Testing, Voluntary Social Distancing, and the spread of an infection

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Introduction

- Testing is one of the most effective ways of combating the pandemic:
    - It alerts infected individuals to treat faster
    - It minimizes the spread of virus with isolation
    - It identifies people who came into contact with infected individuals
        - Reduces asymptomatic transmission

A message from NIH leadership September 04, 2020

Why COVID-19 testing is the key to getting back to normal

HHS.gov **March 17, 2021**

Biden Administration to Invest More Than $12 Billion to Expand COVID-19 Testing

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Is there a downside to increasing testing?

- By increasing the testing, individuals will be less cautious, leading to an increase in their social activity

- This can potentially offset the impact of increasing the testing and can make the individuals worse off

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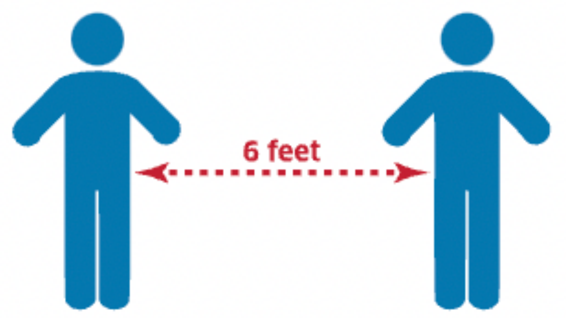
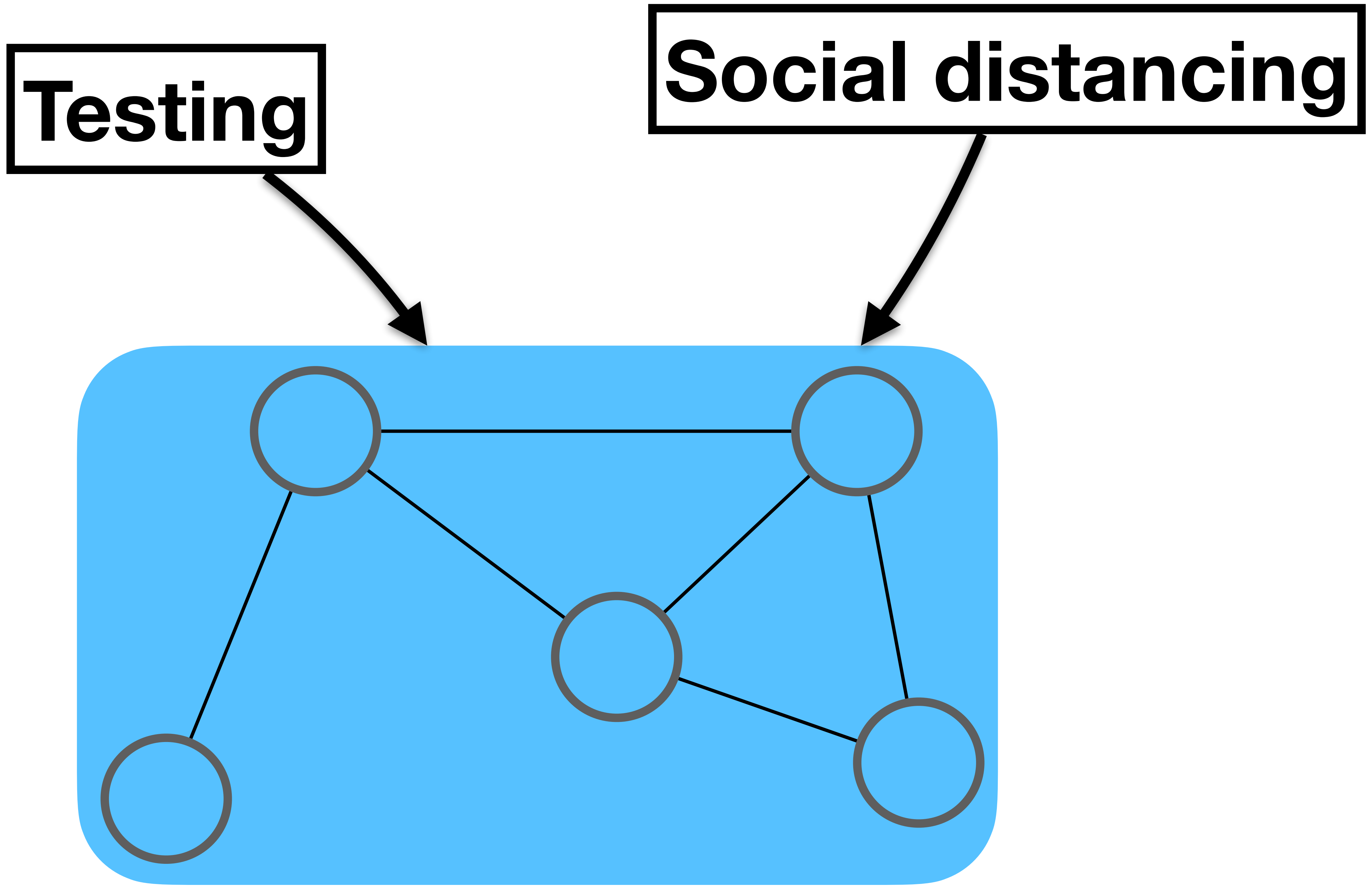
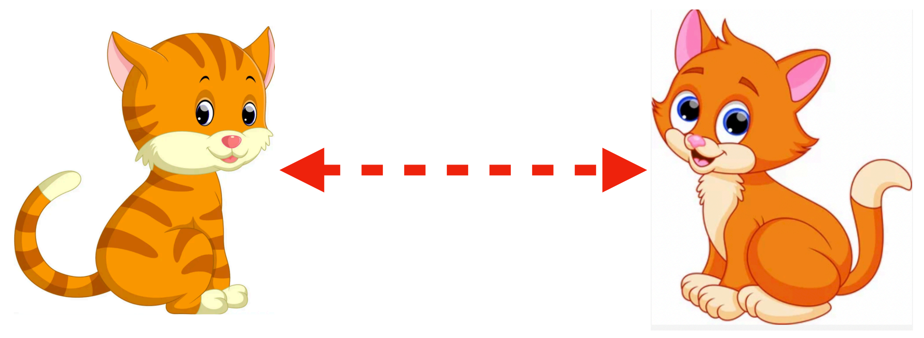




- Another important instrument to control a pandemic is (mandatory) social distancing

How should testing be combined with social distancing?



npr August 17, 2021

Even Moderate COVID Restrictions Can Slow The Spread Of The Virus — If They're Timely

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    - Testing enables the isolation of infected individuals, slowing down the infections

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    - Testing enables the isolation of infected individuals, slowing down the infections
- We characterize the equilibrium activity level of individuals for any given testing policy
- We then study the impact of testing policy on the equilibrium outcome and characterize the optimal testing policy

Main results

- Greater testing can lead to more social activity (less social distancing) and thus a denser social network
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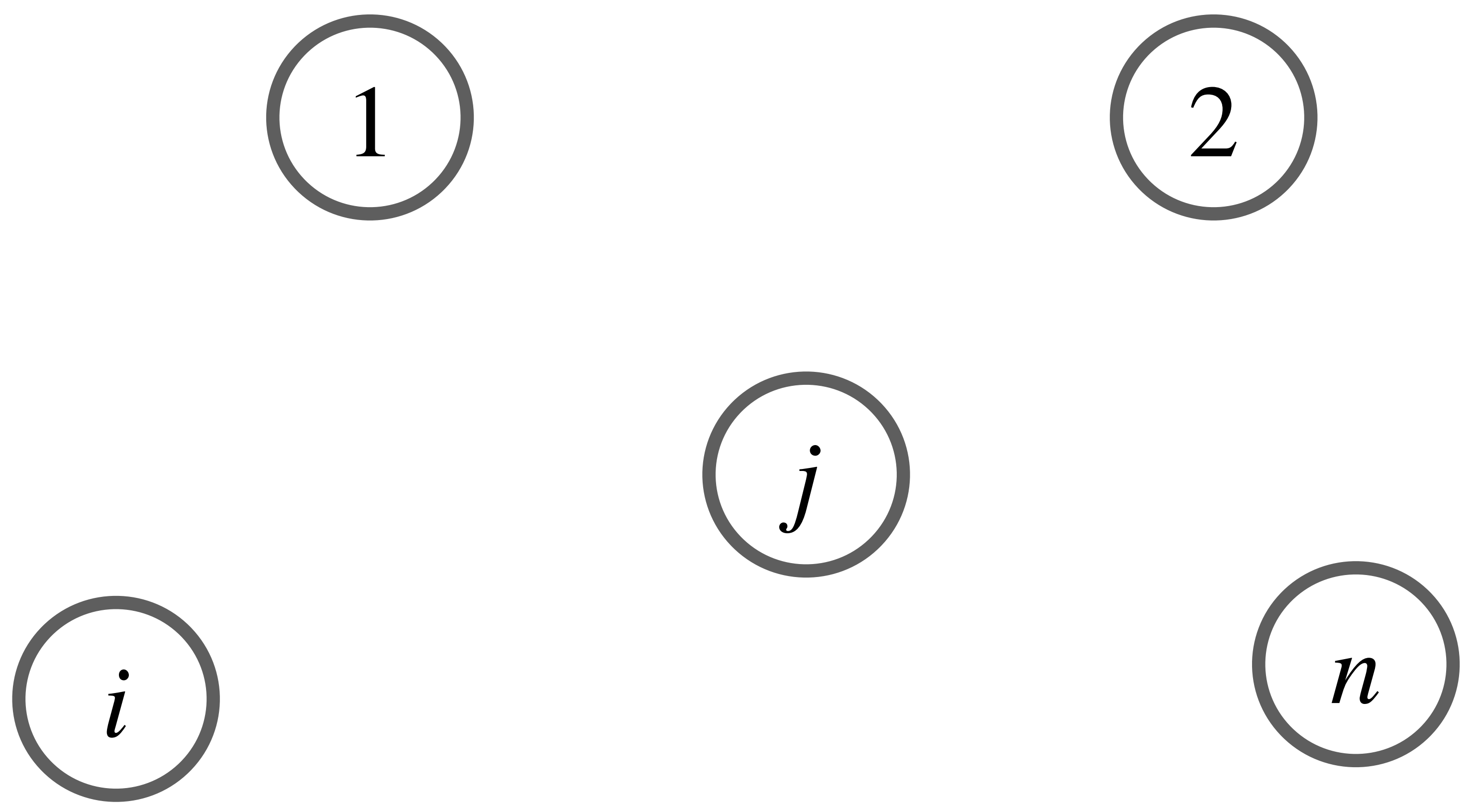
    - For a range of parameters, paradoxically, greater testing increases infections
- The optimal testing policy may leave some of the testing capacity of society unused in order to avoid adverse effects on social distancing
- Testing should be combined with mandatory social distancing to avoid these adverse behavioral effects

Related literature

- Endogenous social network formation
    - Jackson and Wolinsky 96, Bala and Goyal 00, Newman et al. 01
    - We use a simple model in which probability of connection is proportional to the product of social activities
- Precautionary tools increase risk-taking and can have adverse effects
    - Peltzam 75 in the context of hydraulic breaks and Lakdawalla et al. 06 in the context of HIV treatments
- Recent literature on epidemics and COVID-19
    - Farboodi et al. 20, Drakopoulos and Randhawa 20, Birge et al. 20, Zhang and Britton 22, Alimohammadi et al. 22, Bastani et al. 21, etc.

Model

- n individuals represented by $\mathcal{V} = \{1, \cdots, n\}$



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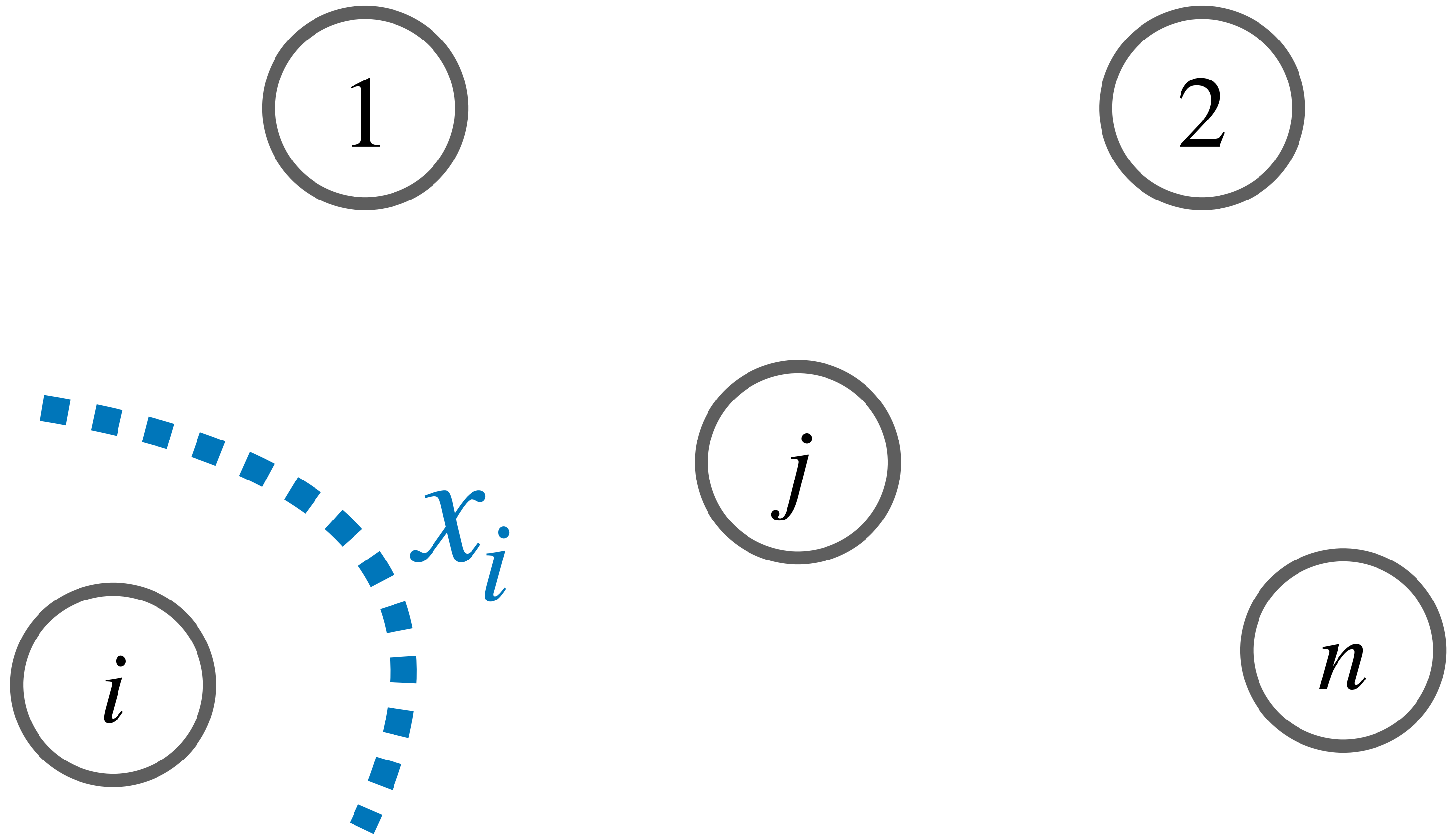
- Each individual i decides about her level of social activity $x_i \in [0,1]$
    - x_i determines the probability of connecting to others



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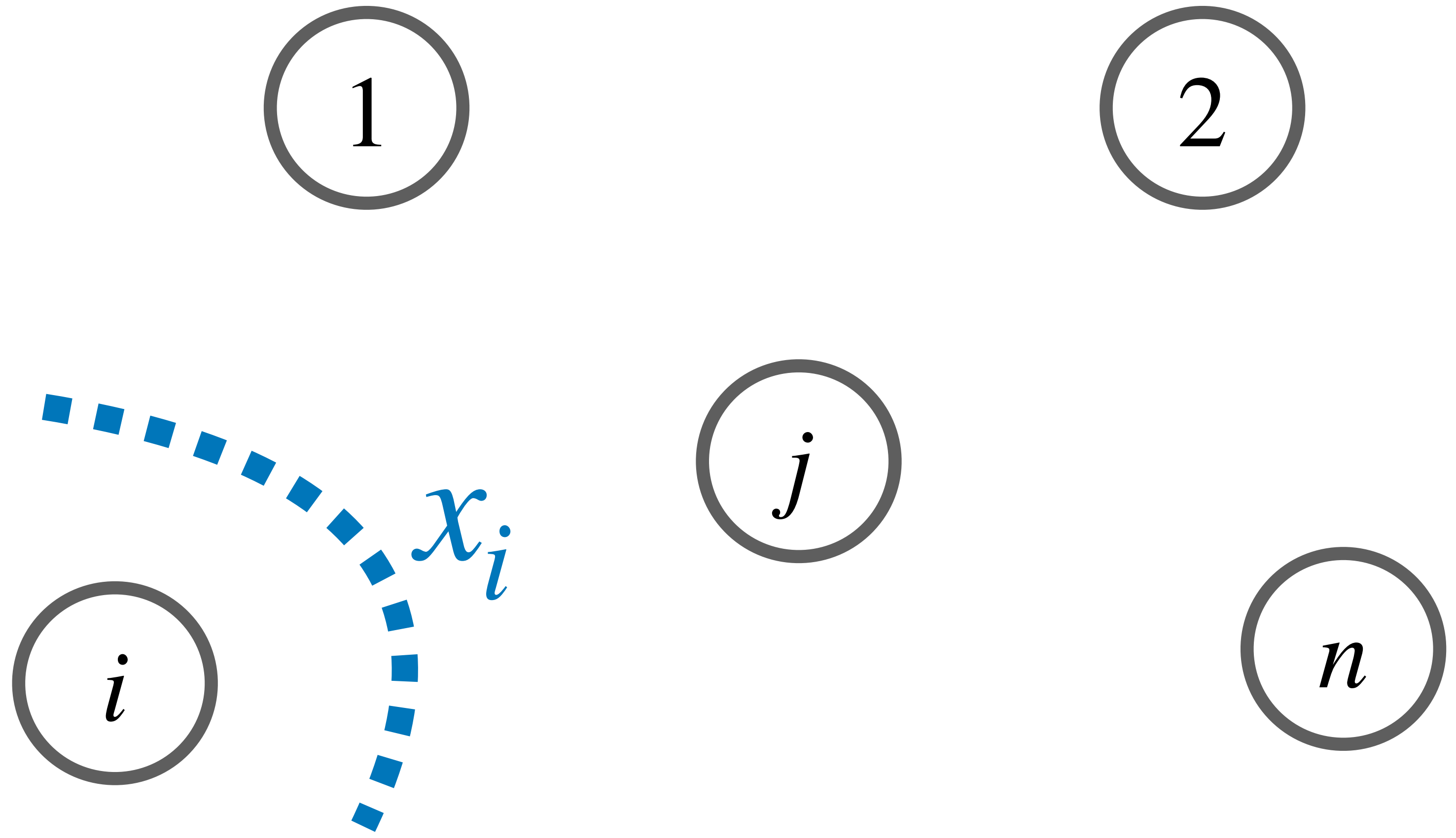
- There are k agent types with different values for social activity
  - Let us consider two types with $0 \leq v_L < v_H \leq 1$, called low-value and high-value types
  - Let us denote the set of high and low-value agents by $\mathscr{H}$ and $\mathscr{L}$



