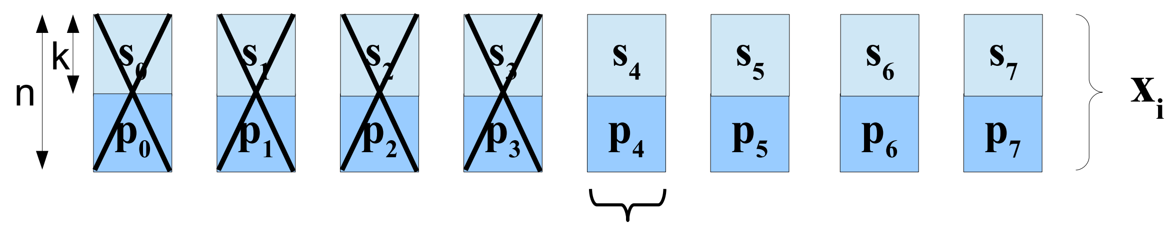

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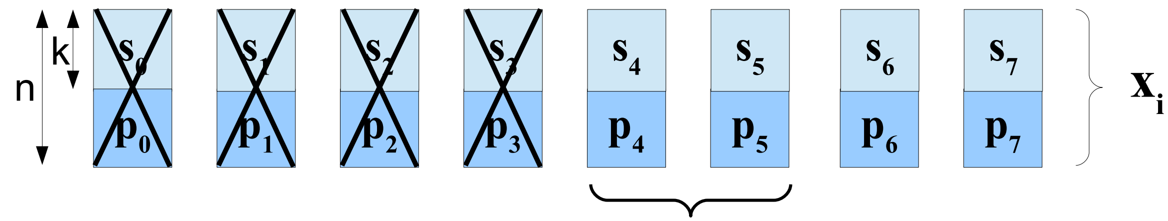

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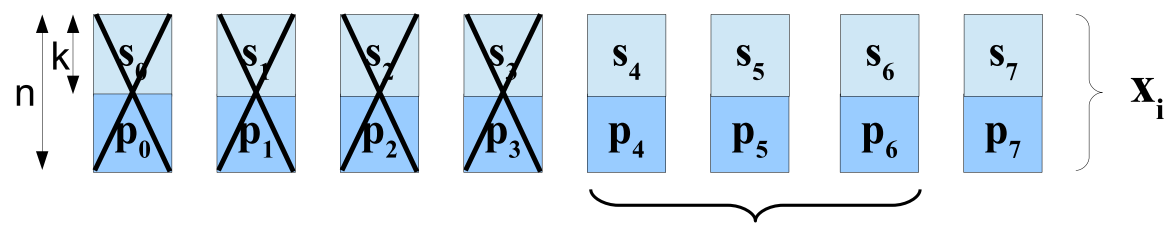

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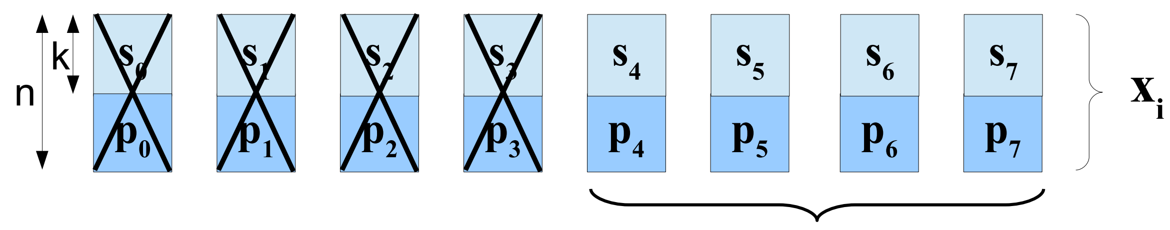

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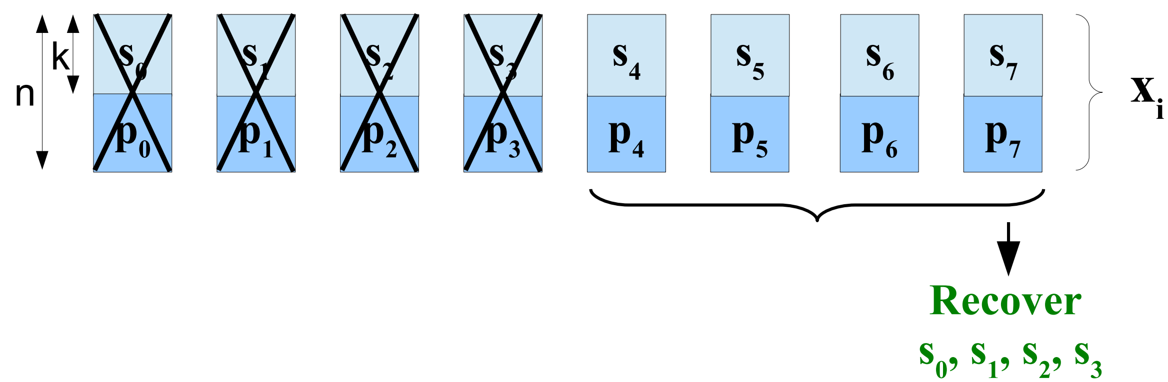

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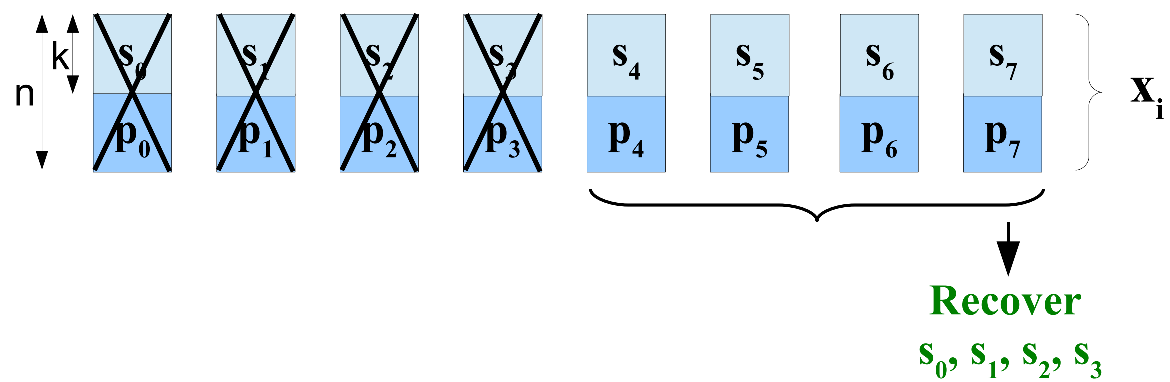

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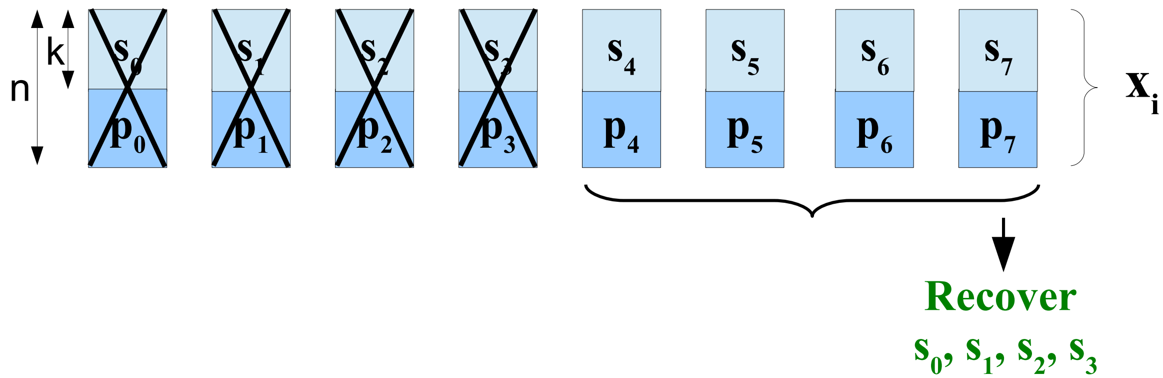

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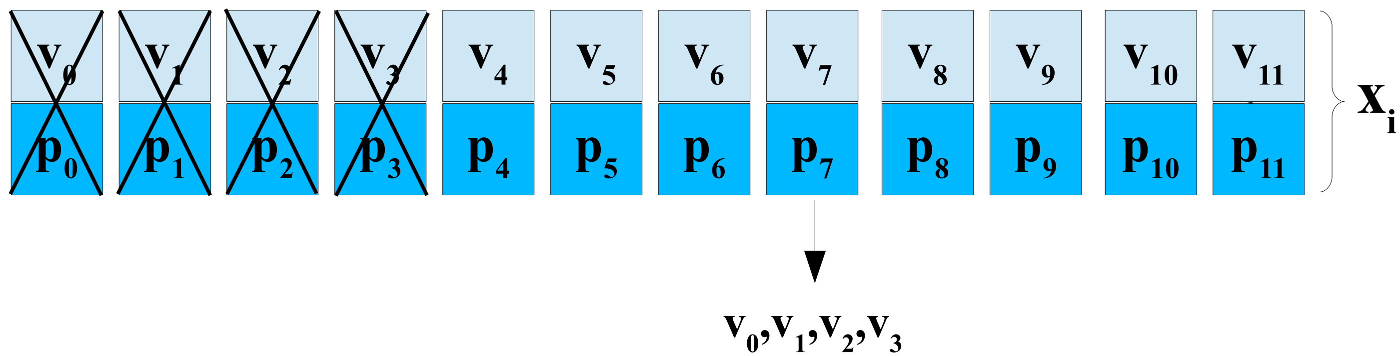
$$\begin{bmatrix} \mathbf{p}_4 \\ \mathbf{p}_5 \\ \mathbf{p}_6 \\ \mathbf{p}_7 \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{H}_4 & \mathbf{H}_3 & \mathbf{H}_2 & \mathbf{H}_1 \\ \mathbf{H}_5 & \mathbf{H}_4 & \mathbf{H}_3 & \mathbf{H}_2 \\ 0 & \mathbf{H}_5 & \mathbf{H}_4 & \mathbf{H}_3 \\ 0 & 0 & \mathbf{H}_5 & \mathbf{H}_4 \end{bmatrix}}_{\text{full rank}} \begin{bmatrix} \mathbf{s}_0 \\ \mathbf{s}_1 \\ \mathbf{s}_2 \\ \mathbf{s}_3 \end{bmatrix}$$

