Towards Characterizing Efficient Boolean Functional Synthesis

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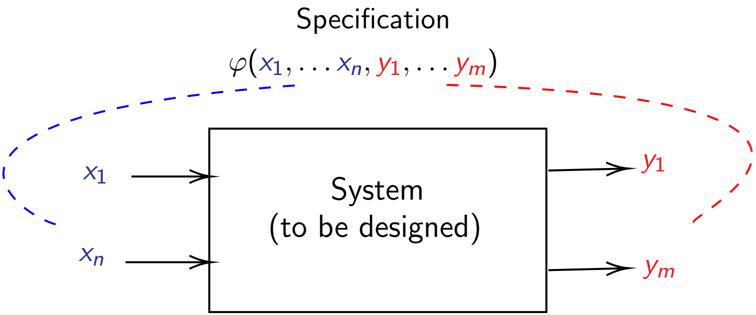
Joint work with S. Akshay, Jatin Arora, Aman Bansal, Divya Raghunathan, Preey Shah, Shetal Shah, Krishna S.

Several interesting discussions with Kuldeep Meel and Dror Fried

SAT: Theory, Practice and Beyond Reunion Workshop, June 2022

Synthesis: Recap from previous talk



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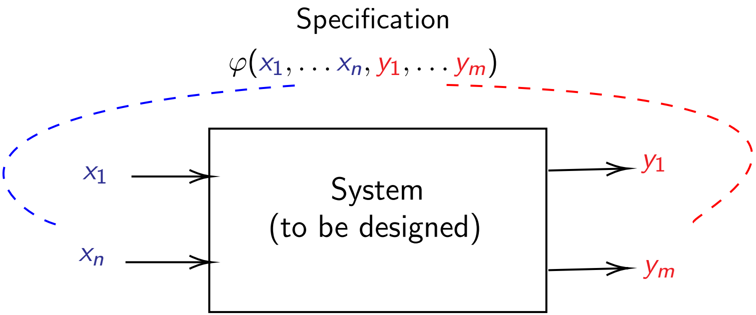

- Goal: Automatically **synthesize system** s.t. it satisfies $\varphi(x_1, .., x_n, y_1, .., y_m)$
  - x_j *input* variables (vector X)
  - y_k *output* variables (vector Y)

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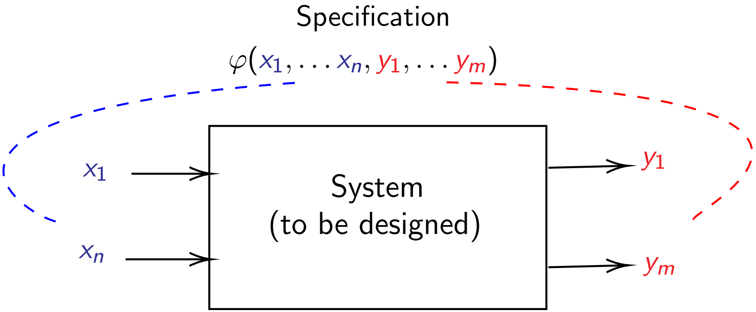

- Goal: Automatically **synthesize system** s.t. it satisfies $\varphi(x_1, .., x_n, y_1, .., y_m)$ **whenever possible**.
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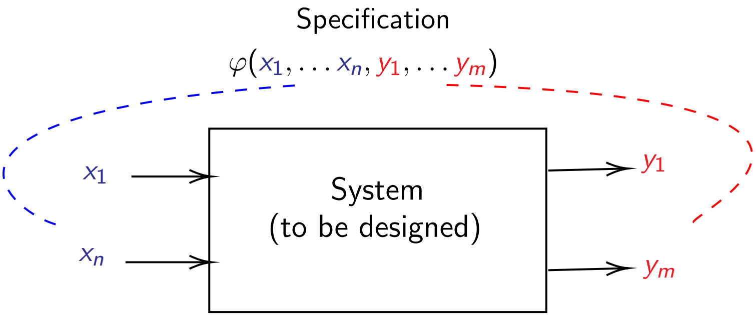

- Goal: Automatically **synthesize system** s.t. it satisfies $\varphi(x_1, .., x_n, y_1, .., y_m)$ **whenever possible**.
  - x_j *input* variables (vector X)
  - y_k *output* variables (vector Y)
- Express $y_1, \ldots y_m$ as $F_1(\mathsf{X}), \ldots F_m(\mathsf{X})$ s.t. $\varphi(\mathsf{X}, \mathsf{F}(\mathsf{X}))$ is satisfied.

Boolean Functional Synthesis (BFnS)

Formal definition

Given Boolean relation $\varphi(x_1, .., x_n, y_1, .., y_m)$

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$$\forall \mathrm{X}(\ \exists y_1 \dots y_m\ \varphi(\mathrm{X}, y_1 \dots y_m) \ \Leftrightarrow\ \varphi(\mathrm{X}, F_1(\mathrm{X}), \dots F_m(\mathrm{X}))\ )$$

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$F_j(X)$ is also called a *Skolem function* for y_j in φ.

