Single-Server Private Information Retrieval with Sublinear Amortized Time

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Alexandra Henzinger — MIT
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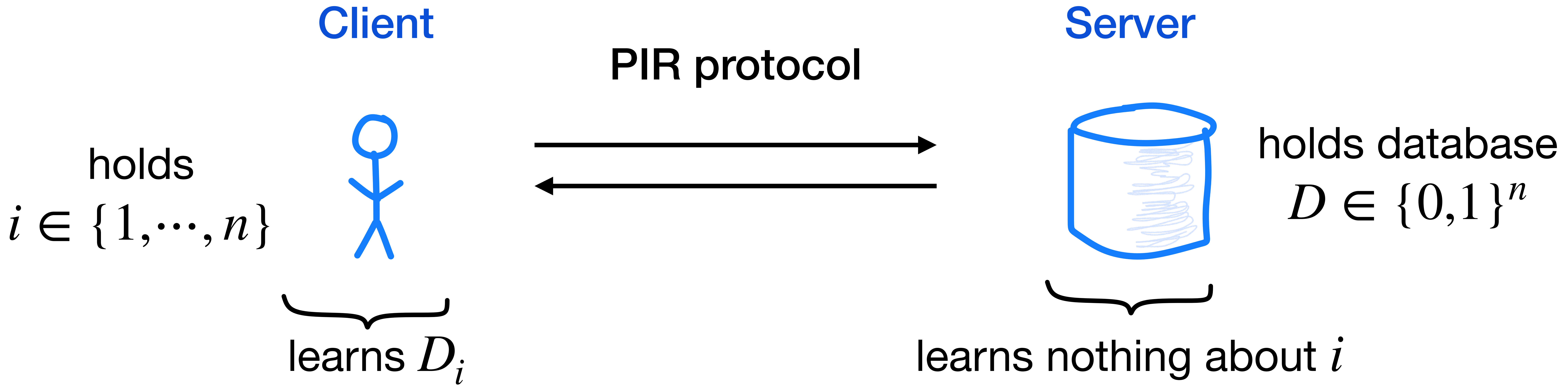
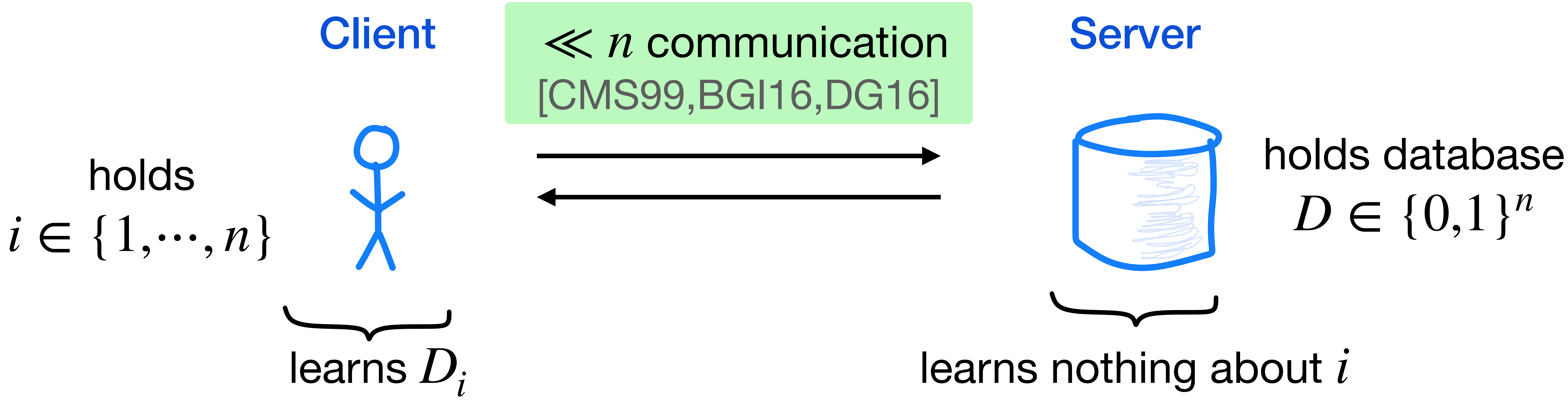
Private information retrieval [CGKS95,KO97]



Applications: private media [GCMSAW16], private e-commerce [HOG11], private ads [J01…], private web browsing [KC21], metadata-hiding messaging [AS16…], …

Private information retrieval [CGKS95,KO97]



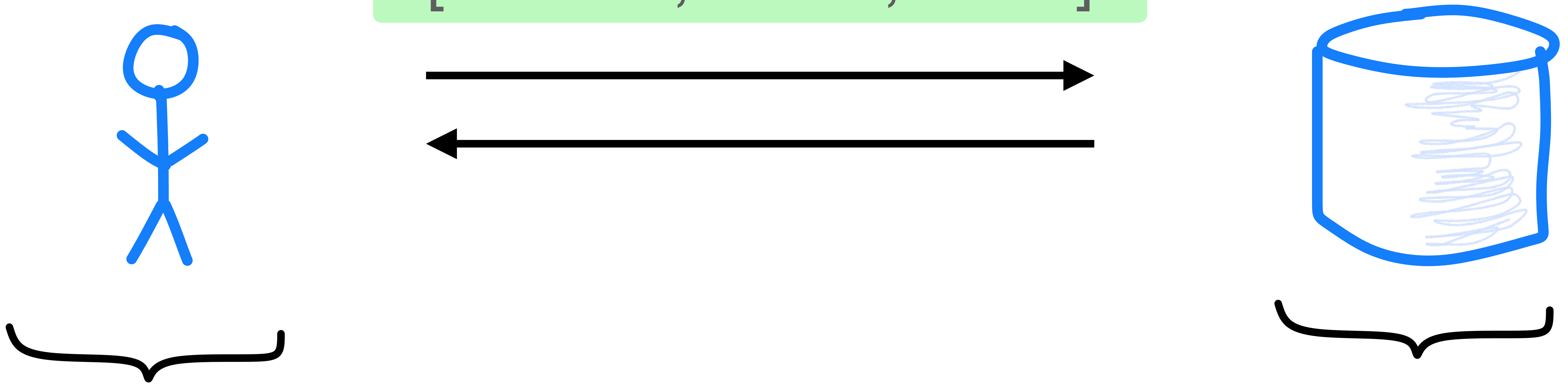
Private information retrieval [CGKS95,KO97]

Client

$\ll n$ communication
[CMS99,BGI16,DG16]

Server

holds
$i \in \{1, \cdots, n\}$



holds database
$D \in \{0,1\}^n$

learns D_i

learns nothing about i

PIR inherently has **high server-side computation** costs [BIM04]:
To answer a single query, the server(s) must run in time n.

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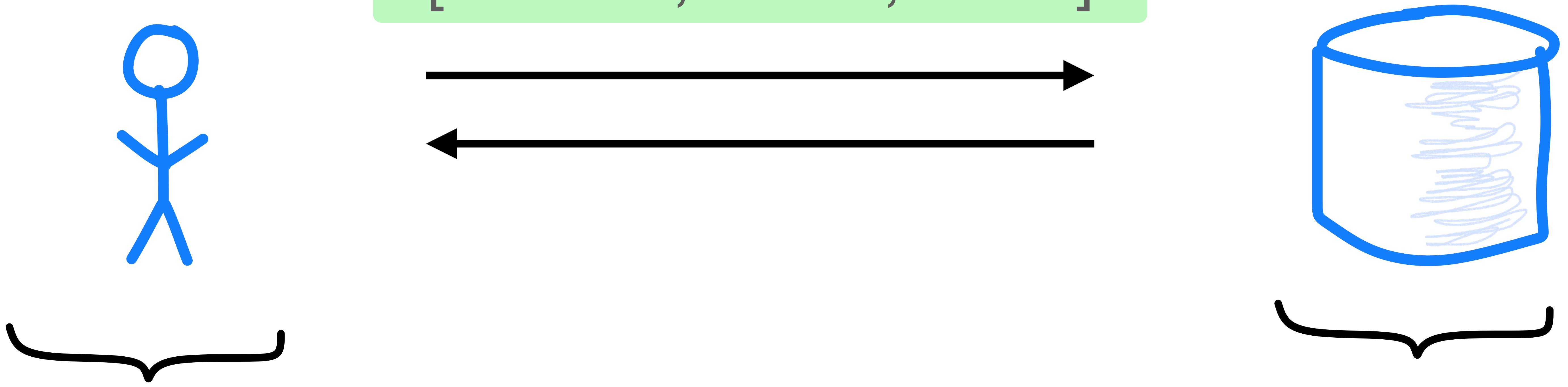

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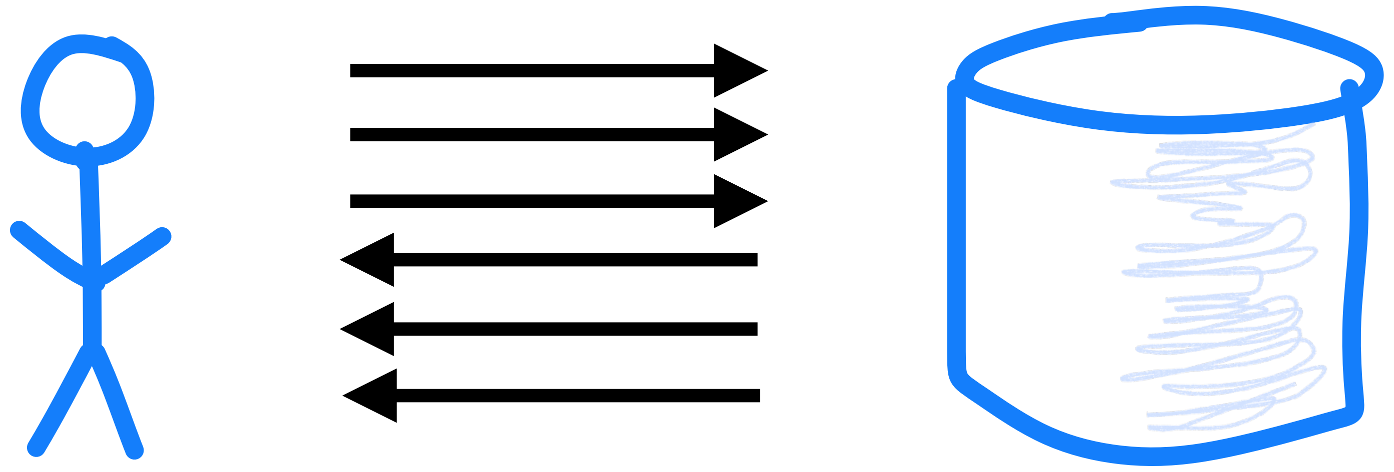
To answer a single query, the server(s) must run in time n.

Idea: Amortize the server time over <u>many</u> queries [BIM04,IKOS04]

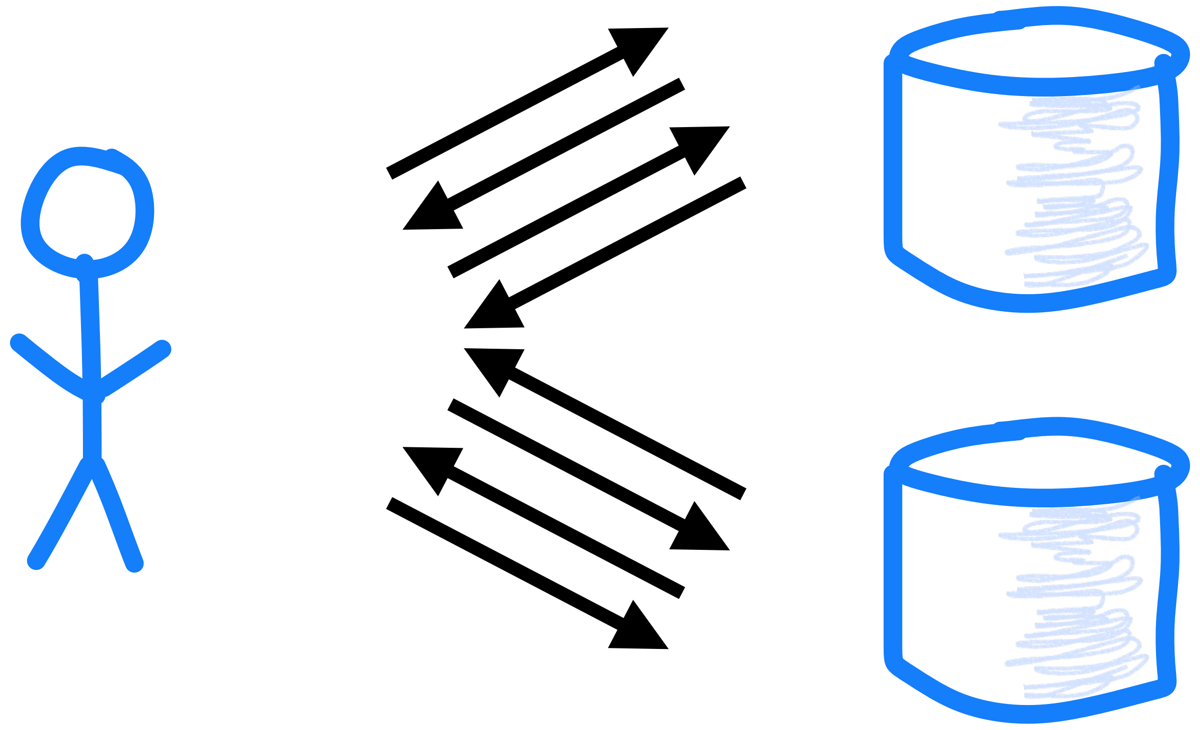
Existing PIR with sublinear time

Batch PIR with non-adaptive queries



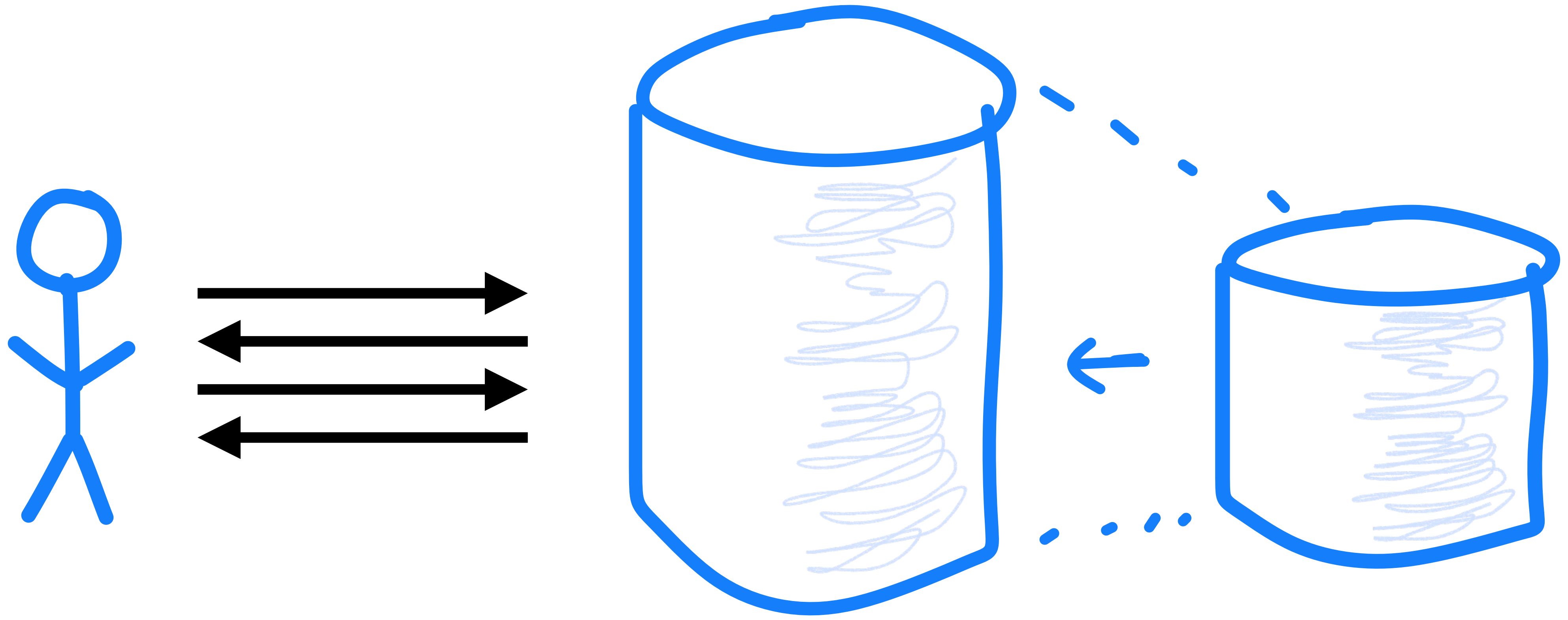
[IKOS04,HHG13,GKL10,AS16,H16,ACLS18,CHLR18]

