Symbolic Computation Techniques in SMT Solving: Mathematical Beauty meets Efficient Heuristics

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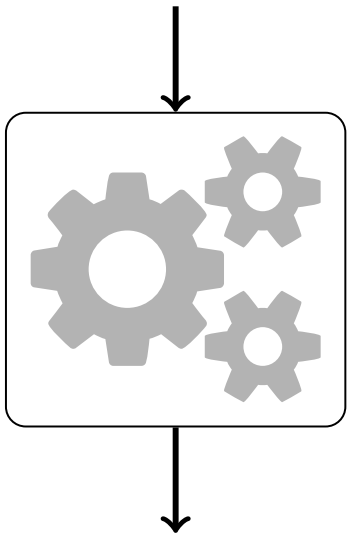
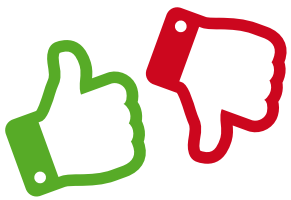
Satisfiability: Theory, Practice, and Beyond
Simons Institute, 21 April 2021

What is this talk about?

$$\neg a \wedge b \vee c$$

$$x^2 + x_2 \quad \sqrt{\varphi}$$





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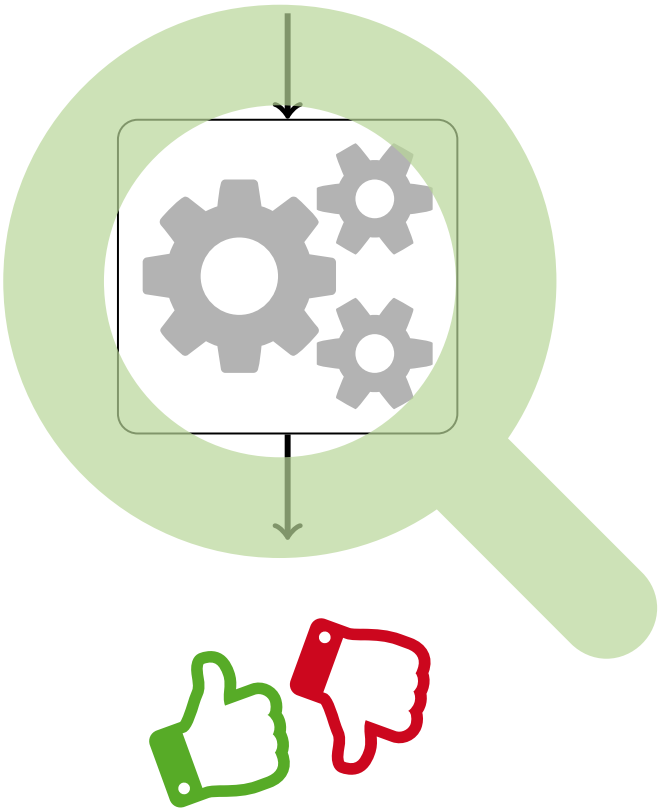

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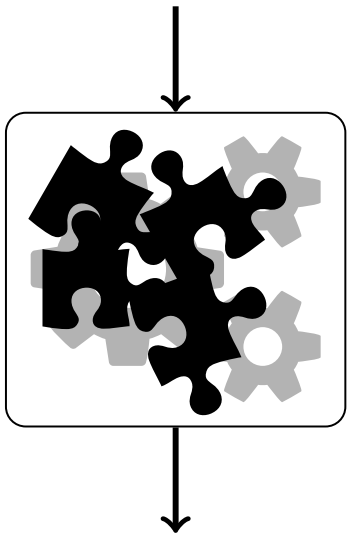

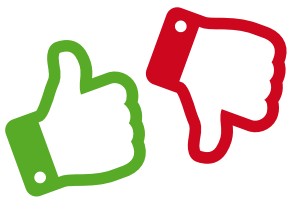

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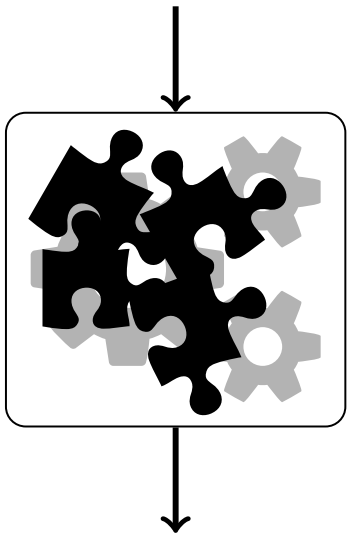

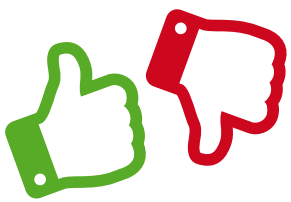

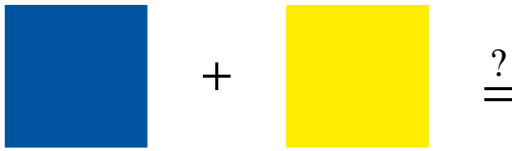

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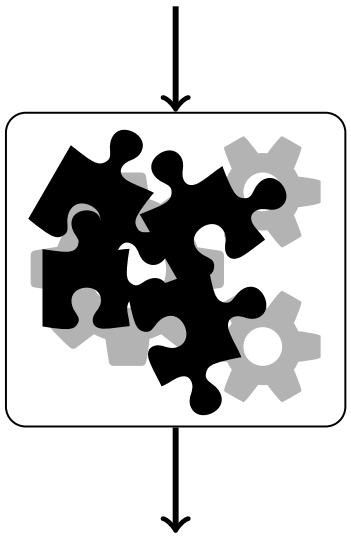

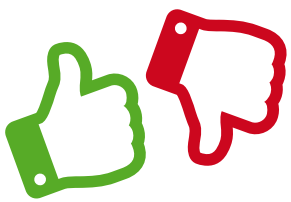

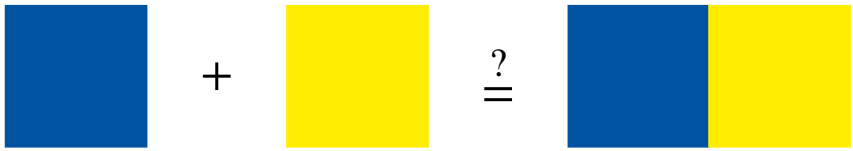

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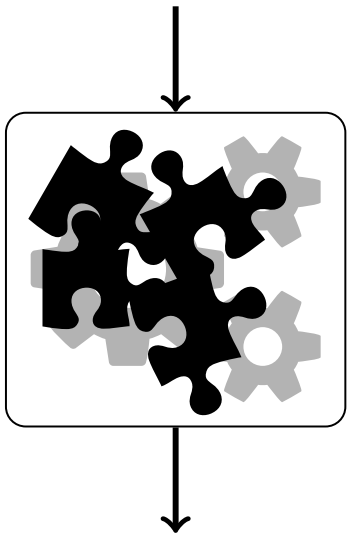

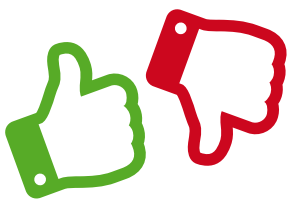

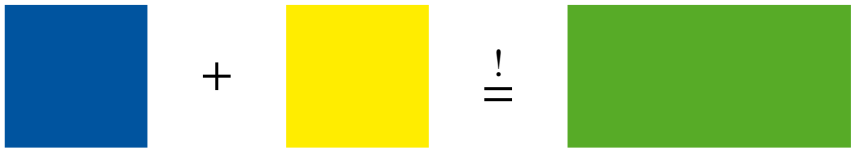

The satisfiability problem

Propositional logic

Formula: $(a \vee \neg b) \wedge (\neg a \vee b \vee c)$

Satisfying assignment: $a = \mathit{true}$, $b = \mathit{false}$, $c = \mathit{true}$

It is perhaps the most well-known NP-complete problem [Cook'71].

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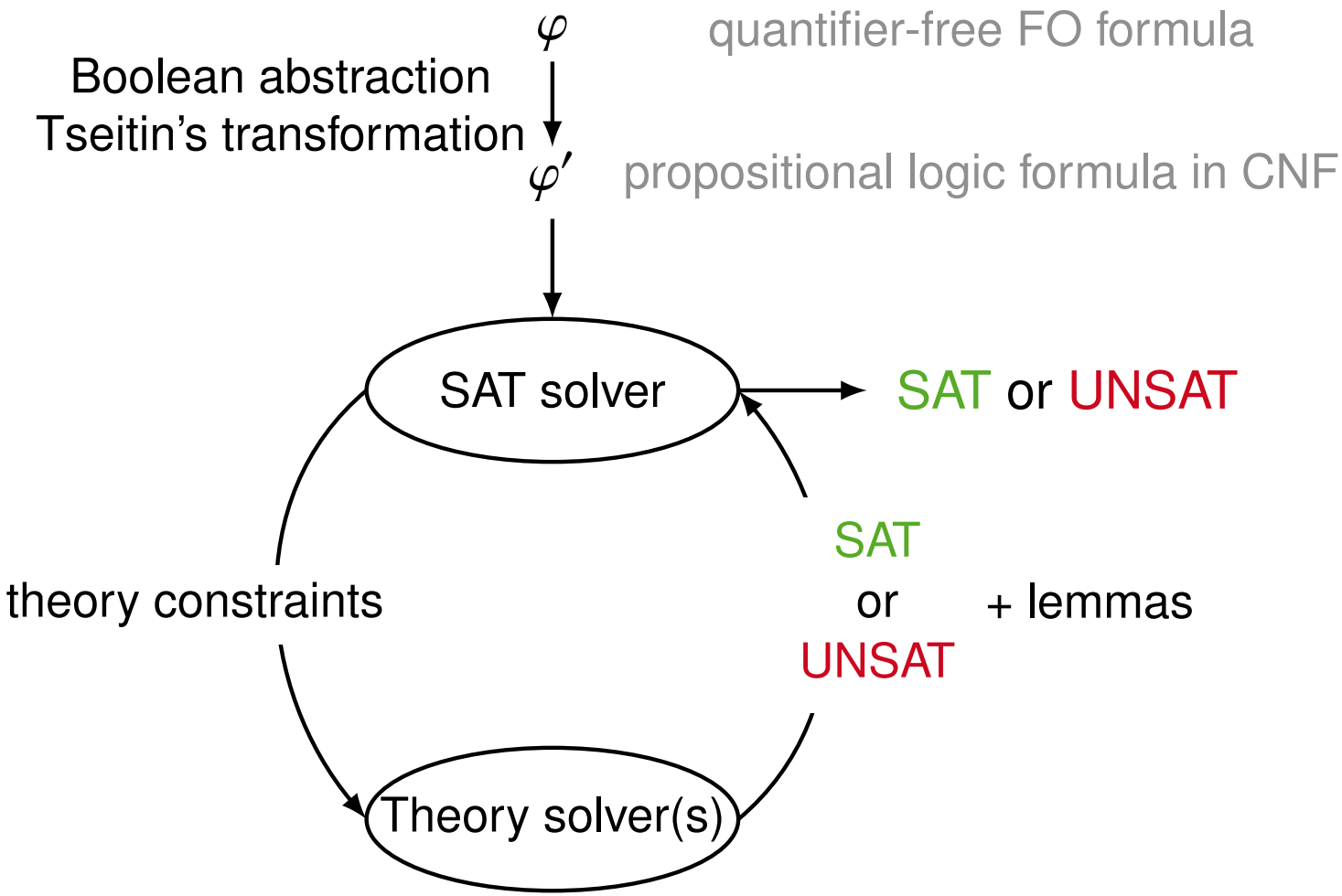
Non-linear real algebra (NRA)

Formula: $(x - 2y > 0 \vee x^2 - 2 = 0) \wedge x^4 y + 2x^2 - 4 > 0$

Satisfying assignment: $x = \sqrt{2}, \quad y = 2$

There are some hard problem classes... non-linear integer arithmetic is even undecidable.

(Full/less) lazy SMT solving



Some theory solver candidates for arithmetic theories

Linear real arithmetic:

- Simplex
- Ellipsoid method
- Fourier-Motzkin variable elimination (mostly preprocessing)
- Interval constraint propagation (incomplete)

Linear integer arithmetic:

- Cutting planes, Gomory cuts
- Branch-and-bound (incomplete)
- Bit-blasting (eager)
- Interval constraint propagation (incomplete)

Non-linear real arithmetic:

- Cylindrical algebraic decomposition
- Gröbner bases (mostly preprocessing/simplification)
- Virtual substitution (focus on low degrees)
- Interval constraint propagation (incomplete)

Non-linear integer arithmetic:

- Generalised branch-and-bound (incomplete)
- Bit-blasting (eager, incomplete)
- Interval constraint propagation (incomplete)

Problem solved?

Can we simply plug in available implementations of such methods as theory solvers into an SMT solver?

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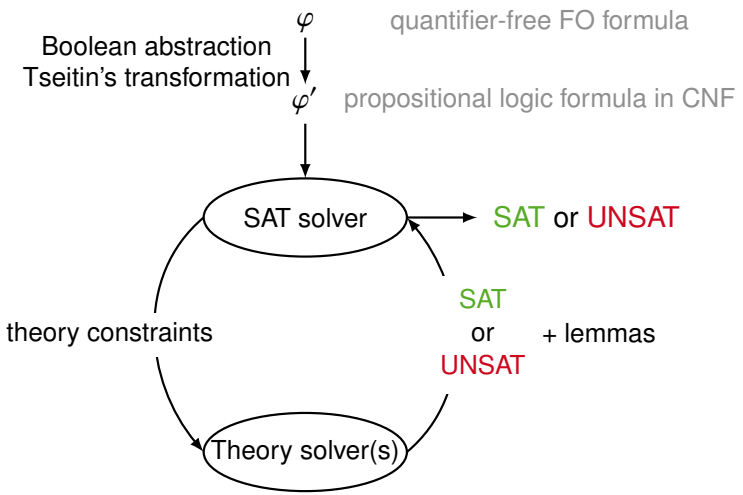
Theory solvers should be SMT-compliant, i.e., they should

- work incrementally,
- generate lemmas explaining inconsistencies, and
- be able to backtrack.



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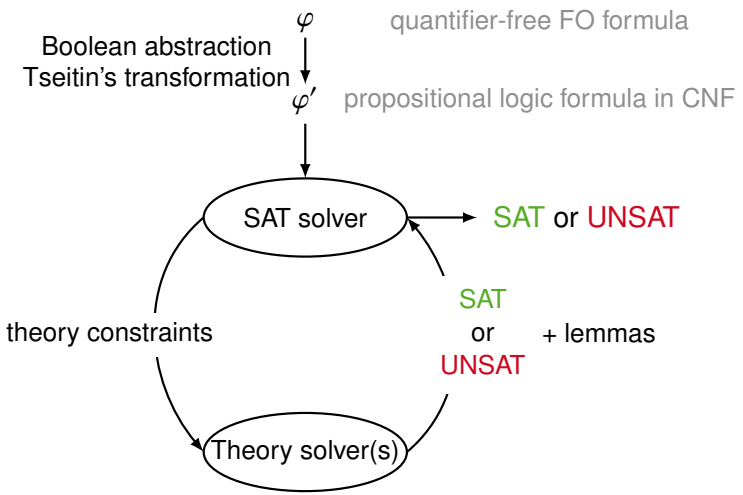
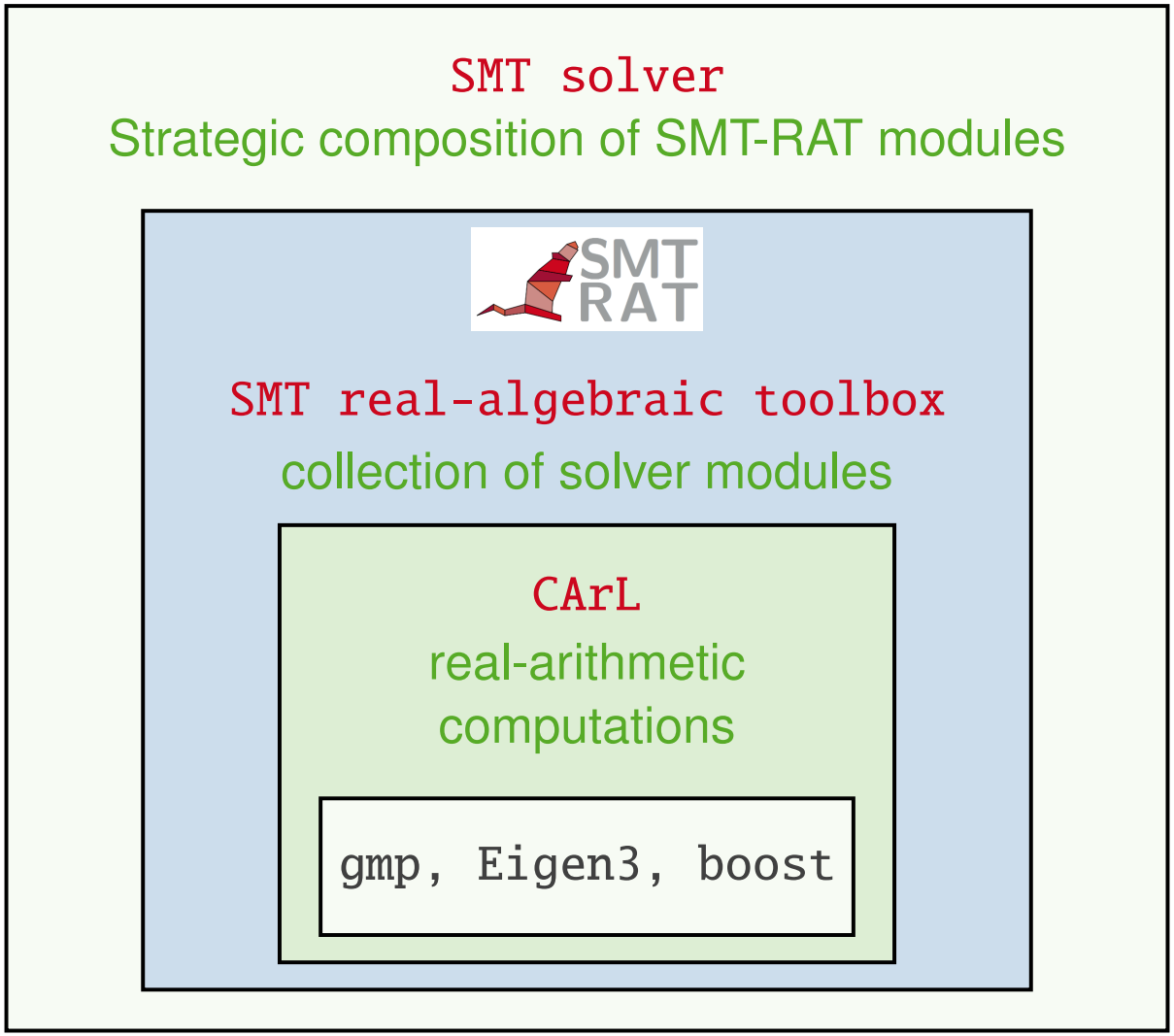


Originally, the mentioned methods are not SMT-compliant.

SMT-adaptations can be tricky, but can lead to beautiful novel algorithms.

Our SMT-RAT library SAT'12, SAT'15



- MIT licensed source code: `github.com/smtrat/smtrat`
- Documentation: `smtrat.github.io`

Solver modules in SMT-RAT [SAT'12, SAT'15]

CArL library for basic arithmetic datatypes and computations [Sapientia'18, NFM'11, CAI'11]

Basic modules

- SAT solver
- CNF converter
- Preprocessing/simplifying modules

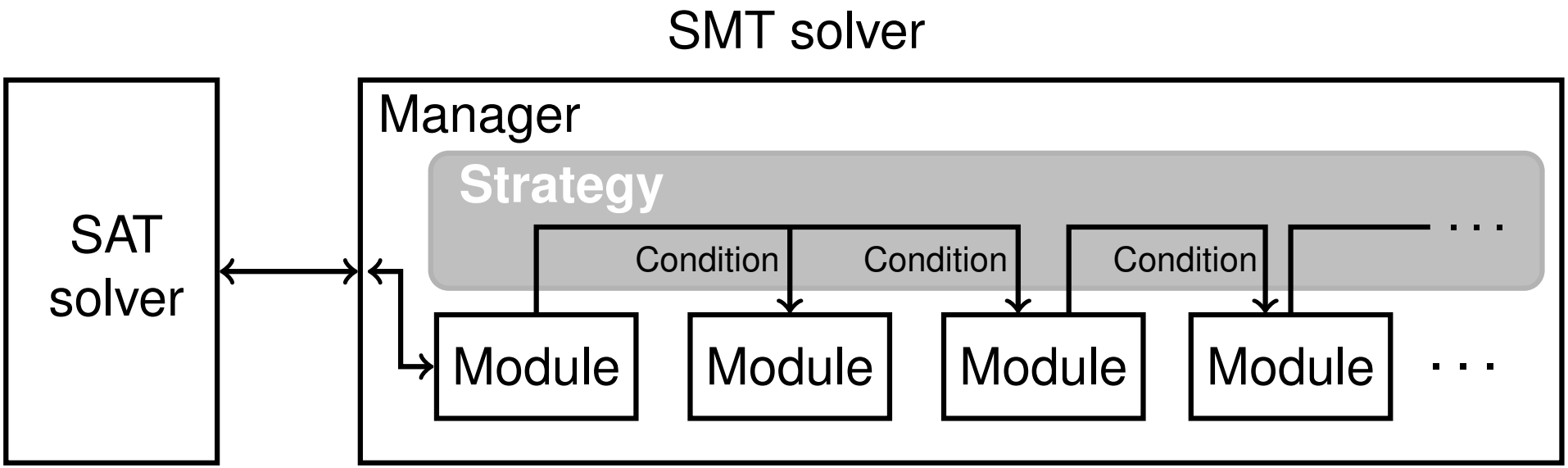
Non-algebraic decision procedures

- Equalities and uninterpreted functions
- Bit-vectors
- Bit-blasting
- Interval constraint propagation
- Pseudo-Boolean formulas

Algebraic decision procedures

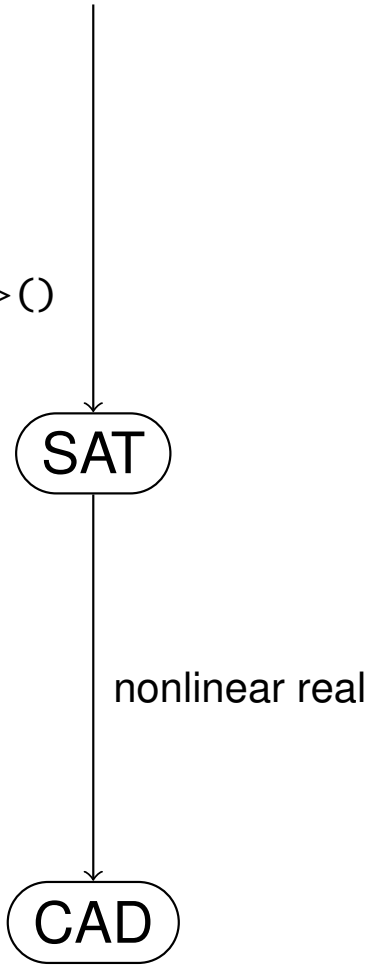
- Fourier-Motzkin variable elimination
- Simplex [SCSC'21]
- Gröbner bases [CAI'13]
- MCSAT (FM,VS,CAD) [2xSC^2'19]
- Cylindrical algebraic decomposition [SCSC'21,JLAMP'20, CADE-24, JSC'19, SC^2'17, 3 PhDs]
- Virtual substitution [FCT'11, SC^2'17, 1 PhD]
- Subtropical satisfiability
- Generalized branch-and-bound [CASC'16]
- Cube tests
- Linearization

Strategic composition of solver modules in SMT-RAT



SMT-RAT strategies

```
class myStrategy:  public Manager {
  myStrategy():  Manager() {
    setStrategy(
      addBackend<SATModule<SATSettings>>(
        addBackend<CADModule<CADSettings>>()
      )
    );
  }
};
```