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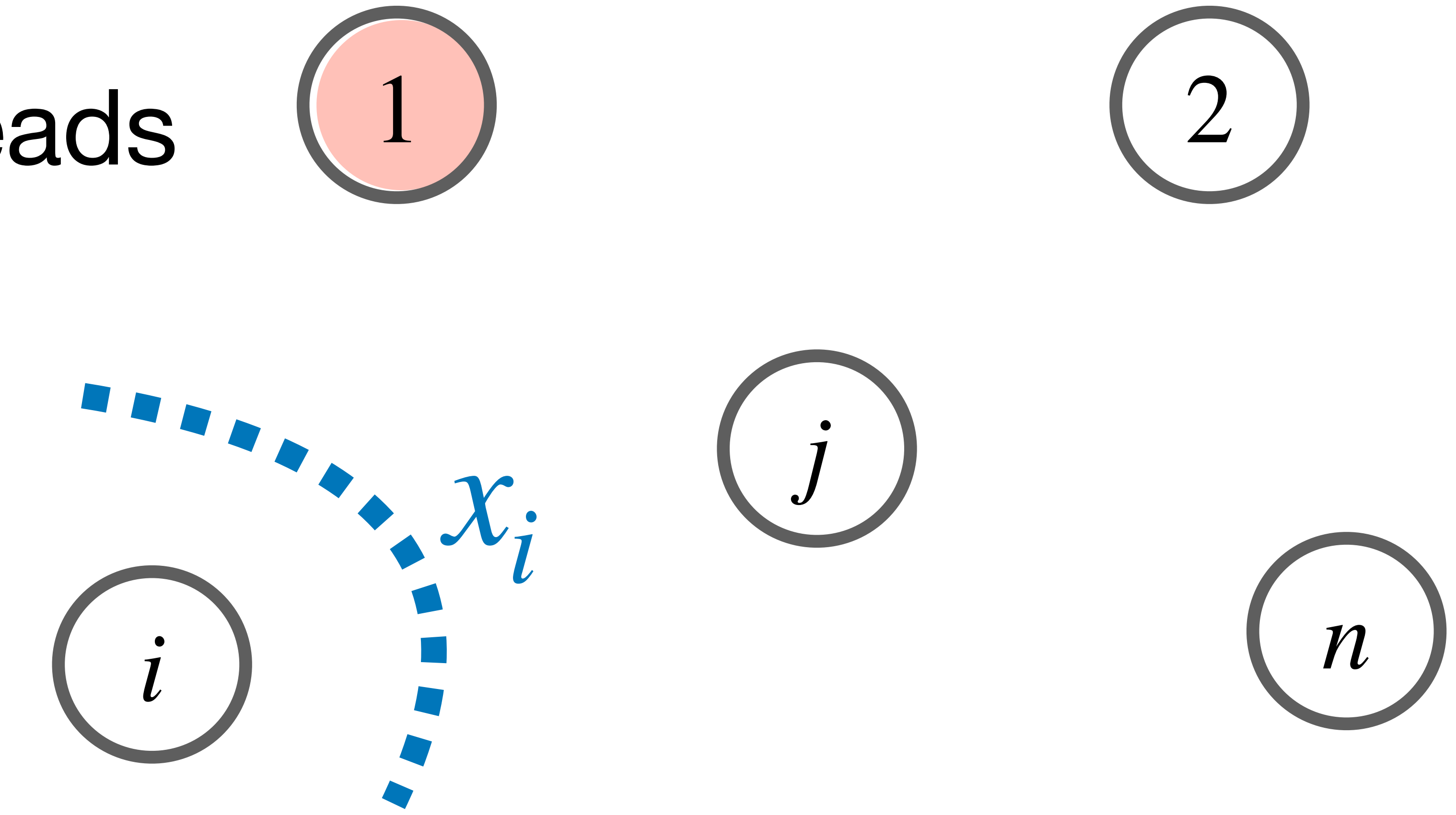
- A virus infects a uniformly random individual and then spreads according to an extended independent cascade model



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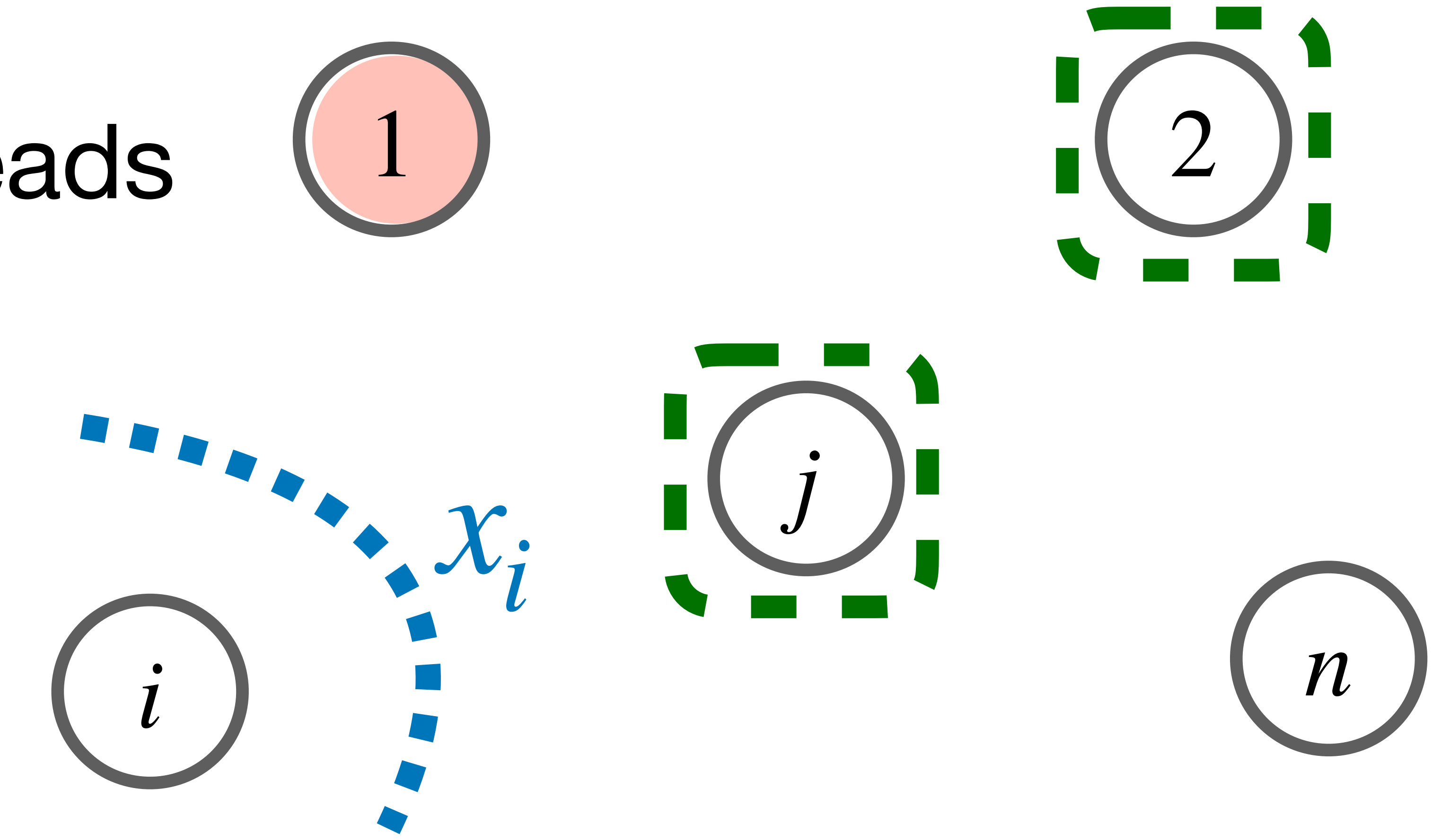
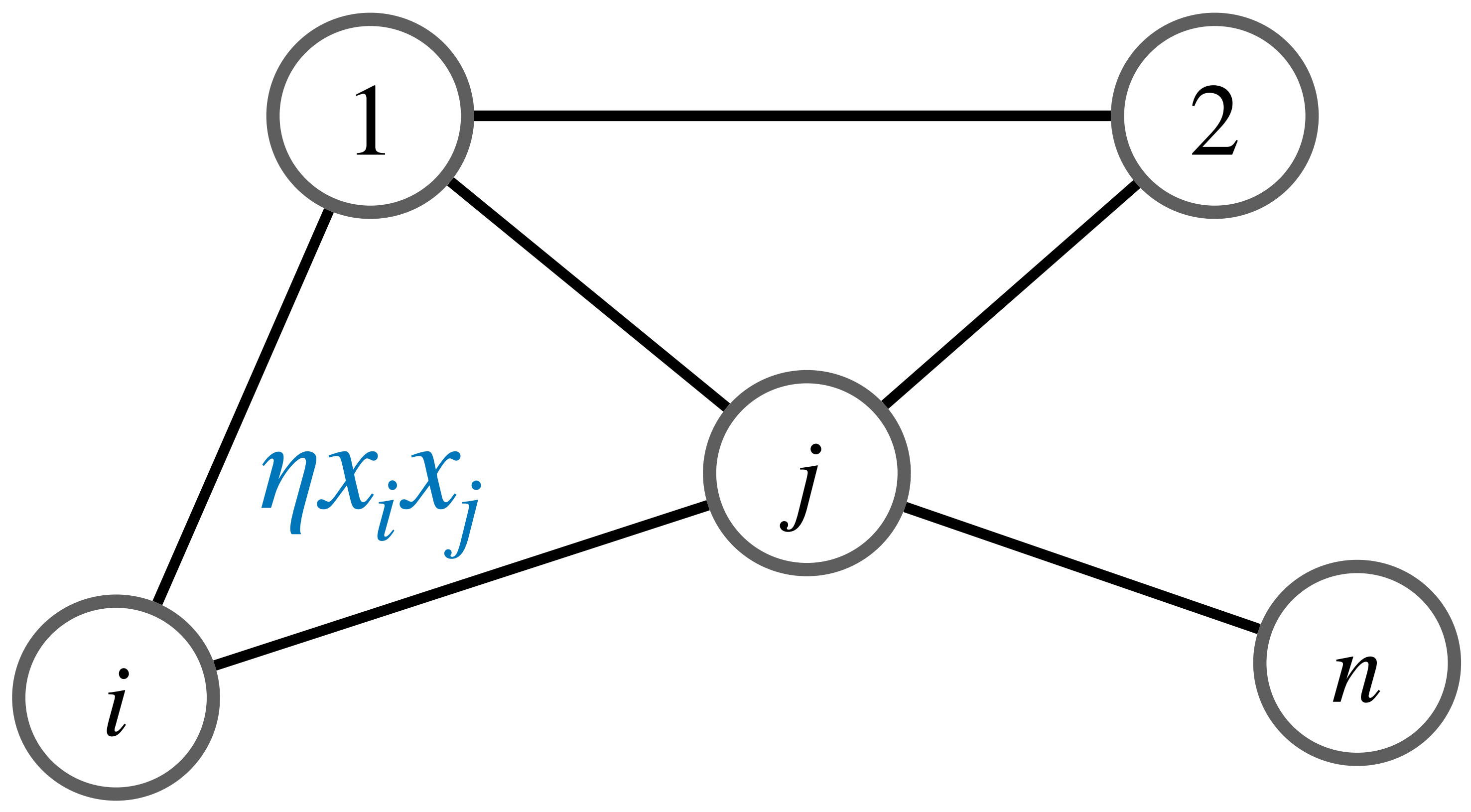
- Testing policy (α_L, α_H): High and low-value individuals are tested with probabilities α_L and $\alpha_H \in [0,1]$, respectively



Contact network

- The social activity profile $\mathbf{x} = (x_1, \ldots, x_n)$ generates a contact network $G = (\mathscr{V}, \mathbf{E})$ where

$$\mathbb{P}(E_{ij} = E_{ji} = 1) = \eta x_i x_j$$



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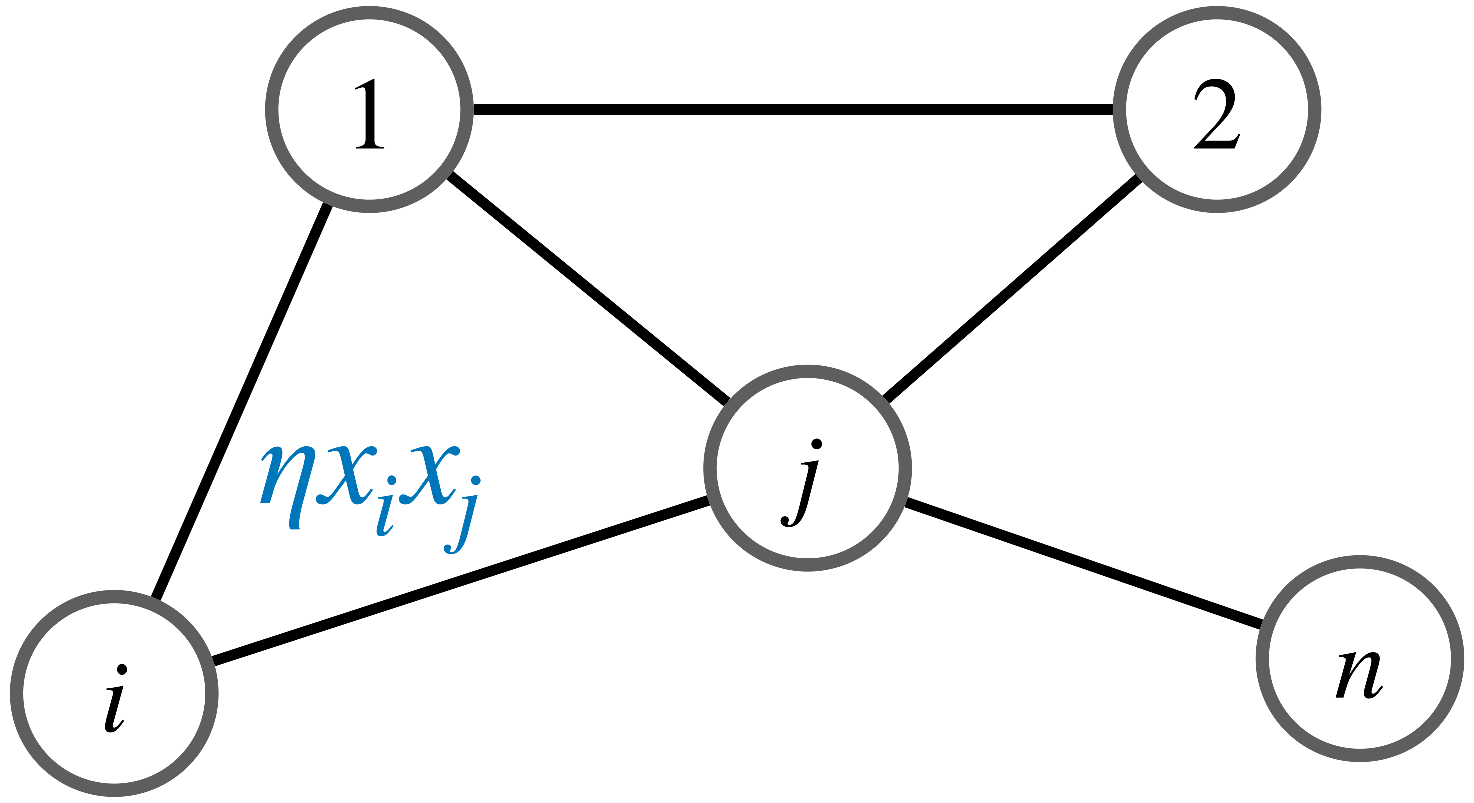
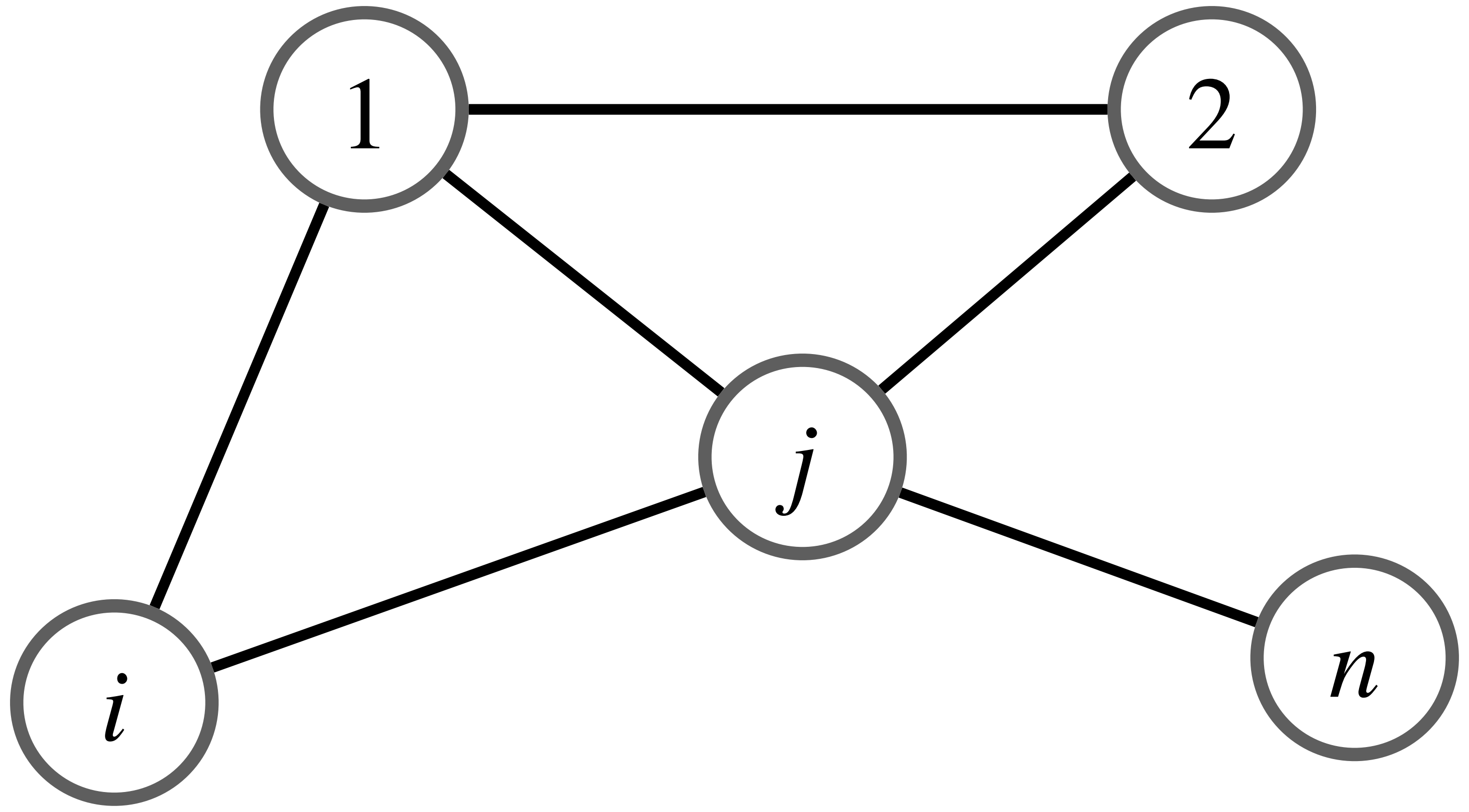


- This captures activities in which individuals interact with each other such as playing basketball, going to a restaurant, going to the office, shopping, etc.

