Automated Verification and Control Synthesis of Complex Models

Alessandro Abate

Department of Computer Science, University of Oxford

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Formal verification: successes and frontiers

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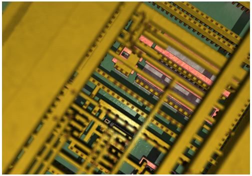
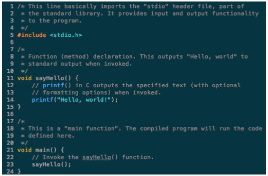
- industrial impact in checking correctness of

protocols, hardware circuits, and software





```c
/* This line basically imports the "stdio" header file, part of
 * the standard library. It provides input and output functionality
 * to the program.
 */
#include <stdio.h>

/*
 * Function (method) declaration. This outputs "Hello, world" to
 * standard output when invoked.
 */
void sayHello() {
    // printf() in C outputs the specified text (with optional
    // formatting options) when invoked.
    printf("Hello, world!");
}

/*
 * This is a "main function". The compiled program will run the code
 * defined here.
 */
void main() {
    // Invoke the sayHello() function.
    sayHello();
}
```

- model-based, automated, and sound guarantees (formal certificates)

Formal verification: successes and frontiers

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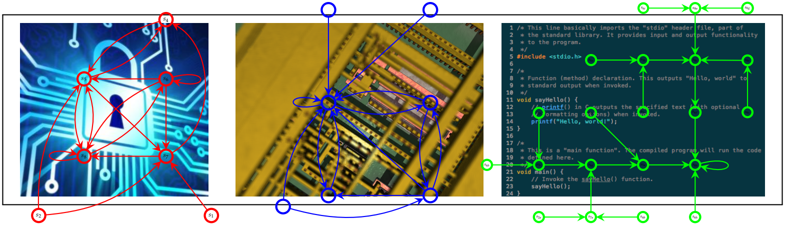

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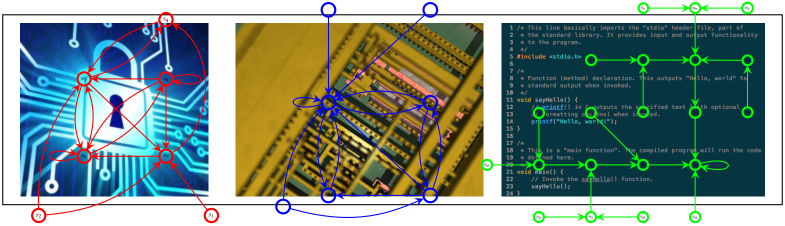

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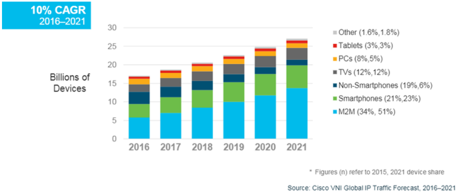
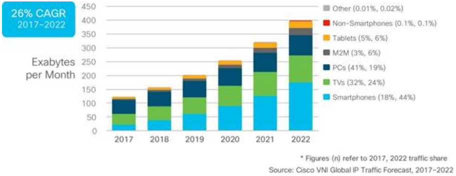
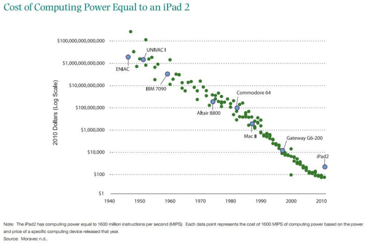
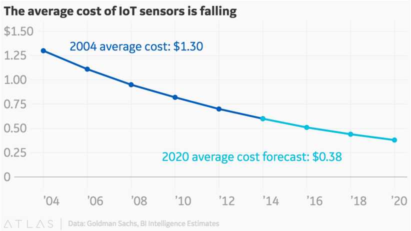
Formal verification and control in the real world





[courtesy M. Zamani]





- tech trends: advances in sensing, networking and embedded computation

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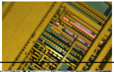
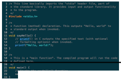
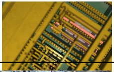
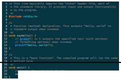




