Transparent Time- and Space-Efficient Arguments From Groups of Unknown Order

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Pre-Quantum Cryptography with Lattices

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Please make me "Post-".

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for an NP relation R with corresponding language L

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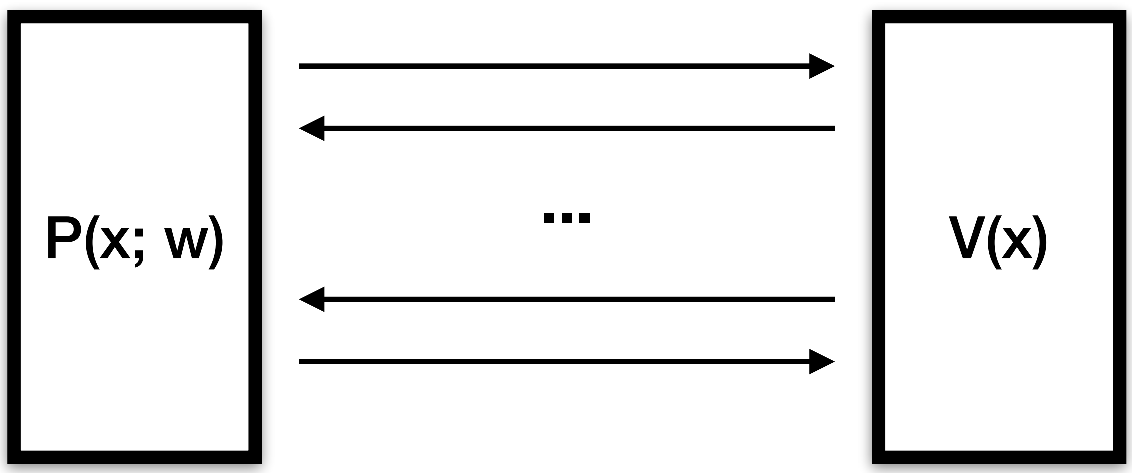

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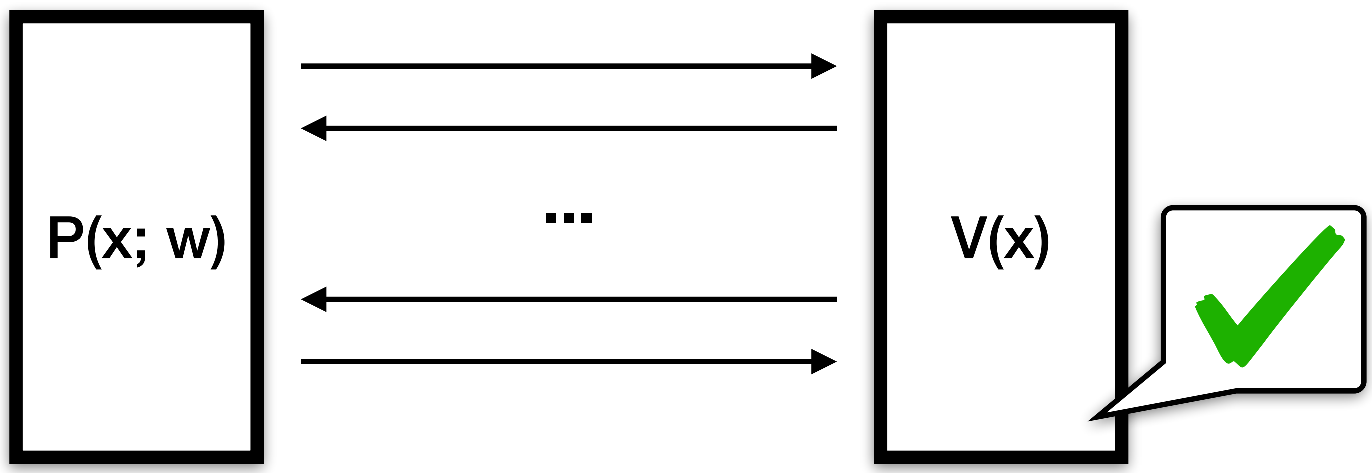

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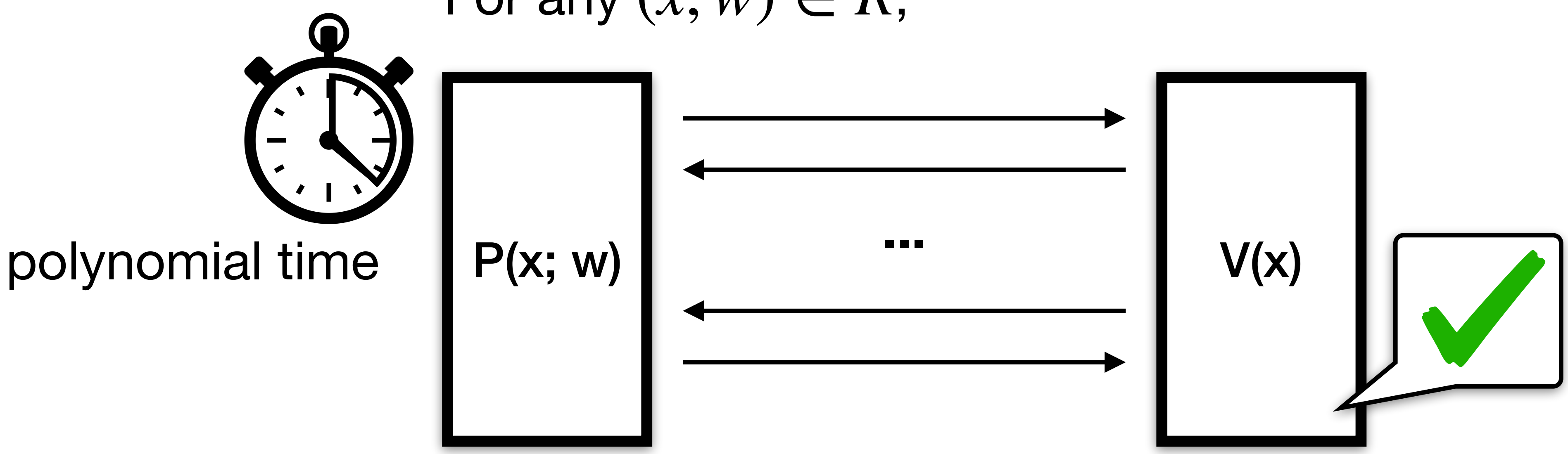

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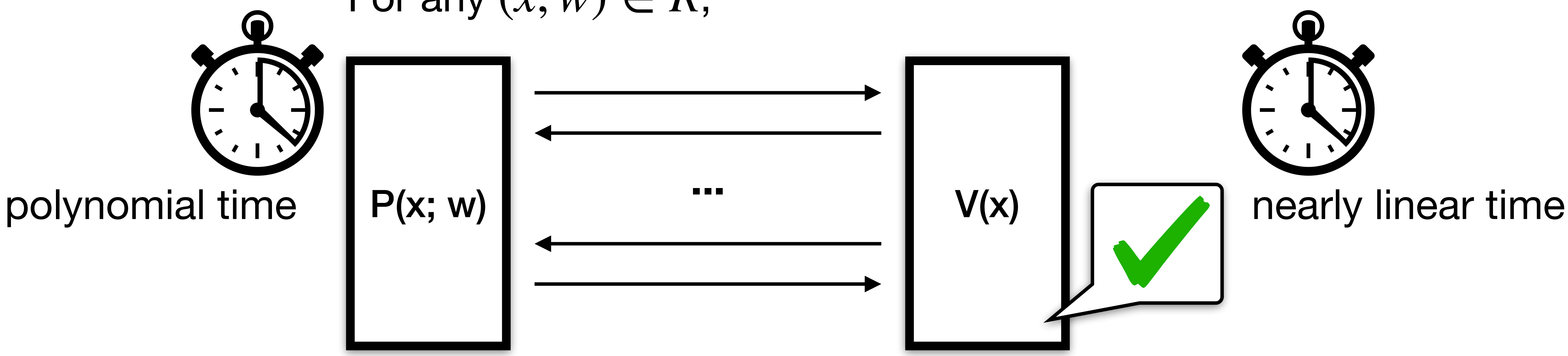

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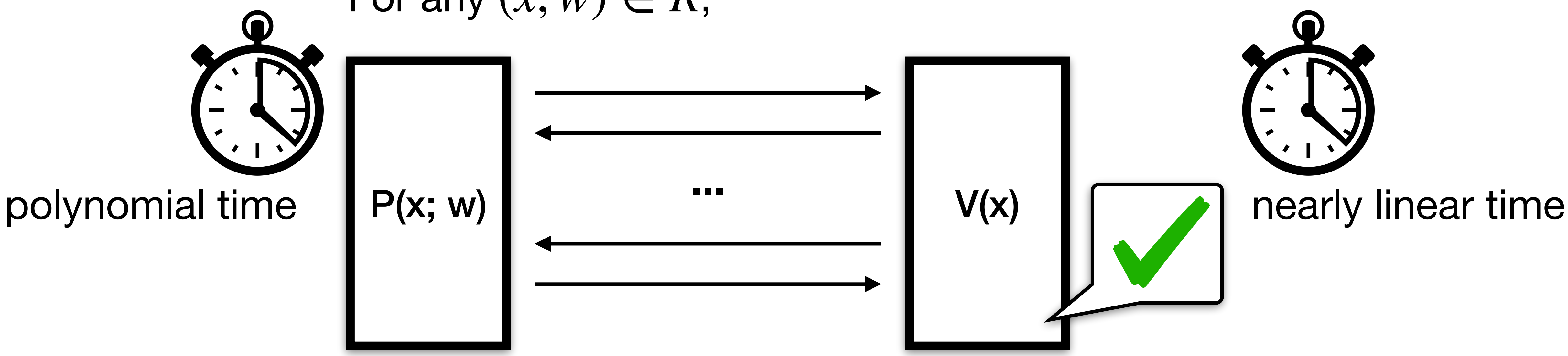

