

The background of the slide features a large, light gray watermark of the McGill University crest. The crest is a shield with a crown at the top, an open book in the center with the Latin motto 'IN DOMINO CONFI DO', and three birds (swallows) below. The shield is divided into four quadrants by a white 'X' shape.

Are Polar Codes Practical?

Prof. Warren J. Gross
McGill University, Montreal, QC.

Coding: from Practice to Theory
Berkeley, CA.
Feb. 10th, 2015.

Achieving the Channel Capacity

Successive cancellation → **low throughput.**

Mediocre performance at moderate code-length.
Becomes excellent for $N > 2^{20}$.



Polar Codes

2008

0 dB

using SC

as $n \rightarrow \infty$

Channel Polarization

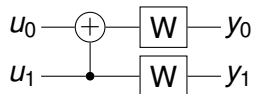


$$I^0 = I(W) = I^1$$

Channel Polarization



$$I^0 = I(W) = I^1$$

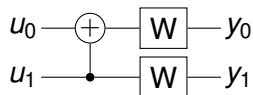


$$I^0 < I(W) < I^1$$

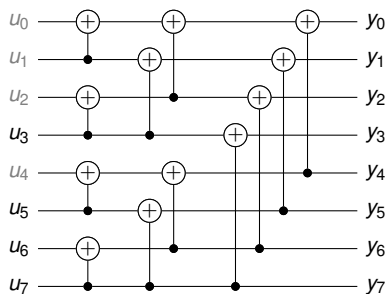
Channel Polarization



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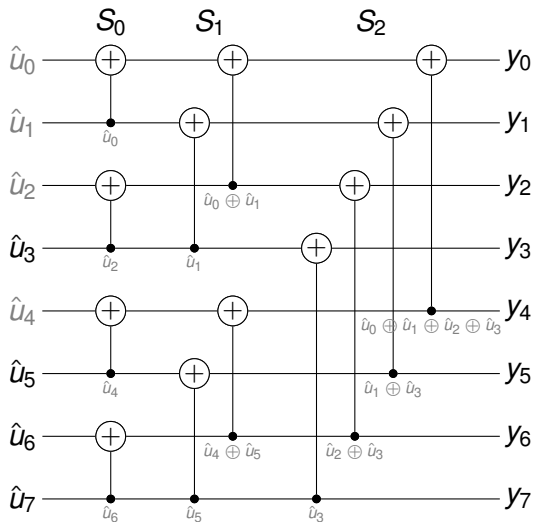


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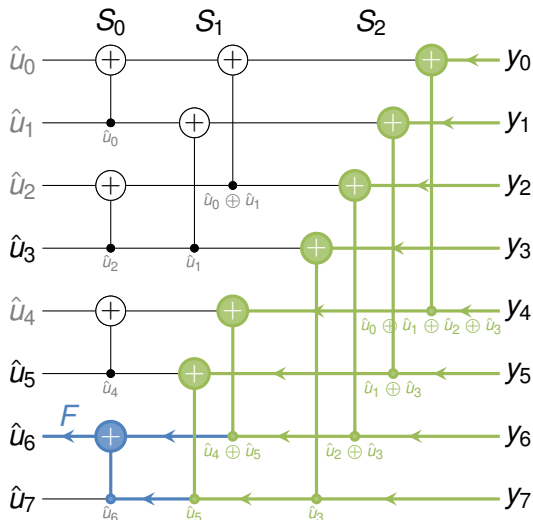


Proportion of reliable bits $\rightarrow I(W)$.

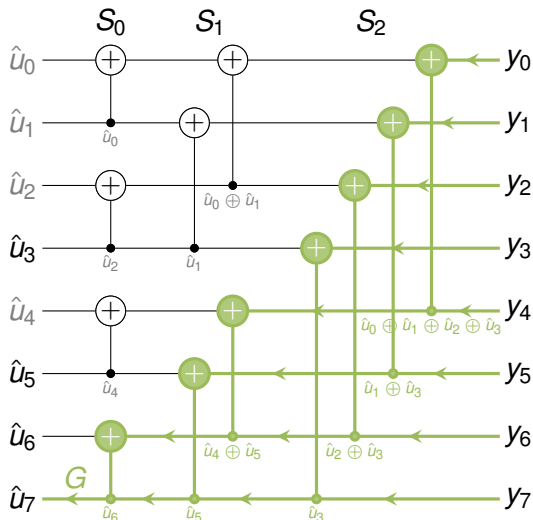
Successive Cancellation Decoding



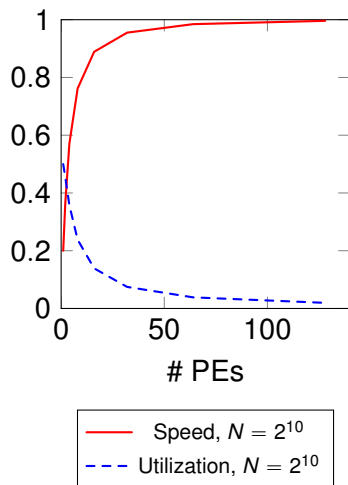
Successive Cancellation Decoding



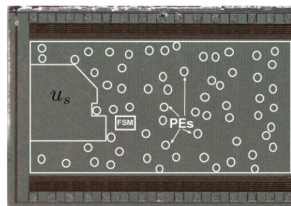
Successive Cancellation Decoding



Semi-Parallel SC Decoder Implementation



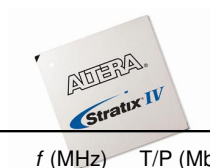
> 95% speed with 64 PEs.