Streaming Code - Burst Erasure Channel

$B = 4,\ T = 8,\ R = \frac{T}{T+B} = \frac{2}{3}$

Rate $1/2$ Baseline Erasure Codes, $T = 7$



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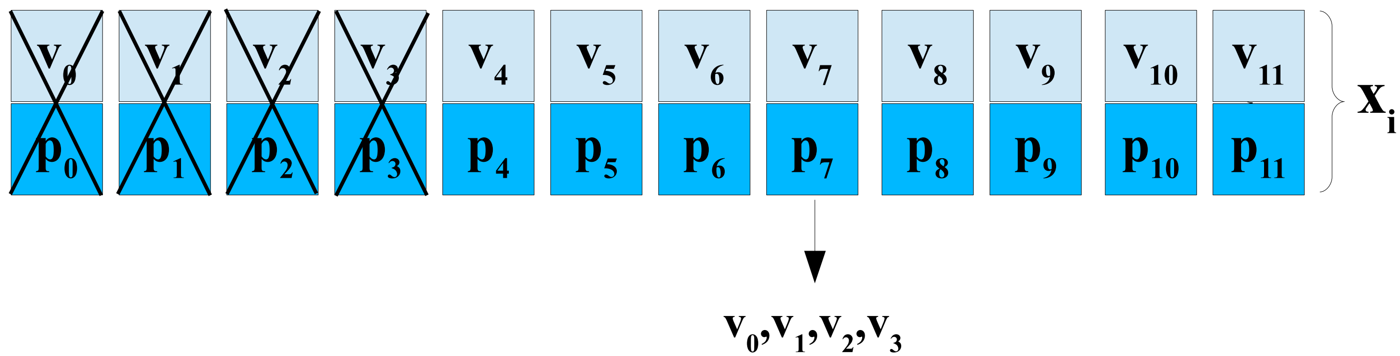
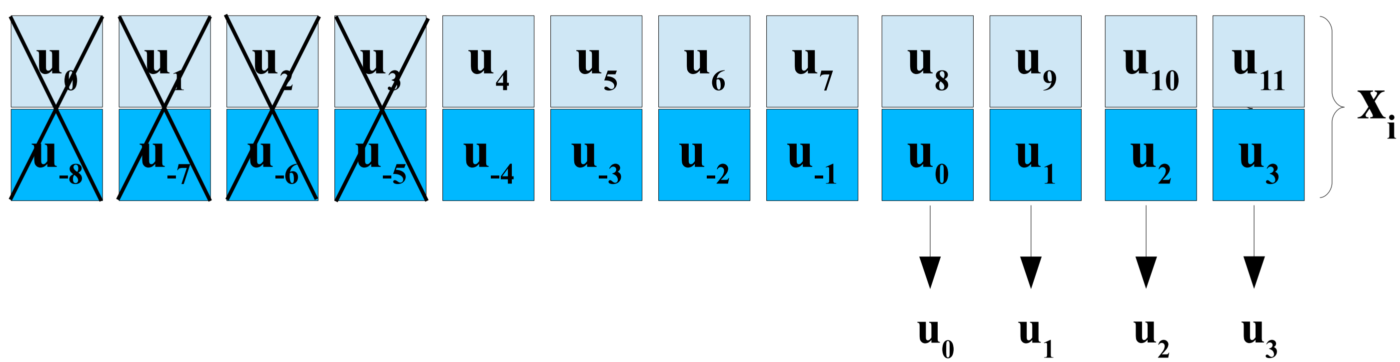


Rate $1/2$ Repetition Code, $T = 8$



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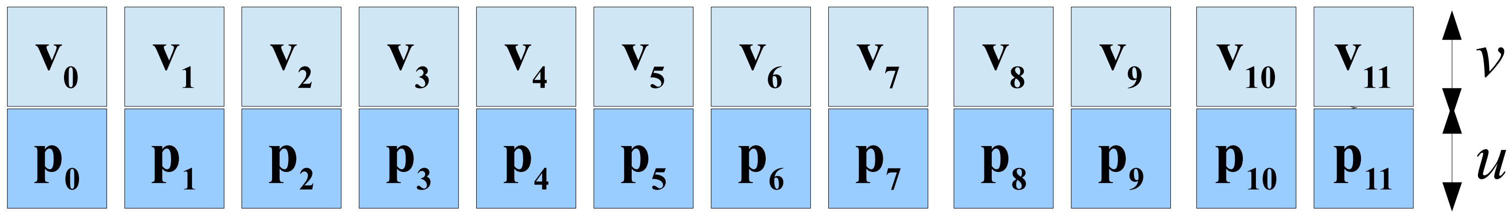

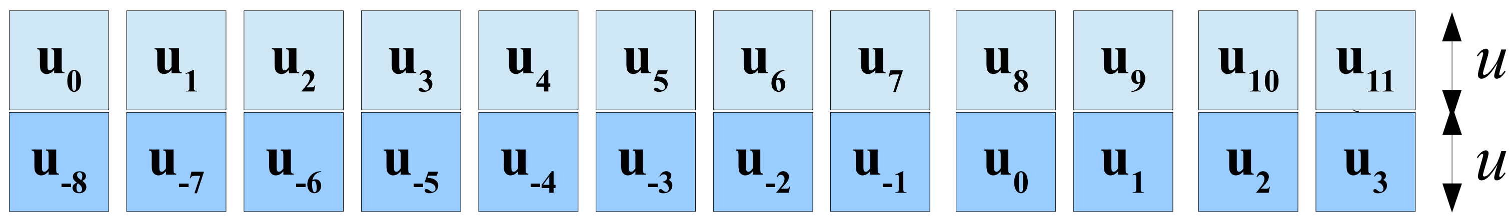

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u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}	u
v_0	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}	v_{11}	v
p_0	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}	p_{11}	u
u_{-8}	u_{-7}	u_{-6}	u_{-5}	u_{-4}	u_{-3}	u_{-2}	u_{-1}	u_0	u_1	u_2	u_3	u

$$R = \frac{u+v}{3u+v} = \frac{1}{2}$$

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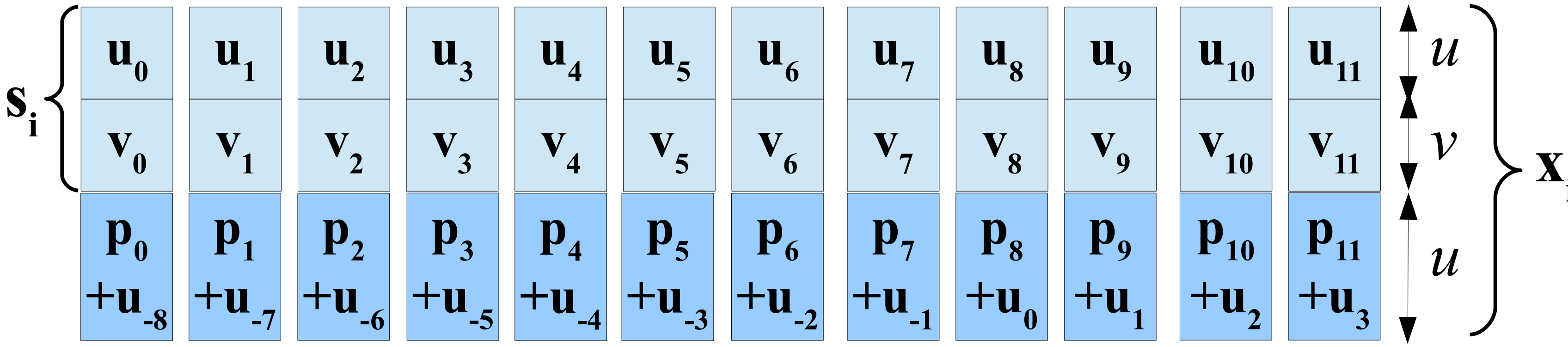

$$R = \frac{u+v}{2u+v} = \frac{2}{3}$$

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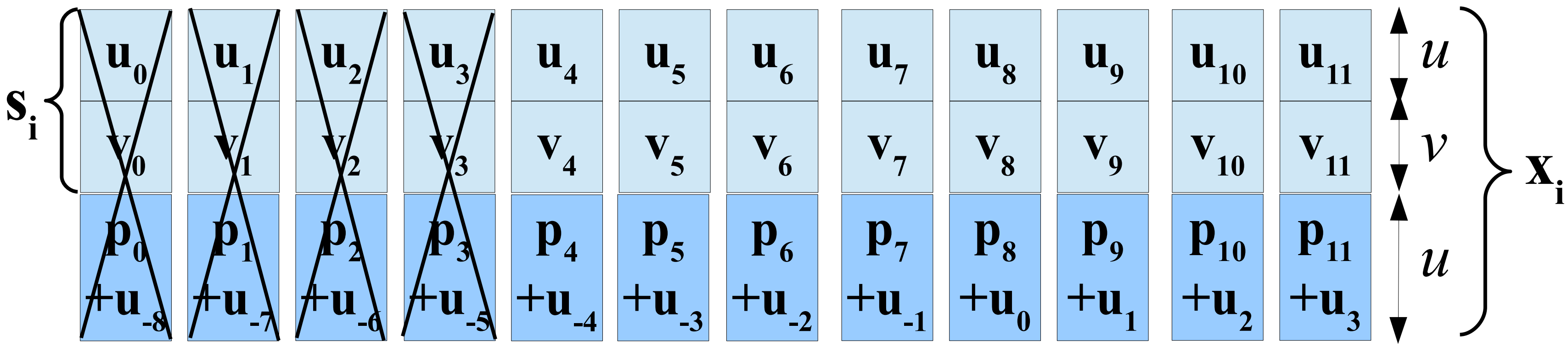

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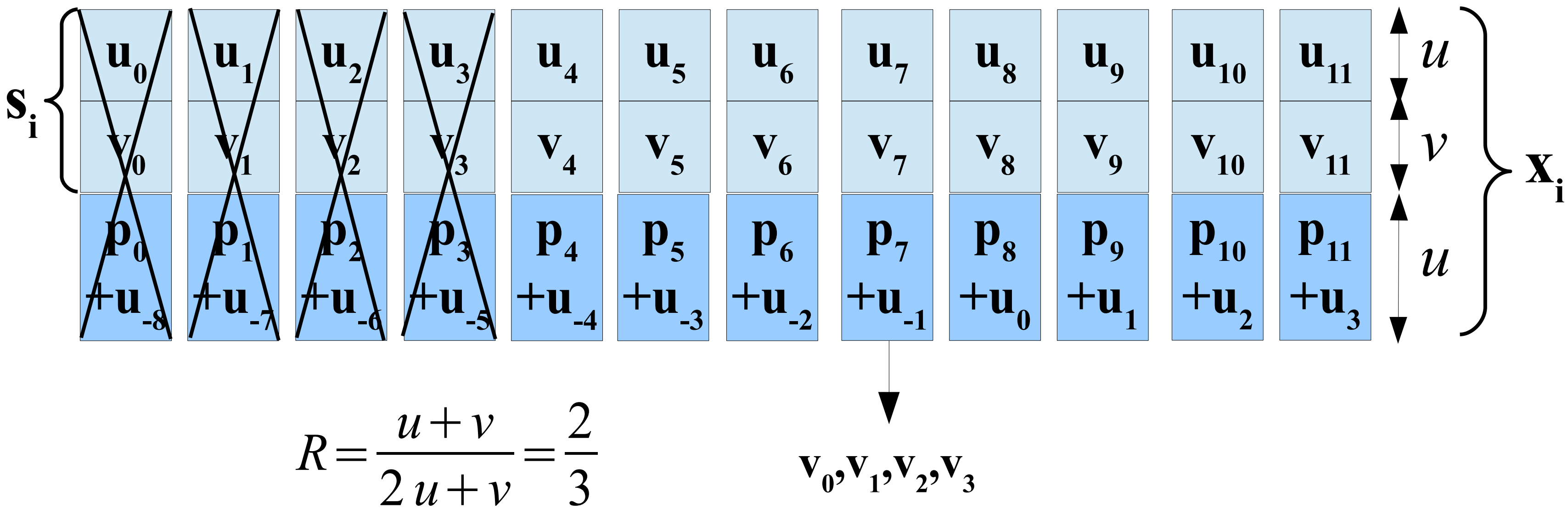