Formal verification and control in the real world

1. integration of learning from data within model-based verification & control
("learning for verification and control")

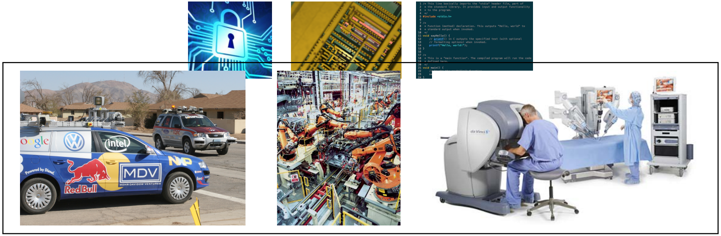
2. certified reinforcement learning for policy synthesis
("certified learning")



Formal verification and control in the real world

- verification and control of complex models
  - dynamical models with uncertainty, noise
  - via formal abstractions



Building automation systems - a CPS exemplar

Building automation system setup in rooms 478/9 at Oxford CS

- advanced modelling for smart buildings
- applications: certifiable energy management
  1. control of temperature, humidity, CO_2
  2. model-based predictive maintenance of devices
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  4. demand-response over smart grids

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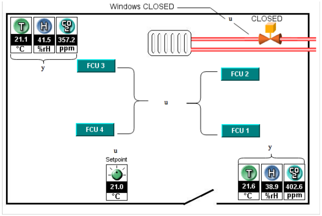

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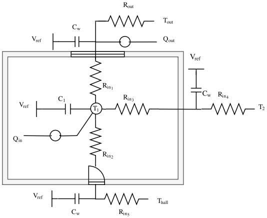

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Building automation systems – a complex model

- model CO_2 dynamics, coupled with temperature evolution

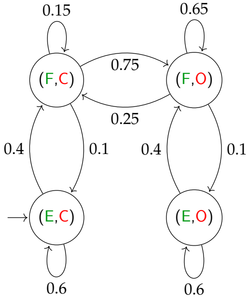
$$x_{k+1} = x_k + \frac{\Delta}{V}\left(-\mathbb{1}_{ON} m x_k + \mu_{\{O,C\}}(C_{out} - x_k)\right) + \mathbb{1}_F C_{occ} + \sigma_x w_k$$

$$y_{k+1} = y_k + \frac{\Delta}{C}\left(\mathbb{1}_{ON} m (T_{set} - y_k) + \mu_{\{O,C\}} \frac{1}{R}(T_{out} - y_k)\right) + \mathbb{1}_F T_{occ,k} + \sigma_y w_k$$

where $T_{occ,k} = \nu x_k + \zeta$

- x - zone CO_2 level
- y - zone temperature

- T_{set} - set temperature (air circulation)
- T_{out} - outside temperature (window)
- T_{occ} - generated heat (occupants)
- $\sigma_{(\cdot)}$ - variance of noise $w_k \sim \mathcal{N}(0,1)$



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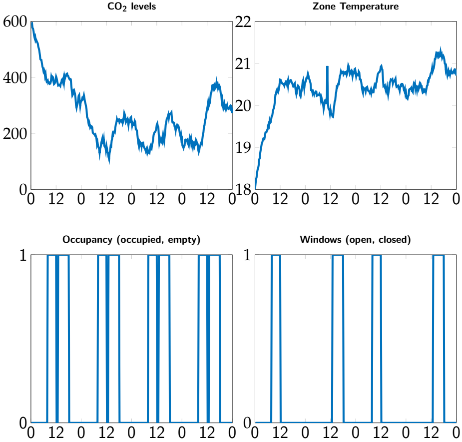


Parameter	Value
C	94.41 J/^{o}C
T_{set}	20 ^{o}C
T_{out}	24 ^{o}C
ν	$2.4 \cdot 10^{-4}$
ζ	0.0107