First polar decoder ASIC.

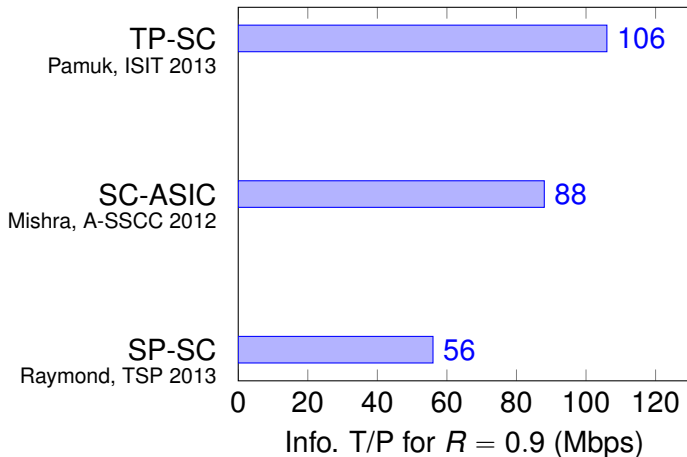
	Code (1024, 512)
Technology	180 nm
Area	1.72 mm ²
Frequency	150 MHz
Info. T/P	49 Mbps
Power	67 mW

FPGA Implementation



N	LUT	FF	RAM (bits)	f (MHz)	T/P (Mbps)
2^{15}	3,263	1,304	411,648	167	62R
2^{16}	3,414	1,316	821,248	157	57R
2^{18}	3,548	1,349	3,278,848	140	51R
2^{20}	5,956	1,366	13,109,248	102	38R

Throughput of SC Decoders

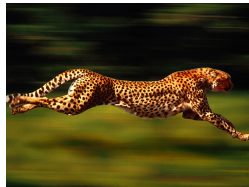


Improving Throughput



100 Mbps
SC

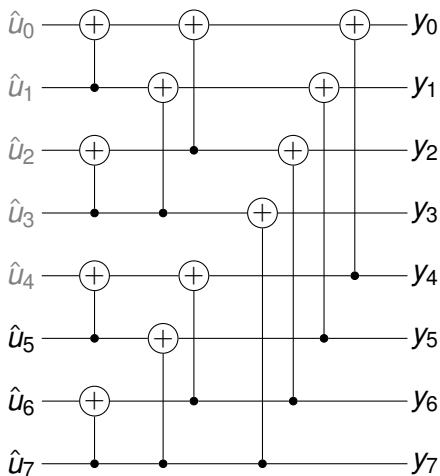
1000×



100 Gbps
Unrolled

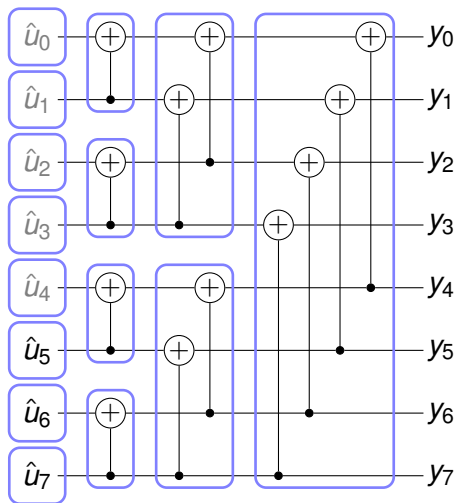
Successive Cancellation Tree

- ▶ View the SC decoder graph as a tree.



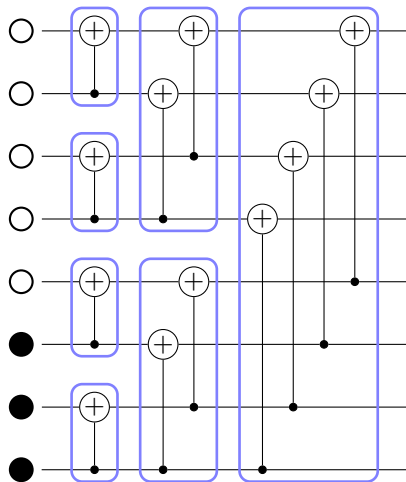
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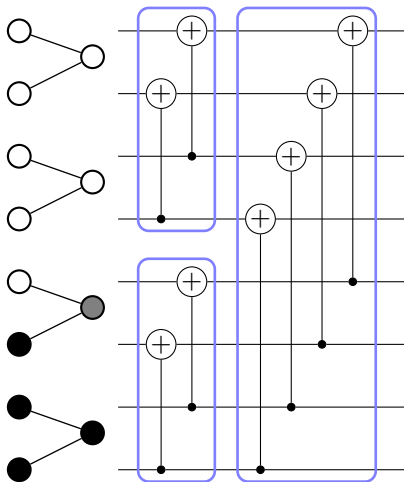
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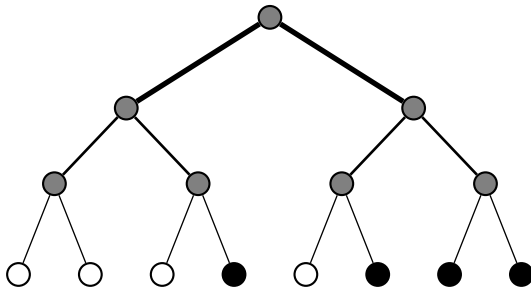


