Quantum Rewinding Tutorial Part 1:

Motivation and Early Quantum Rewinding Techniques

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(Simons & Berkeley)

Based on:

- "Quantum Proofs of Knowledge" by **Dominique Unruh** (2012)
- "Computationally Binding Quantum Commitments" by **Dominique Unruh** (2016)
- "Zero Knowledge Against Quantum Attacks" by **John Watrous** (2005)
- "Quantum Arthur Merlin Games" by **Chris Marriott** and **John Watrous** (2005)
- "Traité des substitutions et des équations algébriques" by **Camille Jordan** (1870)

Today's Goal:

We want *classical* cryptography
secure against *quantum* attacks
(post-quantum cryptography)

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Key point: problem must be hard for quantum computers!

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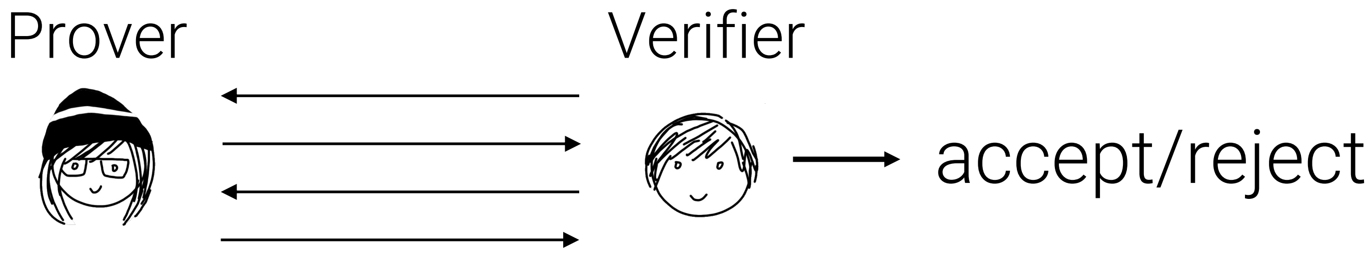
[BCMVV18]
Protocol



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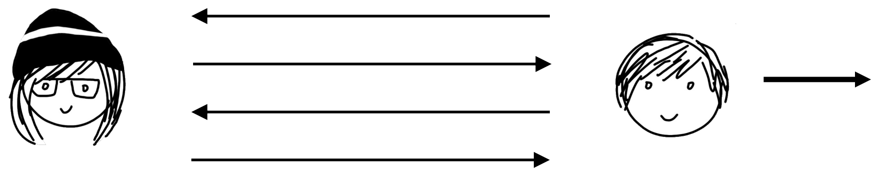

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- Efficient classical P *cannot* make V accept assuming LWE

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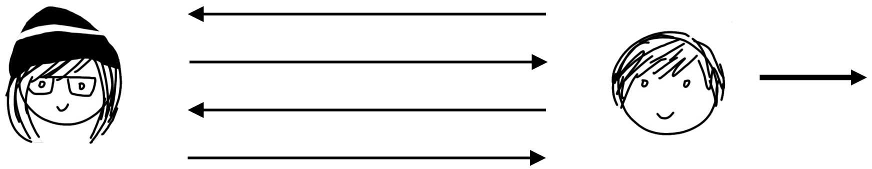

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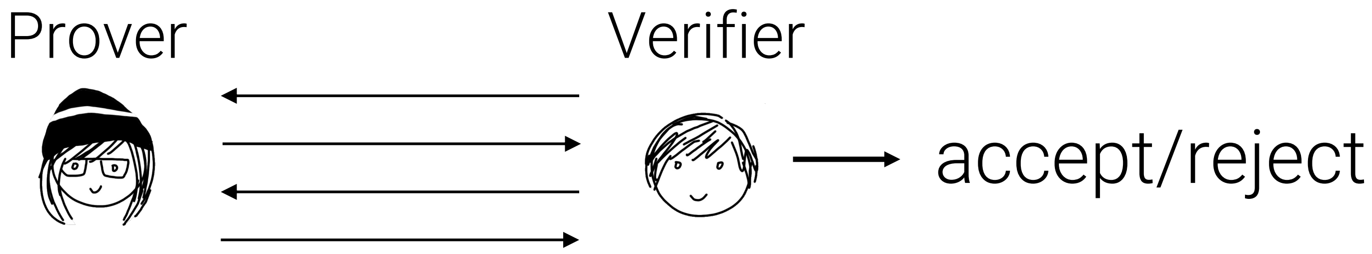

- Efficient classical P *cannot* make V accept assuming LWE.
- Efficient quantum P *can* convince V to accept.

In [BCMVV18] this is presented as a proof of quantumness.

Takeaway

Quantum computers can break classically secure crypto *without* solving the underlying hard problem!

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$\xrightarrow{a}$

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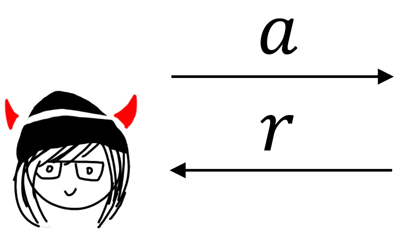

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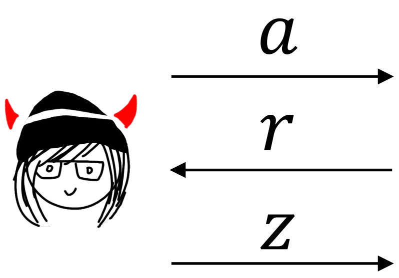

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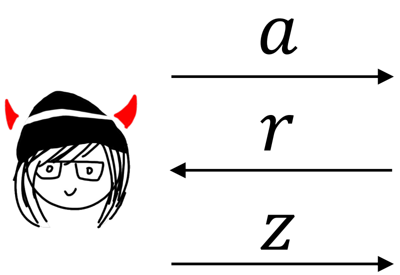

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1) Record (a, r, z)

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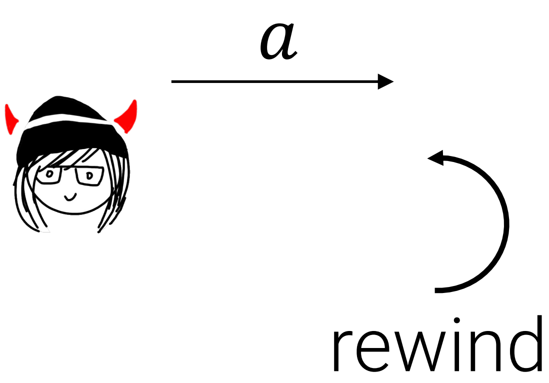

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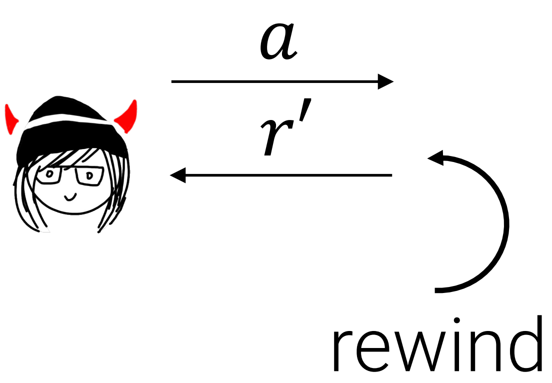

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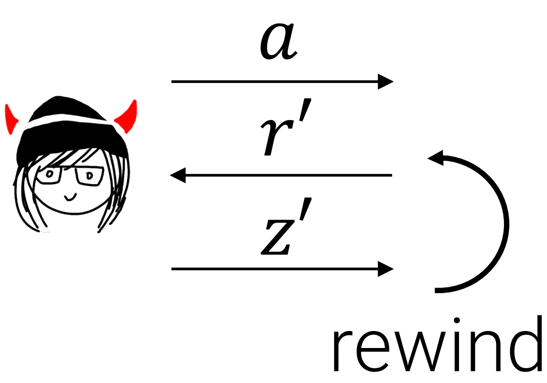

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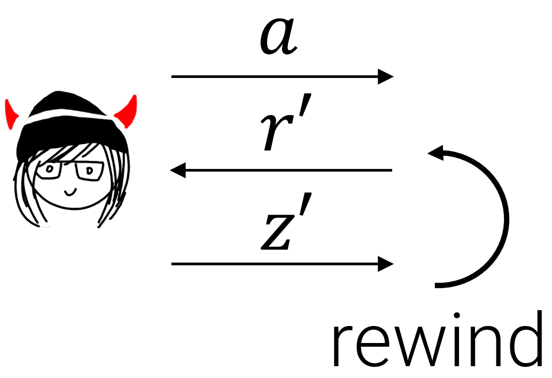

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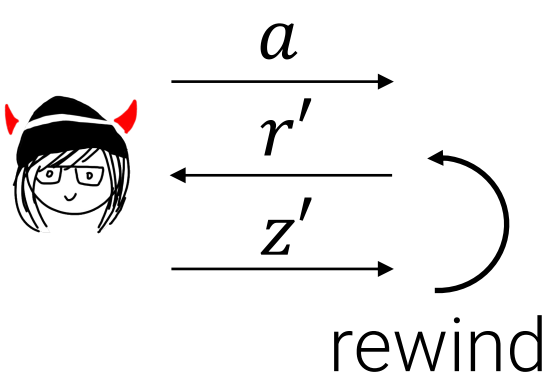

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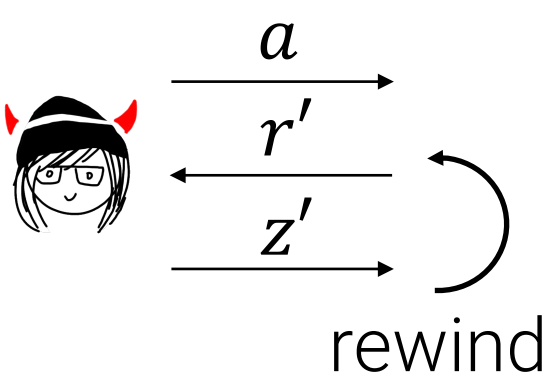

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Reduction doesn't work for quantum adversaries because measuring the response can disturb the adversary's state.

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Classical security of [BCMVV18] relies on *rewinding*

$\xrightarrow{a}$

$\xleftarrow{r'}$

$\xrightarrow{\cancel{z'}}$

rewind

[BCMVV18] Reduction

1) Record (a, r, z)
2) Rewind
3) ~~Record~~ $\cancel{(a, r', z')}$

~~break LWE~~ $\longrightarrow$

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More generally, rewinding-based security proofs are not safe!

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But rewinding is one of the most common techniques in cryptography…

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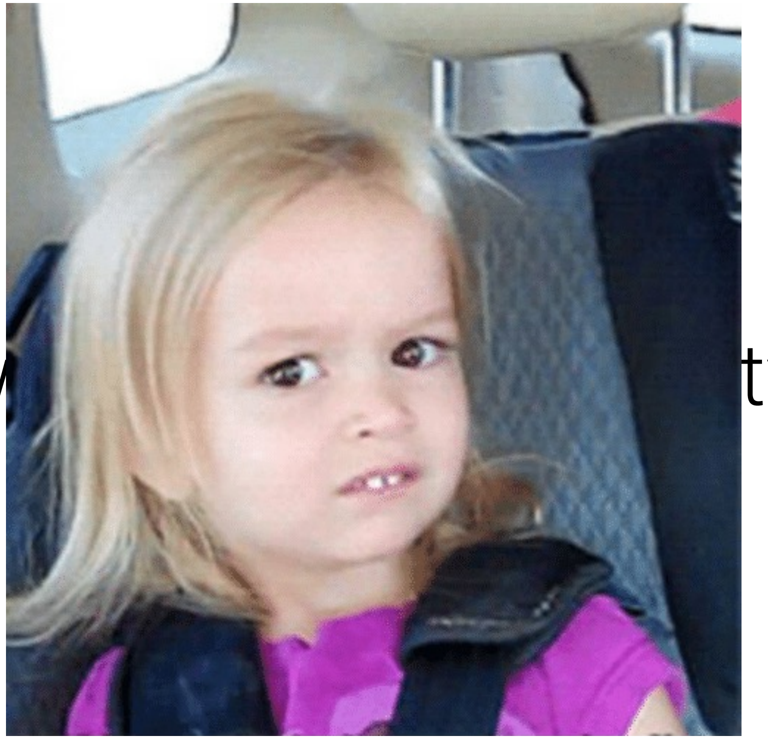

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Question for today:

When is quantum rewinding possible?

Claim: $x \in 3\text{SAT}$

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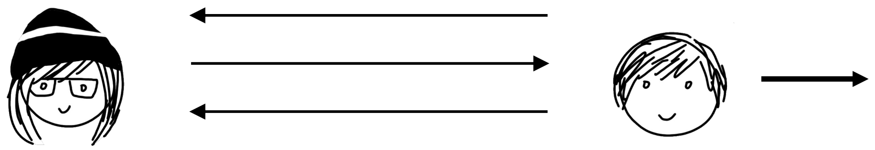

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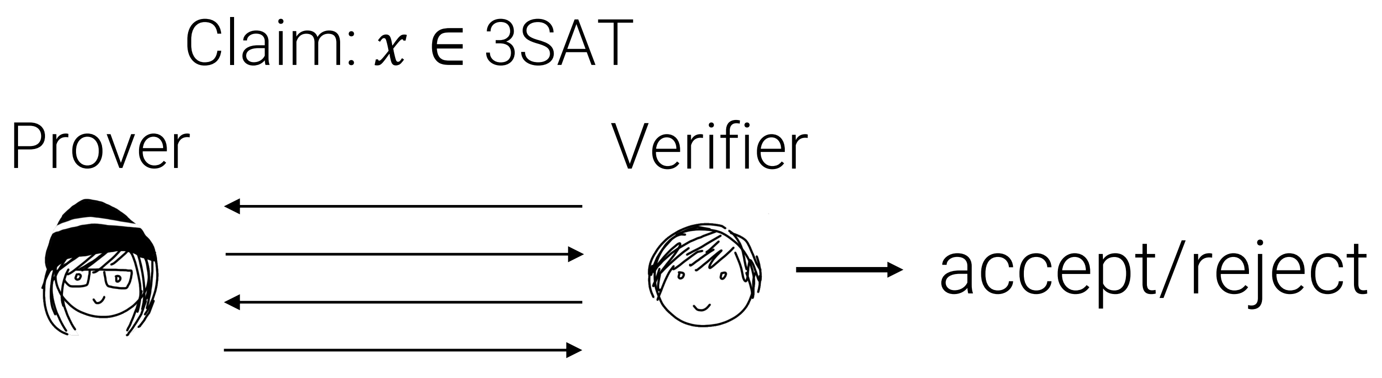
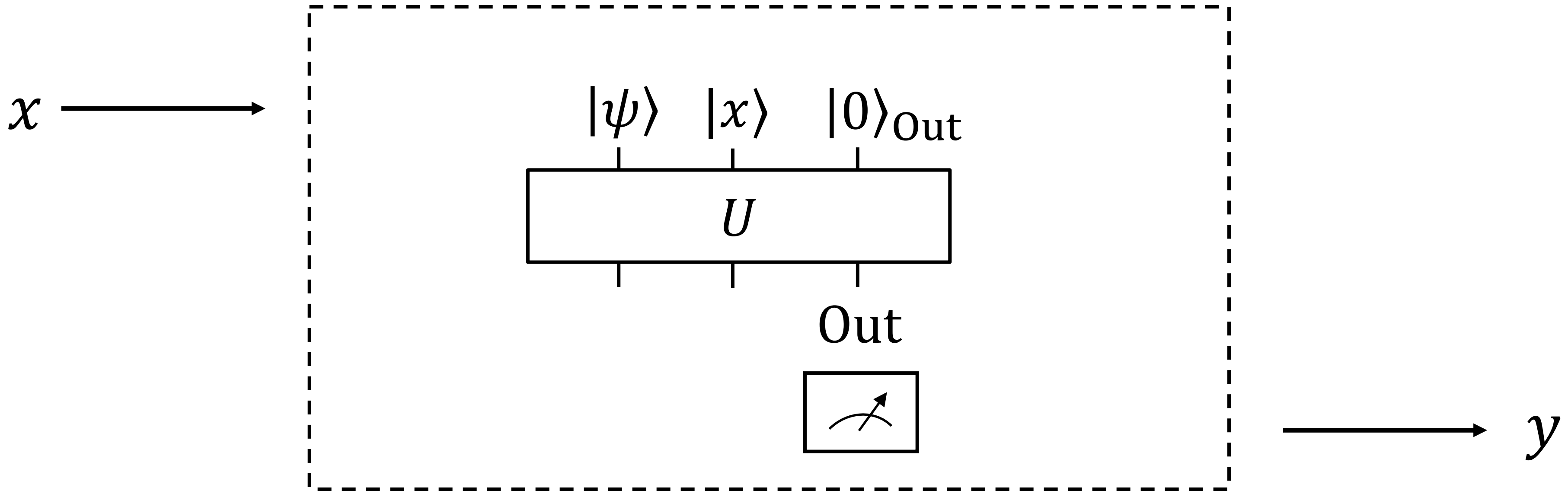


Soundness: Malicious P can't trick V into accepting a false claim.

Zero Knowledge [GMR85]: View of malicious V can be efficiently **simulated** without P.

Preliminaries: Quantum Adversary Model

(Non-Uniform) Quantum Adversary consists of efficiently computable and invertible U along with a measurement in the standard basis



(One-shot case) equivalent to efficient quantum circuit.

Interactive adversary will be stateful.

This Talk

1) Blum's protocol for graph Hamiltonicity

2) Post-Quantum Soundness of Blum

3) Post-Quantum Zero Knowledge of Blum

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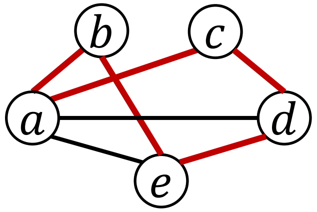
Blum's Protocol for Hamiltonian Cycles

$G =$ 

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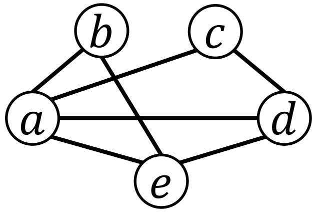
$G =$ 

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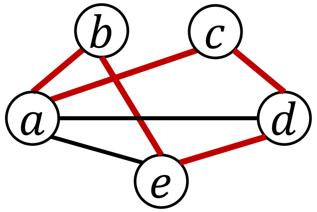
$G =$ 

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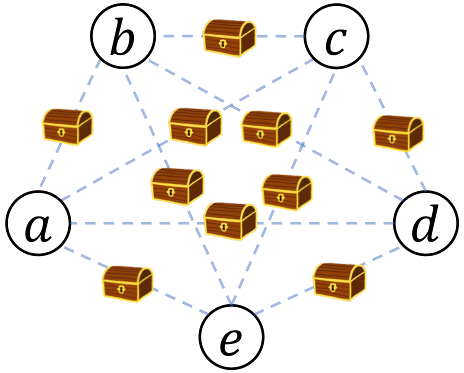
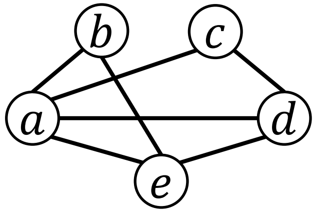


$G =$ 

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Commit to the adjacency matrix of $\pi(G)$

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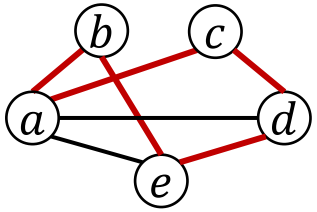
$G =$ 

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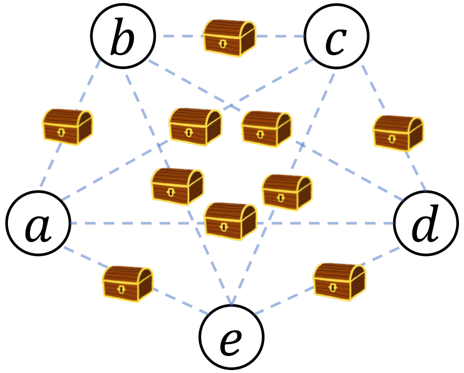
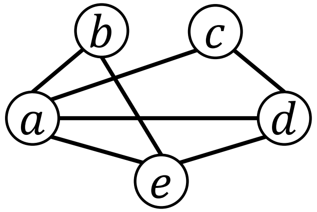


$G =$ 

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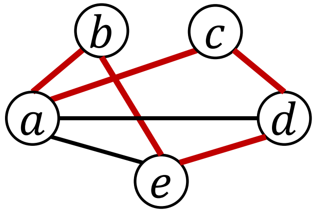
$G =$ 

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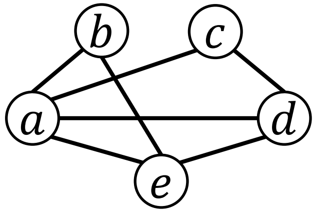
$G =$ 

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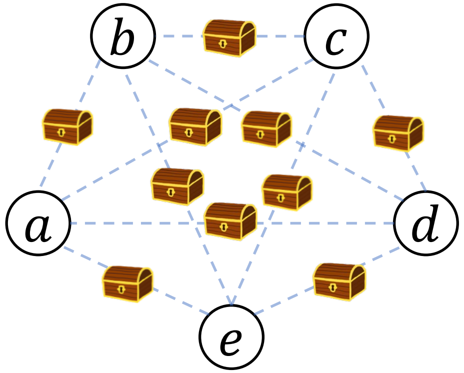

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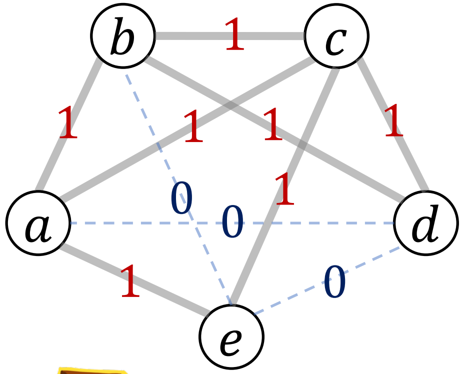
$\pi,$ 

 (if $r = 0$)

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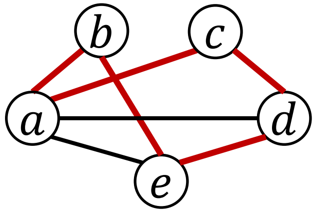
$G =$ 

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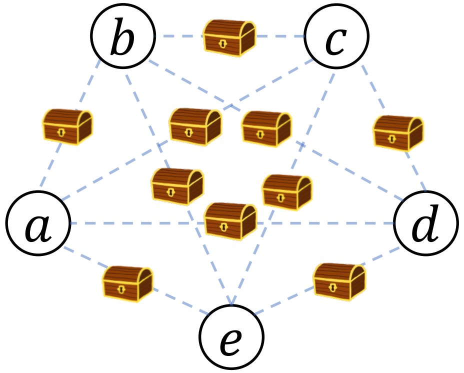
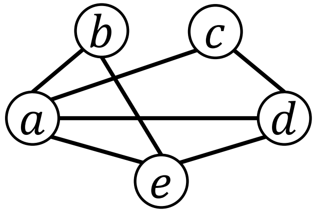