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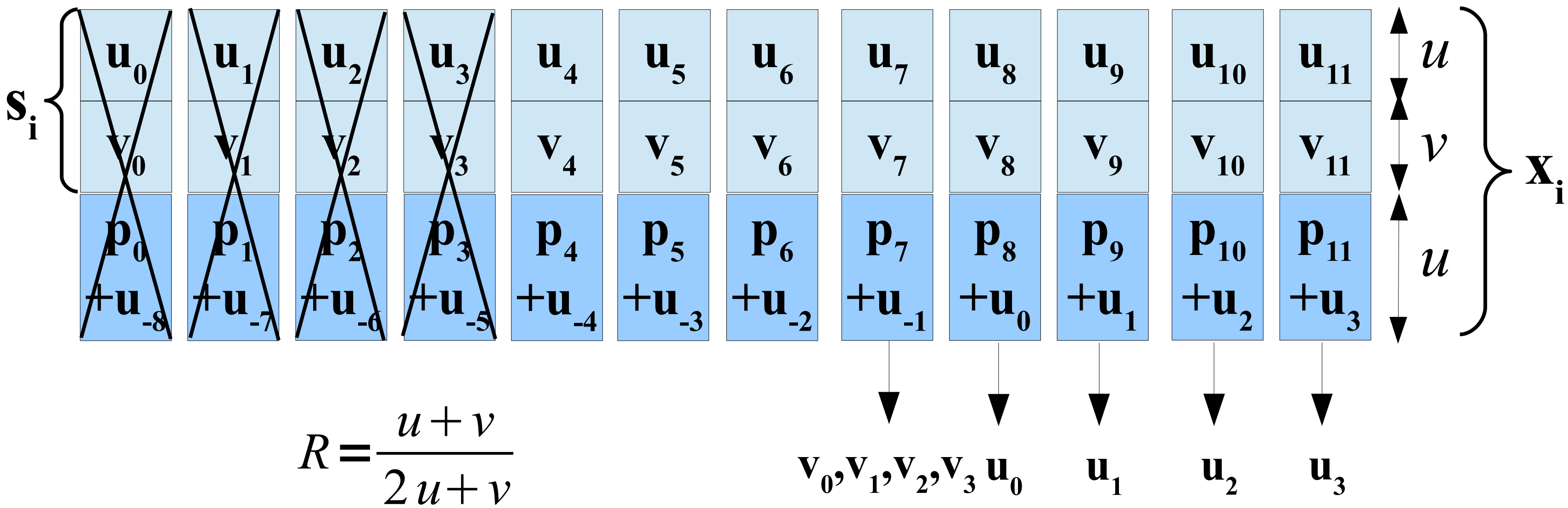

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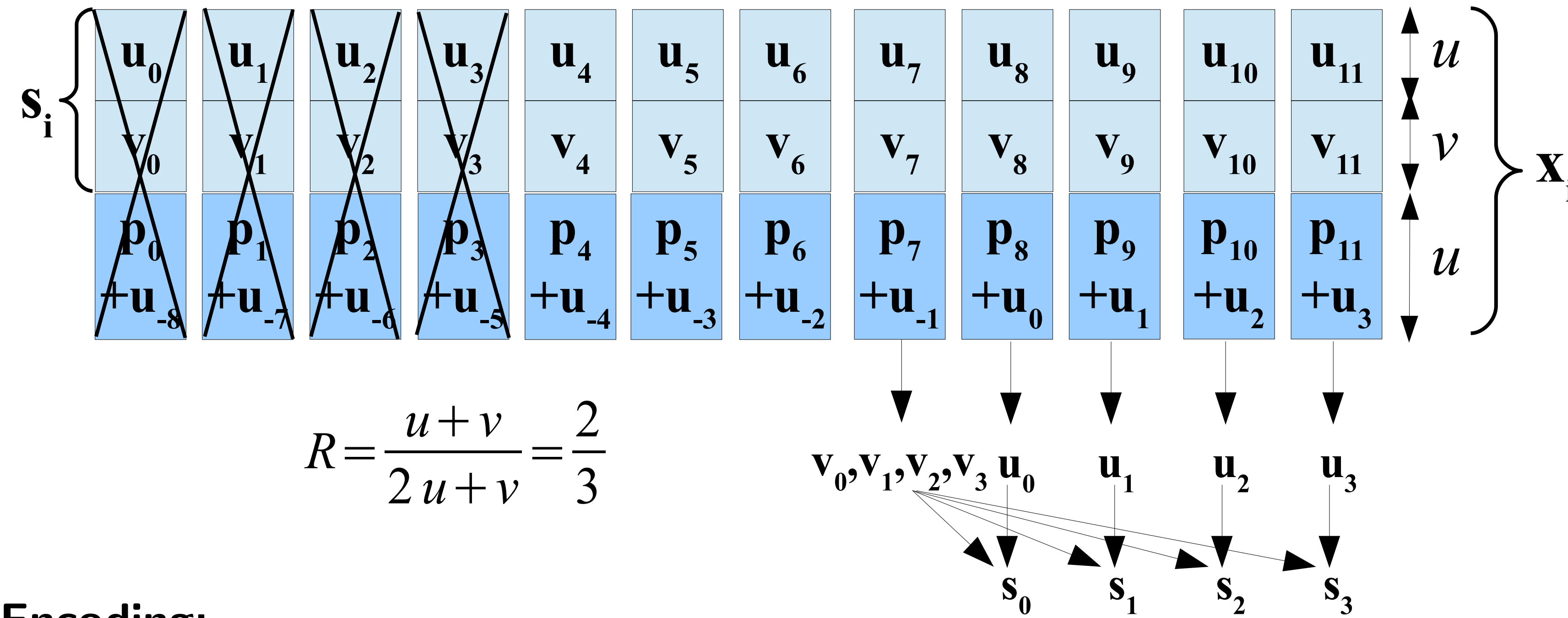

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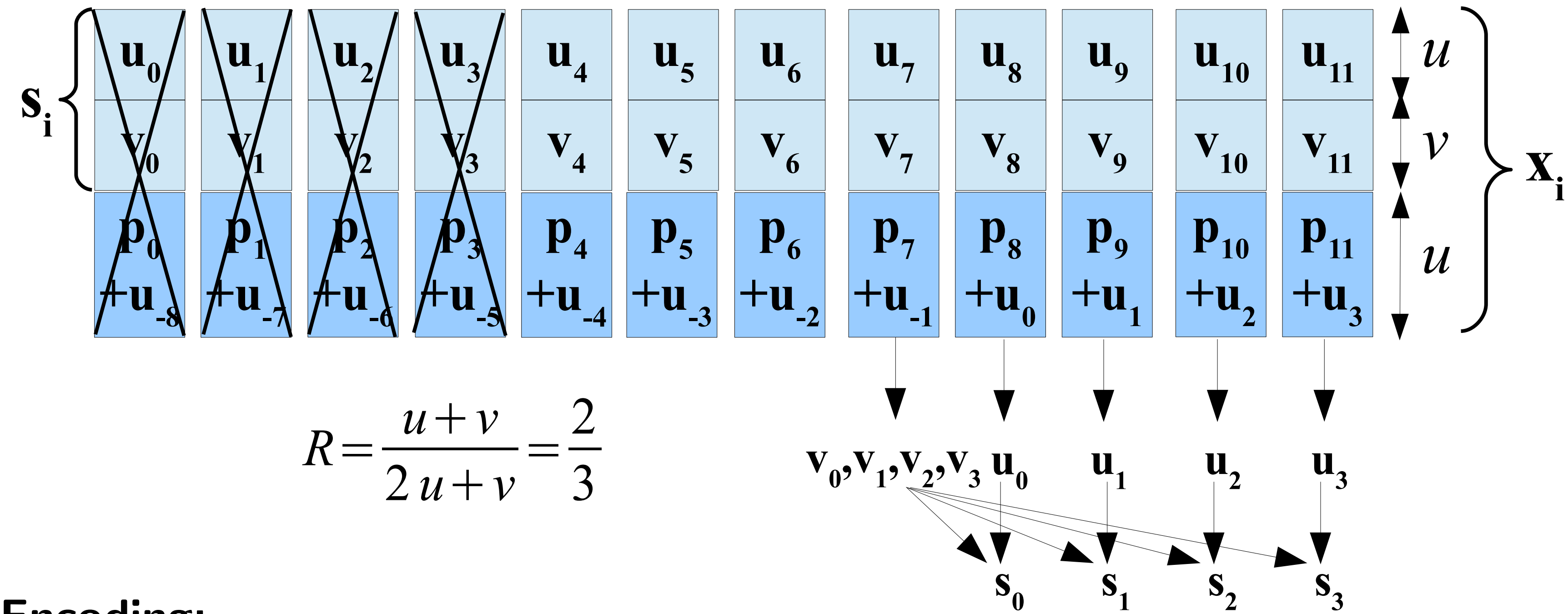

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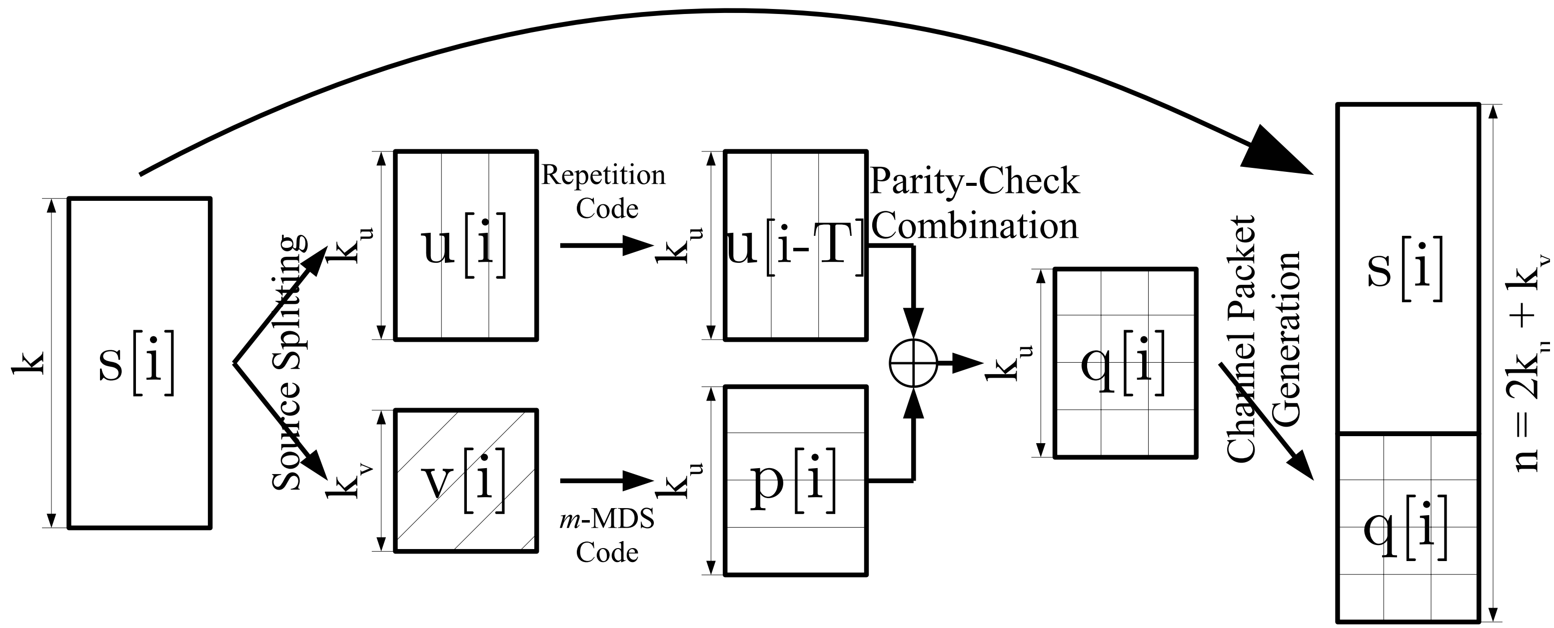
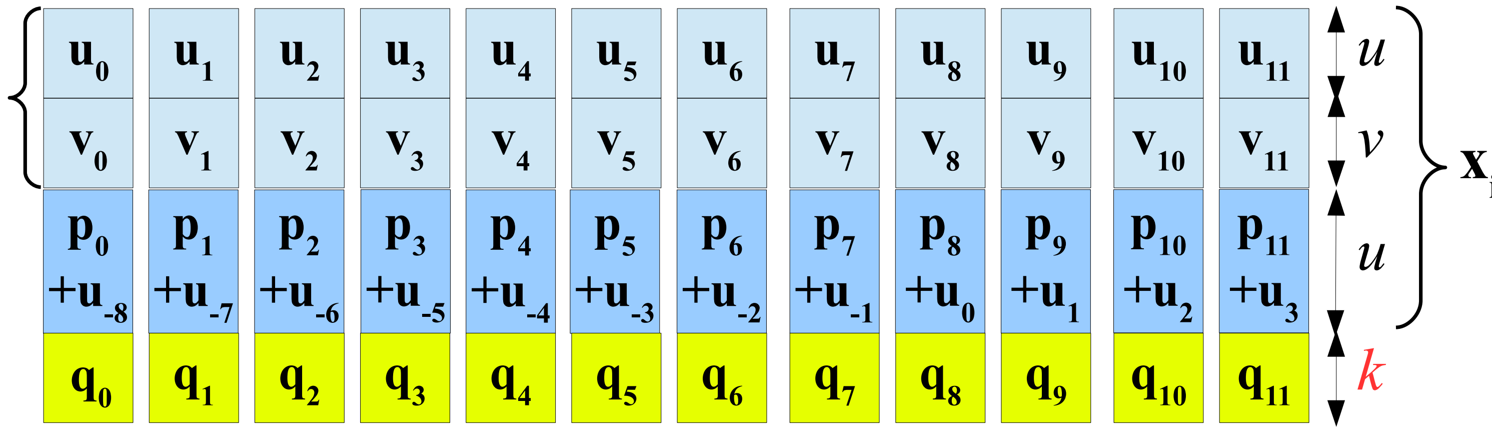
Streaming Code