How Hard is Boolean Skolem Function Synthesis?

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Representation

Spec $\varphi(X, Y)$ & Skolem functions $F(X)$: NNF Boolean circuits

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- Unless $\Pi_2^P = \Sigma_2^P$, there exist $\varphi(X, Y)$ for which Skolem function sizes are super-polynomial in $|\varphi|$.

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Silver lining: [Akshay+'18,'19]

- Solvable in polynomial (in $|\varphi|$) time and space if φ is represented in special normal forms

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- Construct *new spec* $\varphi'(X, y_m) \equiv \exists y_1 \ldots y_{m-1}\ \varphi$
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- Synthesize $F_{m-1}(X, y_m)$ for y_{m-1}; substitute $F_m(X)$ for y_m
- Repeat ...

Why Do Some Representations Help?

Efficient computation of
$\exists y_1 \dots y_i \; \varphi(X, y_1, \dots y_m)$
$\forall i \in \{1, \dots m\}$

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Efficient computation of
Sk fn $F_i(X)$ in $\varphi(X, y_1, \dots y_m)$
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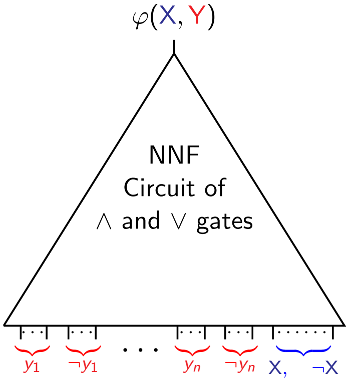
then it also enables this

Existential Quantification with NNF circuits



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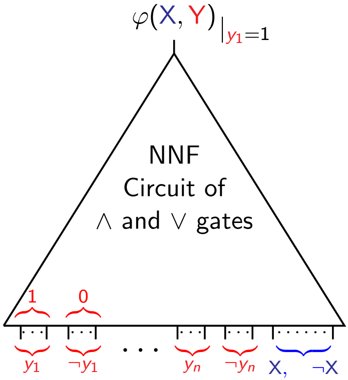

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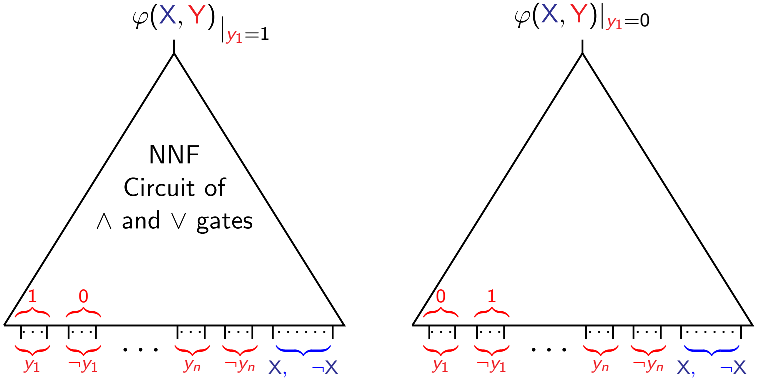

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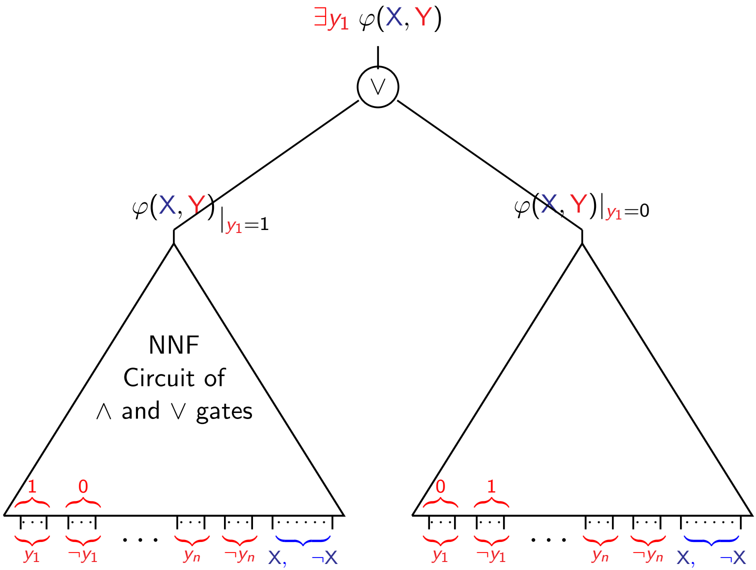