$G =$ 

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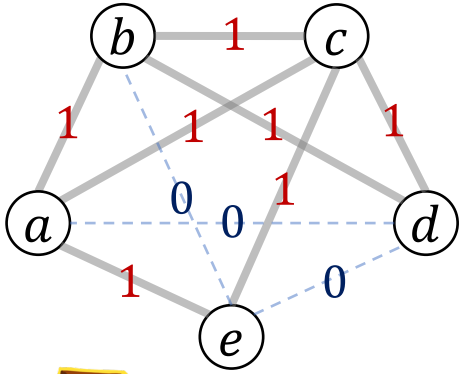
π, 

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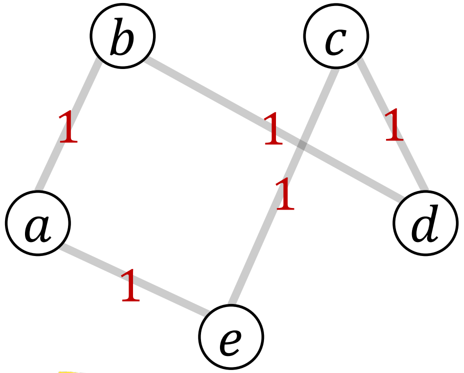
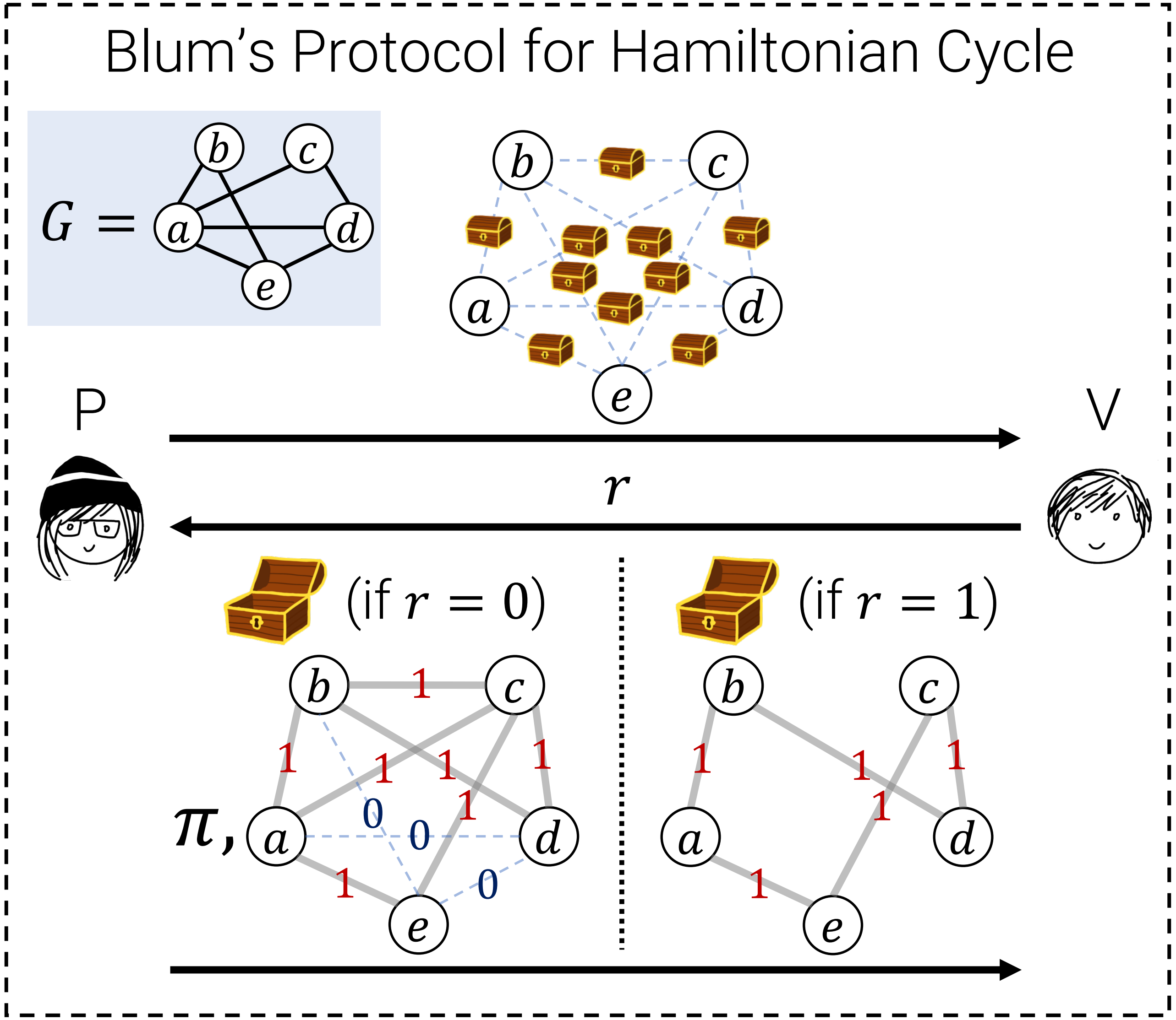
or



 (if $r = 1$)

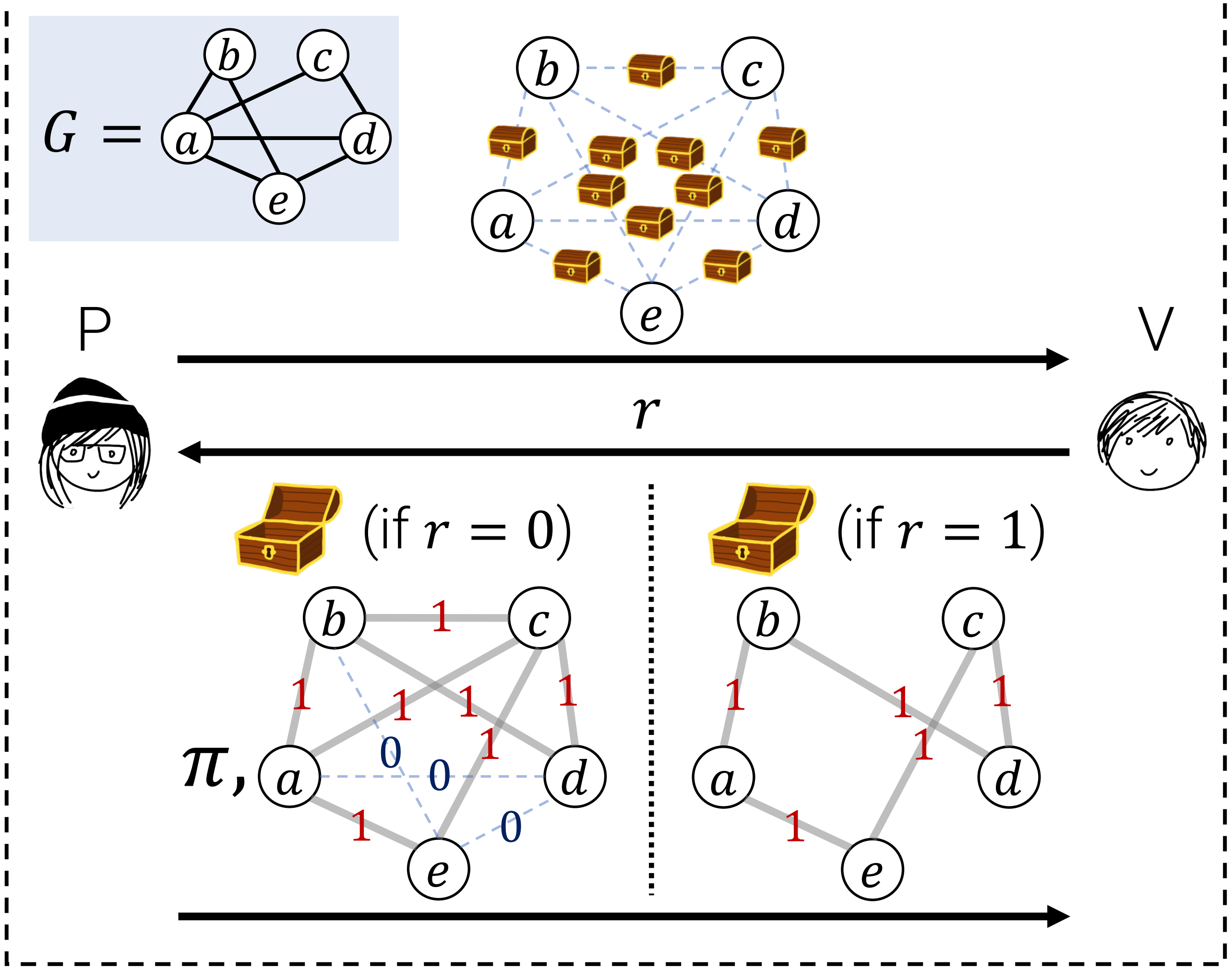


Soundness (intuition)

By *binding*, the first message determines a graph H such that:

- H is a permutation of G
- H contains a Ham cycle

Blum's Protocol for Hamiltonian Cycle



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Zero Knowledge (intuition)

First message reveals nothing since commitments are *hiding*.

Last message also reveals nothing:

- $(r = 0)$ random permutation of G
- $(r = 1)$ random cycle

This Talk

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2) Post-Quantum Soundness of Blum

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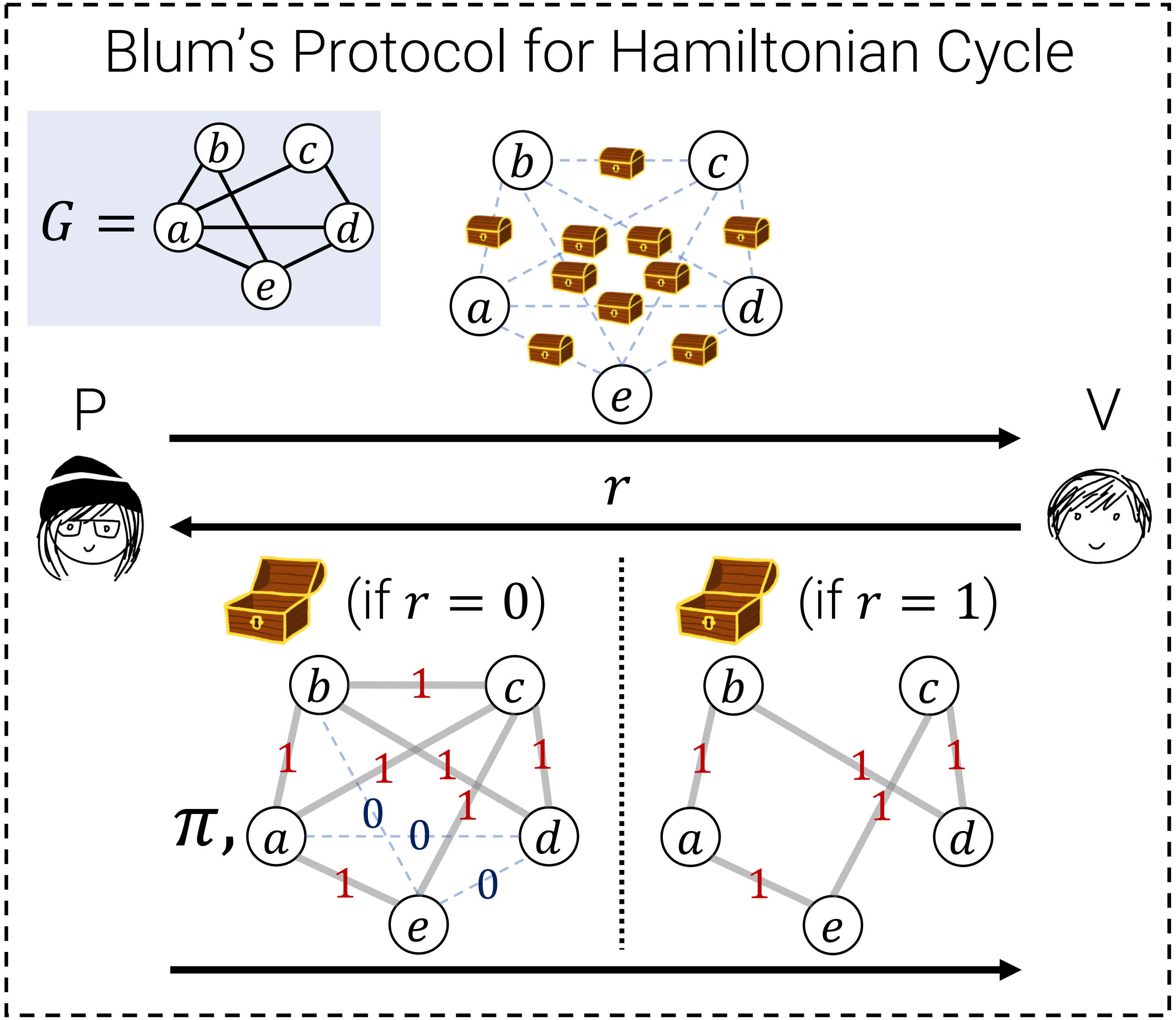
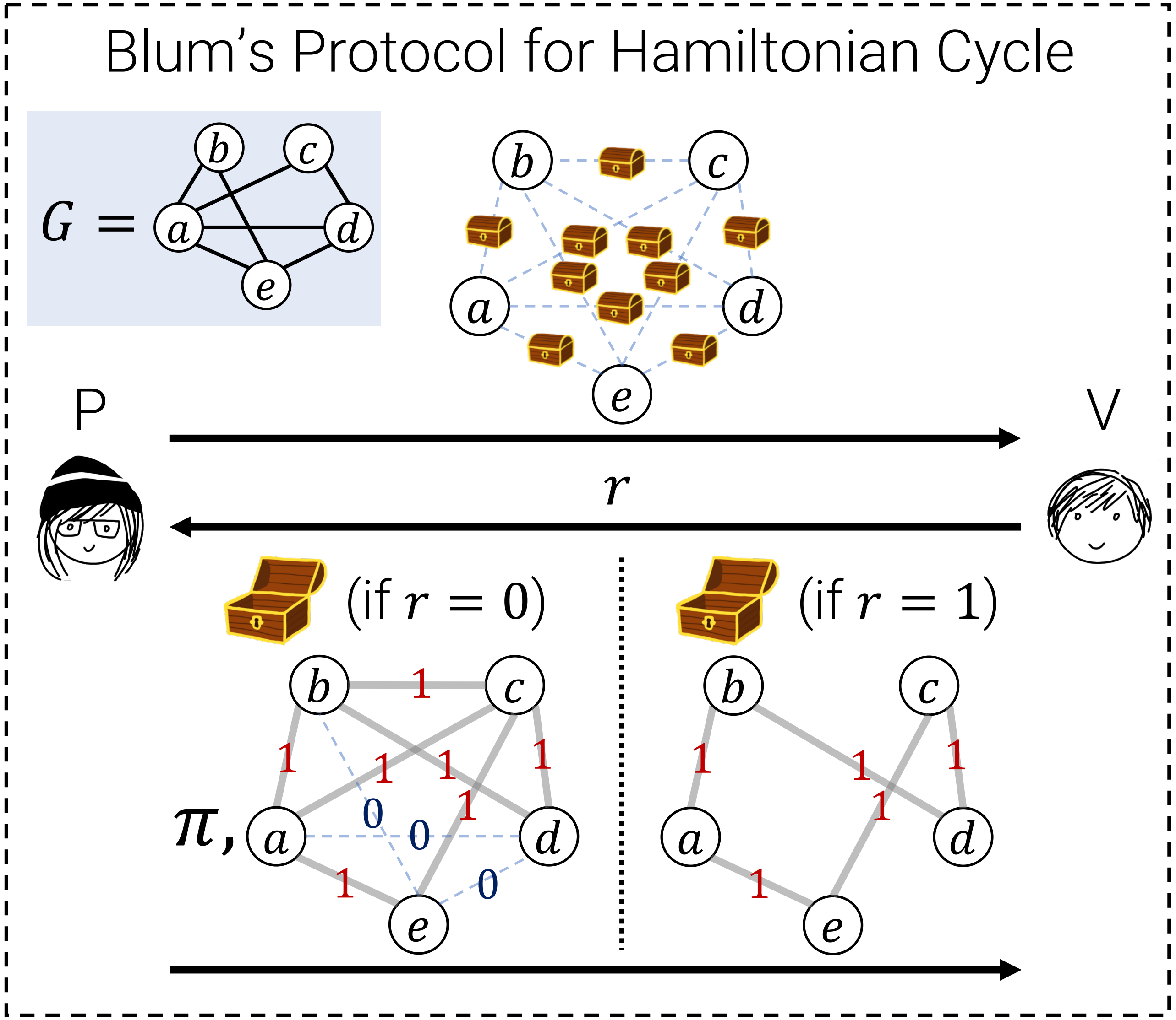
- Classical soundness
- Collapse-binding commitments
- Unruh's rewinding lemma

3) Post-Quantum Zero Knowledge of Blum



Classical Soundness

Soundness: If efficient classical P* convinces V with prob $\frac{1}{2} + \varepsilon$, then G must have a Ham cycle.



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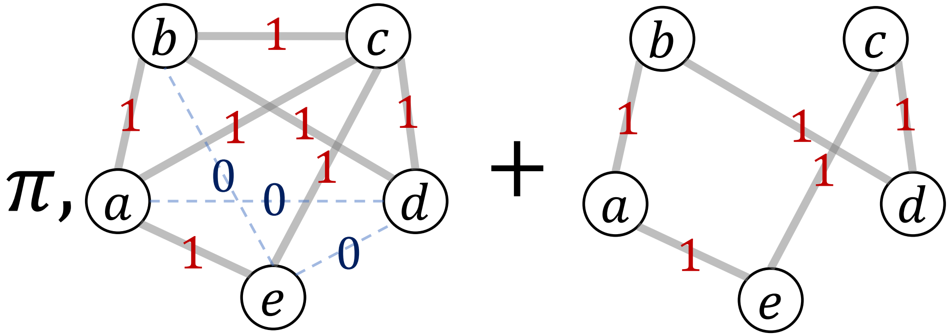
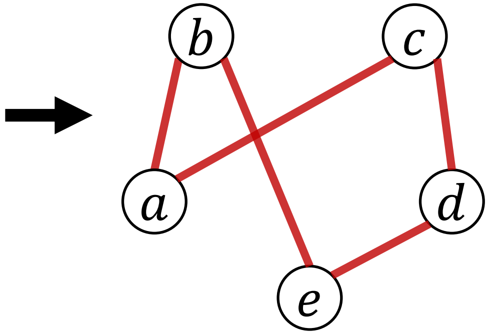
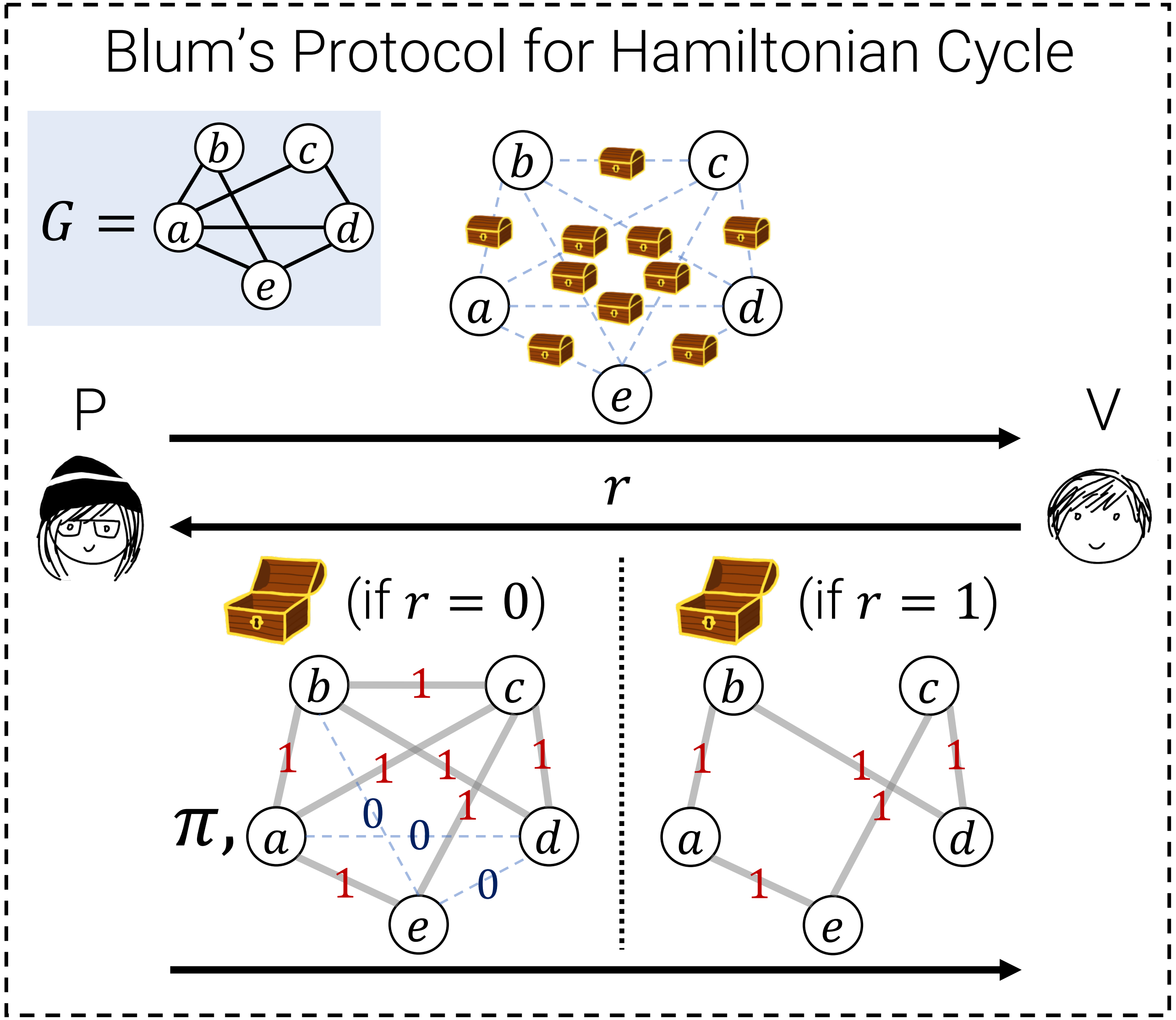
2-special soundness: Two valid transcripts → Ham cycle