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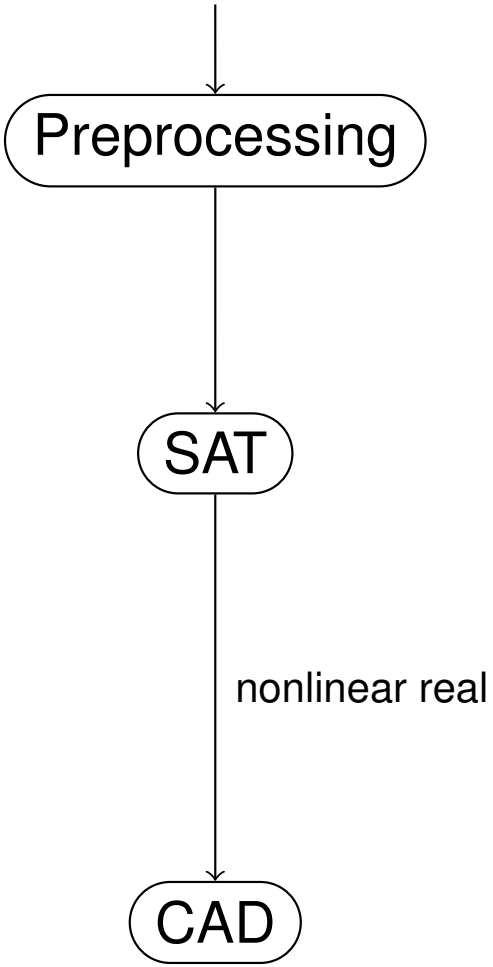

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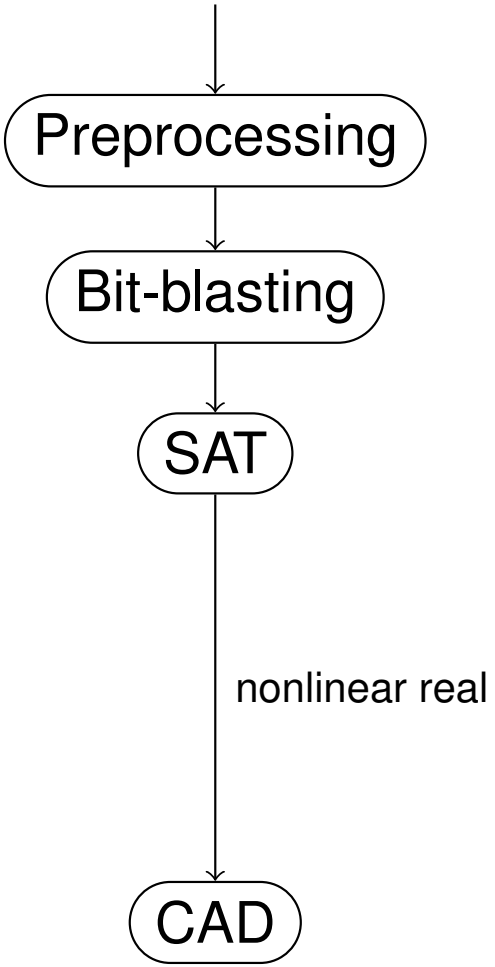

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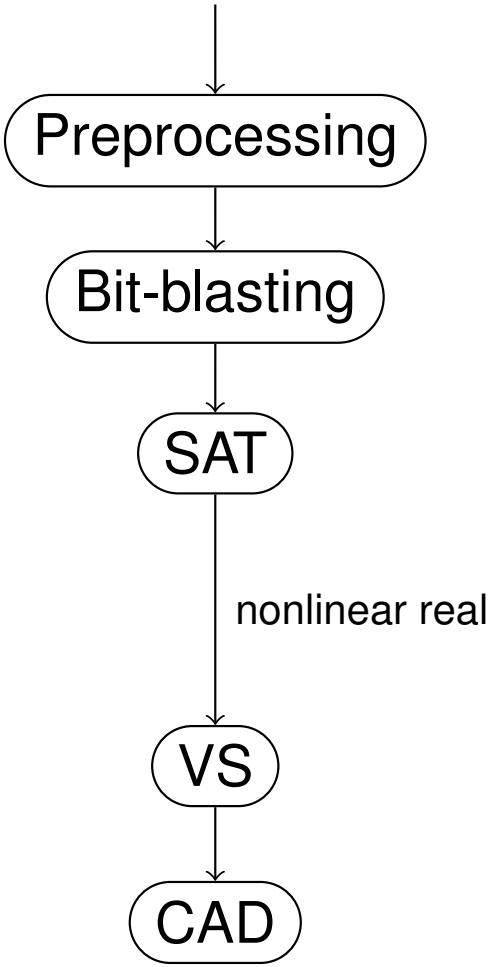

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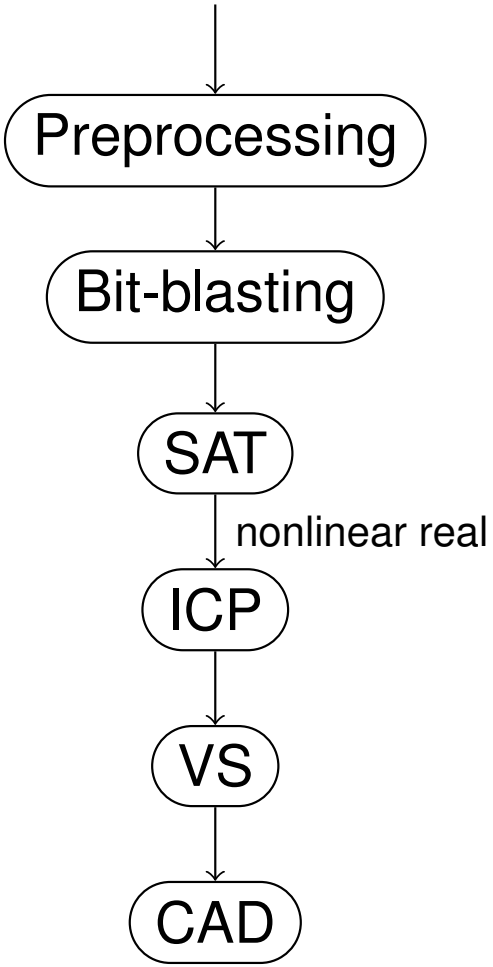

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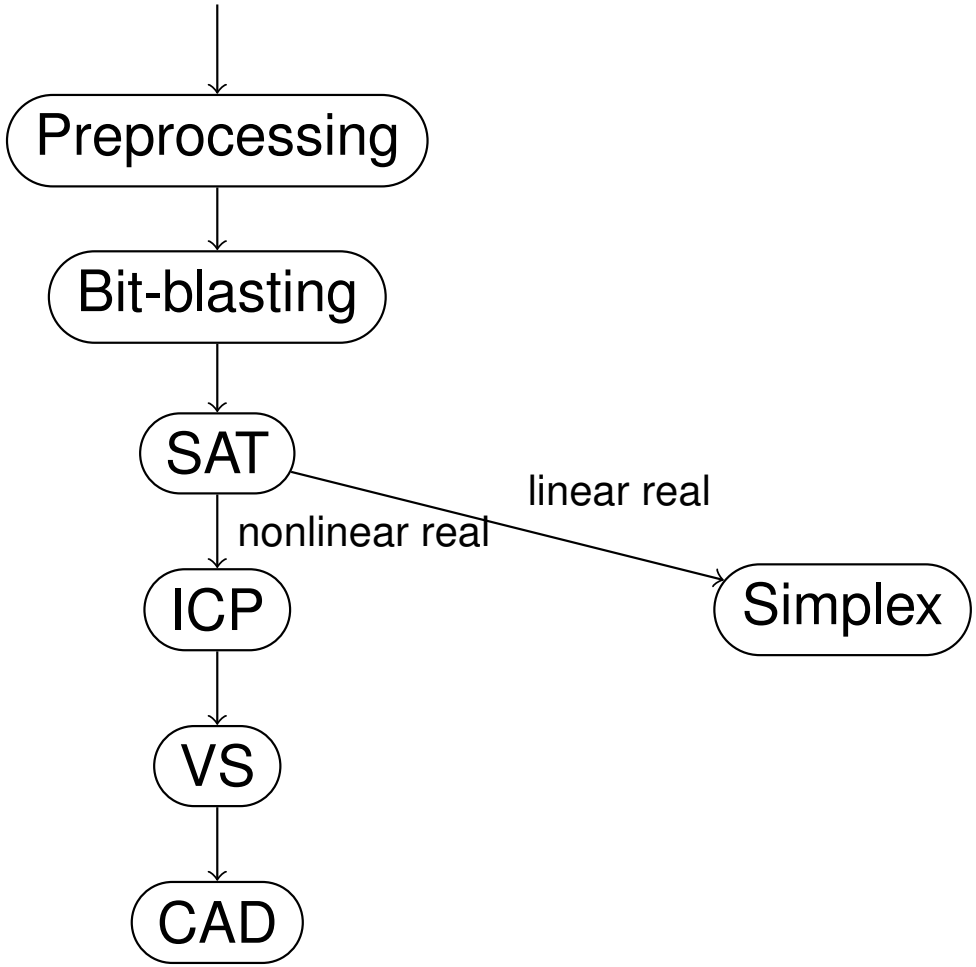

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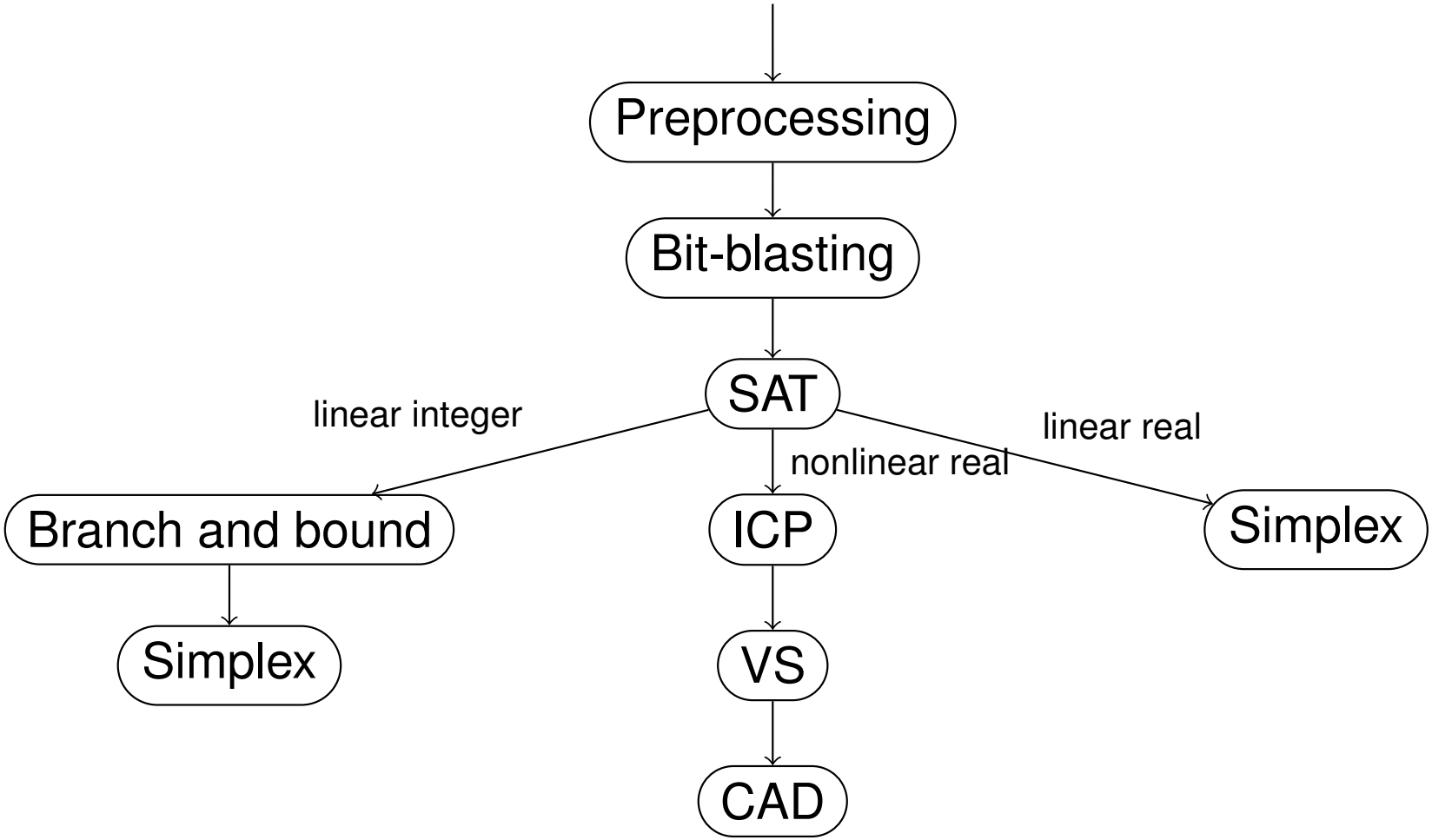

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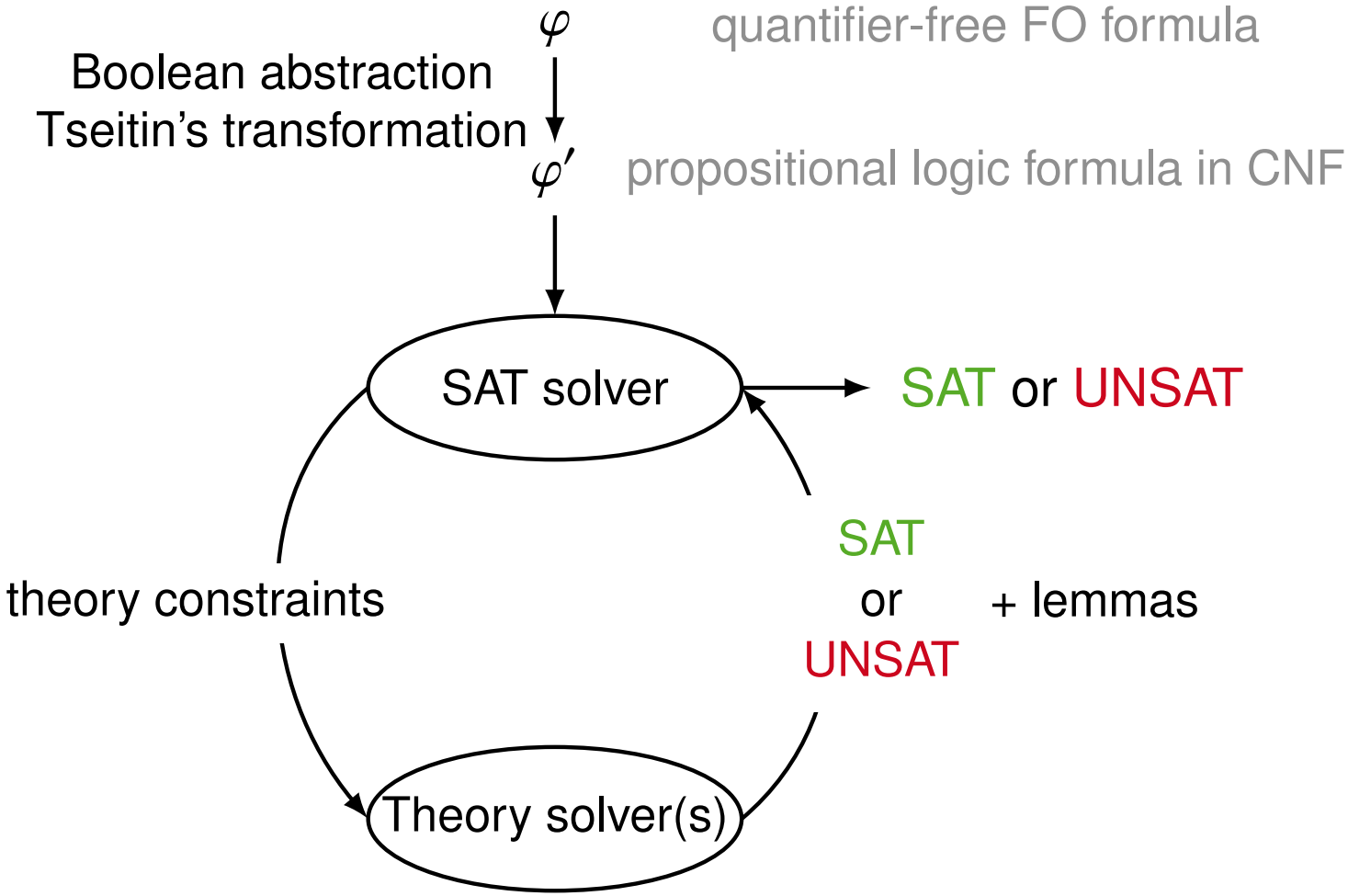

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Assumption: propositional logic formula in conjunctive normal form (CNF)

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$$\exists x.\, C_x \wedge C_{\neg x} \wedge C \quad \leftrightarrow \quad \mathit{Resolvents}\,(C_x, C_{\neg x}) \wedge C$$

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Problem: combinatorial blow-up

The DPLL idea

exploration:	$\mathbb{B}$-decision
look-ahead:	$\mathbb{B}$-propagation
proof system:	$\mathbb{B}$-conflict resolution

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$$(a \vee b \vee c) \wedge (a \vee b \vee \neg c)$$

$\mathbb{B}$-propagate	-
$\mathbb{B}$-decide	$a = \mathit{false}$
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$\mathbb{B}$-decide	$b = \mathit{false}$
$\mathbb{B}$-propagate	conflict
$\mathbb{B}$-conflict resolution	$(a \vee b)$

The MCSAT idea [de Moura, Jovanović, VMCAI'13]

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$(a \vee b \vee c) \wedge (a \vee b \vee \neg c)$ $\qquad$ $(\ldots \vee \underbrace{x \cdot y \cdot z^2 < 0}_{d=\mathit{true}} \vee \ldots)$

$\mathbb{B}$-propagate	-	$\mathbb{T}$-propagate	-
$\mathbb{B}$-decide	$a = \mathit{false}$	$\mathbb{T}$-decide	$x = 1$
$\mathbb{B}$-propagate	-	$\mathbb{T}$-propagate	-
$\mathbb{B}$-decide	$b = \mathit{false}$	$\mathbb{T}$-decide	$y = 1$
$\mathbb{B}$-propagate	conflict	$\mathbb{T}$-propagate	conflict
$\mathbb{B}$-conflict resolution	$(a \vee b)$	$\mathbb{T}$-conflict resolution	$(\neg d \vee x < 0 \vee y < 0)$

Fourier-Motzkin variable elimination

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$$\exists x. \begin{pmatrix} l_1 \leq x \wedge & x \leq u_1 \wedge \\ \dots & \dots \\ l_k \leq x \wedge & x \leq u_n \end{pmatrix} \Leftrightarrow \begin{pmatrix} l_1 \leq u_1 \wedge & \dots & l_1 \leq u_n \wedge \\ & \dots & \\ l_k \leq u_1 \wedge & \dots & l_k \leq u_n \end{pmatrix}$$

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Eliminate x_1:

Fourier-Motzkin variable elimination

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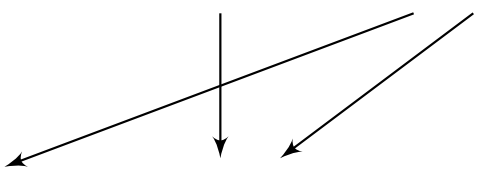

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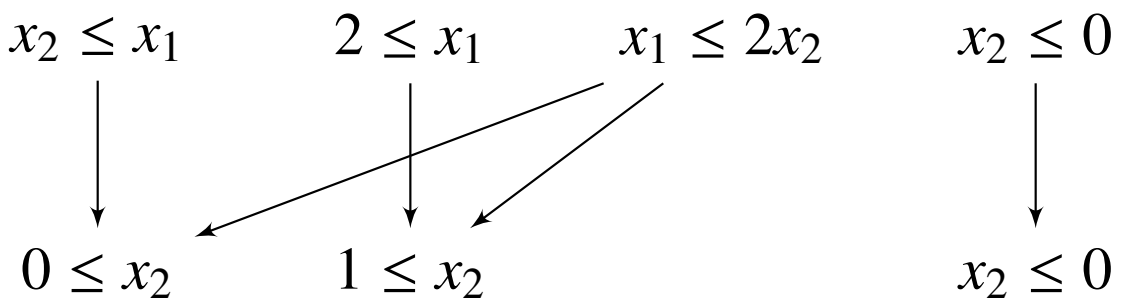

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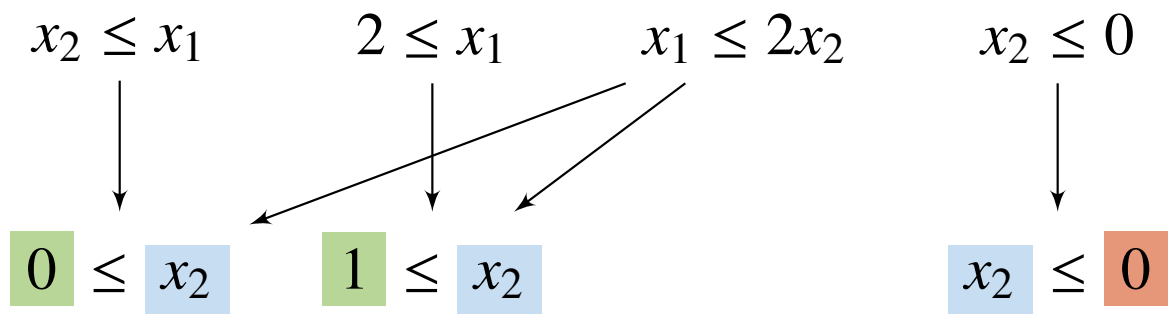

Eliminate x_2:

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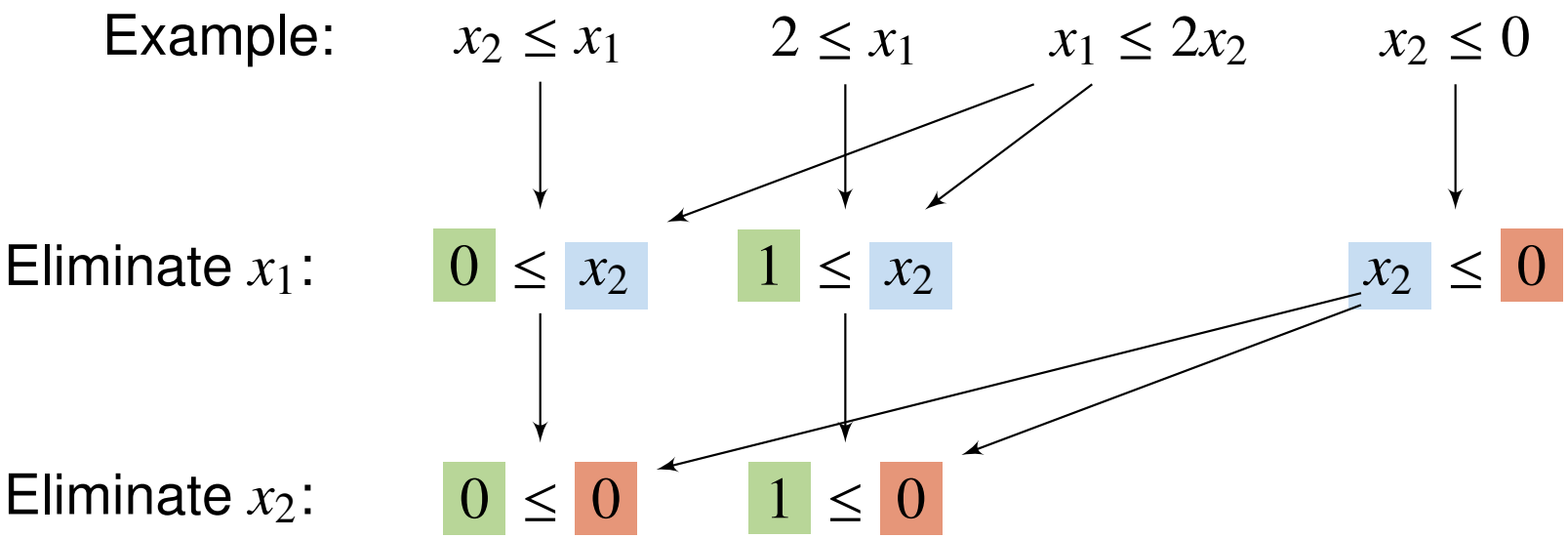