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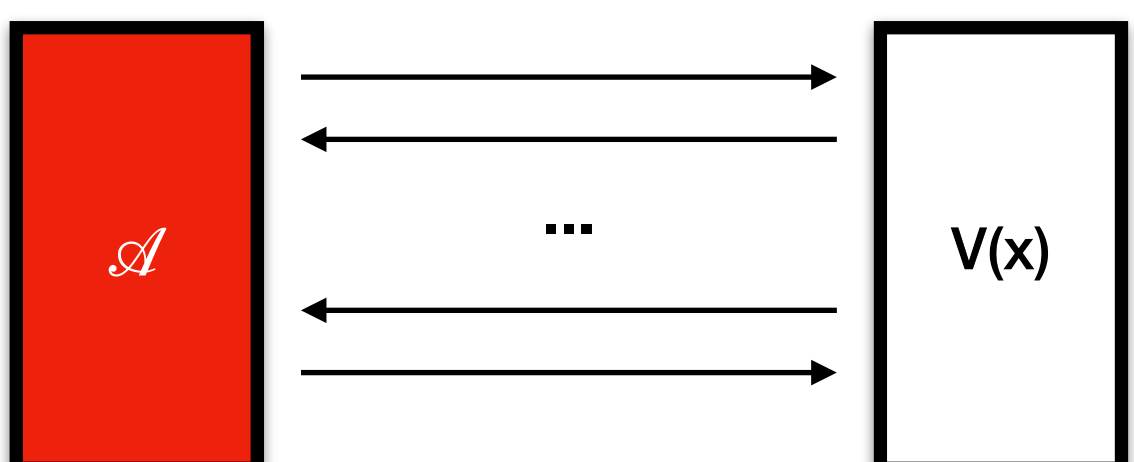

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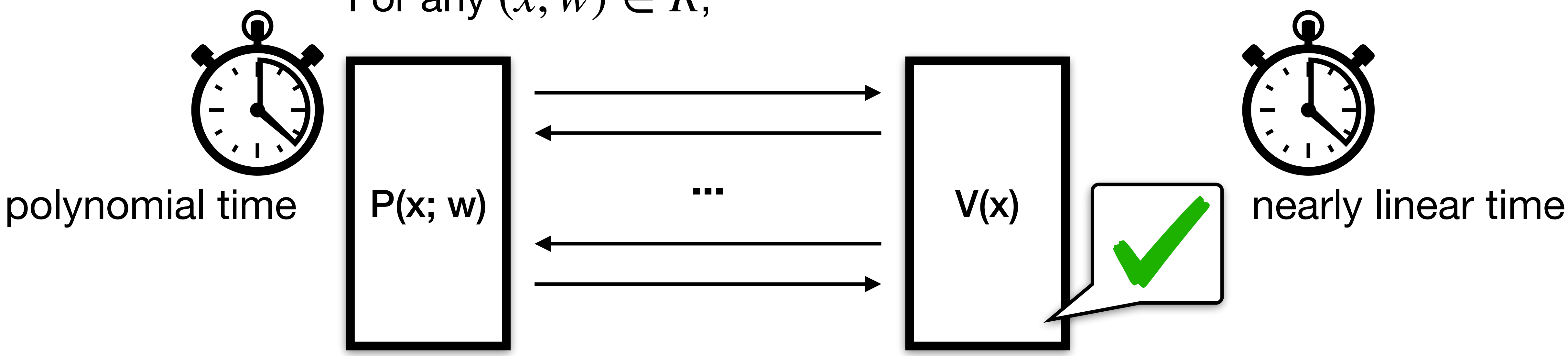

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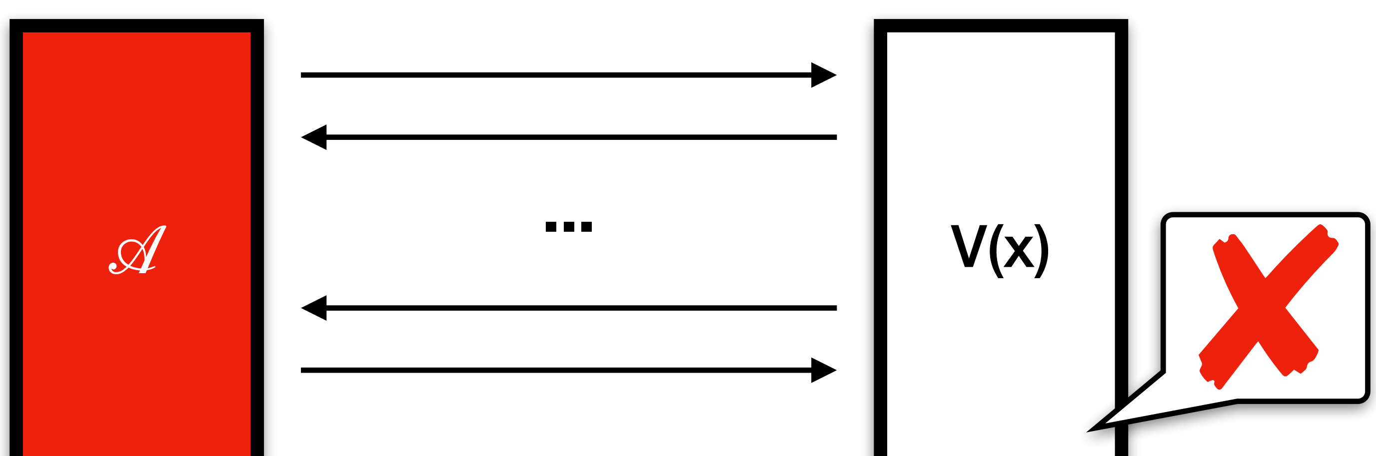

Two Desirable Properties for Interactive Arguments

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Time- and Space-Efficient Prover:

- If $(x; w) \overset{?}{\in} R$ is decidable in time T and space S, then prover runs in time $\approx T$ and space $\approx S$
- Space can be as much of a bottleneck as time, but is often overlooked

Prior Approaches for
Time- and Space-Efficient Proving

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- **This work**: **public-coin** arguments, based on a **simple & falsifiable** "hidden order" assumption

Compiling IOPs to Arguments

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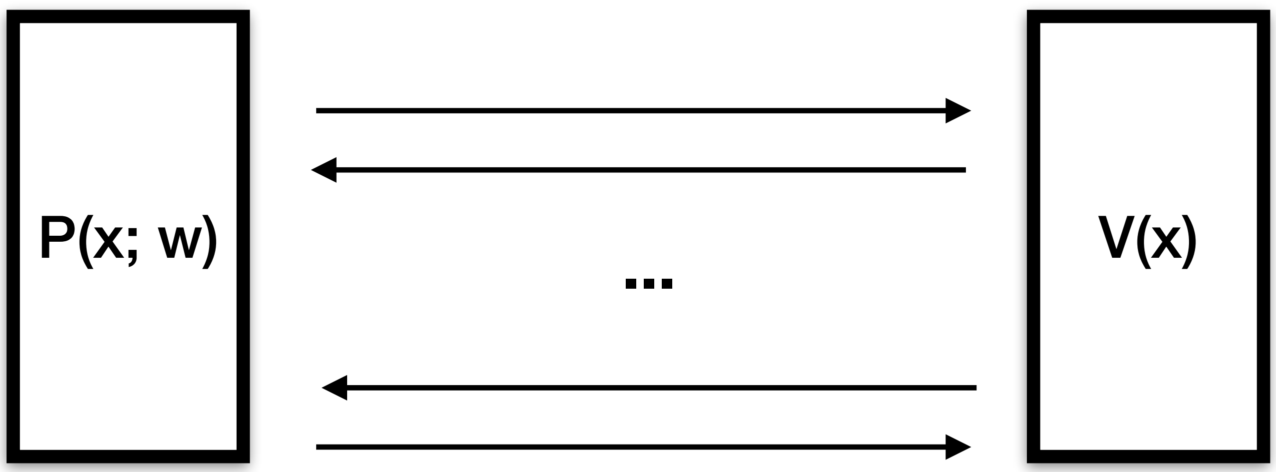

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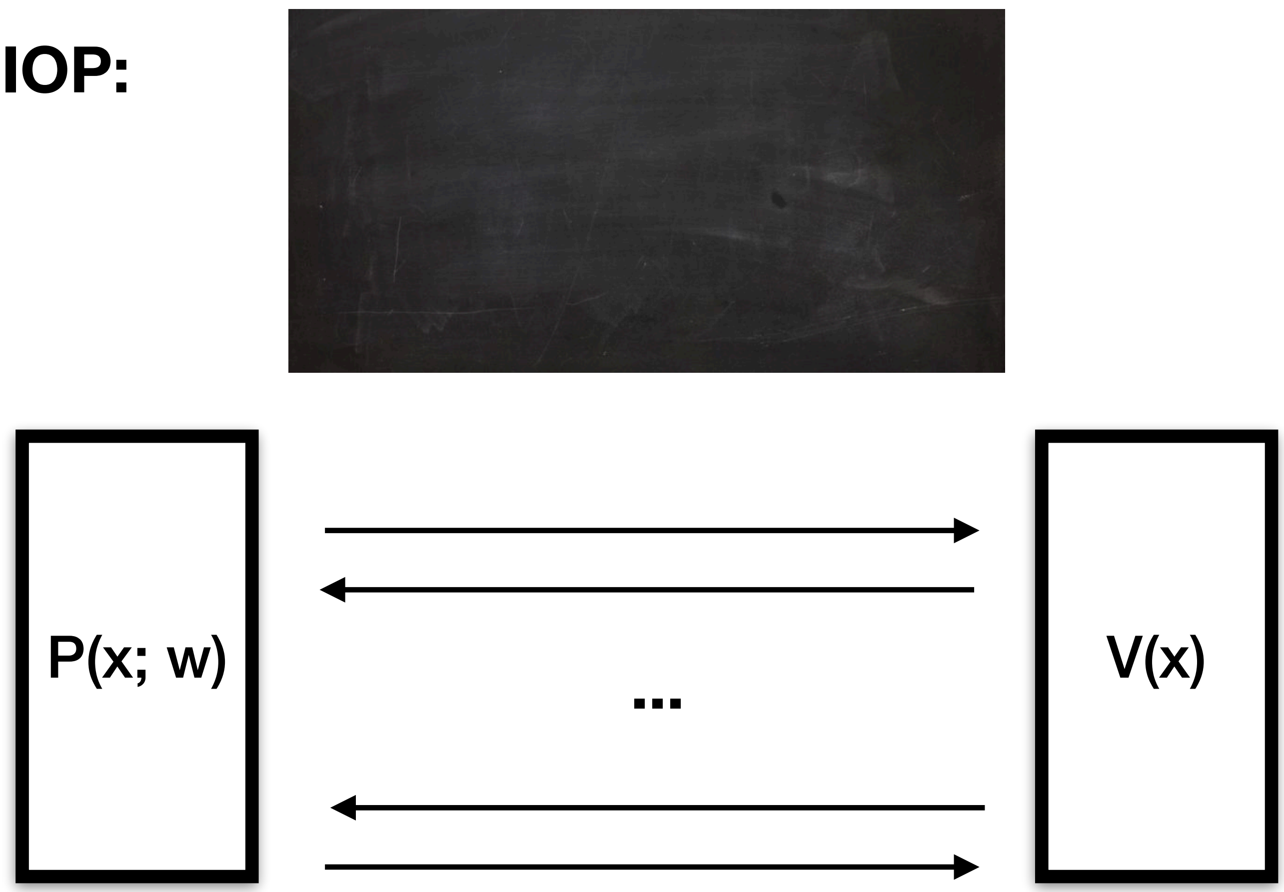