Successive Cancellation Tree

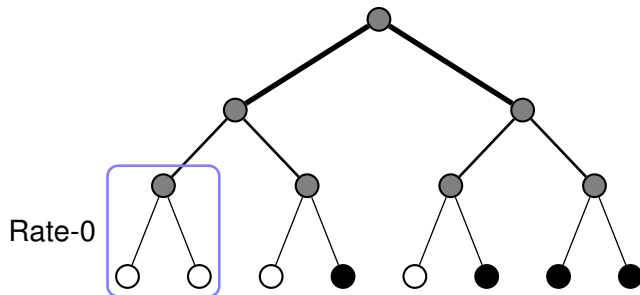
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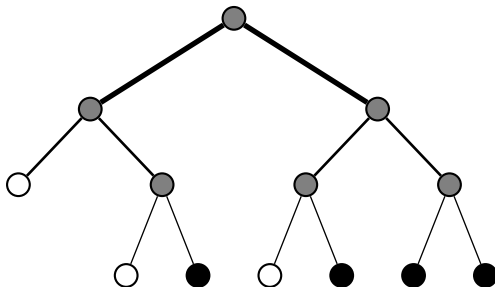
Simplified Successive-Cancellation Decoding



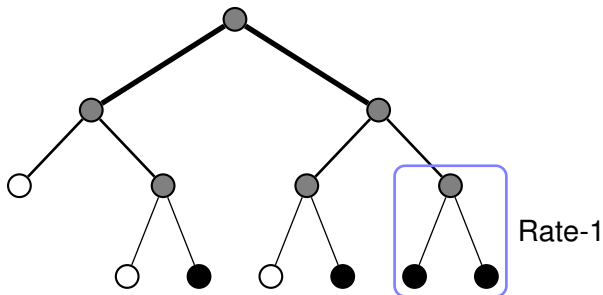
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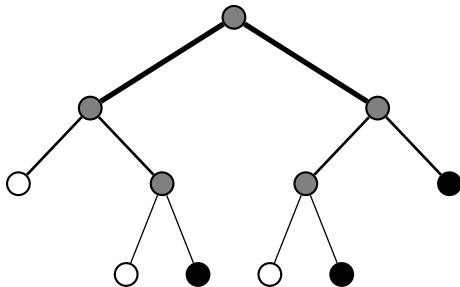
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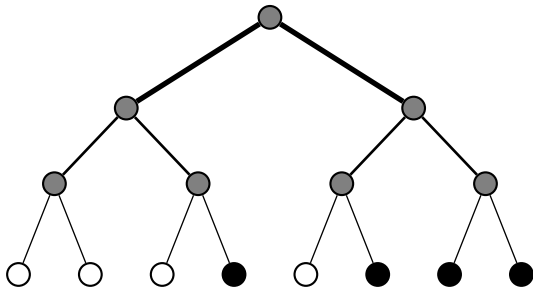
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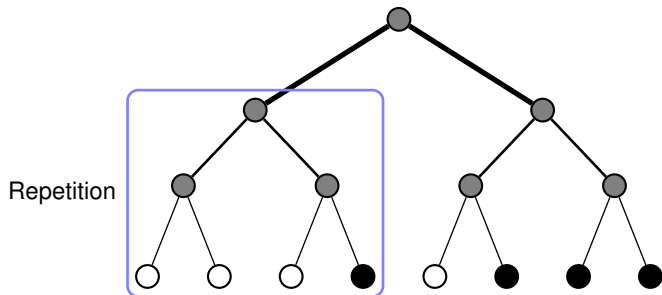
Simplified Successive-Cancellation Decoding



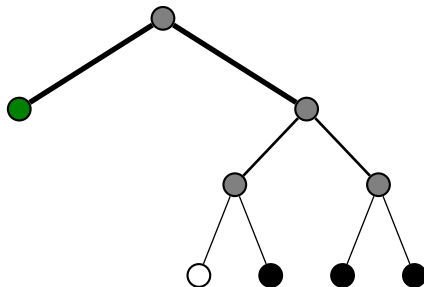
Fast-SSC Decoding



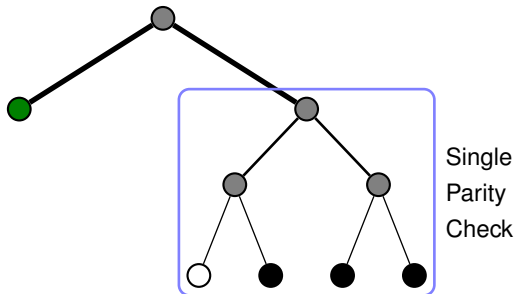
Fast-SSC Decoding



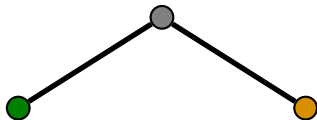
Fast-SSC Decoding



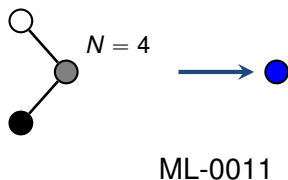
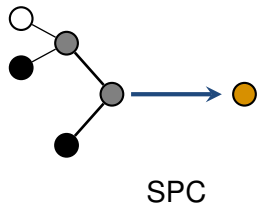
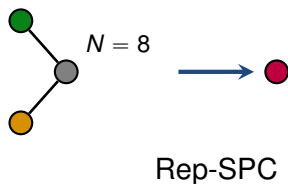
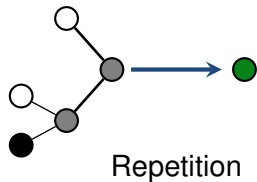
Fast-SSC Decoding