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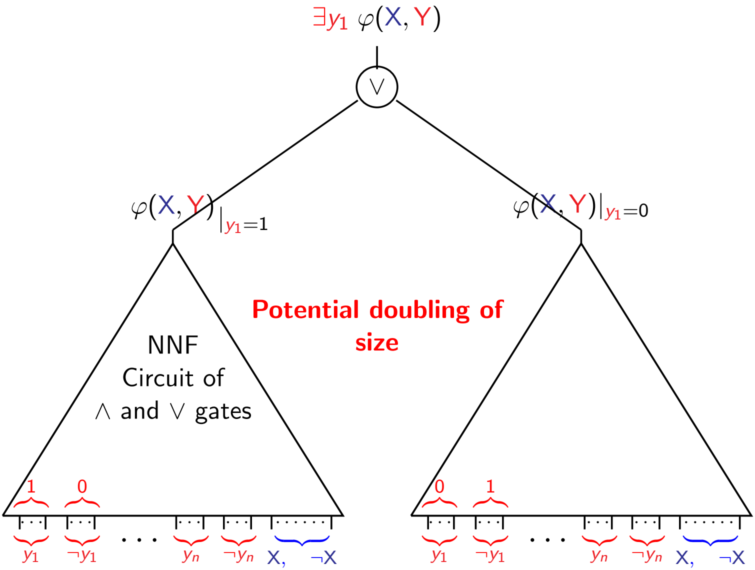
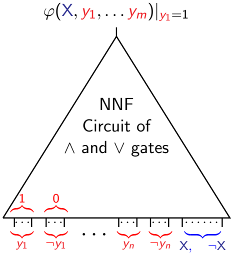
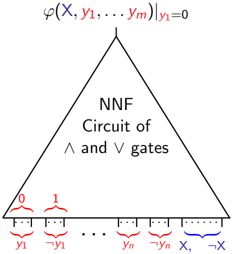


Over-approximating $\exists y_1\ \varphi(X, Y)$ Sans Doubling





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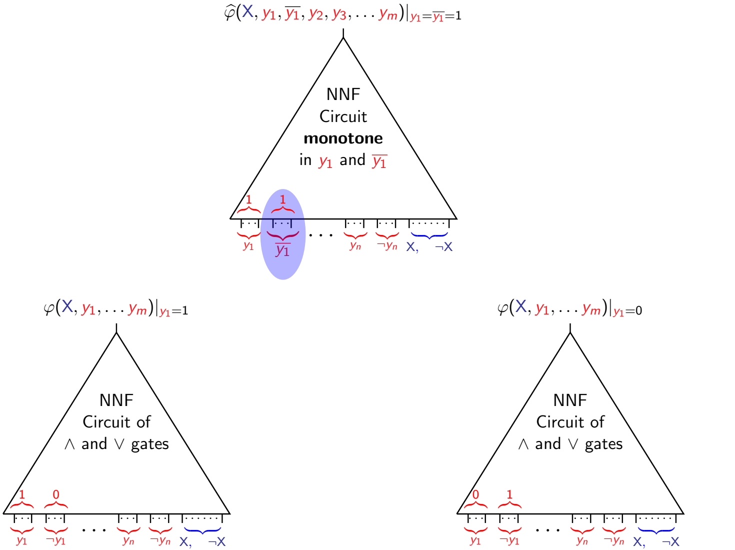

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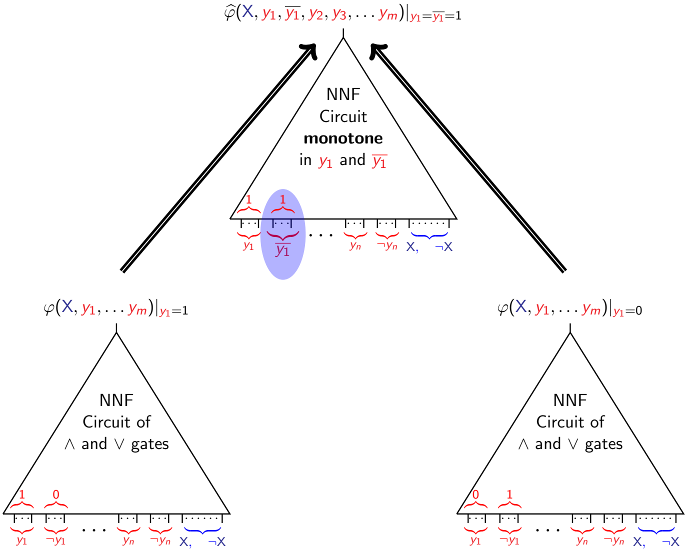