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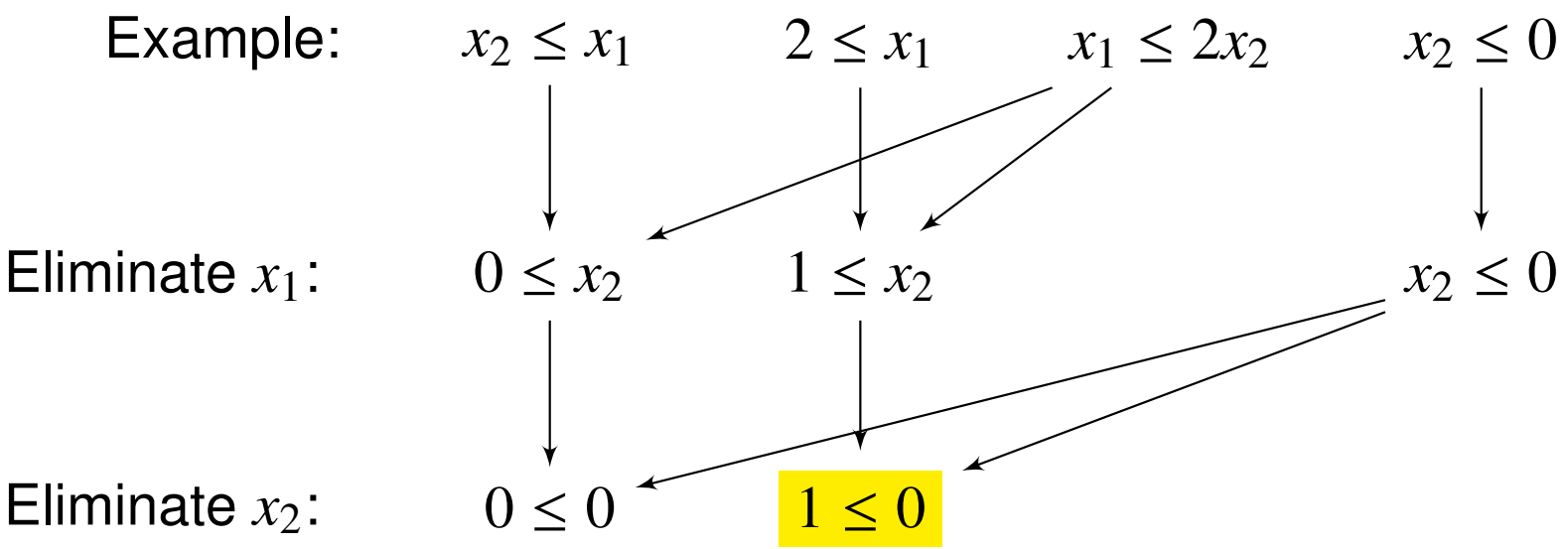

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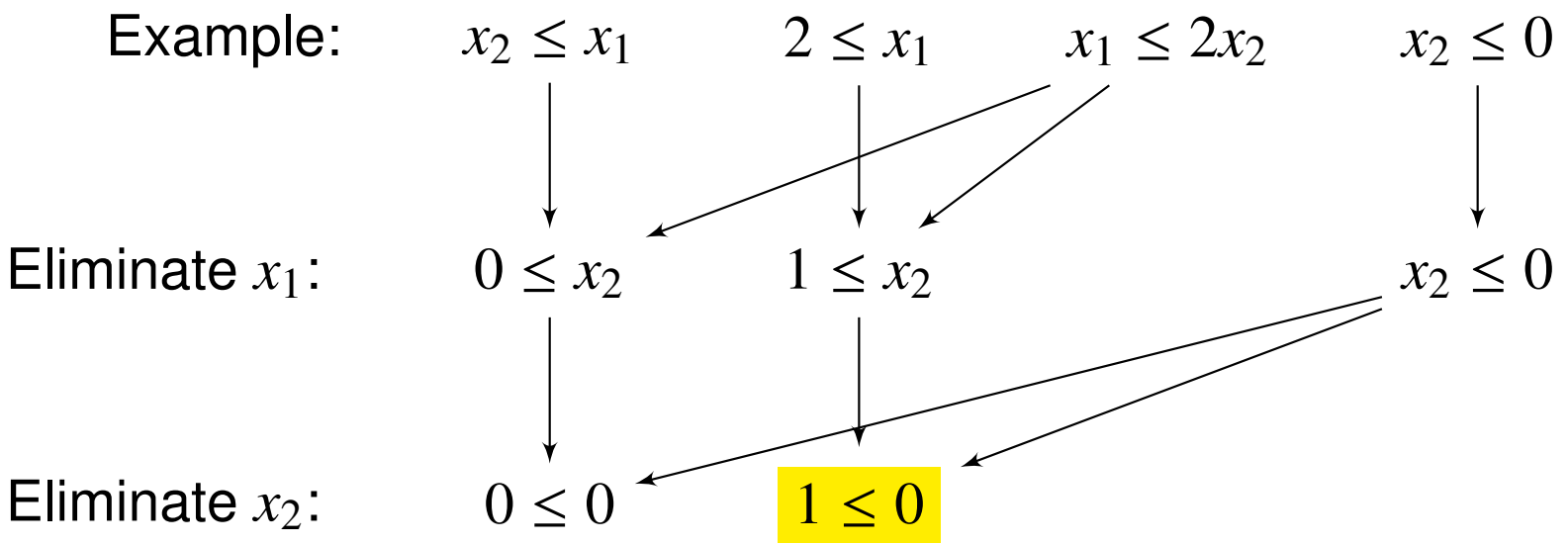

Problem: combinatorial blow-up

Fourier-Motzkin in MCSAT

Variable decision order: x_2, x_1

$$\ldots \underbrace{x_2 \leq x_1}_{a=\mathit{true}} \ldots \underbrace{2 \leq x_1}_{b=\mathit{true}} \ldots \underbrace{x_1 \leq 2x_2}_{c=\mathit{true}} \ldots \underbrace{x_2 \leq 0}_{d=\mathit{true}} \ldots$$

Fourier-Motzkin in MCSAT

Variable decision order: x_2, x_1

$$\ldots \underbrace{x_2 \leq x_1}_{a=\mathit{true}} \ldots \quad \underbrace{2 \leq x_1}_{b=\mathit{true}} \quad \ldots \underbrace{x_1 \leq 2\, x_2}_{c=\mathit{true}} \ldots \quad \underbrace{x_2 \leq 0}_{d=\mathit{true}} \ldots$$

Fourier-Motzkin in MCSAT

Variable decision order: x_2, x_1

$\mathbb{T}$-prop.+$\mathbb{T}$-dec.: $x_2 := 0$

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$$0 \leq x_1 \qquad 2 \leq x_1 \qquad x_1 \leq 0 \qquad 0 \leq 0$$

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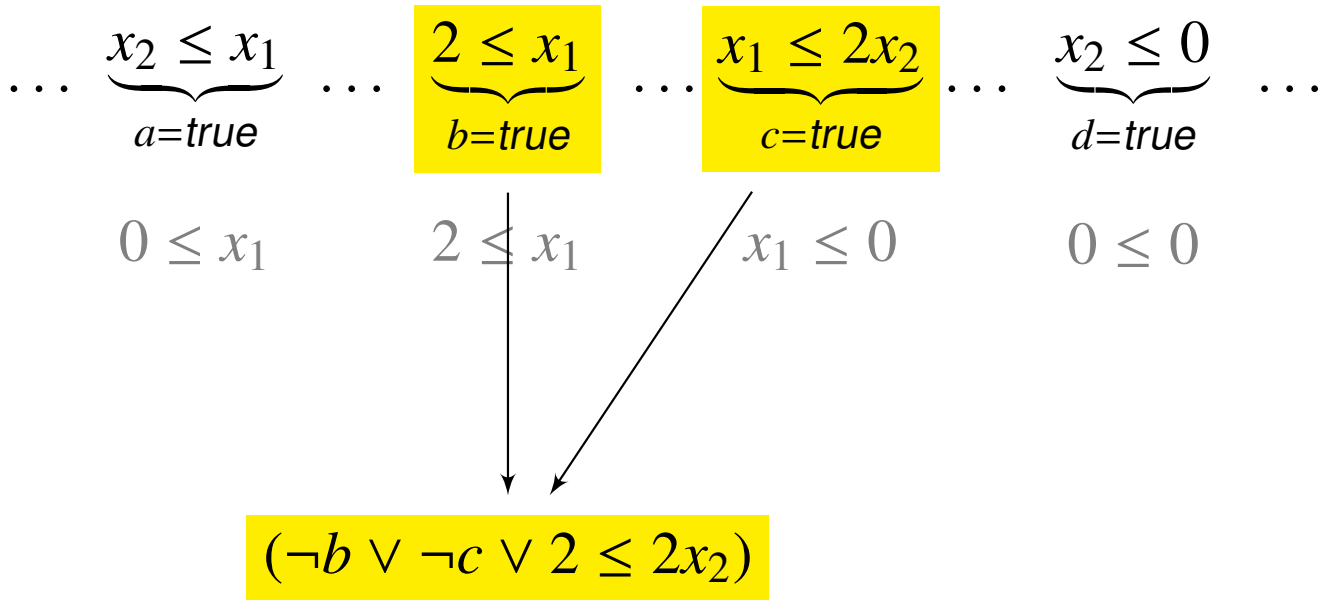
Fourier-Motzkin in MCSAT

Variable decision order: x_2, x_1

$\mathbb{T}$-prop.+$\mathbb{T}$-dec.: $x_2 := 0$

$\mathbb{T}$-prop.: conflict



Exploration-guided Fourier-Motzkin

x_1: $x_2 \leq x_1$ $2 \leq x_1$ $x_1 \leq 2x_2$

x_2: $x_2 \leq 0$

Exploration-guided Fourier-Motzkin

x_1: $x_2 \leq x_1$ $2 \leq x_1$ $x_1 \leq 2x_2$

x_2: ? $x_2 \leq 0$

Exploration-guided Fourier-Motzkin

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Exploration-guided Fourier-Motzkin

x_1: ? $x_2 \leq x_1$ $2 \leq x_1$ $x_1 \leq 2x_2$

x_2: 0 $x_2 \leq 0$

Exploration-guided Fourier-Motzkin

x_1: conflict $\quad x_2 \leq x_1 \quad 2 \leq x_1 \quad x_1 \leq 2x_2$

x_2: 0 $\quad x_2 \leq 0$

Exploration-guided Fourier-Motzkin

x_1: conflict $x_2 \leq x_1$

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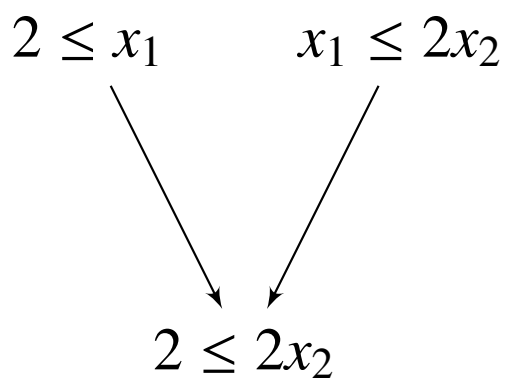
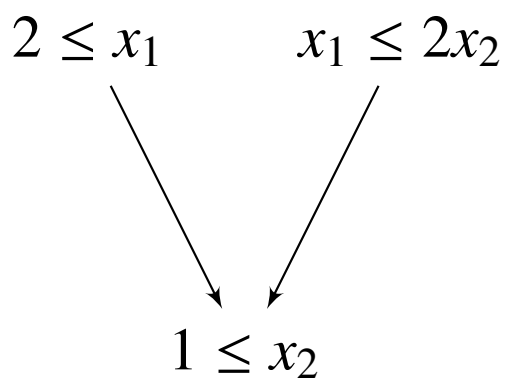


Exploration-guided Fourier-Motzkin

x_1: conflict $x_2 \leq x_1$

x_2: 0



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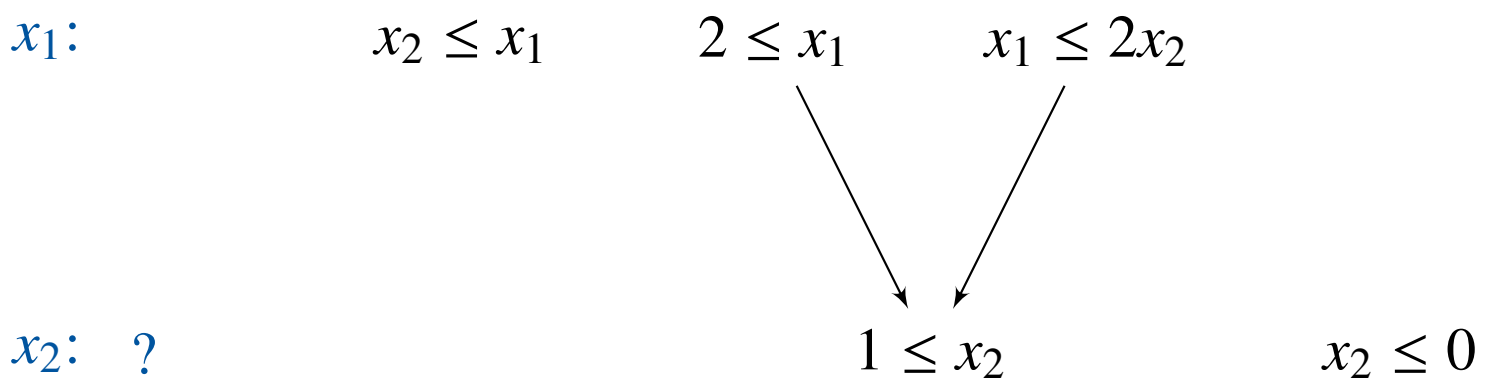

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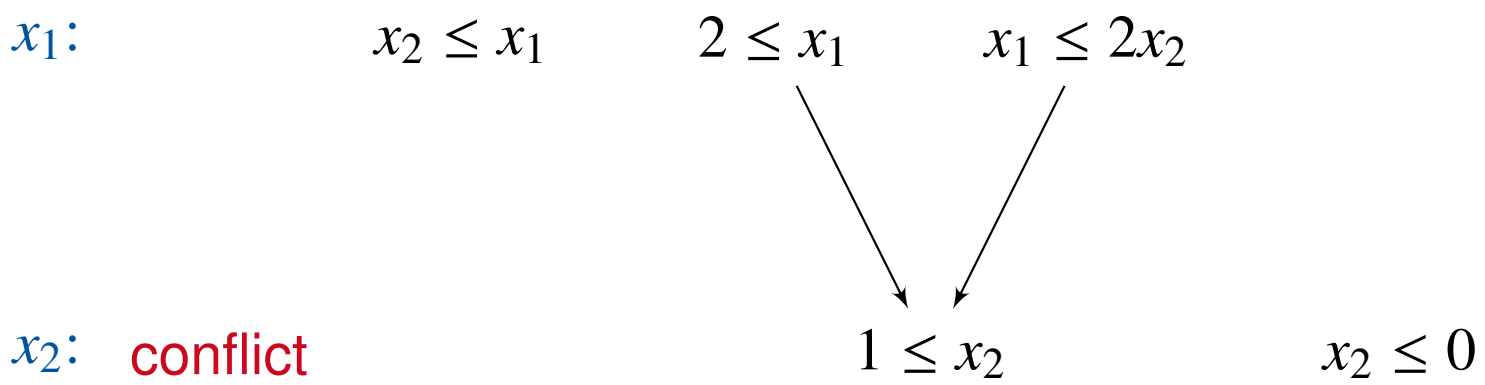
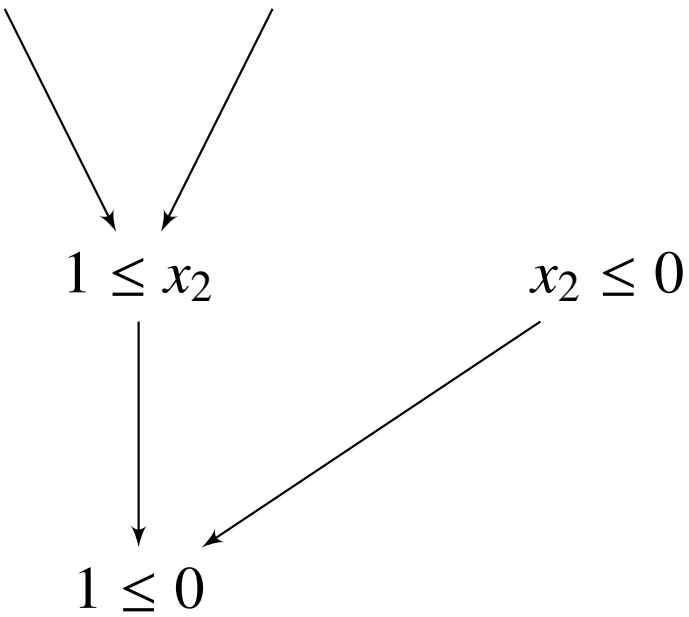


Exploration-guided Fourier-Motzkin

x_1: $x_2 \leq x_1$ $\quad 2 \leq x_1$ $\quad x_1 \leq 2x_2$

x_2: conflict



The key to decidability of NRA: Sign-invariant regions

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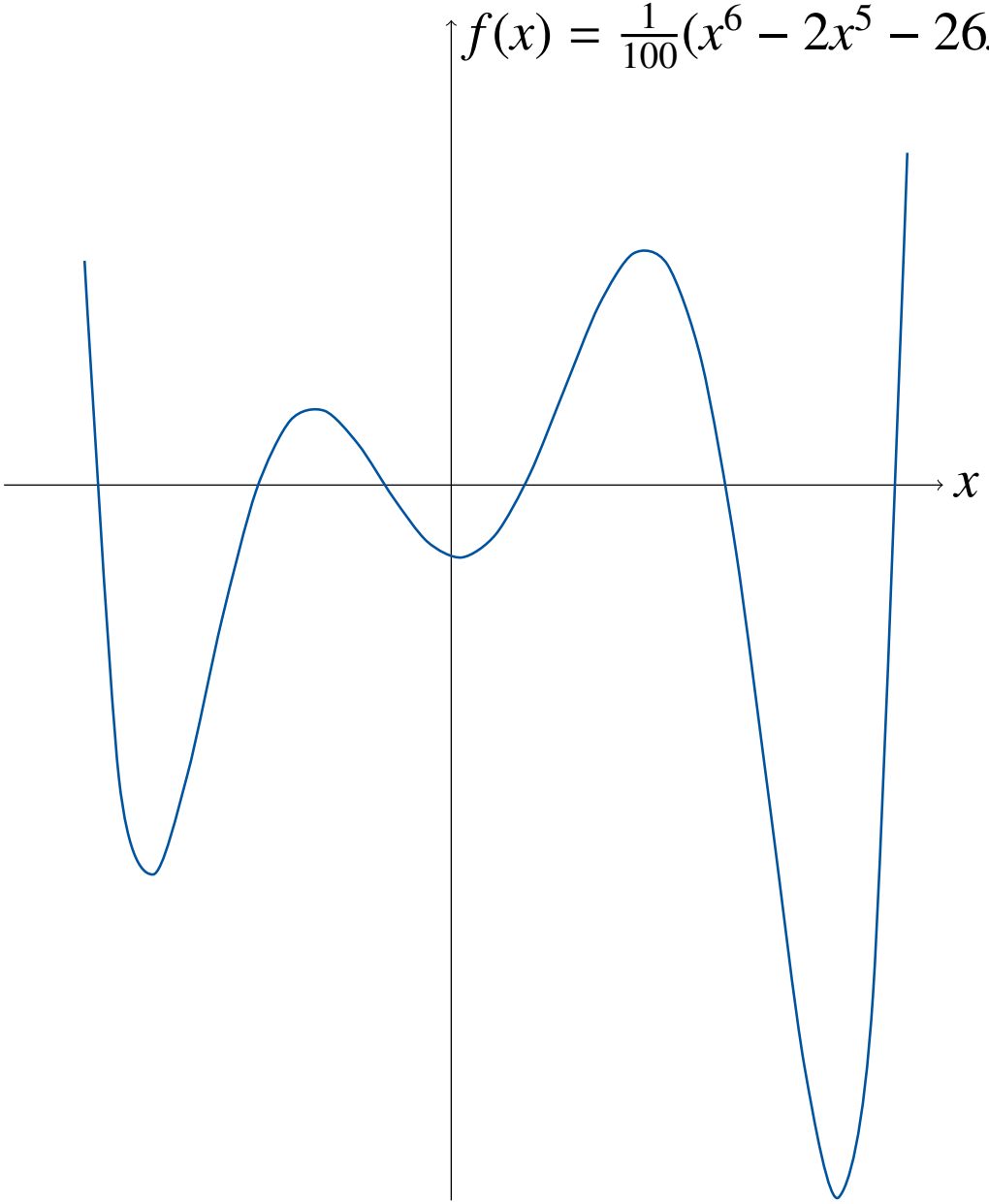
$$f(x) = \frac{1}{100}(x^6 - 2x^5 - 26x^4 + 28x^3 + 145x^2 - 26x - 80)$$



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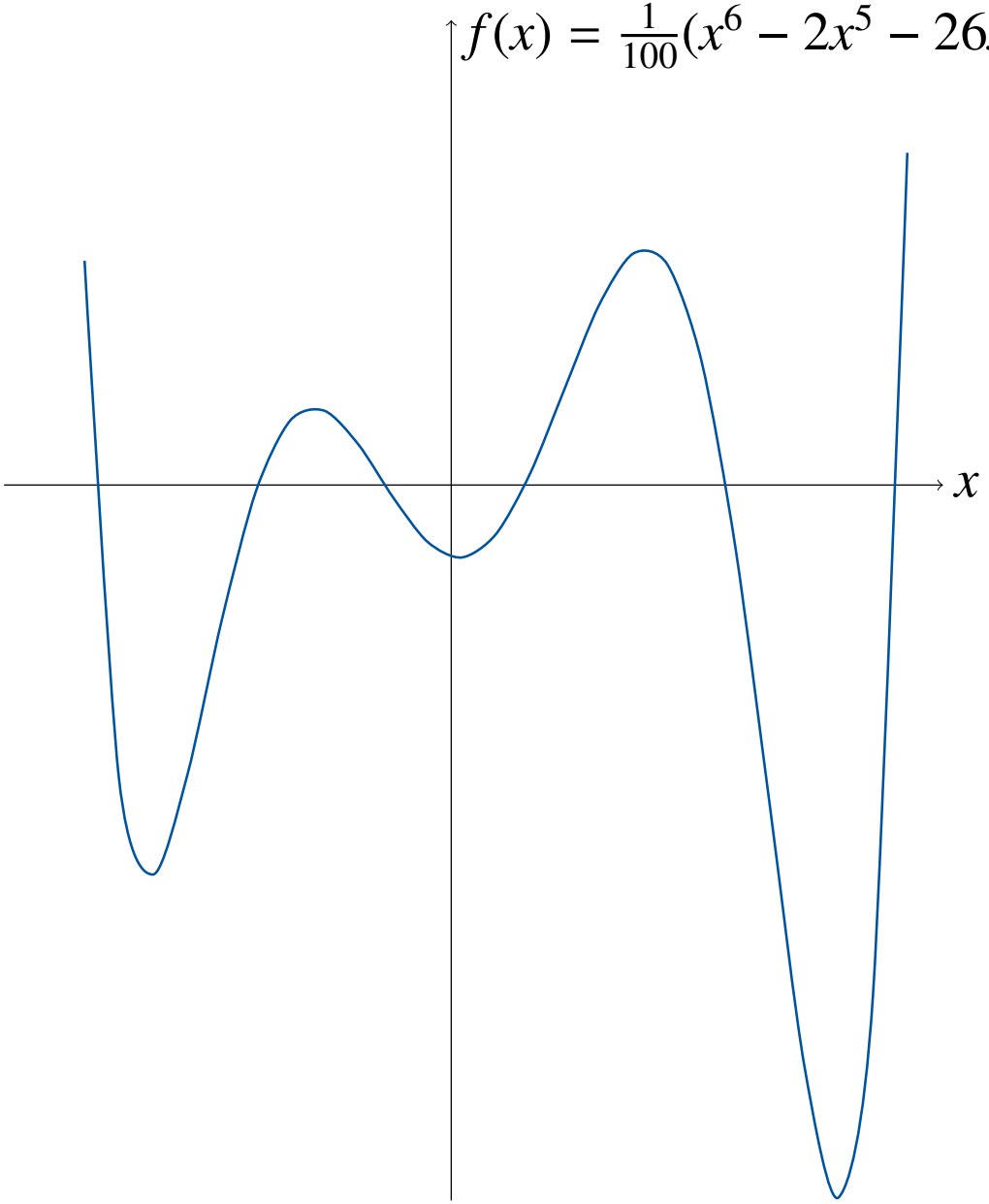