Ham cycle for original G



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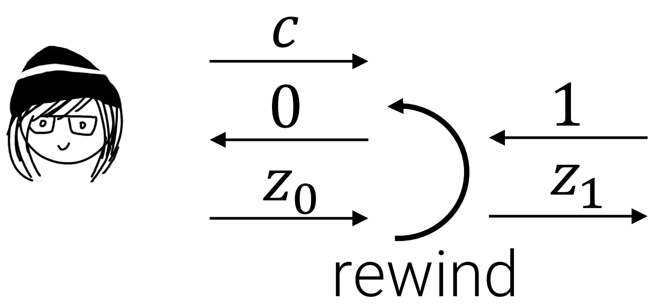
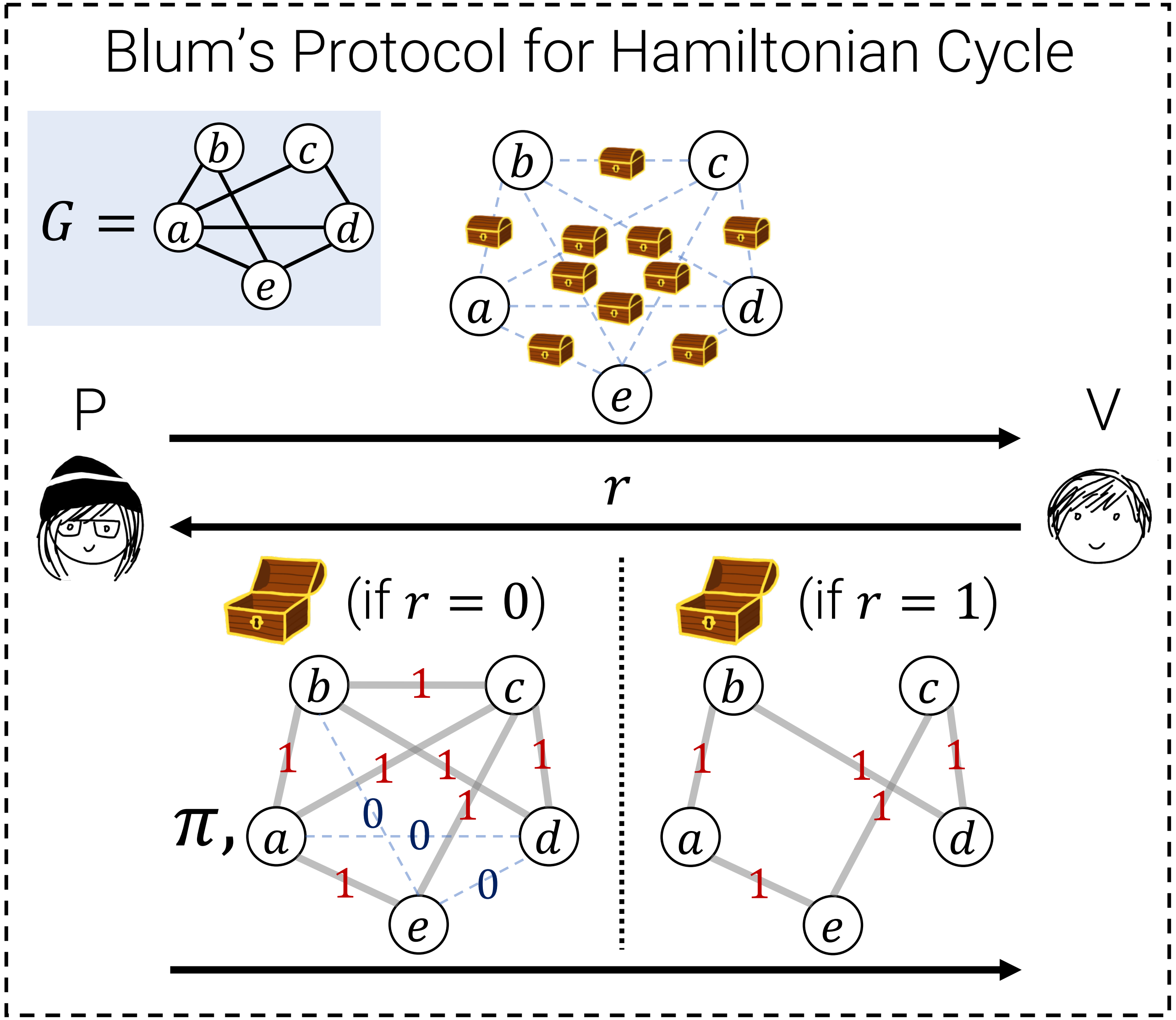
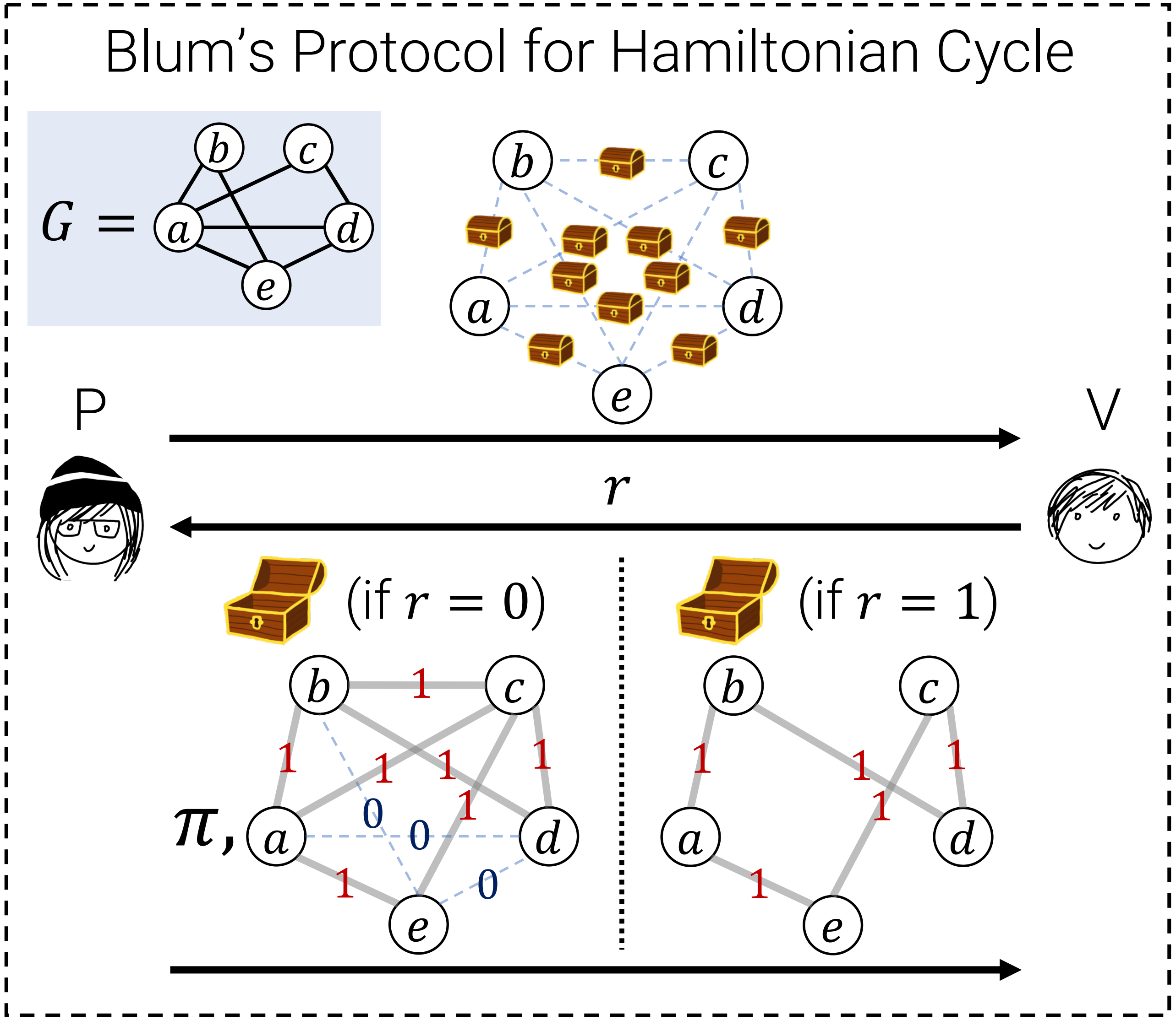
Rewinding argument: query P* once on $r = 0$ and once on $r = 1$



Probability at least $\Omega(\varepsilon)$ of two accepting responses.

Post-Quantum Soundness

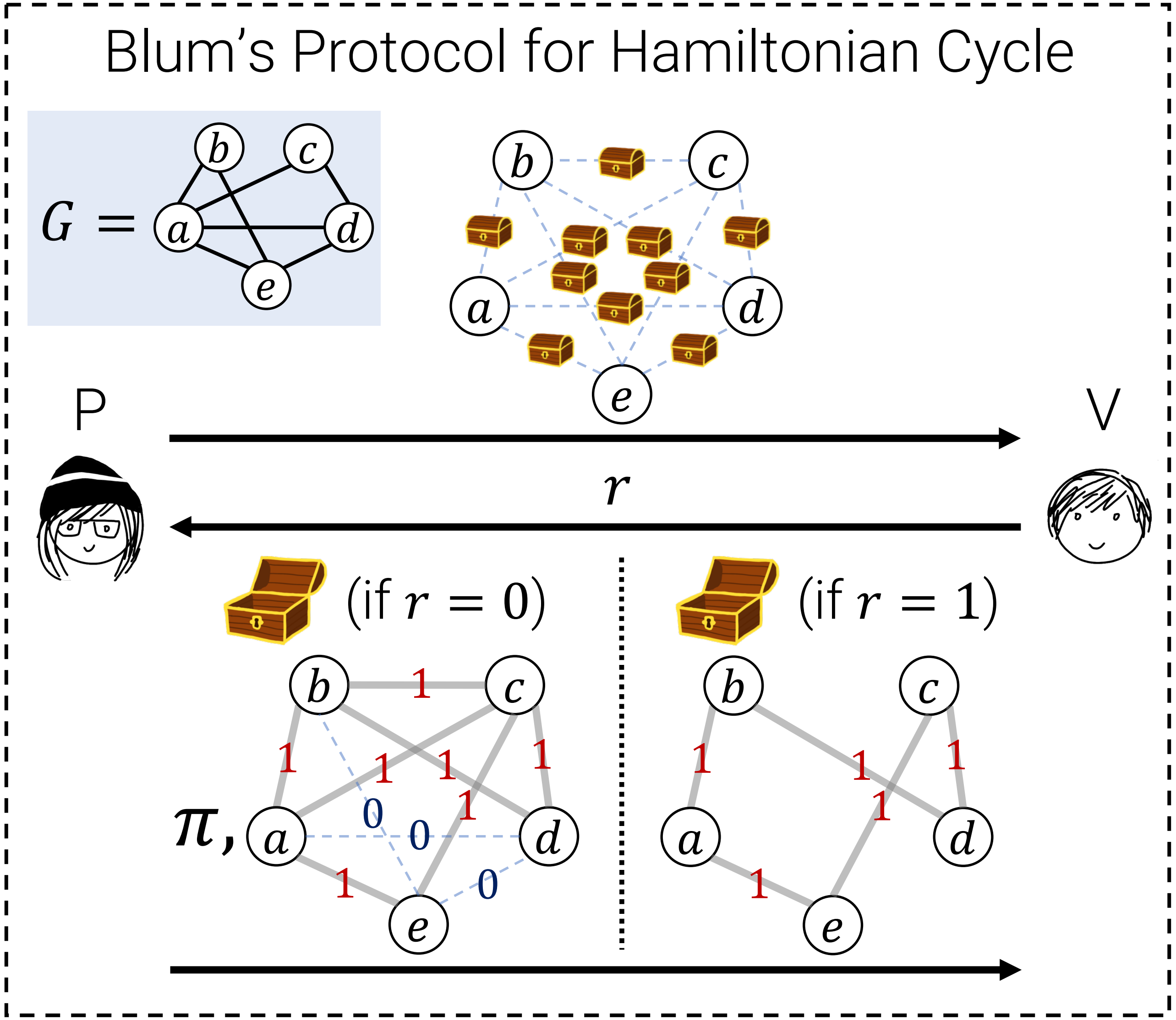




Post-Quantum Soundness

Easy case: statistically binding commitments

Soundness holds against unbounded attackers (and hence quantum)



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Interesting case: what if the commitments are only computationally binding?

Binding Against Quantum Attack

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Commitment Syntax

sender

receiver

$$\xleftarrow{\quad ck \quad}$$

$$\xrightarrow{\quad c \,=\, \mathrm{Com}_{ck}(m;d) \quad}$$

$$\xrightarrow{\quad m,d \quad}$$

Accept if $c = \mathrm{Com}_{ck}(m;d)$.

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$\xleftarrow{\;ck\;}$

$\xrightarrow{\;c \,=\, \mathrm{Com}_{ck}(m;d)\;}$

$\xrightarrow{\;m,d\;}$

Accept if $c = \mathrm{Com}_{ck}(m;d)$.

Classical definition:

PPT adversary can't output c and valid $(m_0, d_0), (m_1, d_1)$ for $m_0 \neq m_1$.

Binding Against Quantum Attack

Before we can analyze soundness, we need to answer a basic question:

What does it mean for a commitment to be *computationally binding*?

Commitment Syntax

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[ARU14]: No!

What's wrong with this definition?

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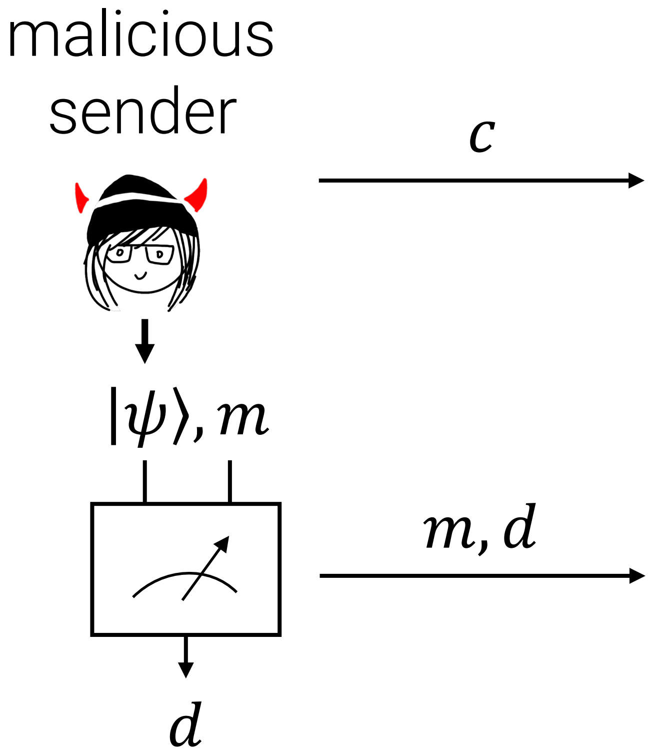

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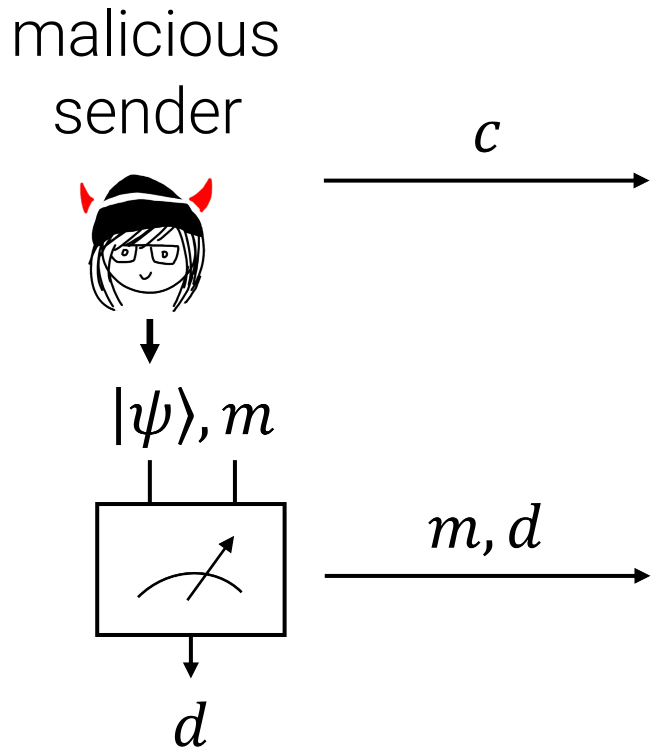

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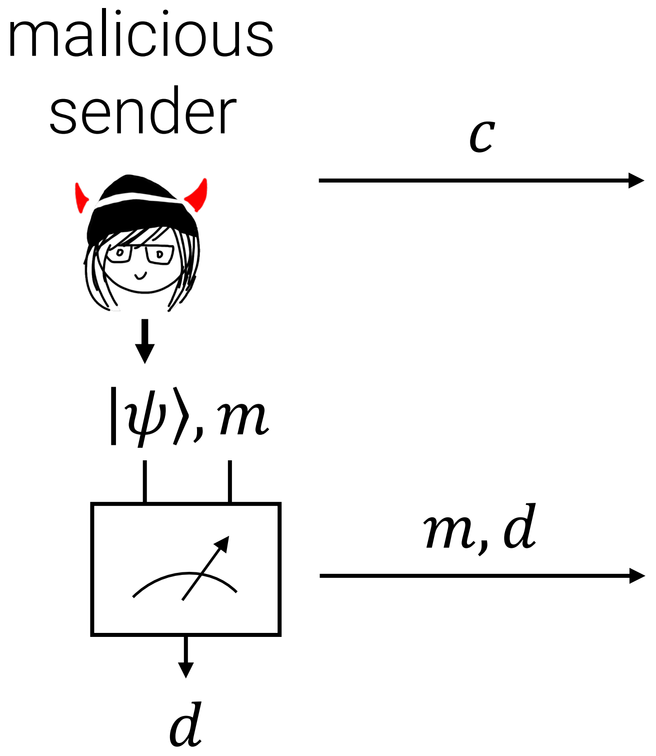

*Caveat: assuming a quantum oracle

**Open: construct example without oracles

A Better Definition: Collapse-Binding

Suppose commitment is *perfectly* binding.

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This will be the notion of binding used throughout today

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What does collapse-binding have to do with soundness?

Lazy measurement of prover responses

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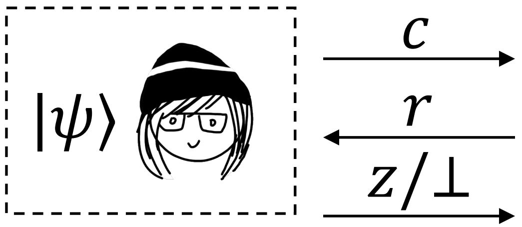

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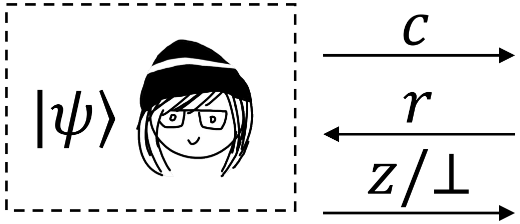

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Thus, we can forget about measuring z and pretend we only measure $(\Pi_r, I - \Pi_r)$

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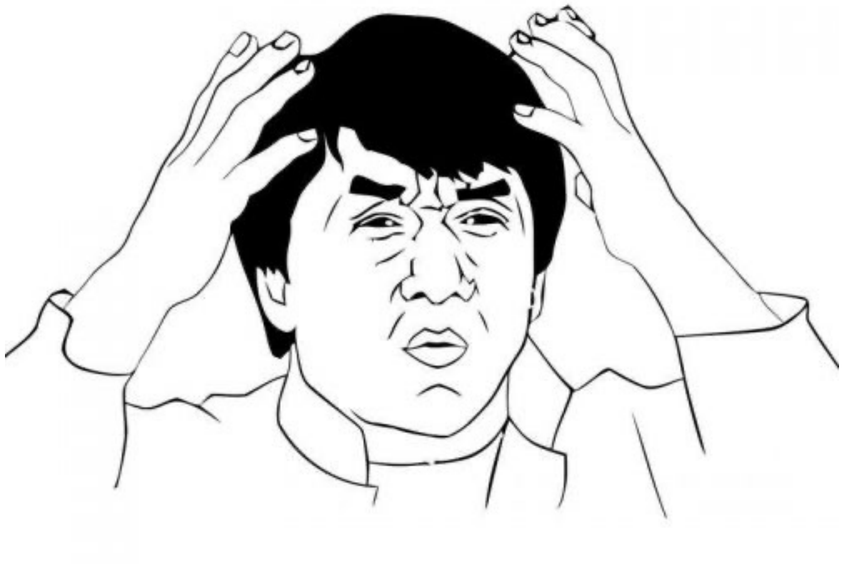

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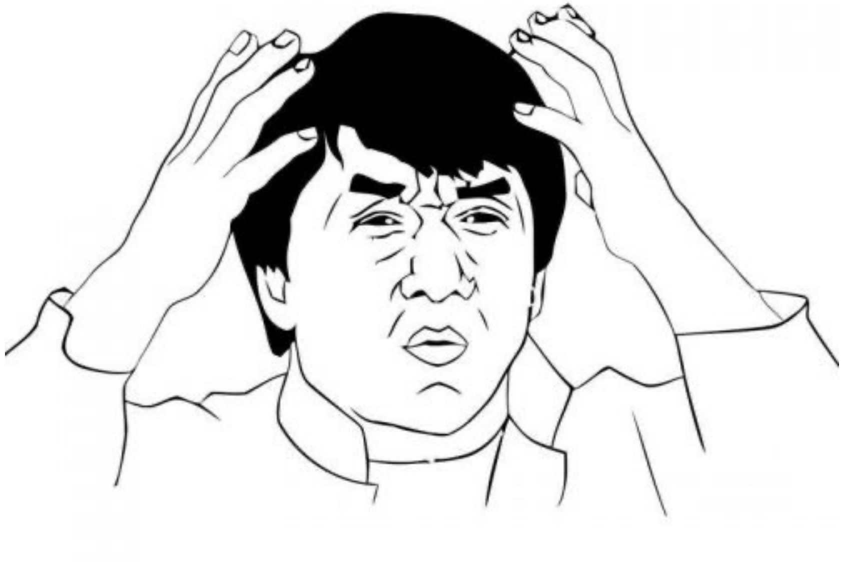

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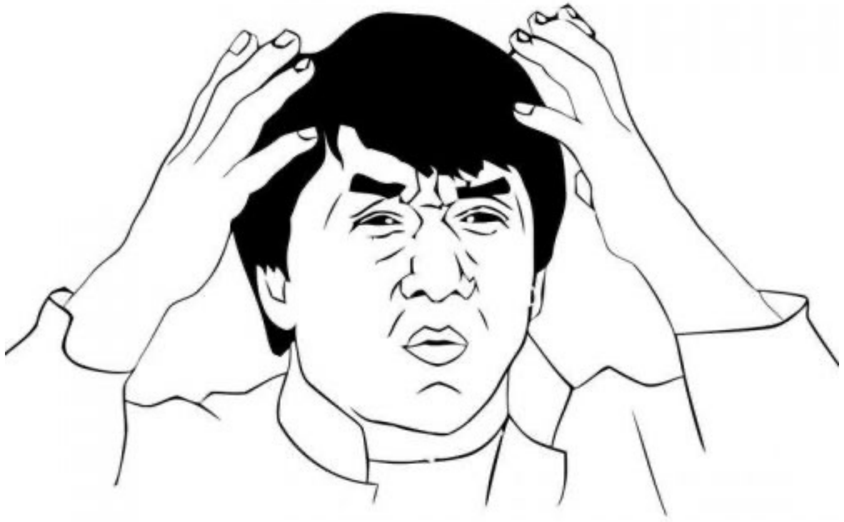