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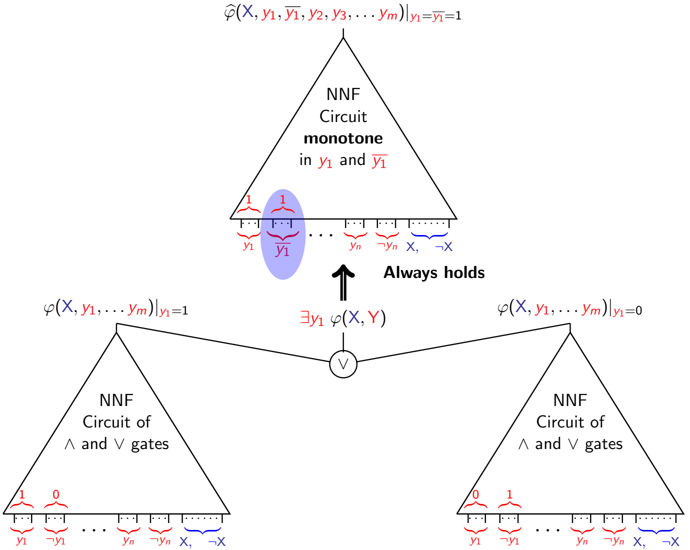
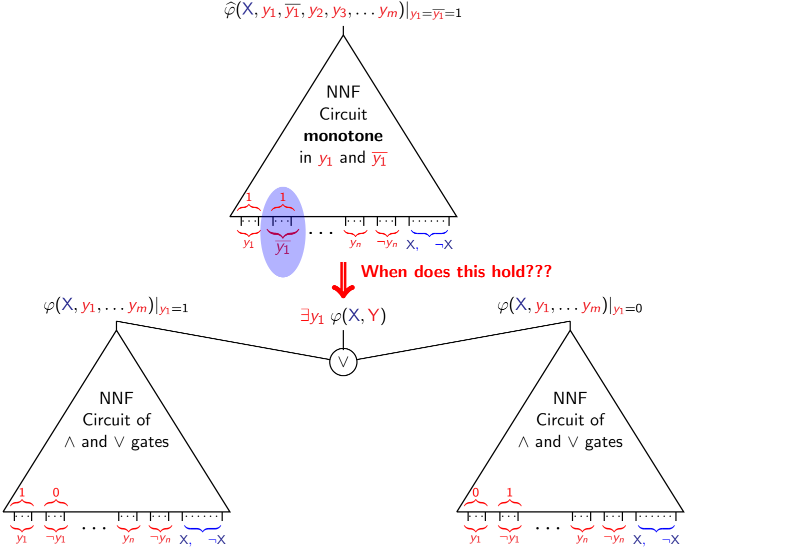


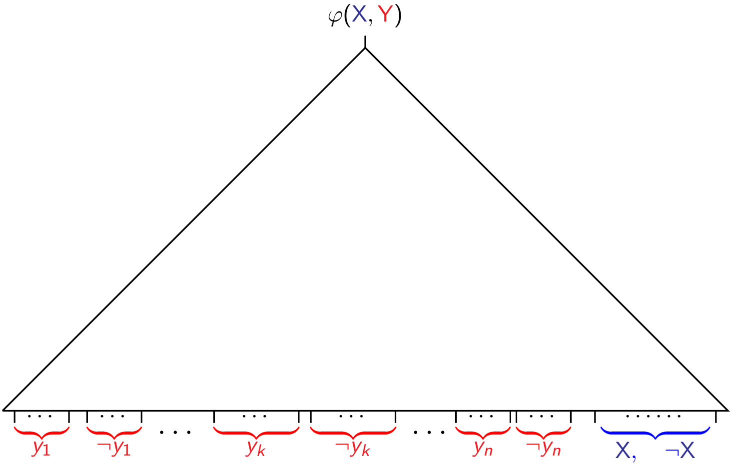
Can We Represent Quantification **Exactly** sans Blow-up?



Special Normal Forms

Decomposable Negation Normal Form (DNNF): Forbidden **structure**

[Darwiche, JACM 2001]



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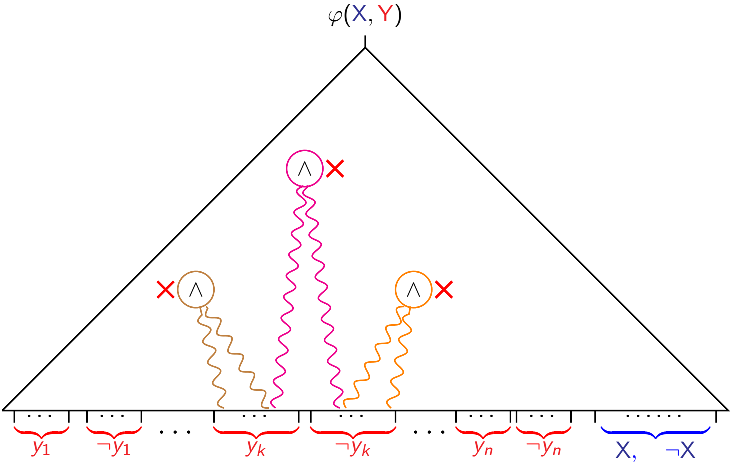

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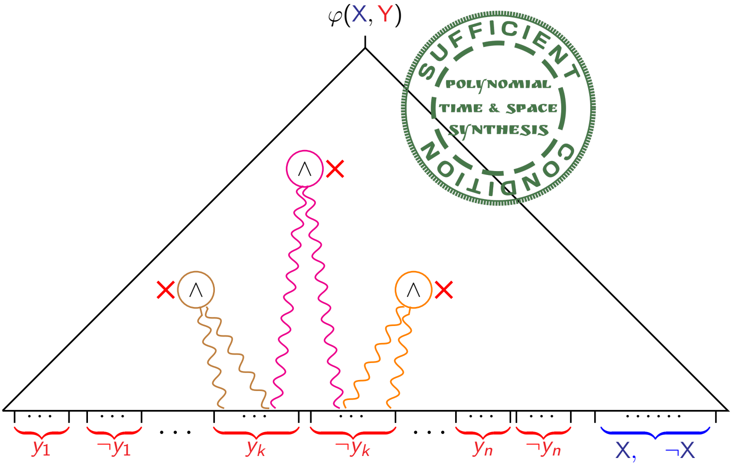