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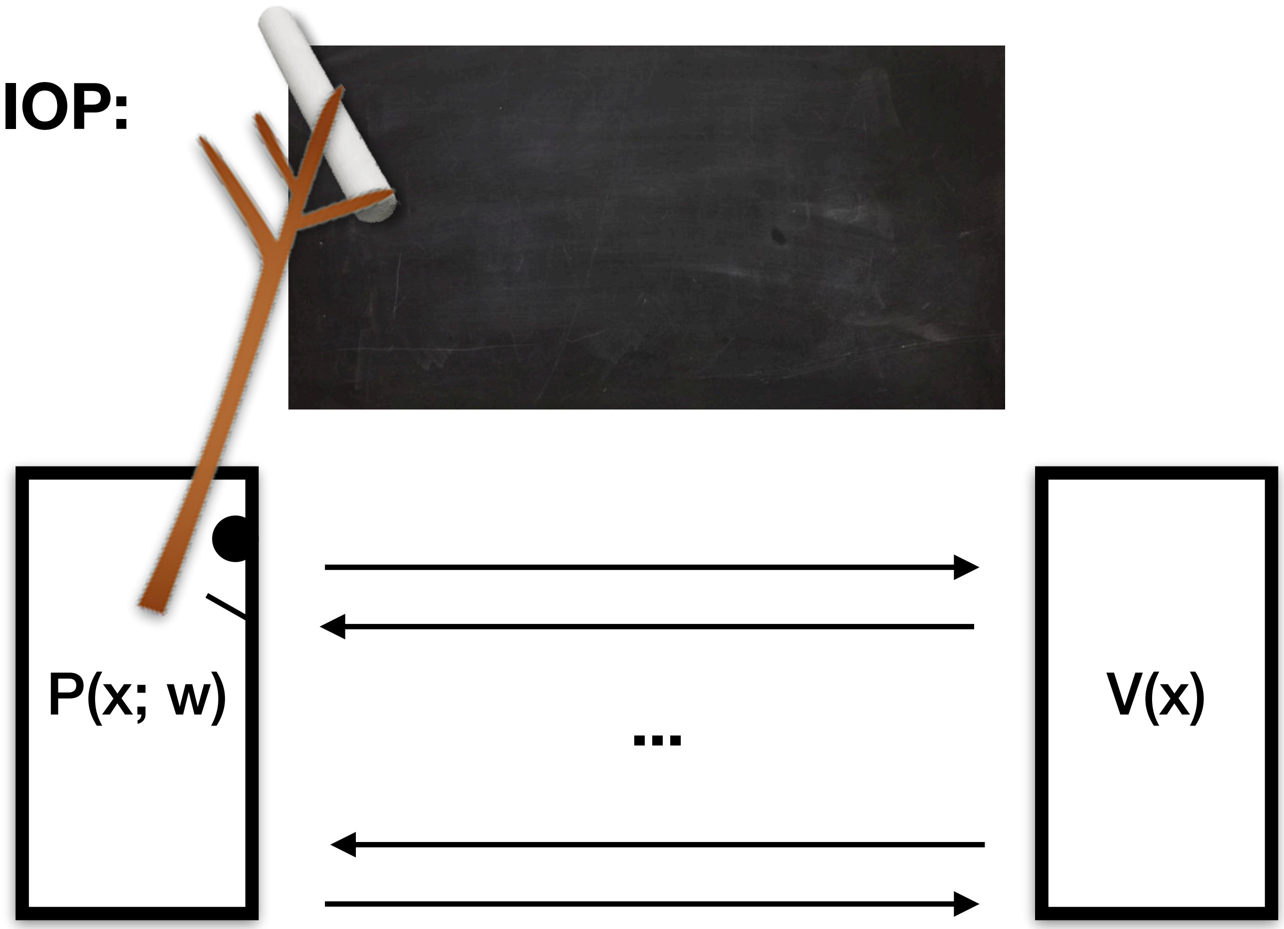

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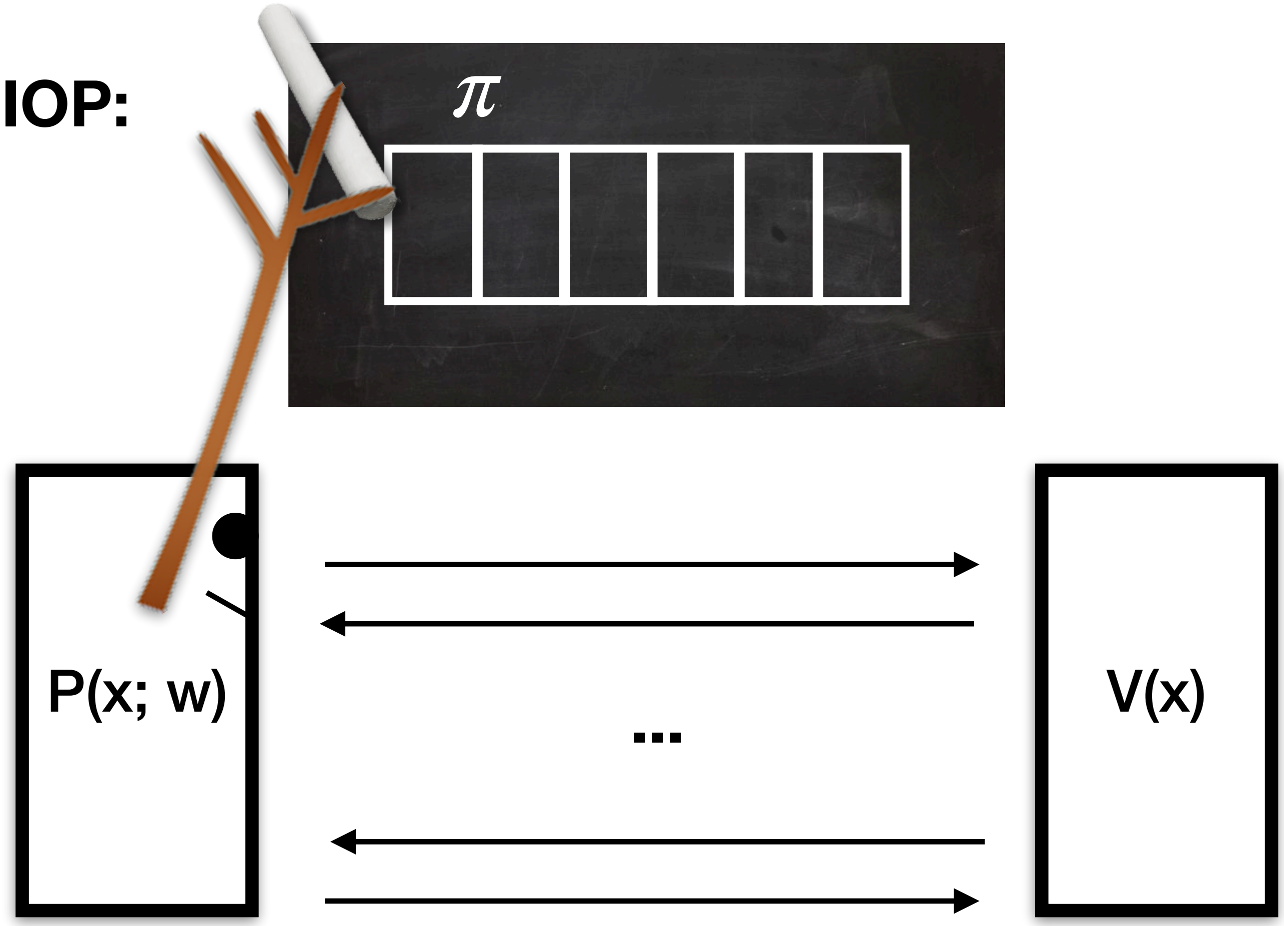