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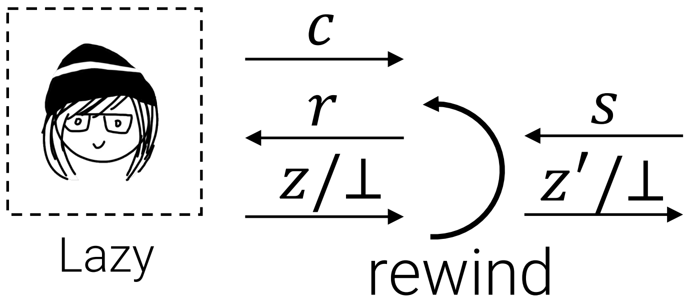

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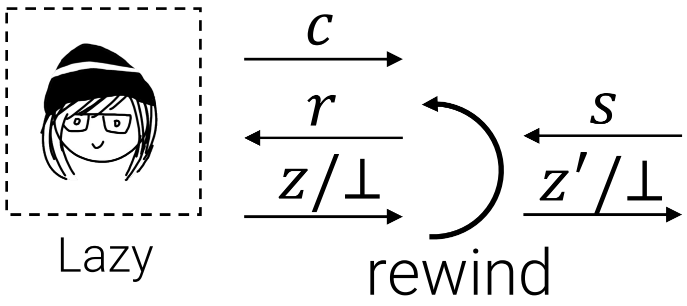

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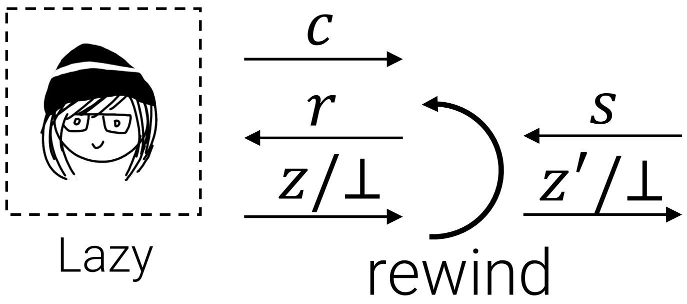

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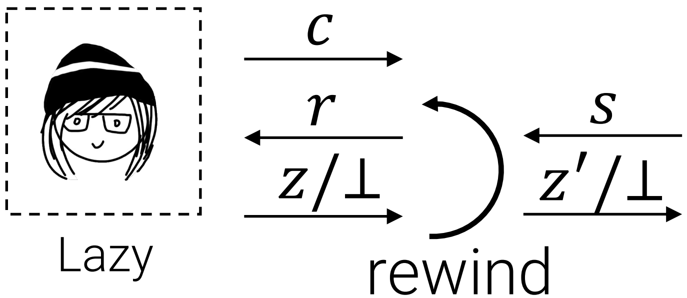

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Proof: Gentle Measurement Lemma

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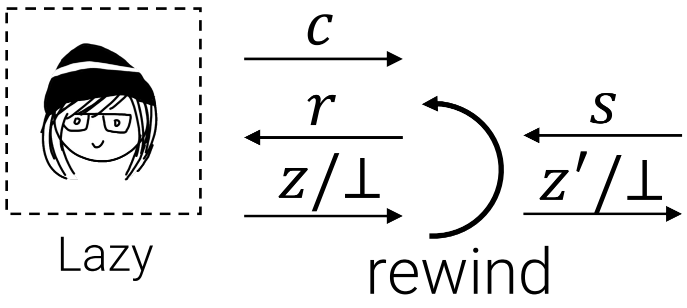

Obtain witness with decent probability by invoking:

1. (**Collapsing**) just need to analyze binary outcome measurements.
2. (**Gentle measurement**) random Π_r, Π_s both accept with good probability.
3. (**Special soundness**) two transcripts reconstruct witness.

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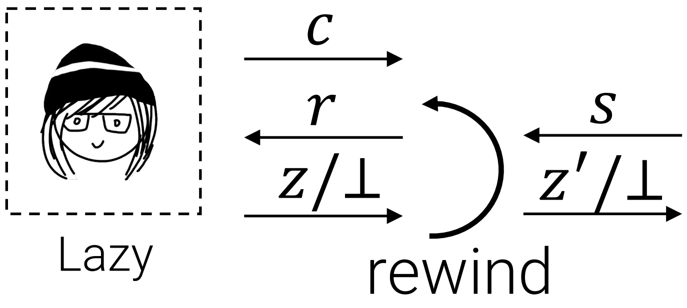

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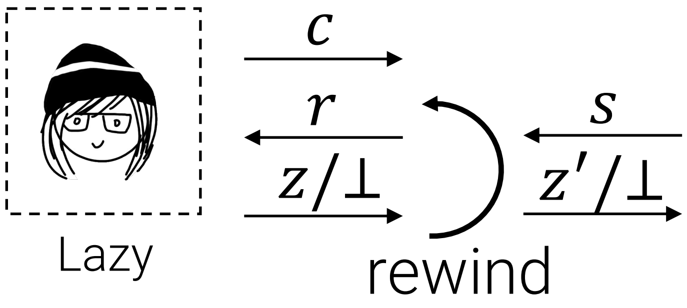

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Open: is there a simple proof that Blum has soundness error ½?

This Talk

1) Blum’s protocol for graph Hamiltonicity ✓

2) Post-Quantum Soundness of Blum ✓

3) Post-Quantum Zero Knowledge of Blum

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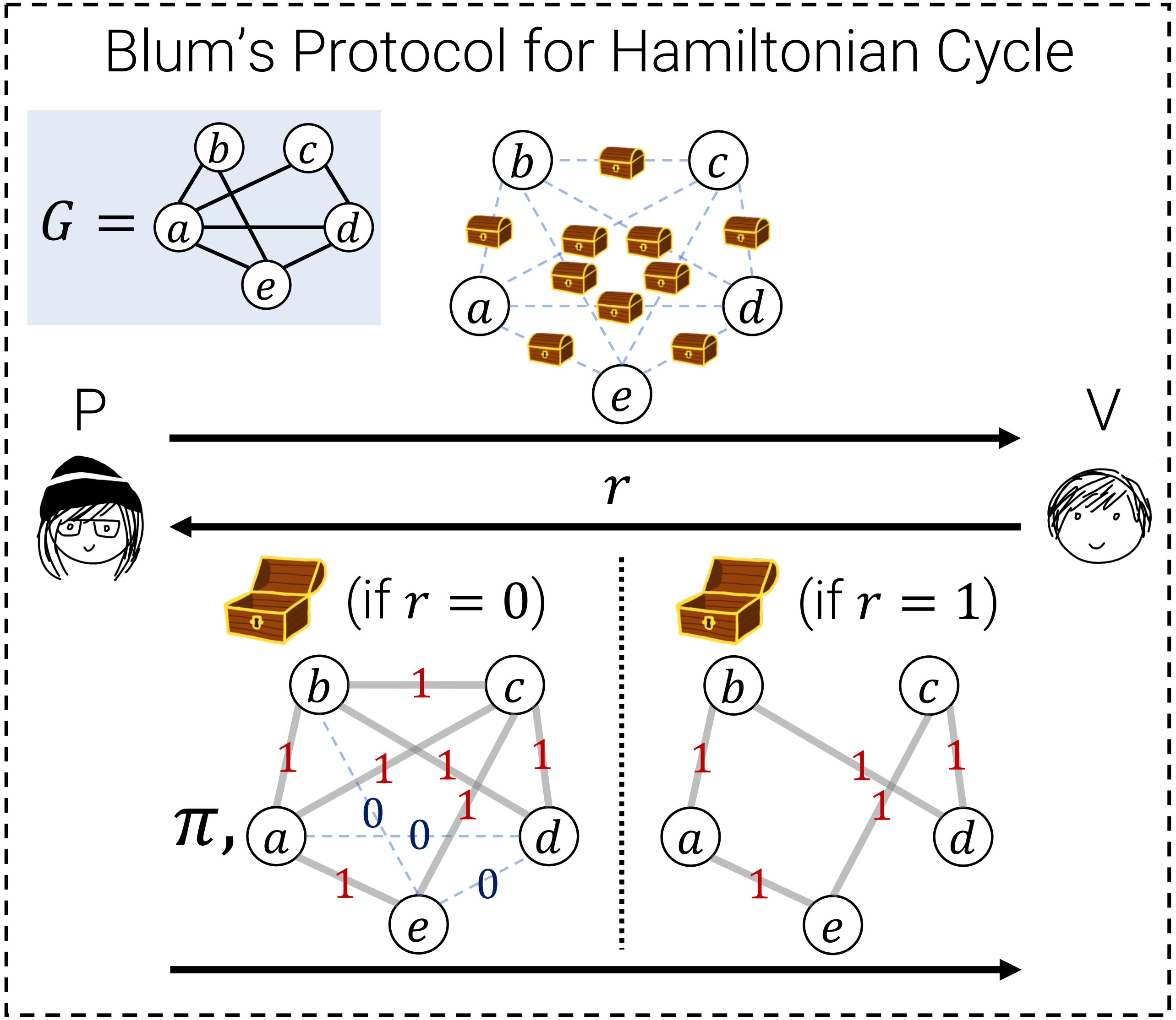
- Classical zero knowledge
- Watrous rewinding with alternating measurements
- Analysis: Jordan's lemma

Classical Zero Knowledge



Key Property: can simulate honest verifier that sends random bit

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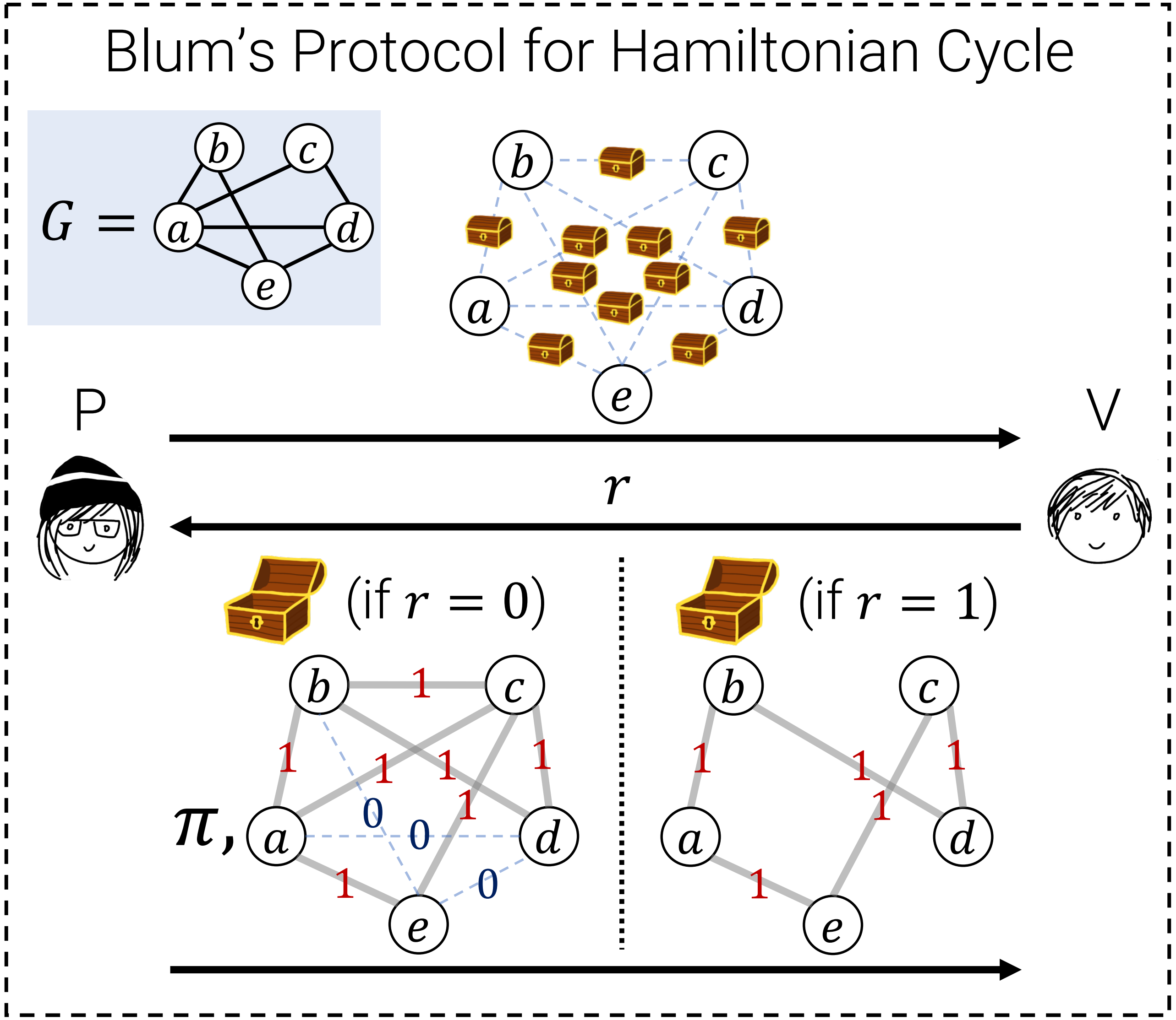

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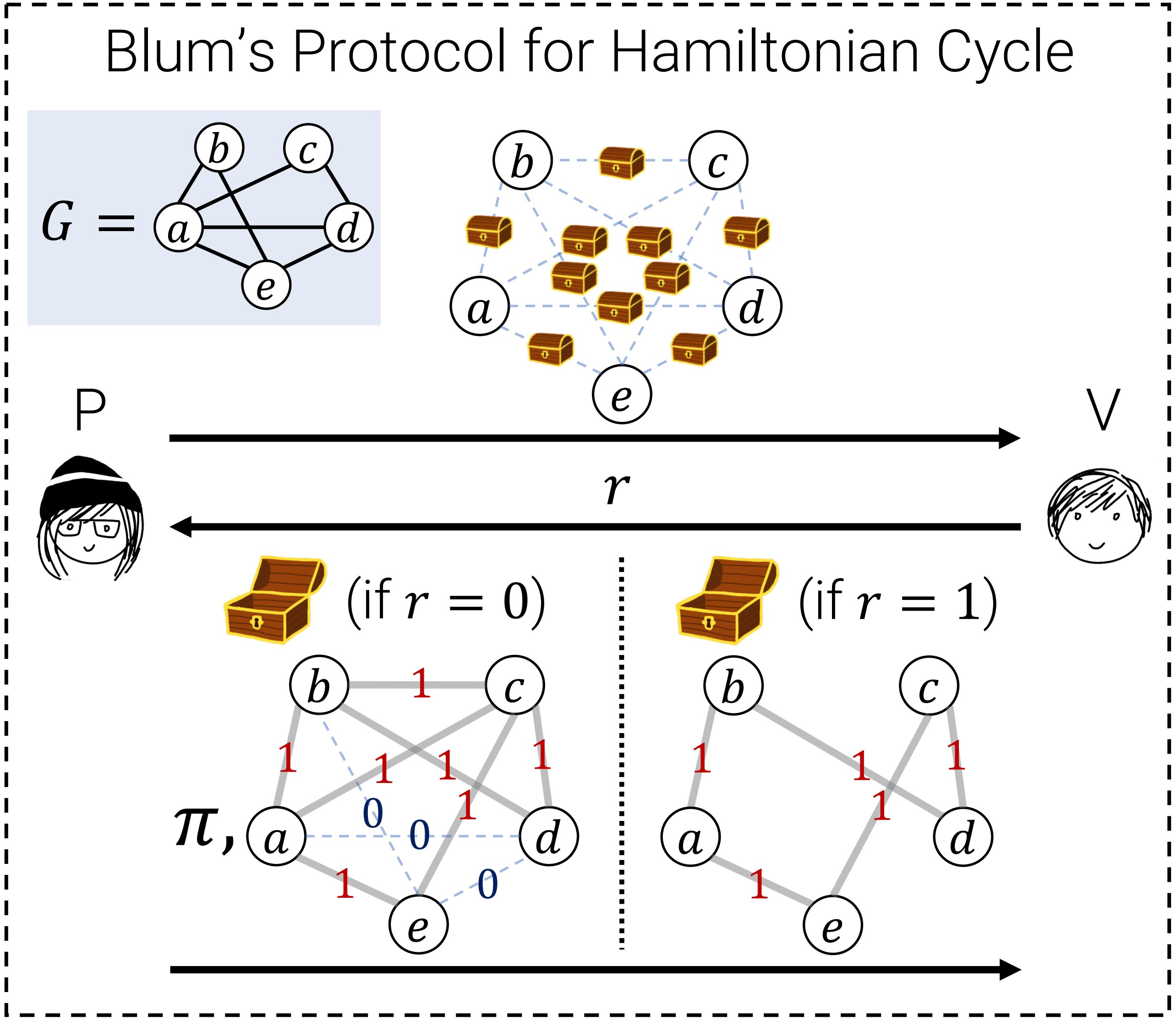

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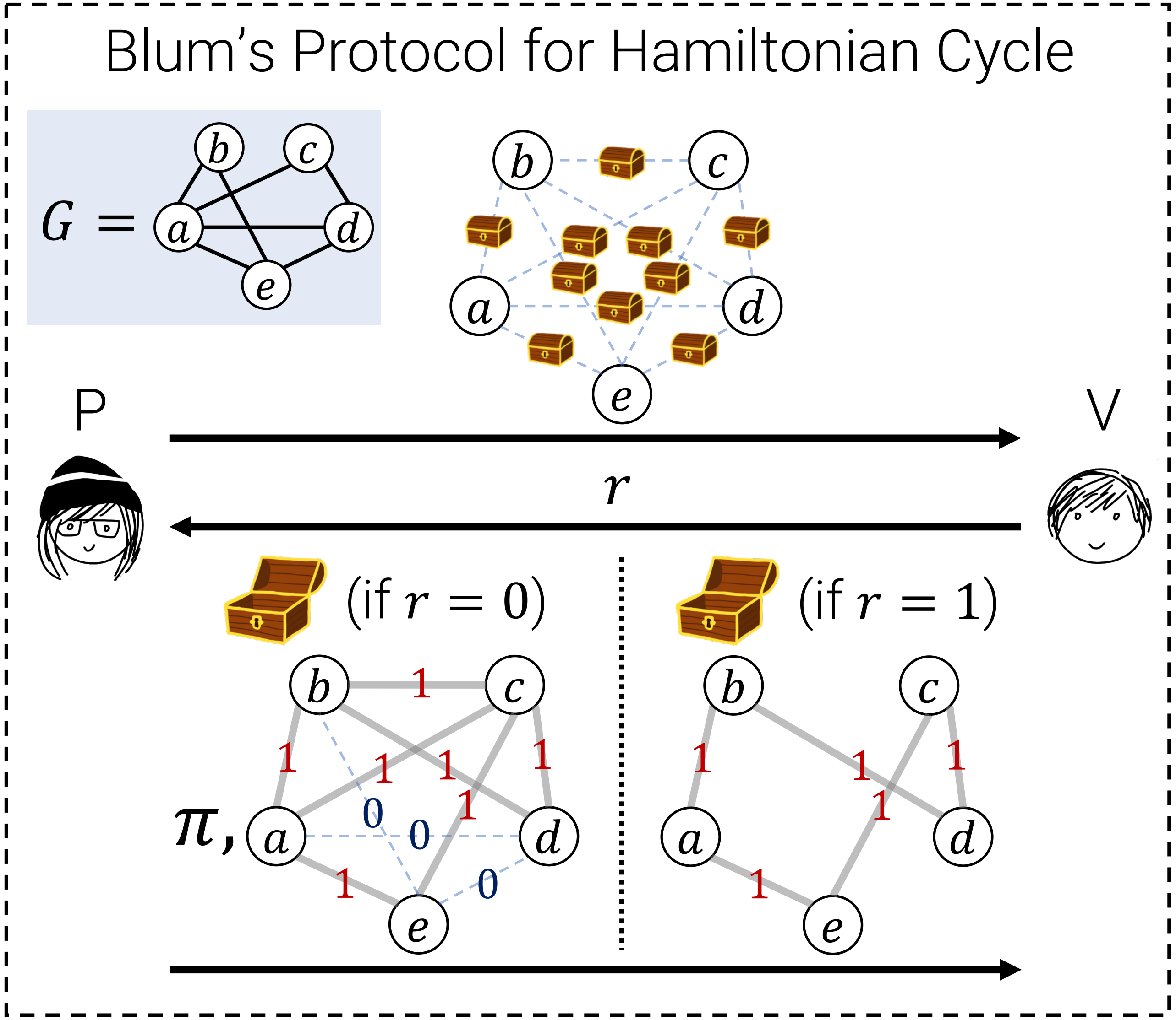