Is
$f(x) < 0$
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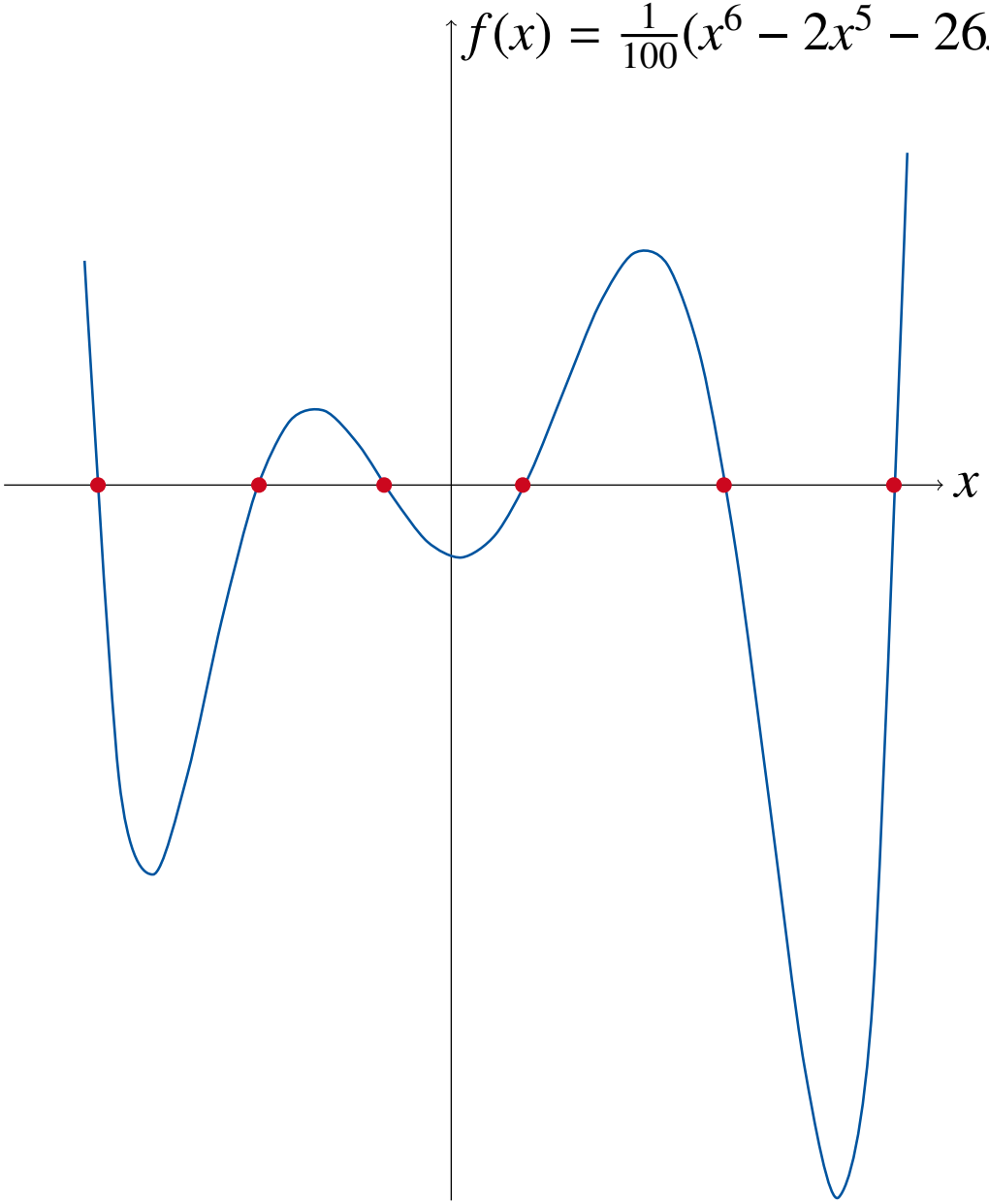

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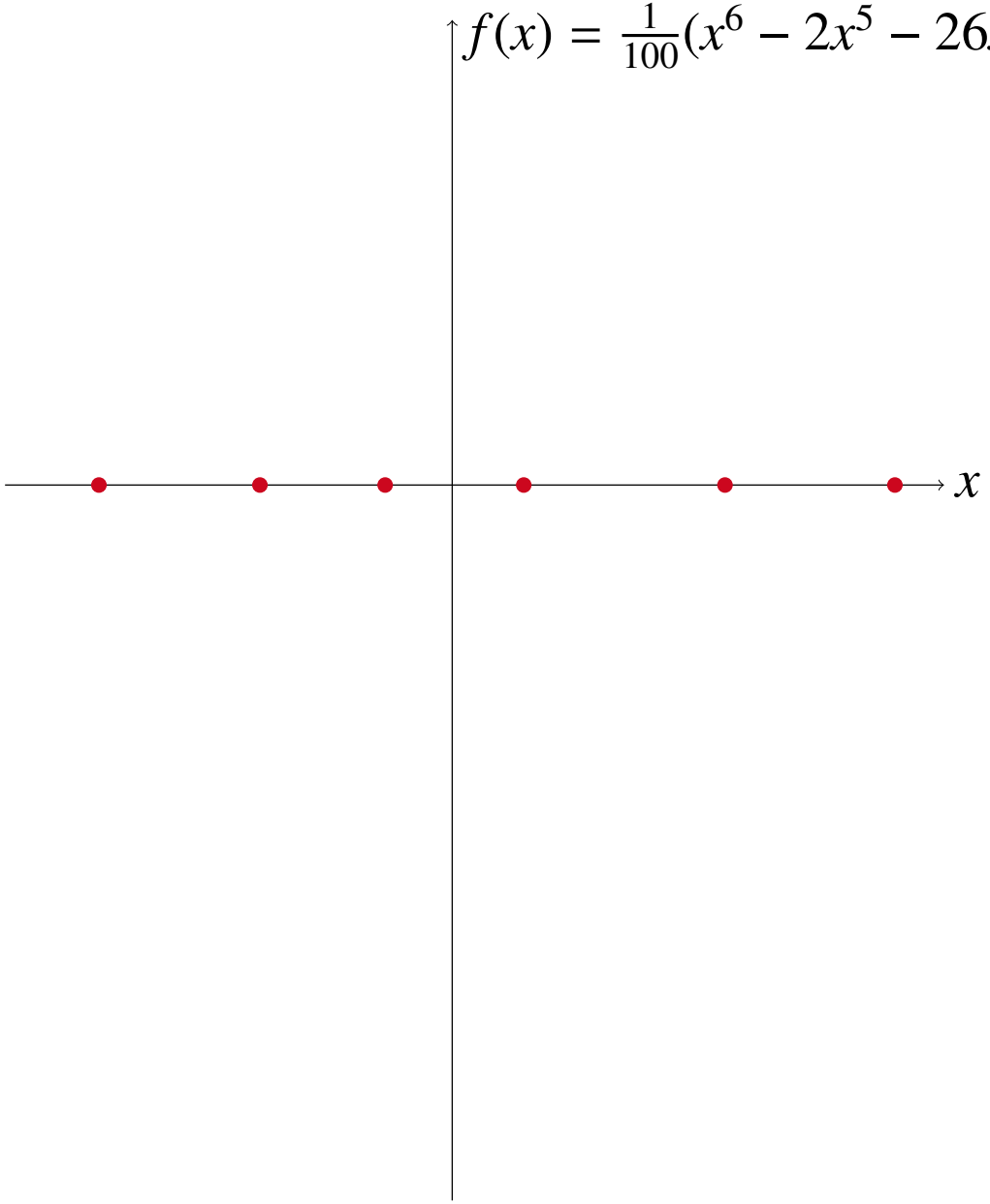

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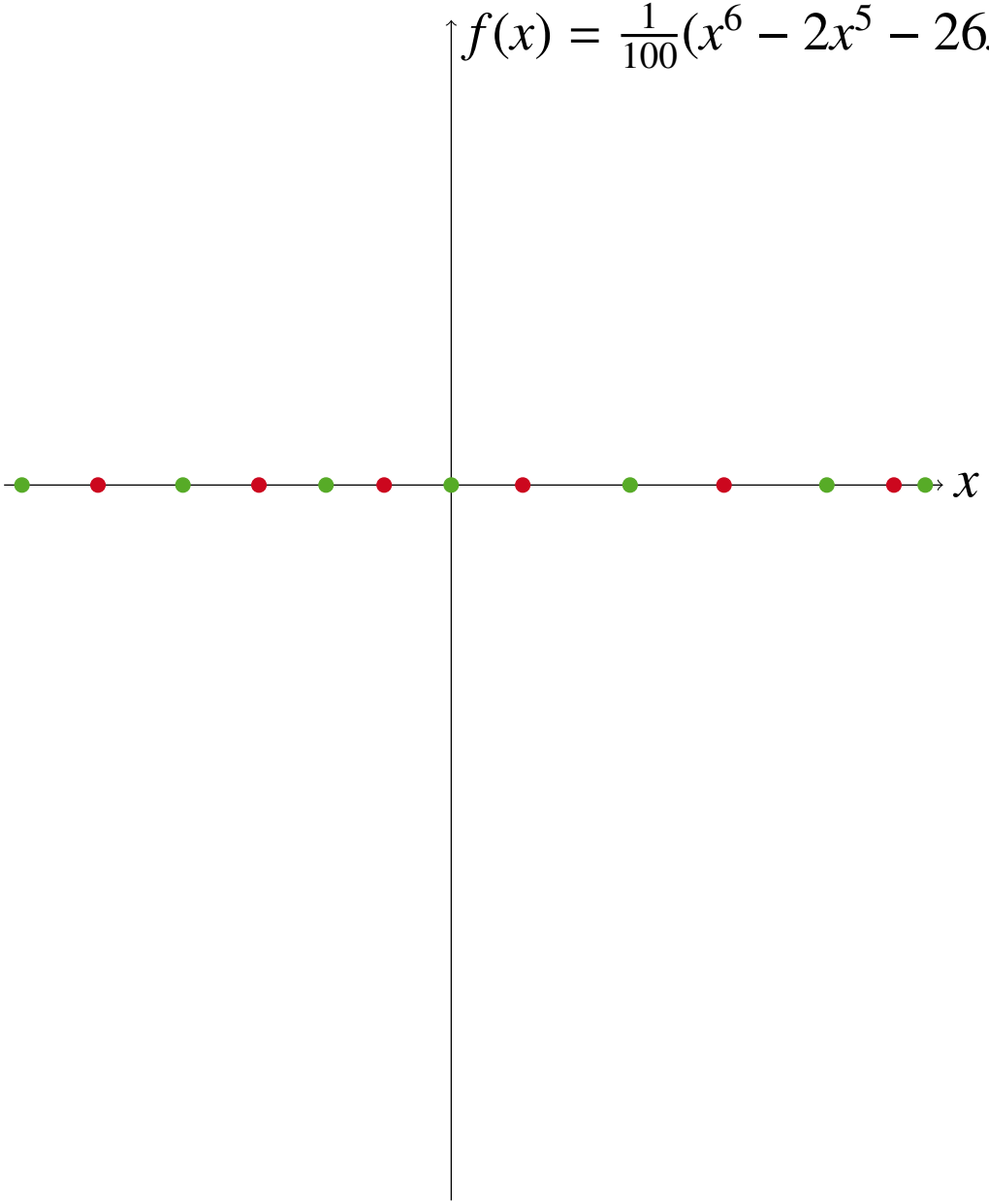

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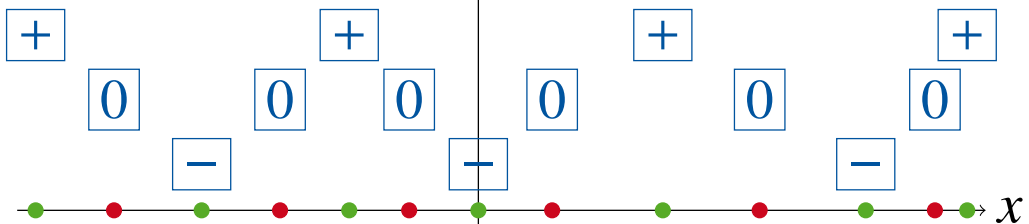

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Cylindrical algebraic decomposition example

$$(x-2)^2 + (y-2)^2 - 1 < 0 \wedge x - y = 0$$

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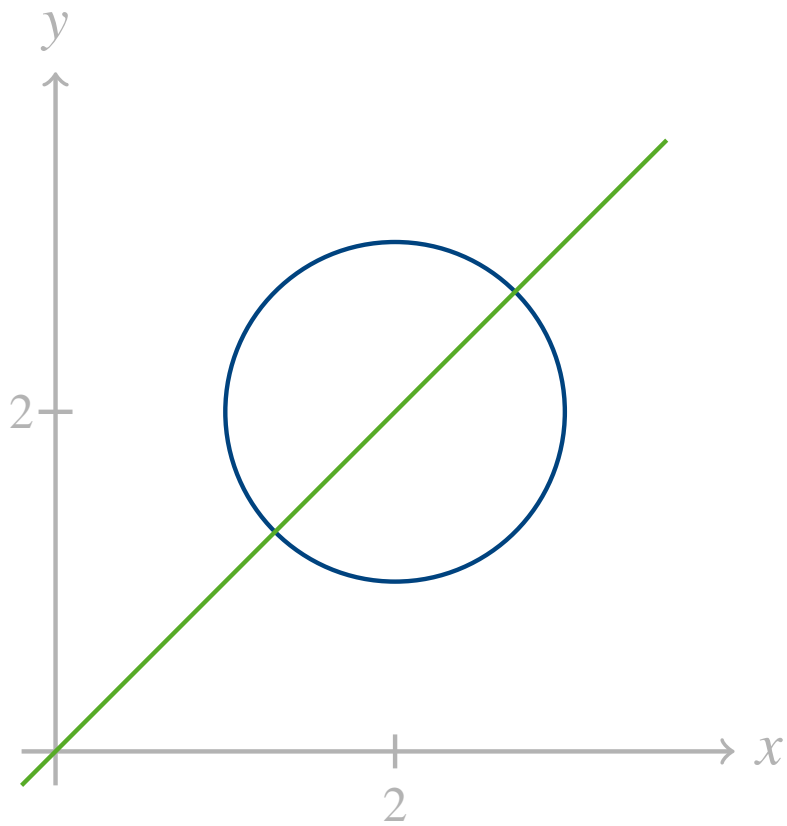

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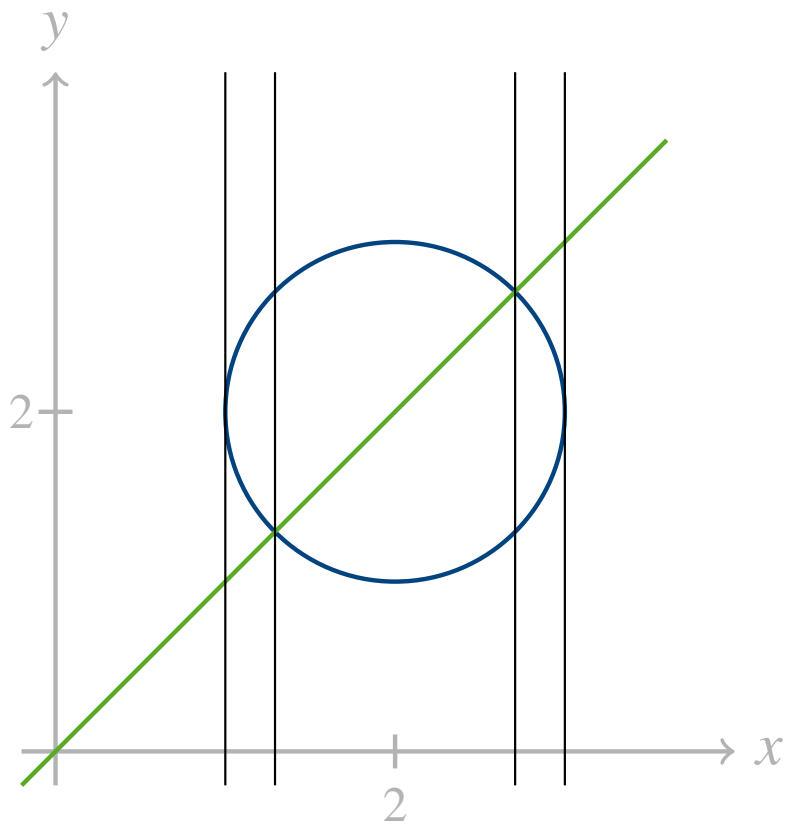

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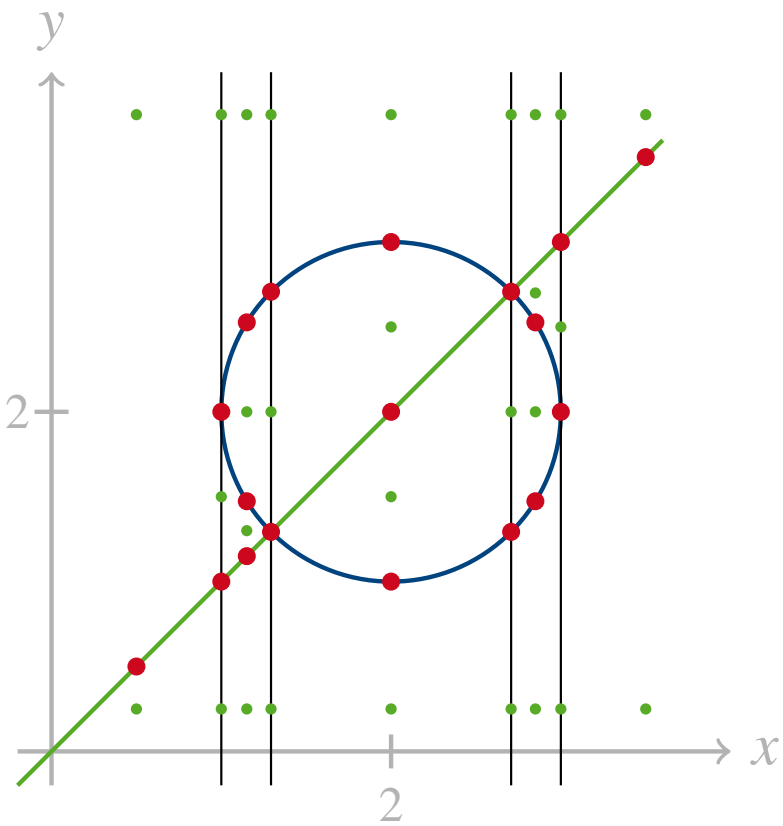
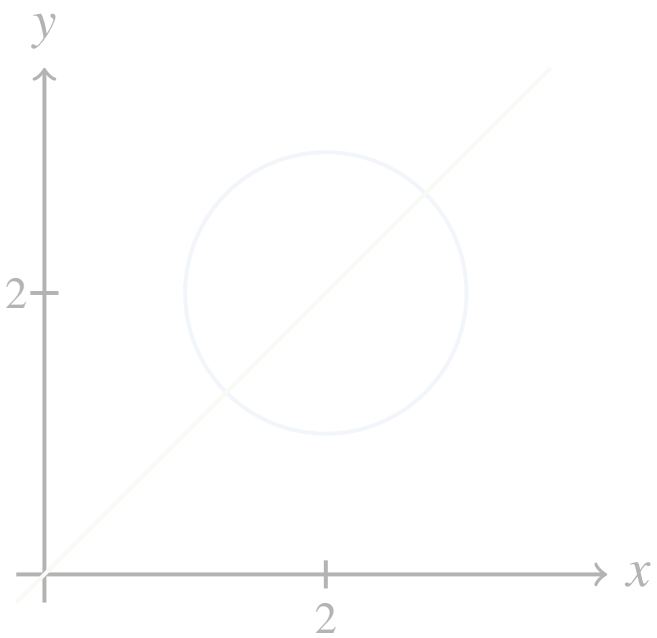


CAD in MCSAT [Jovanović, de Moura, IJCAR'12]

$$\varphi = \underbrace{(x-2)^2 + (y-2)^2 - 1 < 0}_{c_1} \wedge \underbrace{x - y = 0}_{c_2}$$



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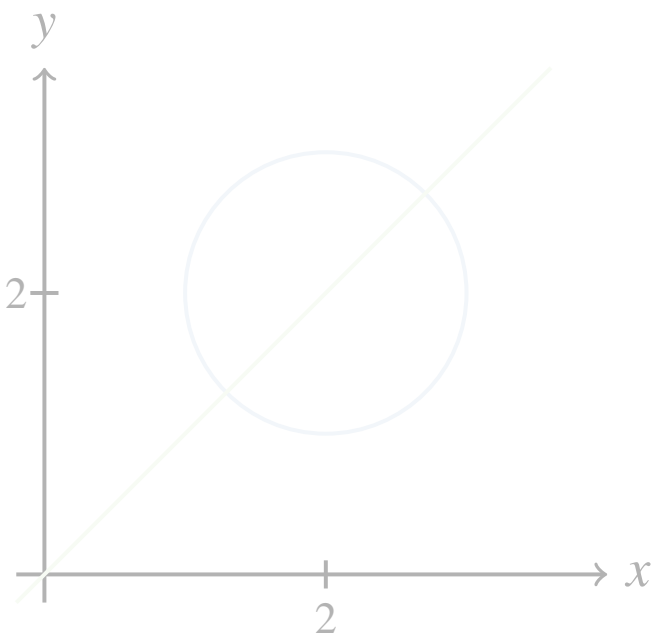
$x := 0.5$



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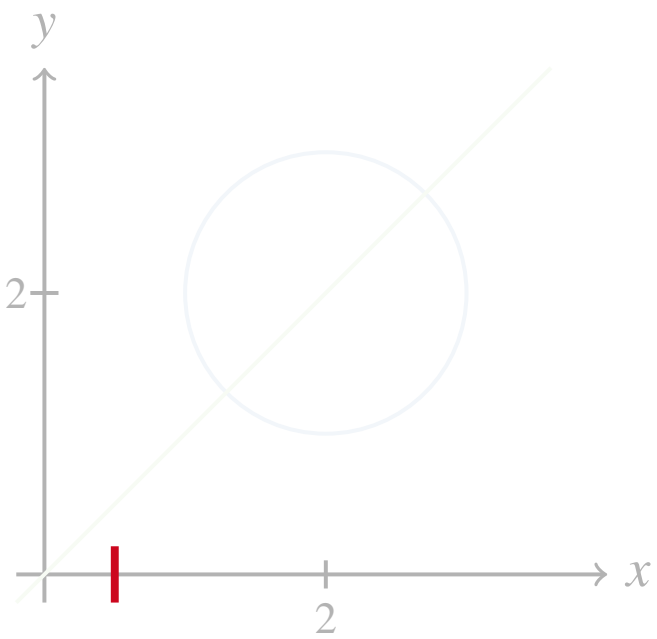