Burst Erasure Channel, $R = \frac{T}{T+B}$



1. **Source Splitting**: $\mathbf{s}_i = (\mathbf{u}_i, \mathbf{v}_i)$, $\mathbf{u}_i \in \mathbb{F}_q^B$, $\mathbf{v}_i \in \mathbb{F}_q^{T-B}$
2. **Erasure Code on $\mathbf{v}_i$**: Generate $\mathbf{v}_i \to (\mathbf{v}_i, \mathbf{p}_i)$ where $\mathbf{p}_i \in \mathbb{F}_q^B$ is obtained from a Strongly-MDS code.
3. **Repetition Code on $\mathbf{u}_i$**: Repeat the $\mathbf{u}_i$ symbols with a shift of T
4. **Merging**: Combine the repeated $\mathbf{u}_i$'s with the $\mathbf{p}_i$'s



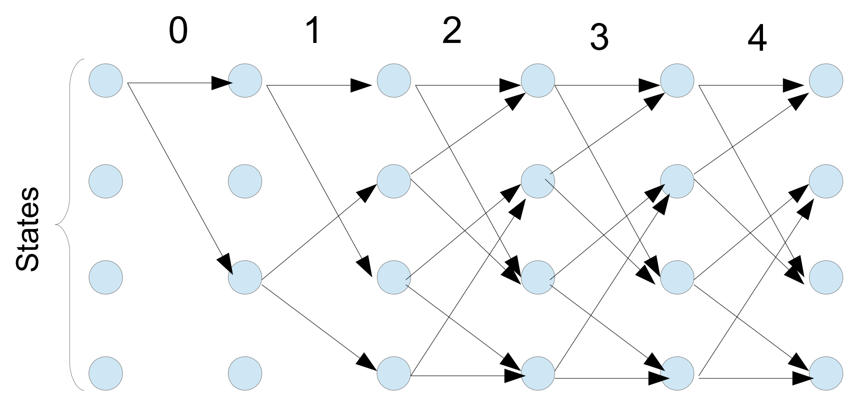
- **Burst-Erasure Streaming Code**: $(\mathbf{u}_i, \mathbf{v}_i, \mathbf{p}_i + \mathbf{u}_{i-T})$
- **Erasure Code**: $\mathbf{q}_i = \sum_{t=1}^{M} \mathbf{u}_{i-t} \cdot \mathbf{H}_t^u, \qquad \mathbf{q}_i \in \mathbb{F}_q^k$
- **Concatenation**: $(\mathbf{u}_i, \mathbf{v}_i, \mathbf{p}_i + \mathbf{u}_{i-T}, \mathbf{q}_i)$

$$R = \frac{T}{T + B + k}$$

- Attains the lower bound

Distance and Span Properties

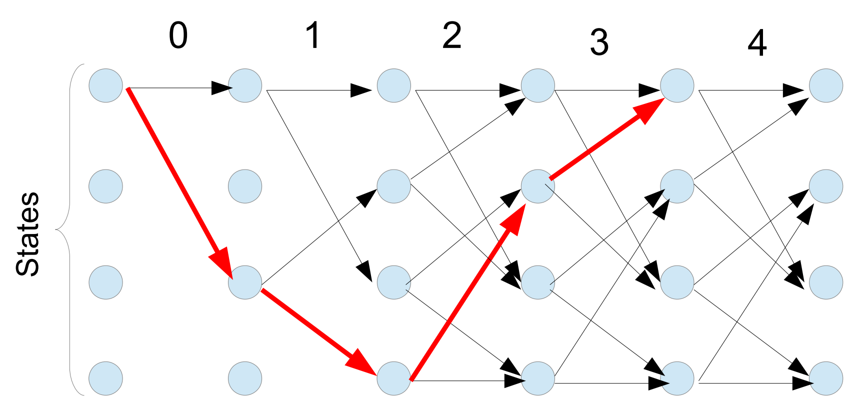
Consider (n, k, m) Convolutional code: $\mathbf{x}_i = \sum_{j=0}^{m} \mathbf{s}_{i-j} \mathbf{G}_j$



Trellis Diagram

Distance and Span Properties

Consider (n, k, m) Convolutional code: $\mathbf{x}_i = \sum_{j=0}^{m} \mathbf{s}_{i-j} \mathbf{G}_j$



Trellis Diagram – Free Distance

Distance and Span Properties

Consider (n, k, m) Convolutional code: $\mathbf{x}_i = \sum_{j=0}^{m} \mathbf{s}_{i-j}\mathbf{G}_j$

Column Distance: d_T

$$d_T = \min_{\substack{[\mathbf{s}_0,\ldots,\mathbf{s}_T] \\ \mathbf{s}_0 \neq 0}} \mathrm{wt}\left(\begin{bmatrix} \mathbf{s}_0 & \ldots & \mathbf{s}_T \end{bmatrix} \begin{bmatrix} \mathbf{G}_0 & \mathbf{G}_1 & \ldots & \mathbf{G}_T \\ 0 & \mathbf{G}_0 & \ldots & \mathbf{G}_{T-1} \\ \vdots & & \ddots & \vdots \\ 0 & & \ldots & \mathbf{G}_0 \end{bmatrix} \right)$$

Distance and Span Properties

Consider (n, k, m) Convolutional code: $\mathbf{x}_i = \sum_{j=0}^{m} \mathbf{s}_{i-j} \mathbf{G}_j$

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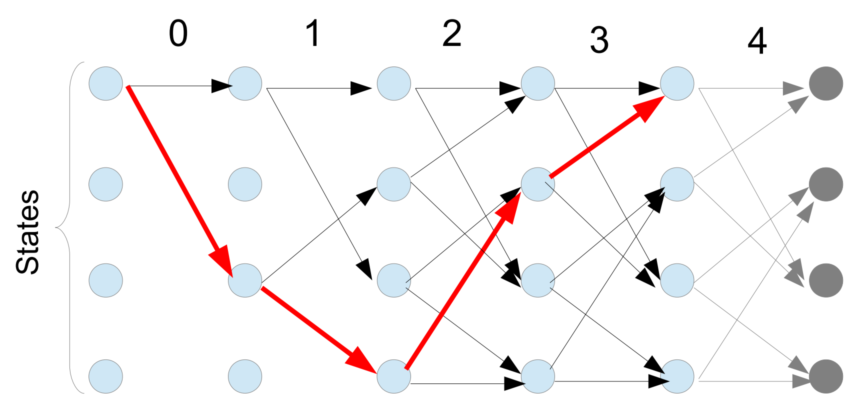

Column Span in [0,3]

Distance and Span Properties

Consider (n, k, m) Convolutional code: $\mathbf{x}_i = \sum_{j=0}^{m} \mathbf{s}_{i-j}\mathbf{G}_j$

Column Distance: d_T

$$d_T = \min_{\substack{[\mathbf{s}_0,\ldots,\mathbf{s}_T] \\ \mathbf{s}_0 \neq 0}} \mathrm{wt}\left(\begin{bmatrix} \mathbf{s}_0 & \ldots & \mathbf{s}_T \end{bmatrix} \begin{bmatrix} \mathbf{G}_0 & \mathbf{G}_1 & \ldots & \mathbf{G}_T \\ 0 & \mathbf{G}_0 & \ldots & \mathbf{G}_{T-1} \\ \vdots & & \ddots & \vdots \\ 0 & & \ldots & \mathbf{G}_0 \end{bmatrix} \right)$$

Column Span: c_T

$$c_T = \min_{\substack{[\mathbf{s}_0,\ldots,\mathbf{s}_T] \\ \mathbf{s}_0 \neq 0}} \mathrm{span}\left(\begin{bmatrix} \mathbf{s}_0 & \ldots & \mathbf{s}_T \end{bmatrix} \begin{bmatrix} \mathbf{G}_0 & \mathbf{G}_1 & \ldots & \mathbf{G}_T \\ 0 & \mathbf{G}_0 & \ldots & \mathbf{G}_{T-1} \\ \vdots & & \ddots & \vdots \\ 0 & & \ldots & \mathbf{G}_0 \end{bmatrix} \right)$$

Column-Distance & Column Span Tradeoff

Badr-Patil-Khisti-Tan-Apostolopoulos (To Appear T-IT 2015)

Theorem

Consider a $\mathcal{C}(N, B, W)$ channel with delay T and $W \geq T+1$. A streaming code is feasible over this channel if and only if it satisfies: $d_T \geq N+1$ and $c_T \geq B+1$

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Theorem

For any rate R convolutional code and any $T \geq 0$ the Column-Distance d_T and Column-Span c_T satisfy the following:

$$\left(\frac{R}{1-R}\right) c_T + d_T \leq T + 1 + \frac{1}{1-R}$$

There exists a rate R code (MiDAS Code) over a sufficiently large field that satisfies:

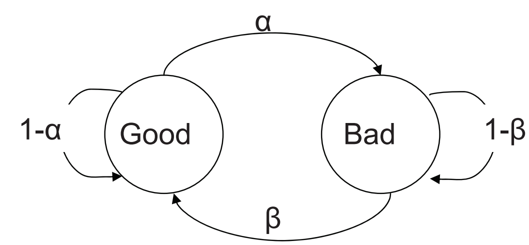
$$\left(\frac{R}{1-R}\right) c_T + d_T \geq T + \frac{1}{1-R}$$