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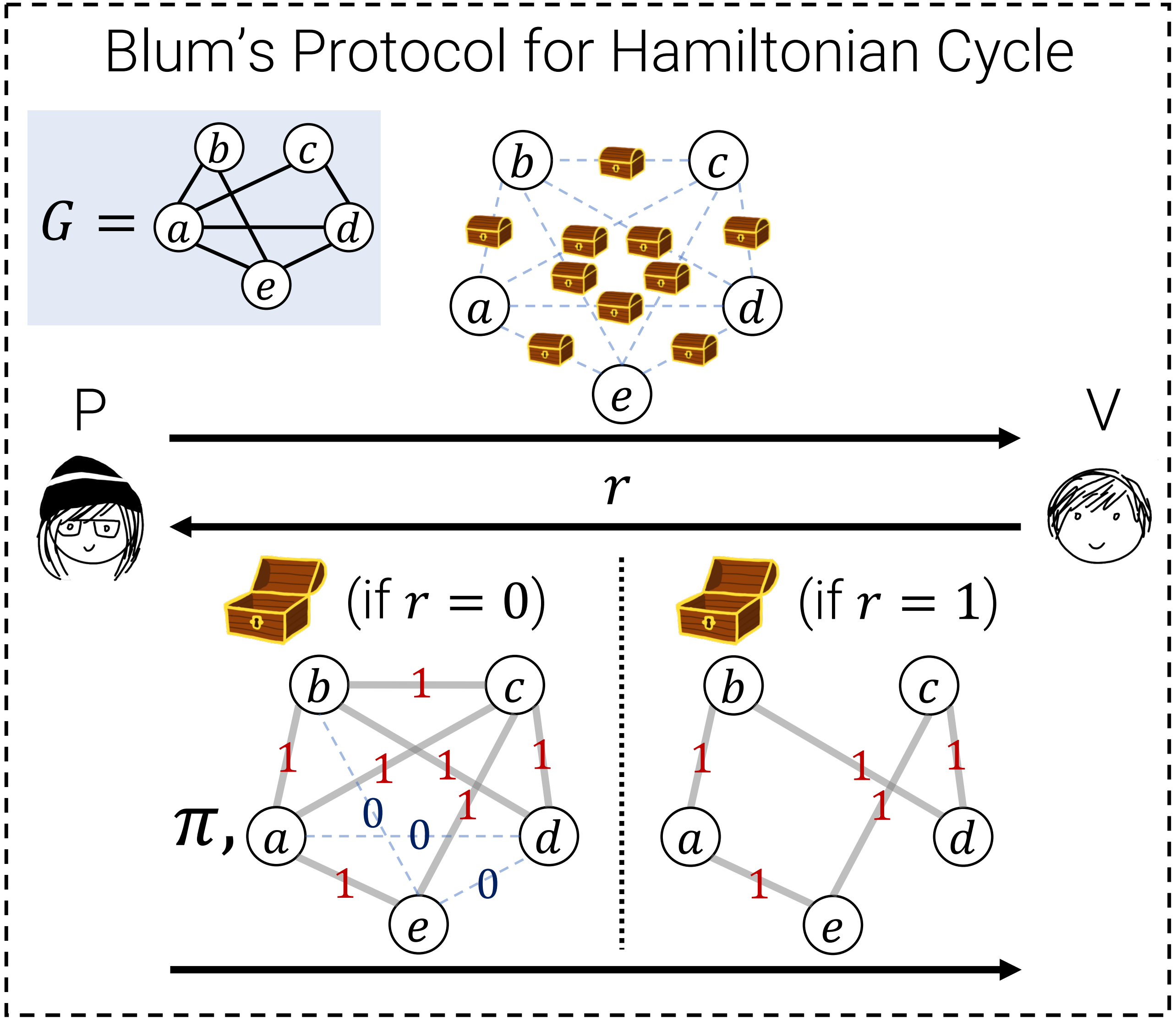

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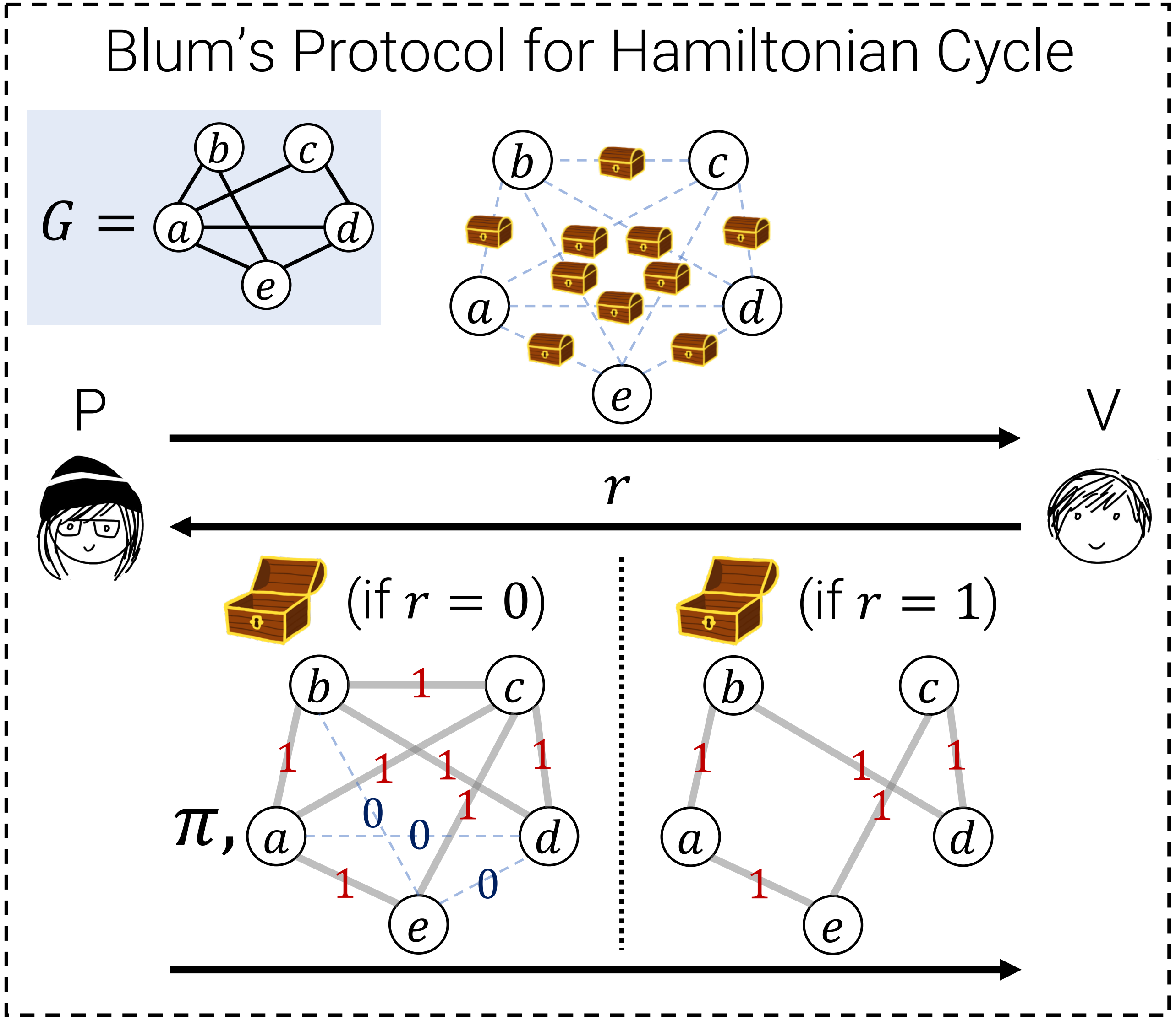

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By hiding, $(c, r', z) \leftarrow \mathrm{HVSim}$ looks like honest-verifier view.

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This leads to the complete ZK simulator:

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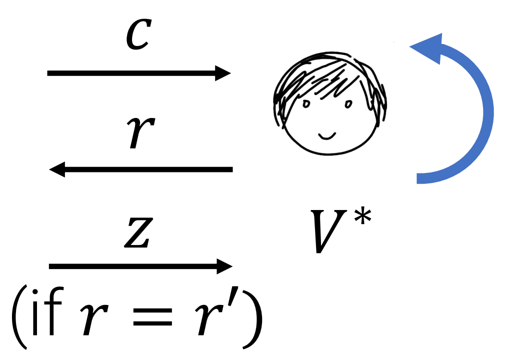

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Unfortunately, this simulator won't suffice for post-quantum ZK! If a malicious V^* has an unknown initial state $|\psi\rangle$ running $\mathrm{Guess}(V^*, |\psi\rangle)$ may irreversibly disturb it.

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But there is a different simulator due to [Watrous05] that works.

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[Watrous05]: If commitment scheme is hiding, then the Blum protocol is post-quantum ZK.

Post-Quantum ZK of Blum [Watrous05]

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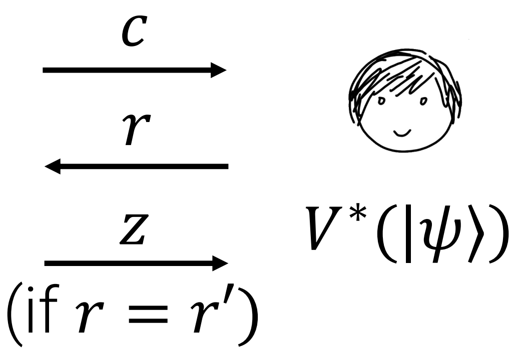

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$V^*(|\psi\rangle)$

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We'll write this process as a quantum circuit on $|\psi\rangle$.

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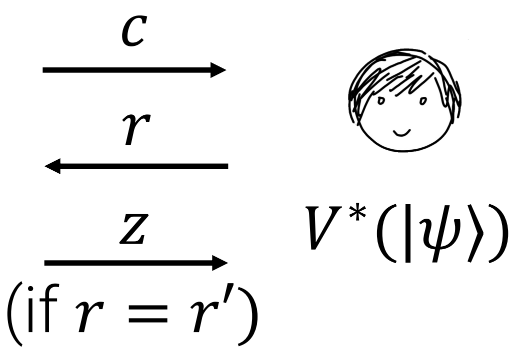

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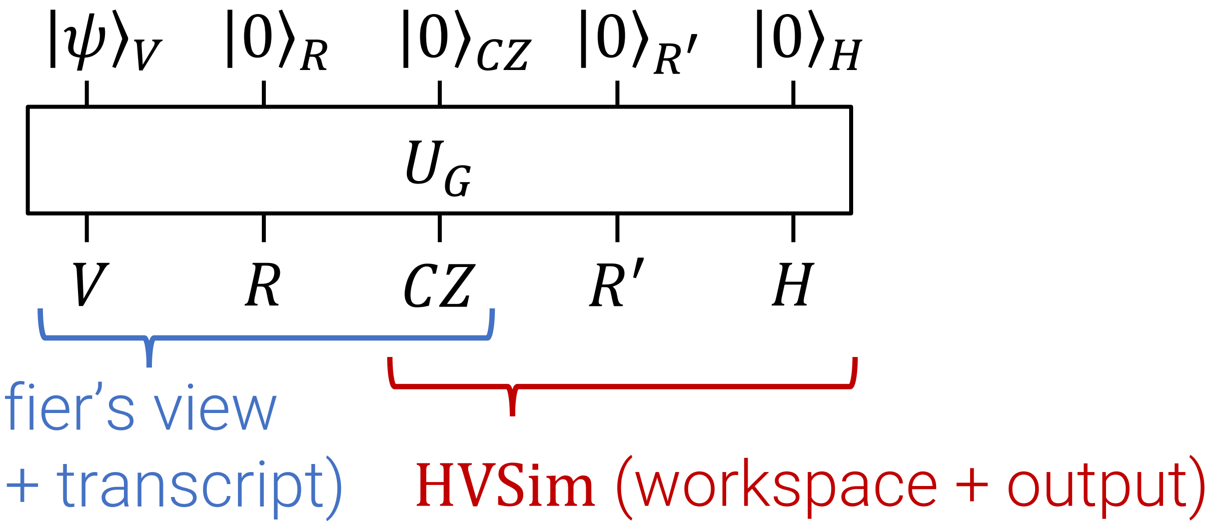

- Computing $U_G|\psi\rangle|0\rangle$ and checking if $R = R'$ is the same as running $\mathbf{Guess}(V^*, |\psi\rangle)$.

Post-Quantum ZK of Blum [Watrous05]

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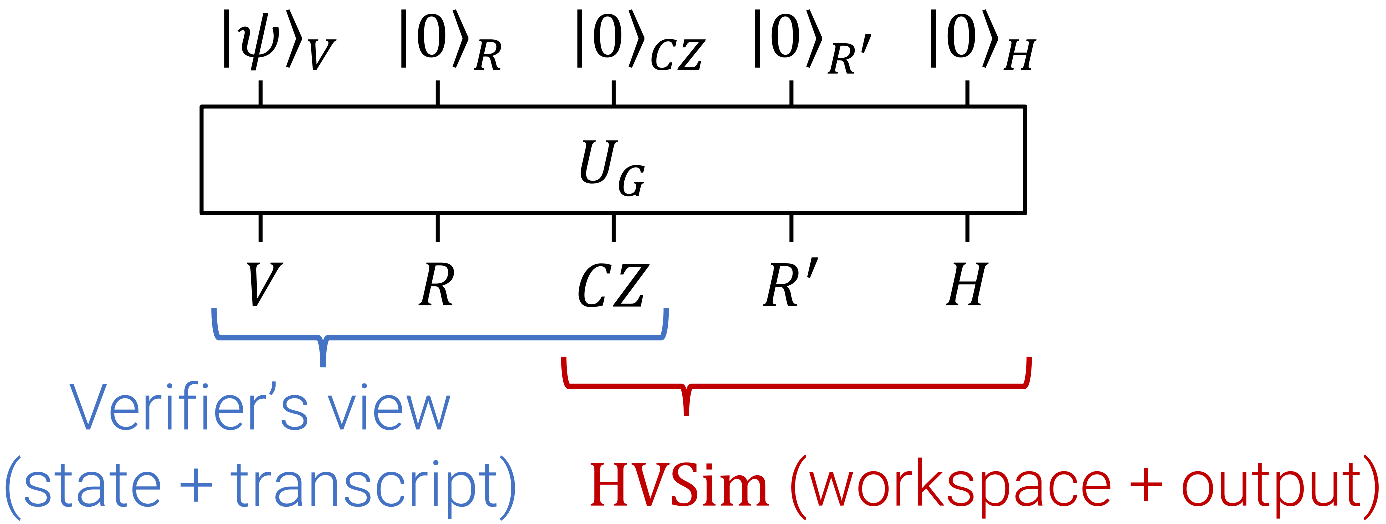

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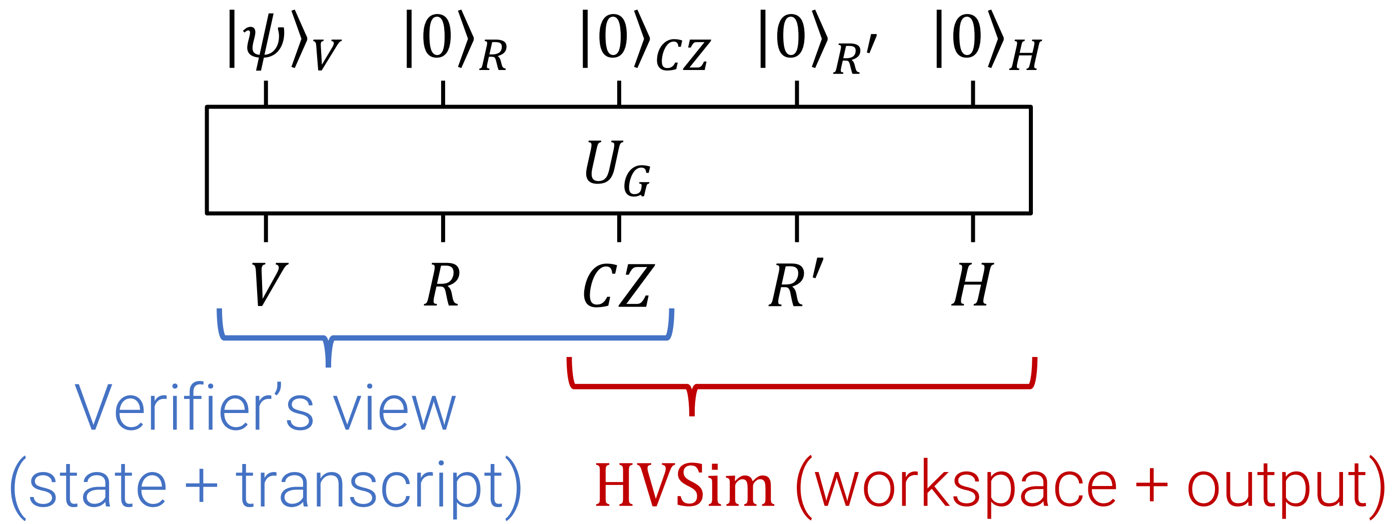

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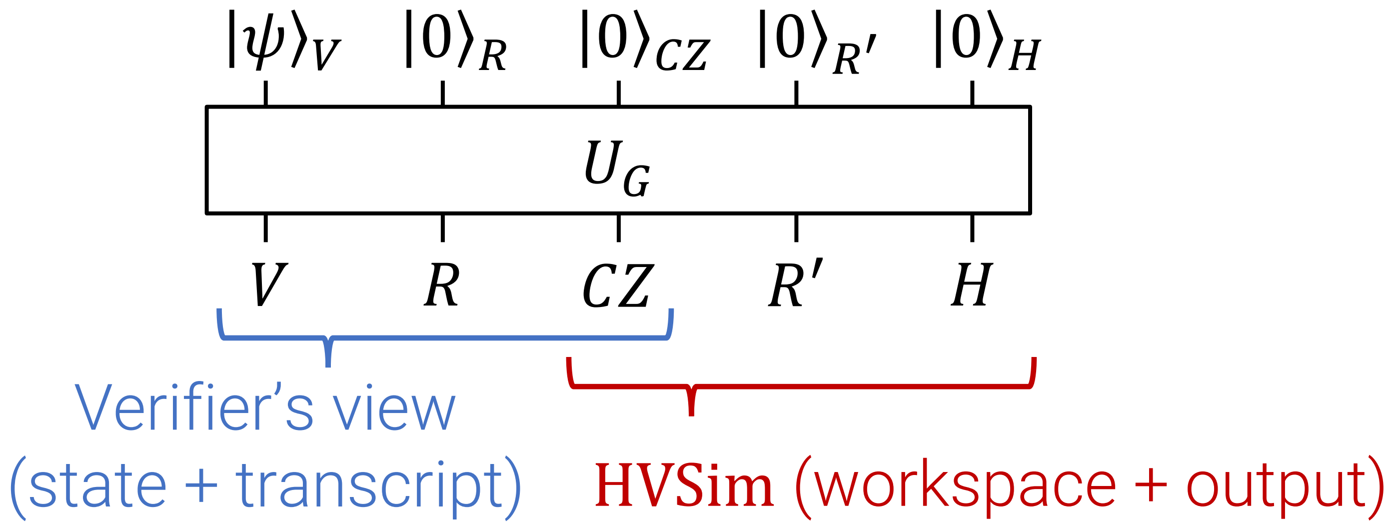

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Rough Intuition:

- Each $(\Pi_G, \mathbb{I} - \Pi_G)$ measurement is one simulation attempt.
- Applying $(\Pi_G, \mathbb{I} - \Pi_G)$ *twice in a row* gives the same outcome (no help).
- We'll write down an M_0 measurement to "reset" each attempt.

The Post-Quantum ZK Simulator [MW05, W05]

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$|\psi\rangle_V$ –

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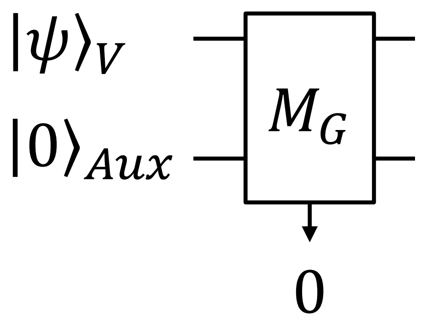

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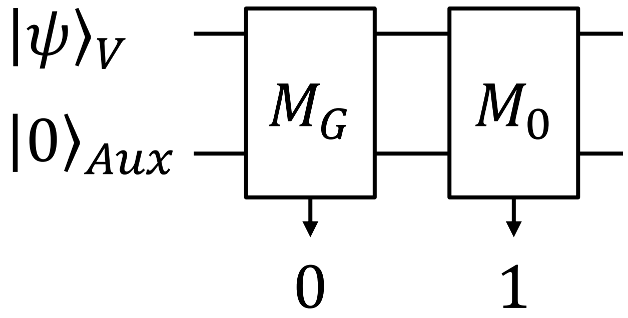

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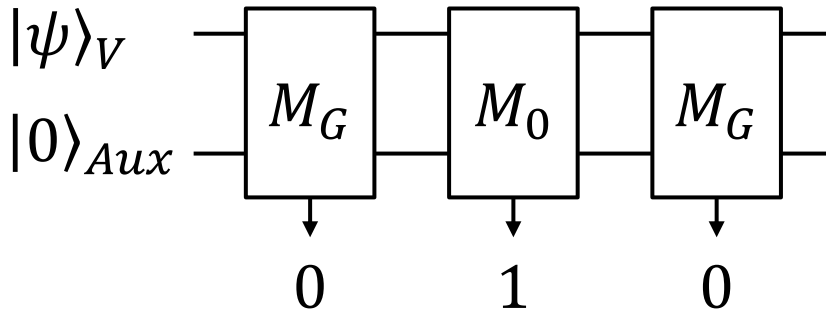

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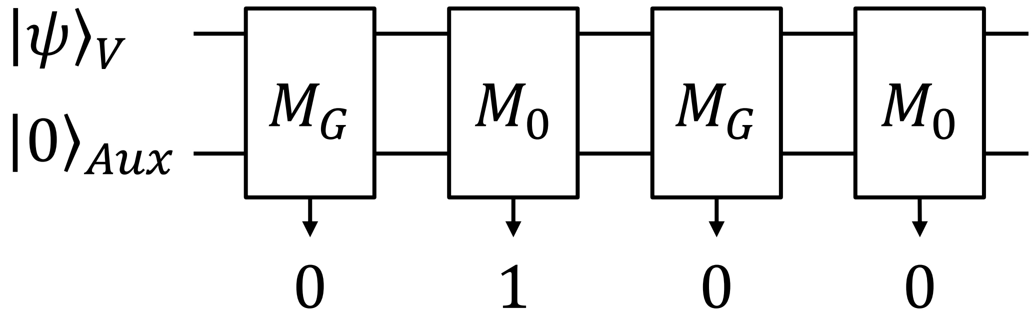

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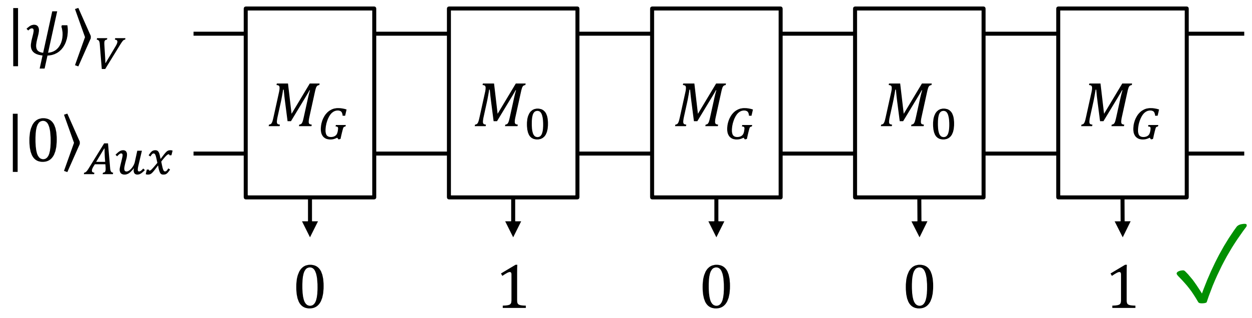