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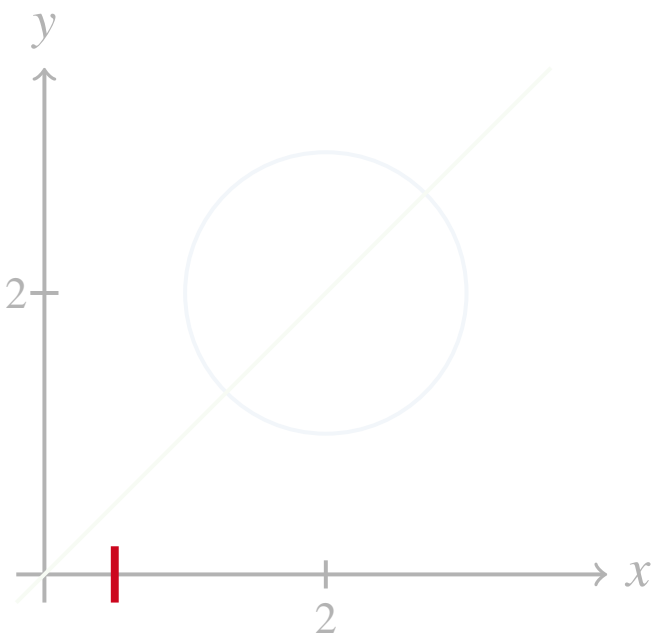

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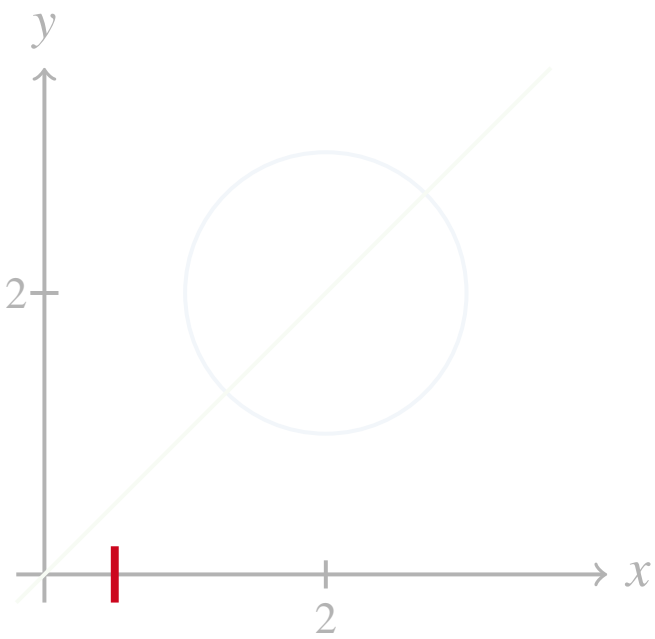

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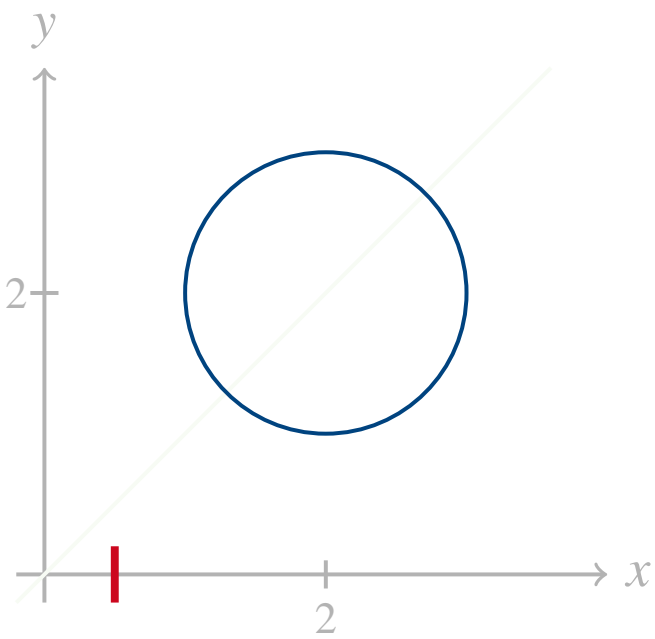

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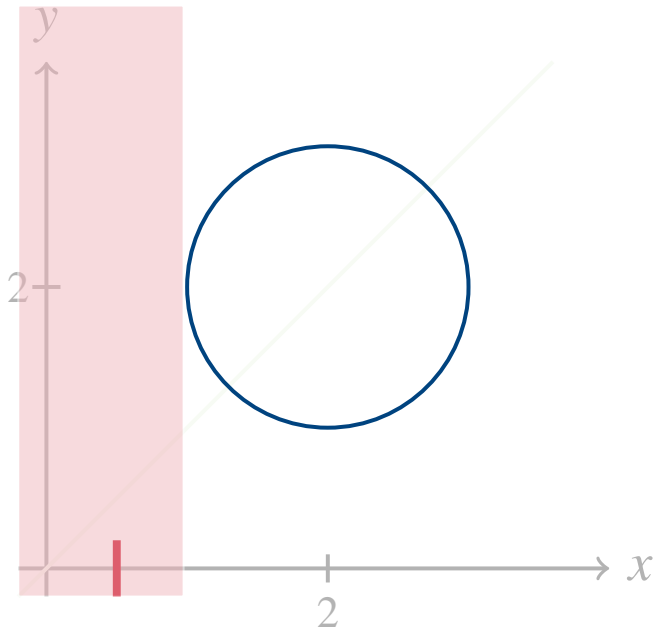
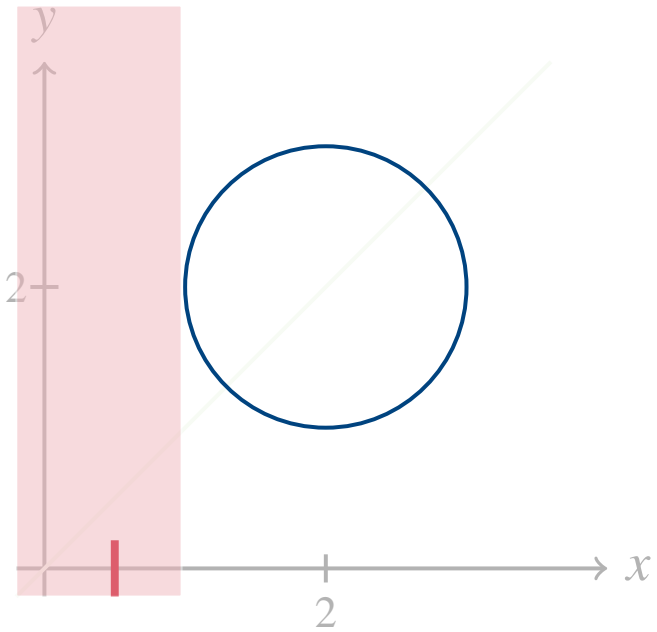


CAD in MCSAT [Jovanović, de Moura, IJCAR'12]

$$\varphi = (x-2)^2 + (y-2)^2 - 1 < 0 \wedge x - y = 0 \wedge$$
$$(c_1 \rightarrow \neg(x < \mathit{zero}(1, x^2 - 4x + 3)))$$



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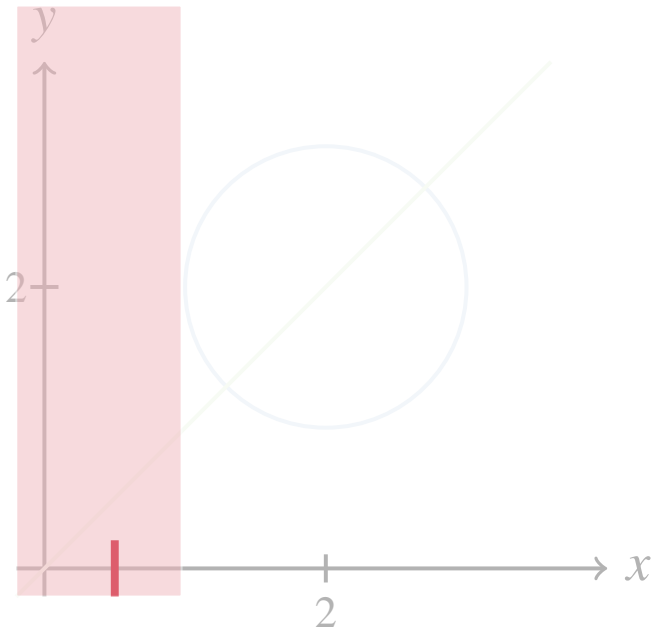

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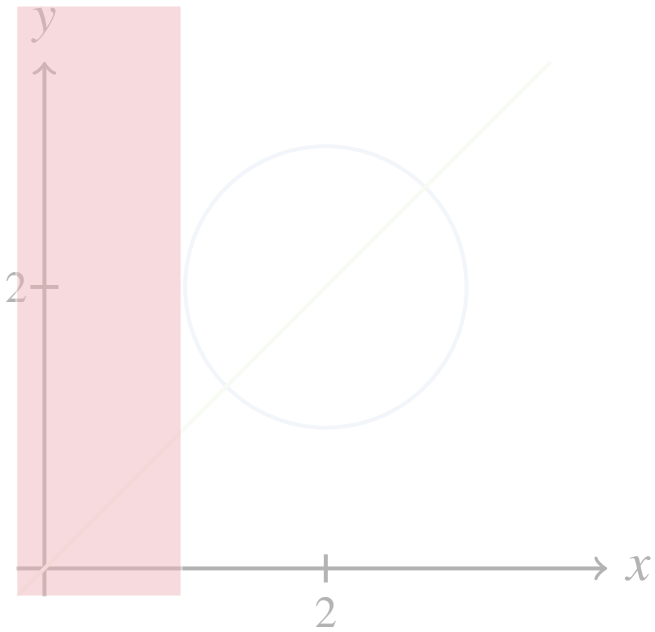

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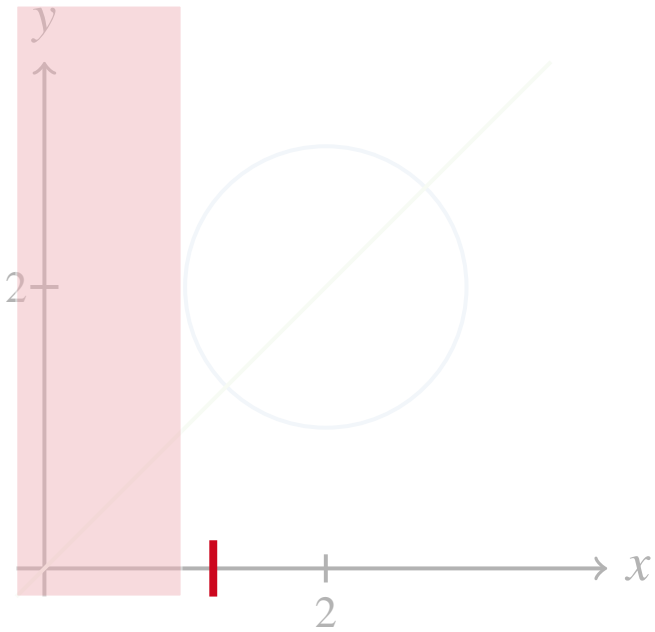

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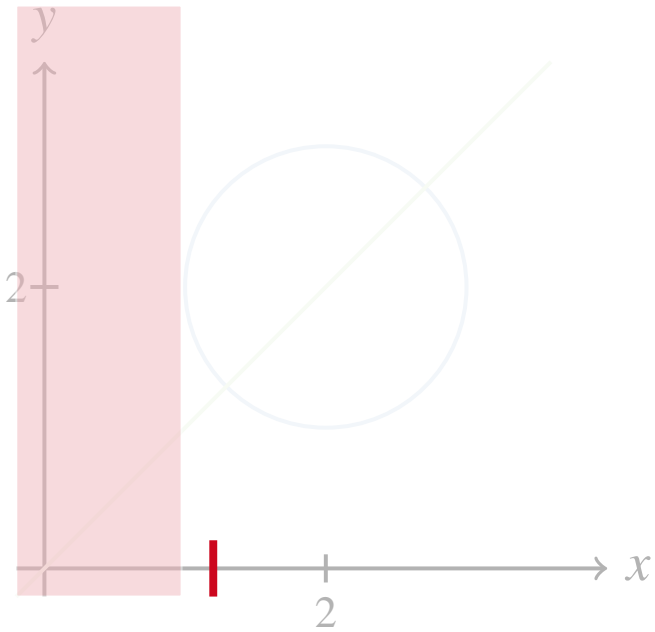

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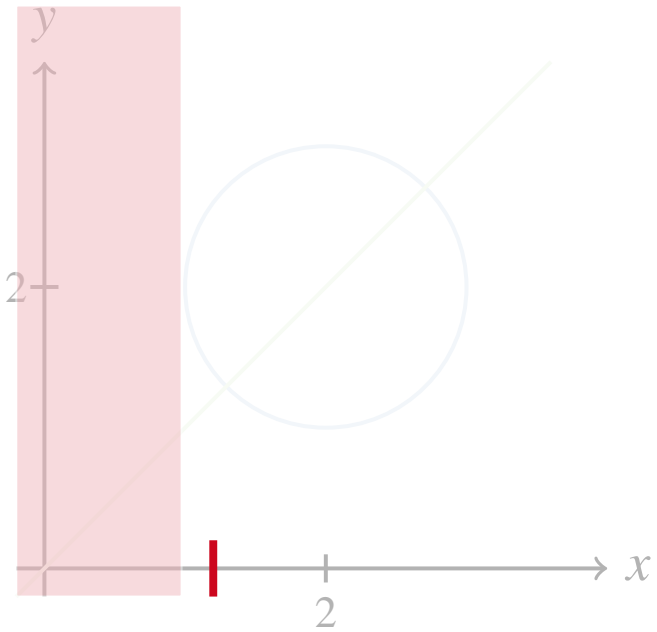

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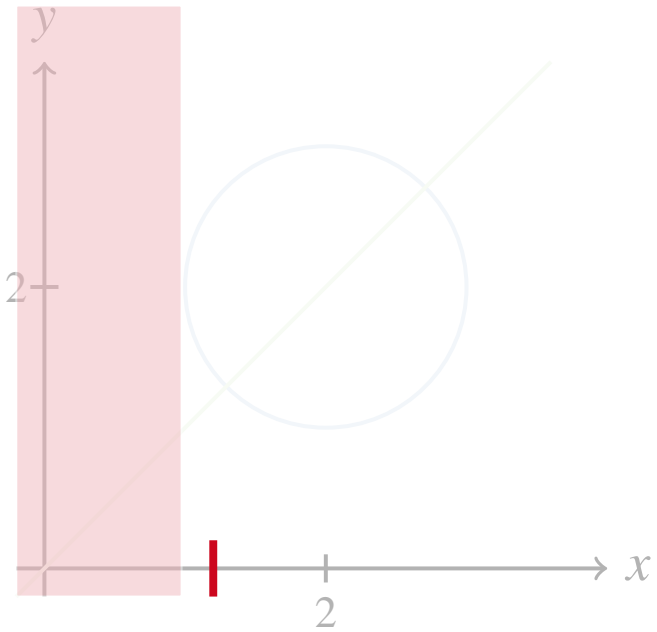

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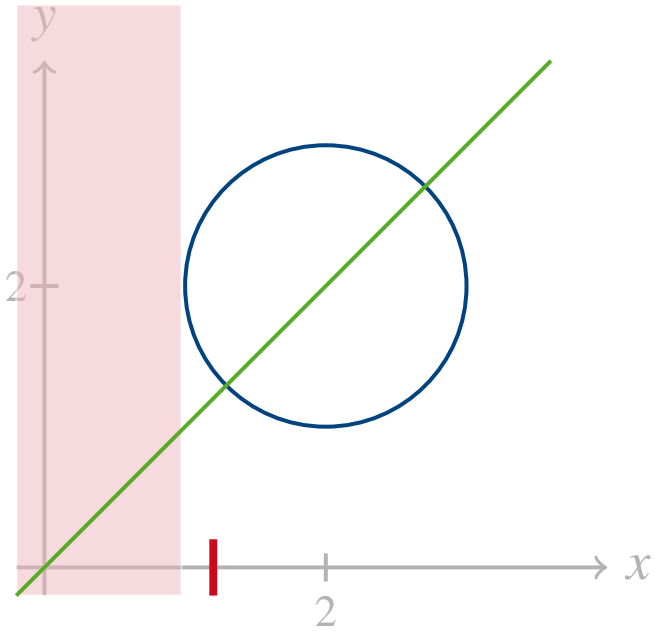

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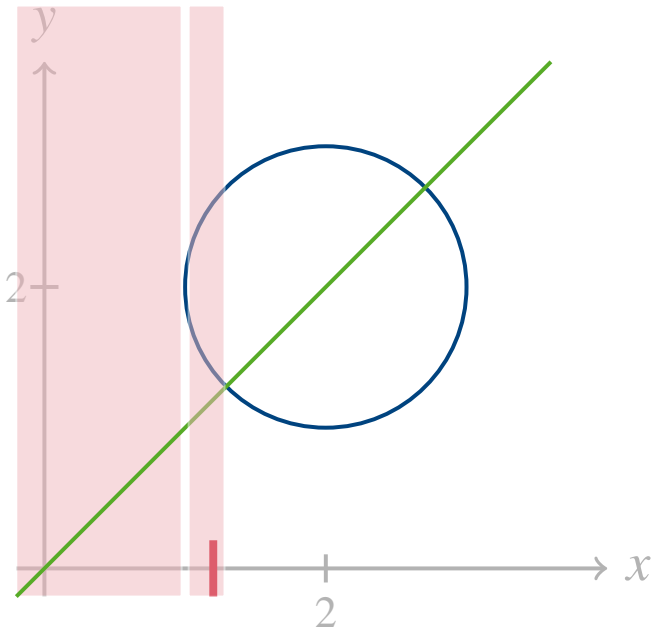

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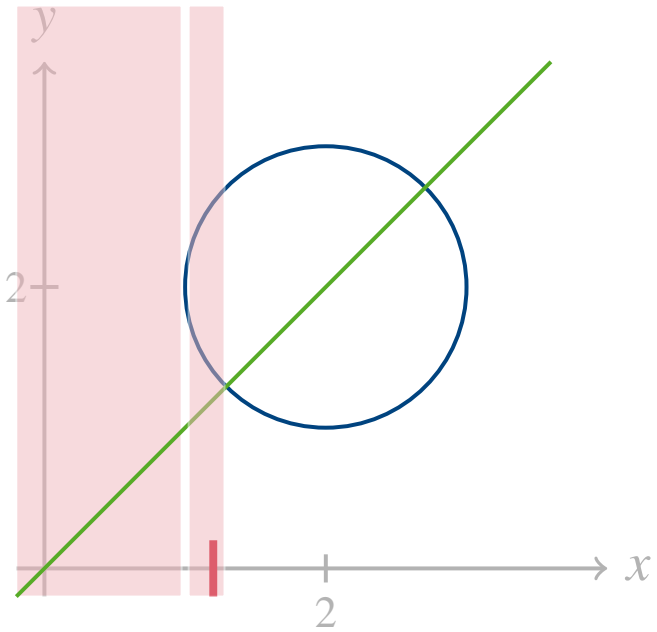
$$((c_1 \wedge c_2) \rightarrow \neg(\mathit{zero}(1, x^2 - 4x + 3) < x < \mathit{zero}(1, 2x^2 - 8x + 7)))$$



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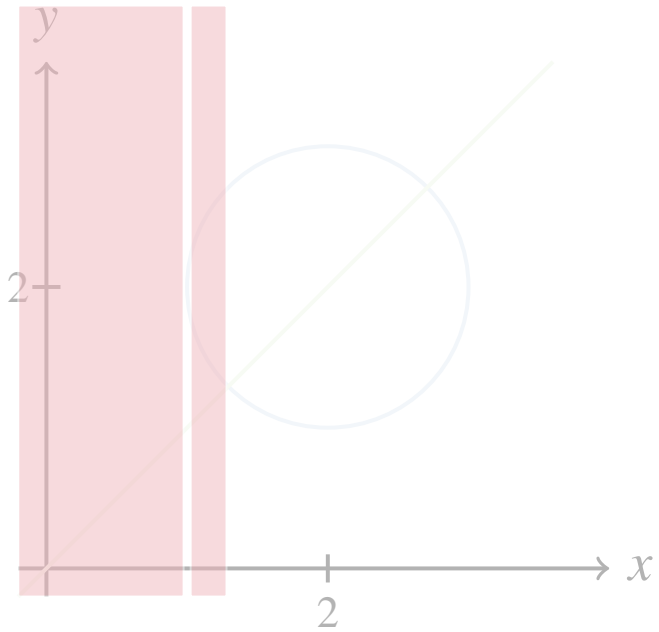

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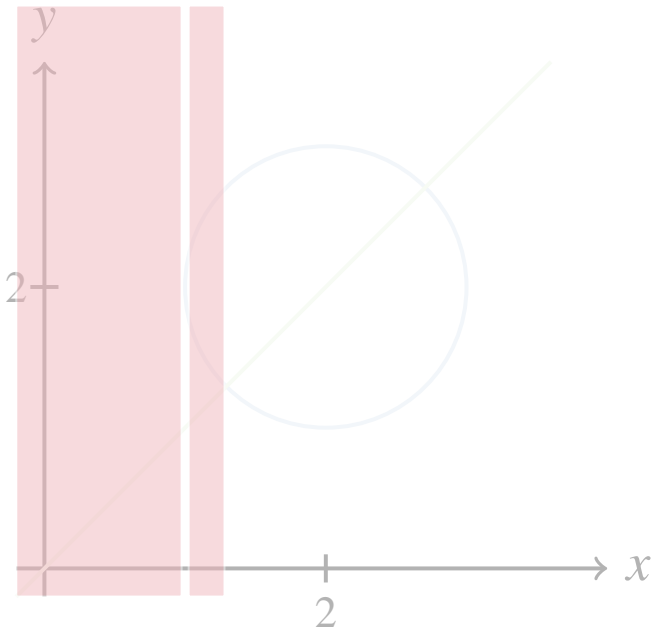

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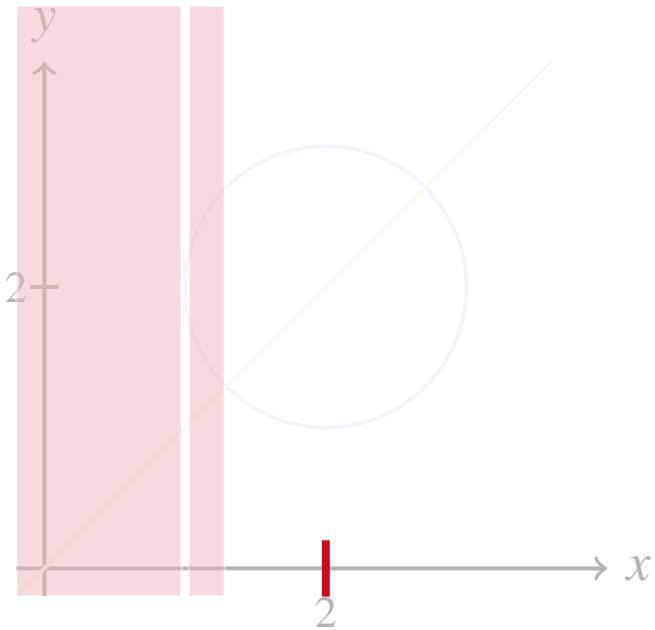