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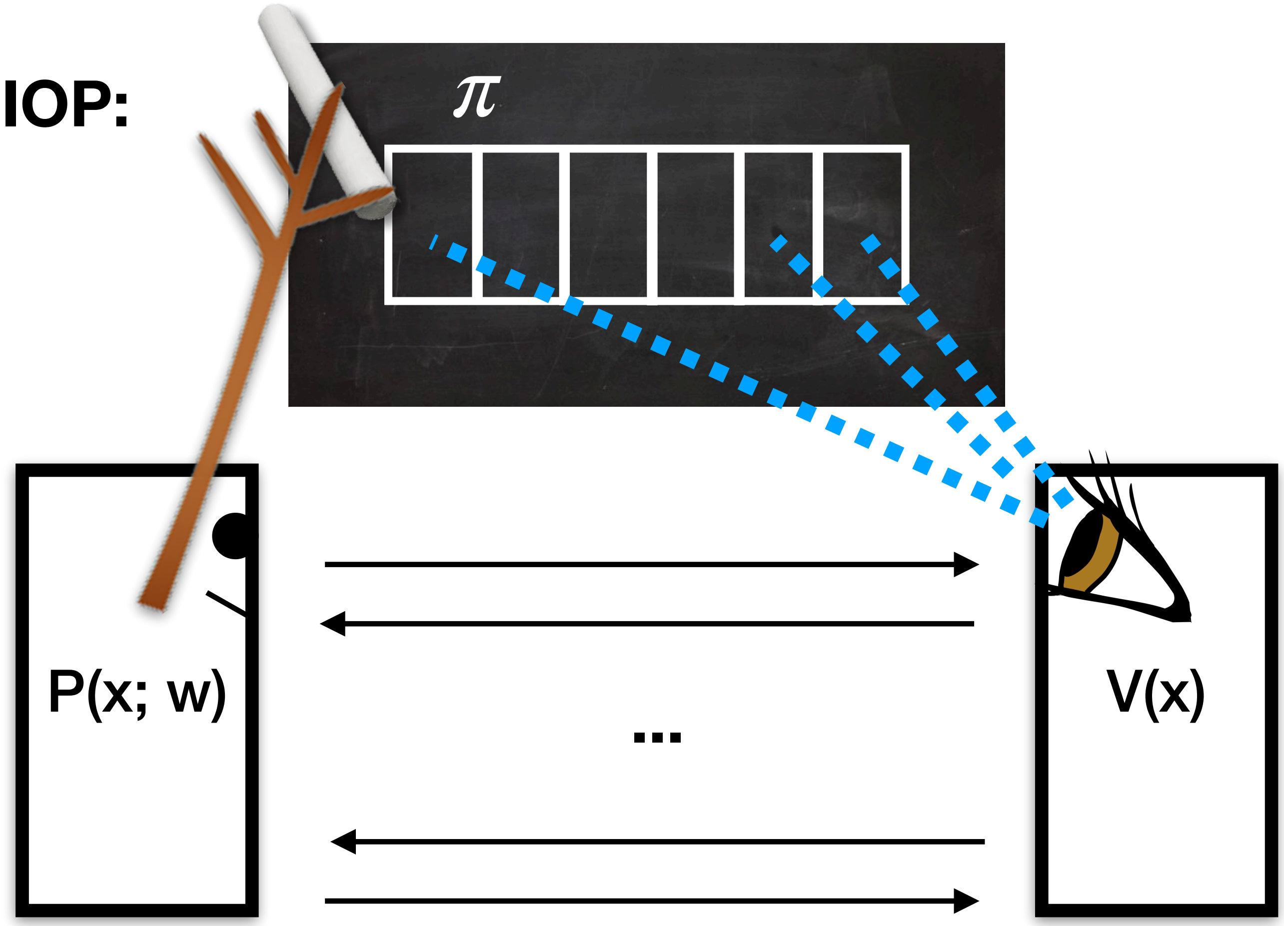

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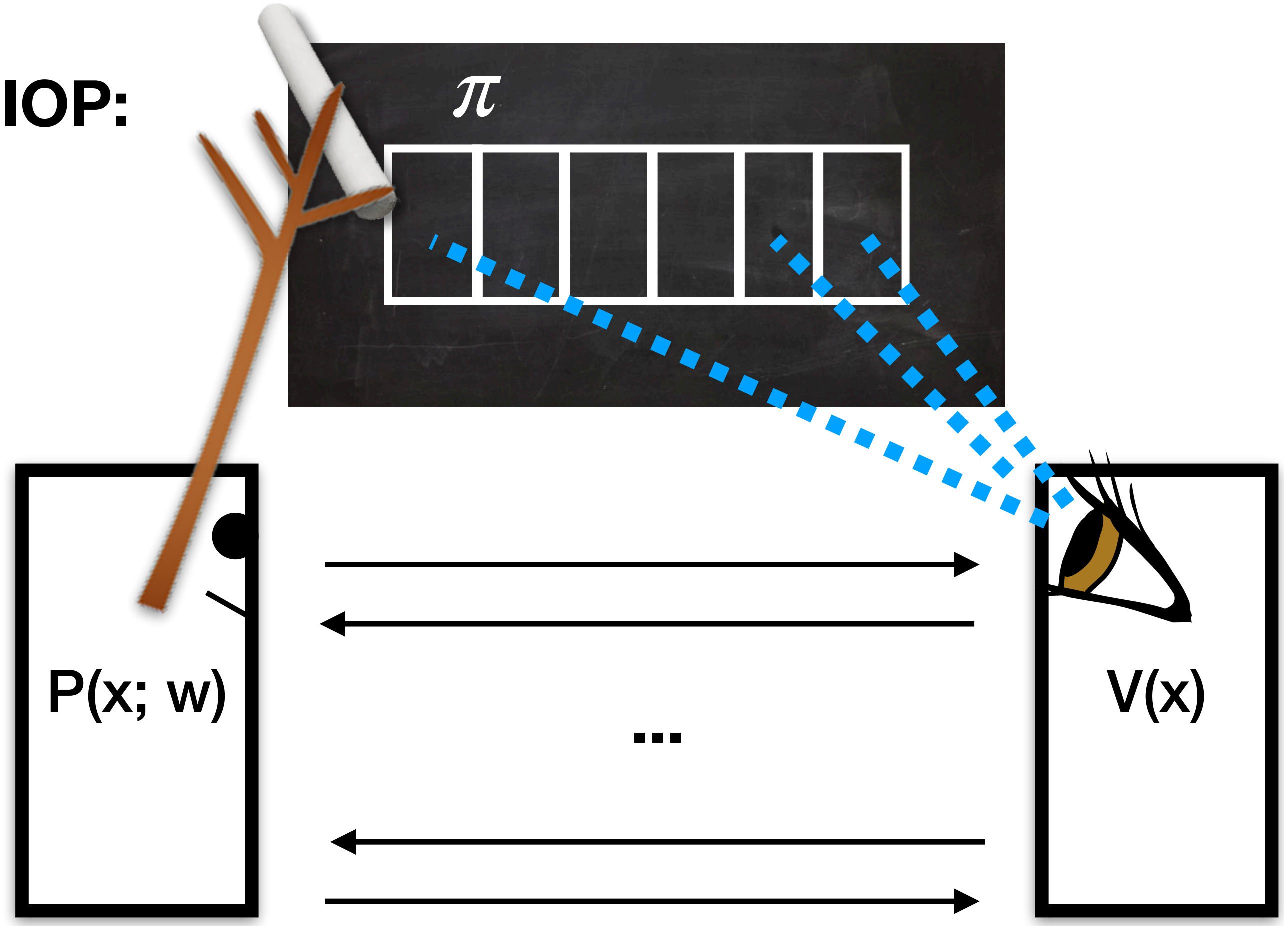

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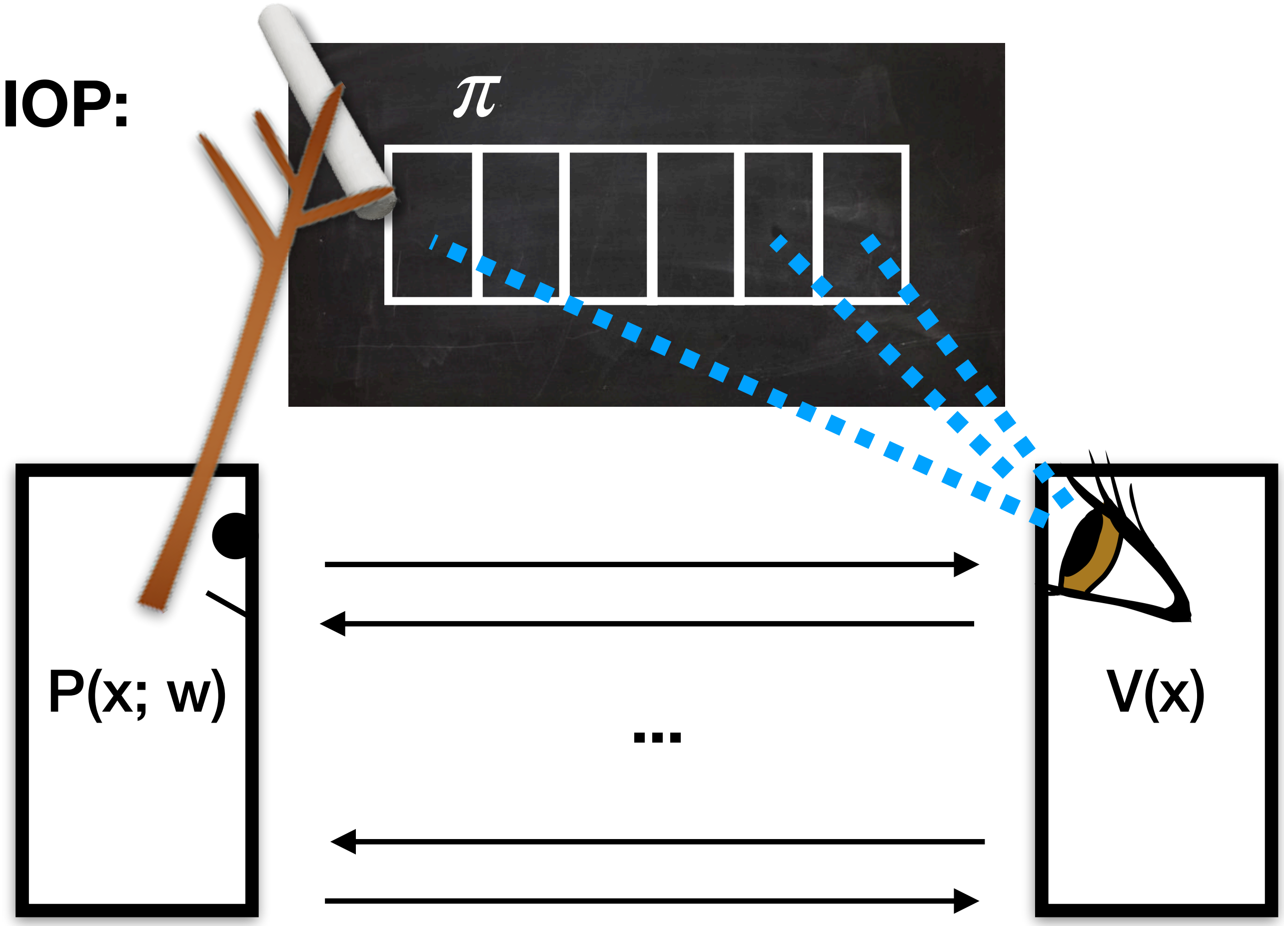