Offline/online PIR with 2 servers



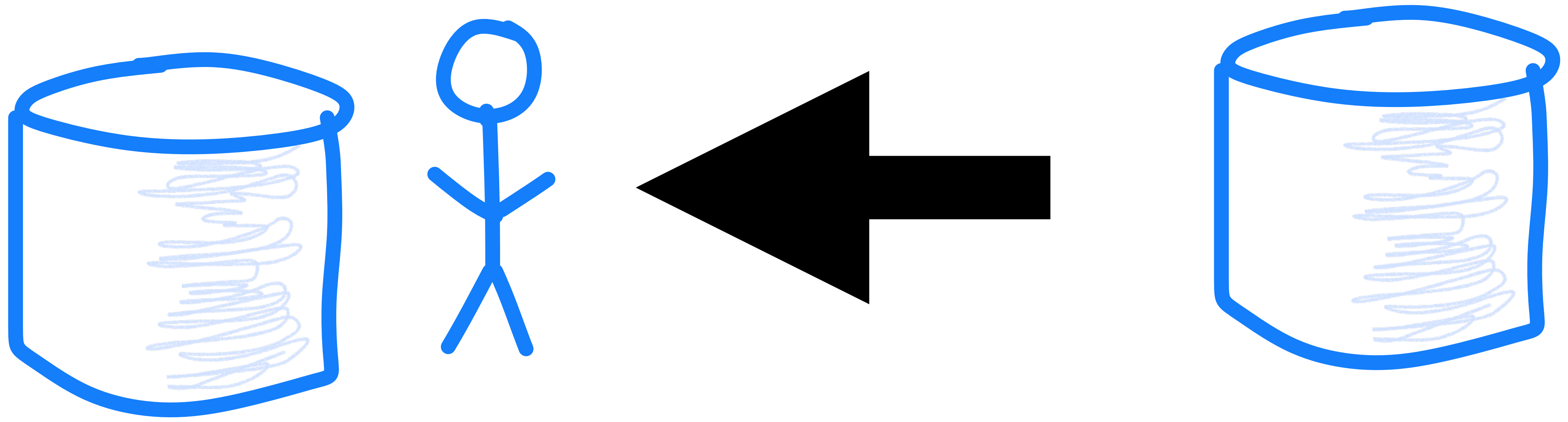
[CK20,SACM21,KC21]

PIR with preprocessing



[BIPW17,CHR17,HOWW18]

Download the database



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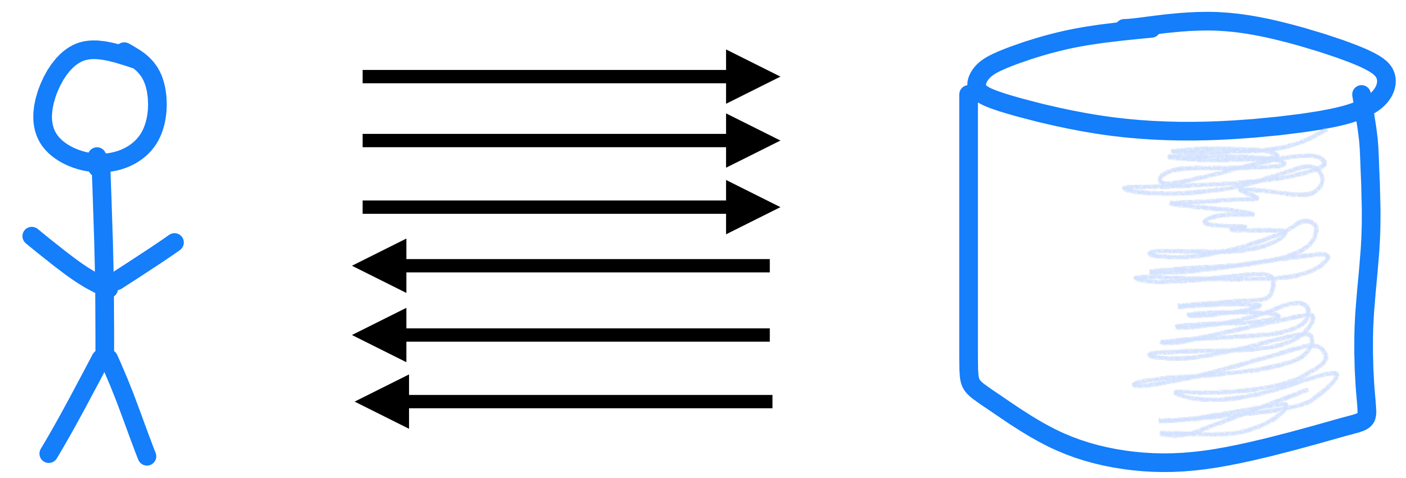

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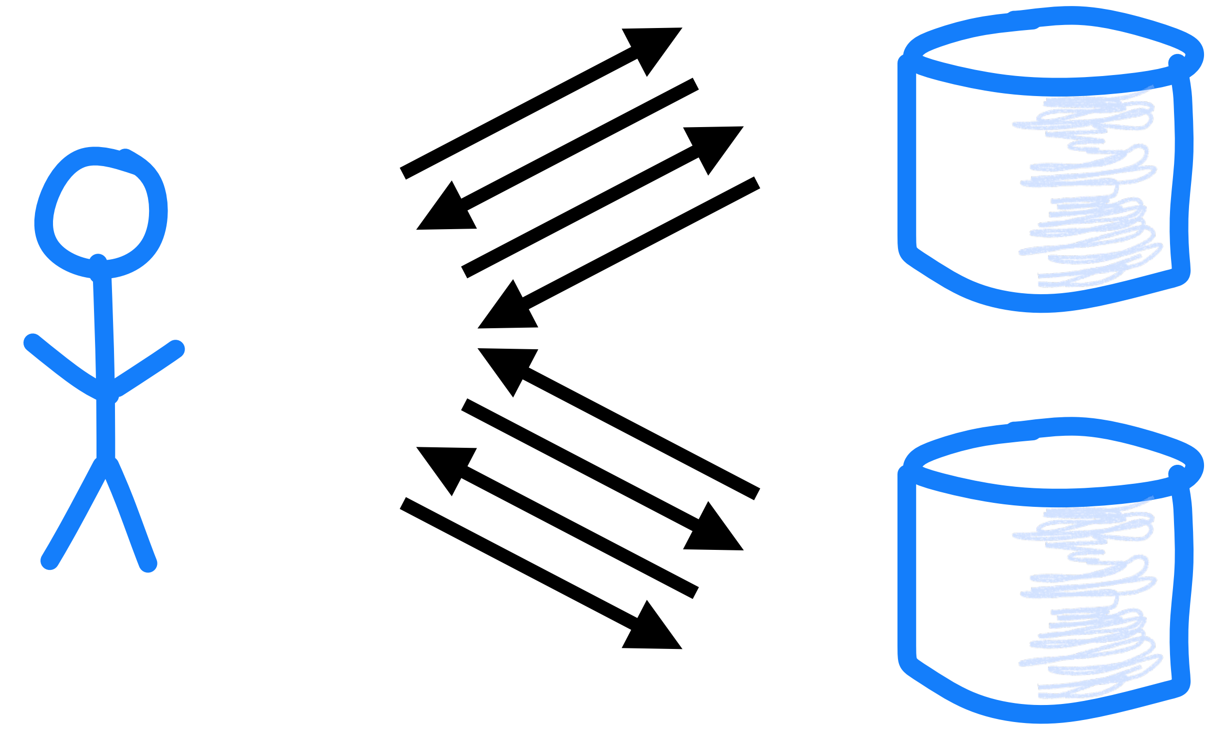

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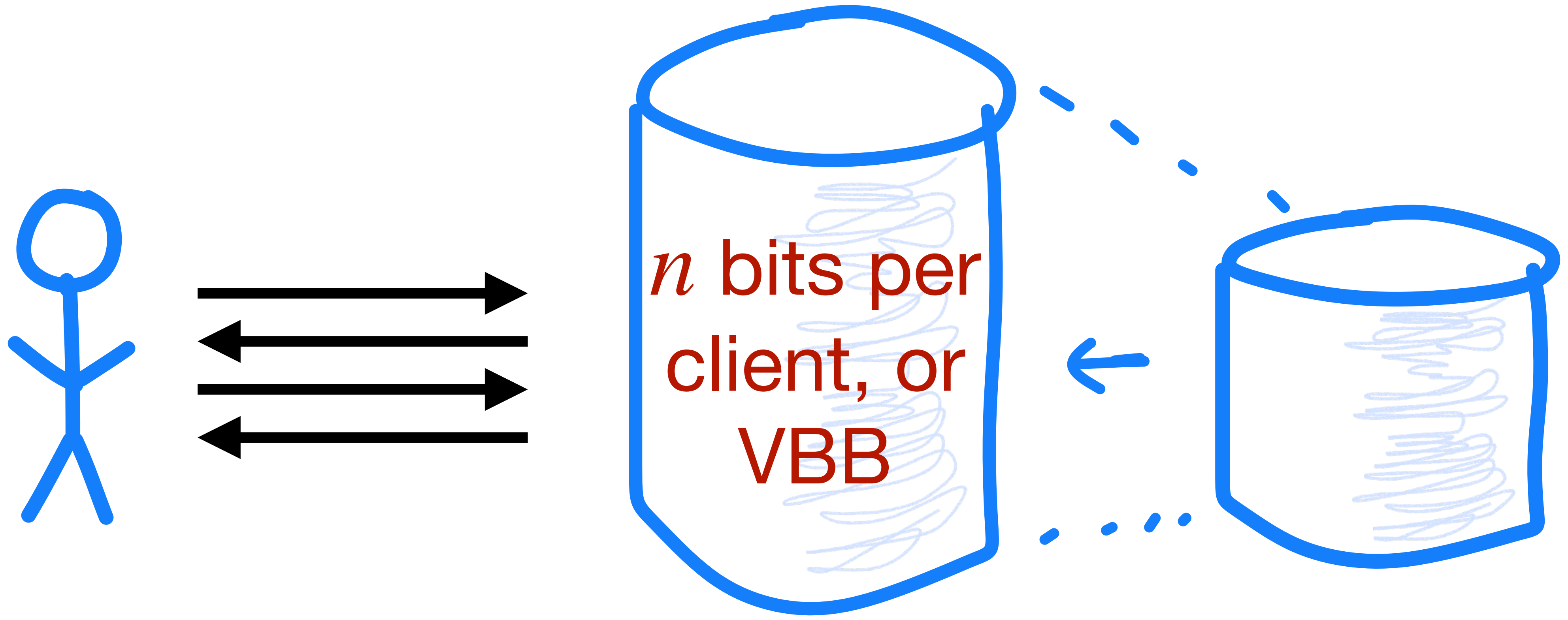

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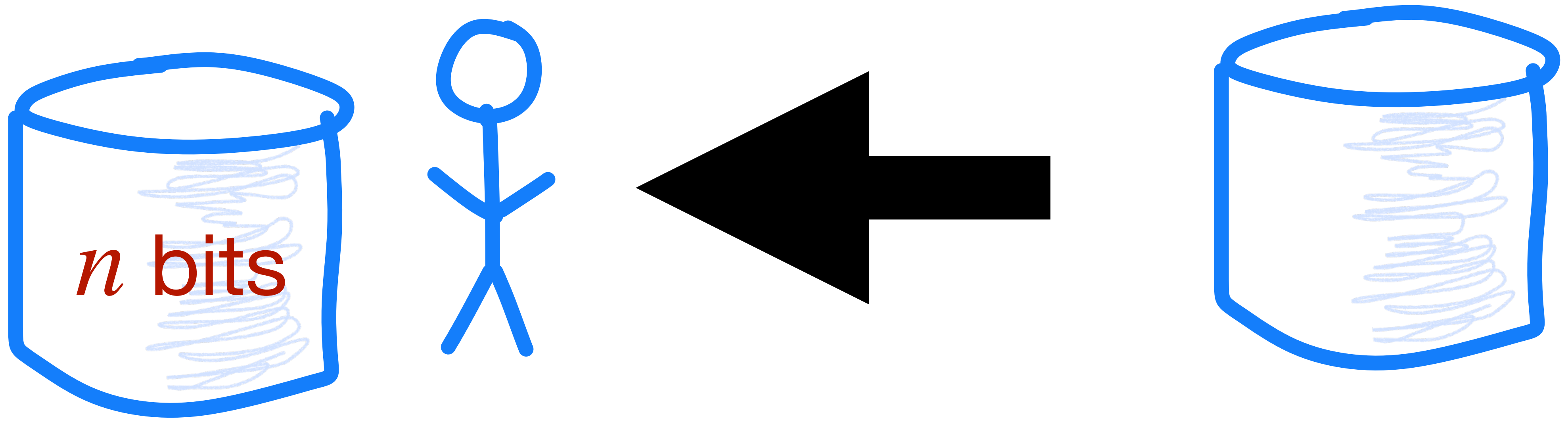
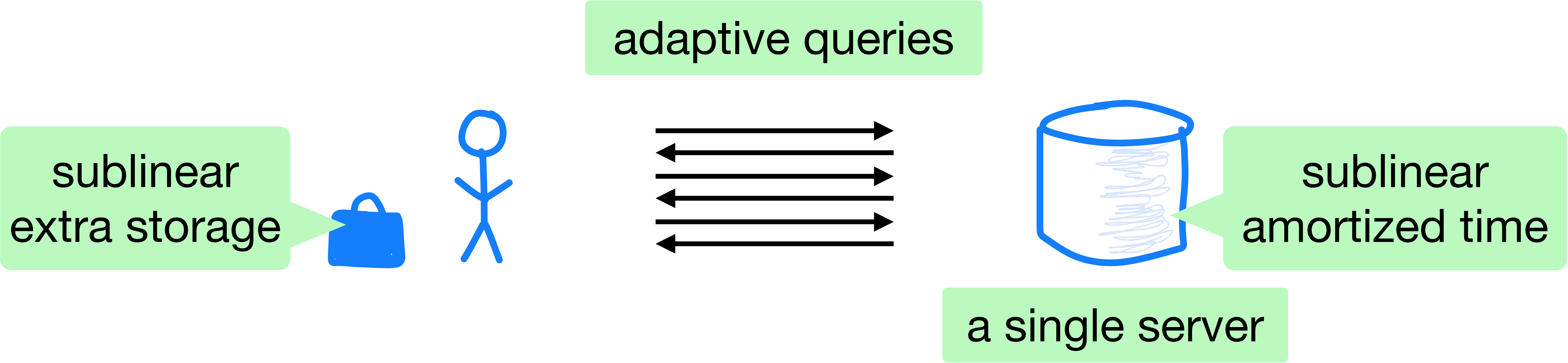


This work

1. **The first PIR schemes** to have:



2. **Matching lower bounds** that relate server time and client storage.