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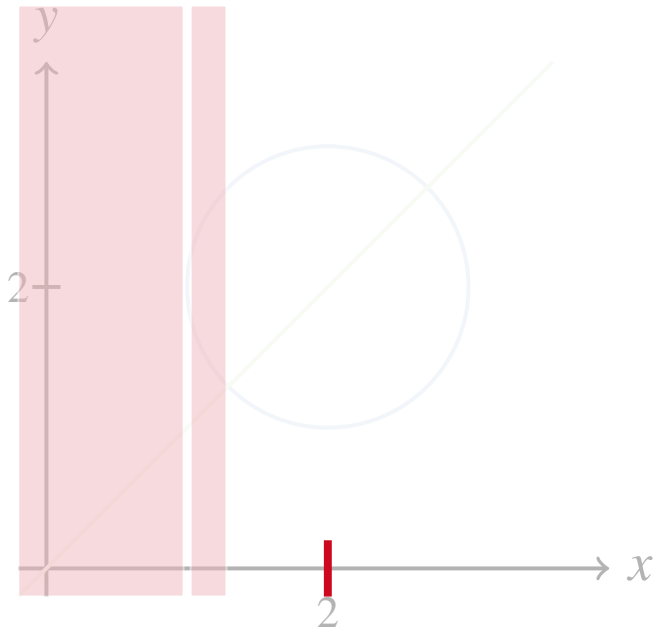

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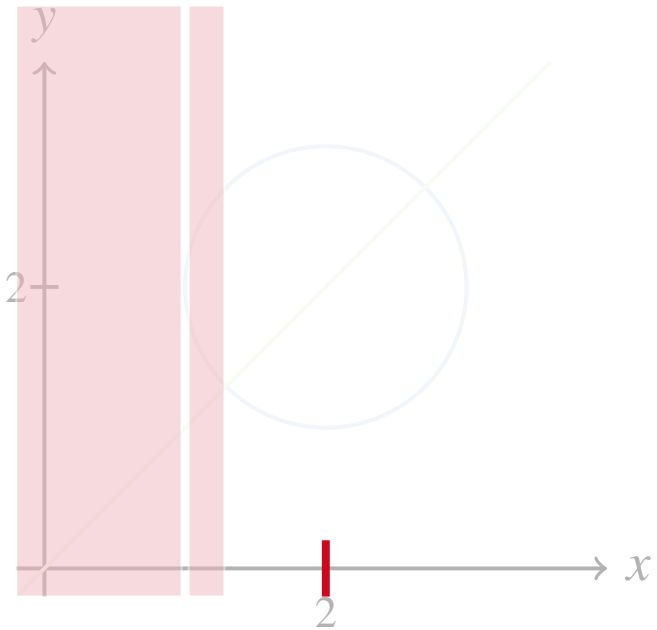

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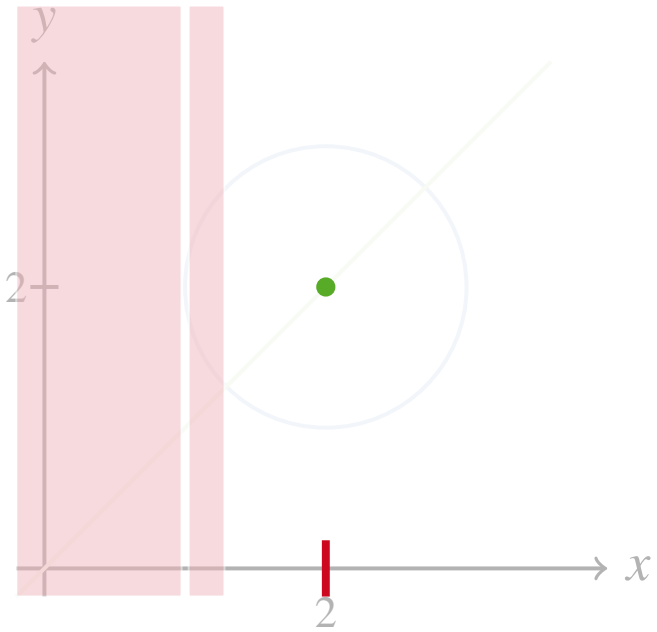

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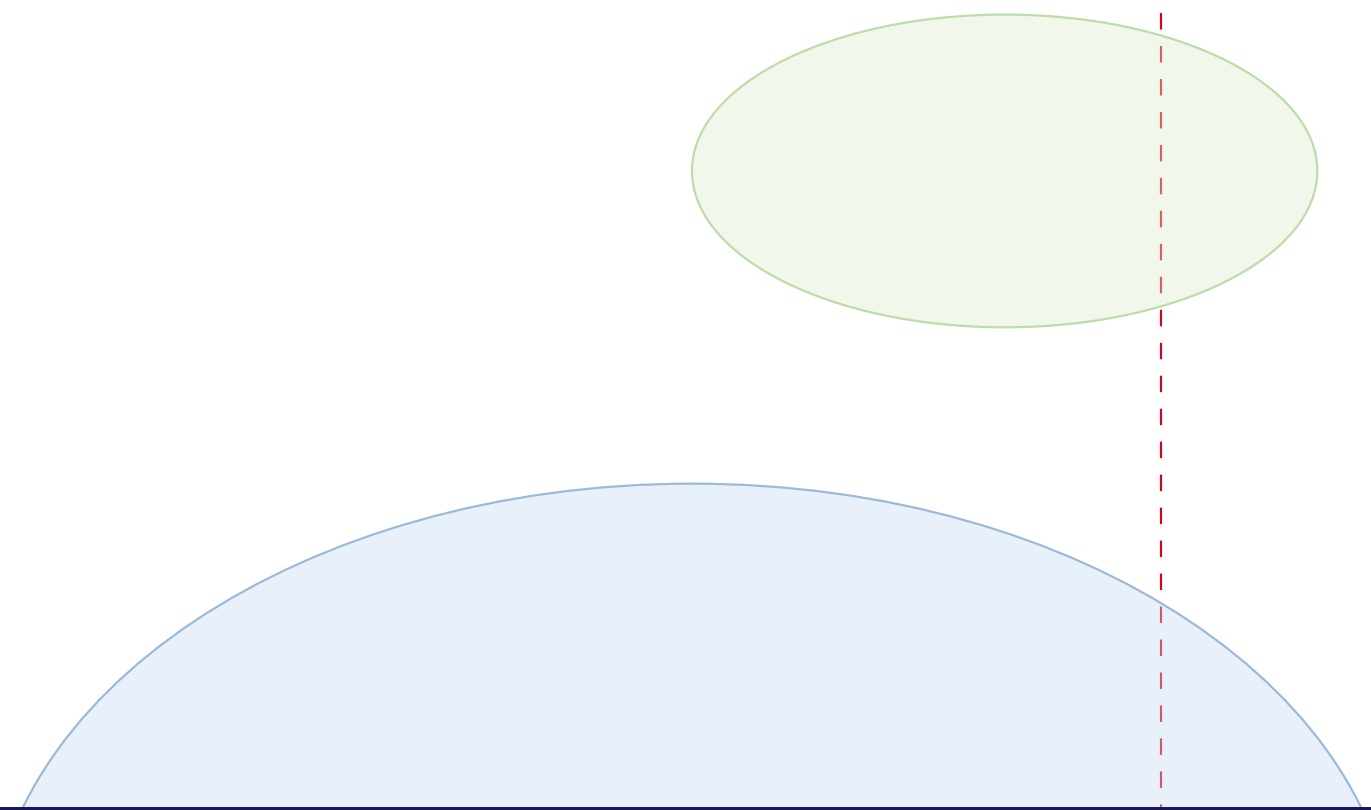

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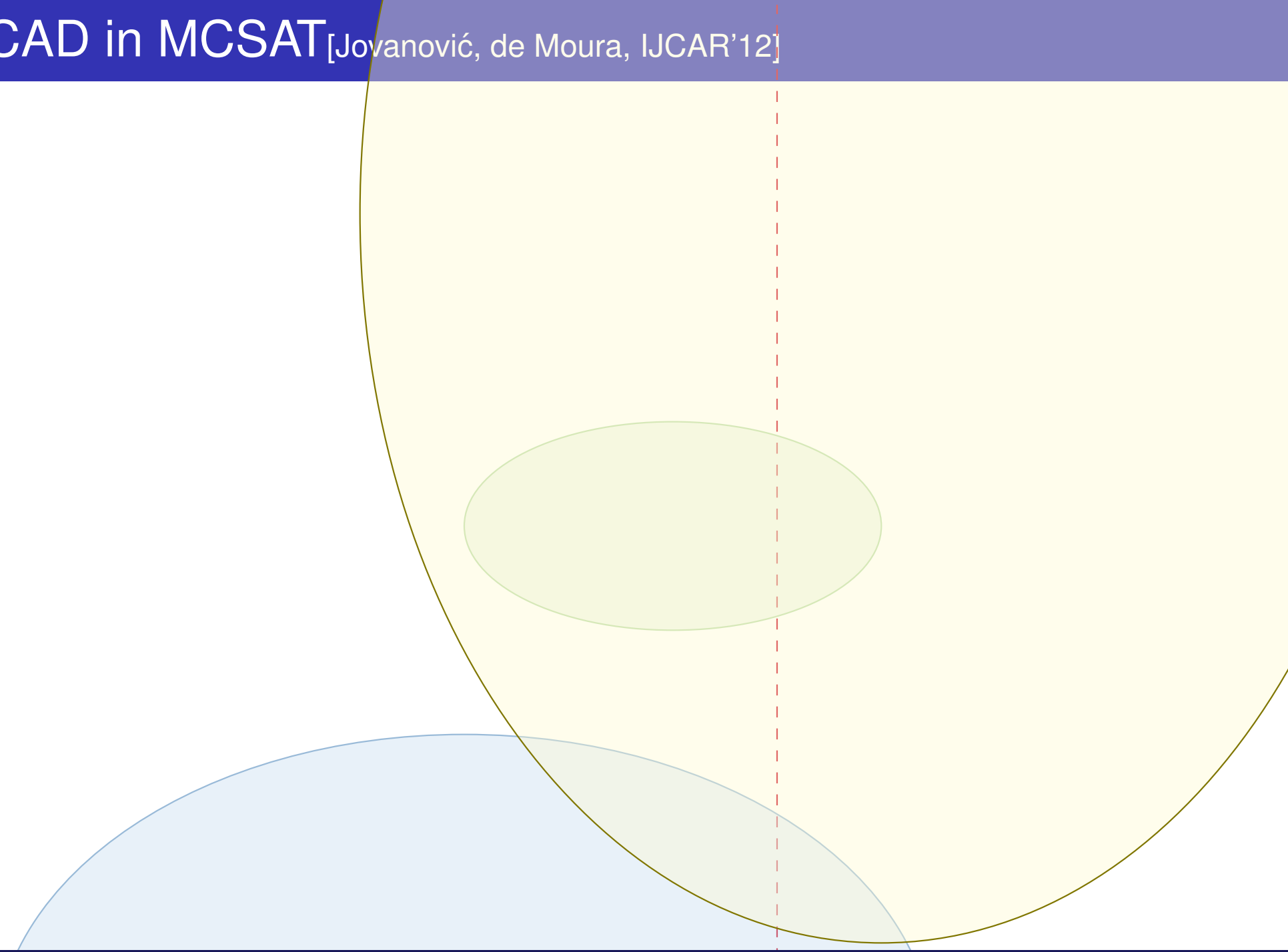

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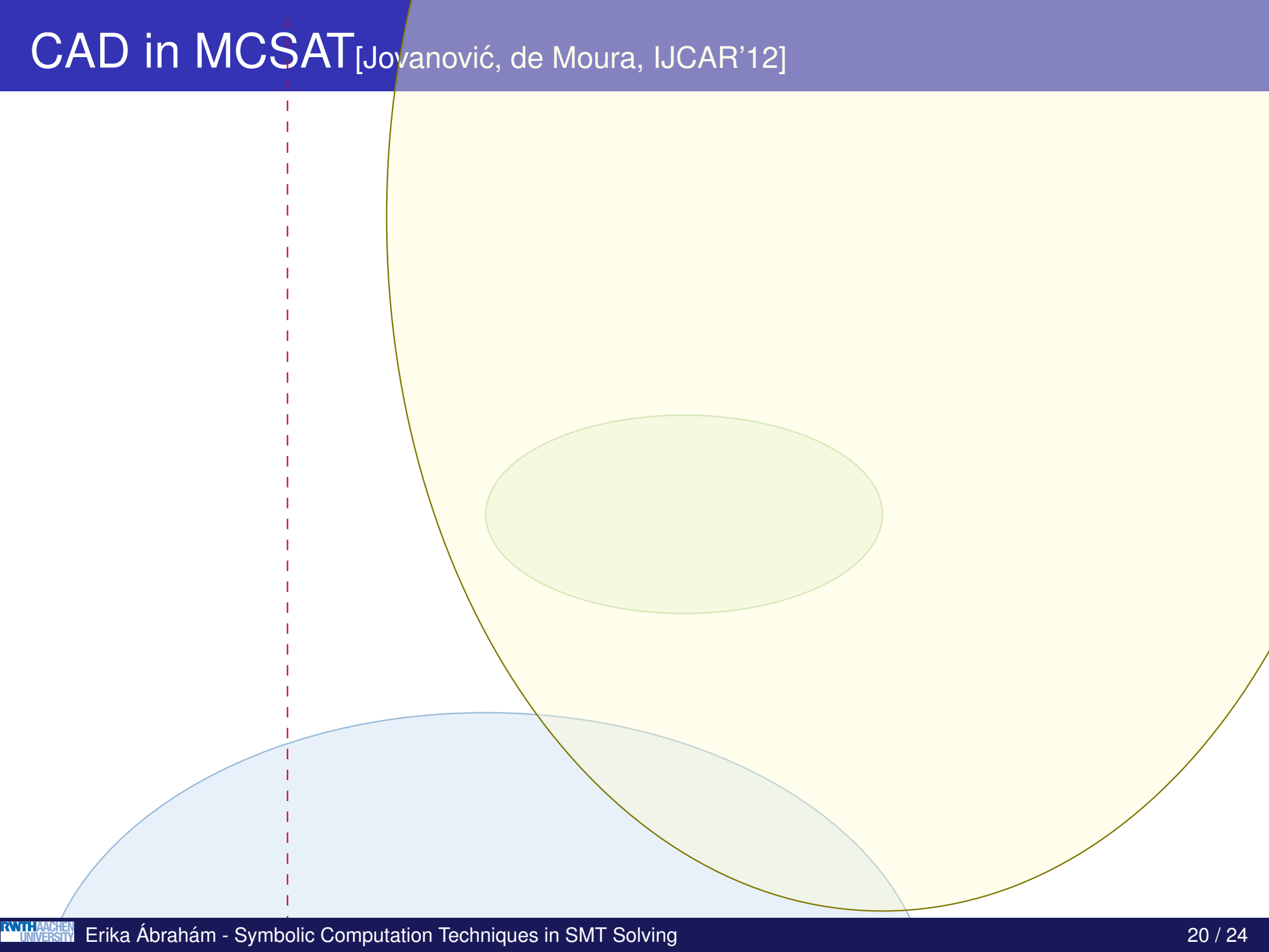

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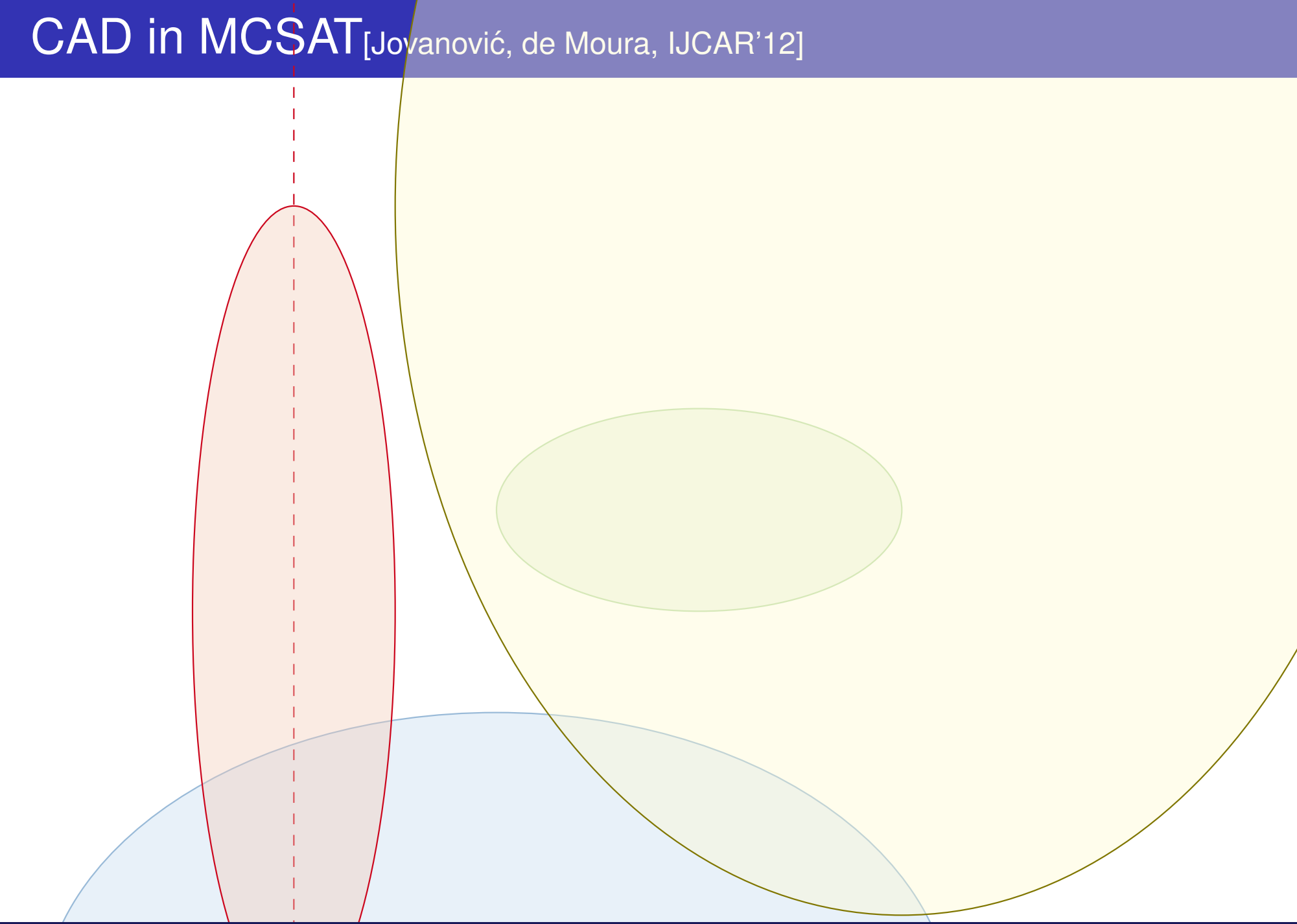

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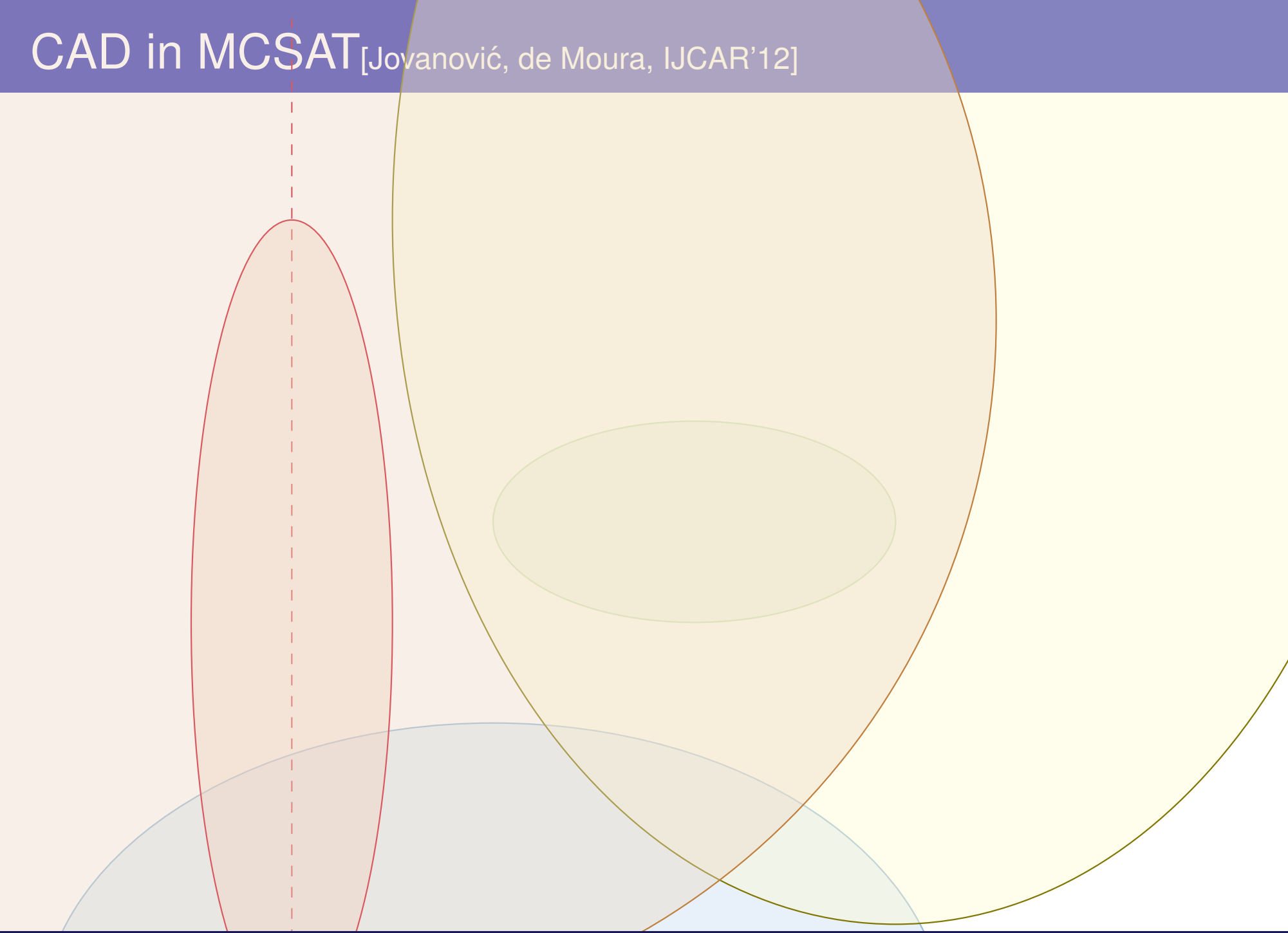