- It does not capture individuals going for activities such as hiking, biking, etc.

Spread of infection

- One of the individuals uniformly at random becomes infected and the infection spreads to others over the contact network via an extension of independent cascade model of Kempe et al. 2015:



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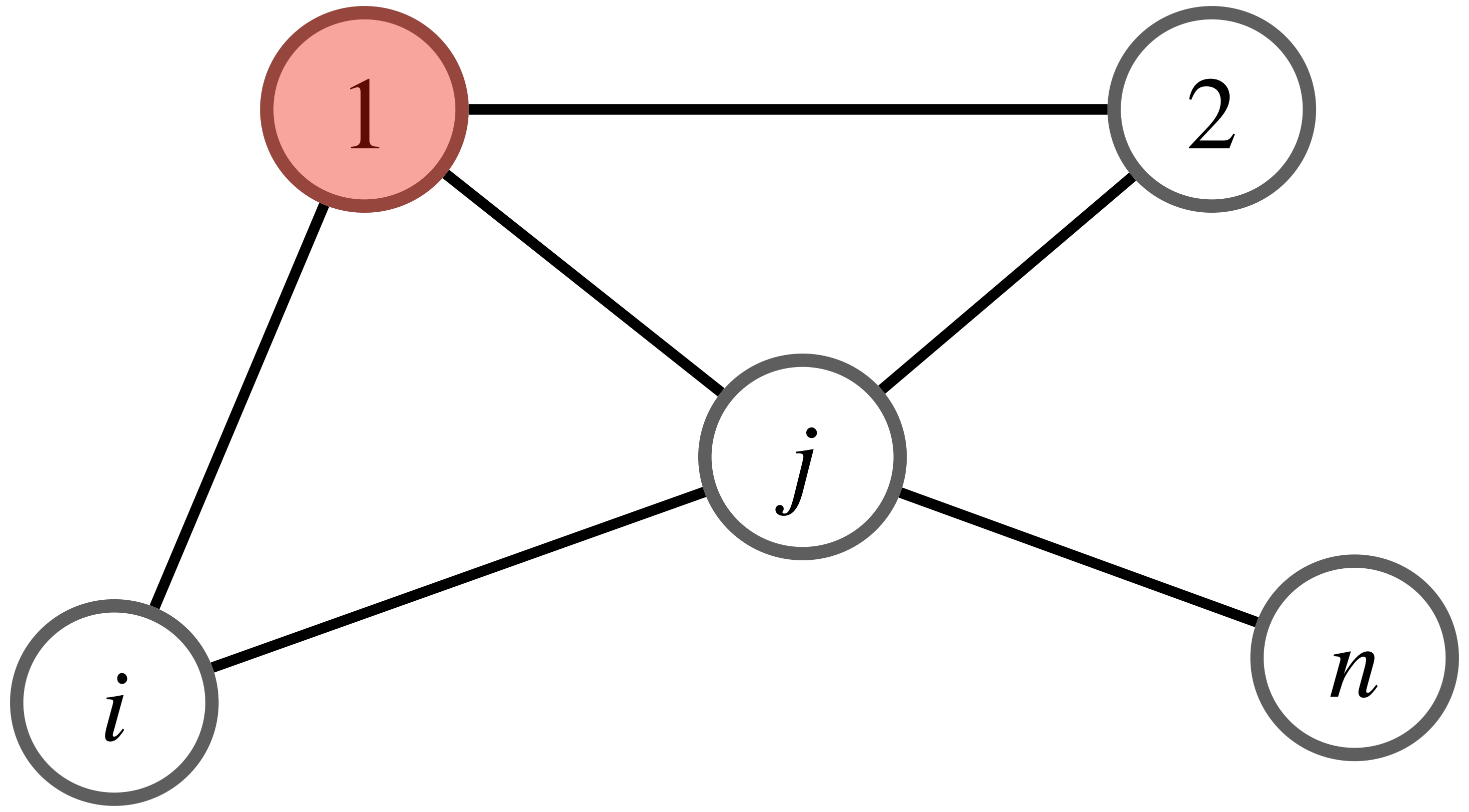

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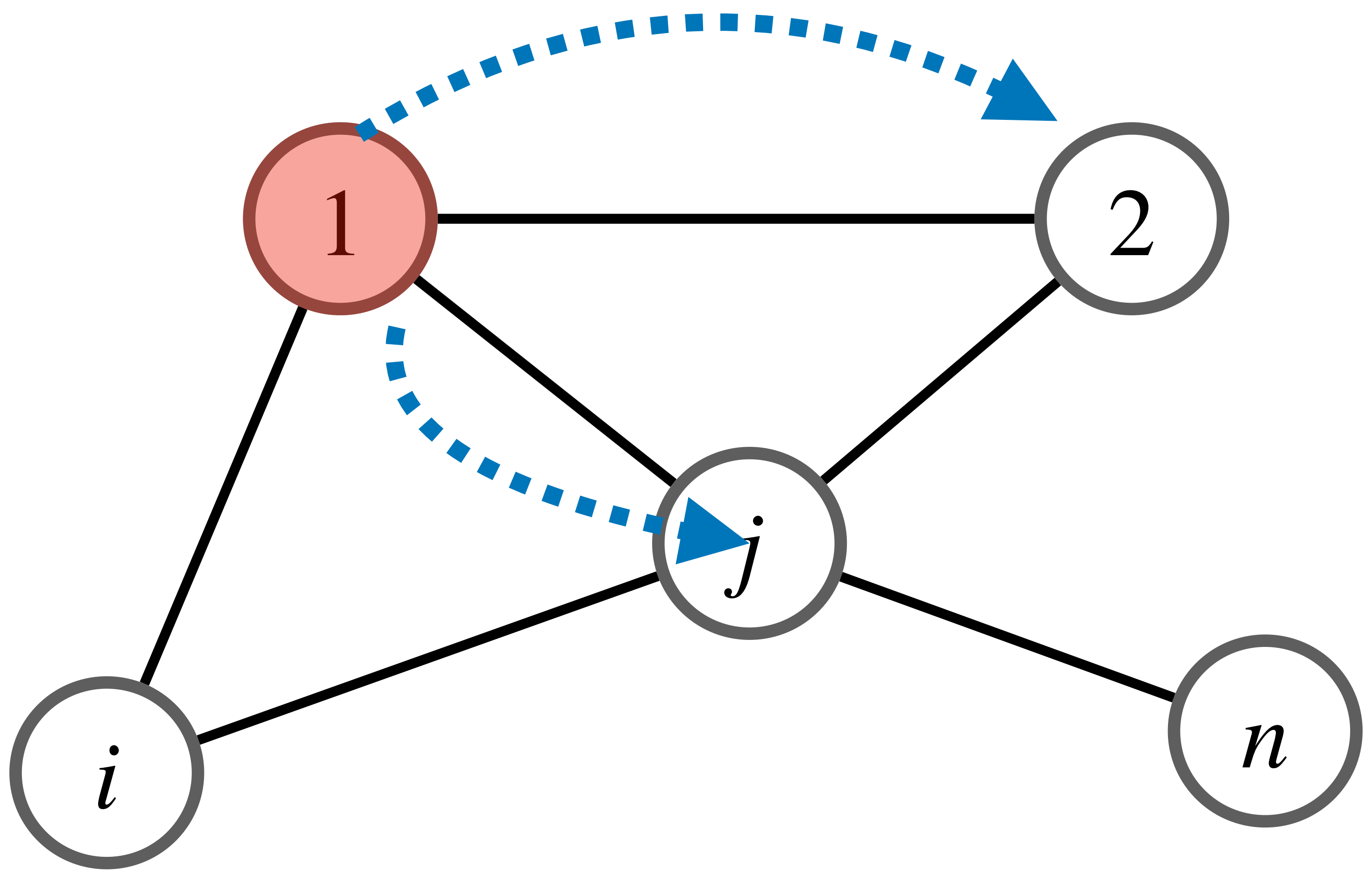
- An infected and not tested (and therefore not isolated) individual is active for one round and transmits infection to her neighbors with transmission probability $\beta \in (0,1]$



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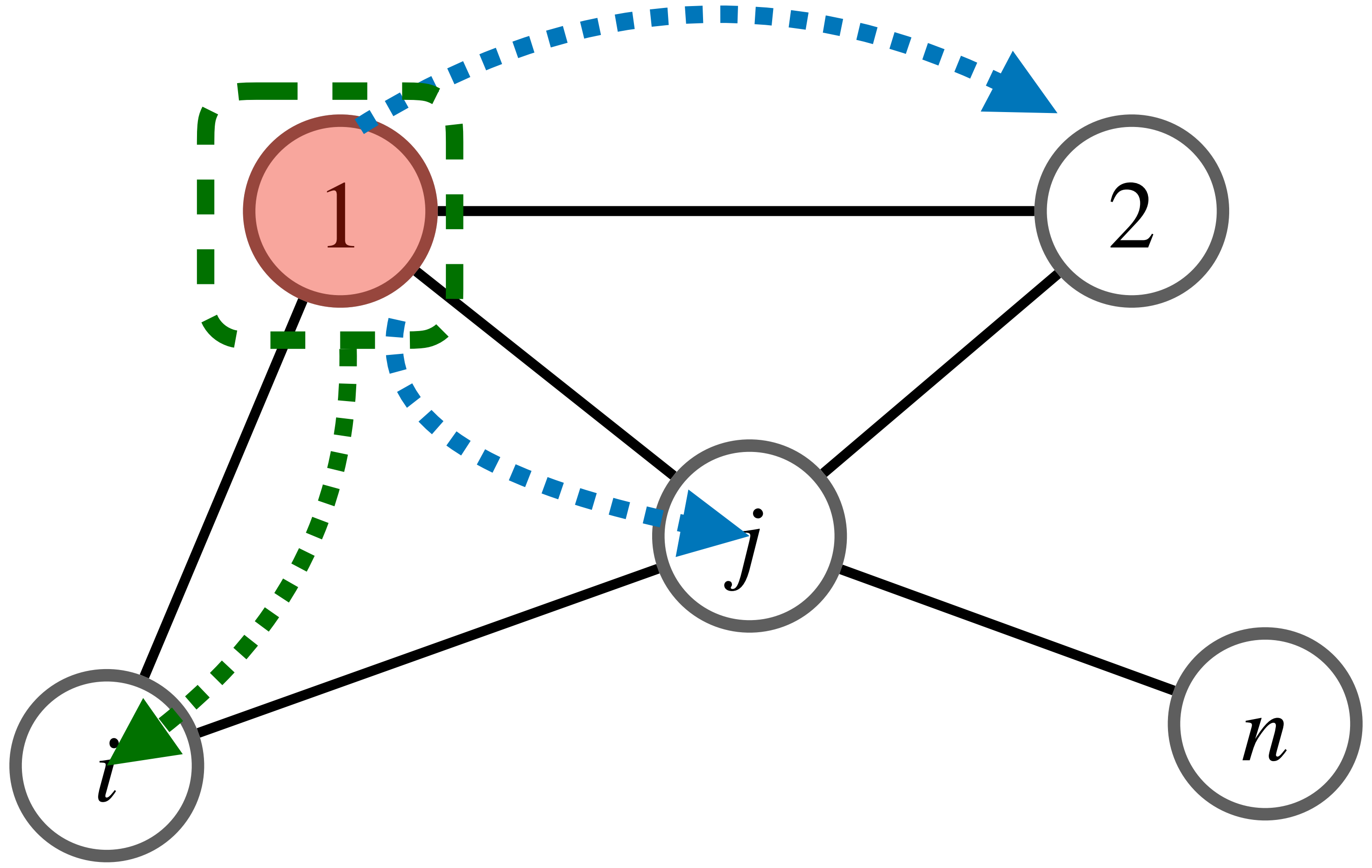

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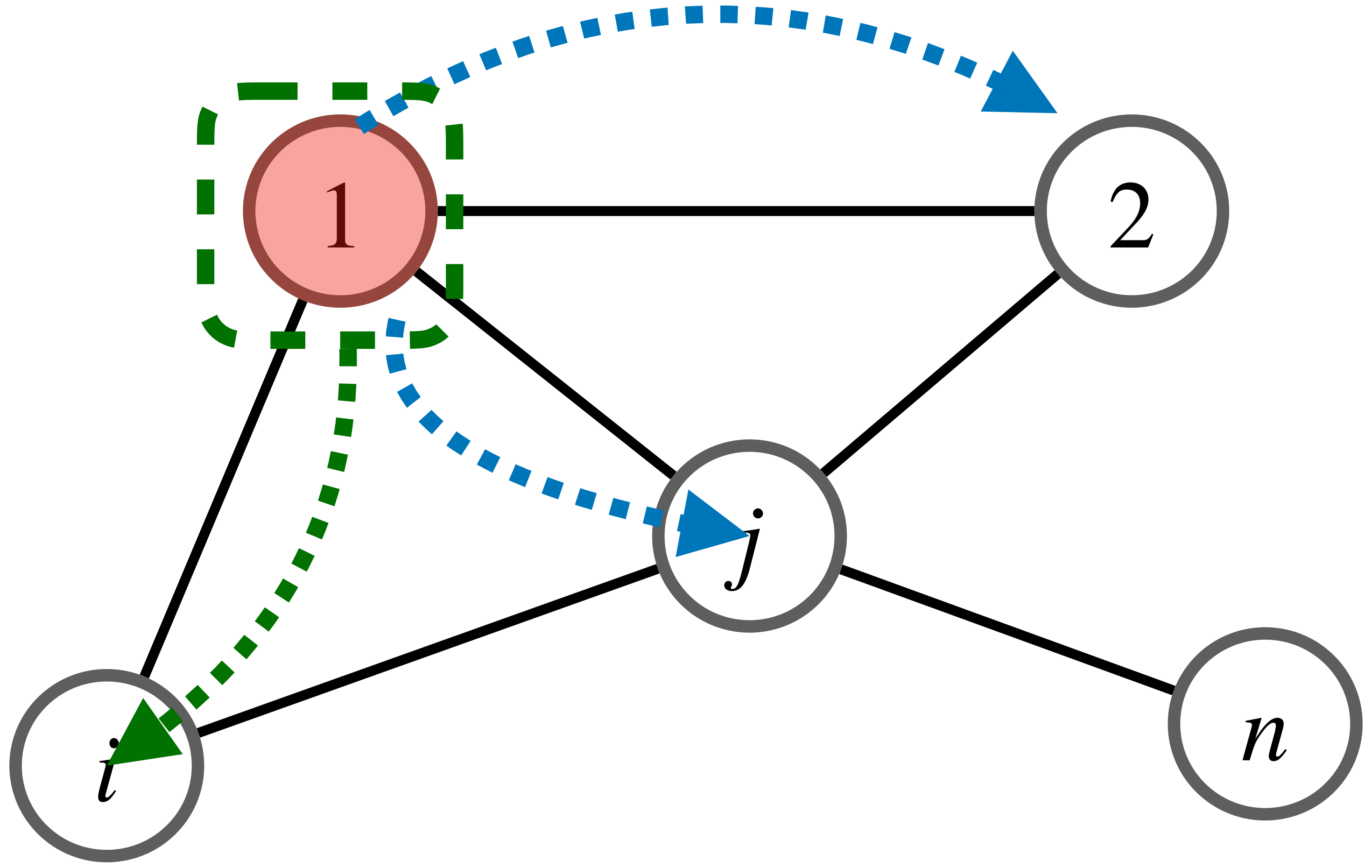
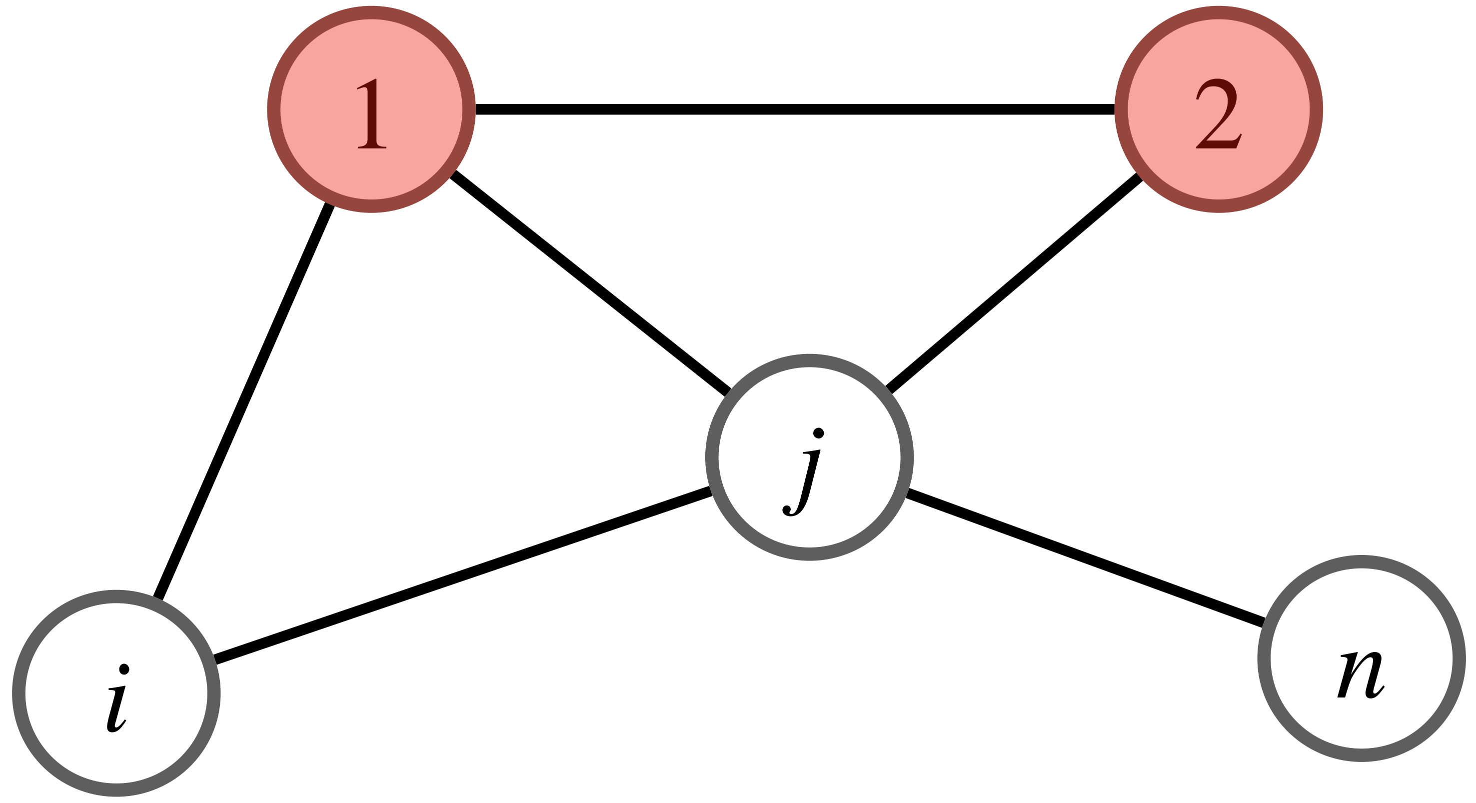
- An active individual remains active for one round