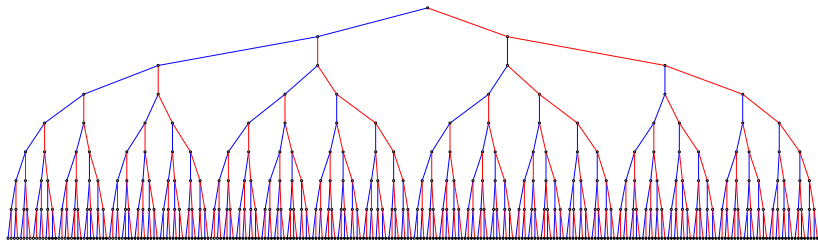
Fast-SSC Decoding



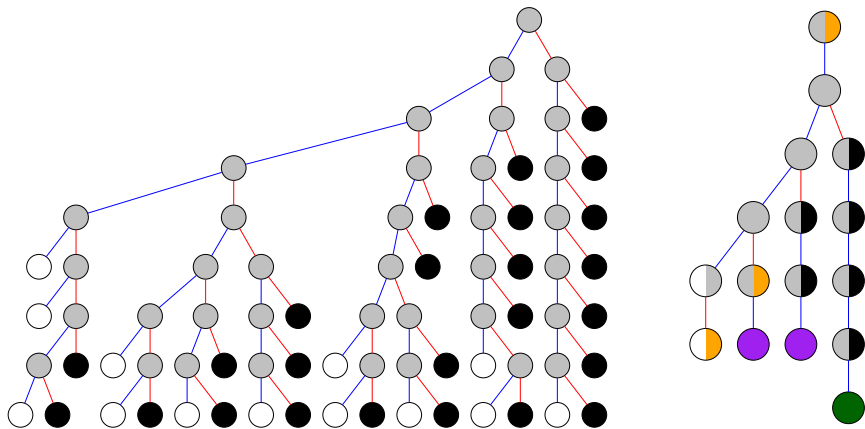
Fast-SSC Decoding—New Nodes



SC Tree—(256, 230)

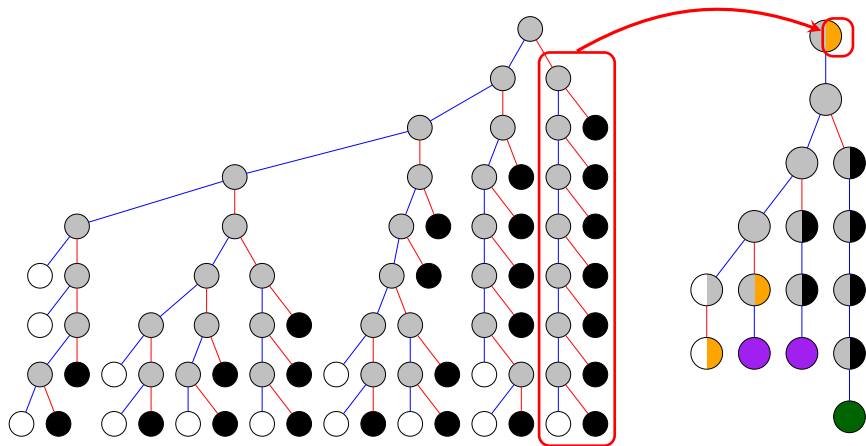


SSC and Fast-SSC Trees—(256, 230)



Developed a compiler to convert the graph to instructions.

SSC and Fast-SSC Trees—(256, 230)



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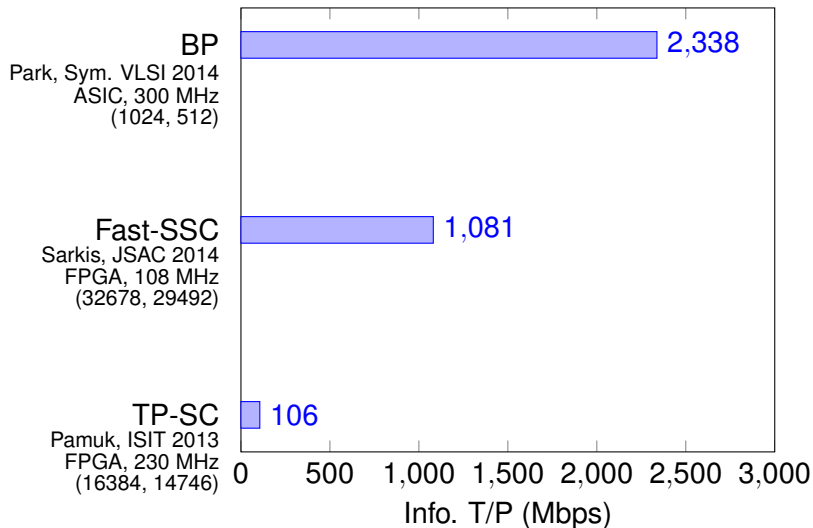
Fast-SSC Decoder Implementation



R	LUT	FF	RAM (bits)	f (MHz)	T/P (Mbps)
Fast-SSC					
0.83	25,866	7,209	536,126	108	791
0.90					1,081
SSC					
0.90					276

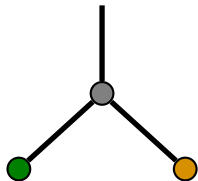
For any code of length = 32,768.