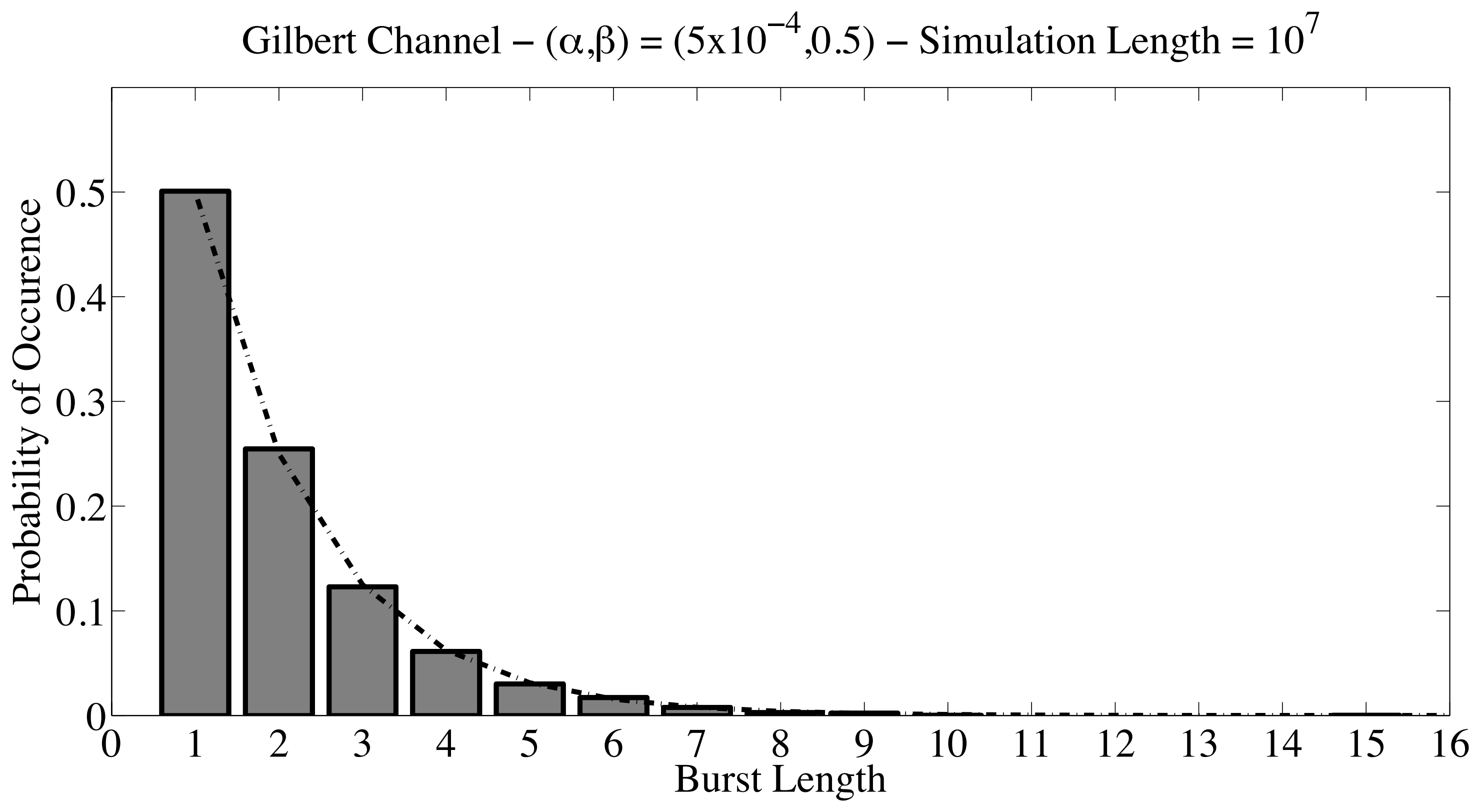
Simulation Result

Gilbert-Elliott Channel $(\alpha, \beta) = (5 \times 10^{-4}, 0.5)$, $T = 12$ and $R \approx 0.5$

Gilbert Elliott Channel

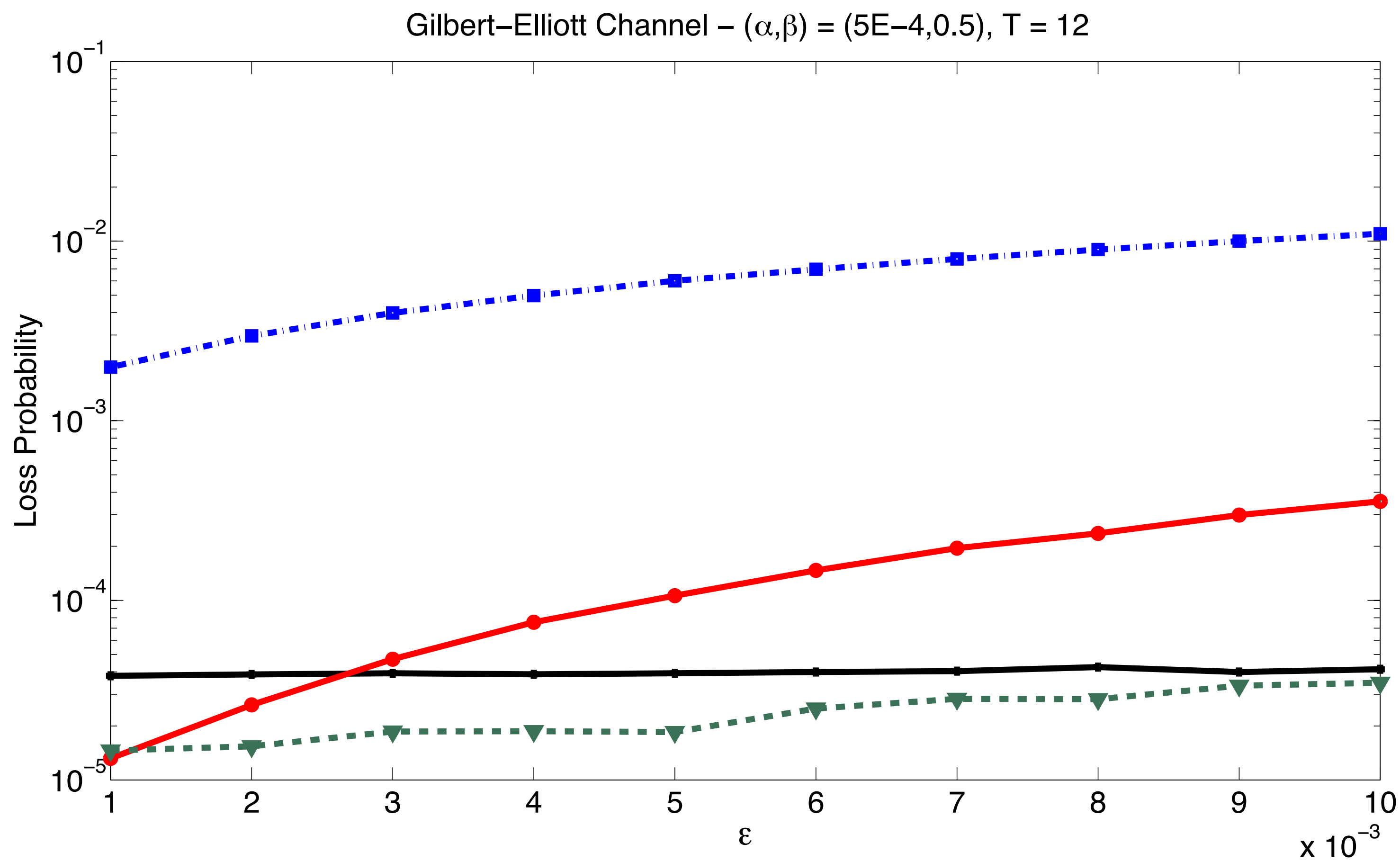
- Good State: $\Pr(\text{loss}) = \varepsilon$
- Bad State: $\Pr(\text{loss}) = 1$





Simulation Results

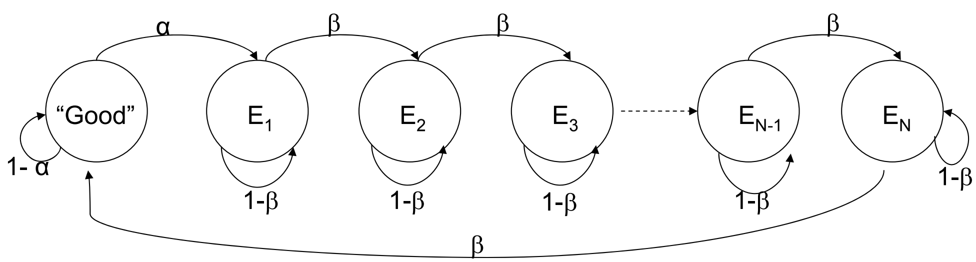
Gilbert-Elliott Channel $(\alpha, \beta) = (5 \times 10^{-4}, 0.5)$, $T = 12$ and $R \approx 0.5$



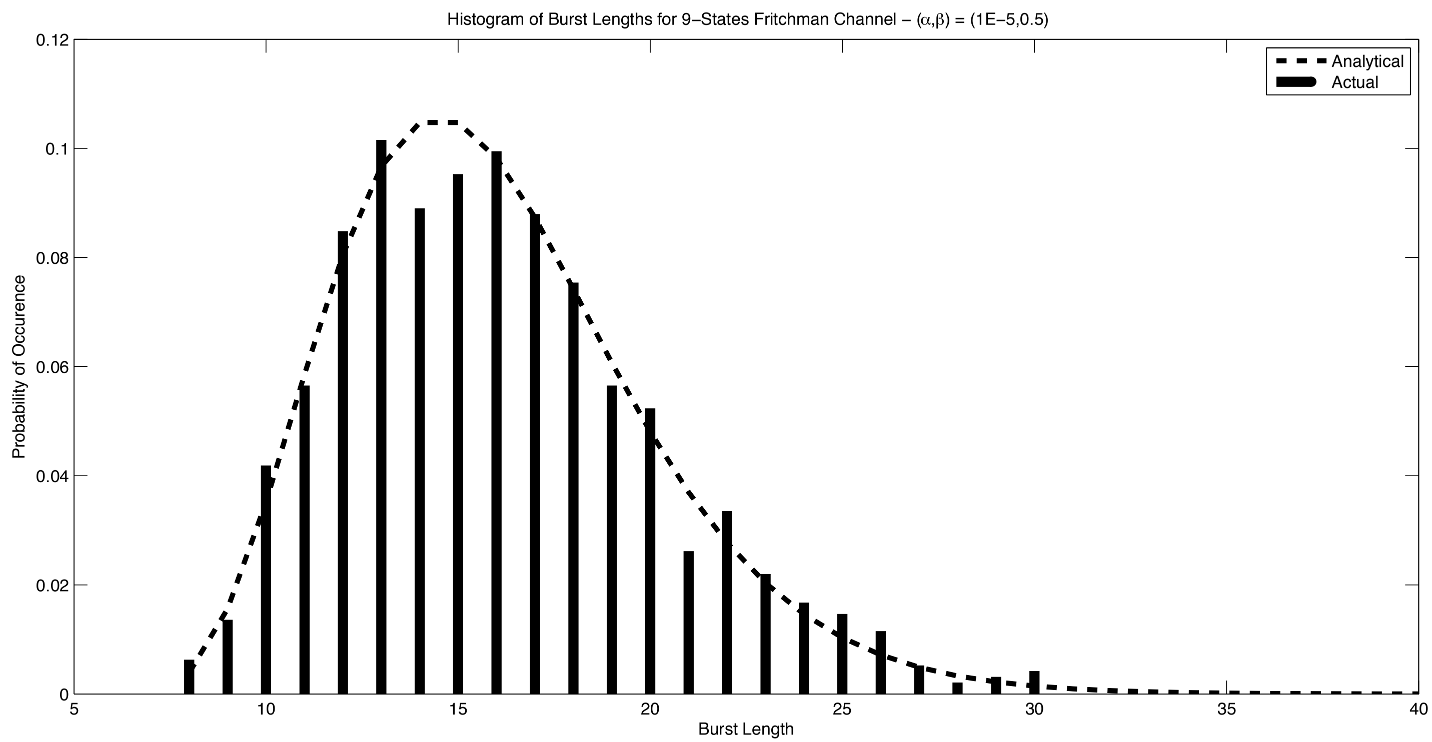
Code	N	B	Code	N	B
Strongly MDS	6	6	MiDAS	2	9
Burst-Erasure	1	11			