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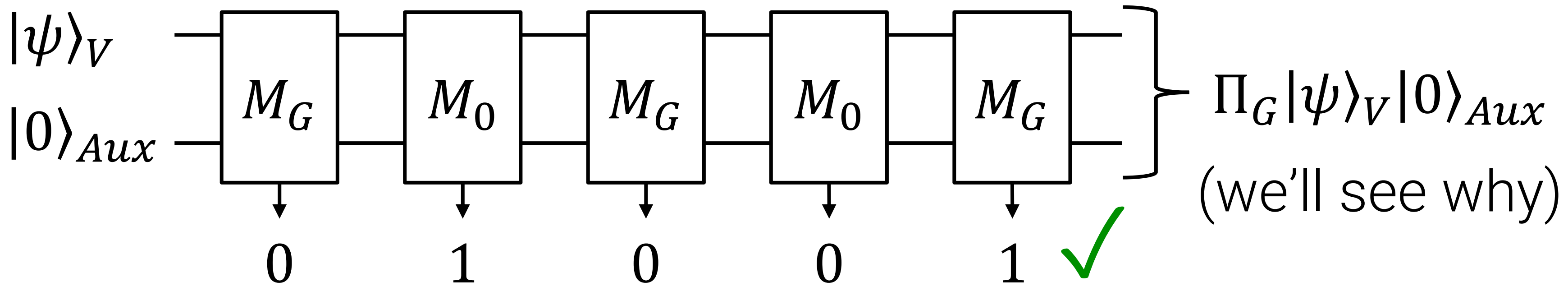

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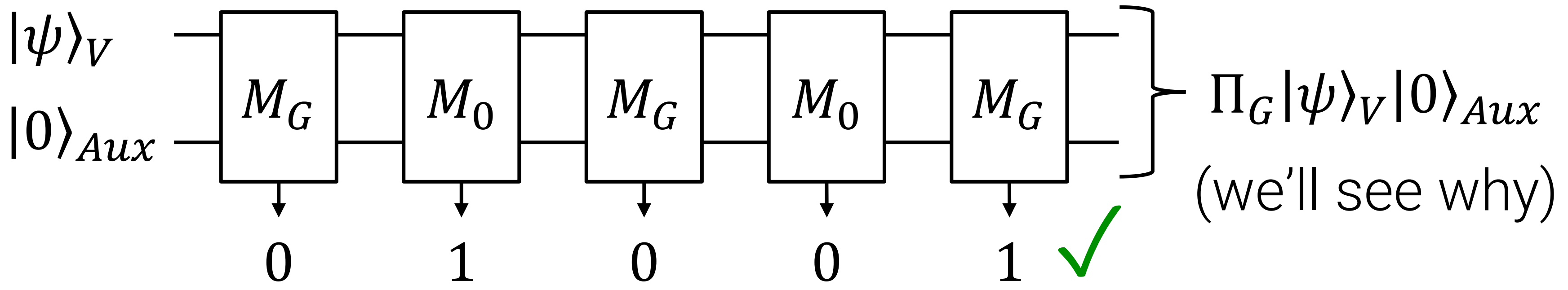

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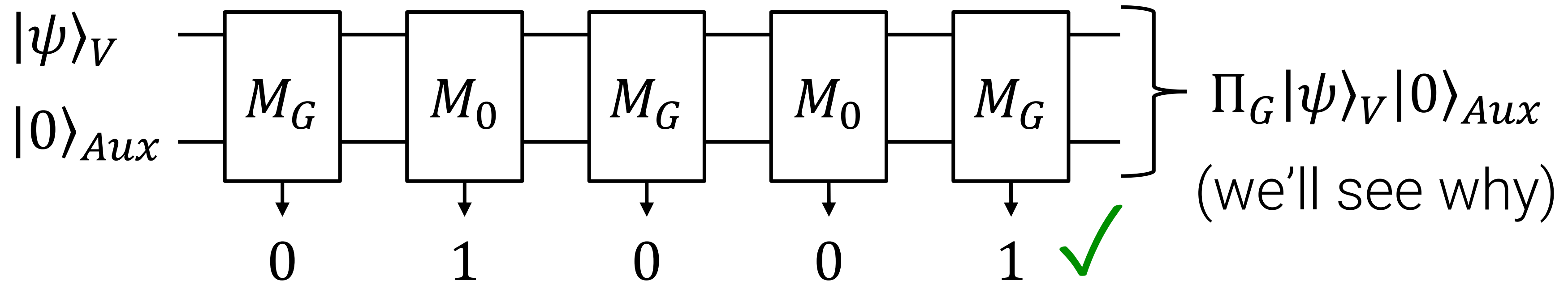

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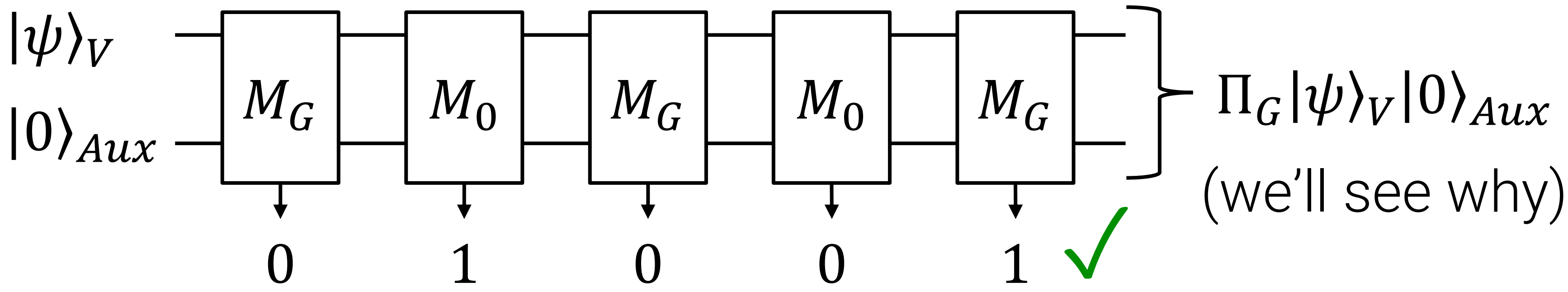

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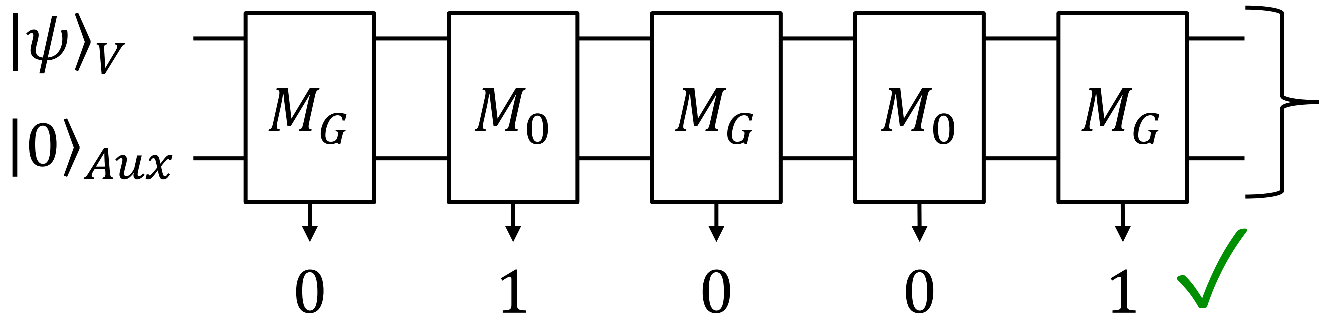

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(we'll see why)

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- **Efficiency**: How long (if ever) until $M_G \to 1$?
- **Simulation:** After $M_G \to 1$, why is the state is $\Pi_G |\psi\rangle_V |0\rangle_{Aux}$?

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What happens if we start at $|v\rangle \in \mathbf{image}(\Pi_A)$ and alternate the measurements $(\Pi_A, \mathbb{I} - \Pi_A)$ and $(\Pi_B, \mathbb{I} - \Pi_B)$?

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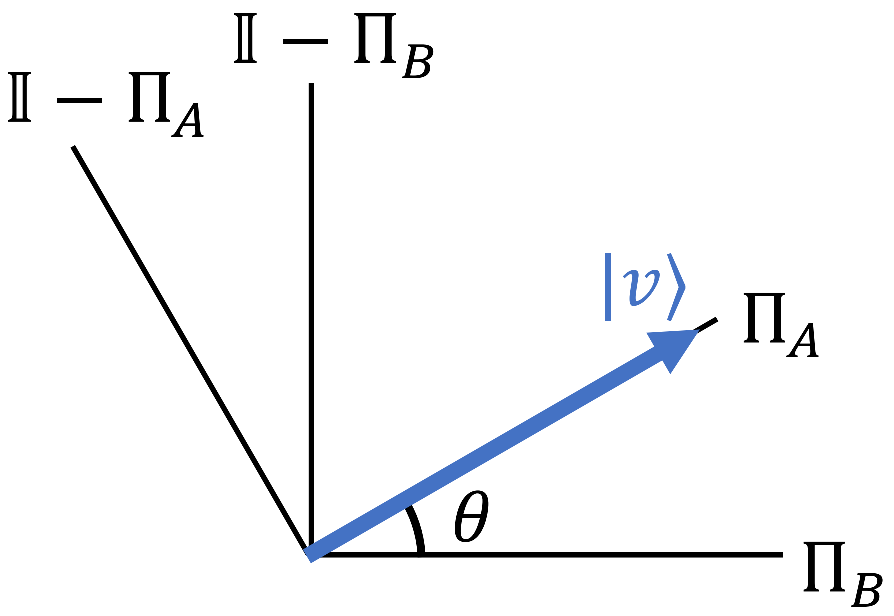

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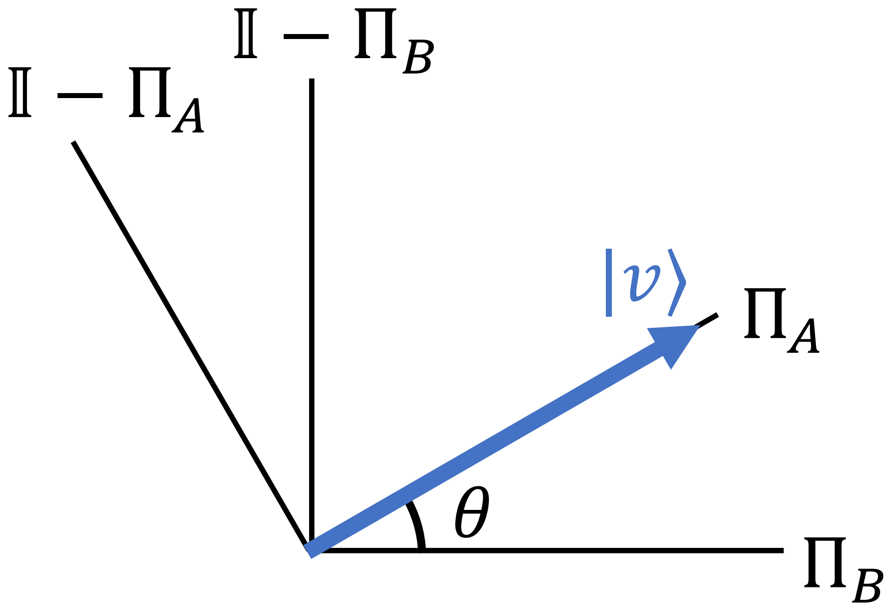

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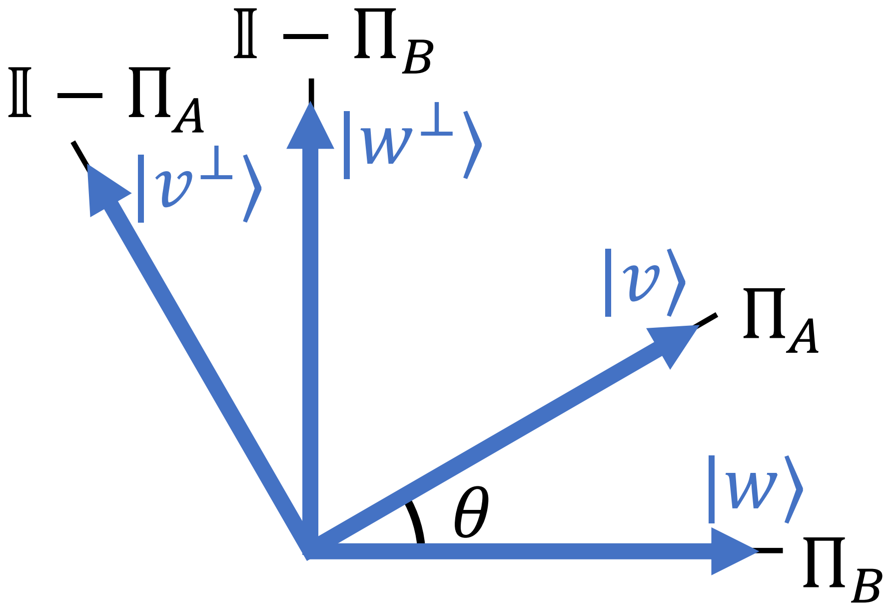

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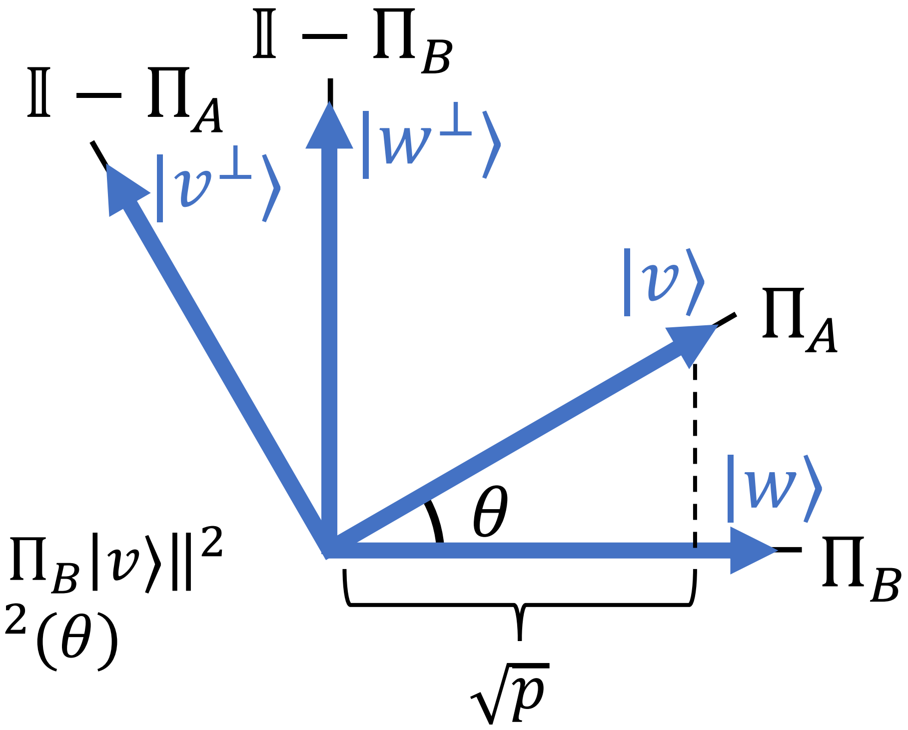

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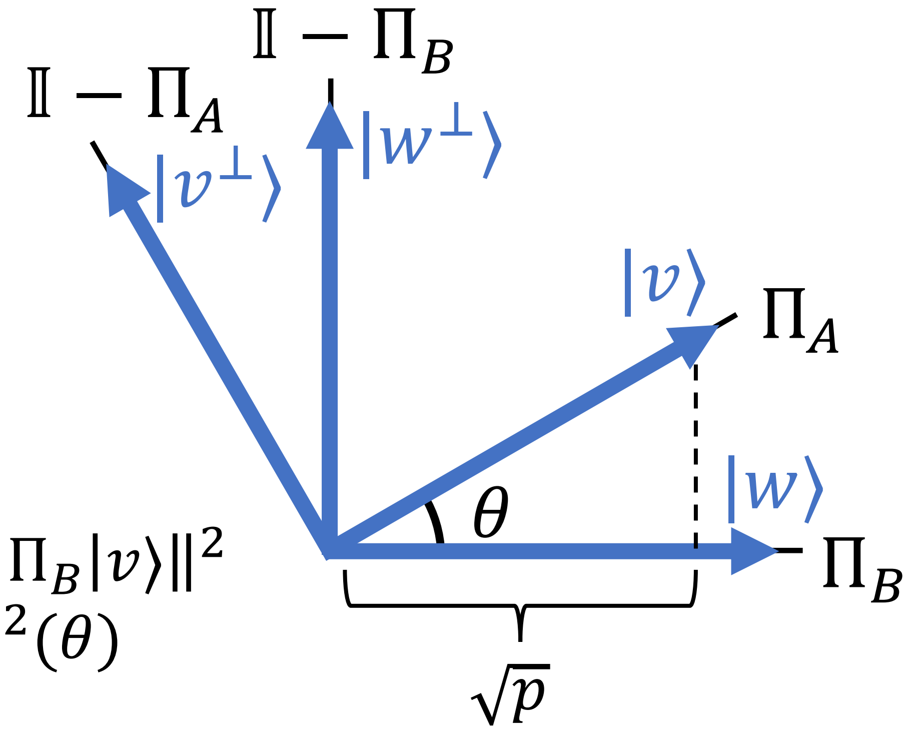

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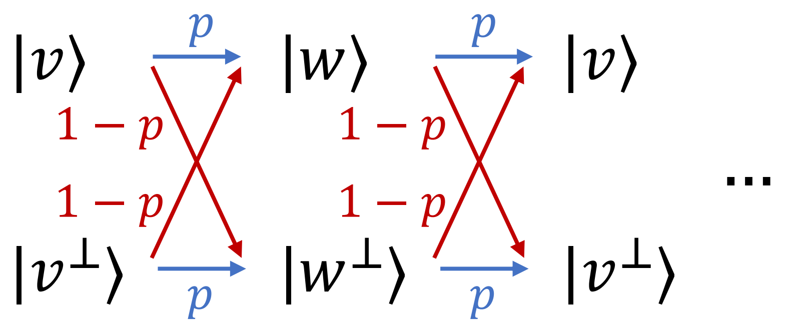

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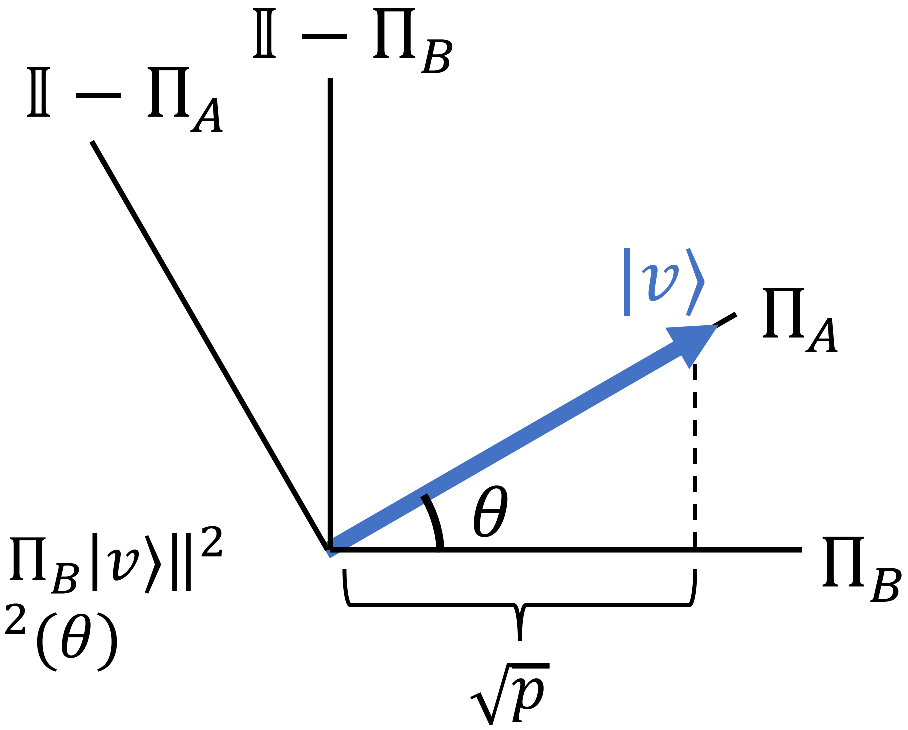

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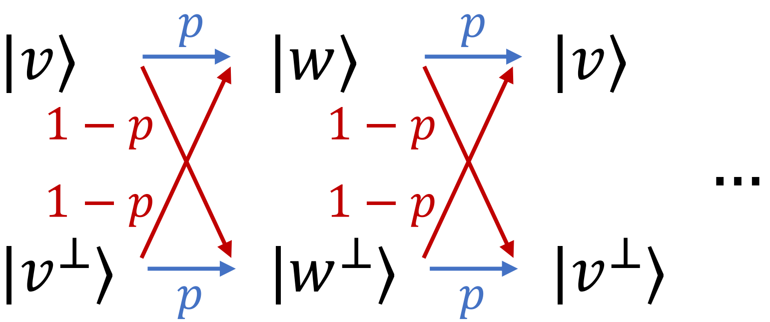

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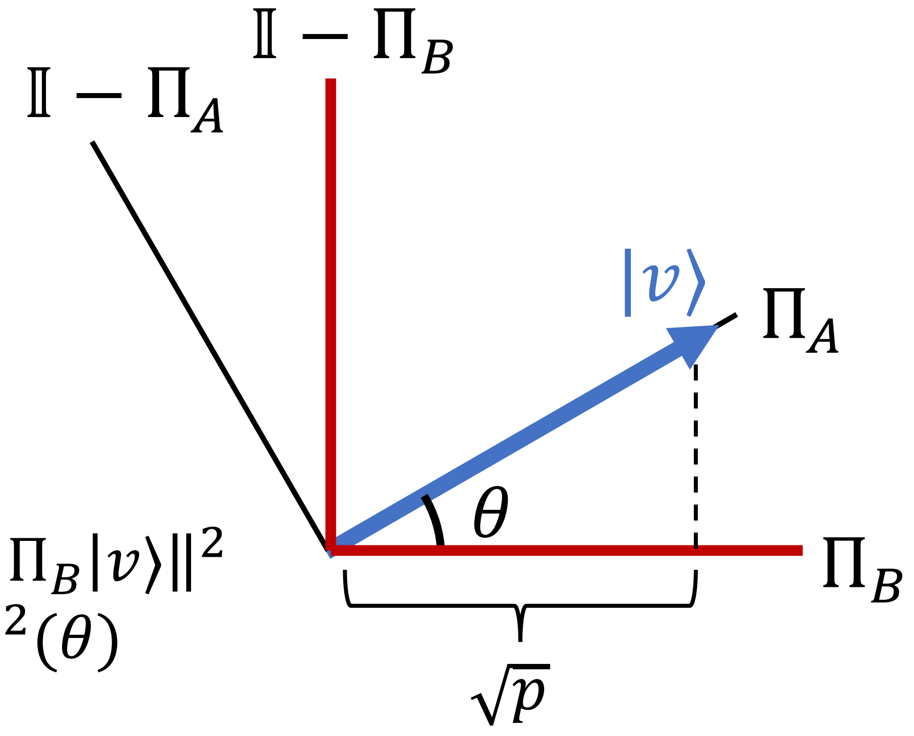