Throughput of Hardware Decoders



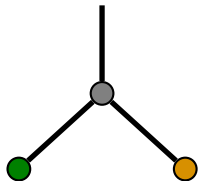
The Unrolled Decoder

- ▶ (8, 4) polar code:

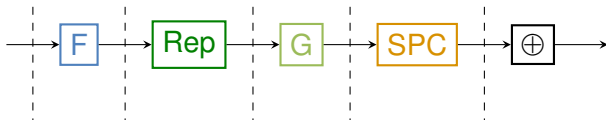


The Unrolled Decoder

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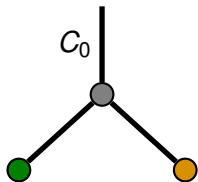


- ▶ Unroll and pipeline all operations.

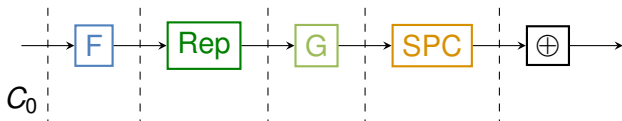


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- ▶ (8, 4) polar code:

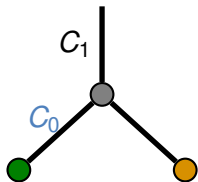


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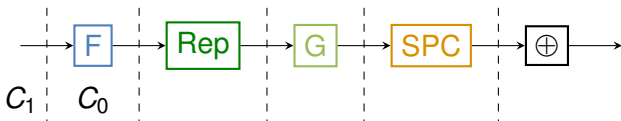


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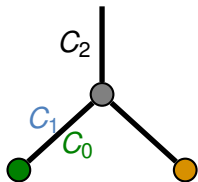


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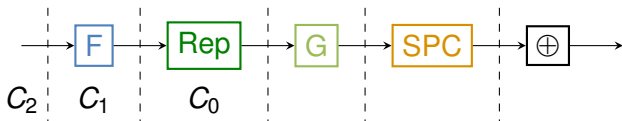


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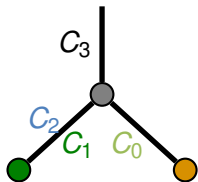


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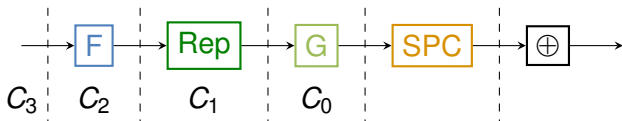


The Unrolled Decoder

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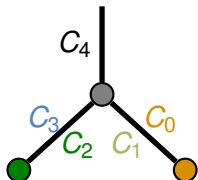


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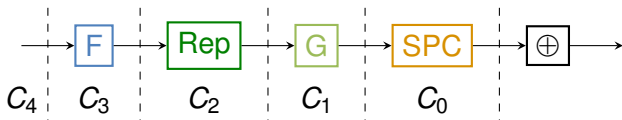


The Unrolled Decoder

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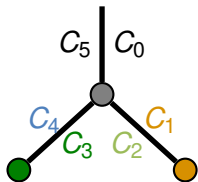


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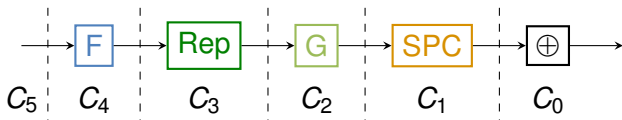


The Unrolled Decoder

- ▶ (8, 4) polar code:



- ▶ Unroll and pipeline all operations.



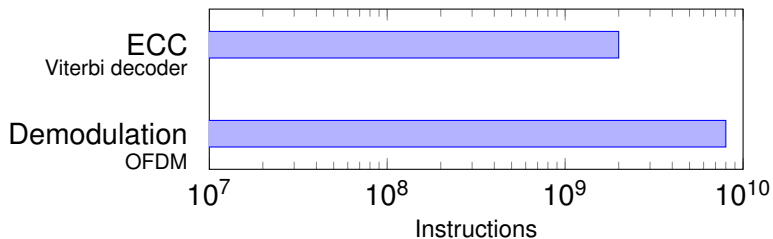
The Unrolled Decoder Implementation



LUT	FF	RAM (bits)	f (MHz)	Info. T/P (Gbps)
SP-SC				
4,130	1,388	11,904	196	0.04
Unrolled				
155,858	158,185	285,120	231	118.5
BP (ASIC)				
			300	2.3

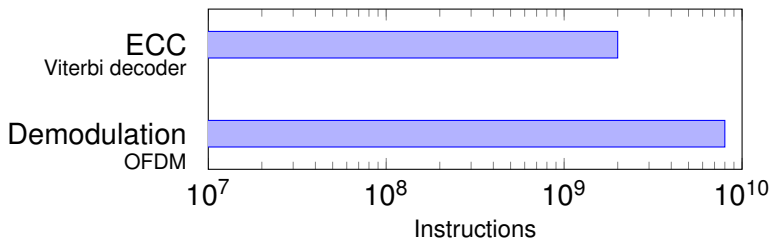
For a (1024, 512) polar code.