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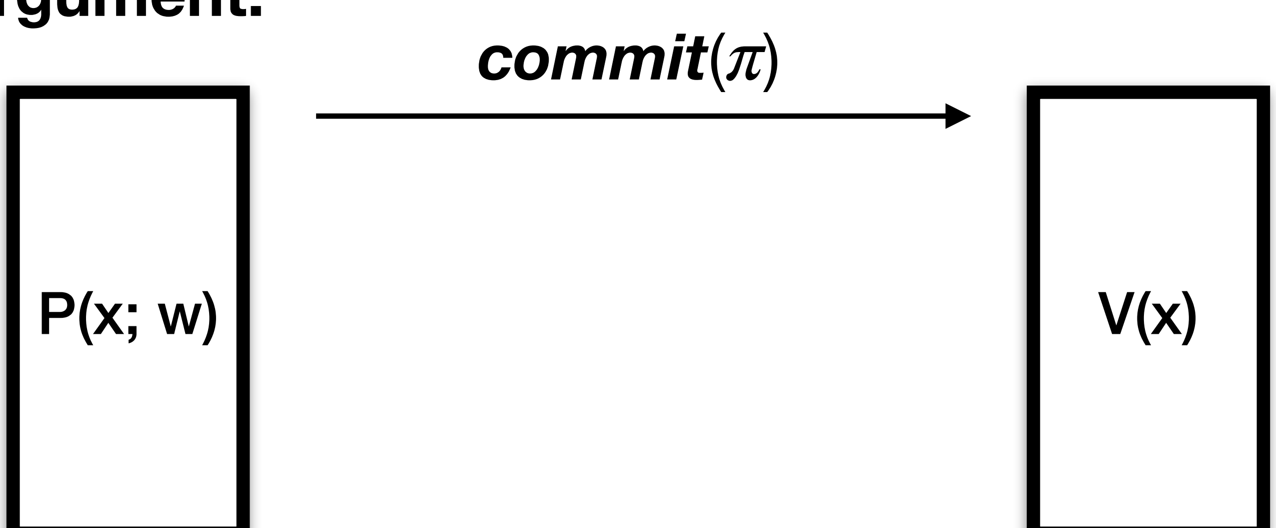

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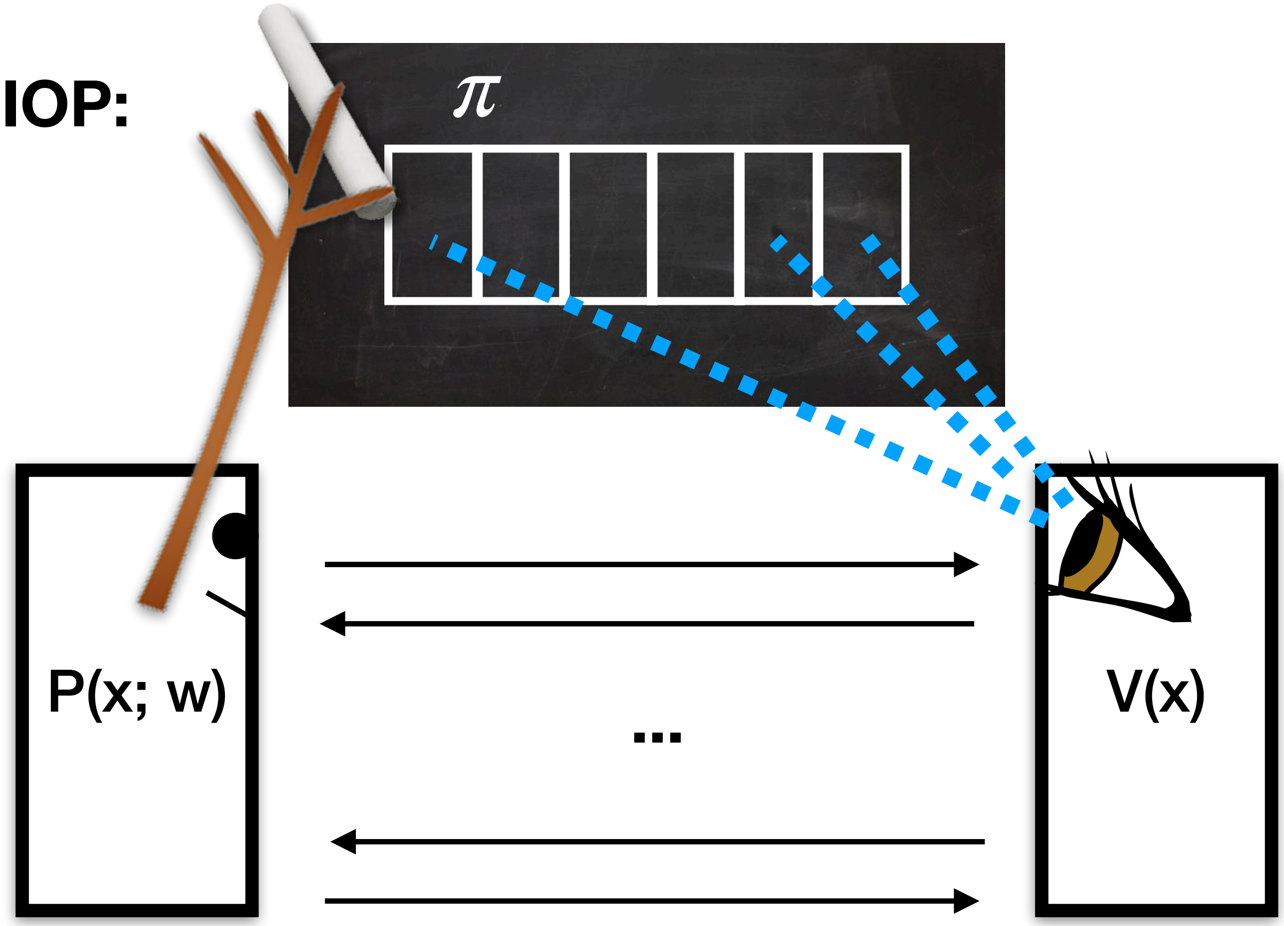

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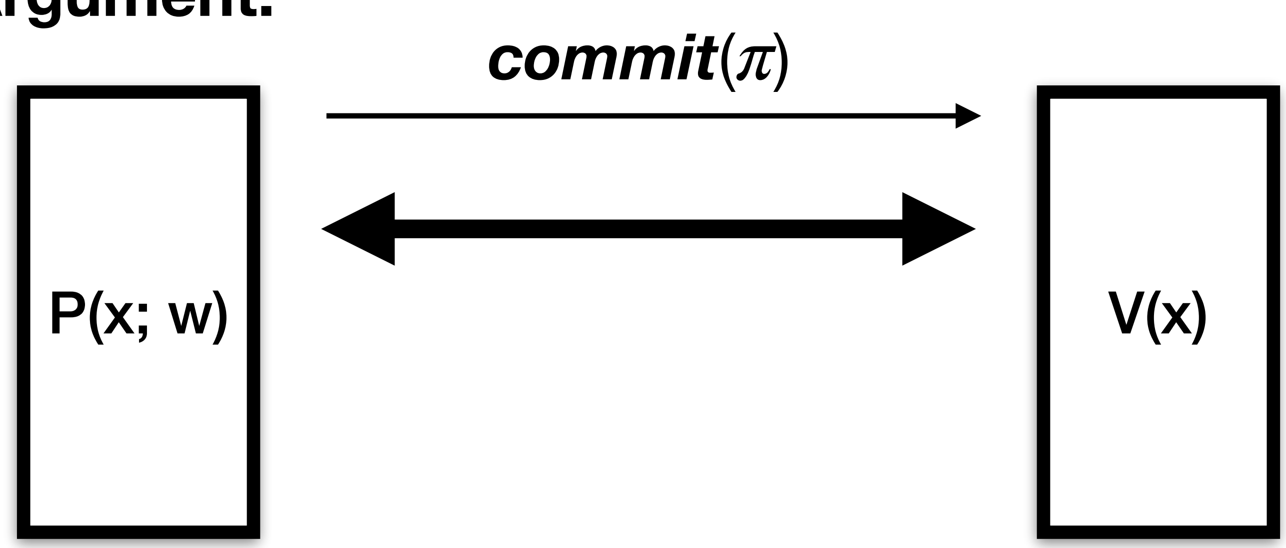

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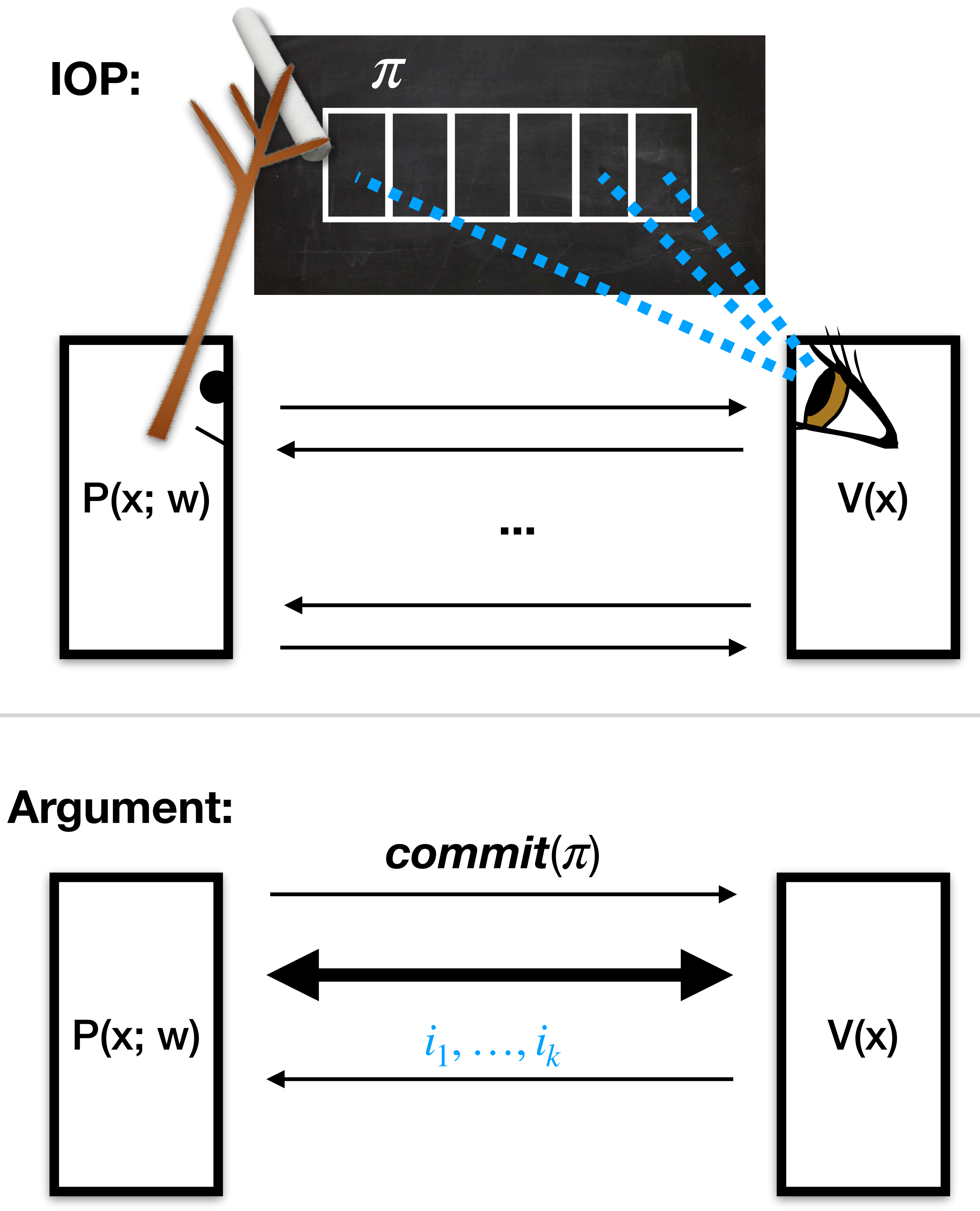