This work

Results preview:
$n^{3/4}$ time + storage from DDH
$n^{1/2}$ time + storage from FHE

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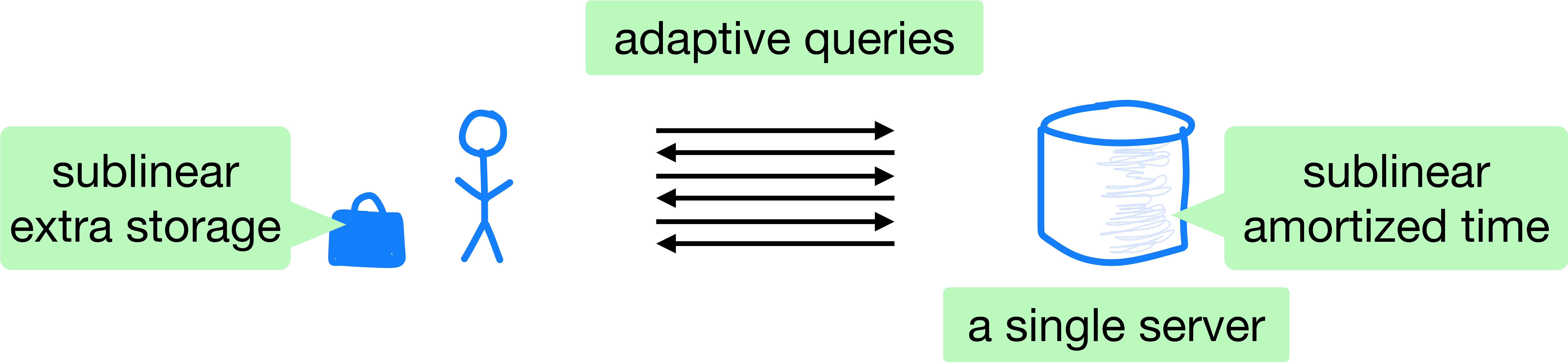

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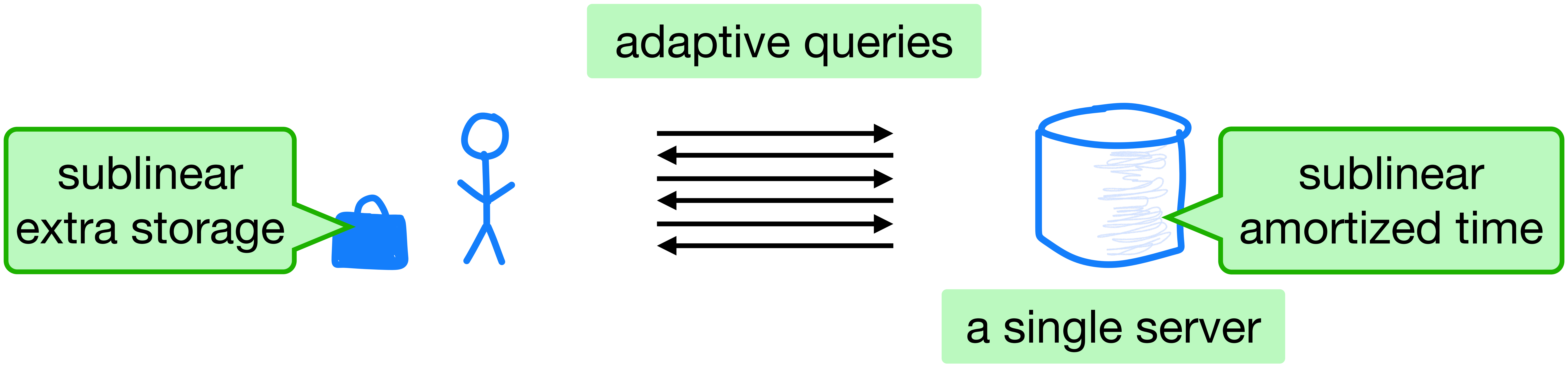

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This talk

1. Background: The offline/online PIR model

2. Our results

   - New PIR schemes with sublinear time
   - New lower bounds on many-query PIR

3. Open questions

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Goal: build PIR for Q adaptive queries, with **sublinear amortized time**

Our approach: build PIR with two phases

1. Once, run a linear-time “offline” phase.

2. For each of the Q queries, run a sublinear-time “online” phase.

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If $Q \geq n^{\epsilon}$, the per-query amortized time is sublinear

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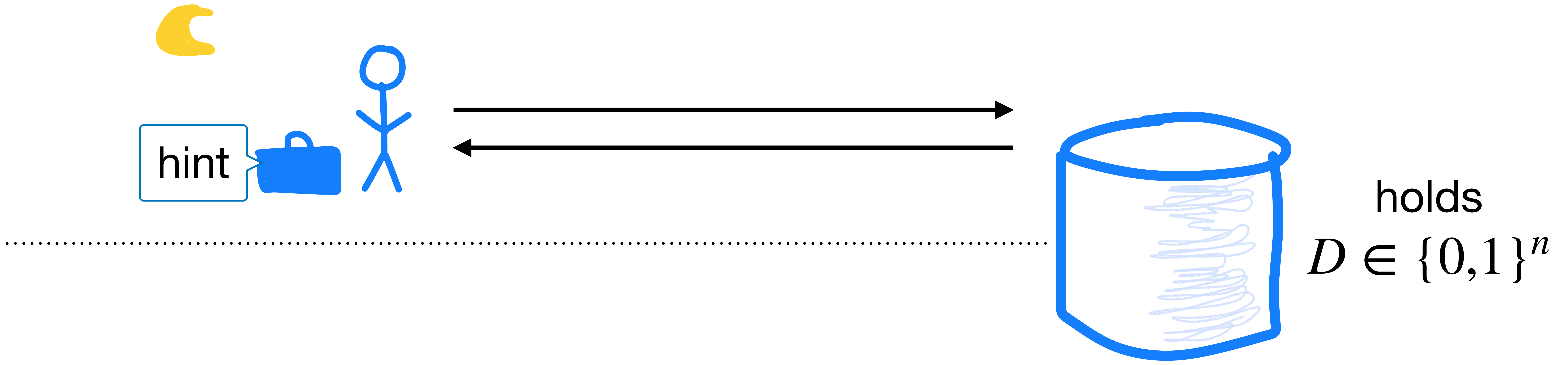
Related work [CK20,SACM21,KC21] supports **1 query per offline phase** in the single-server setting → cannot give sublinear amortized time

Many-query offline/online PIR

holds
$D \in \{0,1\}^n$

Many-query offline/online PIR



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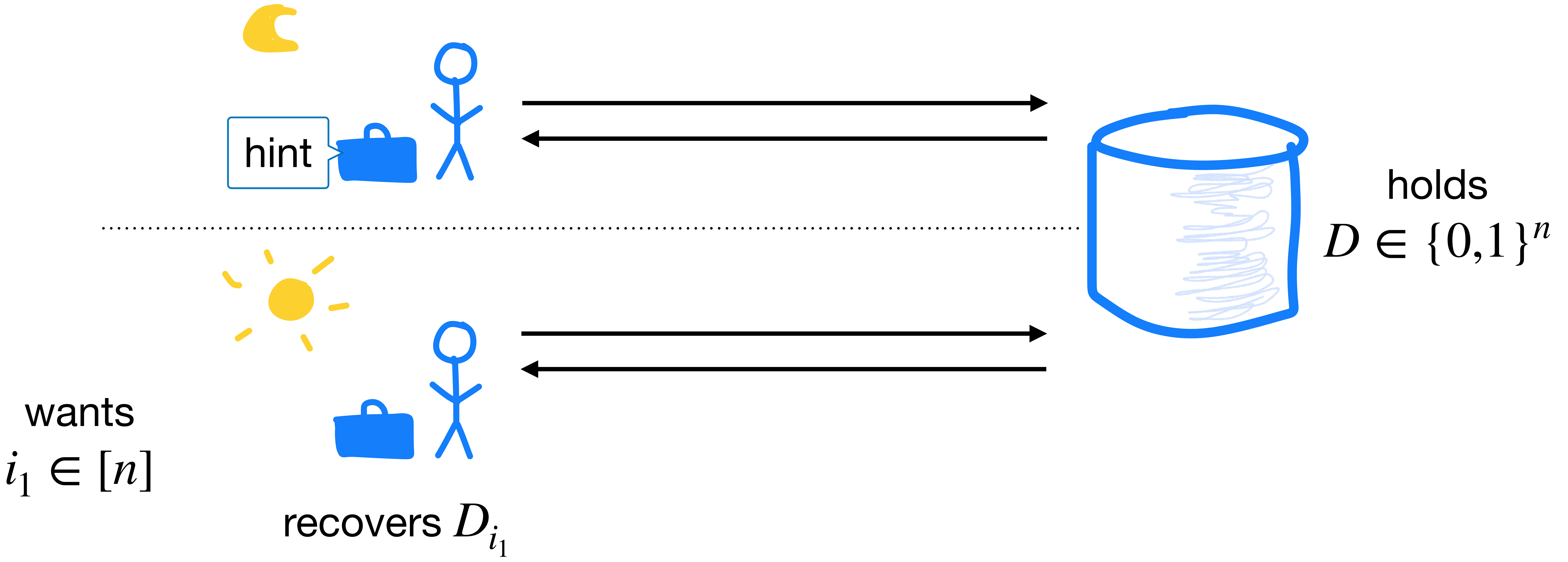

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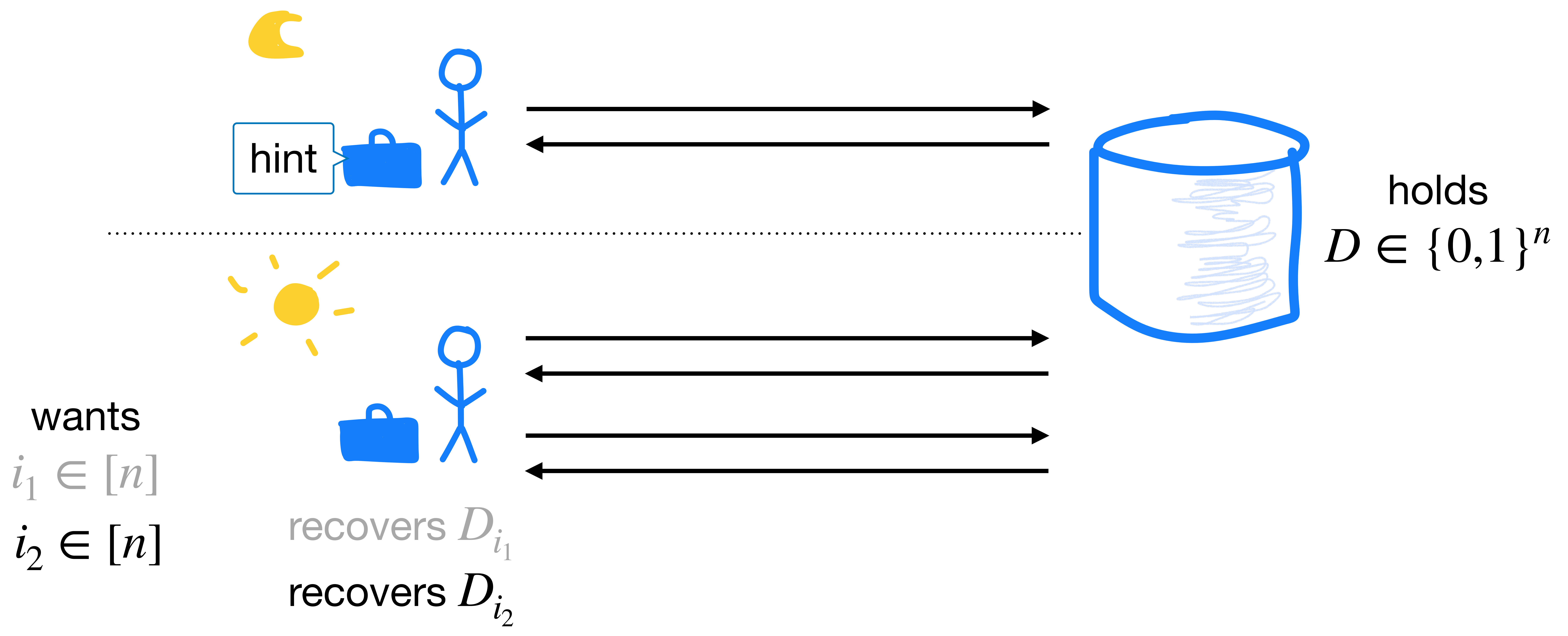