Spread of infection

- From active agents, the infection simultaneously and independently transmits to each of their uninfected neighbors
    - If an uninfected agent is neighbor to multiple infected individuals, then the infection is transmitted to this agent in an order-independent fashion



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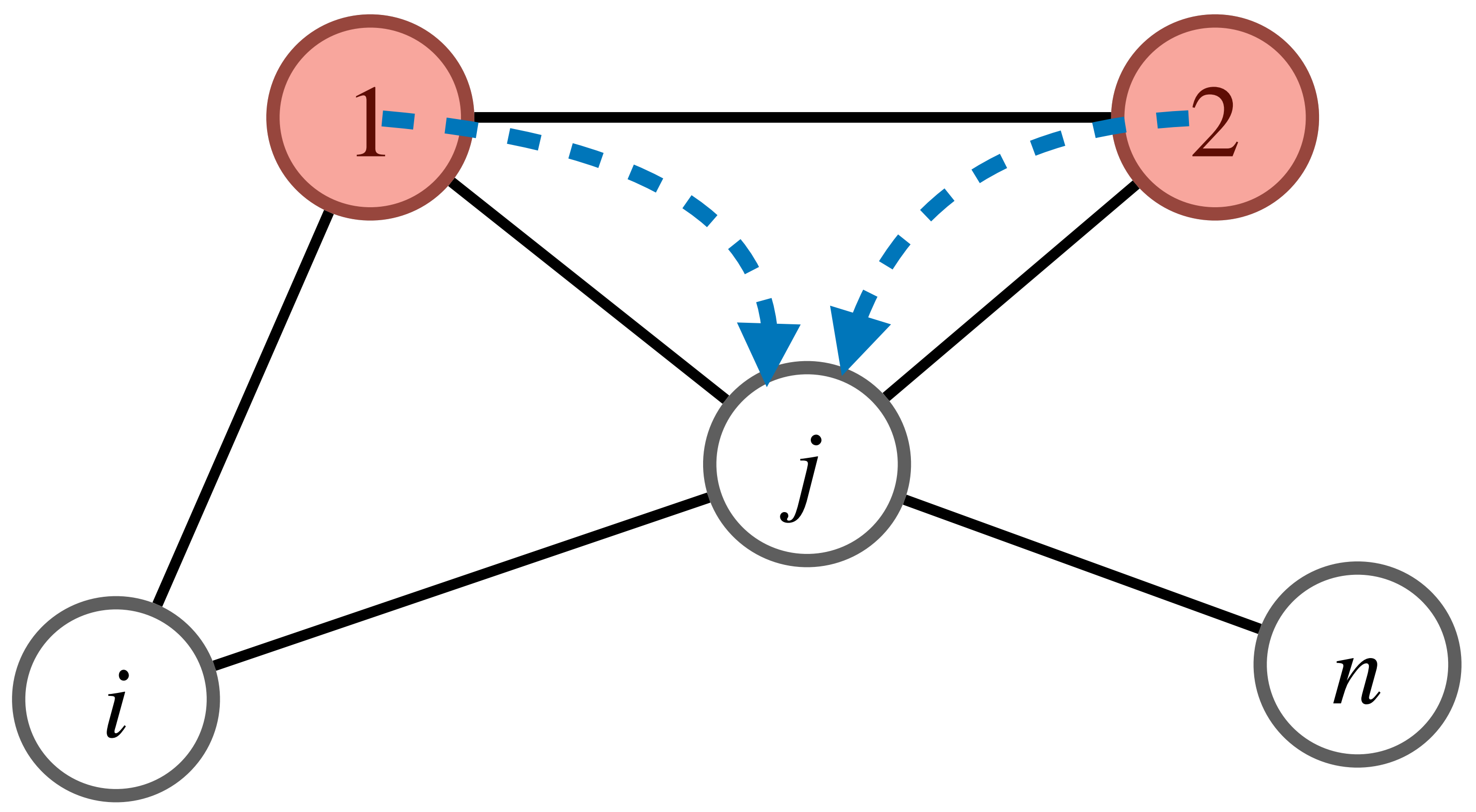
$$1 - (1 - \beta)^2$$



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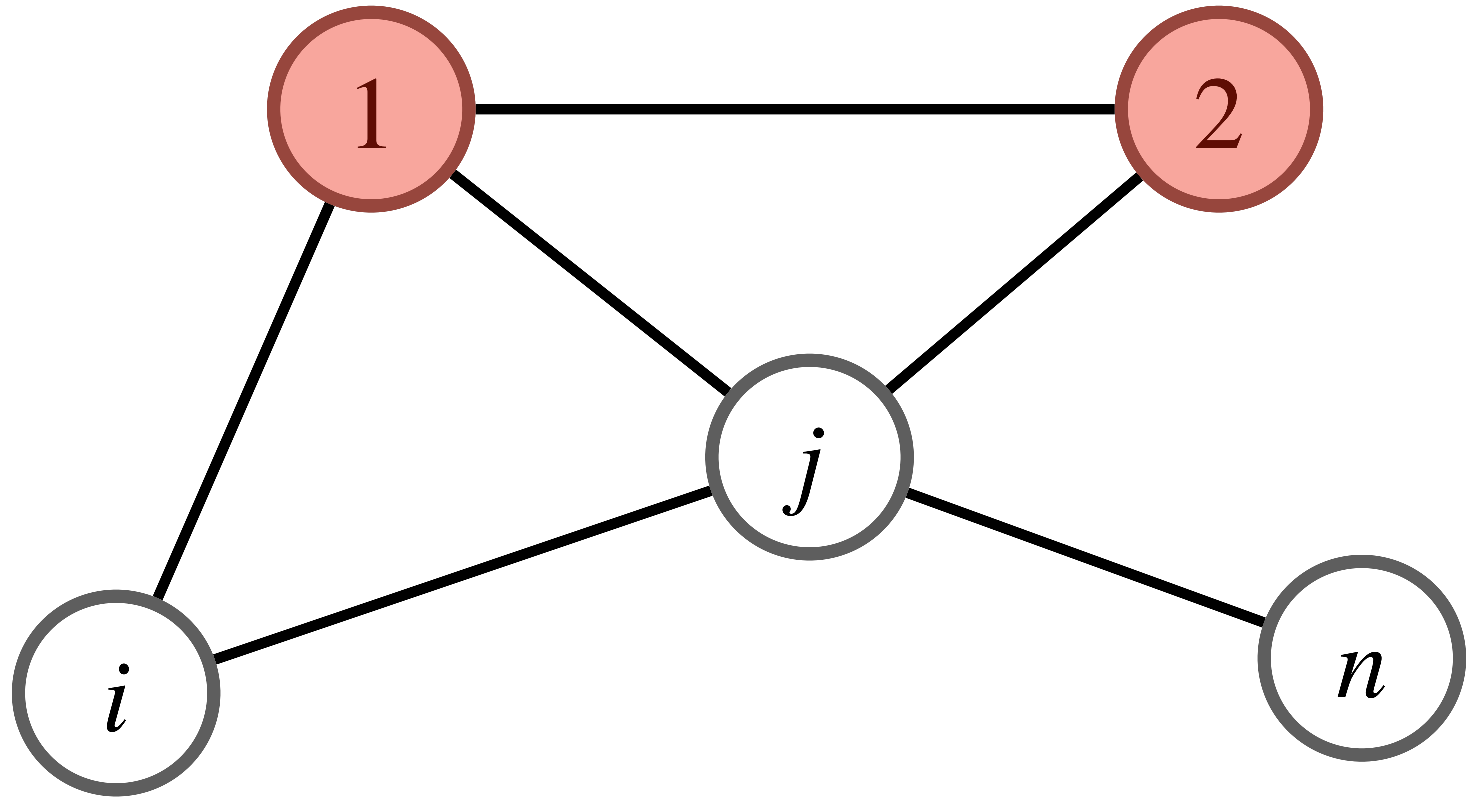

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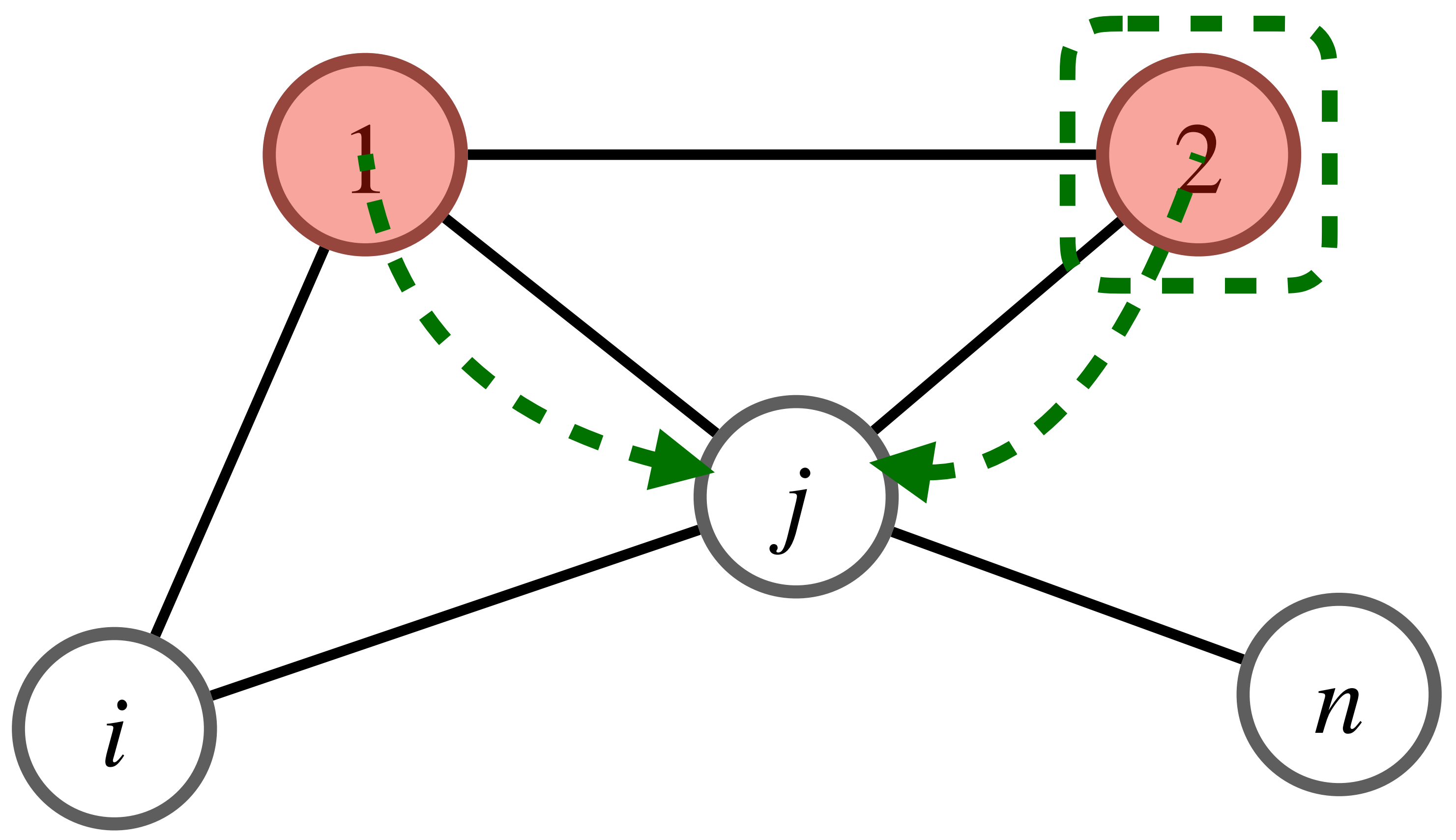

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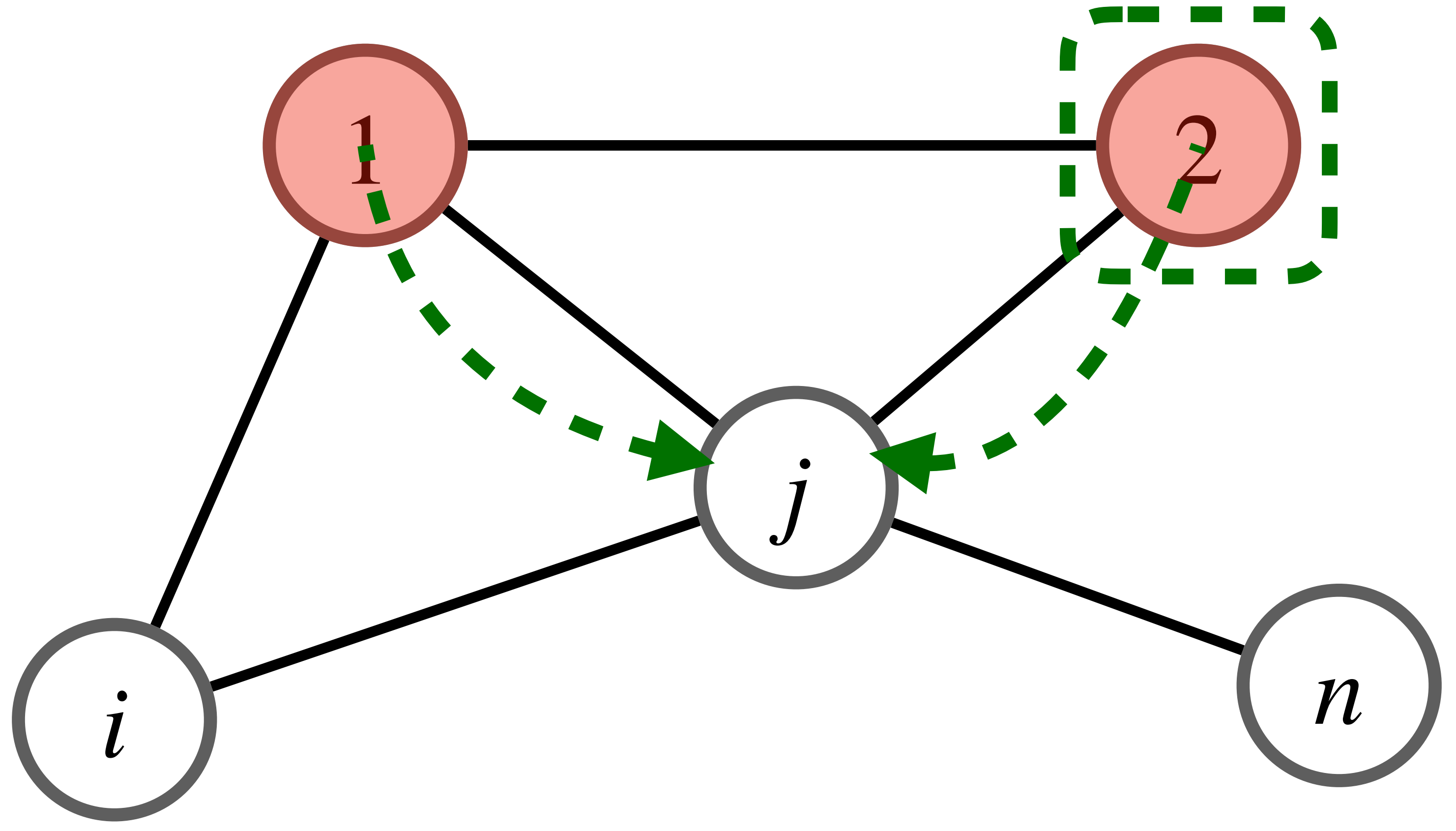
- $\mathbb{P}_i^{\text{inf}}(\mathbf{x}, \alpha_L, \alpha_H)$: The expected **infection probability of individual i** in the random network of contacts for social activity profile $\mathbf{x}$ and testing policy (α_L, α_H)

$$1 - (1 - \beta)(1 - \beta p)$$



Utility of agents

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$u_i(\mathbf{x}, \alpha_L, \alpha_H) = v_i x_i - \mathbb{P}_i^{\inf}(\mathbf{x}, \alpha_L, \alpha_H) - c\left(\alpha_L \mathbf{1}\{i \in L\} + \alpha_H \mathbf{1}\{i \in H\}\right)$

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A possibly small cost of testing

Infection probability

Lemma

- **Monotonicity in action profile**: For any $i \in \mathscr{V}$, $\mathbb{P}_i^{\text{inf}}(\hat{\mathbf{x}}, \alpha_L, \alpha_H) \geq \mathbb{P}_i^{\text{inf}}(\mathbf{x}, \alpha_L, \alpha_H)$ for $\hat{\mathbf{x}} \geq \mathbf{x}$

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  $(\alpha'_H, \alpha'_L) \geq (\alpha_H, \alpha_L)$

These three are the main properties that we use in the analysis!