- air circulation: ON

Complex models: from finite to uncountable

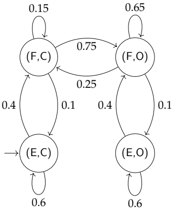
finite-space Markov chain

$(S, \mathbb{T})$

$S = (z_1, z_2, z_3, z_4)$

$$\mathbb{T} = \begin{bmatrix} p_{11} & \cdots & p_{14} \\ \cdots & \cdots & \cdots \\ p_{41} & \cdots & \cdots \end{bmatrix}$$

$\mathbb{P}(z_1, \{z_2, z_3\}) = p_{12} + p_{13}$



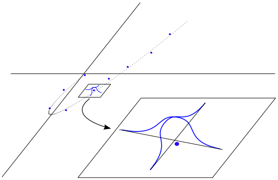
uncountable-space Markov process

$(\mathcal{S}, \mathcal{T})$

$\mathcal{S} = \mathbb{R}^2$

$$\mathcal{T}(dx|s) = \frac{e^{-\frac{1}{2}(x-m(s))^T \Sigma^{-1}(s)(x-m(s))}}{\sqrt{2\pi}|\Sigma(s)|^{1/2}} dx$$

$\mathbb{P}(s, A) = \int_A \mathcal{T}(dx|s), \quad A \subseteq \mathcal{S}$



Probabilistic model checking of complex models

- general specifications expressed as PCTL formulae, e.g.
- simplest instance: probabilistic safety is *the probability that the execution, started at s, stays in safe set A during the time horizon $[0, N]$*

$$\mathcal{P}_s(A) = \mathbb{P}_s(s_k \in A, \forall k \in [0, N])$$

- select $p \in [0, 1]$; probabilistic safe set with safety level p is

$$S(p) = \{s \in \mathcal{S} : \mathcal{P}_s(A) \geq p\}$$

- PCTL formula: $\mathbb{P}_{\leq 1-p} \left(\texttt{true}\ \mathsf{U}^{\leq N} \neg A\right)$

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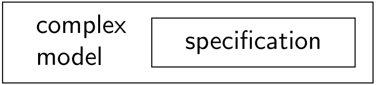
- PCTL formula: $\mathbb{P}_{\leq 1-p}\left(\texttt{true}\ \mathsf{U}^{\leq N} \neg A\right)$

- $\mathcal{P}_s(A)$ can be fully characterised (and optimised)
- issues with computation of $\mathcal{P}_s(A)$ and of $S(p)$

Formal abstractions



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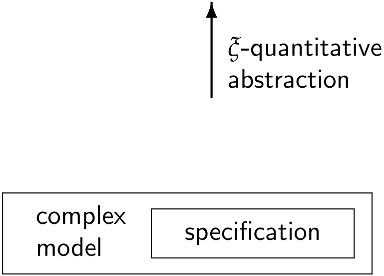

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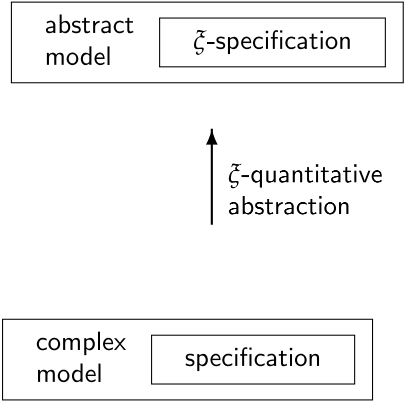

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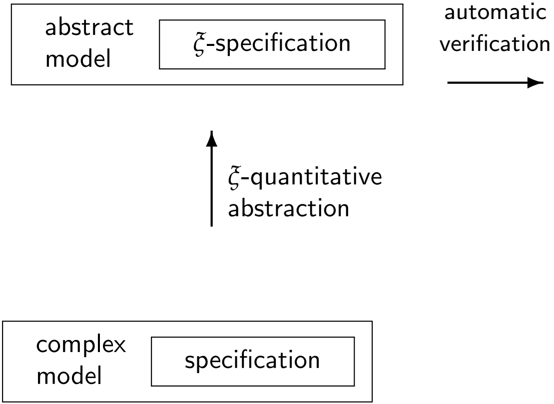