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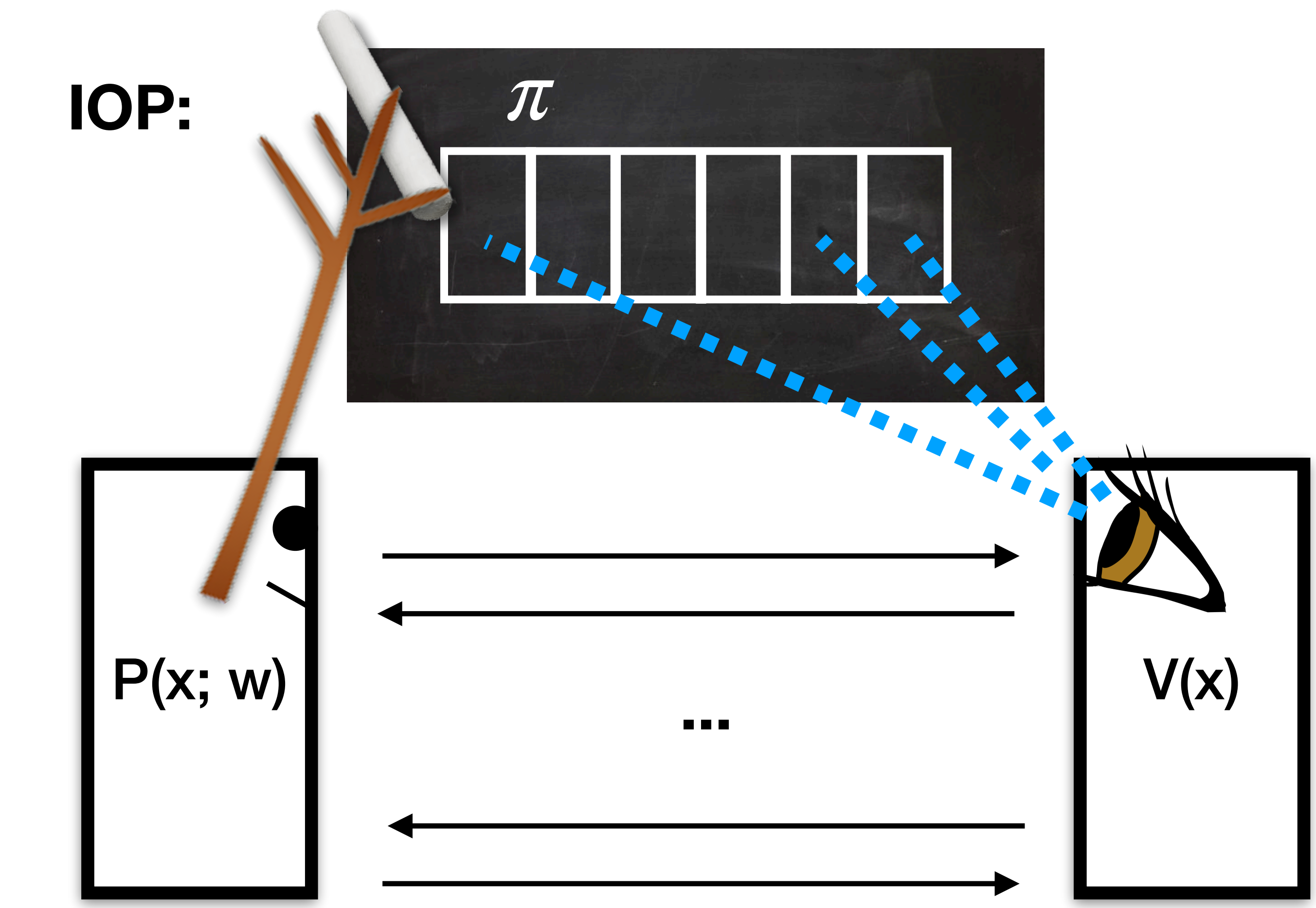