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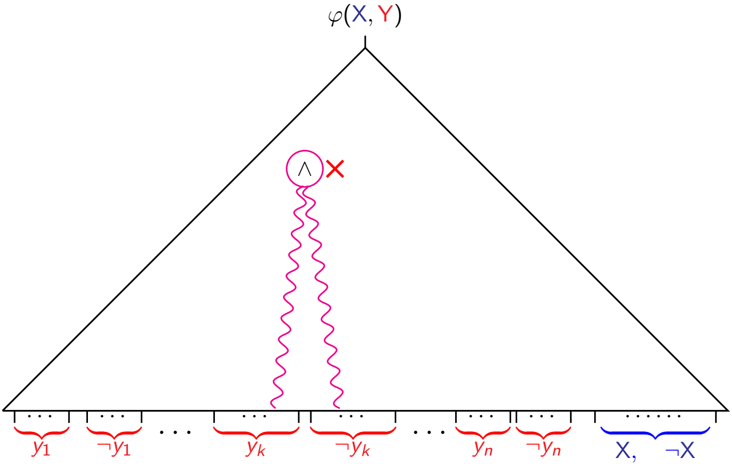
Weak DNNF (wDNNF): Forbidden **structure**

[Akshay,Chakraborty,Goel,Kulal,Shah CAV18, FMSD20]



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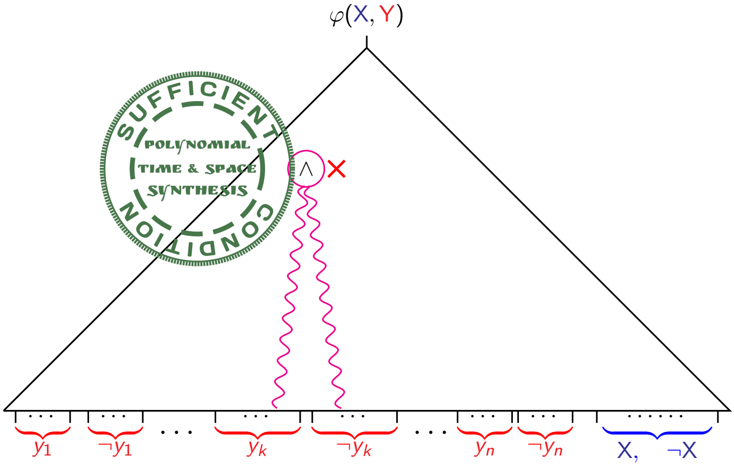
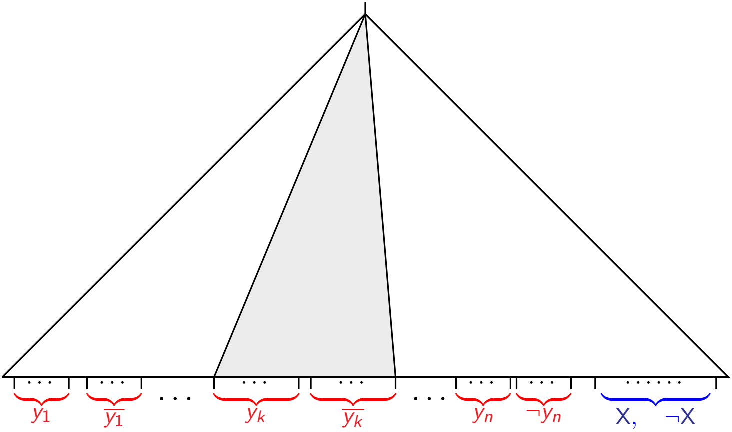


Special Normal Forms

Synthesis Negation Normal Form (SynNNF): Forbidden **semantics**

[Akshay,Arora,Chakraborty,Krishna,Raghunathan,Shah FMCAD19]

$\varphi(X, Y) \not\Leftarrow (y_k \wedge \overline{y_k})$



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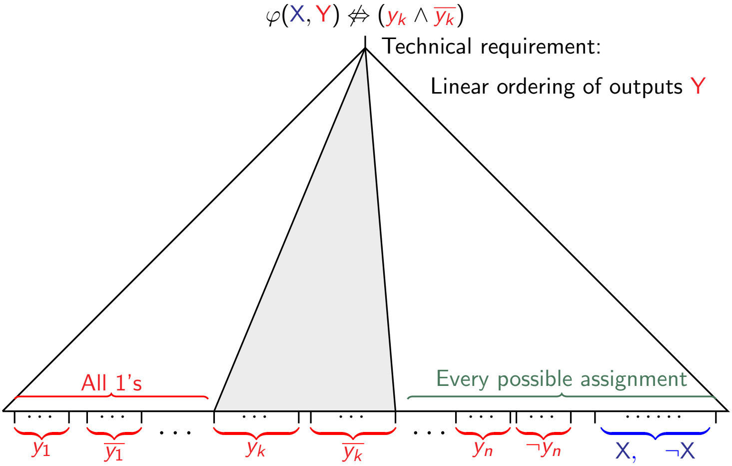