- Higher social activity implies more connections and higher infection probability
- The marginal increase in the infection probability decreases as the social activity increases
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How about in equilibrium when individuals choose their social activity endogenously?

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$$v_i - \mathbb{P}_i^{\text{inf}}(\mathbf{x}_{\mathscr{H}} = \mathbf{1}, \mathbf{x}_{\mathscr{L}} = \mathbf{1}, \alpha_L, \alpha_H) \geq -\frac{1}{n} \text{ for all } i$$

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$$\Rightarrow \text{ for } (\alpha_L, \alpha_H) \in \mathscr{A}_1 = \left\{ (\alpha_L, \alpha_H) \in [0,1]^2 : \mathbb{P}_i^{\inf}(\mathbf{x}_{\mathscr{H}} = \mathbf{1}, \mathbf{x}_{\mathscr{L}} = \mathbf{1}, \alpha_L, \alpha_H) \leq v_i + \frac{1}{n}, i = l, h \right\}$$

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- As we decrease (α_L, α_H), one of the constraints of $\mathscr{A}_1$ will be violated
  - The constraint corresponding to low-value agents violate first
  - There is no symmetric pure equilibrium

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$$\mathscr{A}_2 = \Big\{(\alpha_L, \alpha_H) \in [0,1]^2 : \mathbb{P}_l^{\inf}(\mathbf{x}_{\mathscr{H}} = \mathbf{1}, \mathbf{x}_{\mathscr{L}} = \mathbf{1}, \alpha_L, \alpha_H) \geq v_L + \frac{1}{n}$$

$$\mathbb{P}_l^{\inf}(\mathbf{x}_{\mathscr{H}} = \mathbf{1}, x_l = 1, \mathbf{x}_{\mathscr{L}\backslash l} = \mathbf{0}, \alpha_L, \alpha_H) \leq v_L + \frac{1}{n}$$

$$\mathbb{P}_h^{\inf}(\mathbf{x}_{\mathscr{H}} = \mathbf{1}, \mathbf{x}_{\mathscr{L}} = \mathbf{0}, \alpha_L, \alpha_H) \leq v_H + \frac{1}{n}\Big\}$$

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- The mixed-strategy for low-value agents exists because of the first two constraints (mean-value theorem) and is given by

$$v_L - \mathbb{P}_l^{\inf}(\mathbf{x}_{\mathscr{H}} = \mathbf{1}, x_l = 1, \mathbf{x}_{\mathscr{L}\setminus\{l\}} = \gamma_L(\alpha_L, \alpha_H), \alpha_L, \alpha_H) = -\frac{1}{n}$$

$$x_i = \begin{cases} 1 & \text{w.p. } \gamma_L(\alpha_L, \alpha_H) \\ 0 & \text{w.p. } 1 - \gamma_L(\alpha_L, \alpha_H) \end{cases}$$

Equilibrium characterization

Proposition

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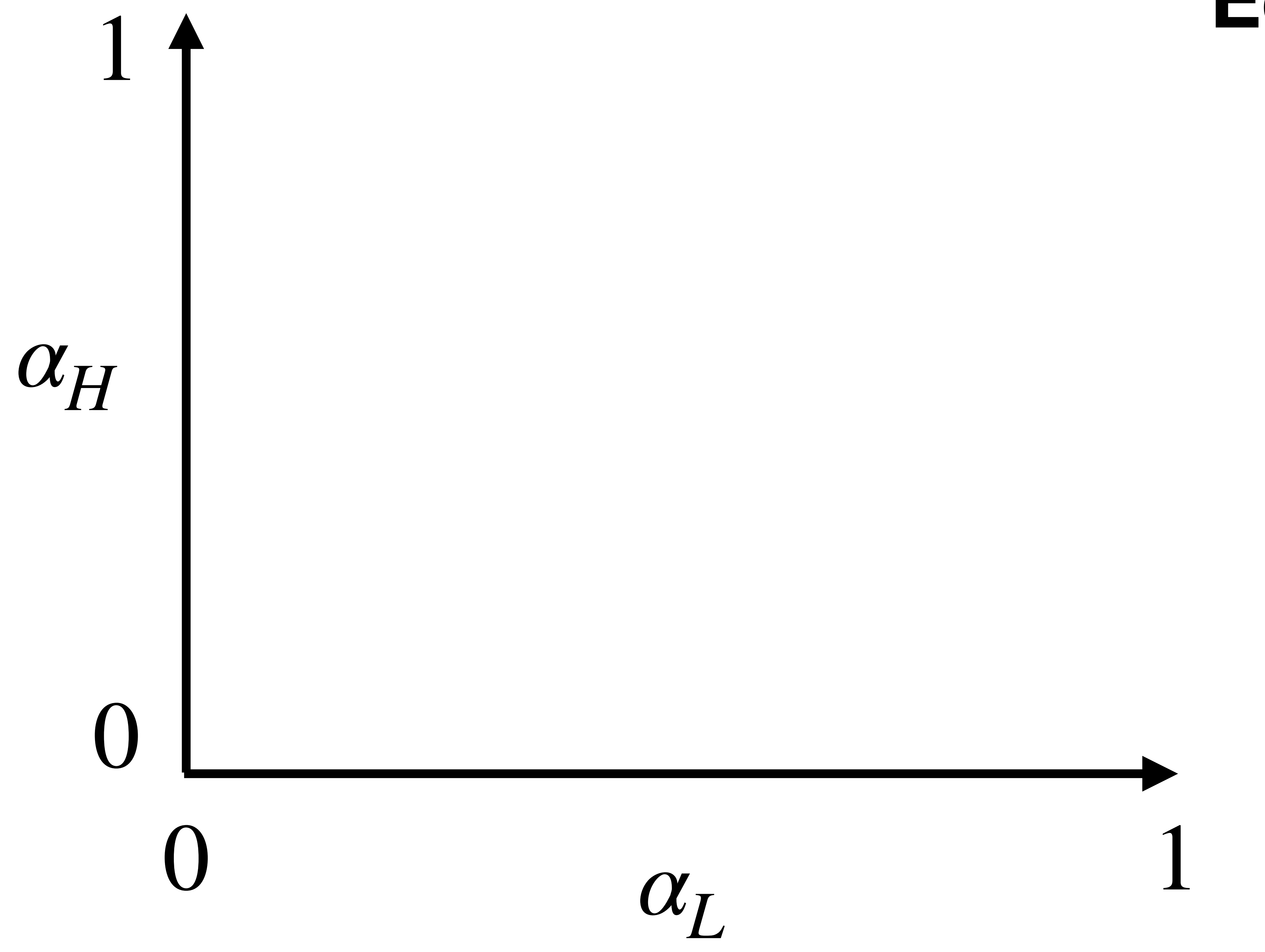

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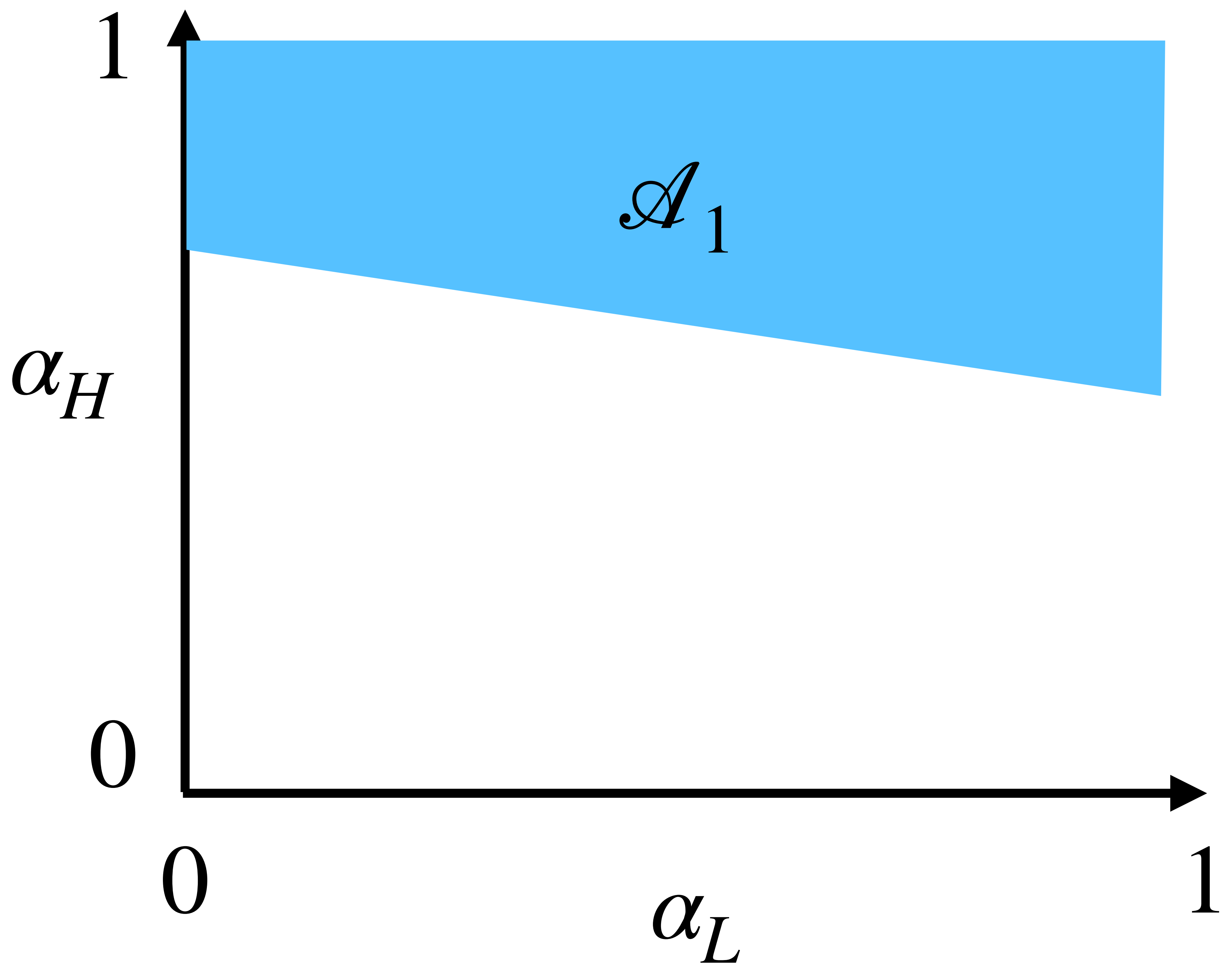

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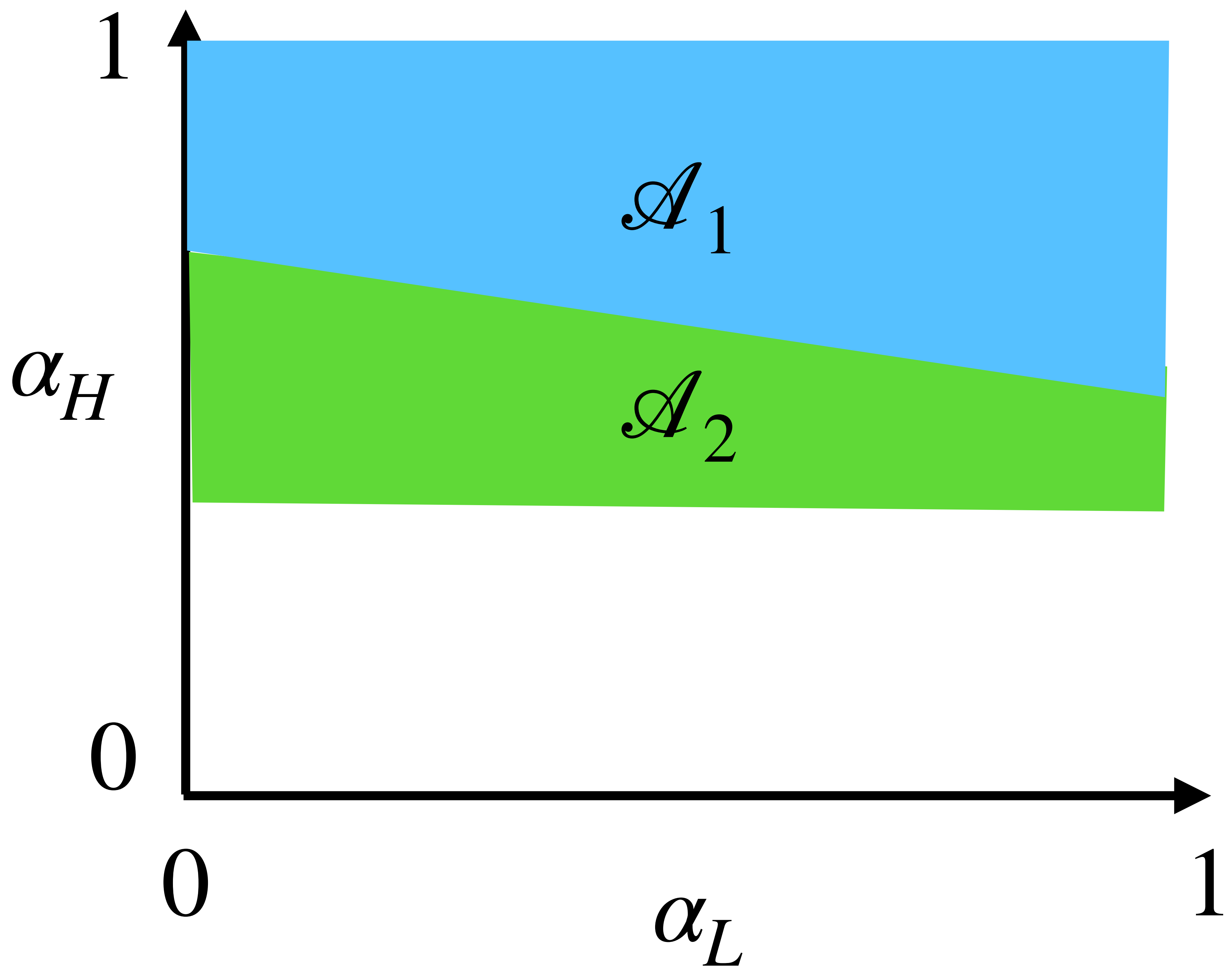

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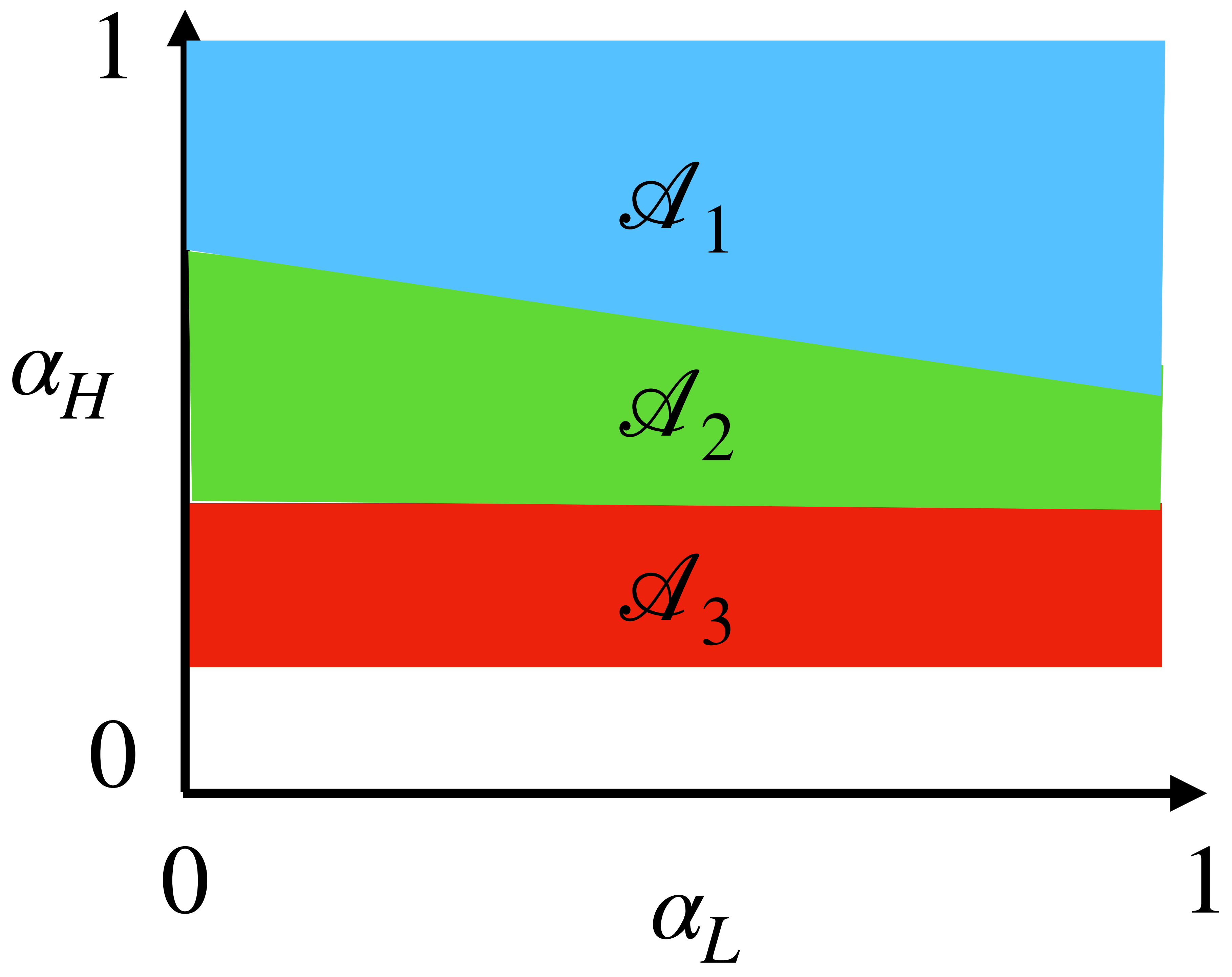