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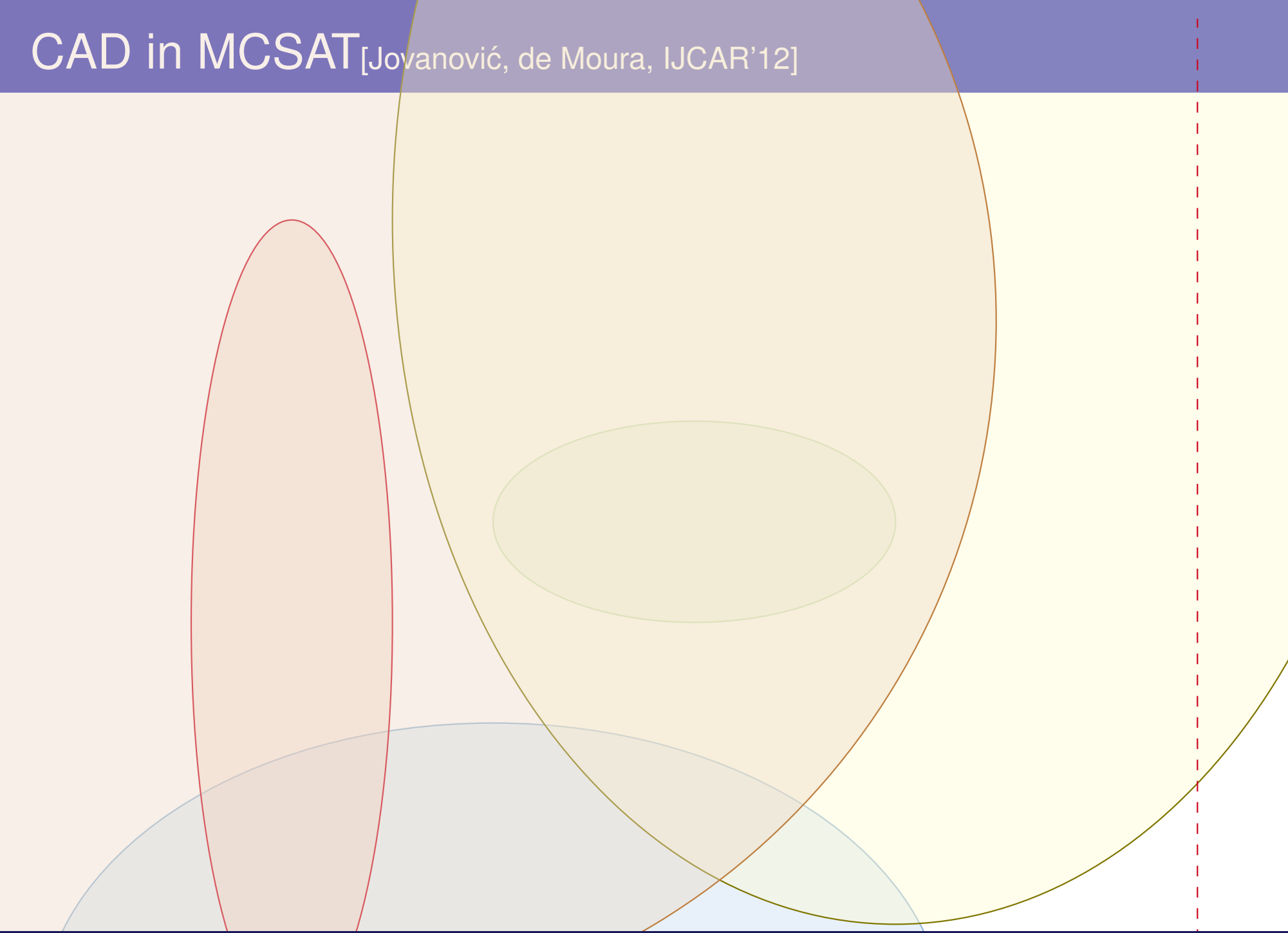

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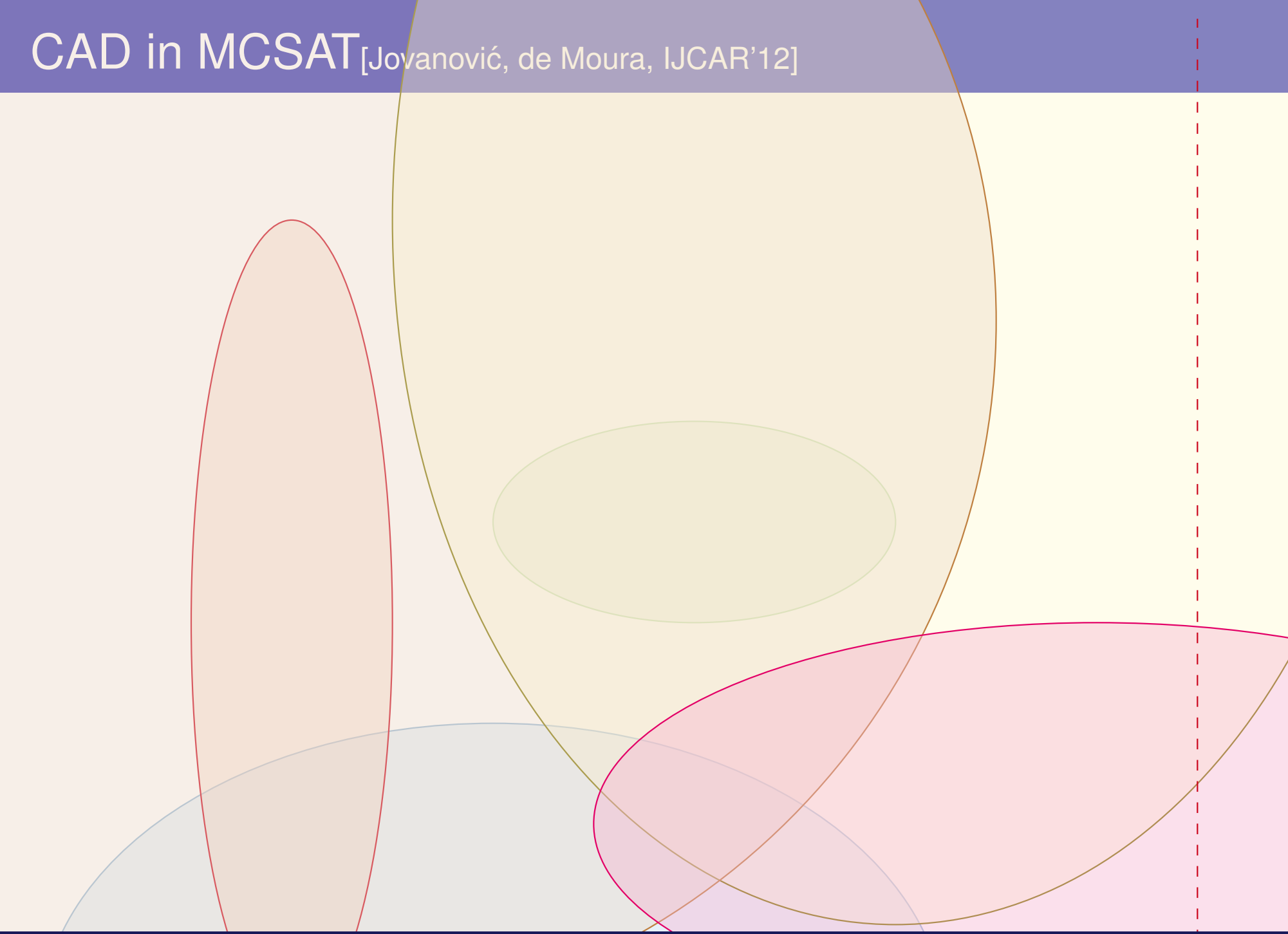

Cylindrical algebraic covering [Ábrahám, Davenport, England, Kremer JLAMP'21]

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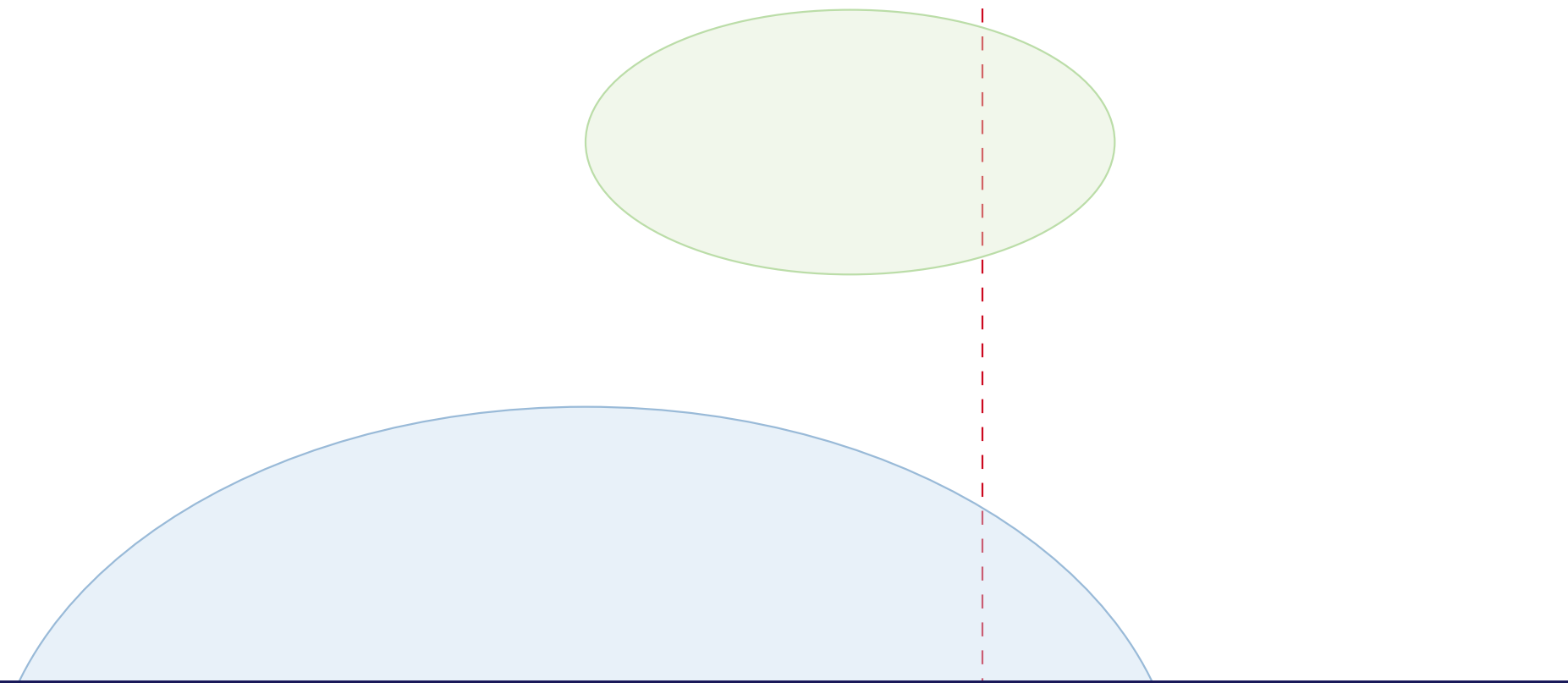

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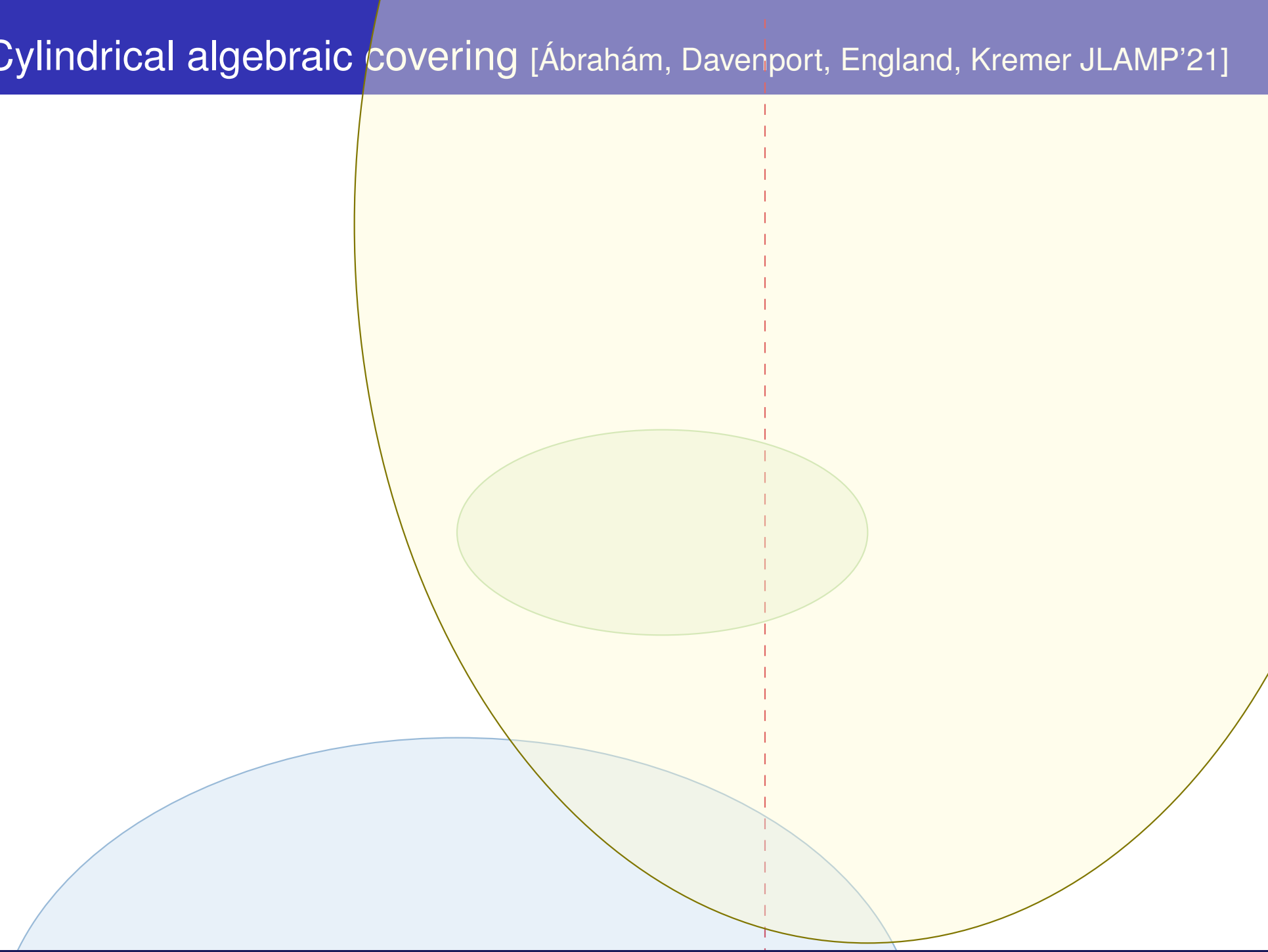

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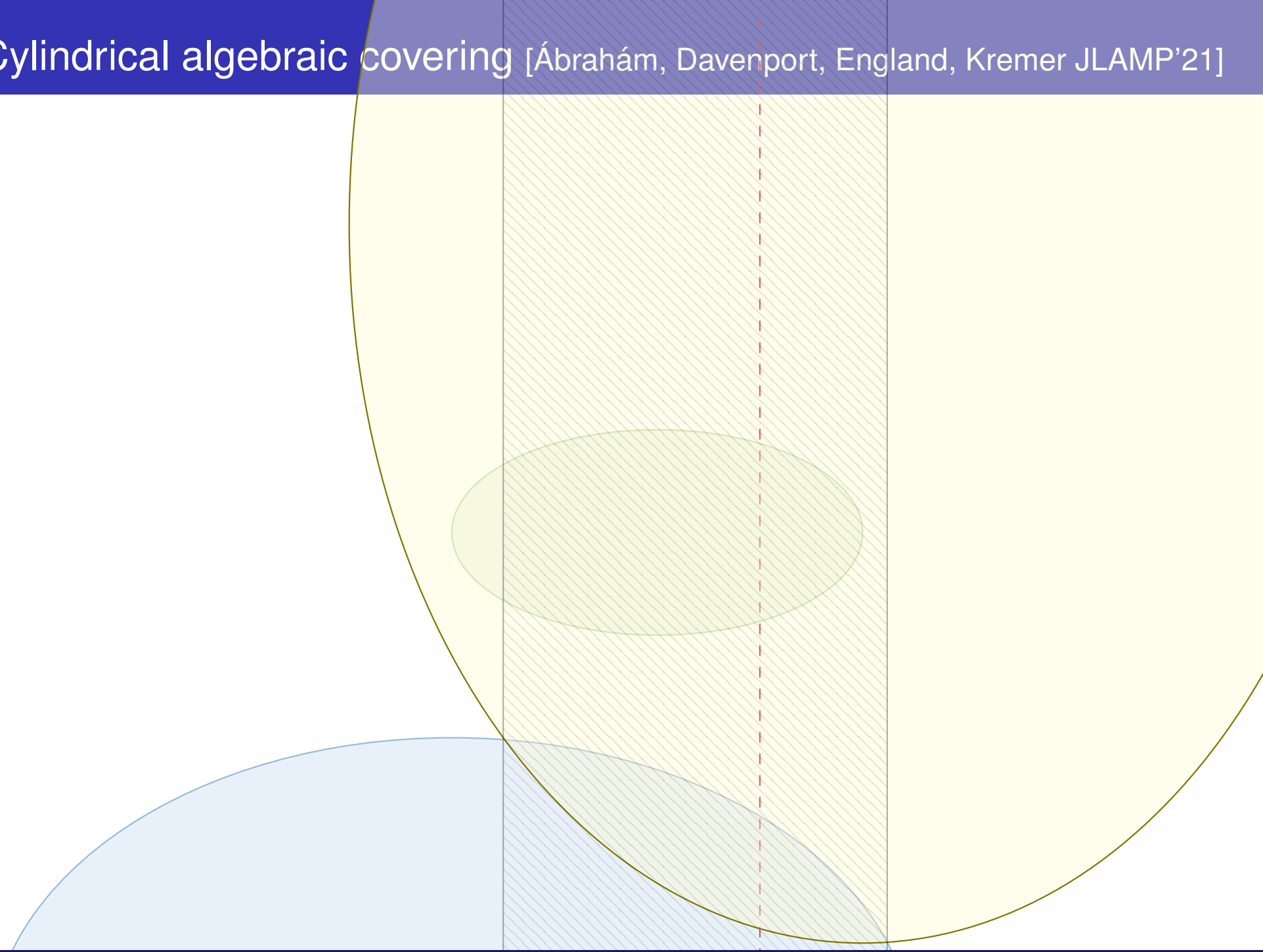

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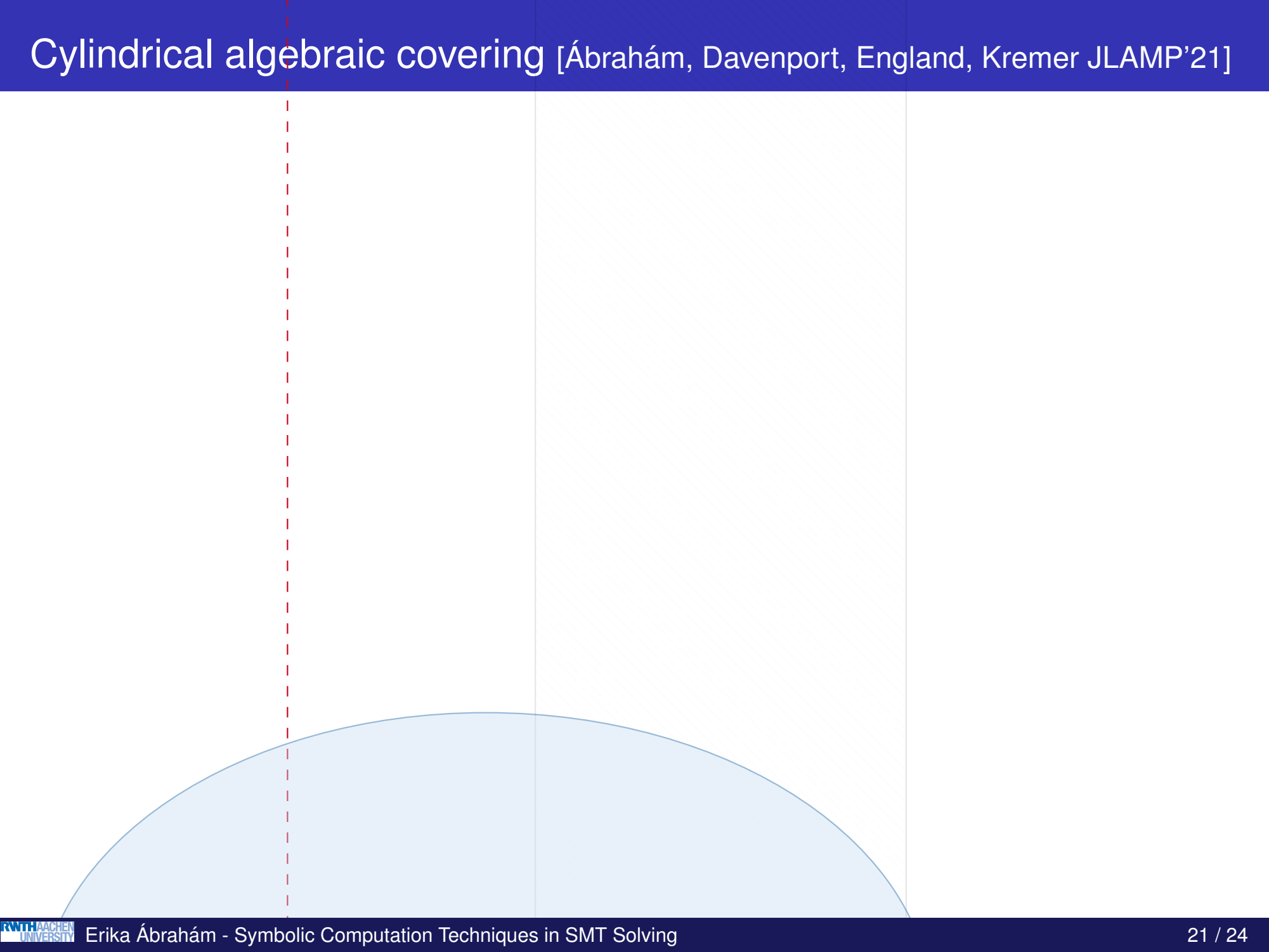

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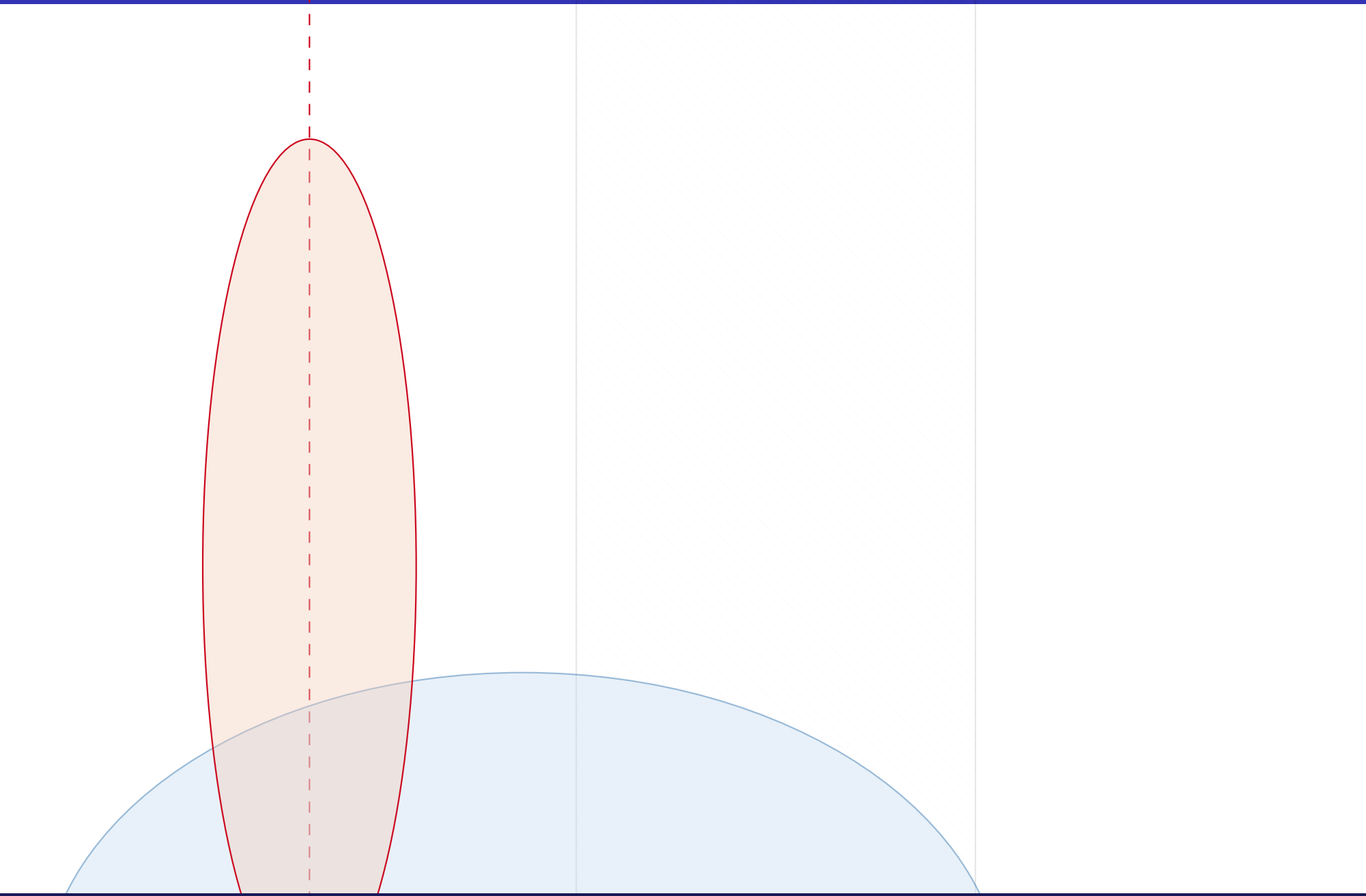