SDR and Modern Software ECC

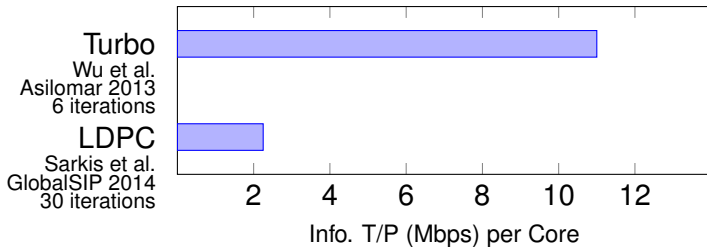


Demel et al. "A LTE Receiver Framework Using GNU Radio," JSPS, 2015.

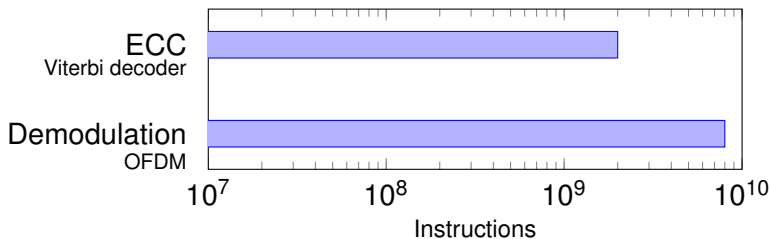
SDR and Modern Software ECC



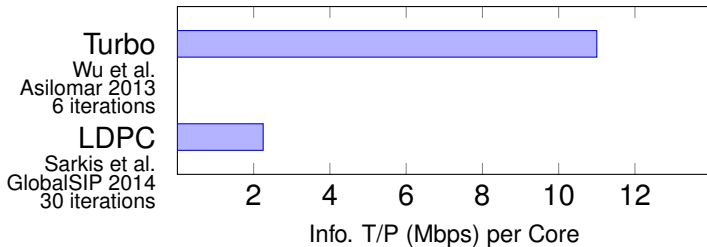
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SDR and Modern Software ECC

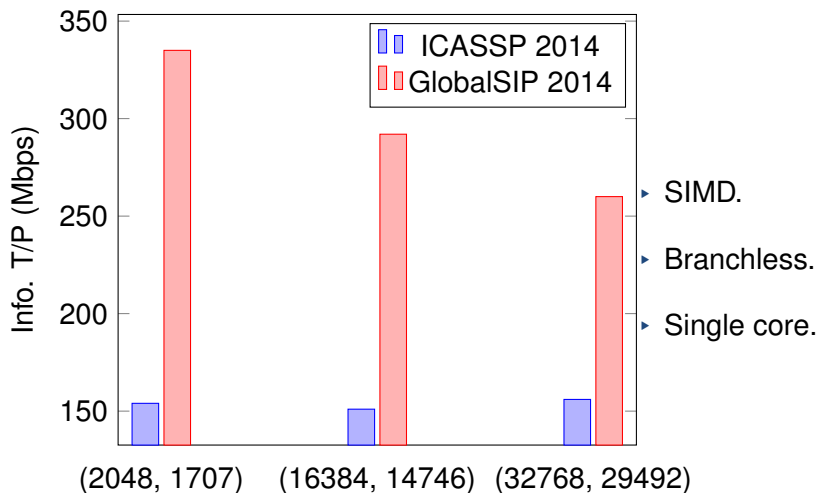


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Goal: 50Mbps!

Floating-Point Throughput

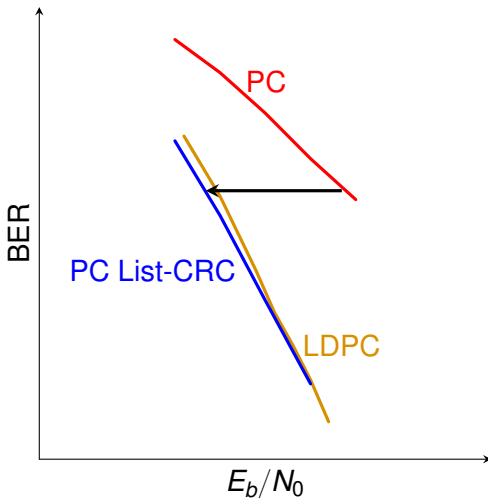


Fixed-Point Software Polar Decoders

Decoder	f (GHz)	Info. T/P (Mbps)	Latency μ s
Le Gal et al.	3.6+	1,557	605
Fast-SSC-int8	3.1+	1,412	21

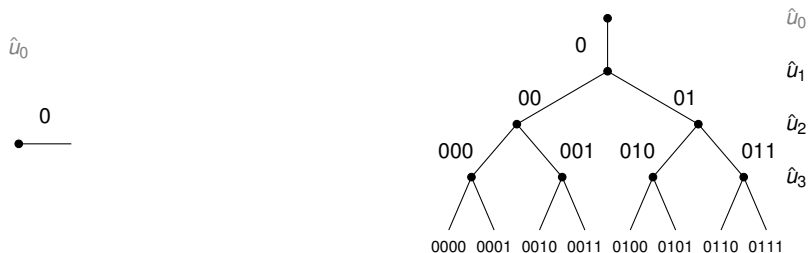
(32768, 29492) polar code.