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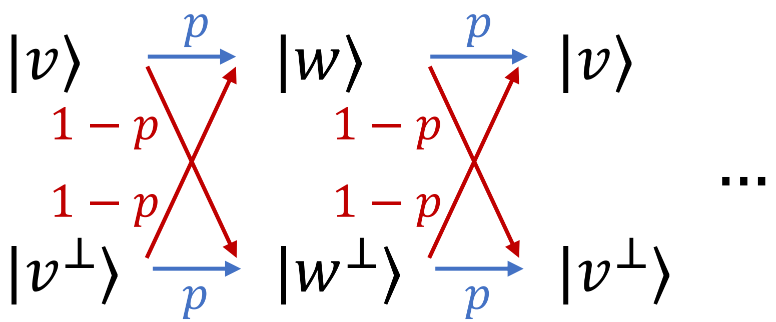

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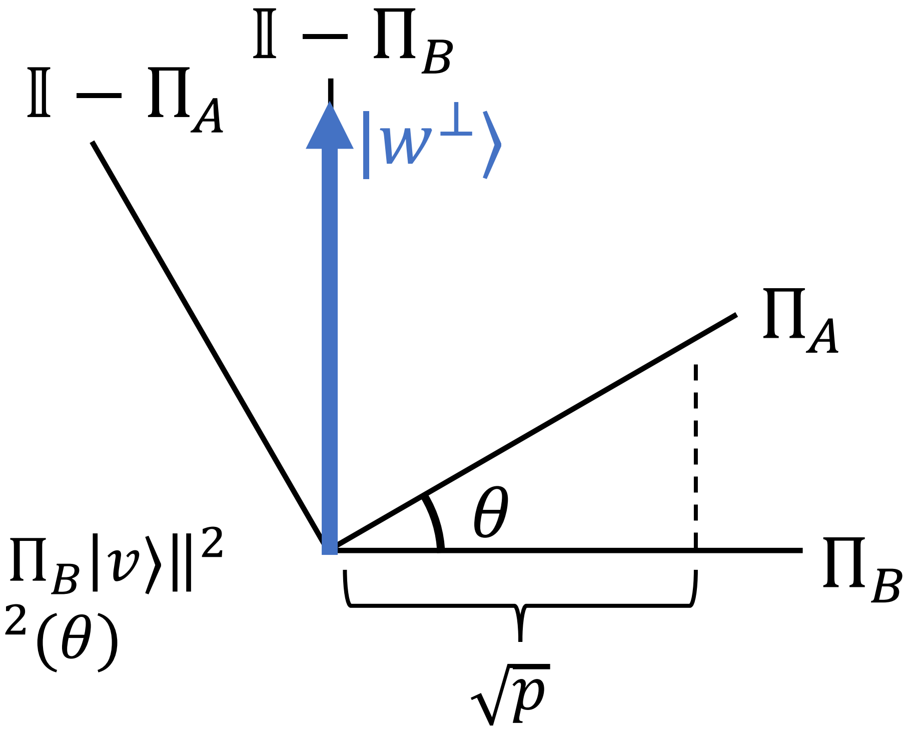

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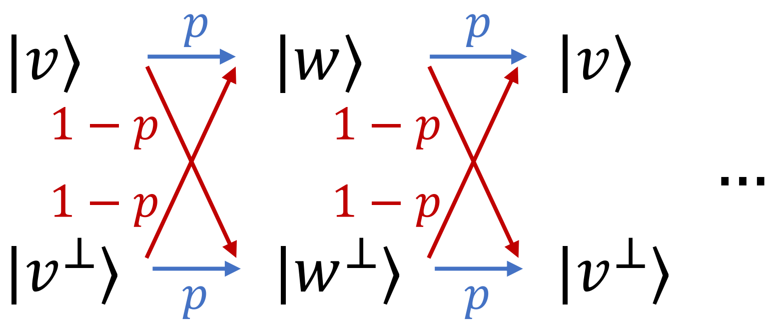

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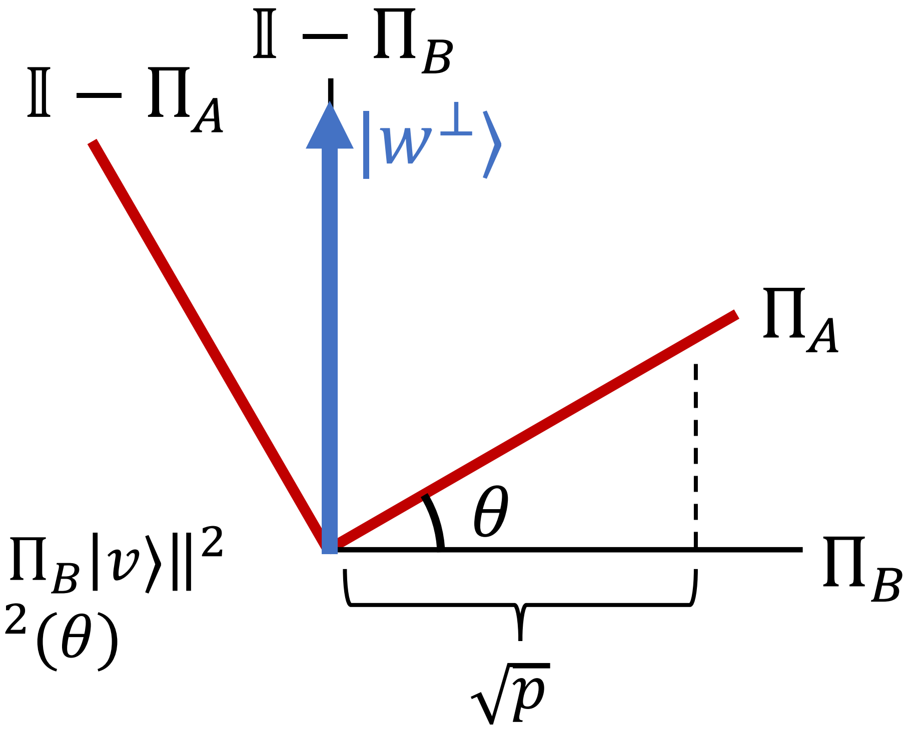

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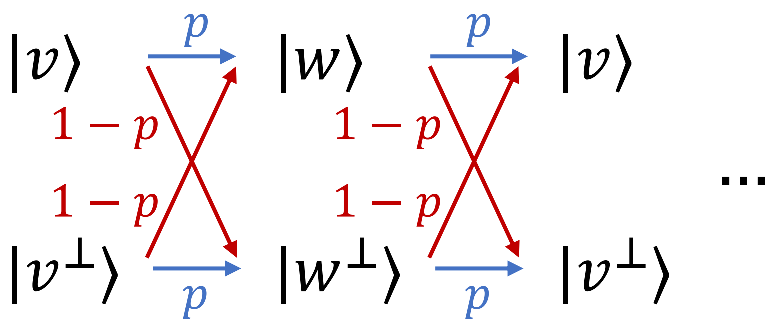

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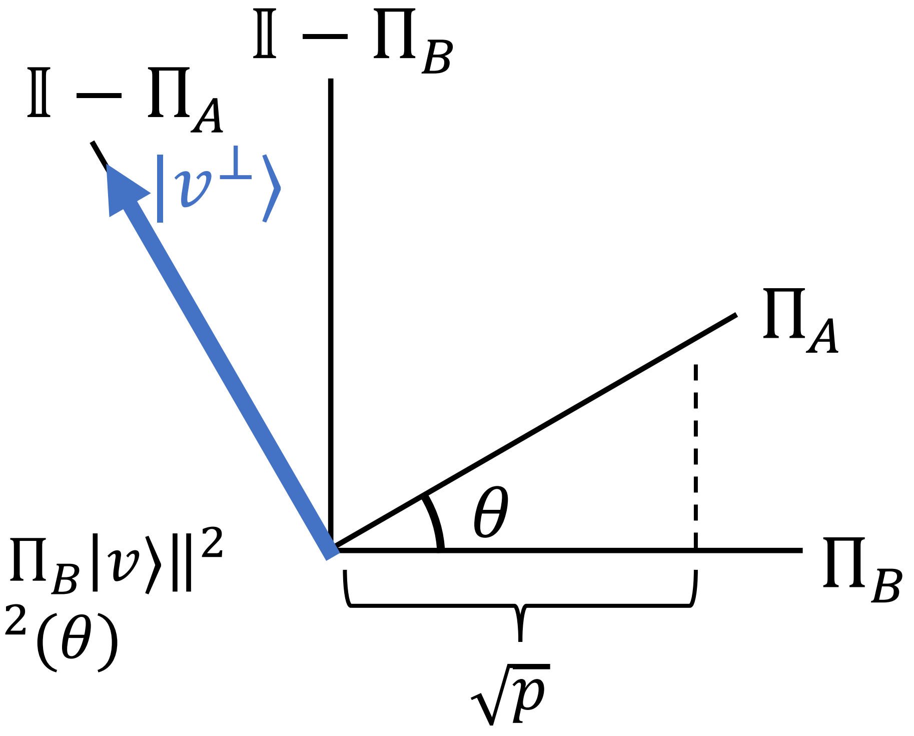

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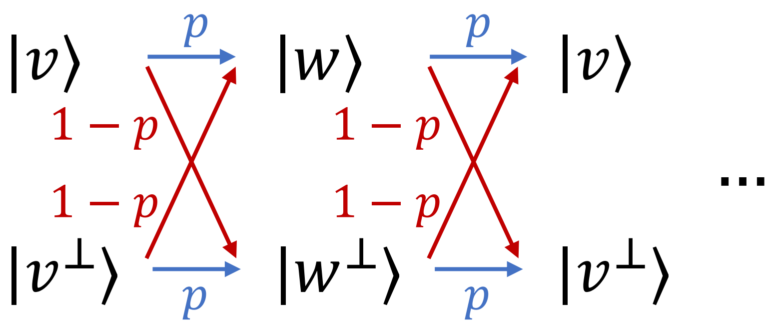

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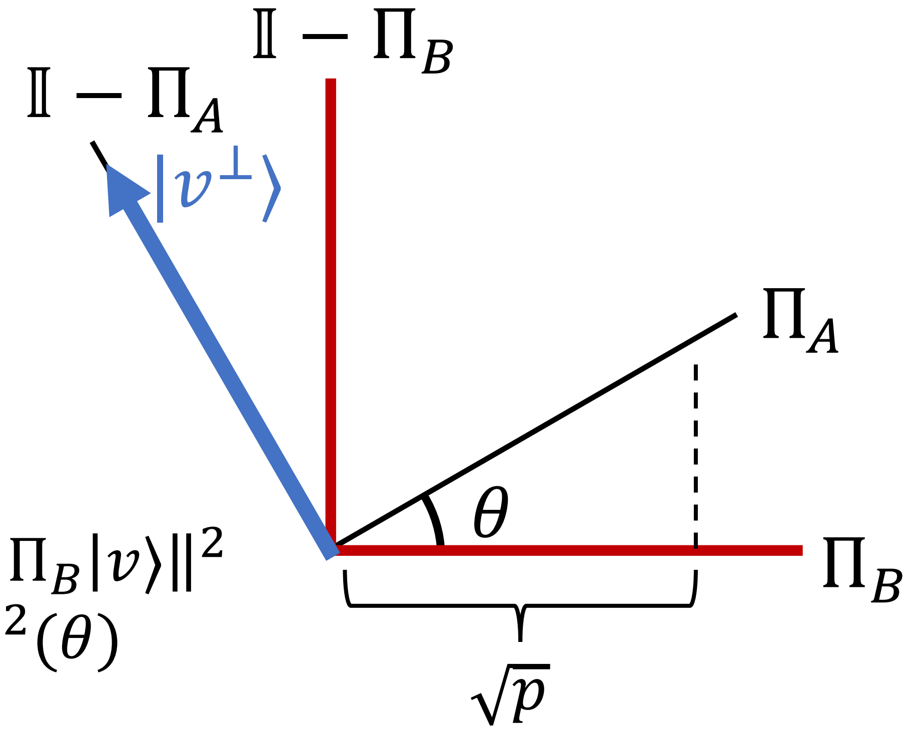

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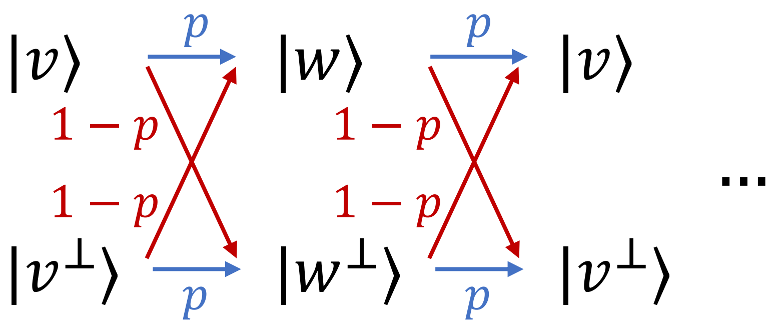

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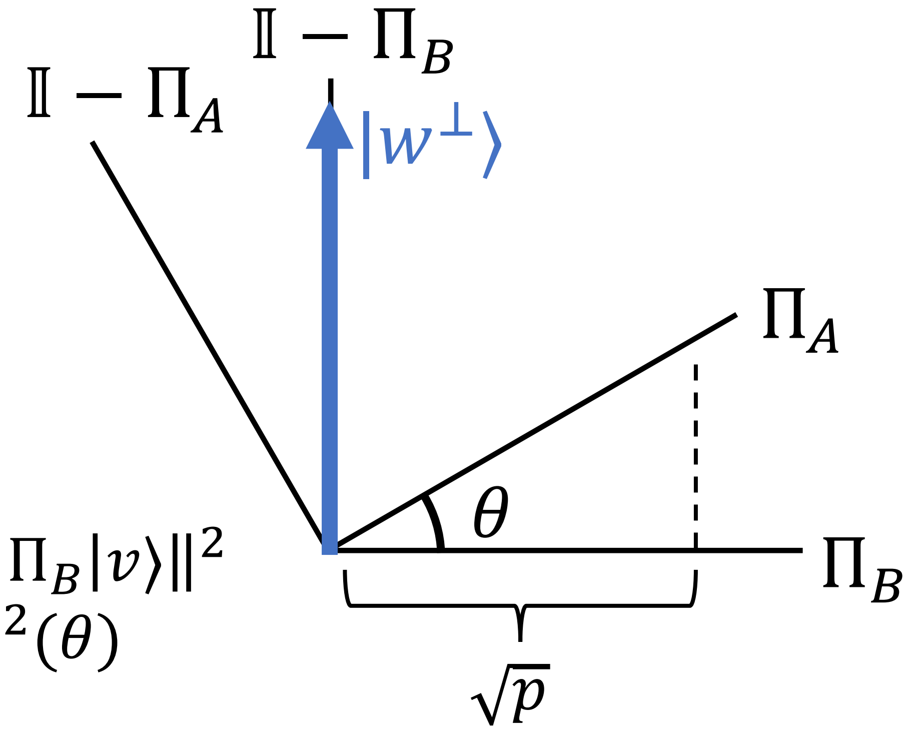

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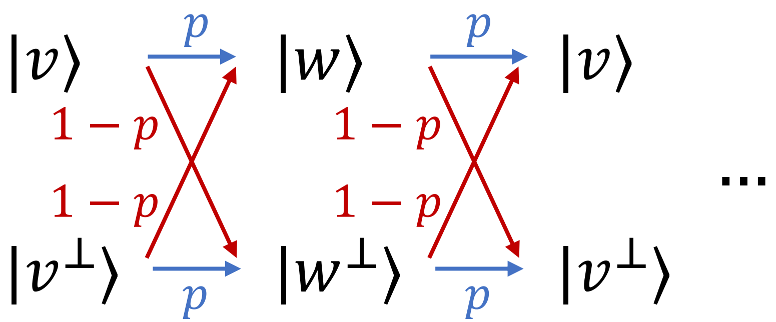

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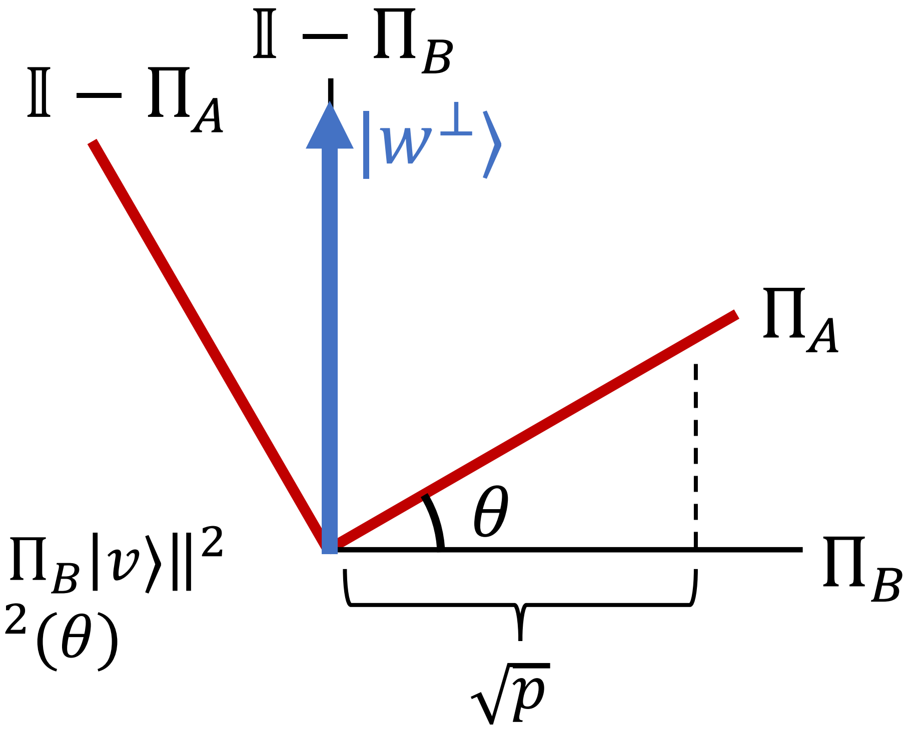

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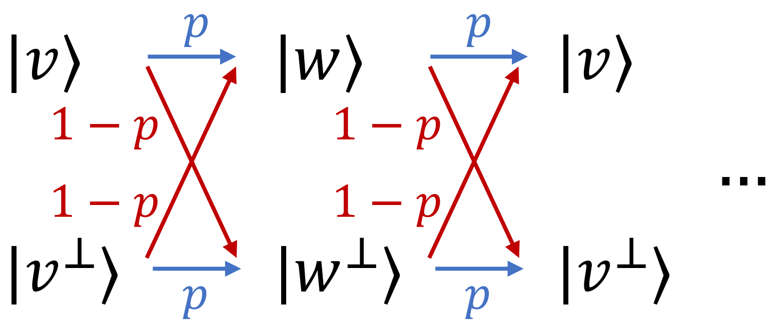

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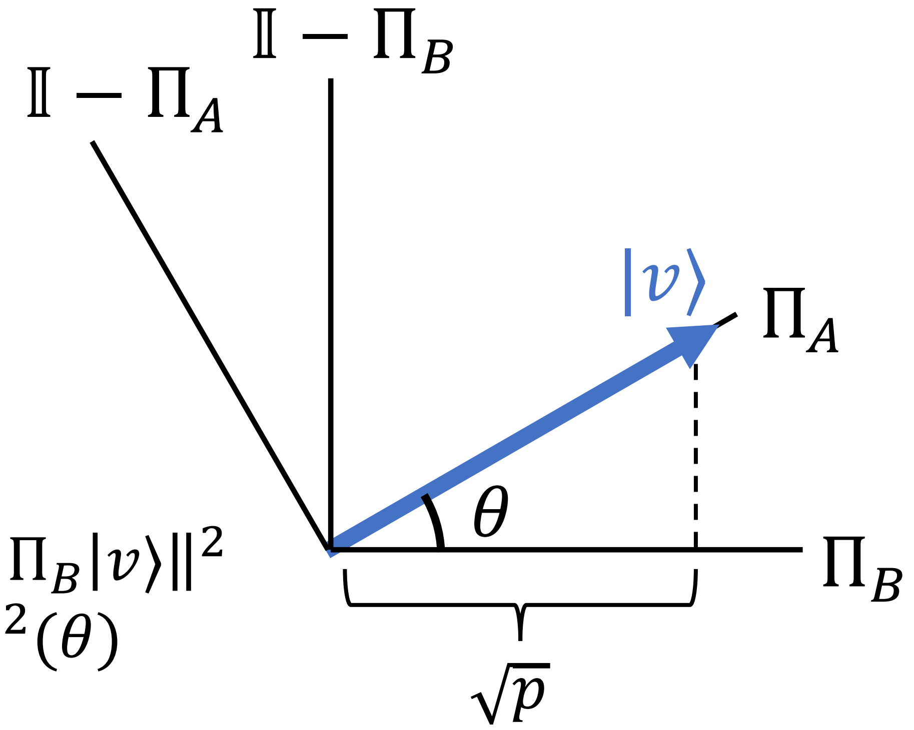

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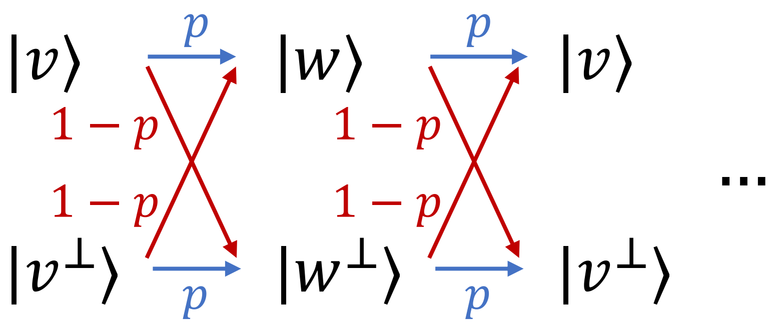

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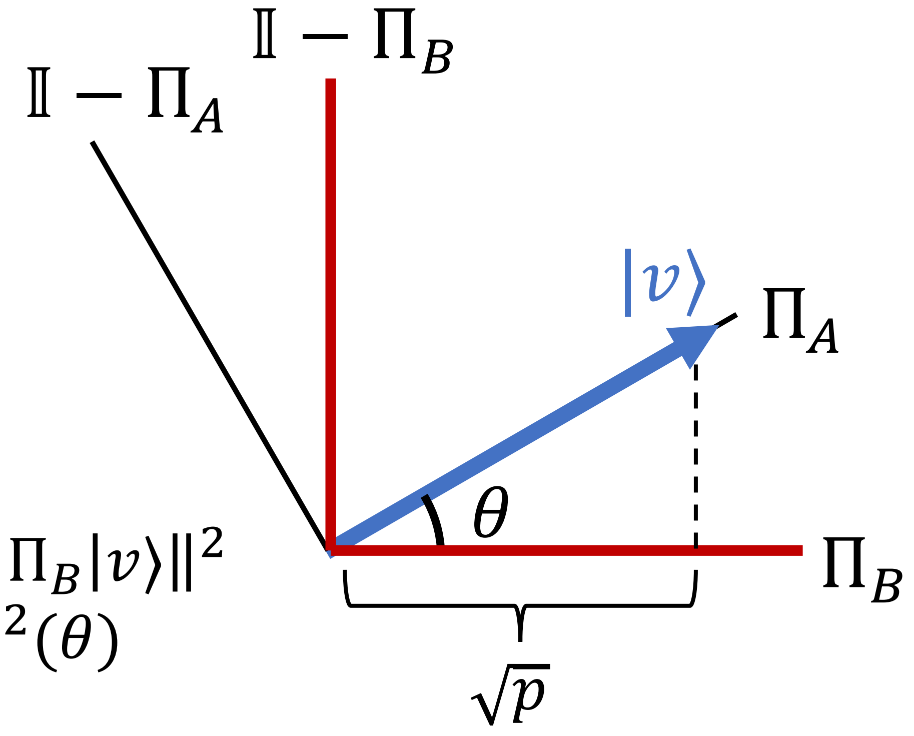