Simulation Results-II

Fritchman Channel $(\alpha, \beta) = (1e-5, 0.5)$ and $T = 40$ and $R = 40/79$, 9 states

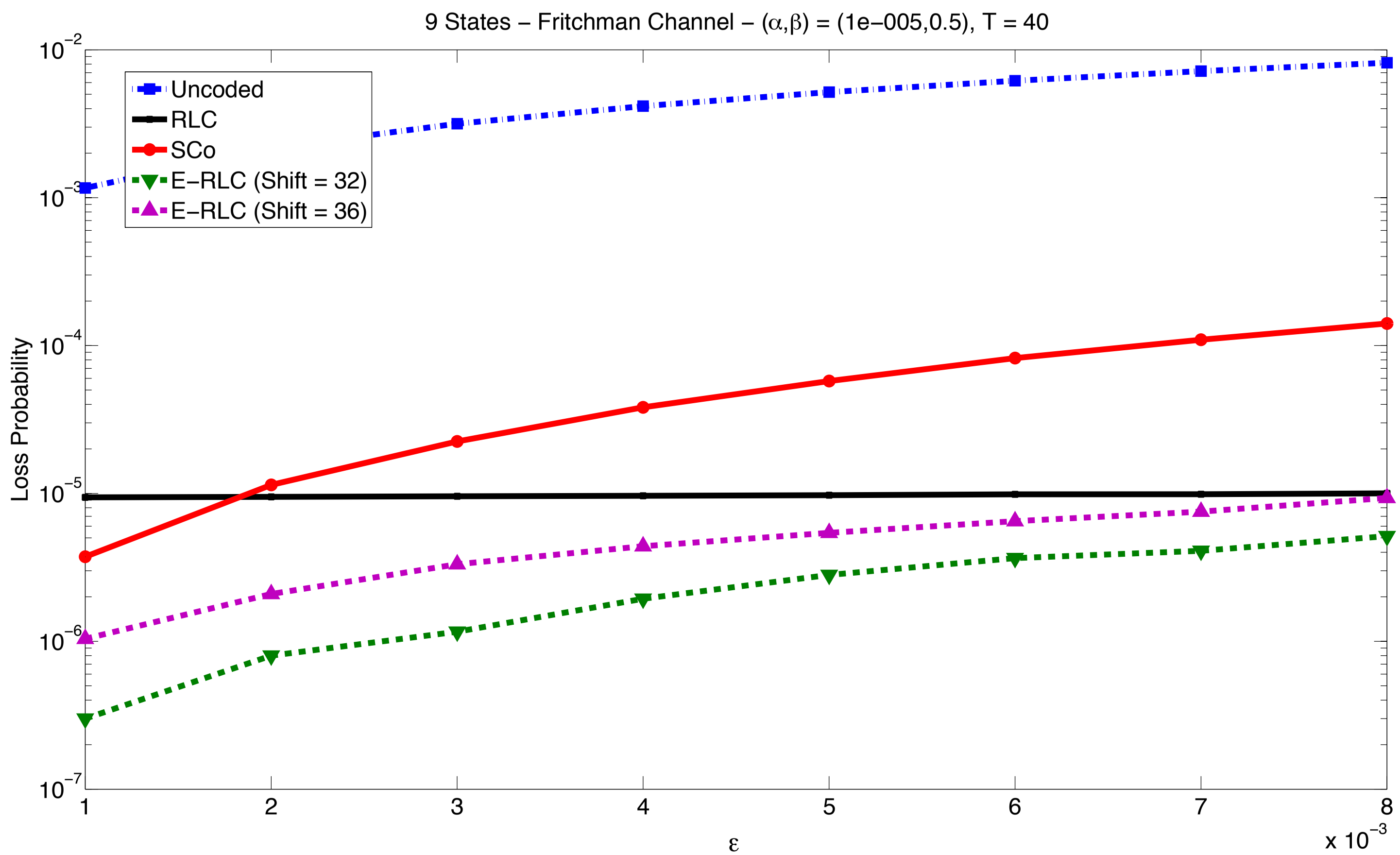


- $\alpha = 1e-5$
- $\beta = 0.5$



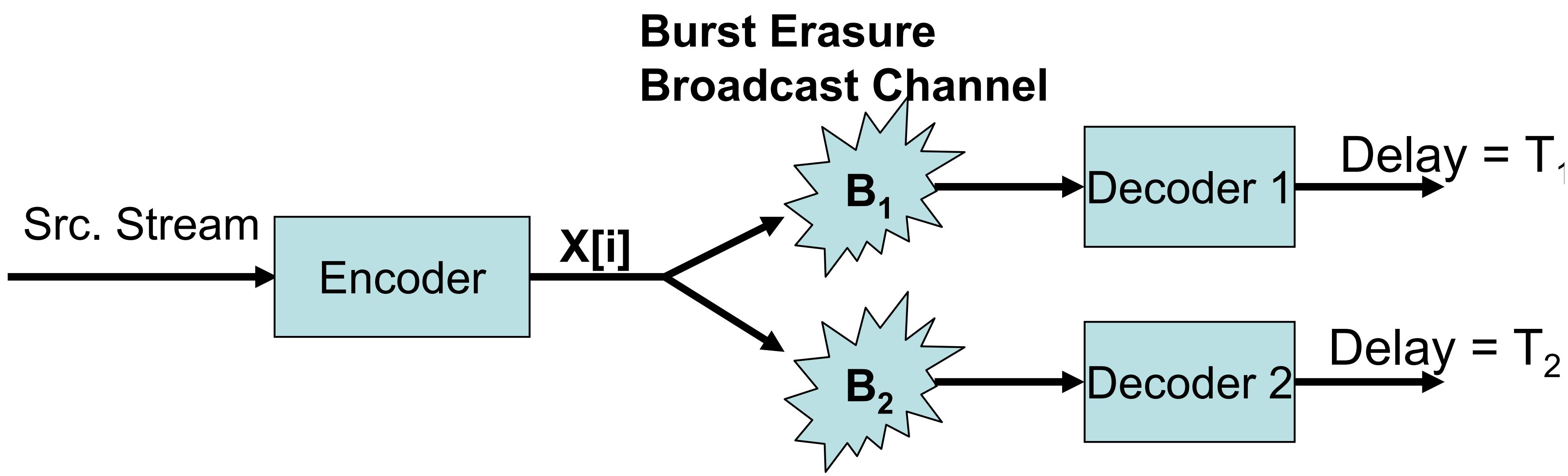
Simulation Results-II

Fritchman Channel $(\alpha, \beta) = (1e-5, 0.5)$ and $T = 40$ and $R = 40/79$, 9 states



Code	N	B	Code	N	B
Strongly MDS	20	20	MiDAS-I	8	31
Burst Erasure	1	39	MiDAS-II	4	35

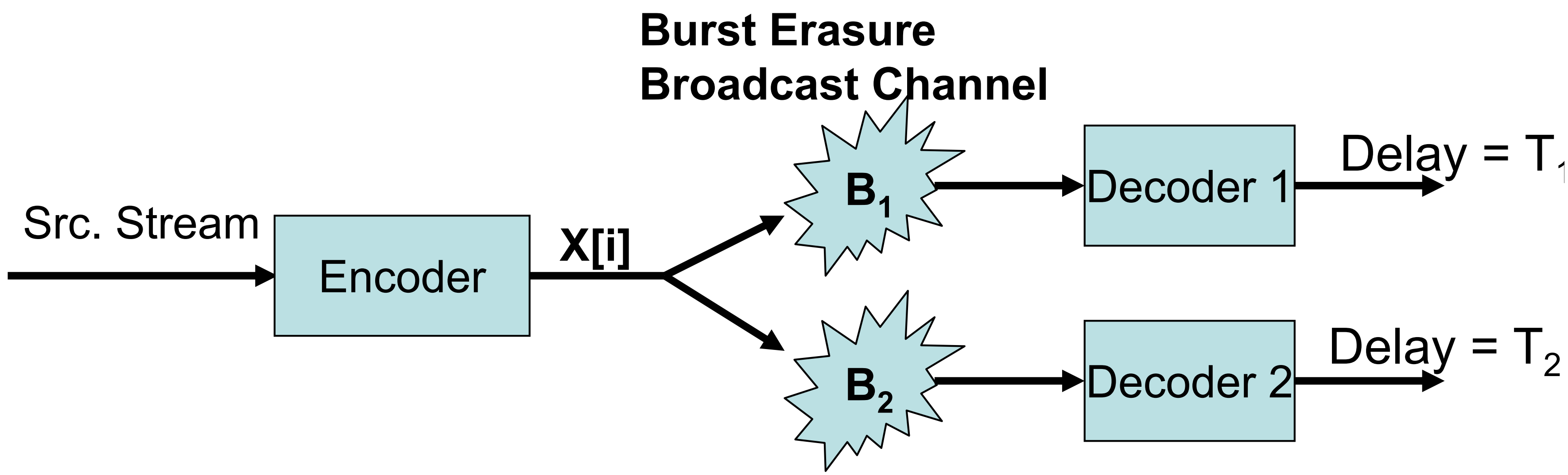
Multicast Streaming Codes



Motivation

- $B_1 < B_2$
- Receiver 1 : Good Channel State
- Receiver 2: Weaker Channel State
- Delay adapts to Channel State