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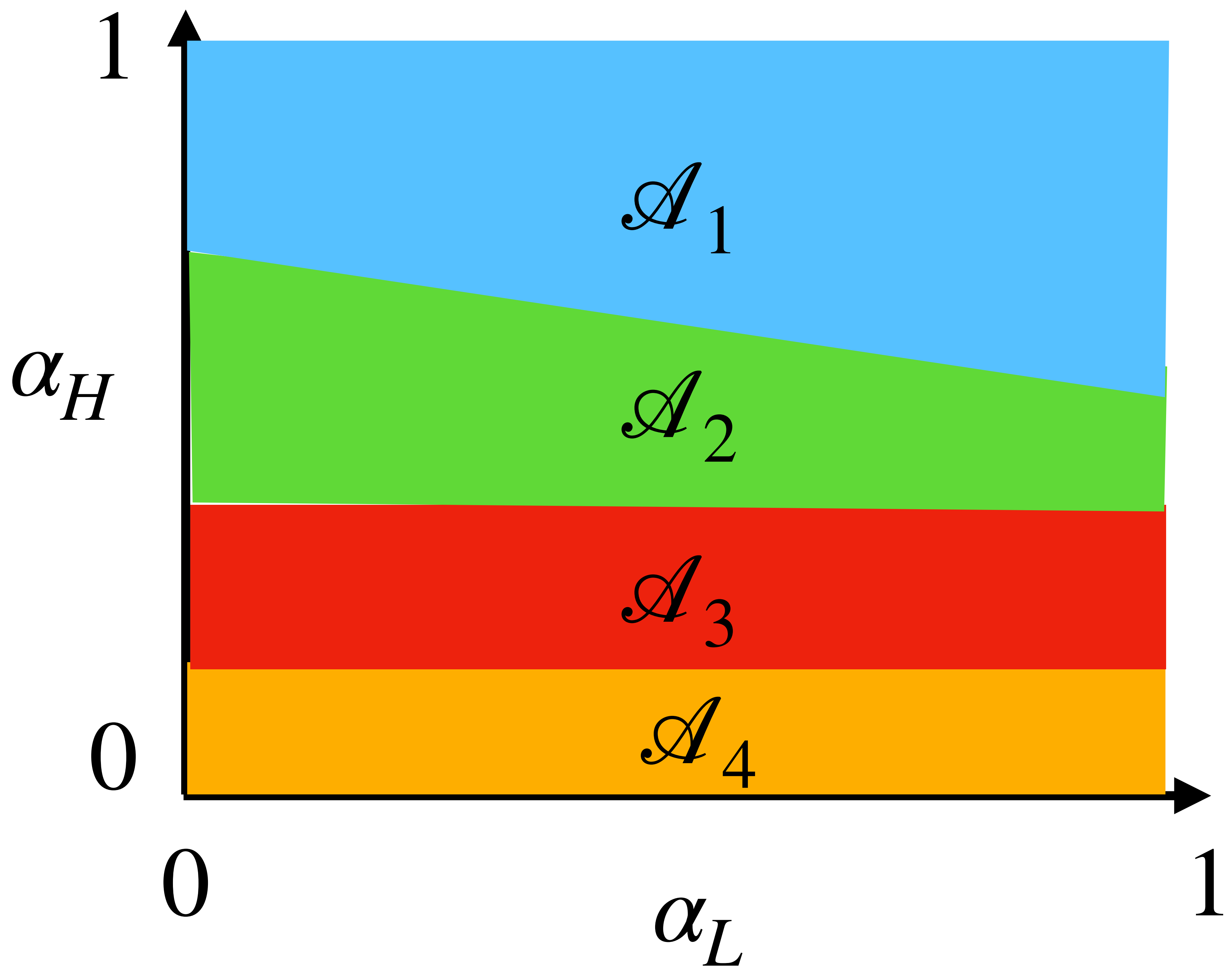

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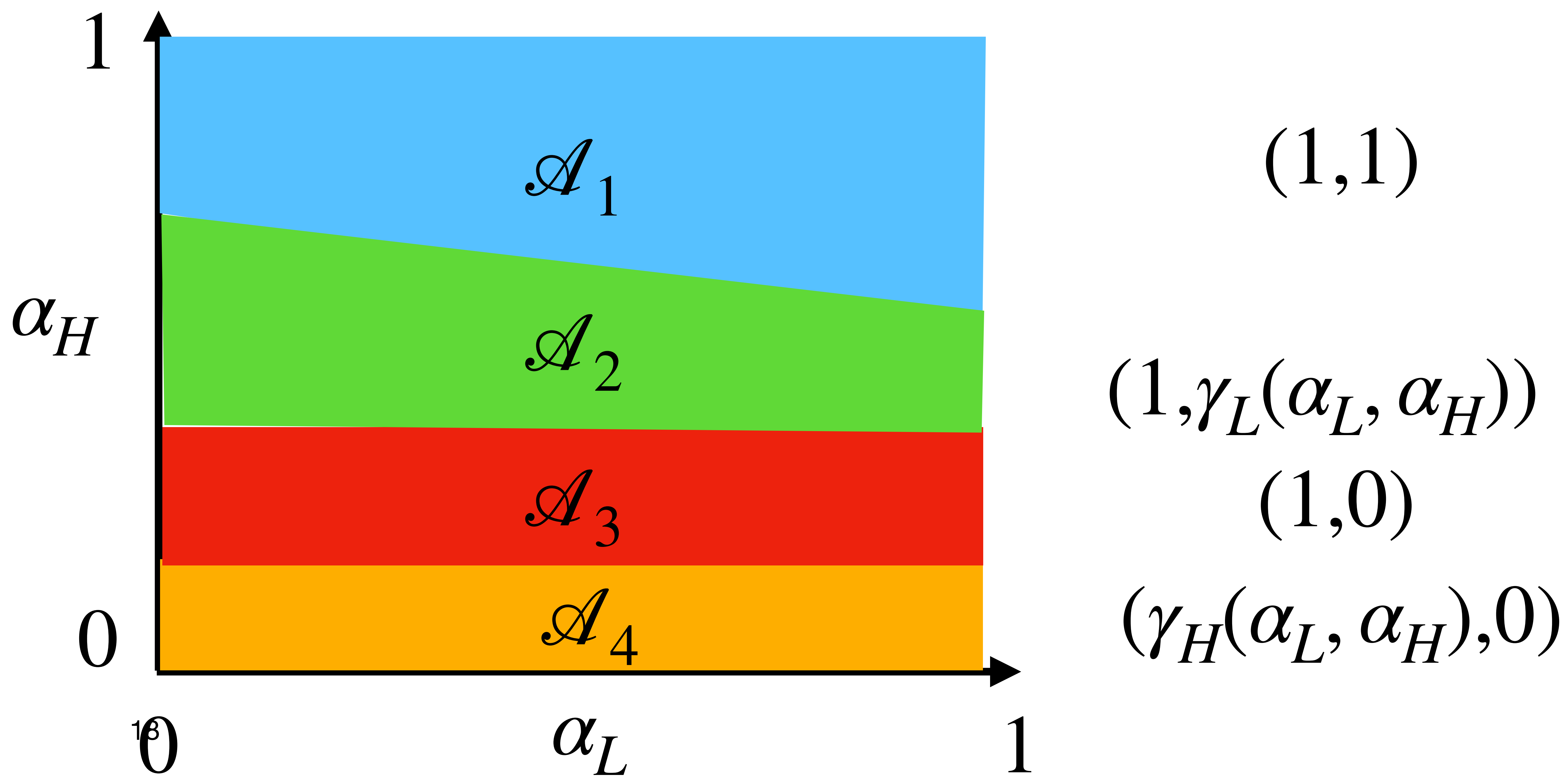
Impact of testing policy on infection probability



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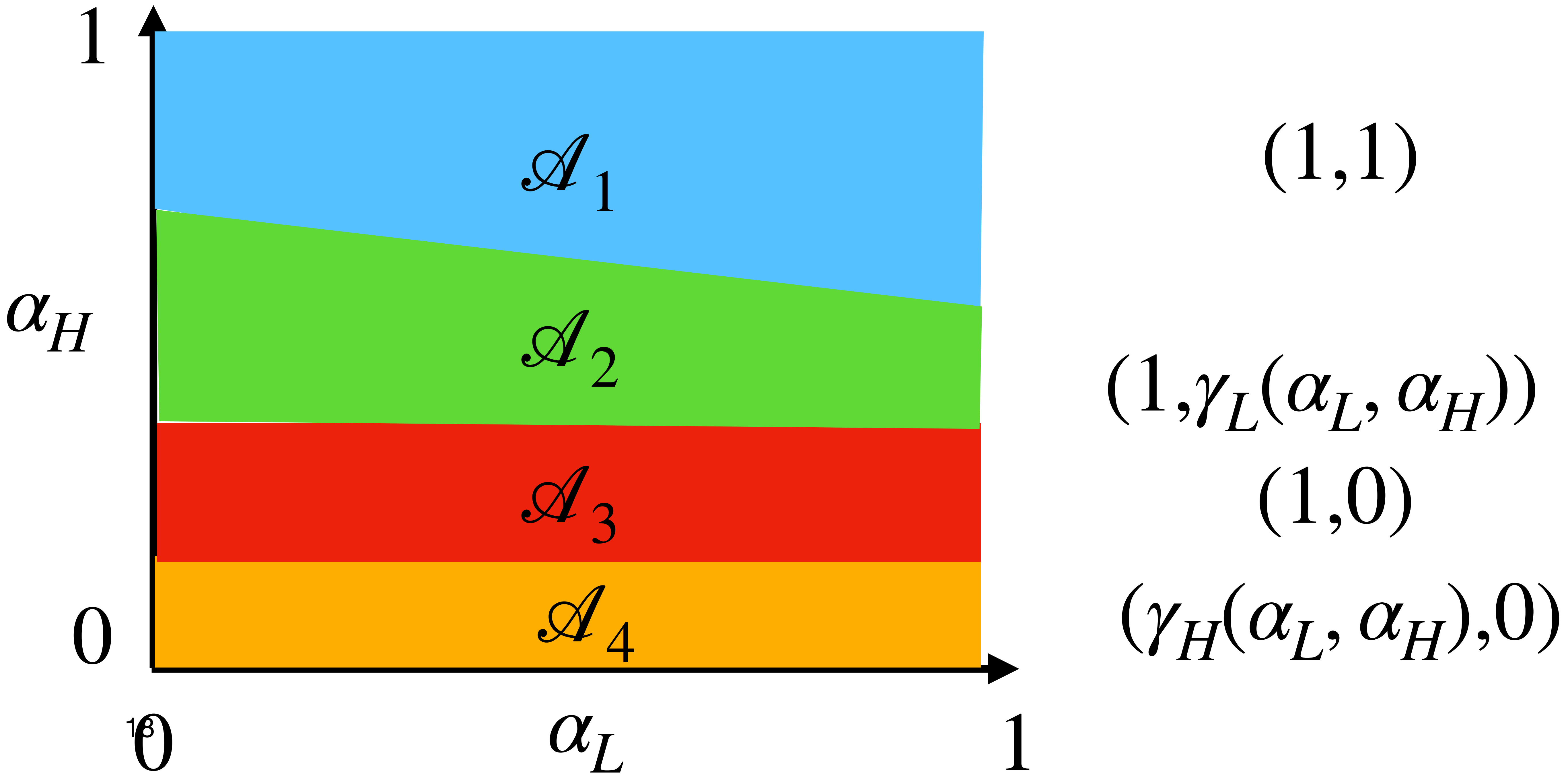

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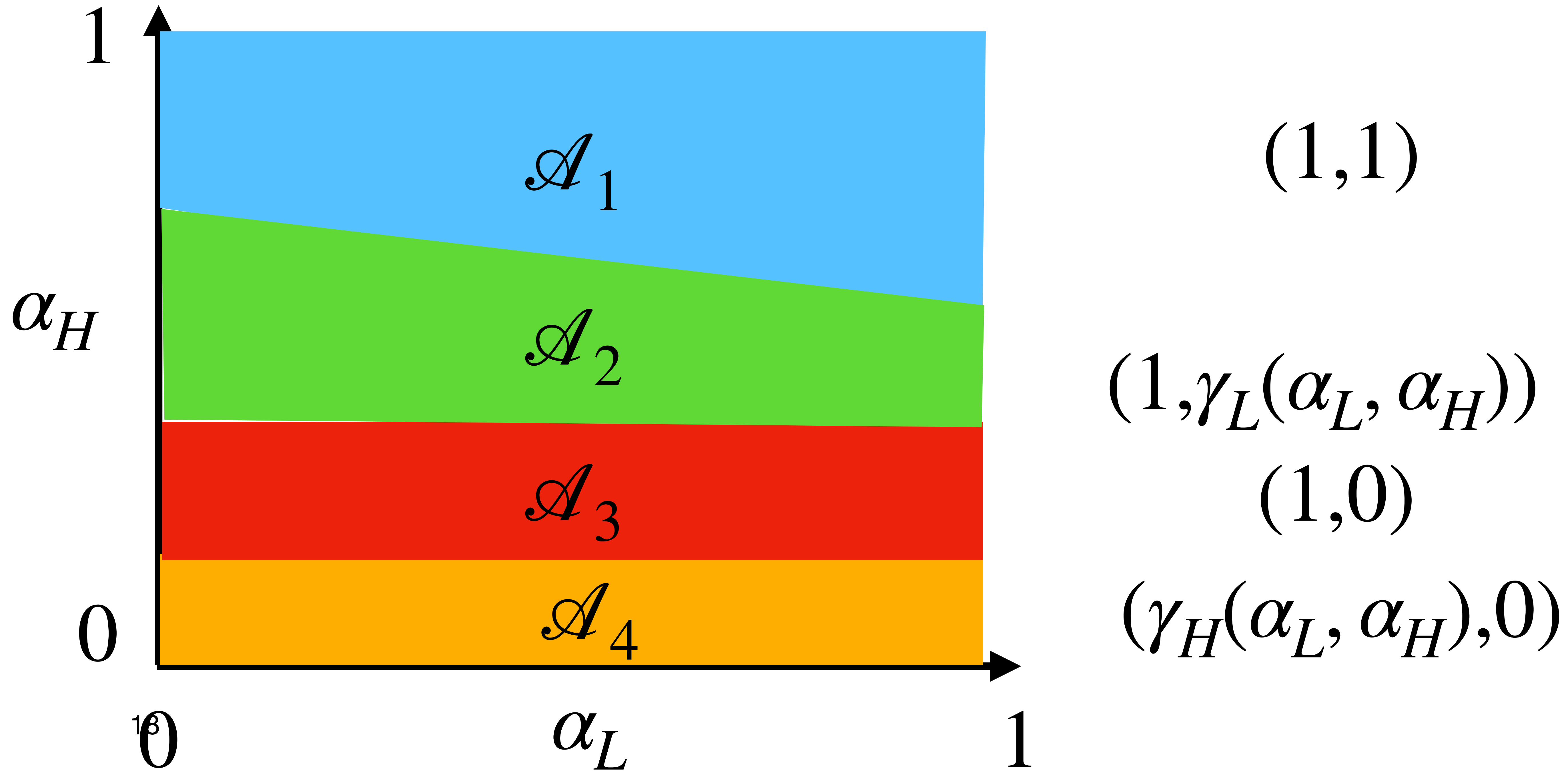

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In the unique symmetric equilibrium, for sufficiently large n, we have:

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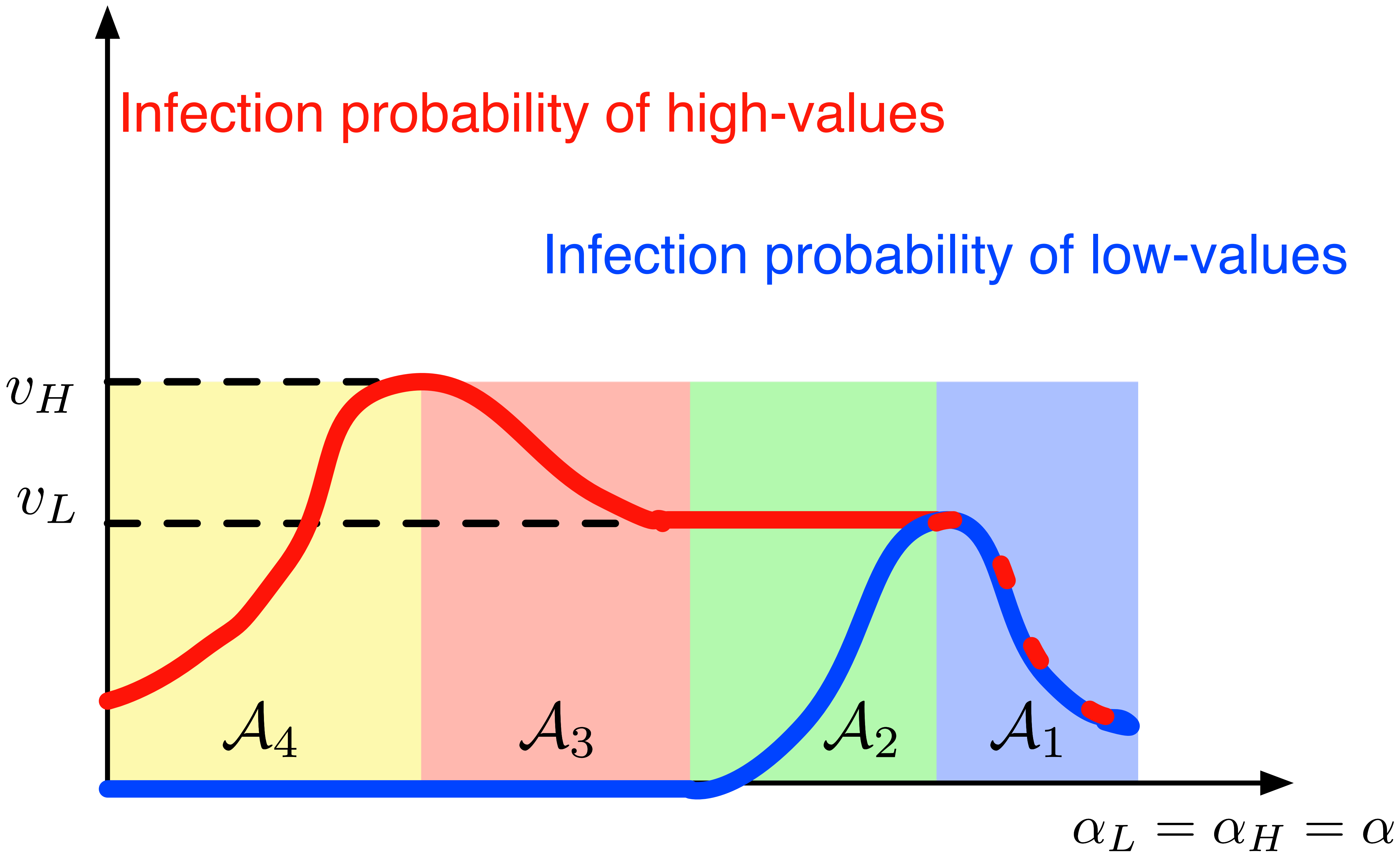
Greater testing increases the infection probability in regions $\mathscr{A}_2$ and $\mathscr{A}_4$ that we have mixed-strategy equilibrium