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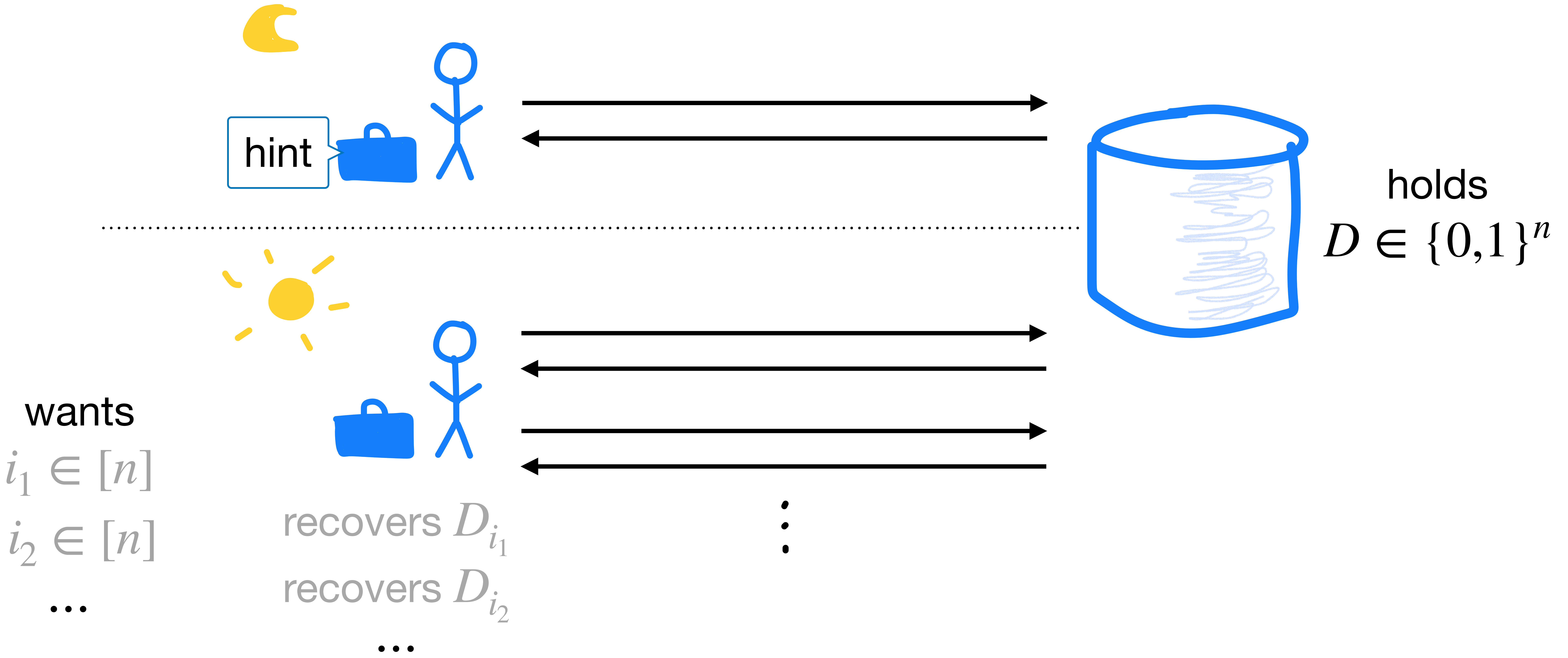

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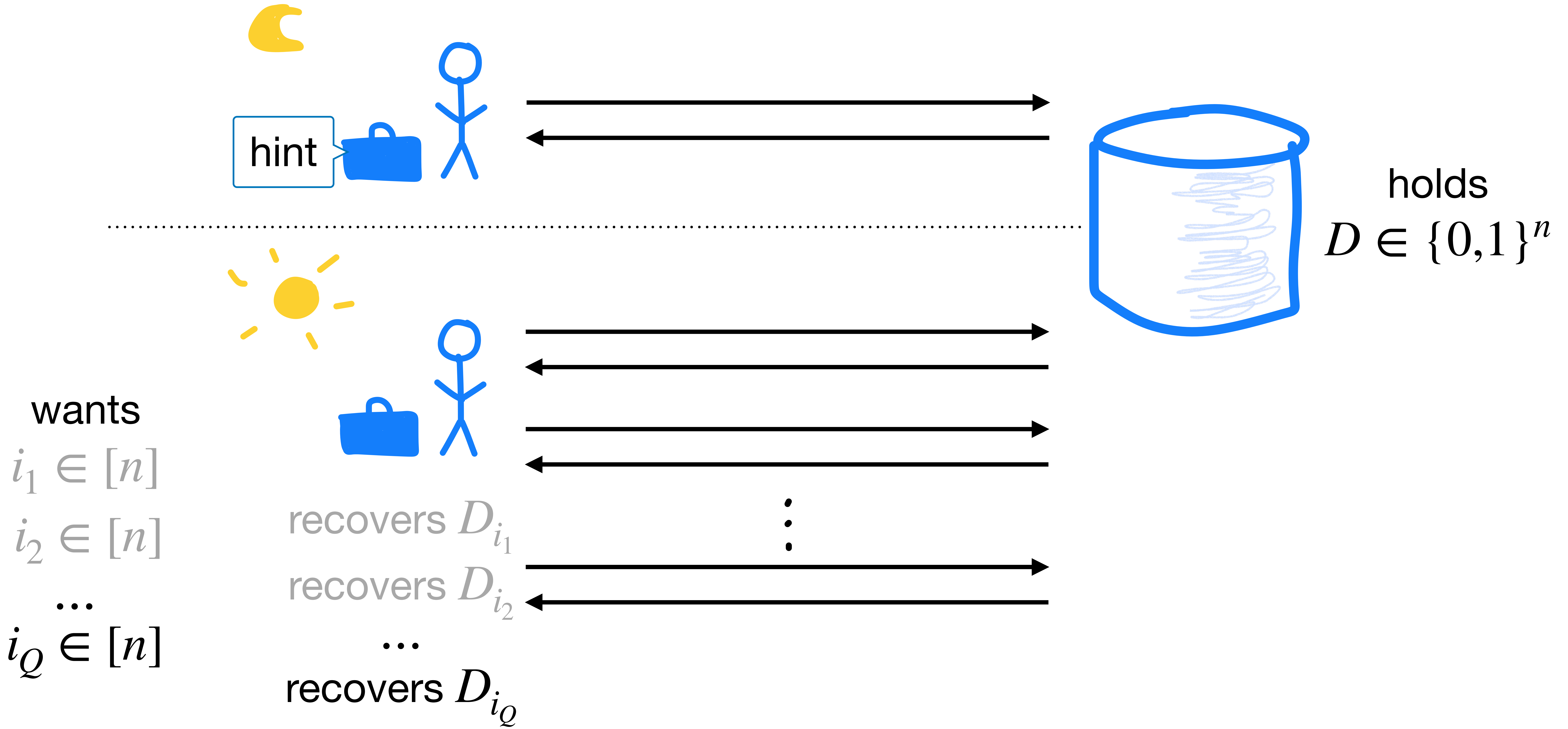

Many-query PIR requirements

- **Correctness**: If the client and server execute the protocol faithfully, for any D, for any $i_1, \cdots, i_Q \in [n]$, the client correctly recovers $D_{i_1}, \cdots, D_{i_Q}$, with overwhelming probability.

- **Malicious security:** Even if the server does not follow the protocol, the server learns nothing about $i_1, \cdots, i_Q$.
  More formally, for any $I, I' \in [n]^Q$,
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In our schemes, the queries are independent of the server's past answers.

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Goal: Minimize communication, computation, and storage costs

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1. Background: The offline/online PIR model

2. Our results
   - ➤ New PIR schemes with sublinear time
   - ➤ New lower bounds on many-query PIR

3. Open questions

Theorem 1: From linearly homomorphic encryption.

Under DDH, QR, DCR, or LWE, there is a single-server PIR scheme that, on database size n, when the client makes $n^{1/4}$ adaptive queries, has:

- amortized server time $n^{3/4}$,
- client storage $n^{3/4}$ and no extra server storage,
- amortized client time $n^{1/2}$, and
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Throughout this talk, we omit $\log(n)$ and $\mathrm{poly}(\lambda)$ factors.

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The amortized time is optimal, given the number of queries. [BIM04]

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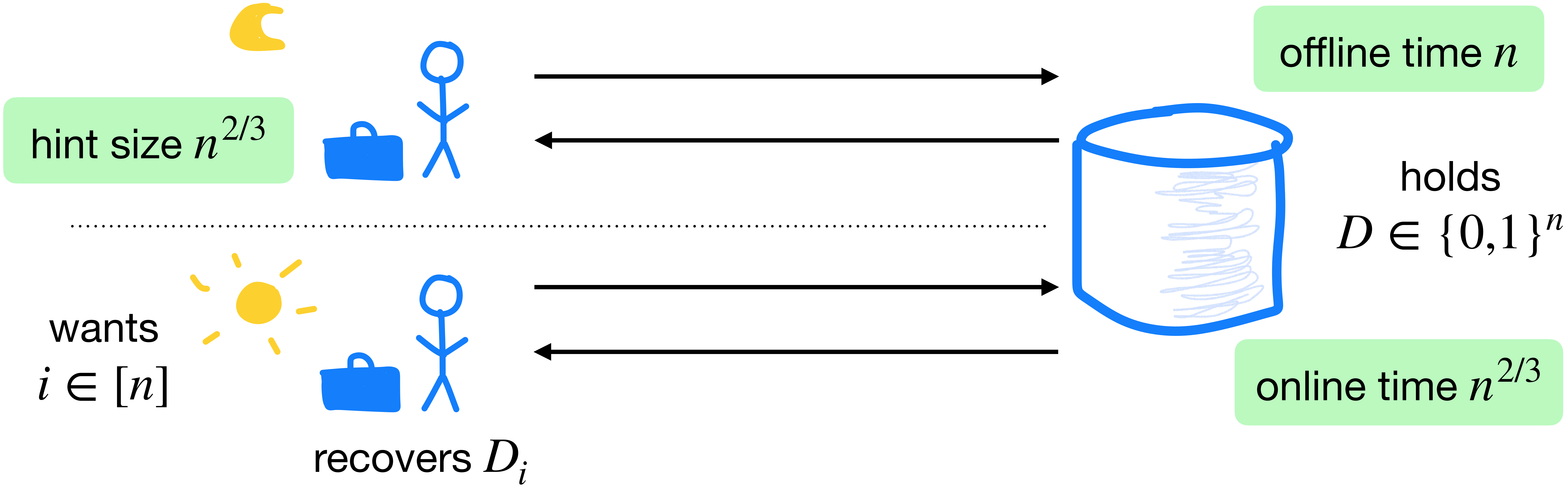
Proof sketch for Theorem 1

Single-query PIR with sublinear online time
[CK20]

New: Many-query PIR with sublinear amortized time

New: generic compiler, applying ideas from batch codes
[IKOS04]

Single-query PIR with sublinear online time [CK20]



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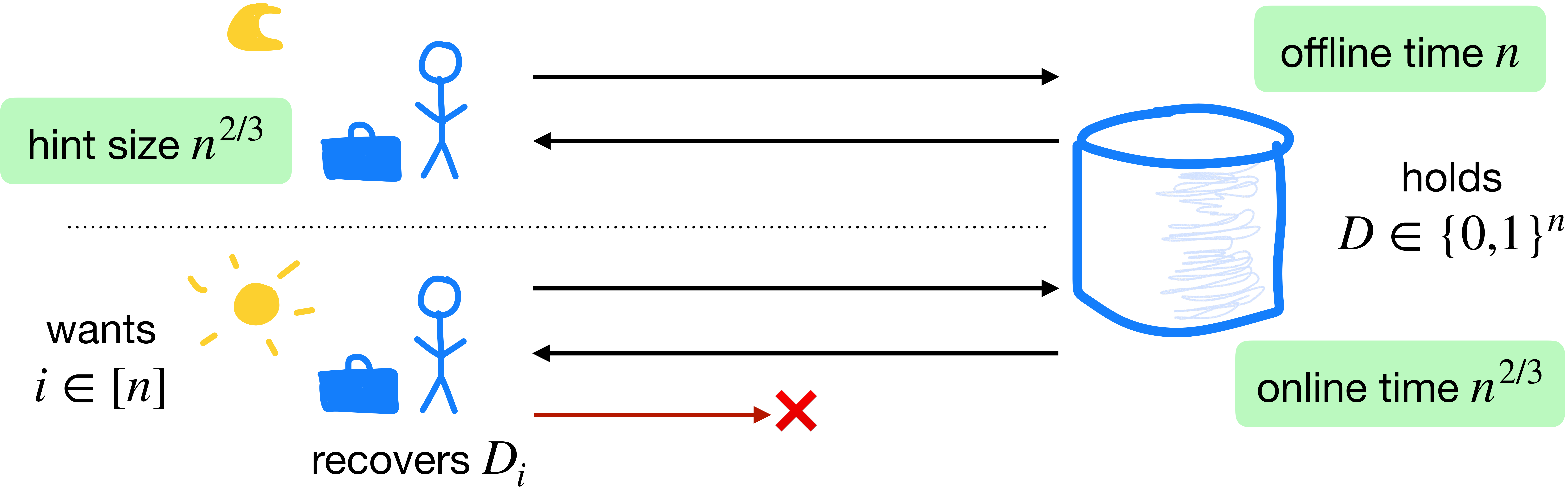

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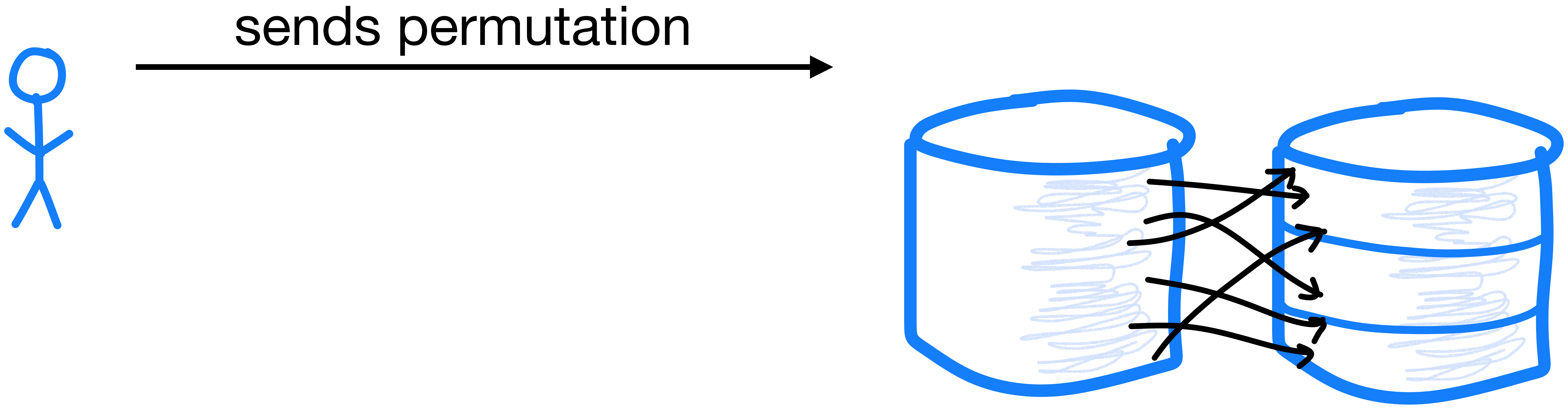