Improving Finite-Length Error-Correction Performance



List Decoding

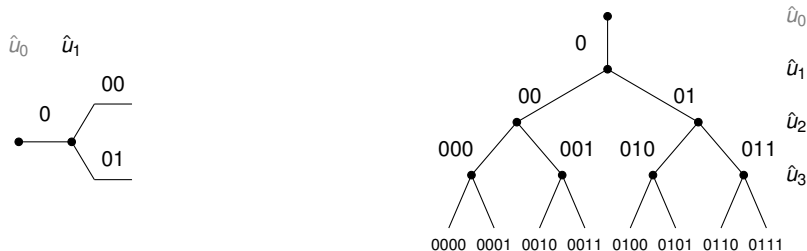
- ▶ For each information bit, continue decoding assuming both 0 and 1.



- ▶ Select paths according to reliability values.

List Decoding

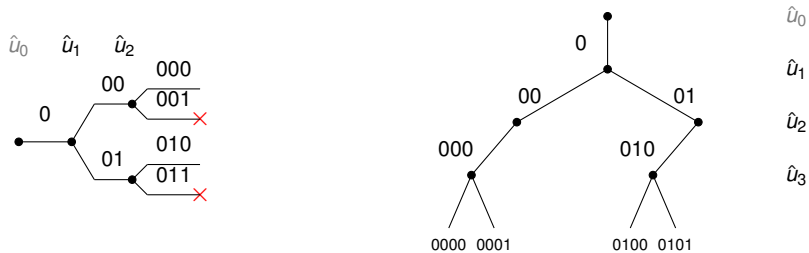
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List Decoding

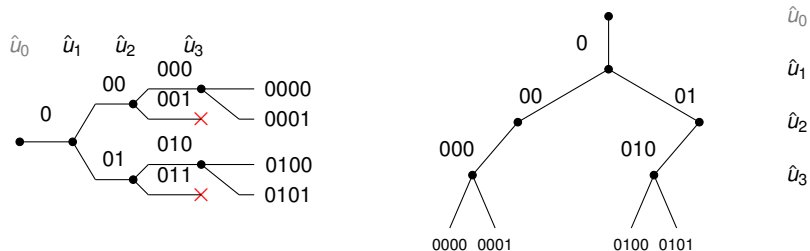
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List Decoding

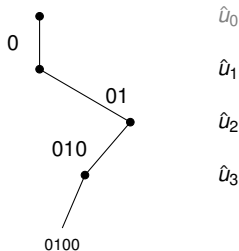
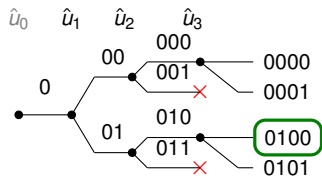
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List Decoding

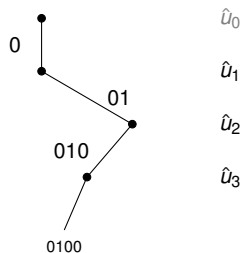
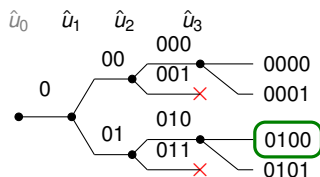
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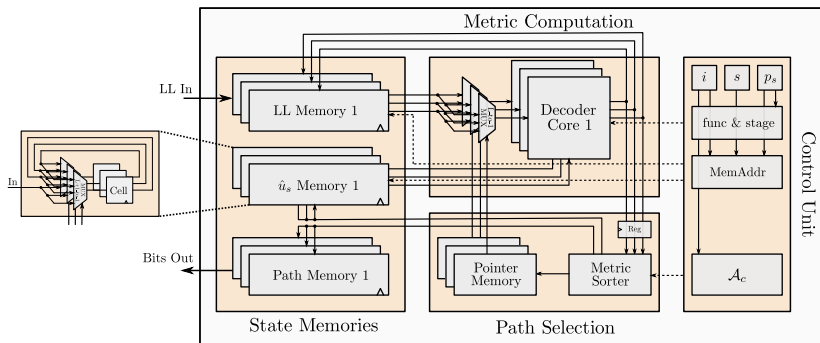
List Decoding

- ▶ For each information bit, continue decoding assuming both 0 and 1.



- ▶ Select paths according to reliability values.
- ▶ Using a CRC to select the output yields a significant improvement.

ASIC Implementation



- Synthesis results for $N = 1024$ using UMC 90nm:

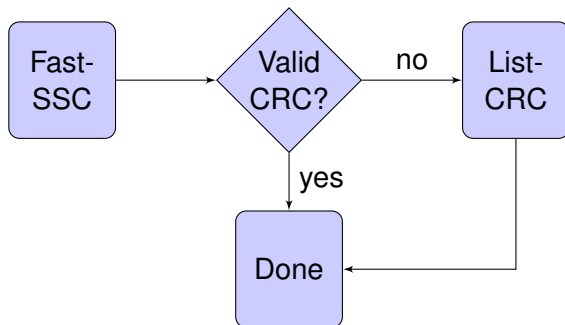
L	Area (mm ²)	f (MHz)	T/P (Mbps)
2	1.60	459	181R
4	3.53	314	89R