Impact of testing policy on infection probability

Example: for $k = 2$

- Let us consider region $\mathscr{A}_4$ in which greater testing increases the infection probability

- In this region, low-values play 0 and high-values play mixed



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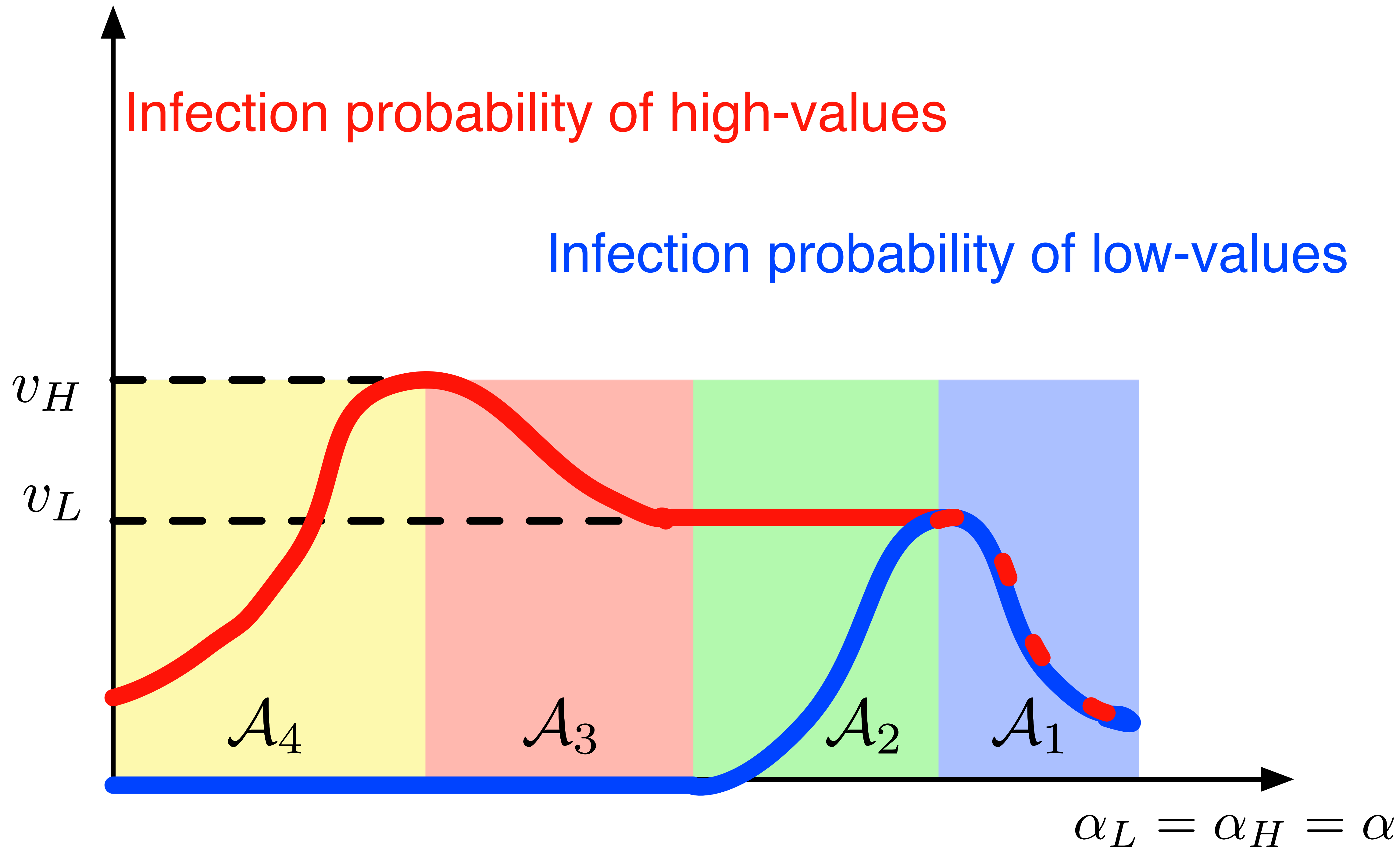

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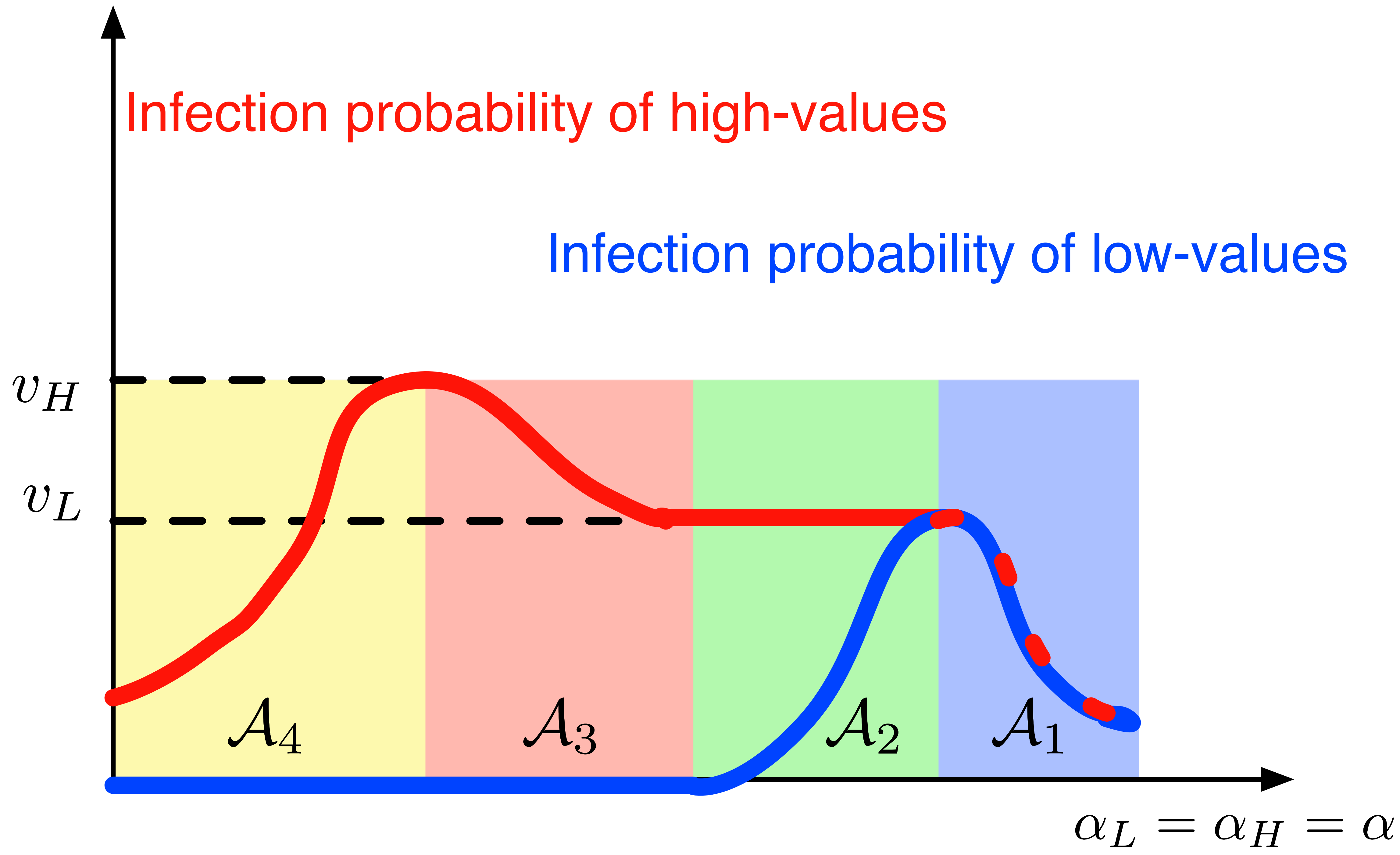

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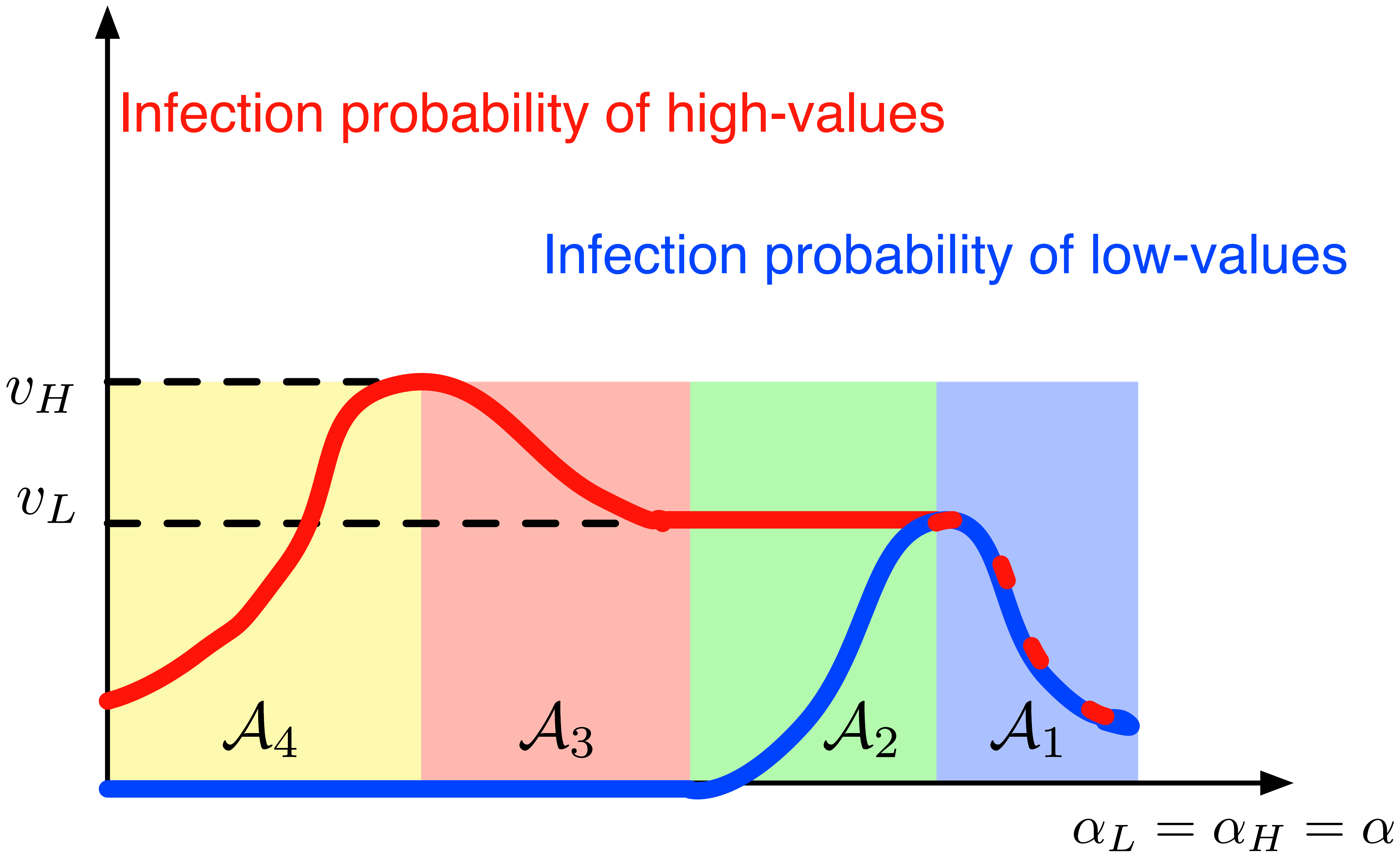

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How about $k > 2$ agent types?

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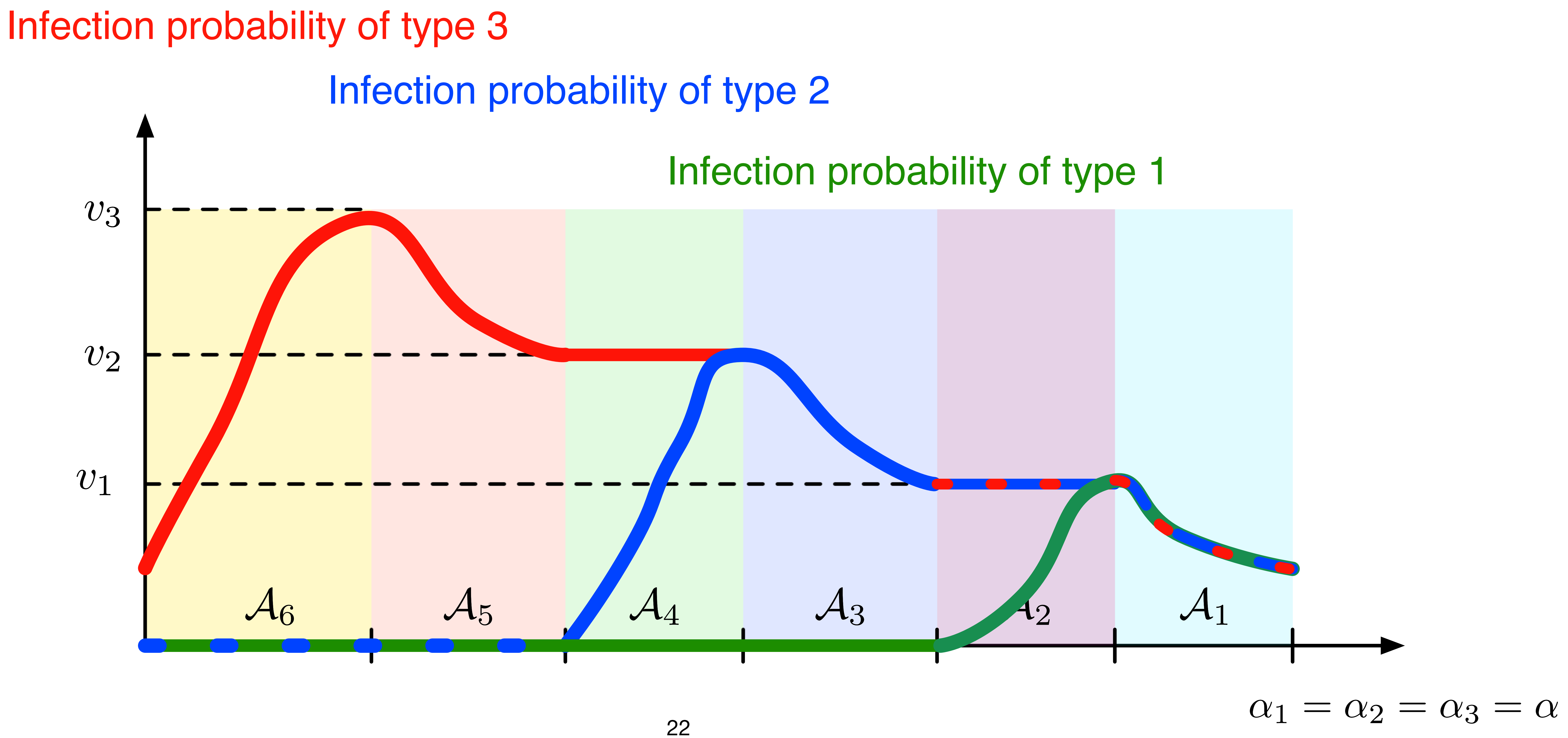
Example: Infection probability as a function of testing probabilities for $k = 3$



So far, we have argued that increasing the testing capacity may increase the infection probability of individuals when we consider their strategic social distancing behavior

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What are the implications of this non-monotonicity for the optimal testing policy?