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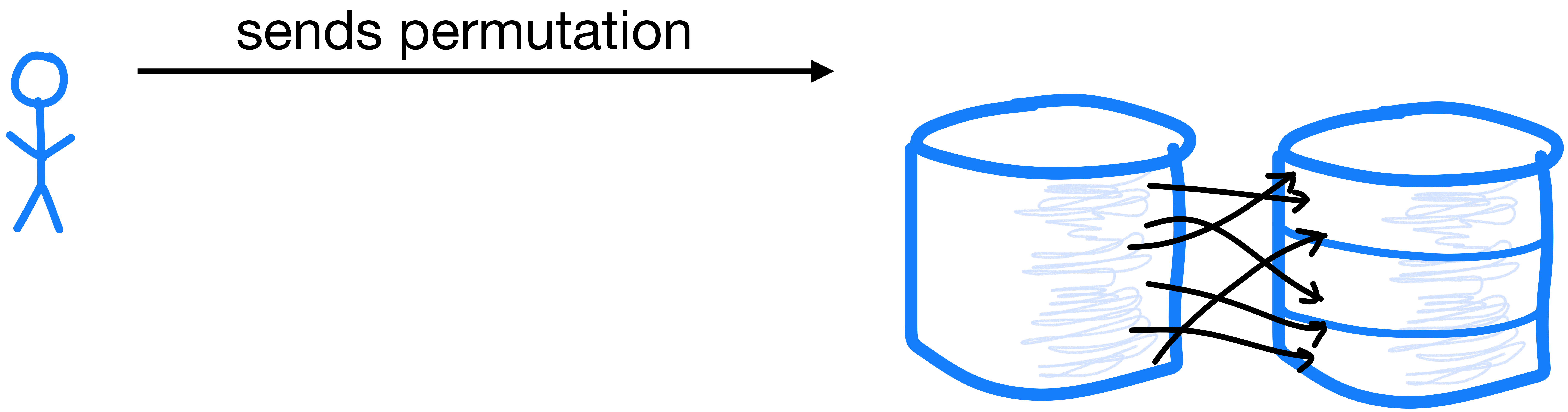

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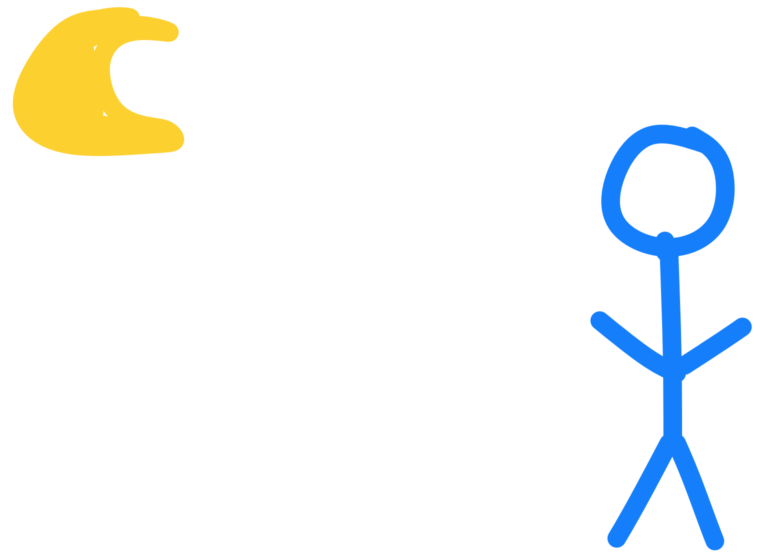
Observation: With probability $1 - \mathrm{negl}(\lambda)$,
at most λ distinct queries fall in any one chunk.

Offline: Permute + partition the database, then run λ offline phases on each chunk.

Online: Run an online phase on each chunk, using a hint matching the index.

The client caches all recovered bits, to never re-query for the same index.



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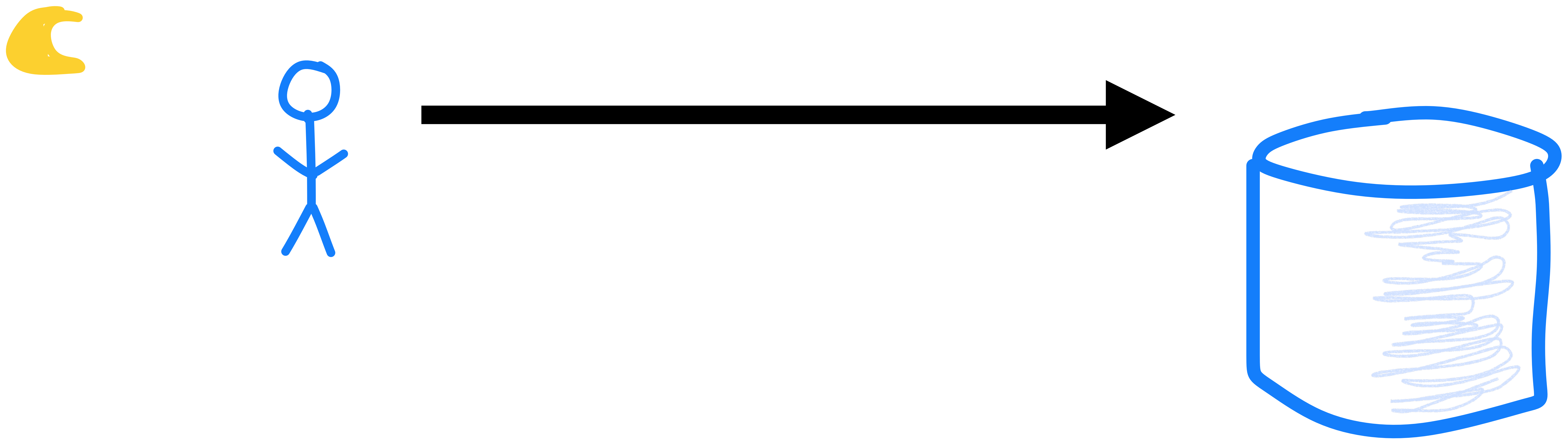

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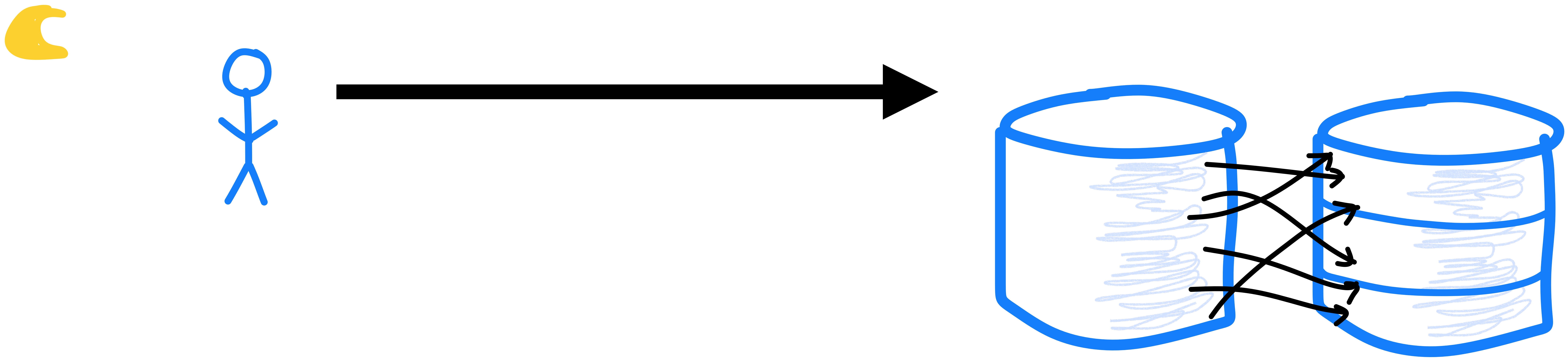

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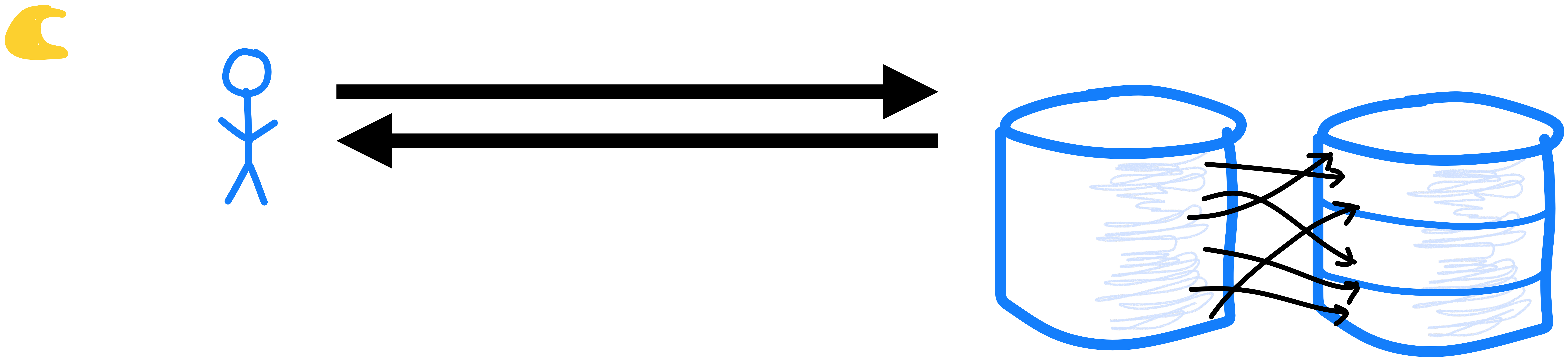

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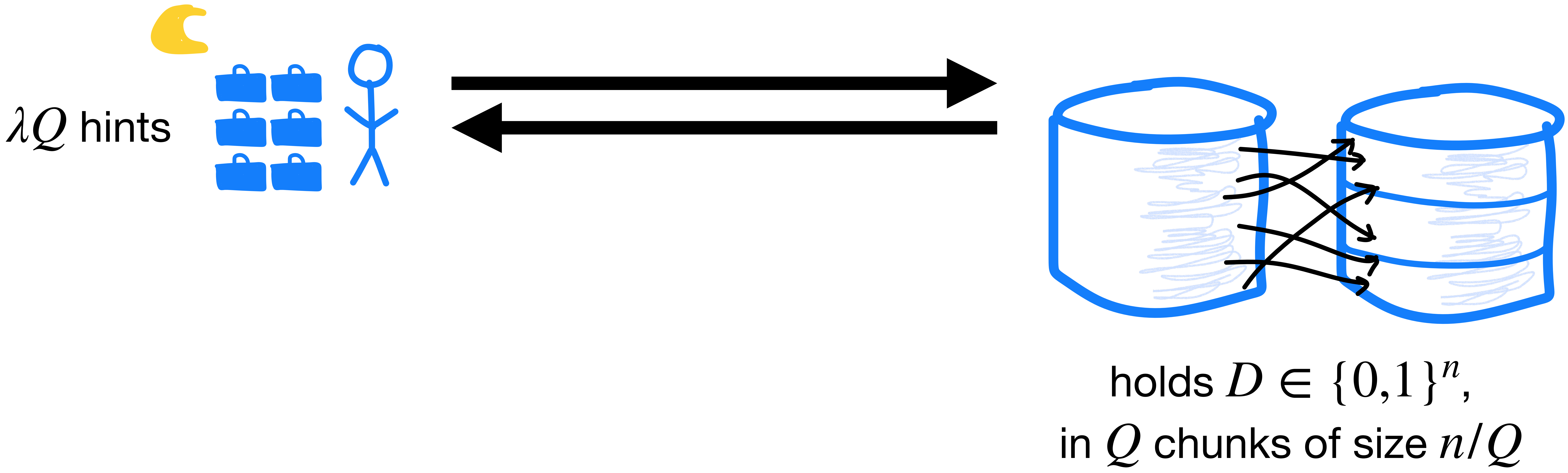

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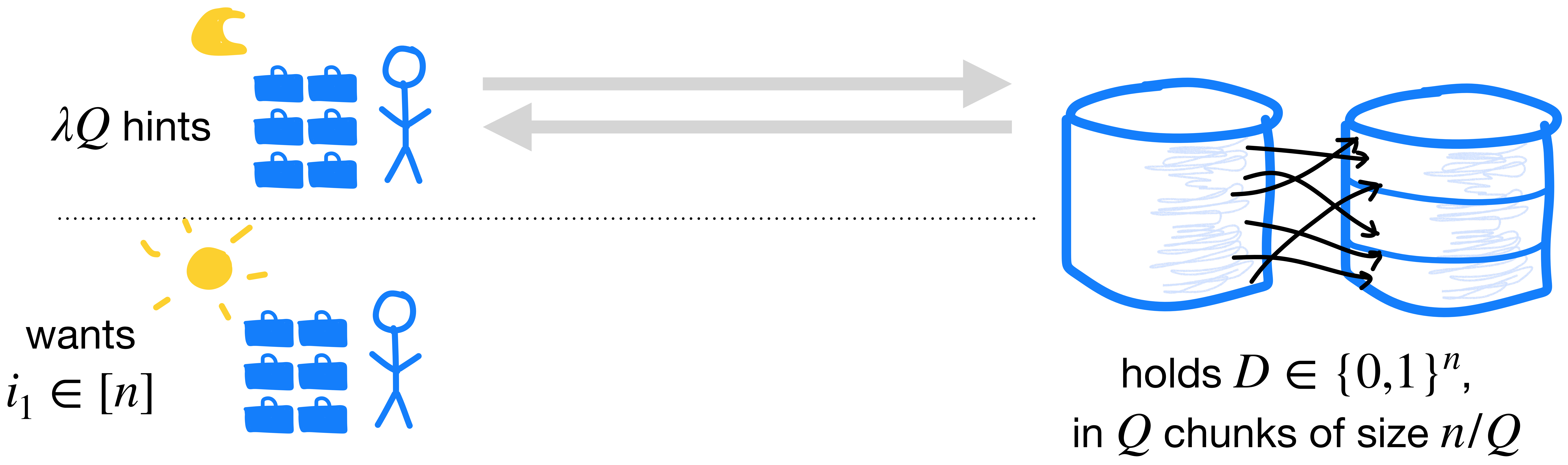