Two-Step SW List Decoder

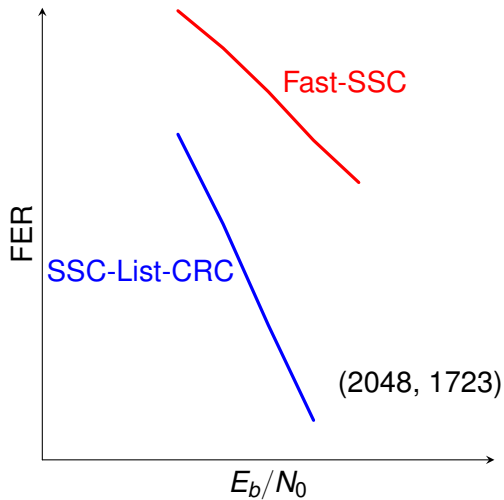
- ▶ List decoders are slower than Fast-SSC decoders.
- ▶ List have better error-correction performance than Fast-SSC decoders.

Two-Step SW List Decoder

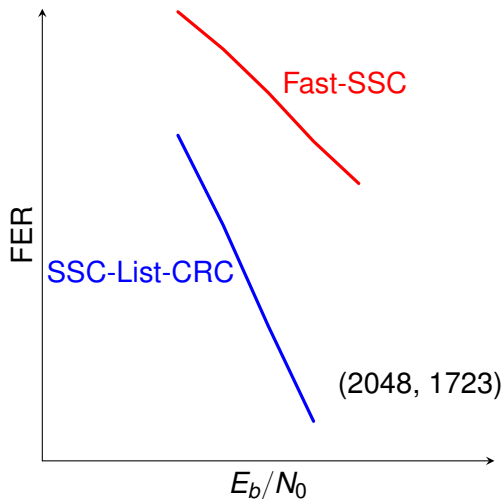
- ▶ List decoders are slower than Fast-SSC decoders.
- ▶ List have better error-correction performance than Fast-SSC decoders.
- ▶ Combine both:



Two-Step SW List Decoder

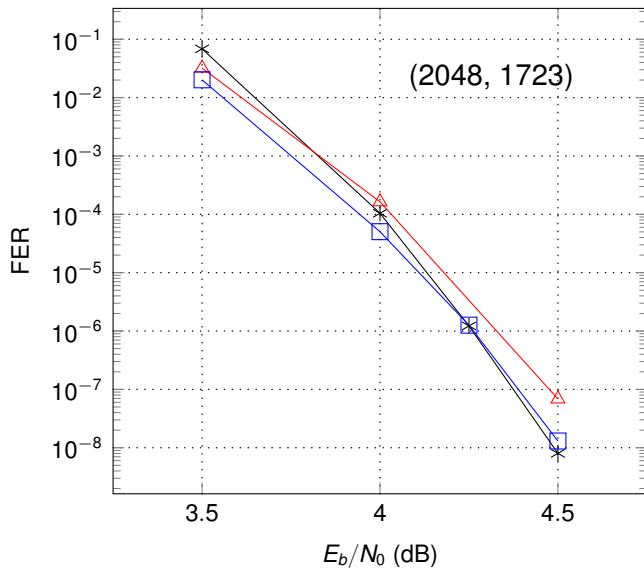


Two-Step SW List Decoder



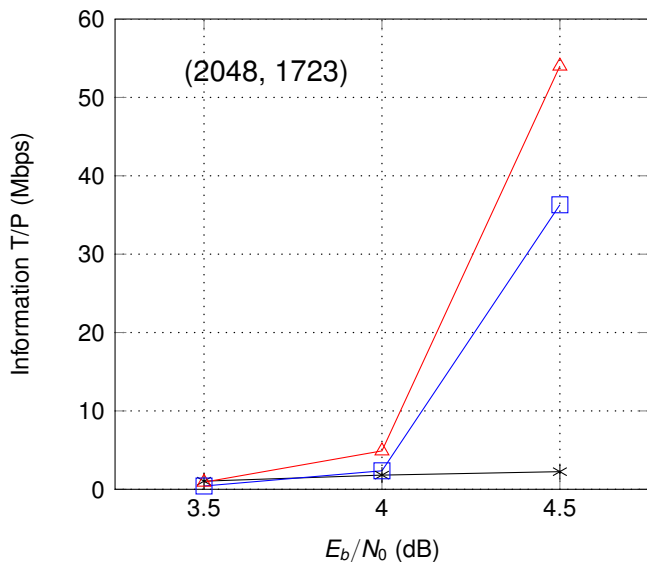
$$\mathcal{T} = \frac{k}{(1 - \text{FER}_F)\mathcal{L}(\text{Fast-SSC}) + \text{FER}_F\mathcal{L}(\text{List})} \text{ bit/s}$$

Two-Step SW List Decoder



—*— LDPC, $I_{\max} = 30$ —△— SSC-List-CRC, $L = 32$ —□— SSC-List-CRC, $L = 64$

Two-Step SW List Decoder



—*— LDPC, $l_{\max} = 30$ —△— SSC-List-CRC, $L = 32$ —□— SSC-List-CRC, $L = 64$

Conclusion

- ▶ Polar codes can have excellent error-correction performance.
- ▶ Polar codes can have throughput in the hundred-Gbps range.
- ▶ Once throughput and performance improvements are combined, we believe that polar codes will be a viable option for multiple applications.

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Thank you!