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There exist three thresholds such that, for sufficiently large n, we have:

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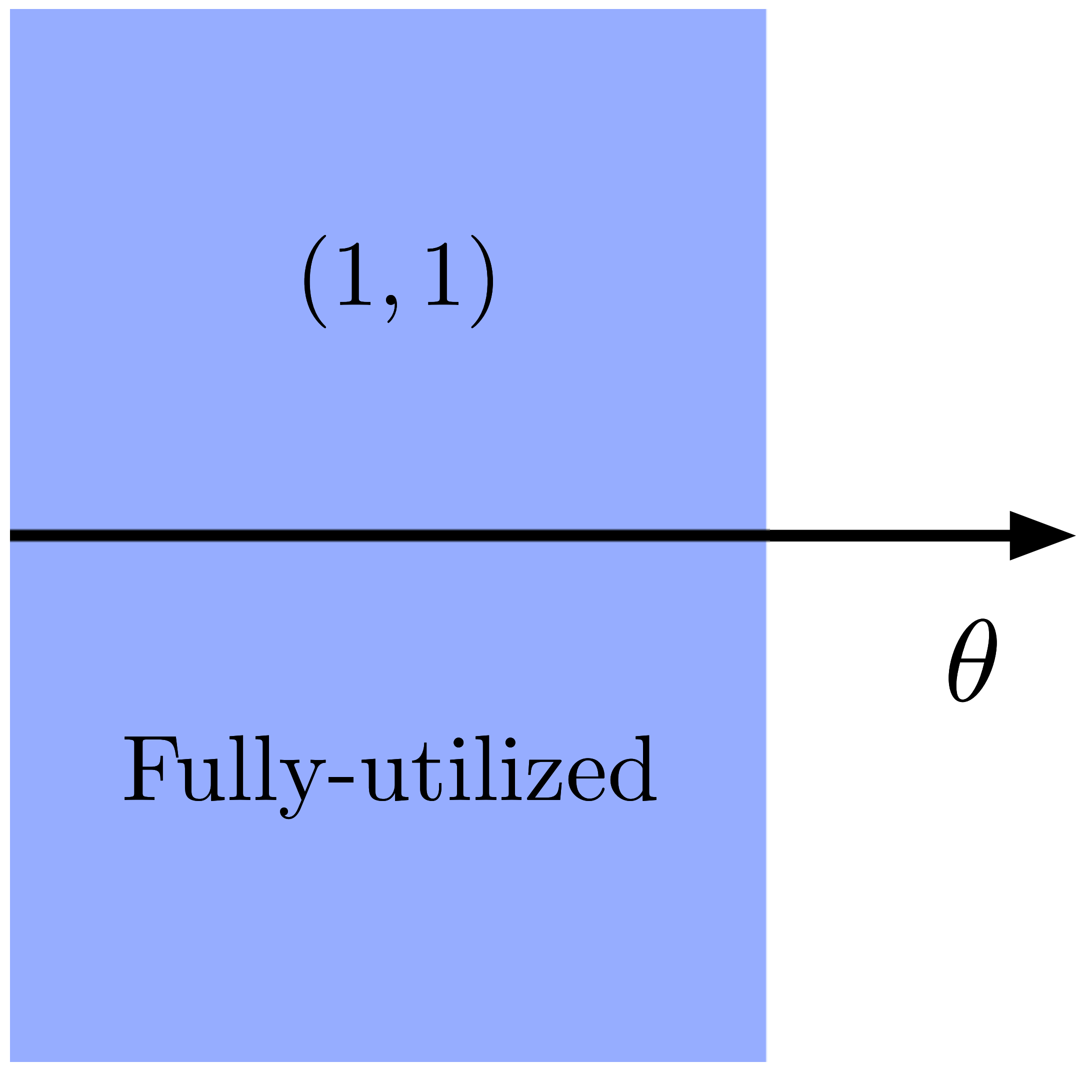

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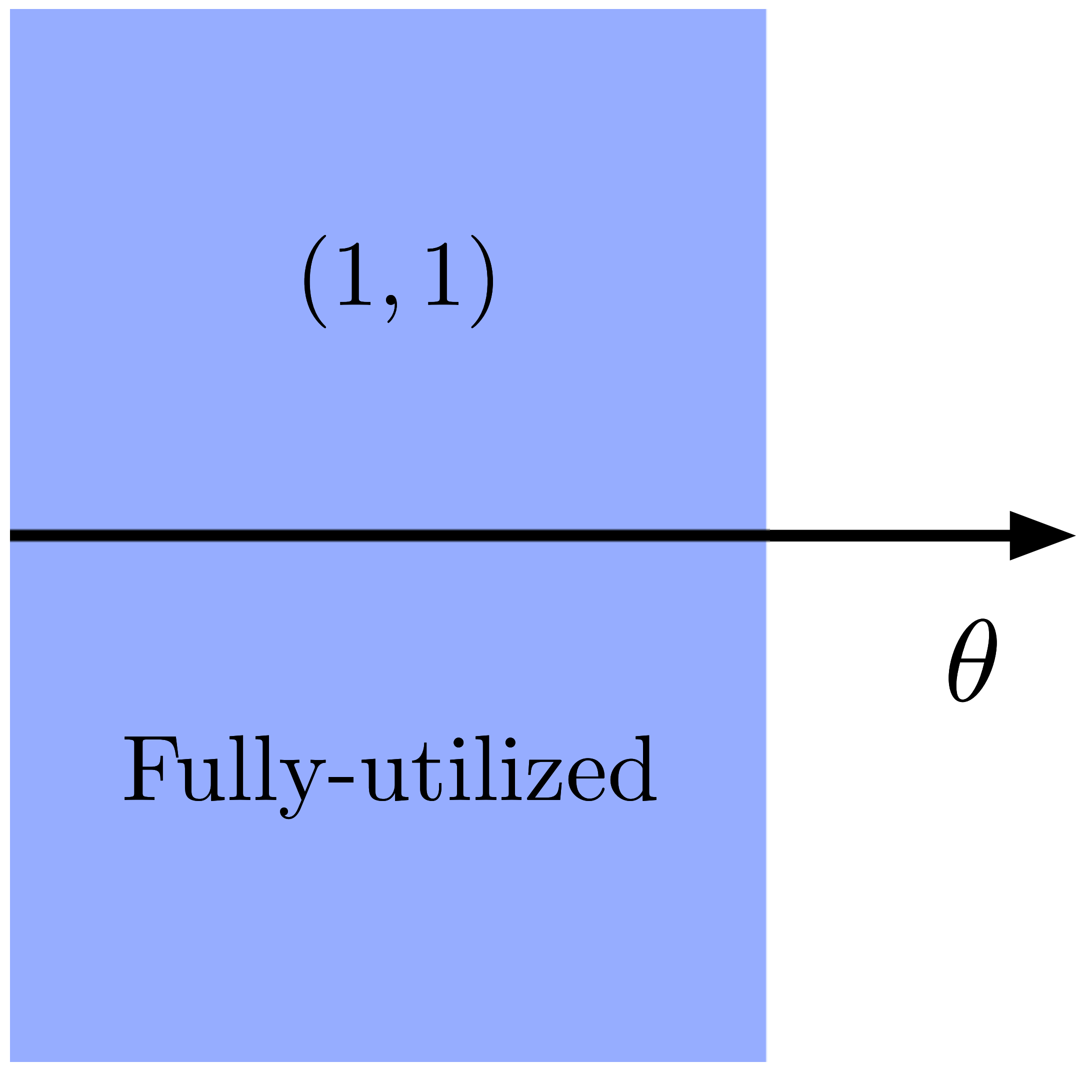

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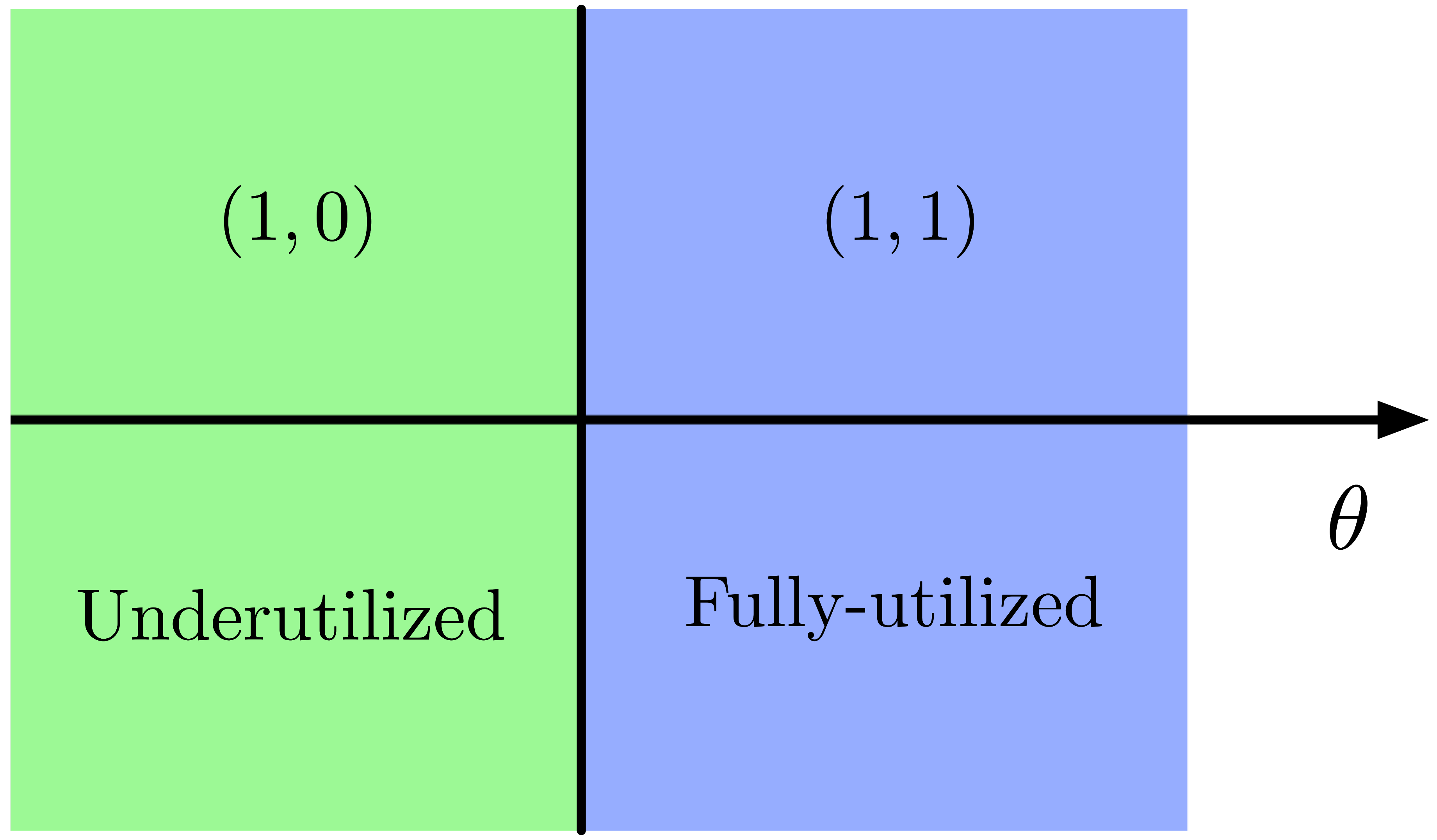

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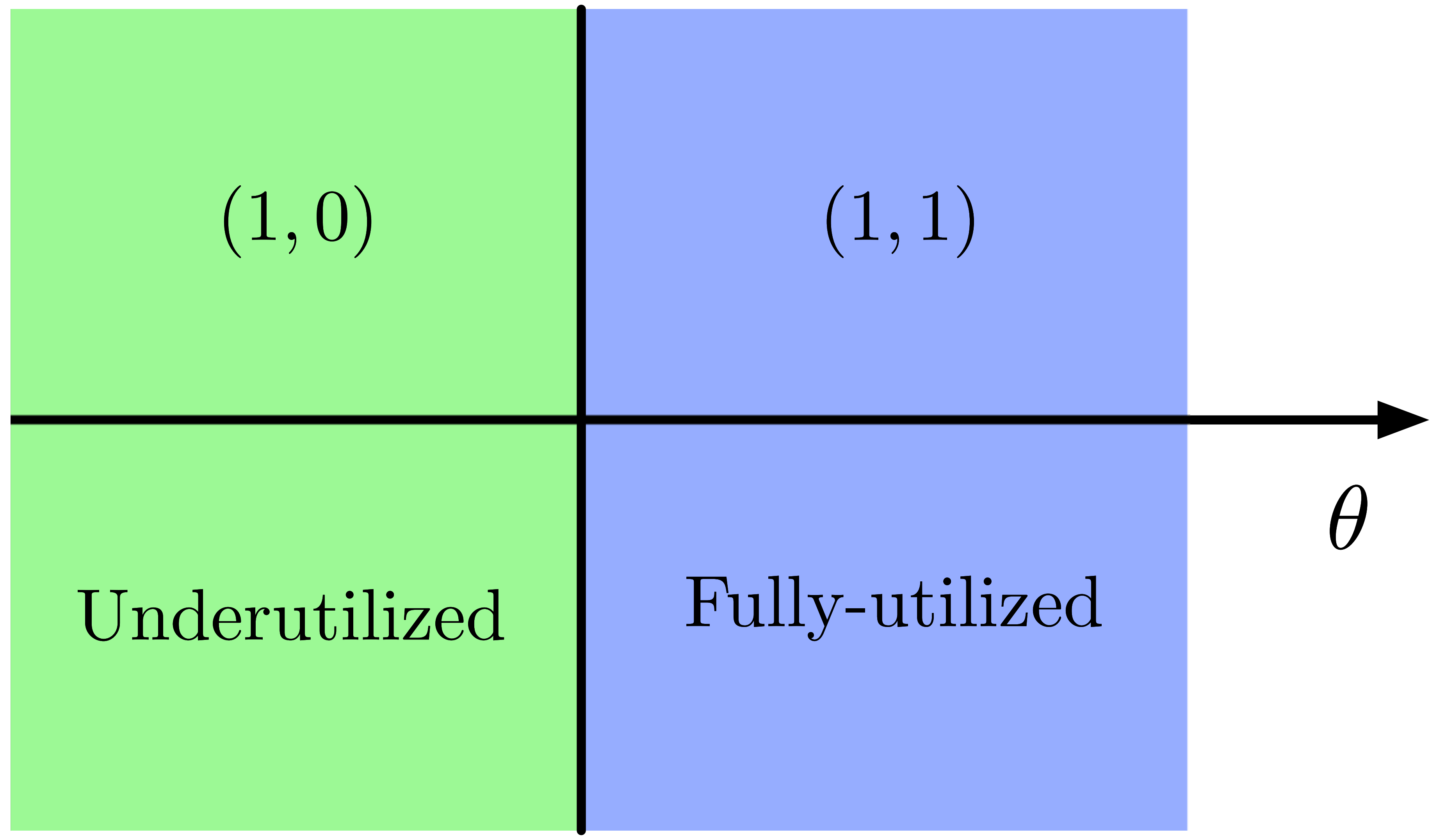

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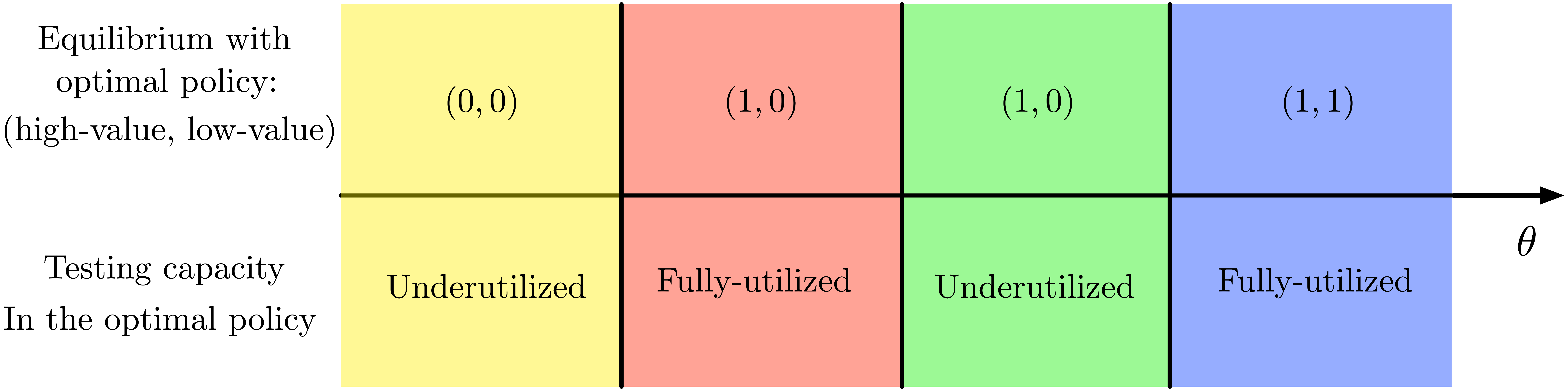

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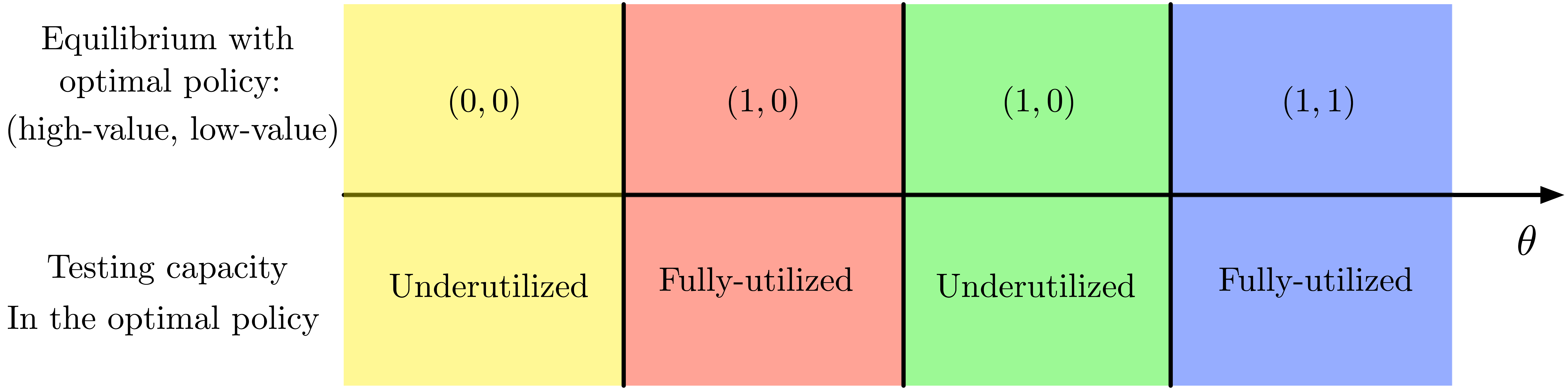

Take away: The optimal testing policy does not necessarily use all tests!

The unwillingness of the social planner to always use all testing capacity is related to the fact that greater testing reduces voluntary social distancing

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This naturally suggests that testing should be combined with mandatory social distancing

Testing policy with mandatory social distancing

- Mandatory social distancing puts an upper bound on the social activity levels

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Proposition

For any testing capacity θ, there exists a testing policy with mandatory social distancing that achieves the social welfare of the first best. Moreover, with this policy the social planner uses all the testing capacity.

Conclusion

- We develop a model of testing, social activity, and voluntary social distancing
  - Social activity levels determine the endogenous contact network over which infection spreads
- Testing enables authorities to identify and isolate infected individuals who spread the virus
- Our analysis, however, shows the impact of testing on the spread of an infection is more complex because, knowing tests lead to isolation of infected ones, agents increase their social activity level
- Because of users strategic behavior greater testing can lead to higher infection probability and the optimal testing policy may not use all testing capacity $\Rightarrow$ testing should be combined with mandatory social distancing
- **Modeling take-aways**
  - To conceptualize the problem of endogenous social distancing behavior as one of social network formation
  - To use the (variation of) independent cascade model to account for a general contact network and testing

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Thanks!

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- Testing enables authorities to identify and isolate infected individuals who spread the virus
- Our analysis, however, shows the impact of testing on the spread of an infection is more complex because, knowing tests lead to isolation of infected ones, agents increase their social activity level
- Because of users strategic behavior greater testing can lead to higher infection probability and the optimal testing policy may not use all testing capacity $\Rightarrow$ testing should be combined with mandatory social distancing
- **Modeling take-aways**
    - To conceptualize the problem of endogenous social distancing behavior as one of social network formation
    - To use the (variation of) independent cascade model to account for a general contact network and testing