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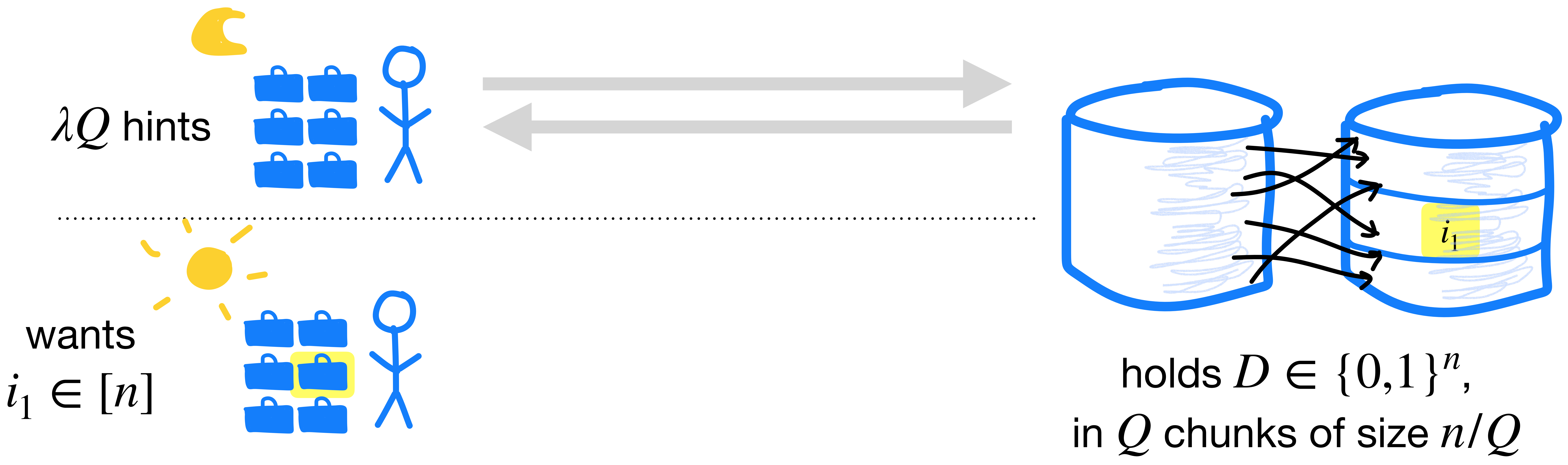

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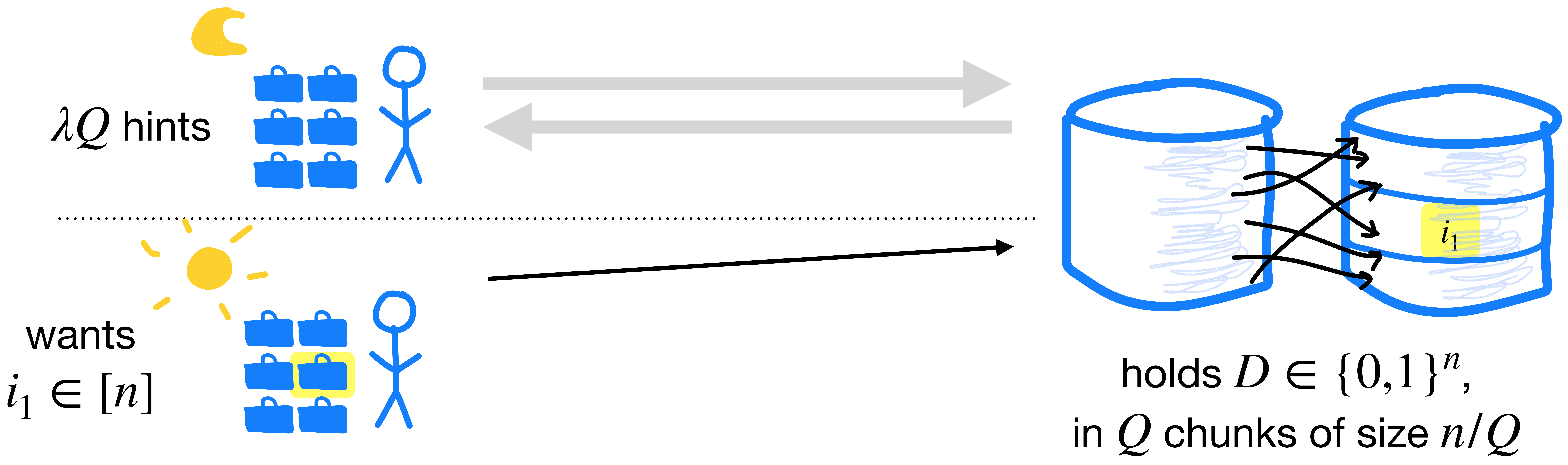

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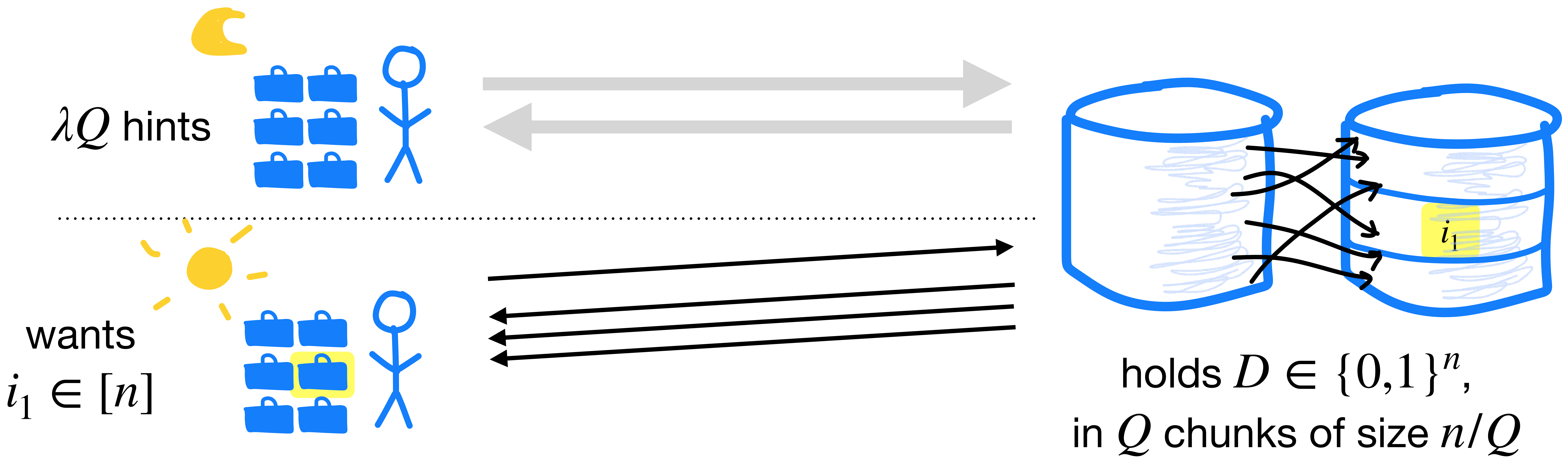

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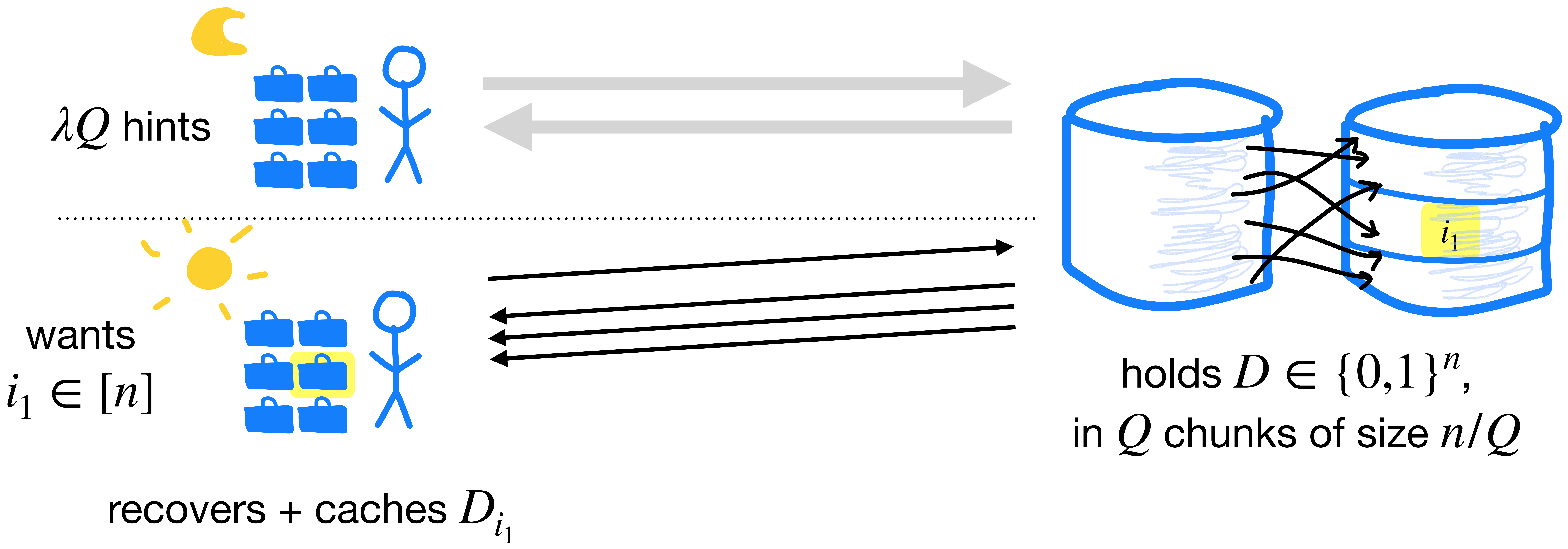

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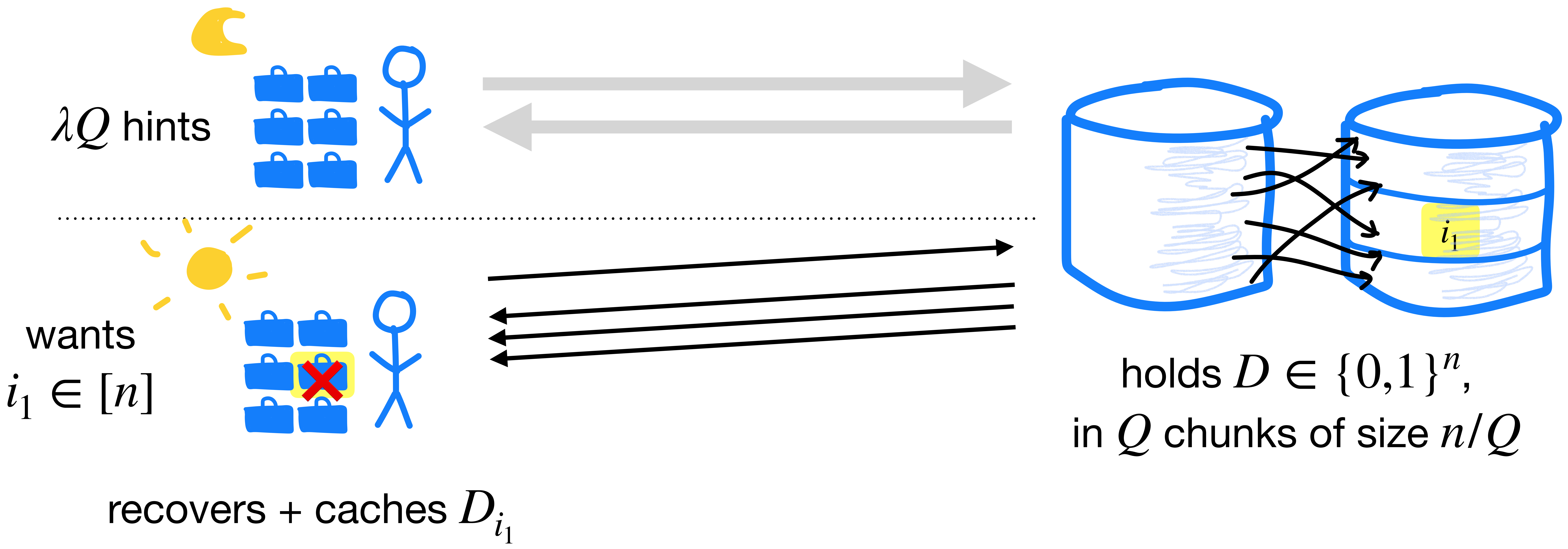

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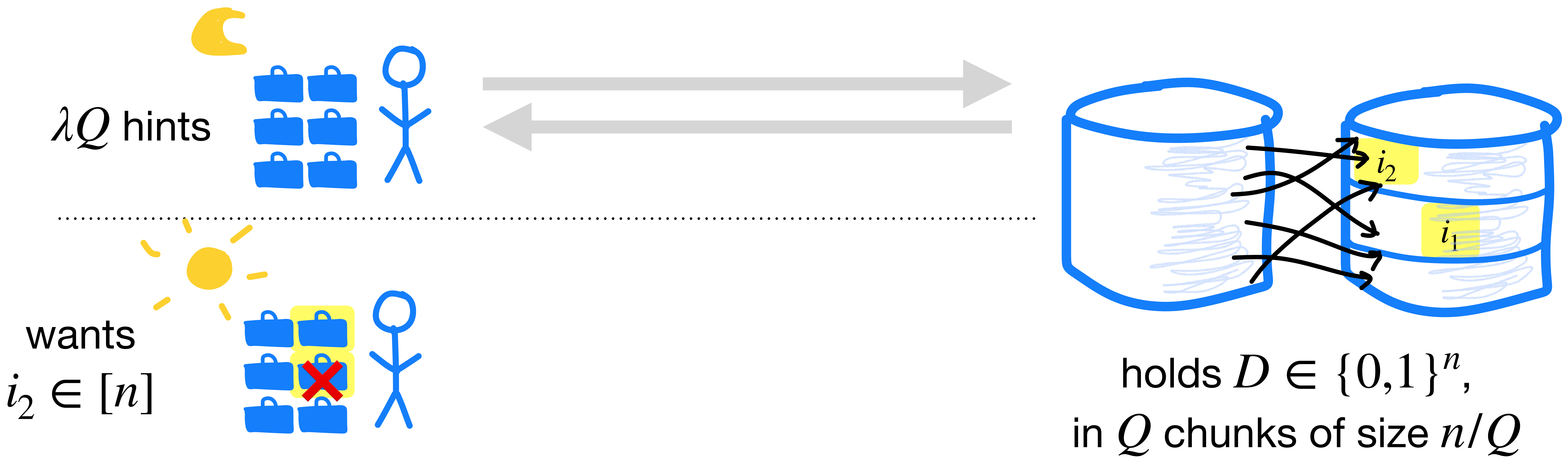