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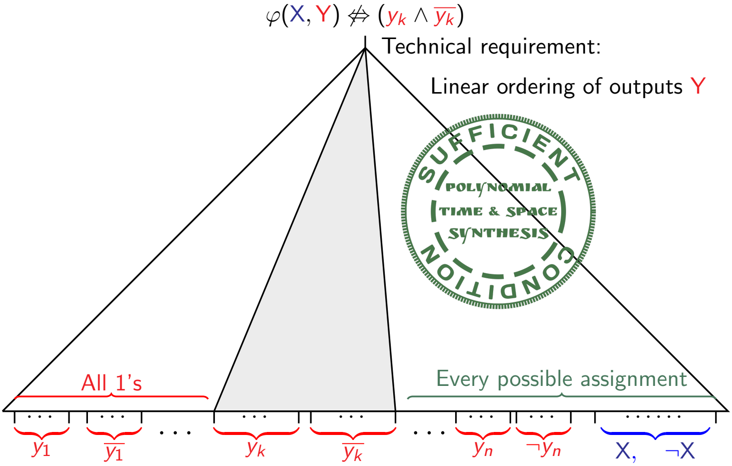

Can we get necessary & sufficient condition?

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Characterizing poly-time and poly-size BFnS

Does there exist a "semantically universal" class $\mathcal{C}^\star$ of ckts s.t.:

P1 : BFnS is poly-time for $\mathcal{C}^\star$

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P2 : For every class $\mathcal{C}$ of ckts:

1. BFnS is poly-time for $\mathcal{C}$ iff $\mathcal{C}$ compiles to $\mathcal{C}^\star$ in poly-time.

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2. BFnS is poly-size for $\mathcal{C}$ iff $\mathcal{C}$ compiles to poly-size ckts in $\mathcal{C}^\star$

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  2. BFnS is poly-size for $\mathcal{C}$ iff $\mathcal{C}$ compiles to poly-size ckts in $\mathcal{C}^\star$

Our Main Result

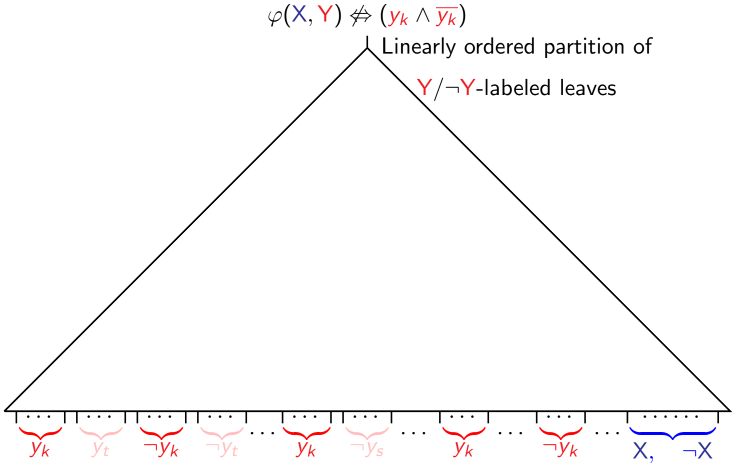
Yes, there exists such a class!

Subset-And-Unrealizable Normal Form (SAUNF)

SAUNF: A Very Special Normal Form

Generalizing forbidden **semantics** of SynNNF

[Shah,Bansal,Akshay,Chakraborty LICS21]



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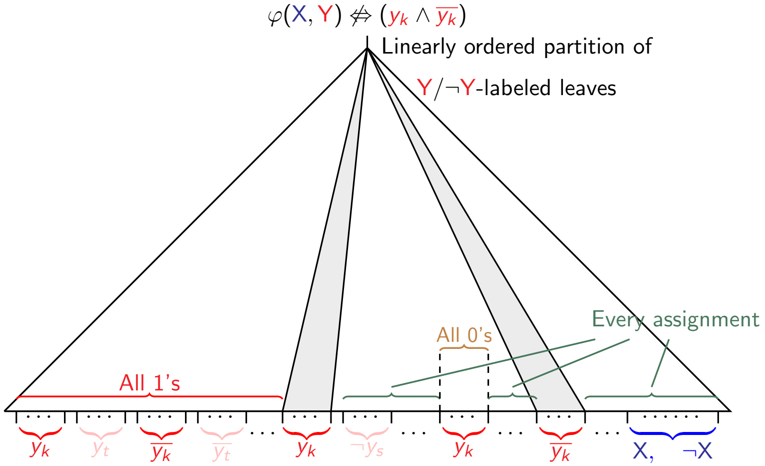