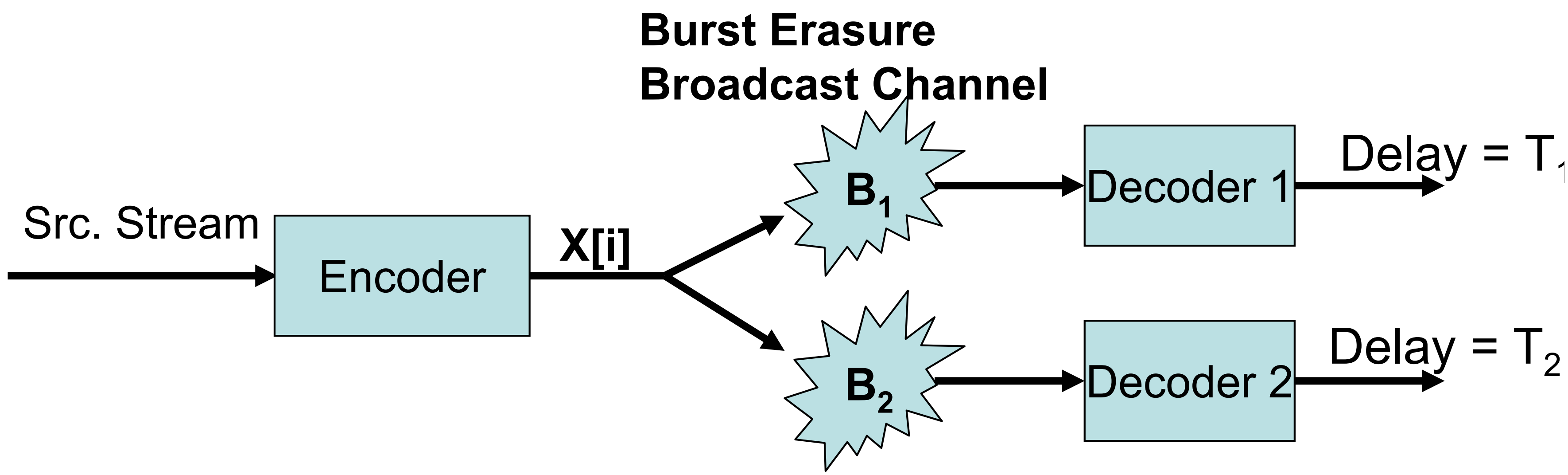
Multicast Streaming Codes



Capacity Function

- Capacity function $C(T_1, T_2, B_1, B_2)$
- Single User Upper Bound: $C \leq \min\left(\frac{T_1}{T_1+B_1}, \frac{T_2}{T_2+B_2}\right)$
- Concatenation Lower Bound: $C \geq \frac{1}{1+\frac{B_1}{T_1}+\frac{B_2}{T_2}}$

Multicast Streaming Setup



Capacity Function

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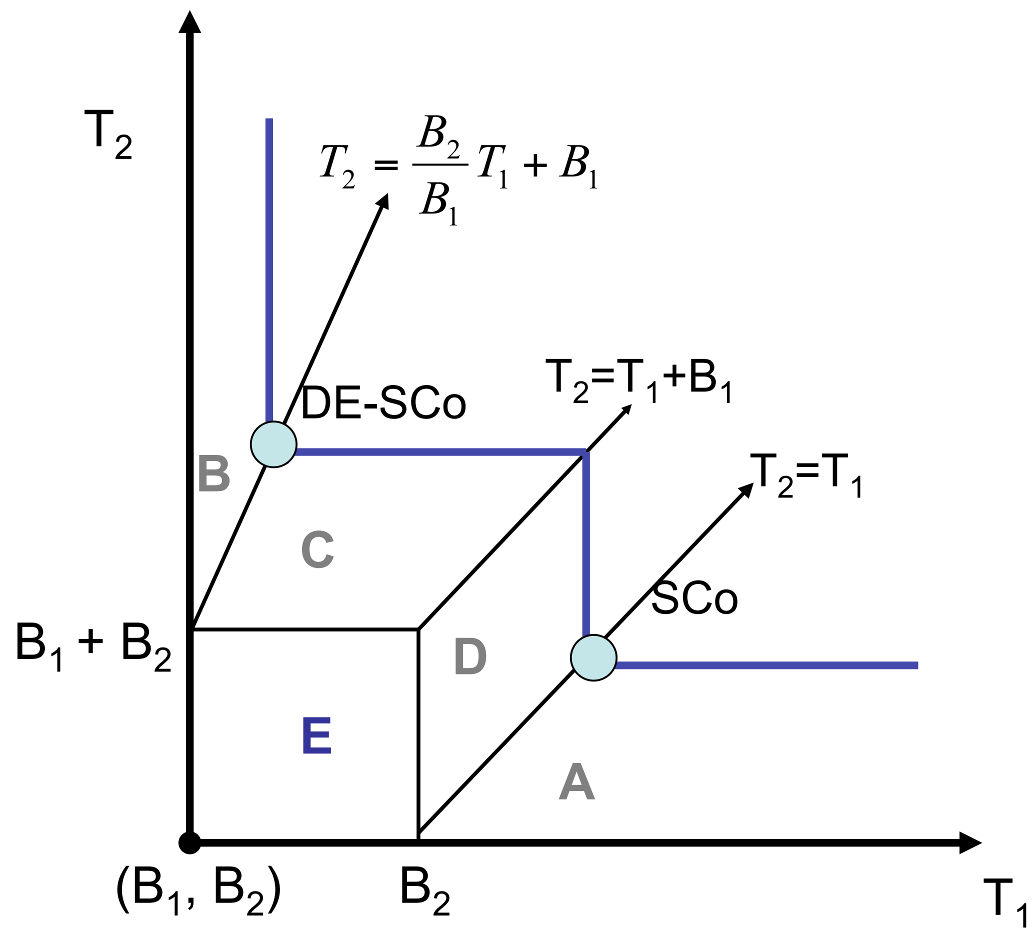
Multicast Streaming Capacity

Badr-Khisti-Lui (IT Trans. To Appear 2015)

Assume w.l.o.g. $B_2 \geq B_1$



Region	Capacity
A	$\frac{T_2}{T_2 + B_2}$
B	$\frac{T_1}{T_1 + B_1}$
C	$\frac{T_2 - B_1}{T_2 - B_1 + B_2}$
D	$\frac{T_1}{T_1 + B_2}$
E	Partial Characterization

Other Extensions

- Mismatched Streaming Codes (Patil-Badr-Khisti-Tan Asilomar 2013)
- Partial Recovery Streaming Codes (Badr-Khisti-Tan-Apostolopoulos JSTSP 2014)
- Multiple Erasure Bursts (Li-Khisti-Girod Asilomar 2011) - Interleaved Low-Delay Codes
- Multiple Links (Lui-Badr-Khisti CWIT 2011) - Layered coding for burst erasure channels
- Multiple Source Streams with Different Decoding Delays (Lui (Unpublished) 2011) - Embedded Codes

Other Results

- Burst Erasure Channels: Martinian and Sundberg (IT-2004)
- Other Recent Results: Leong-Ho (ISIT 2012), Leong-Qureshi-Ho (ISIT 2013)

Conclusions

Real-Time Communication over Channels with Burst and Isolated Erasures

- Sliding Window Erasure Channel Model
- MiDAS Codes: Near Optimal Distance/Span Tradeoff
- Layering Approach
- Distance and Span Metrics

Future Work

- Improved constructions for short-inter burst gaps
- Systems Theoretic Approach (e.g. Dual Codes for MiDAS Codes)
- Analysis of probabilistic channels

References

- A. Badr, P. Patil, A. Khisti, W. Tan and J. Apostolopoulos "Layered Construction for Low-Delay Streaming Codes" To Appear, IEEE Trans. Inf. Theory, 2015
- A. Badr, A. Khisti, W. Tan and J. Apostolopoulos, "Streaming Codes with Partial Recovery over Channels with Burst and Isolated Erasures," IEEE Journal on Selected Topics in Signal Processing (JSTSP), Special Issue on Interactive Media Processing for Immersive Communication, To Appear, Mar. 2015
- A. Badr, D. Lui and A. Khisti, "Low Delay Streaming Codes for Multicast over Burst Erasure Channels," To Appear, IEEE Trans. Inf. Theory, 2015
- A. Badr, A. Khisti and E. Martinian, "Diversity Embedded Streaming Erasure Codes (DE-SCo): Constructions and Optimality," JSAC Special Issue on Trading Rate for Delay at the transport and application layers, Mar. 2011

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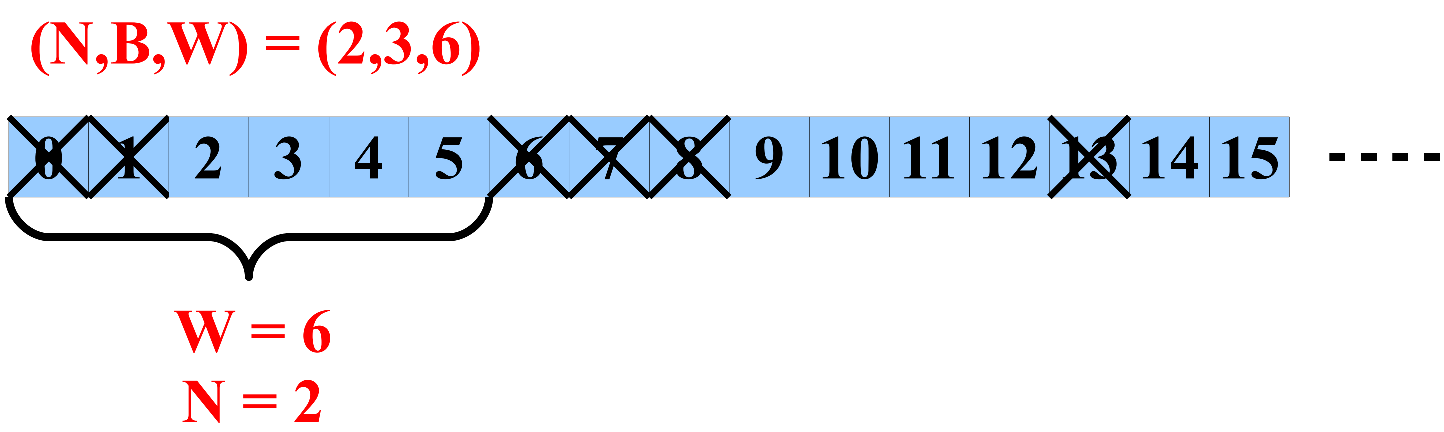
Sliding Window Erasure Channel: Remarks



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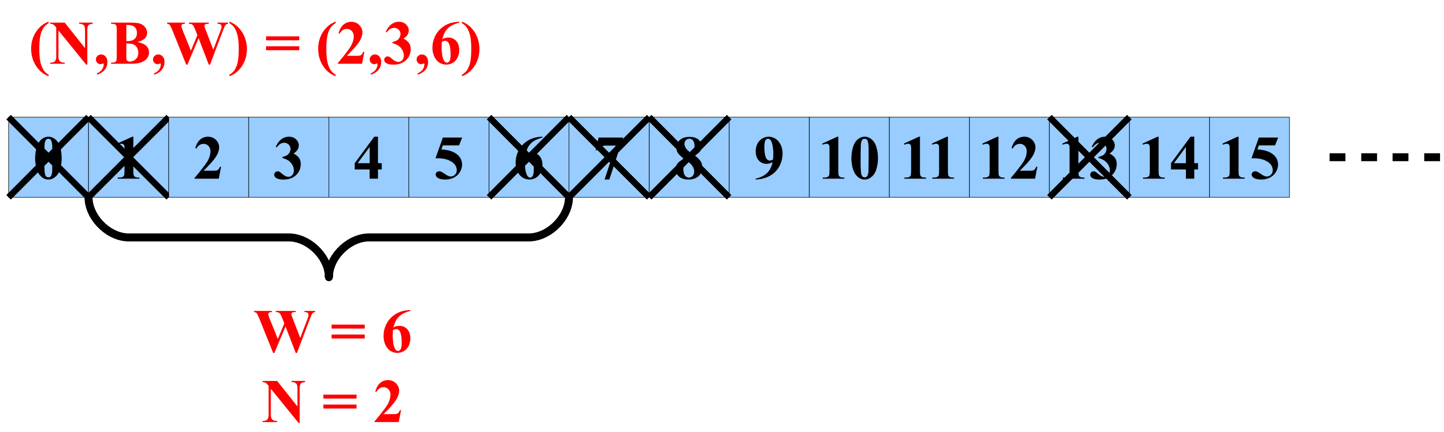