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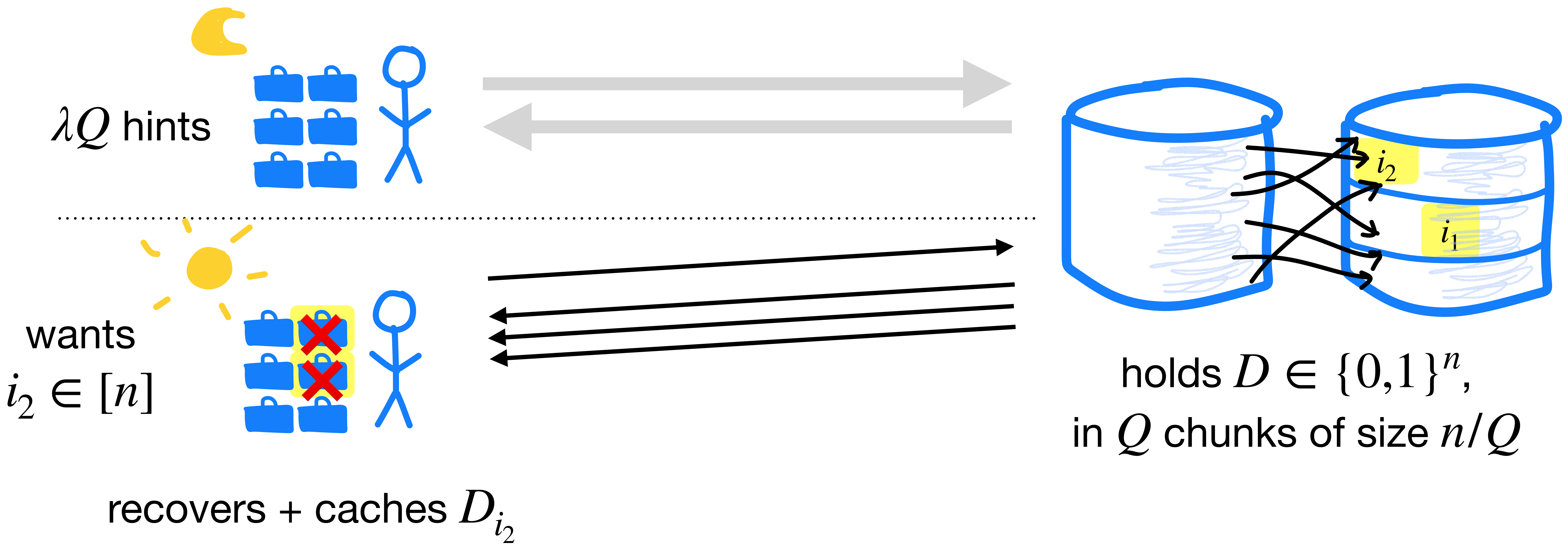

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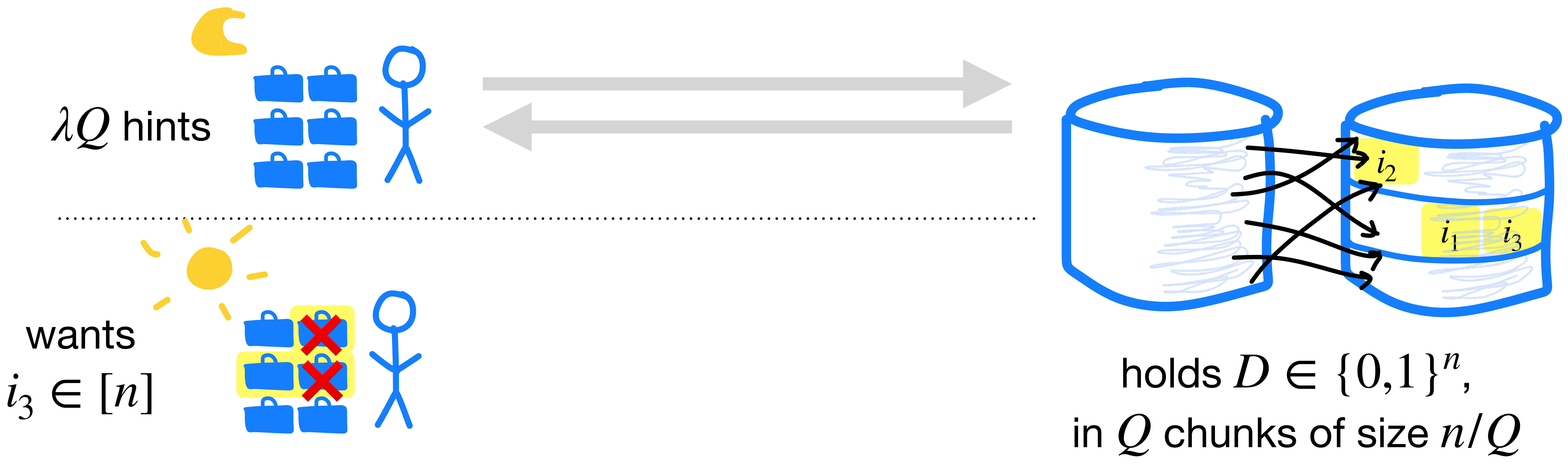

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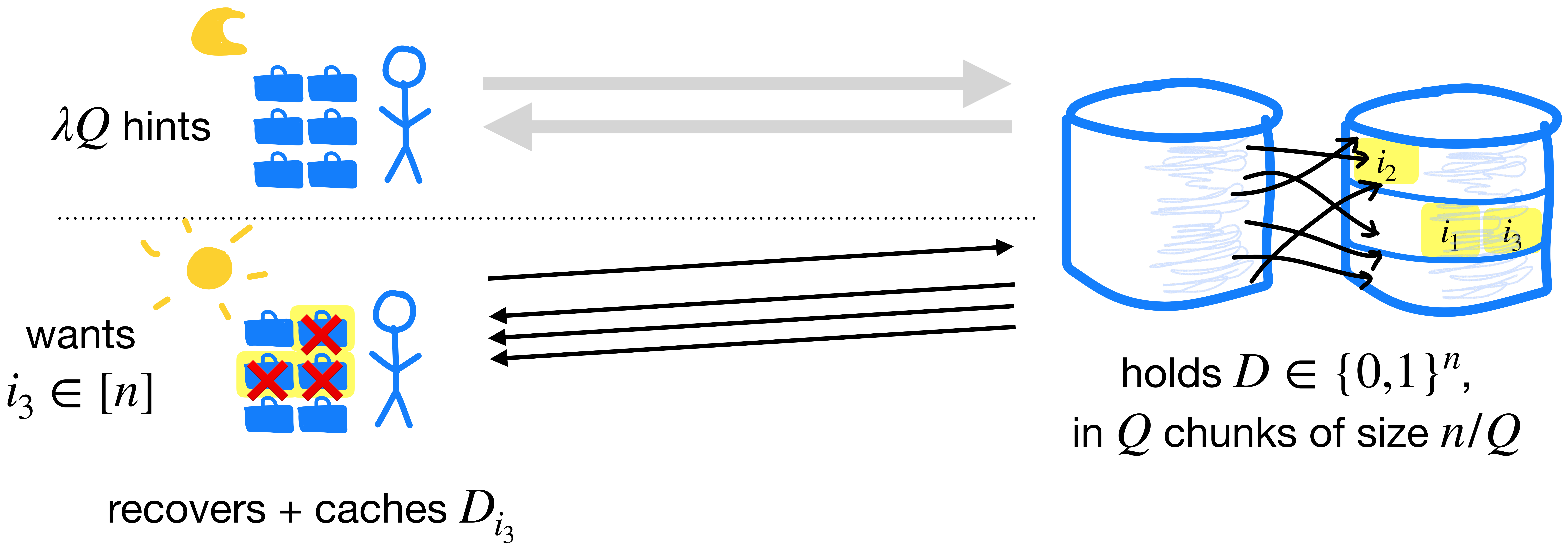

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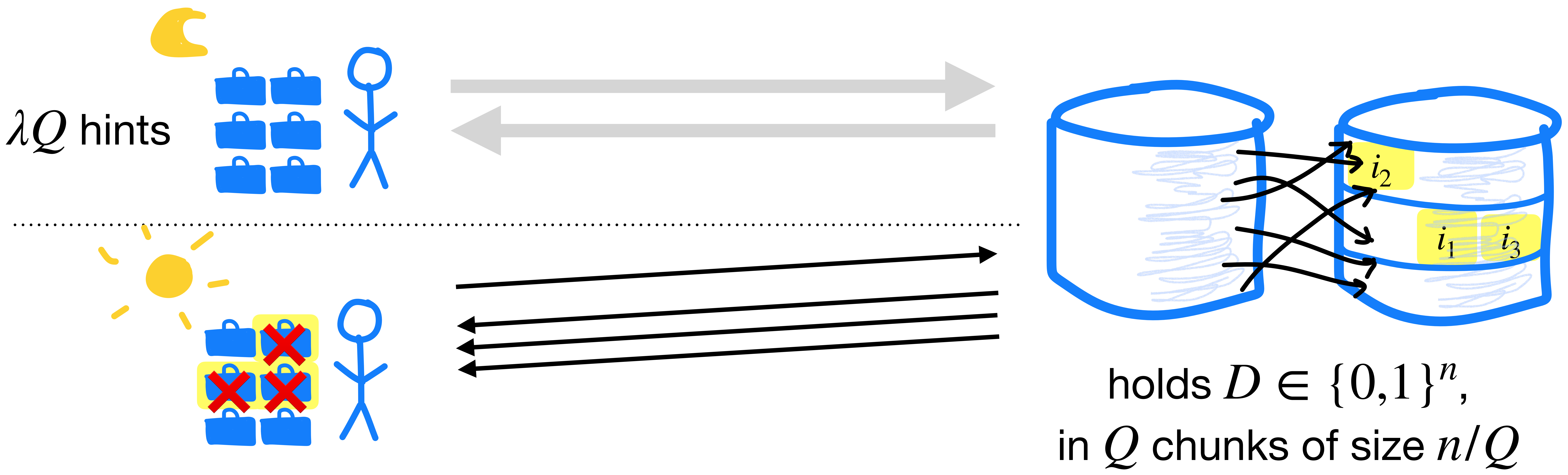

Correctness: for any query sequence, the client does not run out of fresh hints (with overwhelming probability over the choice of permutation).

Offline: Permute + partition the database, then run λ offline phases on each chunk.

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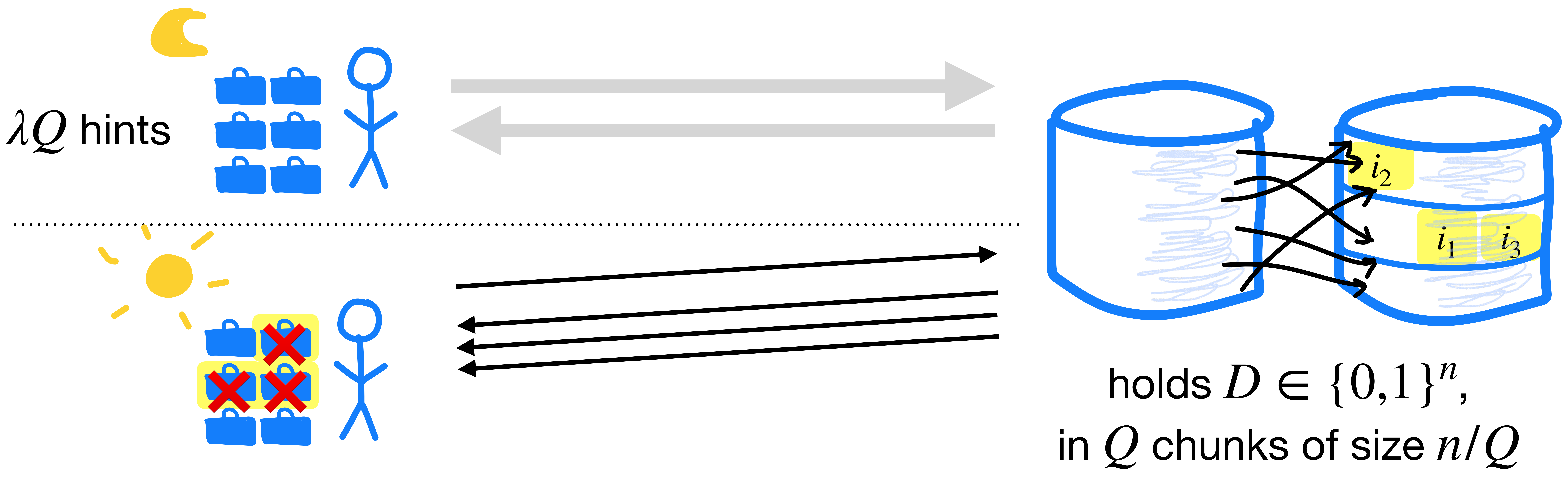

holds $D \in \{0,1\}^n$,

in Q chunks of size n/Q

Security: The client's query does not reveal which chunk it is reading.

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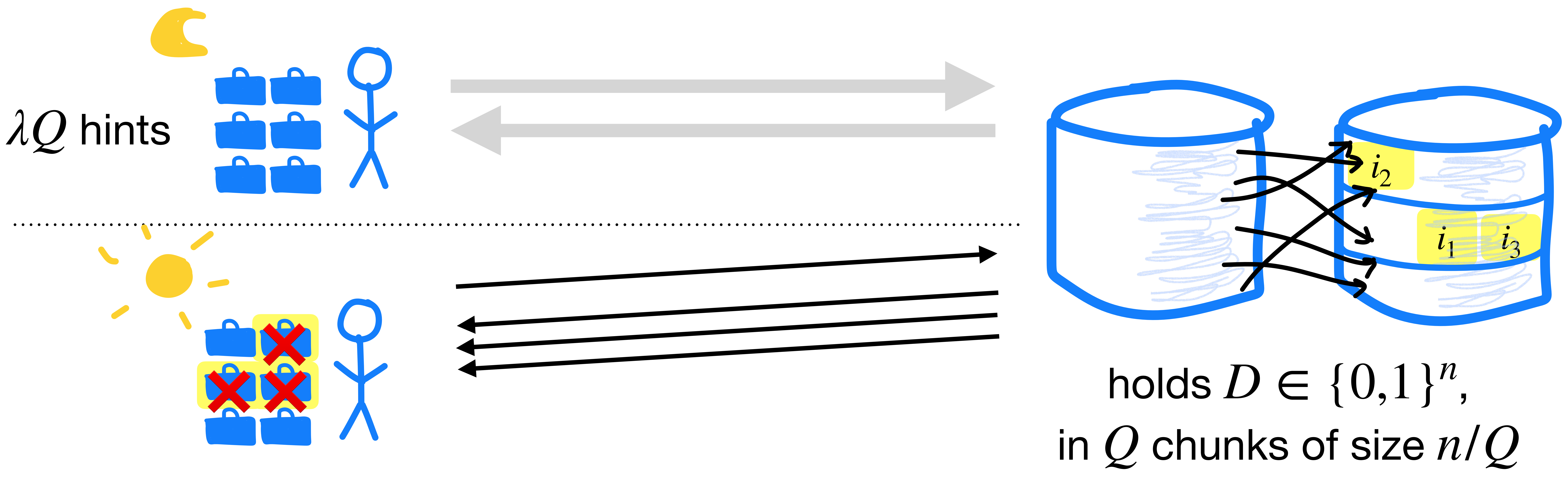

Cost: We ran the underlying PIR λQ times, on database size n/Q.