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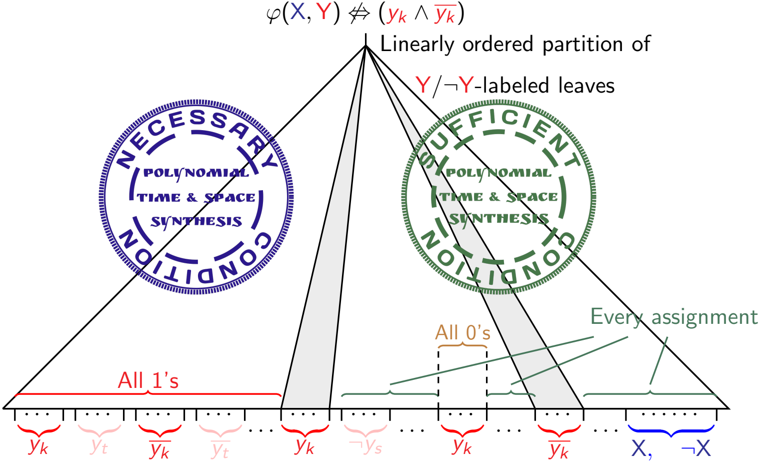

SAUNF vs Existing Popular Normal Forms

Proposition

- Every SynNNF, wDNNF, DNNF circuit is also in SAUNF.
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Proposition

SAUNF is exponentially more succinct than DNNF/dDNNF, which are themselves exponentially more succinct than ROBDDs/FBDD.

Properties of SAUNF

Operations on SAUNF

Given $\varphi_1(X, Y)$ and $\varphi_2(X, Y)$ in SAUNF

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  - Quantifying subset of Y not possible in linear time in general.

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- Computing $\varphi_1 \wedge \varphi_2$ in SAUNF
  - Takes constant time if every pair of Y-labeled leaves of φ_1 and φ_2 are consistent.
  - Otherwise,
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- Existentially quantifying $y_1, \ldots y_m$ takes linear time.
  - Quantifying subset of Y not possible in linear time in general.

Checking if a given specification is in SAUNF

- Is Co-NP complete, given linearly ordered partition of Y-labeled leaves

Properties of SAUNF

Operations on SAUNF

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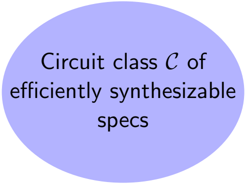
Compiling Other Circuit Classes of Specs to SAUNF



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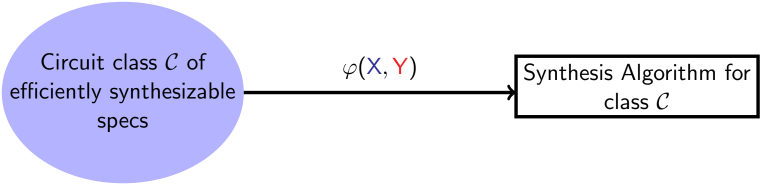

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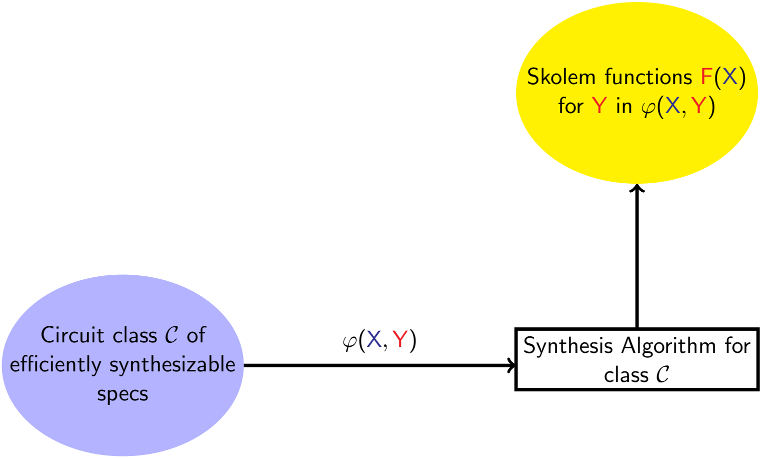

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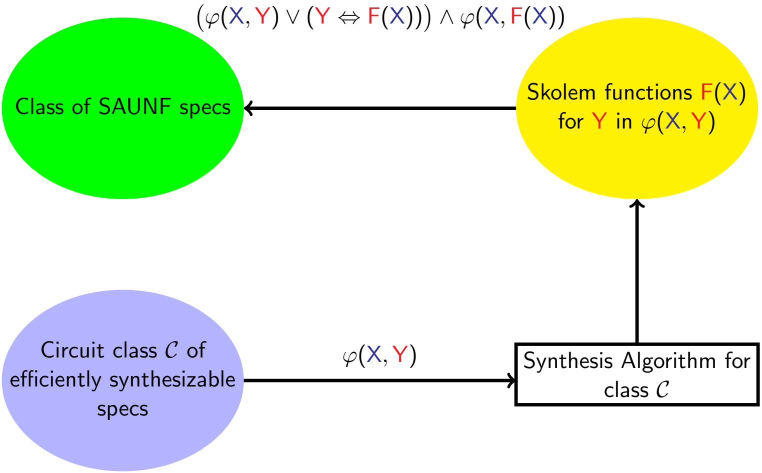