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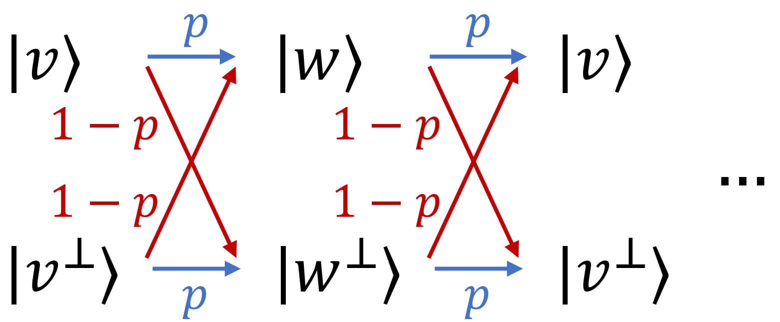

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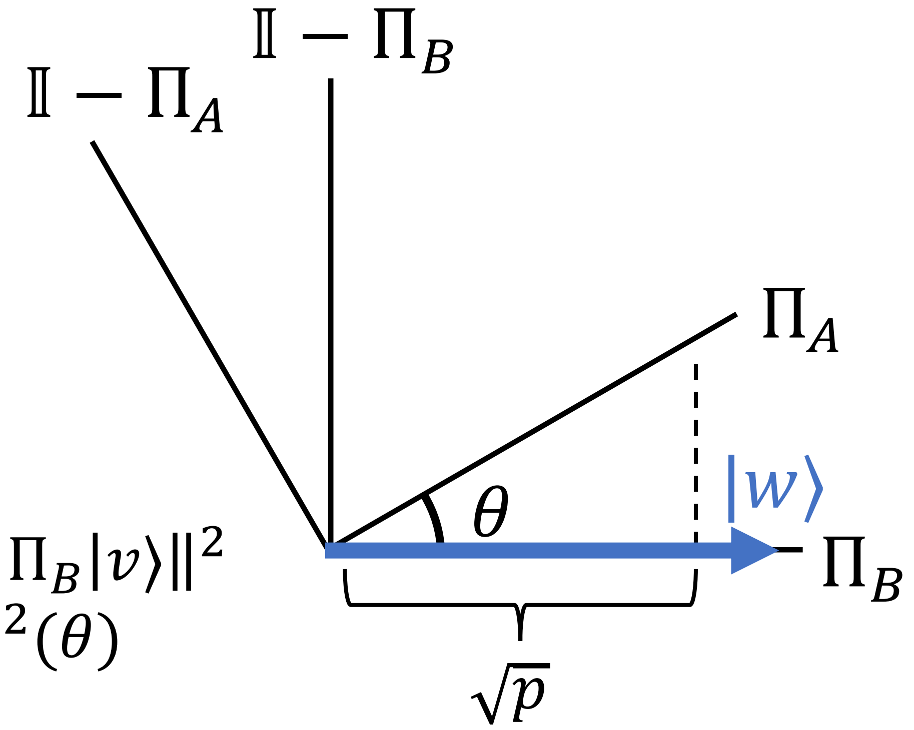

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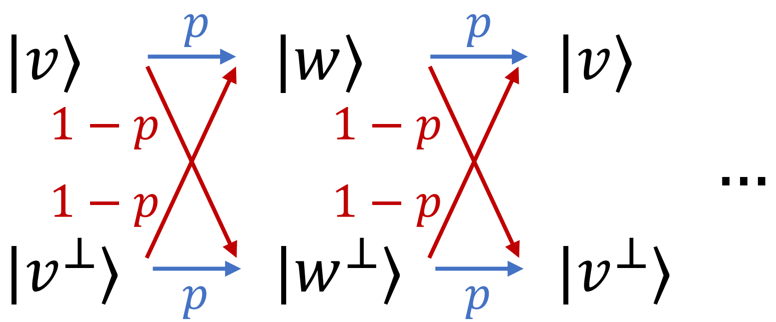

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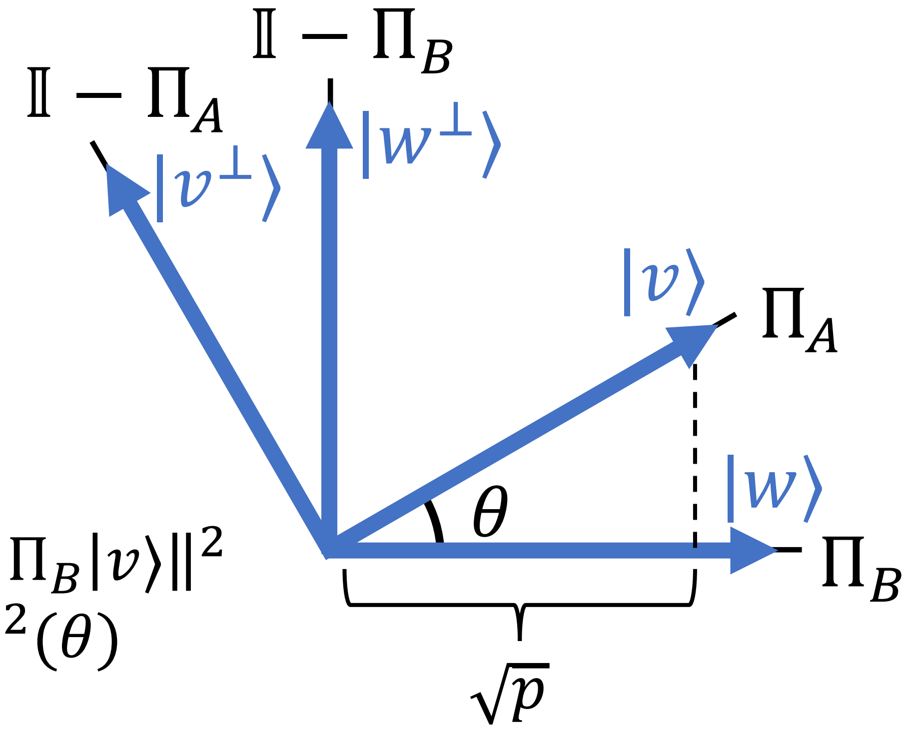

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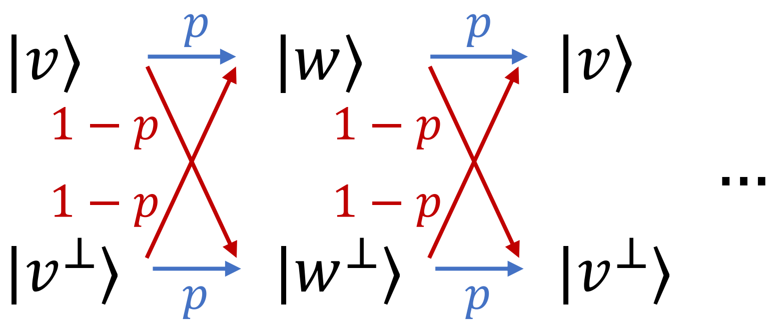

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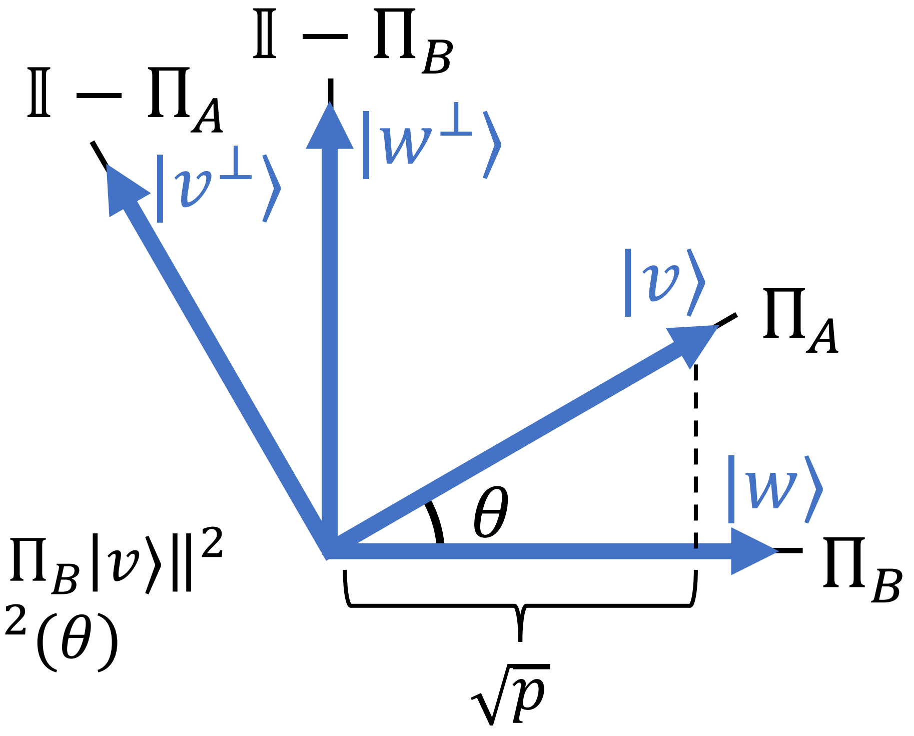

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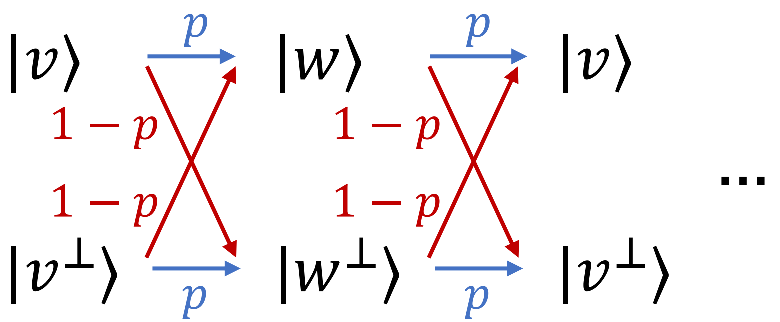

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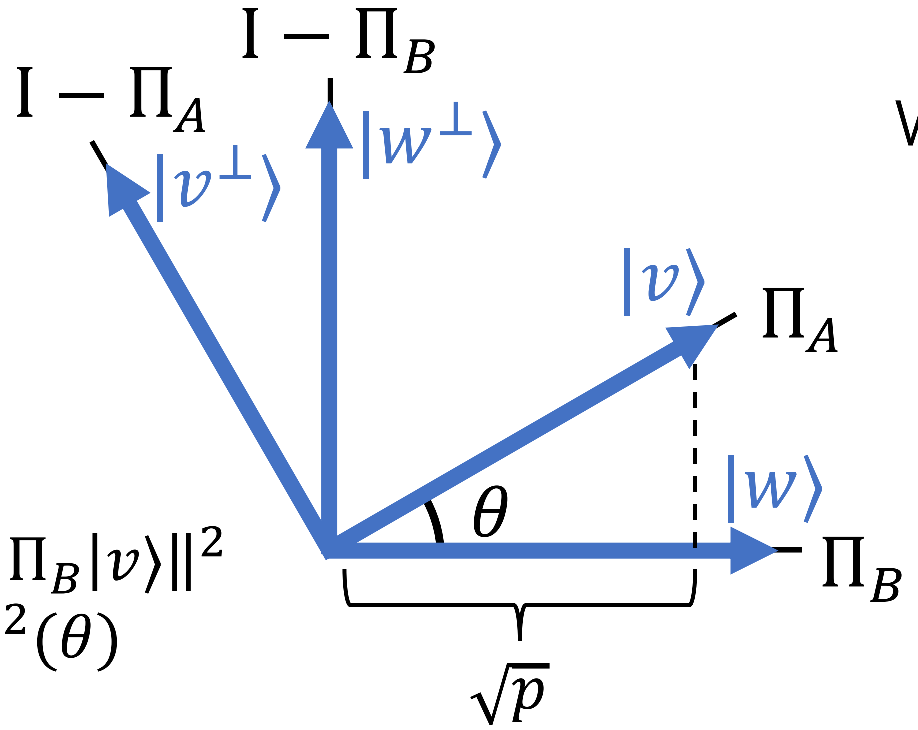

$p \coloneqq \| \Pi_B |v\rangle \|^2 = cos^2(\theta)$

These are the guarantees we want, but Π_0, Π_G don't live in 2D!

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- For general Π_A, Π_B: no!
- For Π_0, Π_G: yes!

Extremely Useful Tool

Jordan's Lemma: For any Π_A, Π_B, we can decompose space into 2-dim invariant subspaces $\{S_j\}$ where Π_A, Π_B are rank-one projectors in each S_j.

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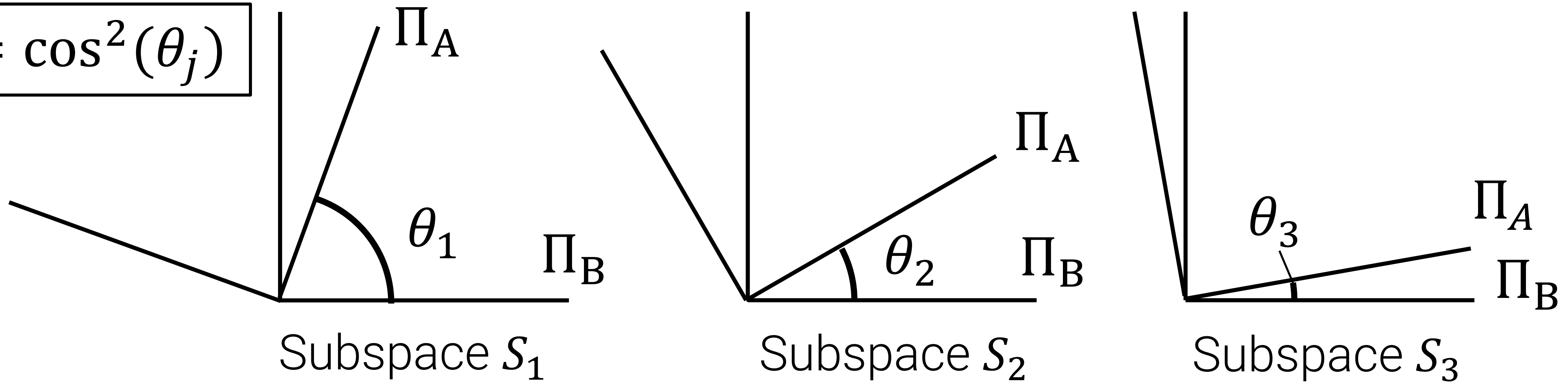

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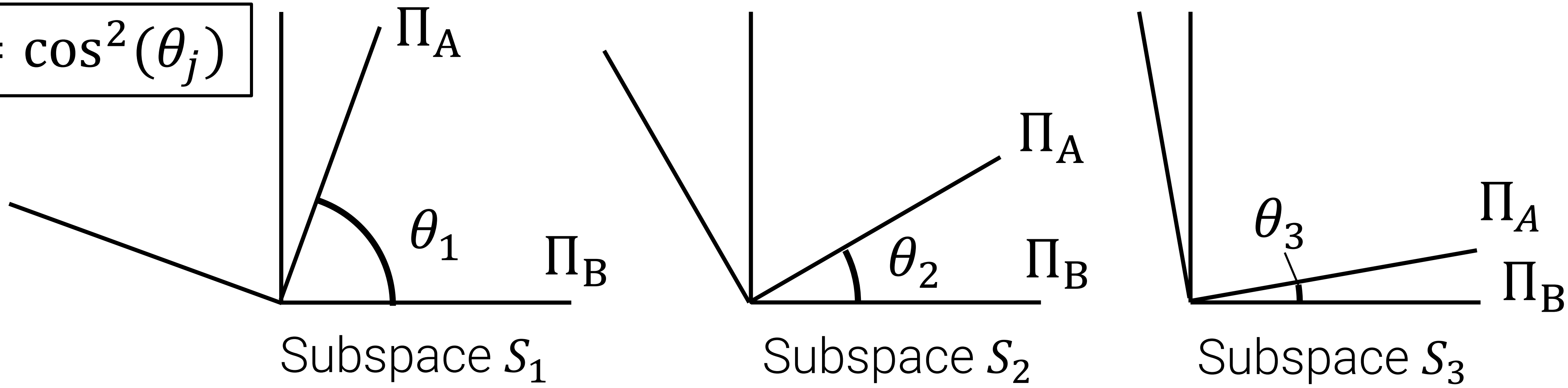

To analyze our simulator, it will be helpful to understand the Jordan subspace decomposition for Π_0, Π_G.

Key Fact: for any $|\phi\rangle \in \mathrm{image}(\Pi_0)$, we have $\| \Pi_G |\phi\rangle \|^2 \approx 1/2$.

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$\mathbf{Guess}(V^*, |\psi\rangle)$:
1) Sample $(c, r', z) \leftarrow \mathbf{HVSim}$
2) If $r = r'$, output (c, r, z). Otherwise $\perp$.

$\xrightarrow{c}$

$\xleftarrow{r}$

$\xrightarrow{z}$
(if $r = r'$)

$V^*, |\psi\rangle$

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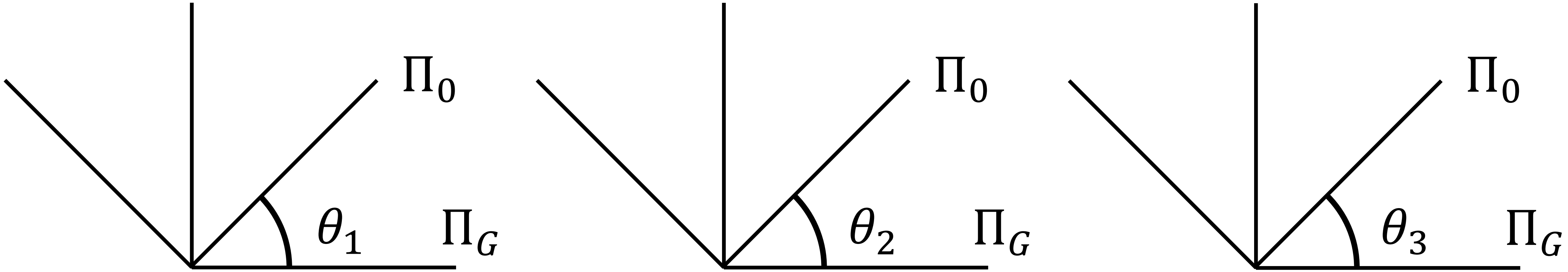

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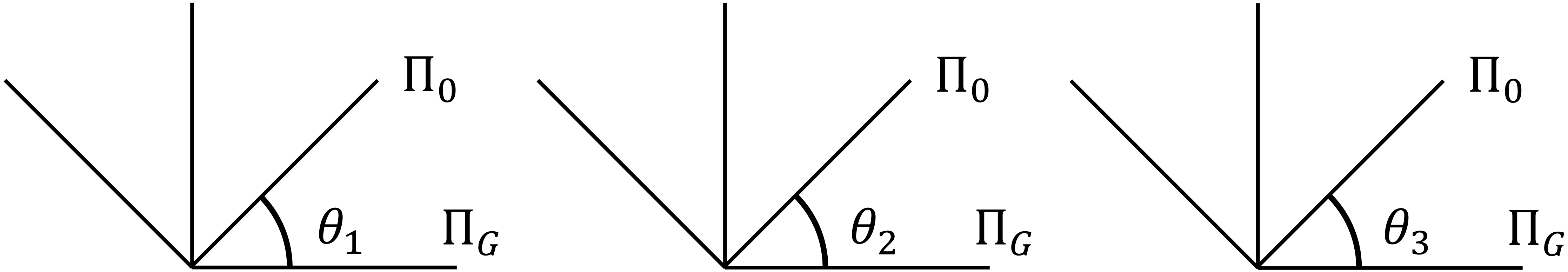

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We can now extend the 2-D analysis to our simulator!

What happens if we start at $|v\rangle \in \mathbf{image}(\Pi_A)$ and alternate the measurements $(\Pi_A, \mathbb{I} - \Pi_A)$ and $(\Pi_B, \mathbb{I} - \Pi_B)$?

Previously, we claimed the following for Π_A, Π_B in 2-D:

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These claims extend to high-dim if all (Π_A, Π_B)-Jordan subspaces have roughly equal p_j.

What happens if we start at $|v\rangle \in \mathbf{image}(\Pi_A)$ and alternate the measurements $(\Pi_A, \mathbb{I} - \Pi_A)$ and $(\Pi_B, \mathbb{I} - \Pi_B)$?

If all (Π_A, Π_B)-Jordan subspaces have $p_j \approx p$, then:

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Intuition for Claim 1: the 2-D runtime analysis extends to higher dimensions because the Π_A, Π_B measurements act independently on each Jordan subspace.

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Intuition for Claim 2:

- Consider $|v\rangle = \sum_j \alpha_j |v_j\rangle$. In each S_j, the state after $(\Pi_B, \mathbb{I} - \Pi_B)$ accepts is $\propto \Pi_B|v_j\rangle$ by our analysis of the 2-D case.

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- Alternating measurement results only depend on p_j, but since all $p_j \approx p$, the measurement outcomes give no signal about j.
- So the final state is $\propto \sum_j \alpha_j \Pi_B |v_j\rangle = \Pi_B|v\rangle$.

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Since Π_0 and Π_G satisfy $p_j \approx 1/2$ in all Jordan subspaces, we can set $\Pi_A = \Pi_0$ and $\Pi_B = \Pi_G$ to analyze the alternating measurements simulator:

- By Claim 1, the simulator is efficient.
- By Claim 2, when $M_G \to 1$, the state is $\propto \Pi_G|\psi\rangle|0\rangle$ as desired.

Recap

We showed that Blum's protocol is post-quantum sound and ZK.

Soundness:

- Collapse-binding commitments enable "lazy" measurement
- Unruh's rewinding: if protocol is collapsing, can extract *two* accepting transcripts from a successful adversary

Zero Knowledge:

- Key tool: obtain a quantum analogue of the classical "repeated-guessing" simulator using alternating projectors.
- Analyze alternating projectors via Jordan's lemma

The next two talks

Part 2: Unruh's rewinding applies to protocols where security requires extracting ***two*** transcripts.

What if we need more transcripts?

Part 3: Watrous' rewinding applies to protocols where simulation entails guessing the verifier's challenge.

What if guessing is impossible?

Thank You!

Questions?