Proof sketch for Theorem 1

Input: Single-query PIR with sublinear online time [CK20]

- hint size $n^{2/3}$
- offline time n
- online time $n^{2/3}$

Generic compiler, with $Q = n^{1/4}$ queries

Output: Many-query PIR with sublinear amortized time

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Throughout this talk, we omit $\log(n)$ and $\mathrm{poly}(\lambda)$ factors.

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Theorem 1: From linearly homomorphic encryption.

Under DDH, QR, DCR, or LWE, there is a single-server PIR scheme that, on database size n, when the client makes $\geq n^{1/4}$ adaptive queries, has:

- amortized server time $n^{3/4}$,
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Theorem 2: From fully homomorphic encryption.

Assuming FHE*, we improve the amortized server time and client storage to $n^{1/2}$, if the client makes $n^{1/2}$ adaptive queries.

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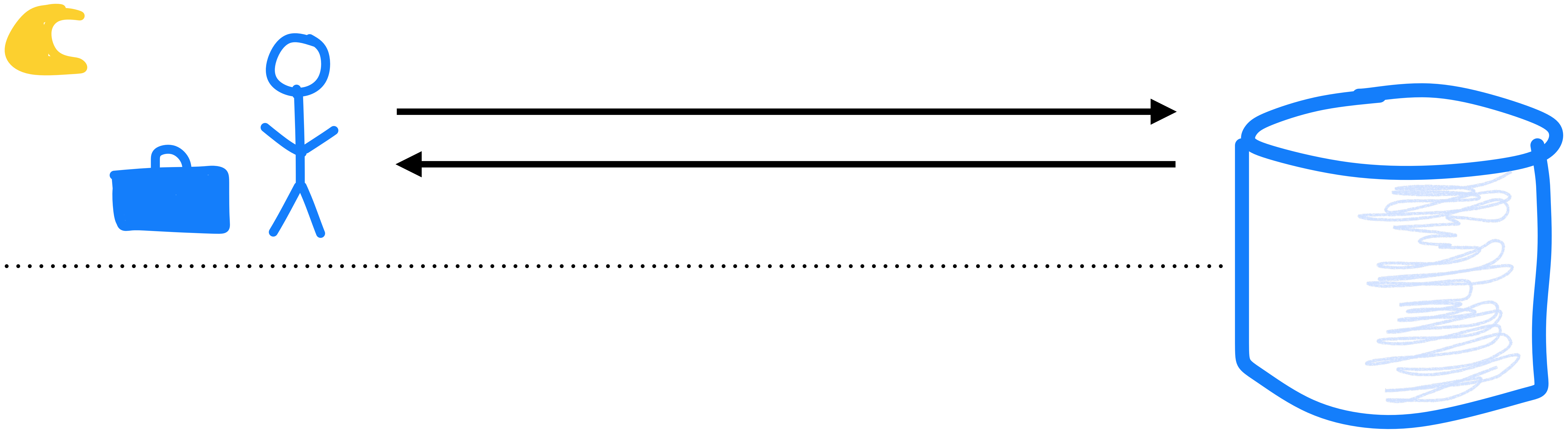
In the offline phase, the server runs the circuit under FHE in linear time.

(1) Sample $O(Q)$ subsets of $\{1,\cdots,n\}$, each of size n/Q.

(2) For each subset S, send $\mathrm{Enc}(S)$ to the server and get its parity, $\sum_{j\in S} D_j$.



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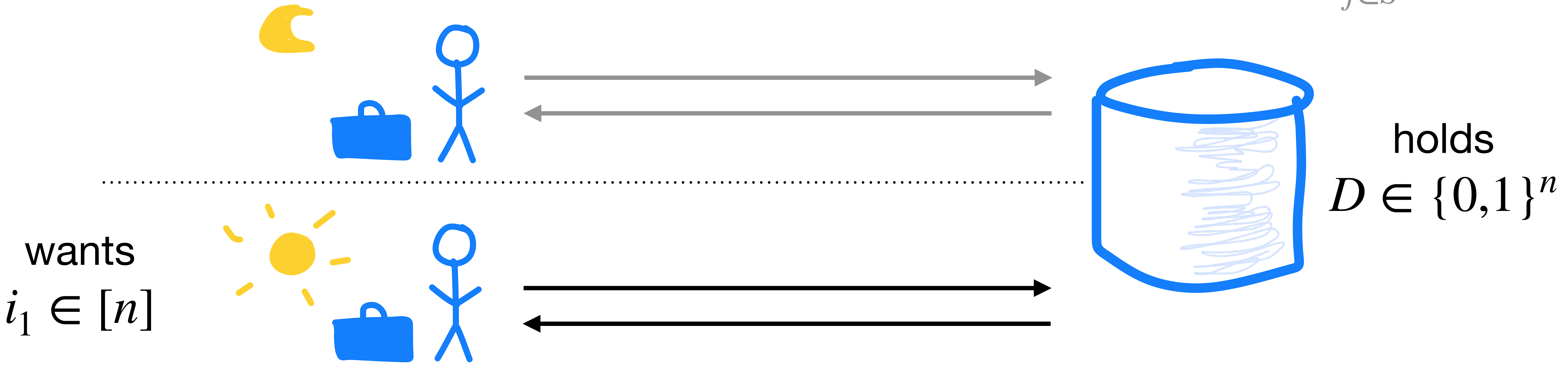

If some subset S contains i_1: then, with *good* probability,

- ask the server for the parity of $S - \{i_1\}$ and recover D_{i_1},
- discard S and "refresh" the distribution of subsets.

Else: send a random subset (that, with some probability, contains i_1).

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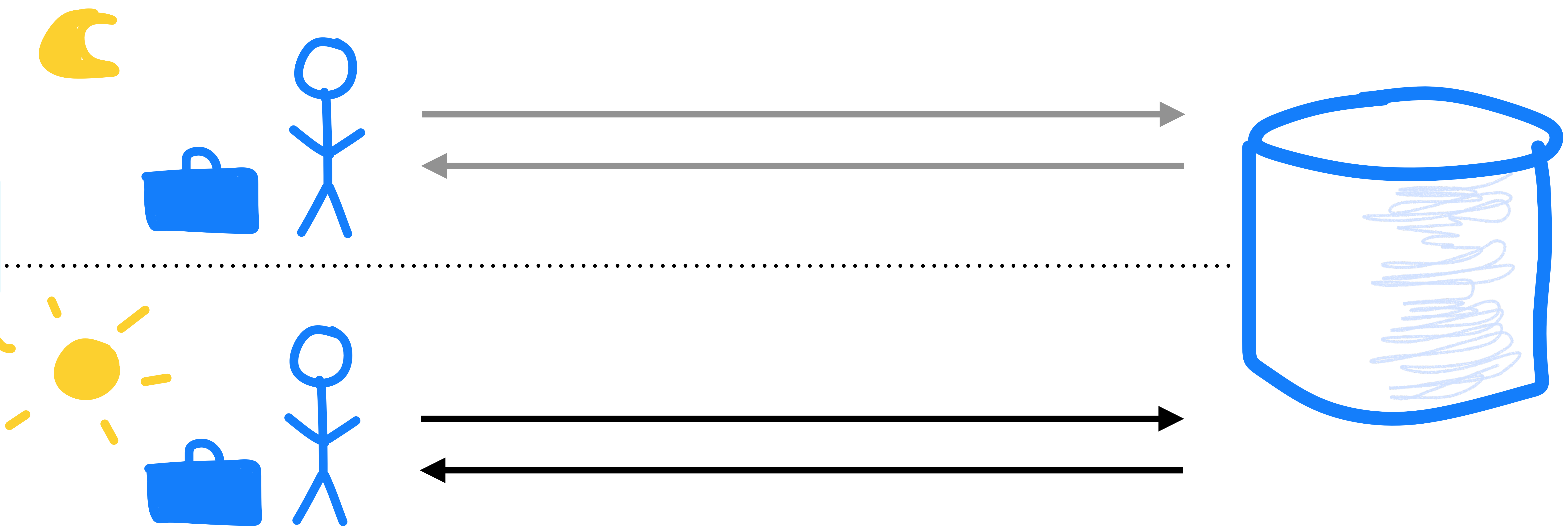

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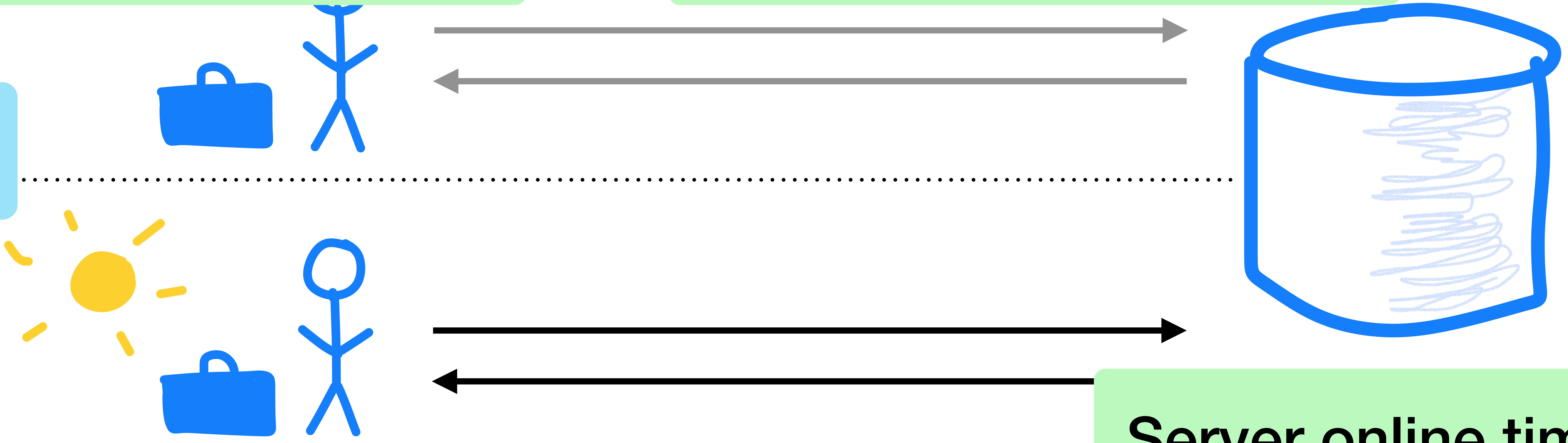

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2. Our results
    - New PIR schemes with sublinear time
    - New lower bounds on many-query PIR
3. Open questions

Theorem 3: Lower bound for adaptive queries

Every single-server PIR scheme for adaptive queries, where:

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Bounds on **PIR with preprocessing**, where the server stores B extra bits:

- With any number of servers, $B \cdot T \geq n$ [BIM04].
- With a single server, if $B \geq \log n$, then $B \cdot T \geq n \log n$ [PY22].

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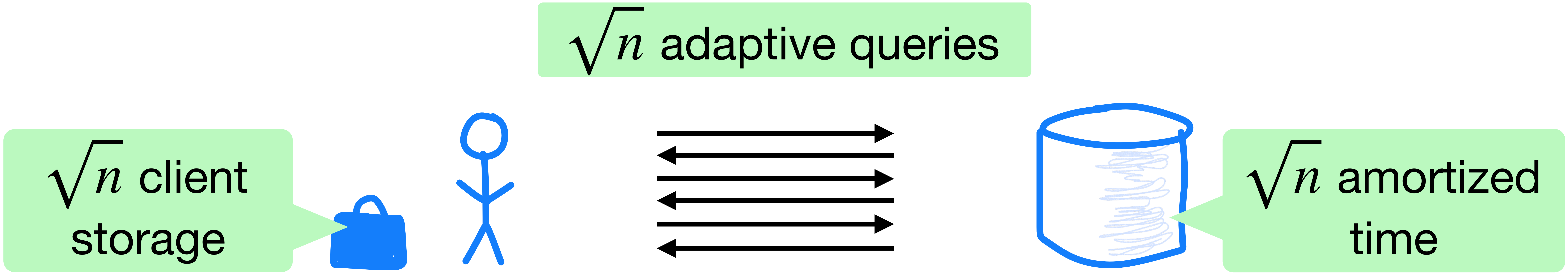

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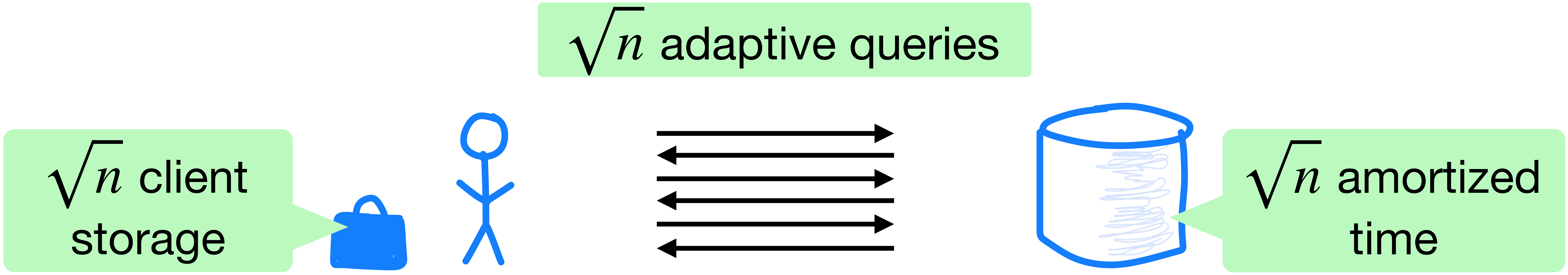

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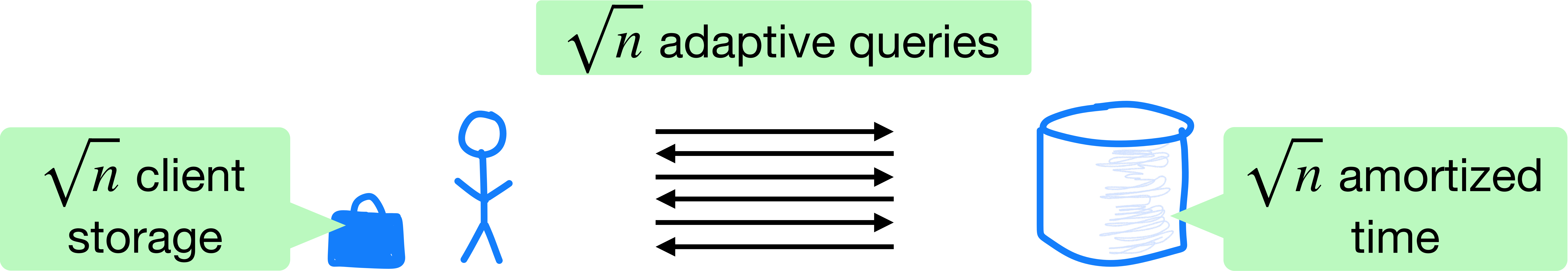

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