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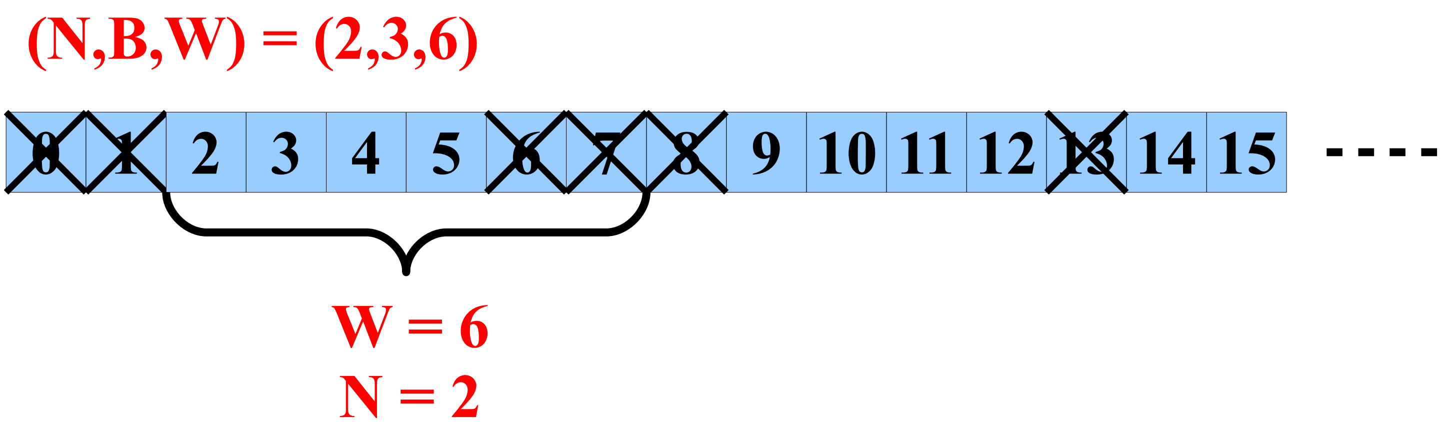

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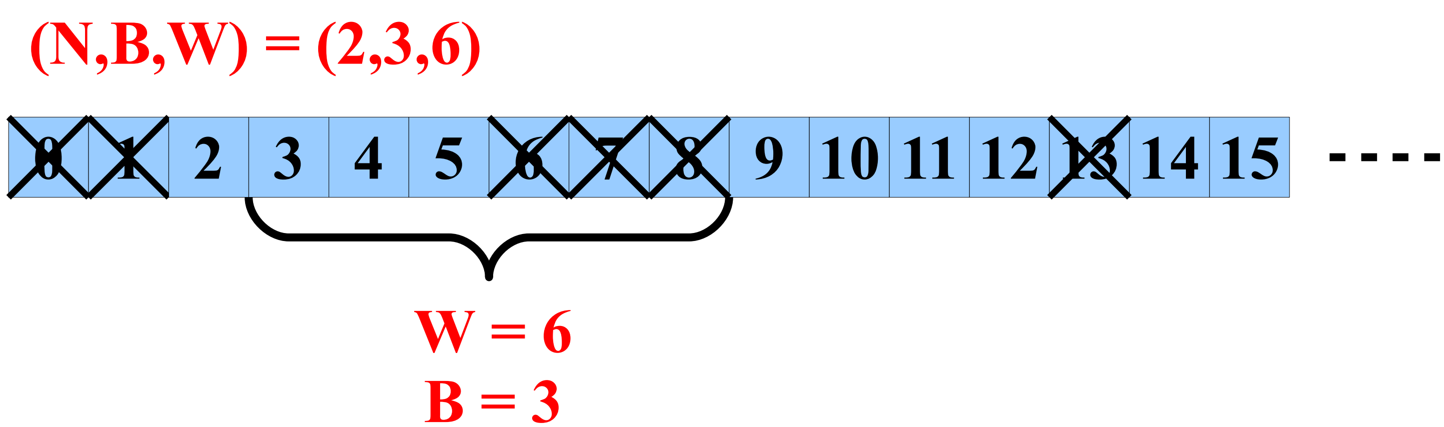

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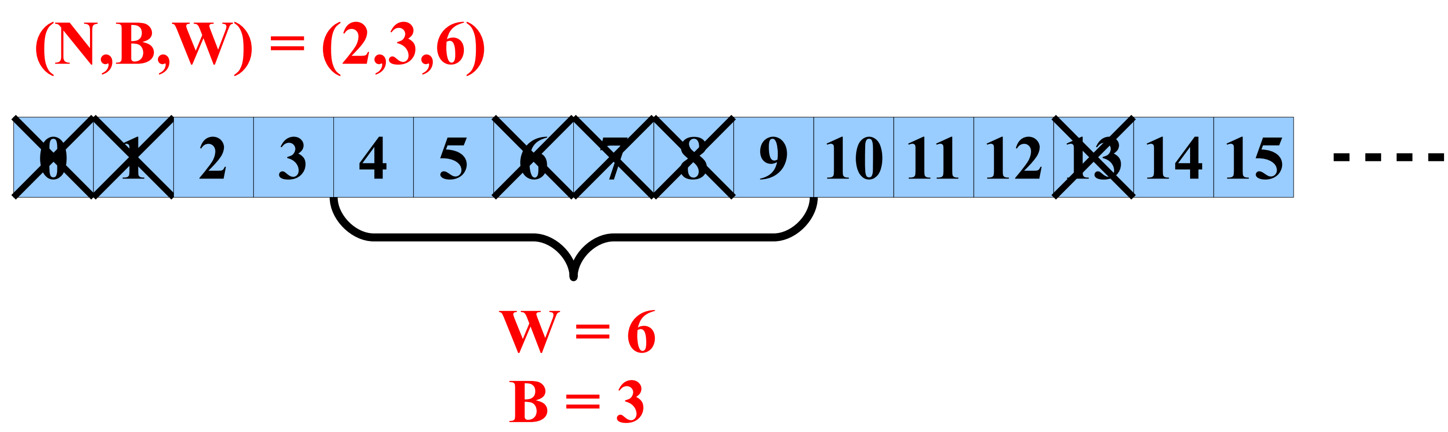
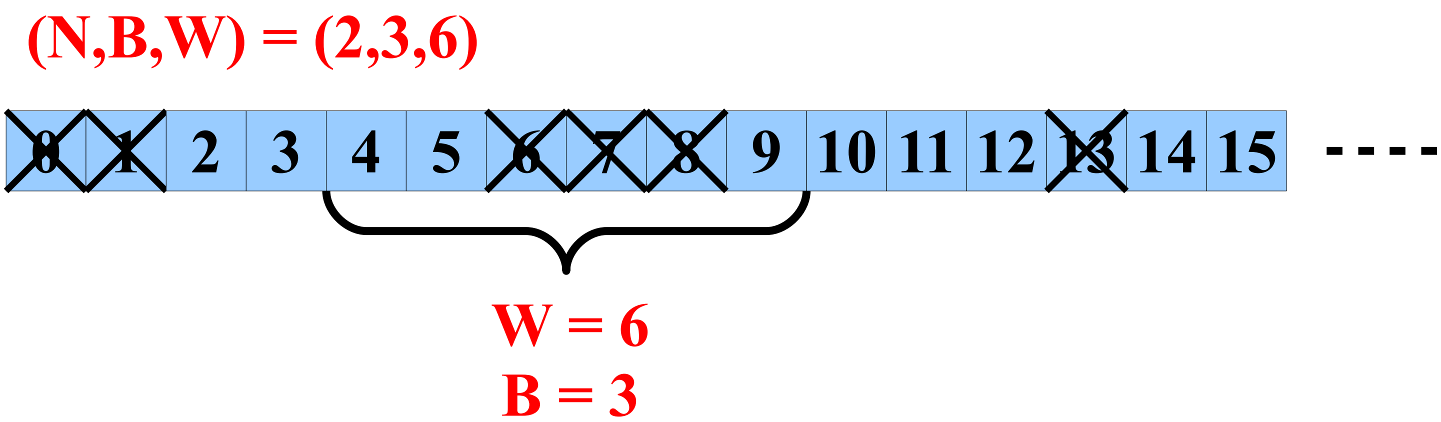


- $\mathcal{C}(N = 1, B, W)$: Burst-Erasure Channel

Sliding Window Erasure Channel: Remarks



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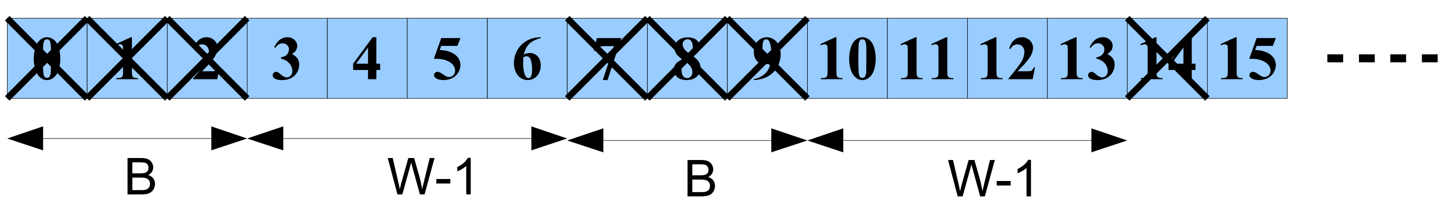

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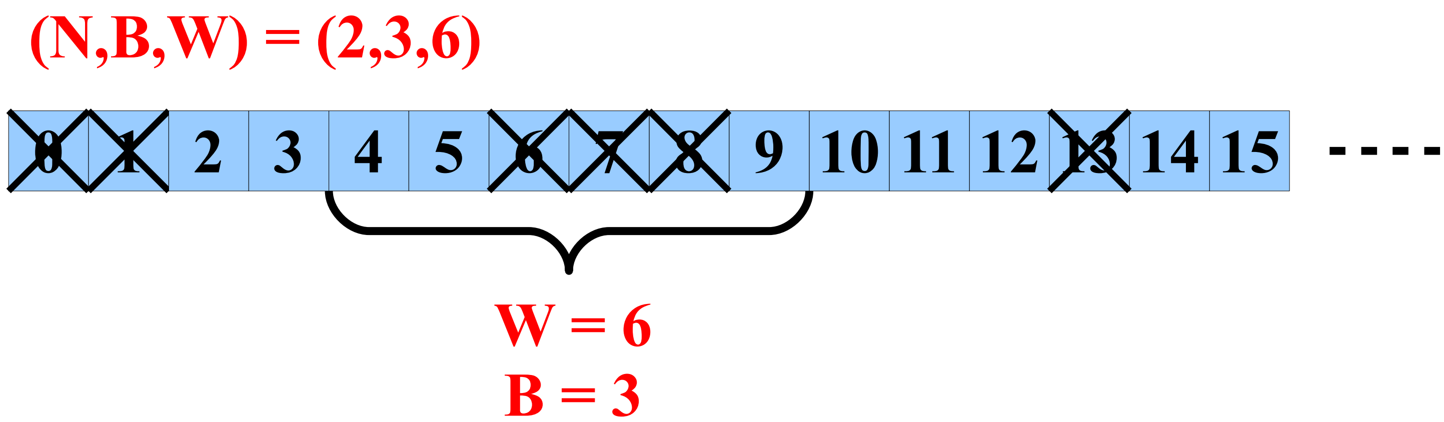

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