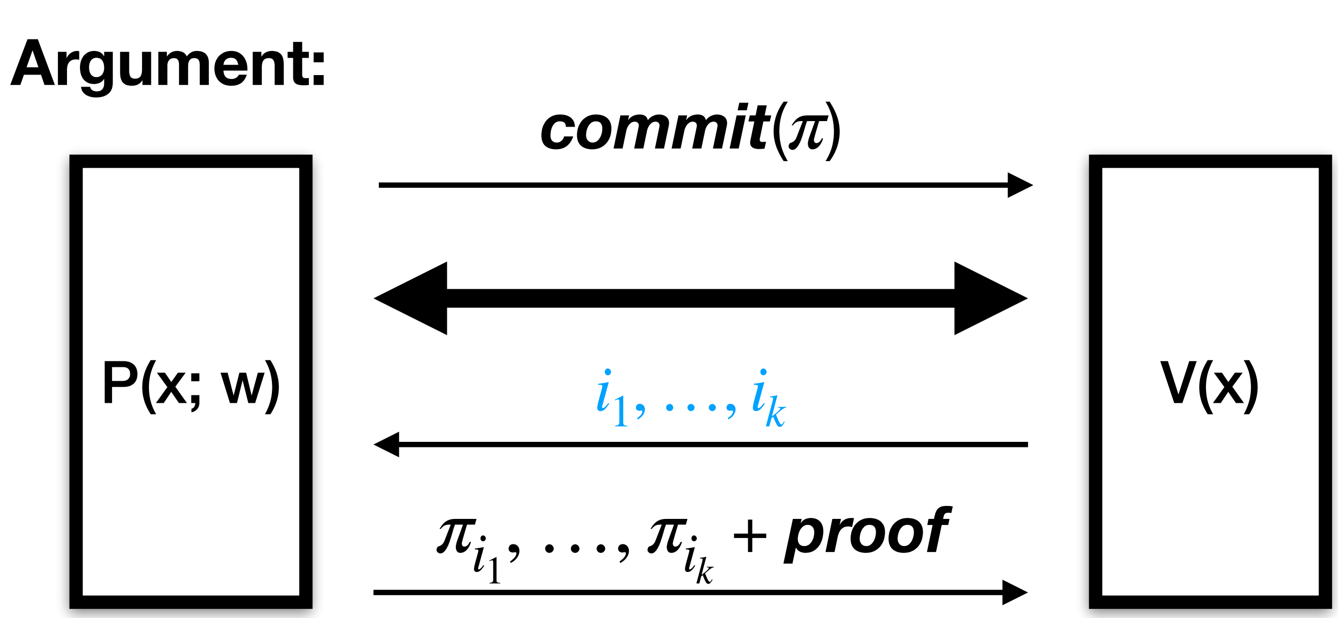

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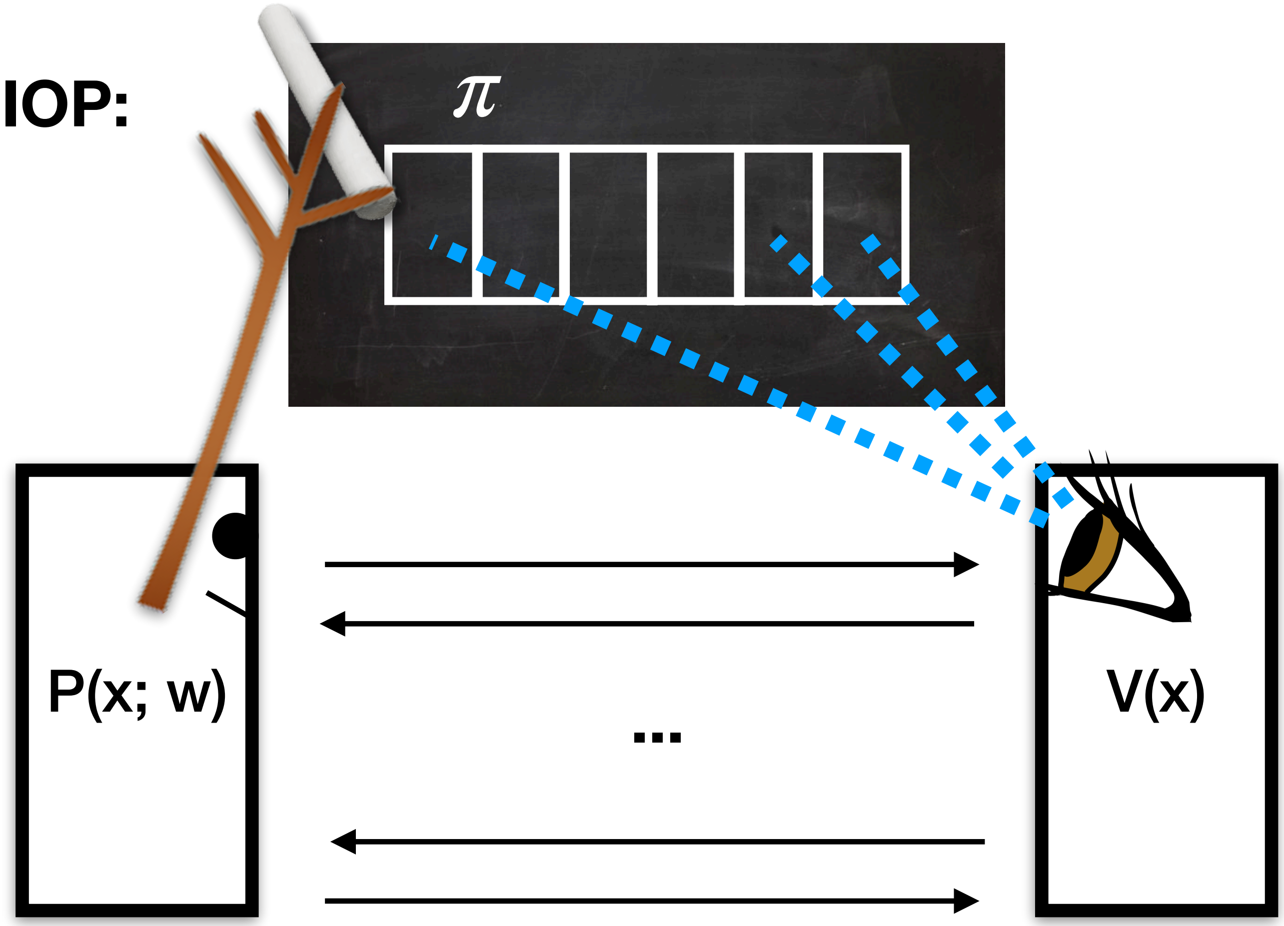

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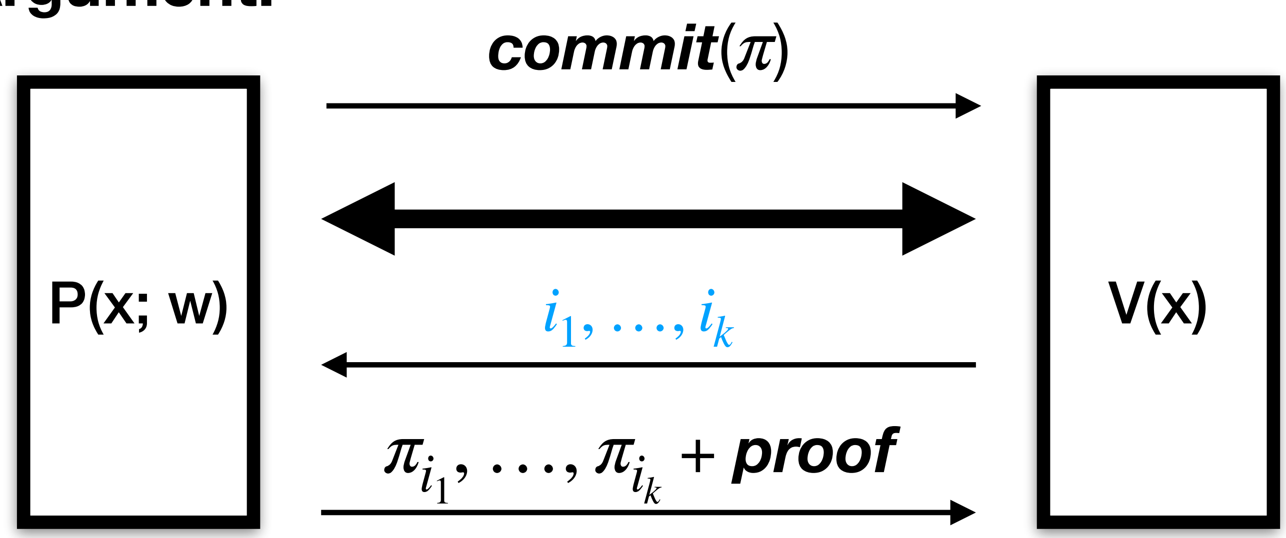

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Which IOP prover cost is most relevant to argument prover?

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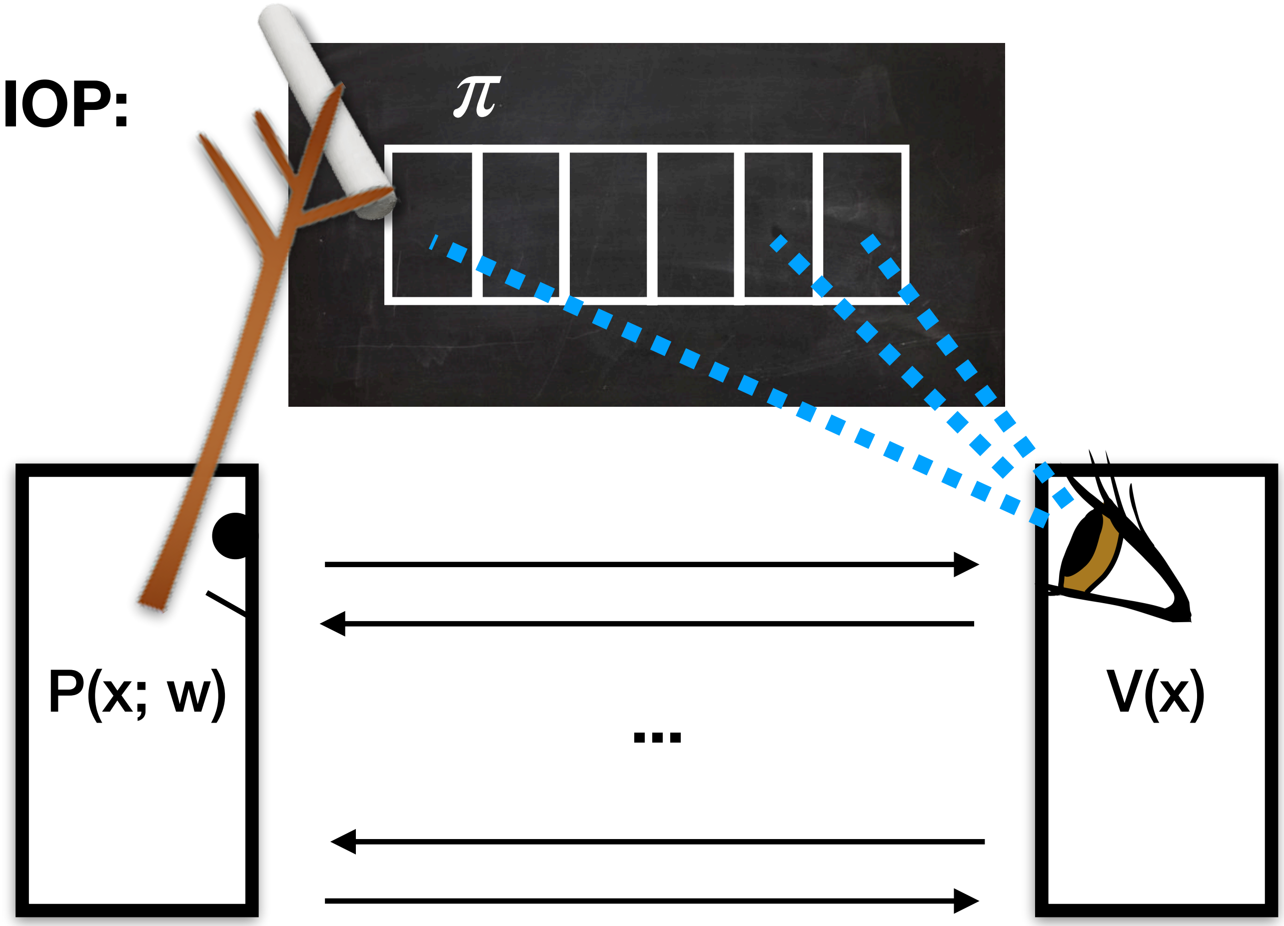

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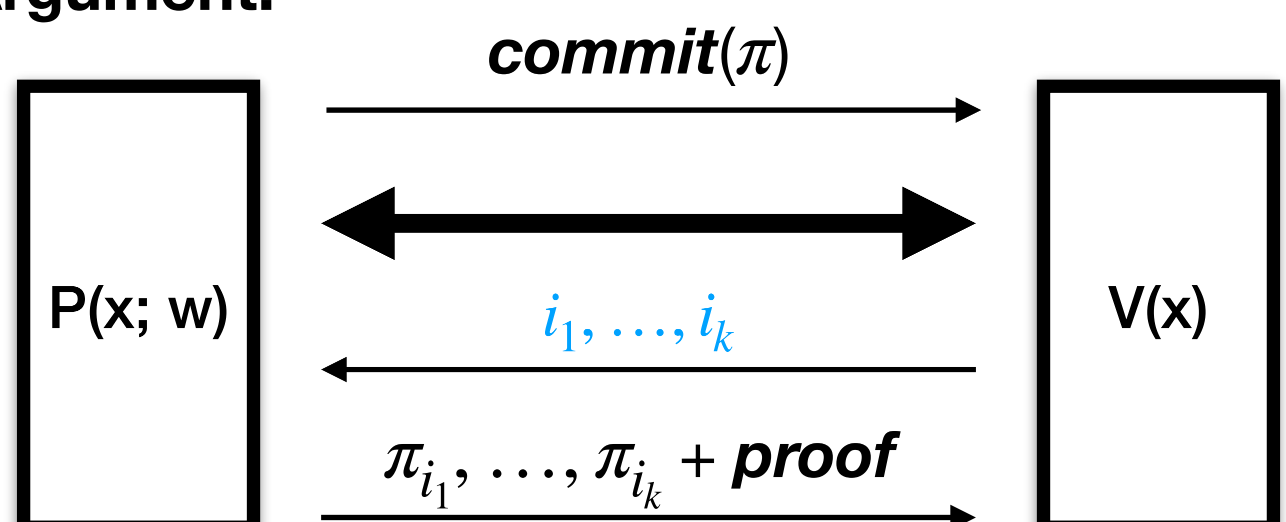