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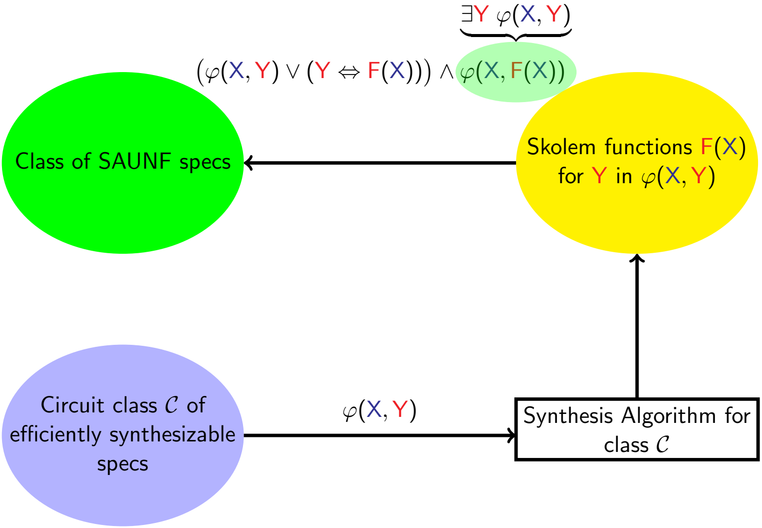

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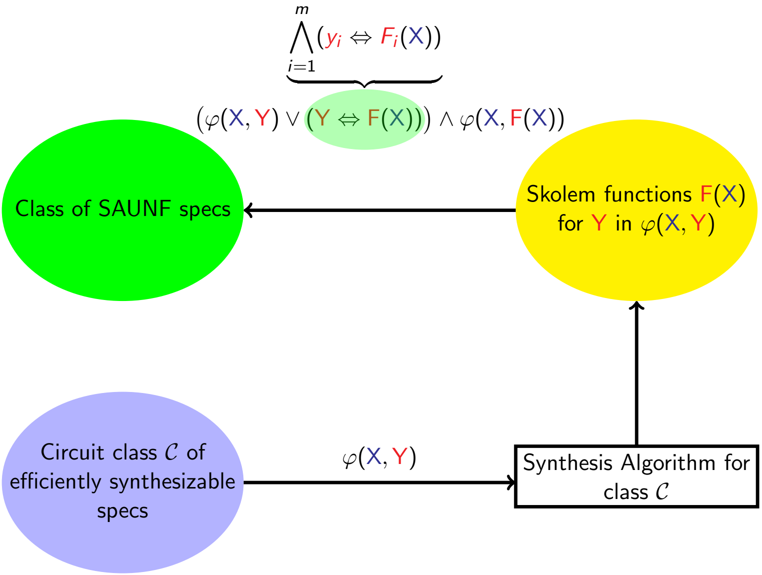

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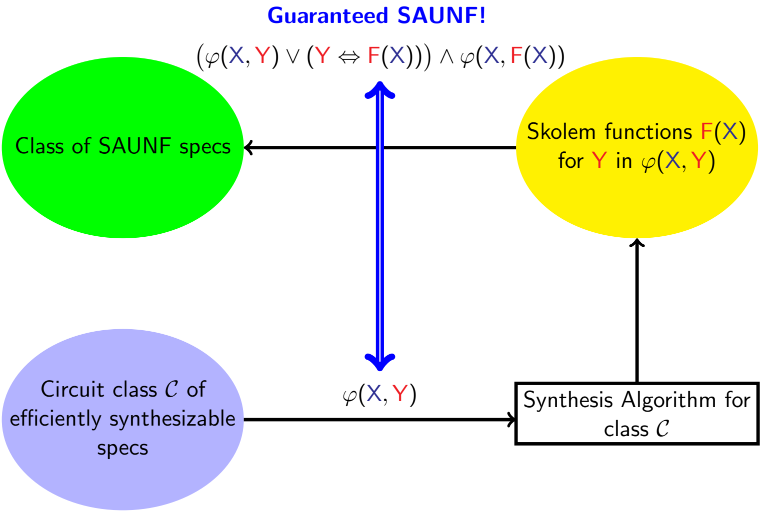

More about compilation to SAUNF

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Compiling CNF to SAUNF [Shah et al LICS21]

- We give an algorithm to compile a CNF formula into SAUNF
- Worst-case exponential-time and space
  - Unavoidable due to hardness results
- Future work: Implementation and comparisons!

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- For $1 \leq \ell \leq j \leq 2n$ and $j - \ell < n$, the spec $\varphi_{\ell,j}(X, Y_1, Y_2)$ representable by a poly-sized SAUNF circuit.
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- Does not solve factorization (yet!)

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- A new normal form (SAUNF) that characterizes poly-time/size Boolean Skolem function synthesis.
- Many beautiful and useful properties.
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- Going beyond the Boolean case!

Thank you!