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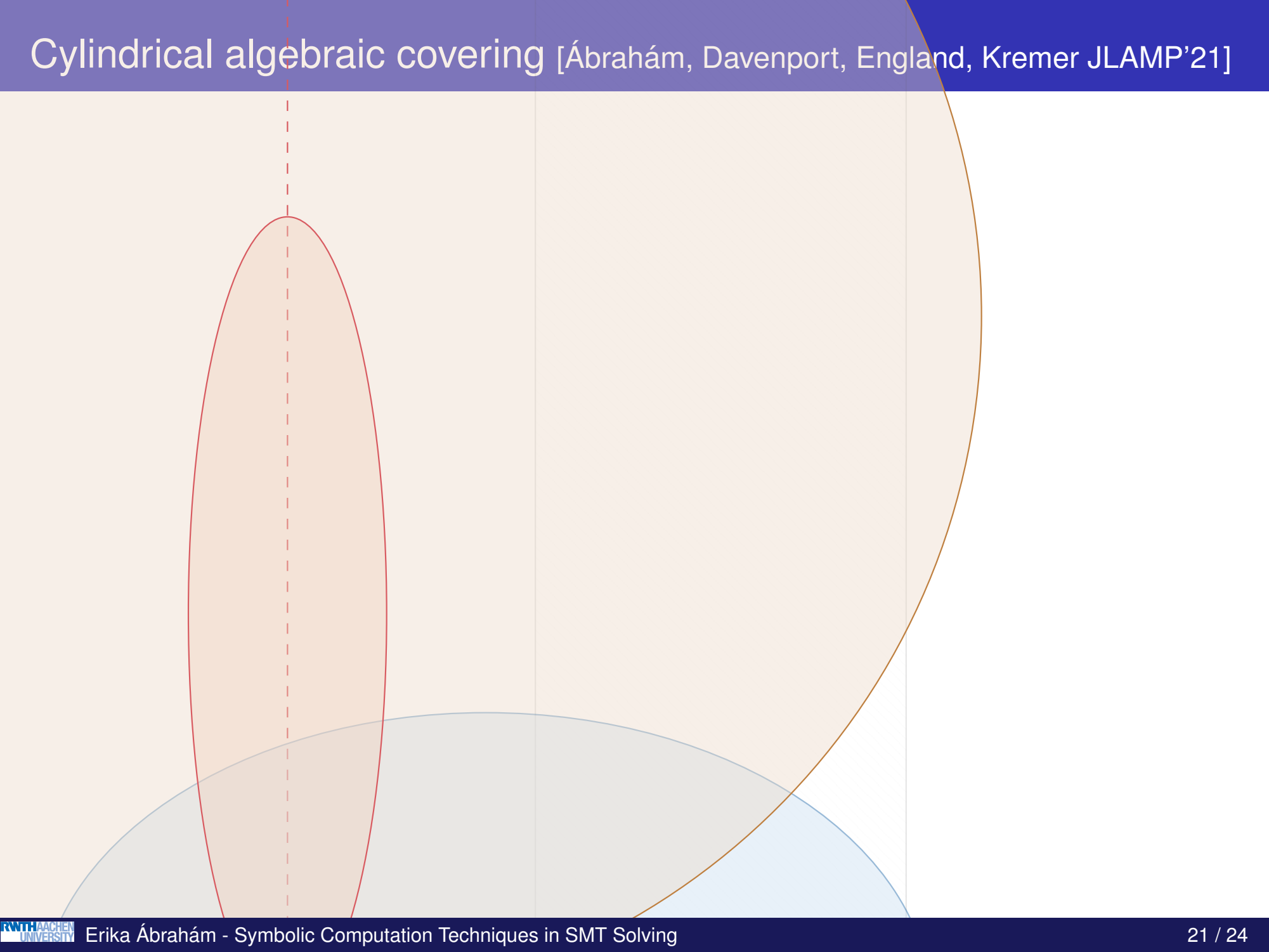

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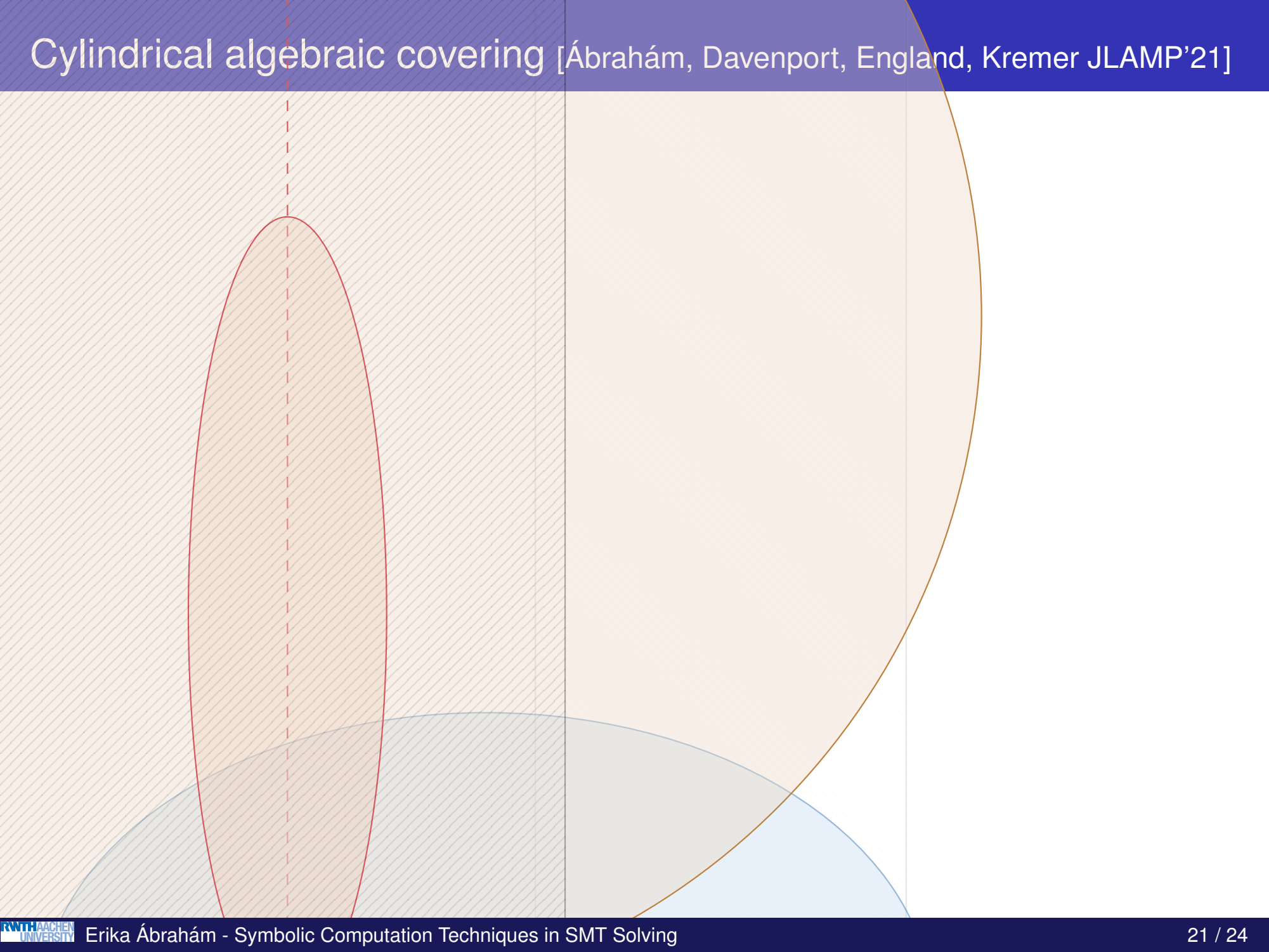

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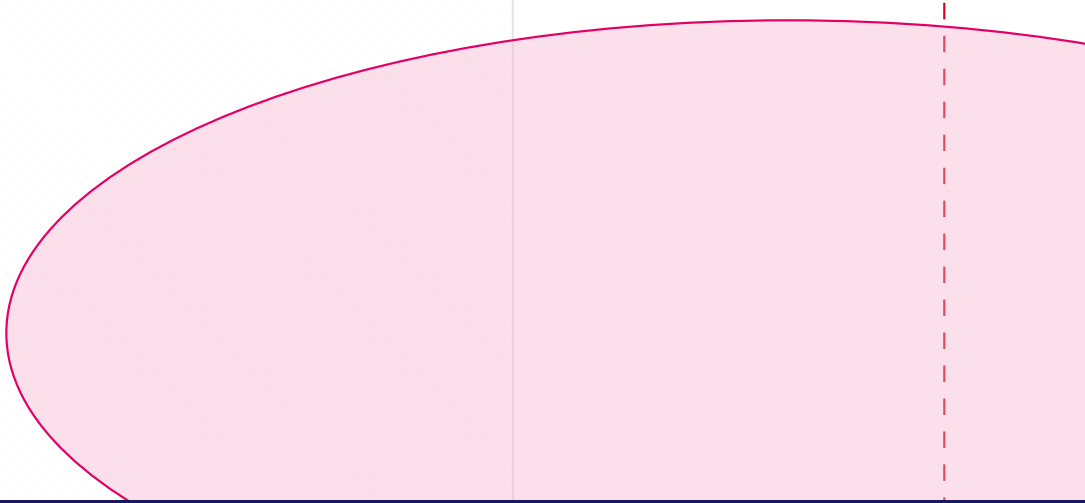

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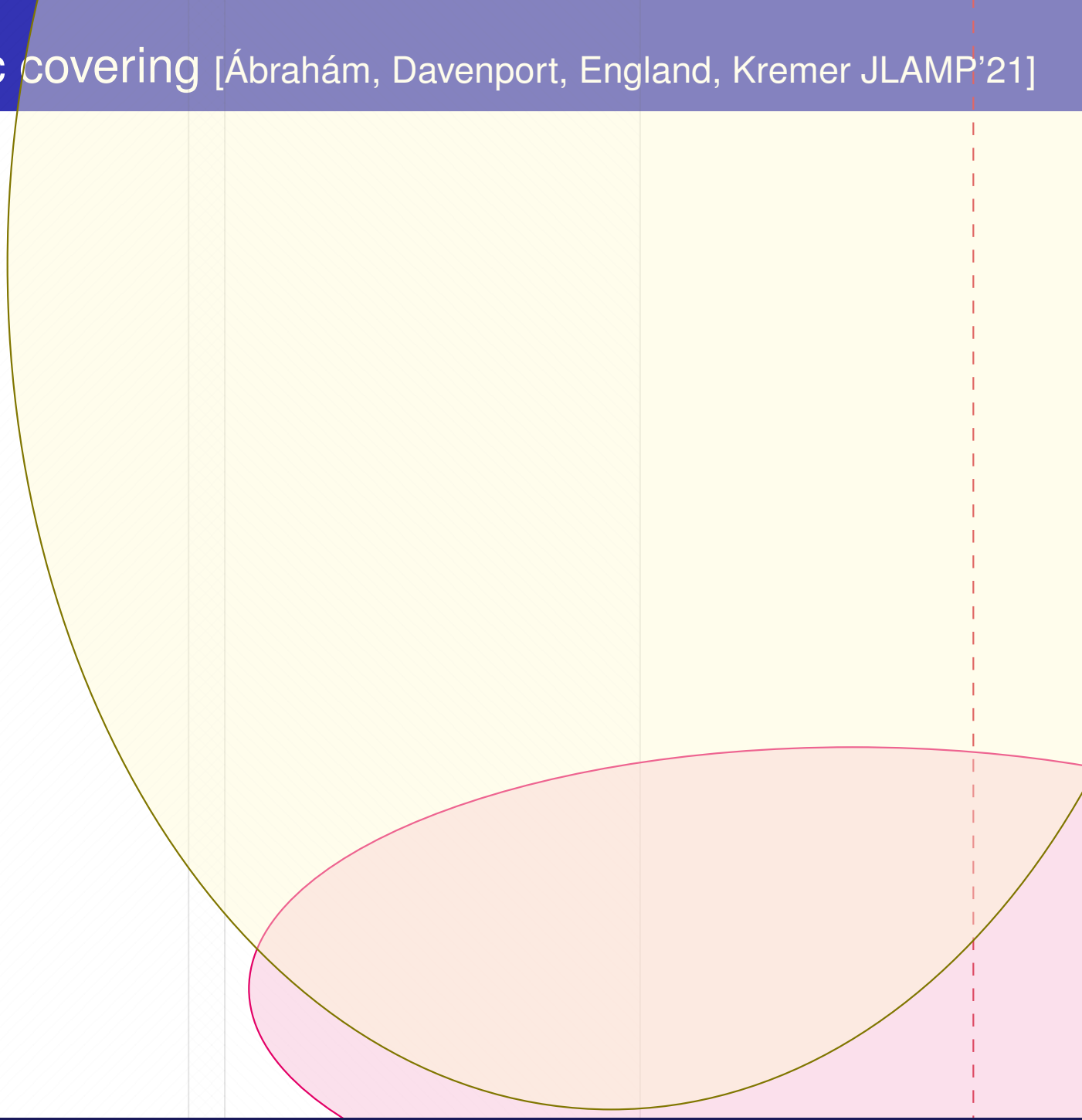

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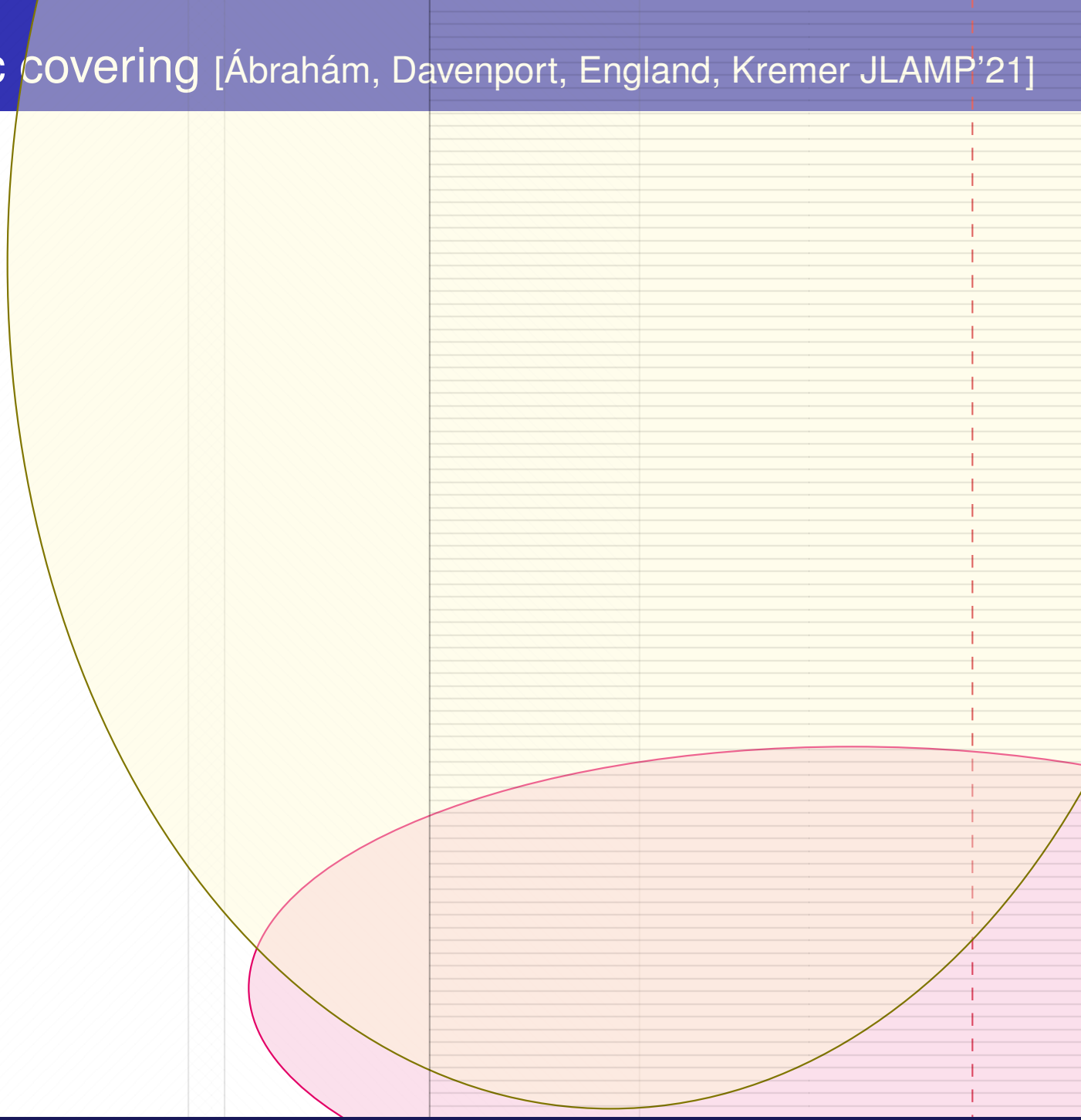

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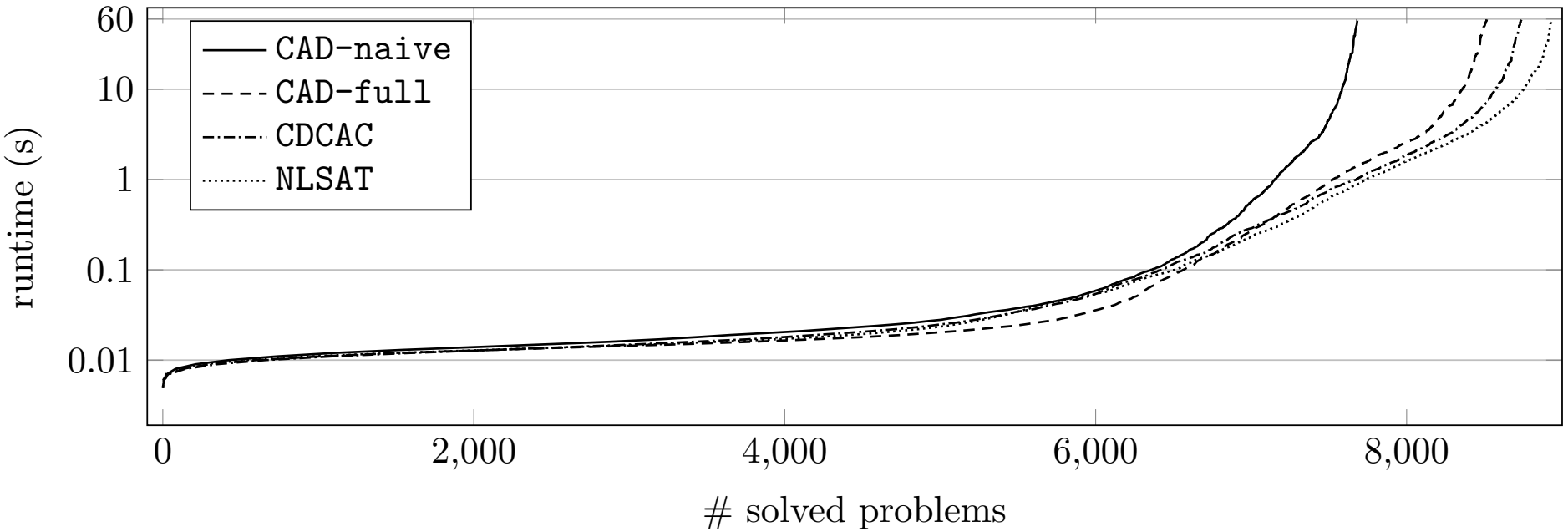
Some experimental results



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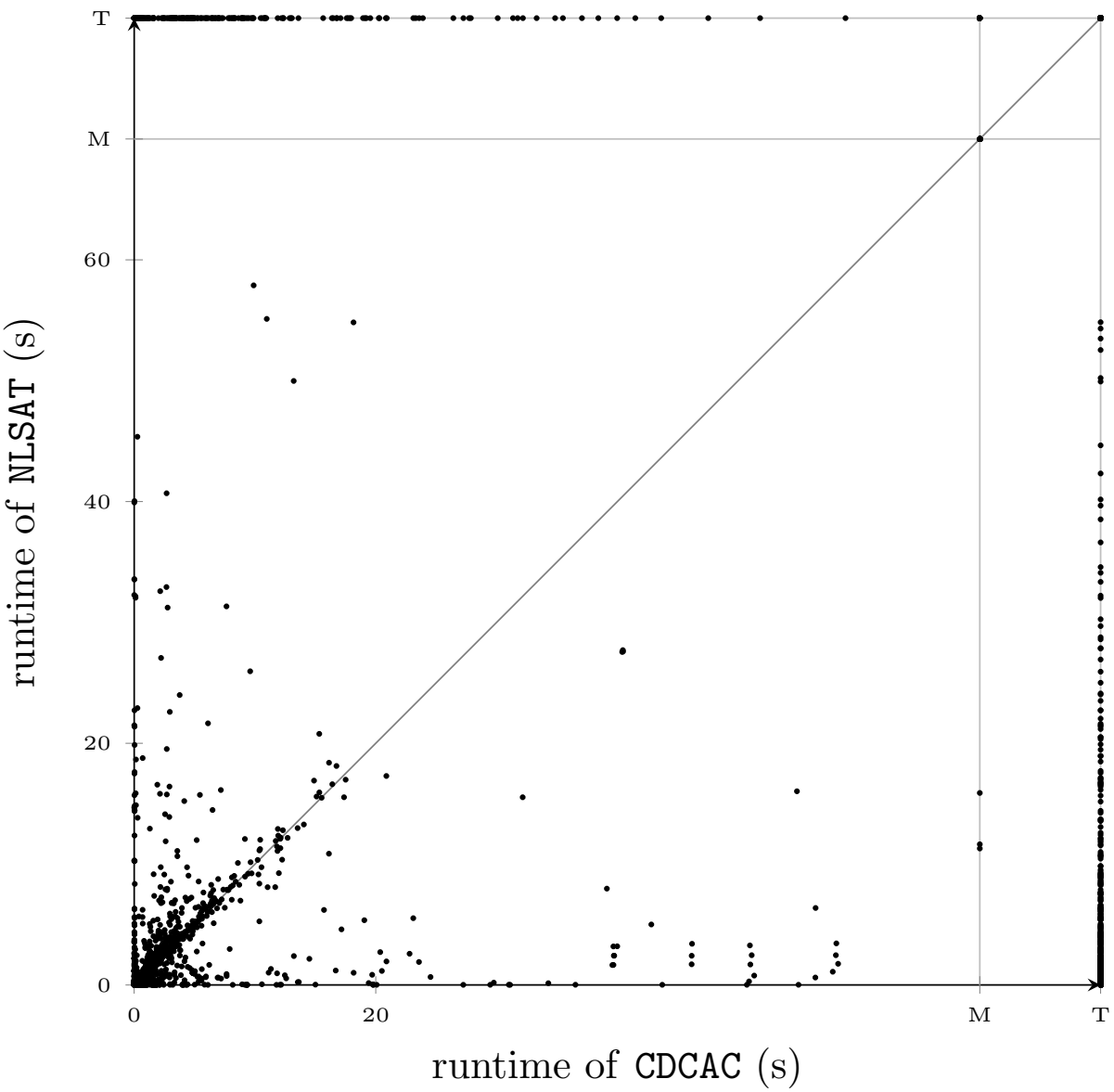

Questions? Contact us!

- MIT licensed source code: `github.com/smtrat/smtrat`
- Documentation: `smtrat.github.io`