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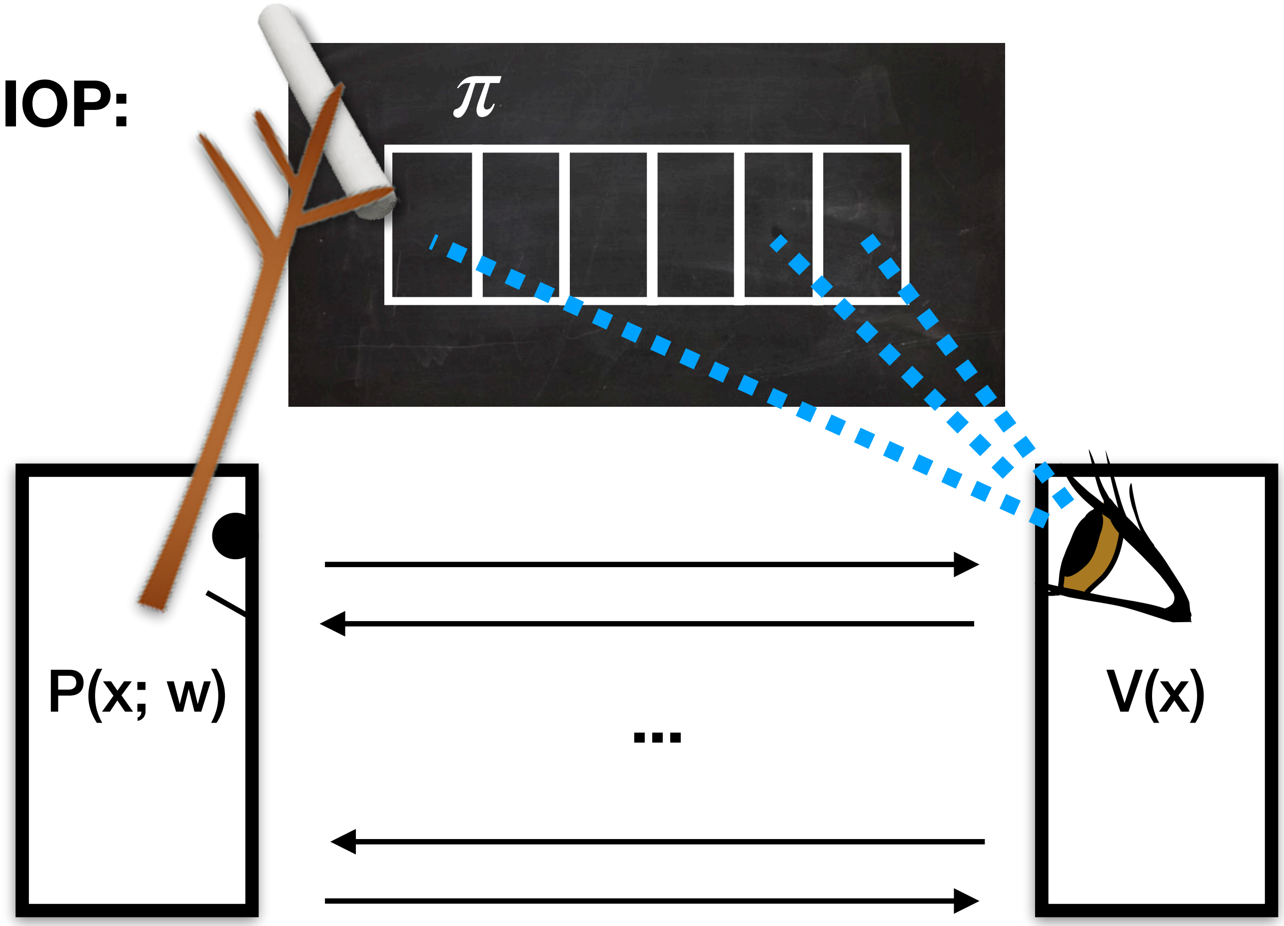

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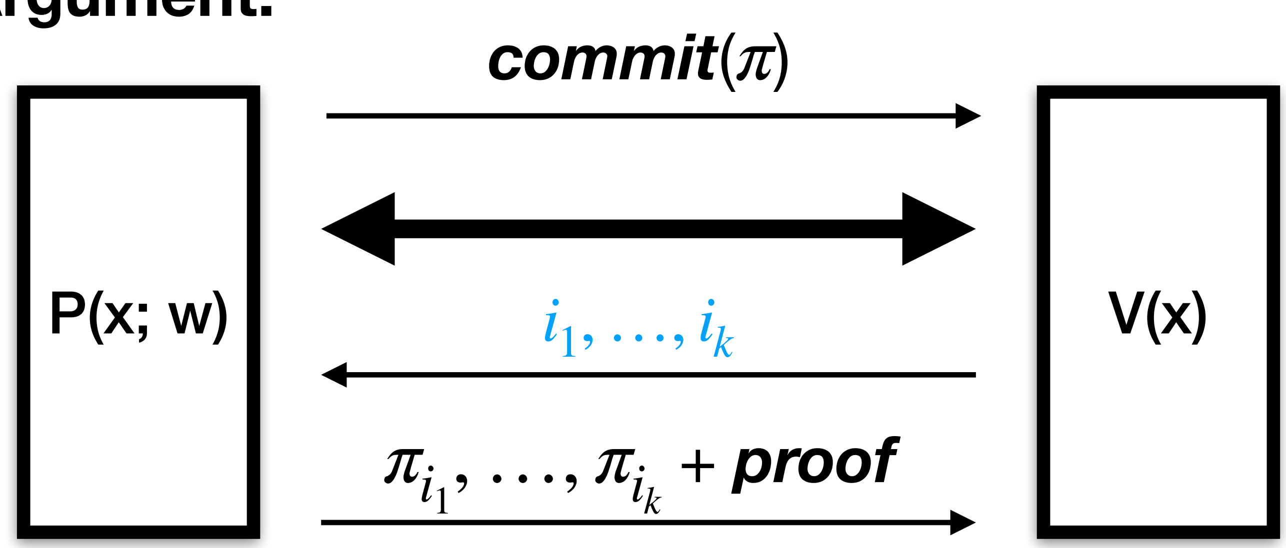

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Why does this matter?

We know IOPs with time- & space-efficient provers in the sense of **(B)** but not **(A)**.

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     (time- and space-) efficient for known IOPs (e.g. Clover [BTVW14])

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No More Talking About (Fine-Grained) Efficiency

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~~Moreover, the committer/prover on input p is efficient (in both time and space) given multi-pass *streaming access* to values of p on $\{0,1\}^n$.~~

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Polynomial Commitment Blueprint / Sketch

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[BFS19] show ***computational*** soundness for a specific f.

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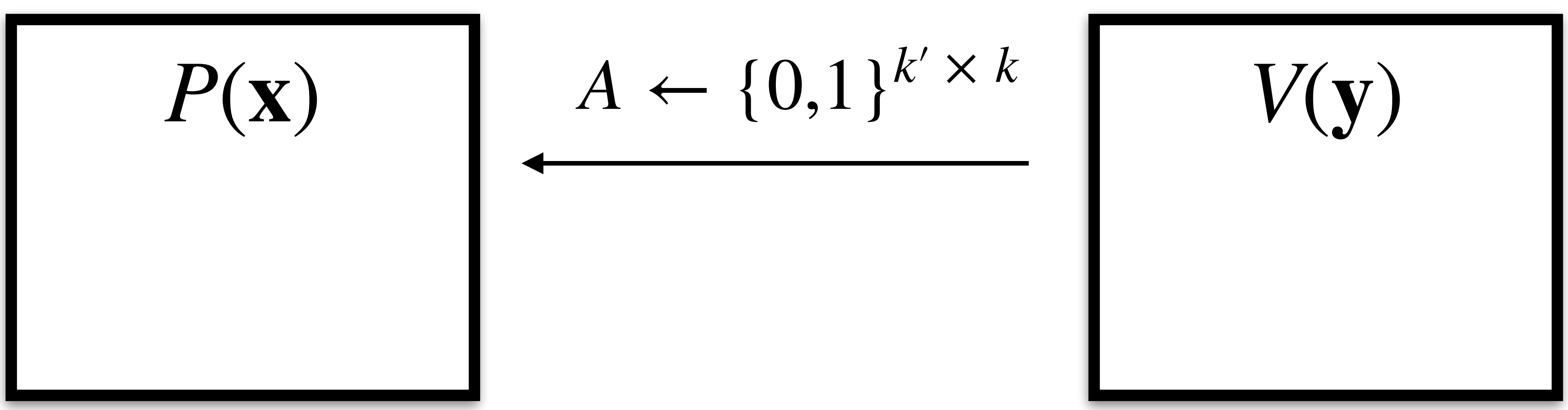

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	$\xrightarrow{\mathbf{x}' := A \cdot \mathbf{x}}$	$\mathbf{y}' := A \cdot \mathbf{y}$	**Accept if** $y'_i = f(x'_i)$ **for all** $i \in [k']$

Soundness: Our Batch Extractor

$P(\mathbf{x})$

$A \leftarrow \{0,1\}^{k' \times k}$ ($V \to P$)

$\mathbf{x}' := A \cdot \mathbf{x}$ ($P \to V$)

$V(\mathbf{y})$

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:-) with all but negl(k) **probability if** $5k/k' \leq B \leq O(1)$

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We want to extract CRHF pre-images, but...

Homomorphic CRHFs

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  - Hardness holds in generic group of unknown order
  - Concrete candidates:
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- Then $h(x) = g^x$ is a homomorphic CRHF from $\mathbb{Z}$ to $\mathbb{G}$

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    - Thus $\mathbb{Z}[x_1, \ldots, x_n] \to \mathbb{G}$ composition is a CRHF only on D.

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  - Equivalently, $|\det(L_i)| \to 1$
  - We analyze prime factorization of $\det(L_i)$, show that each step kills enough prime powers with enough probability to deduce the lemma.

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  - **Open:** from lattice assumptions, or in random oracle model
- Found and fixed bug in DARK polynomial commitment
- Techniques likely more broadly applicable: we also improve Pietrzak's proof of exponentiation protocol to achieve statistical soundness in arbitrary groups

Questions?