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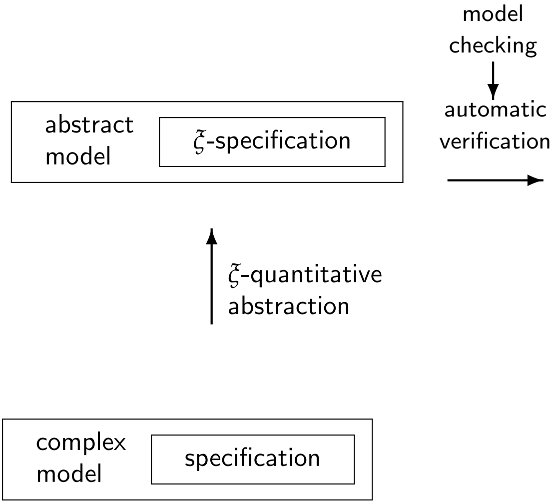

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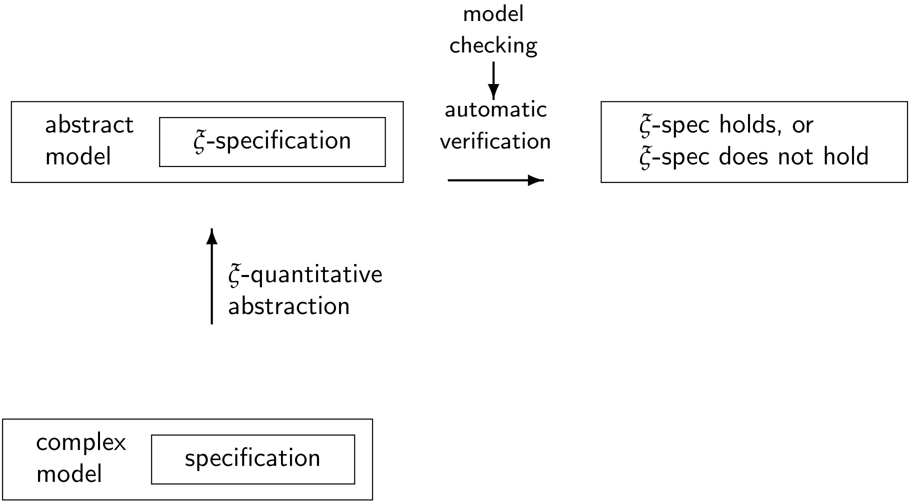

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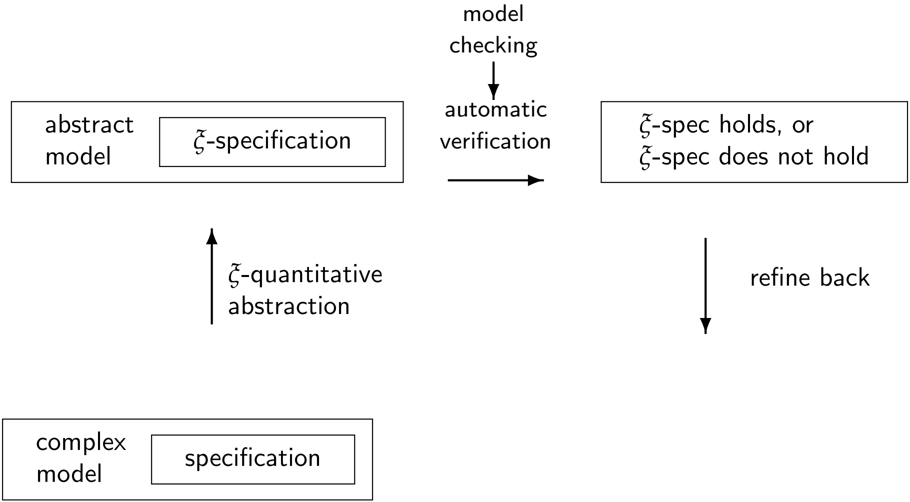

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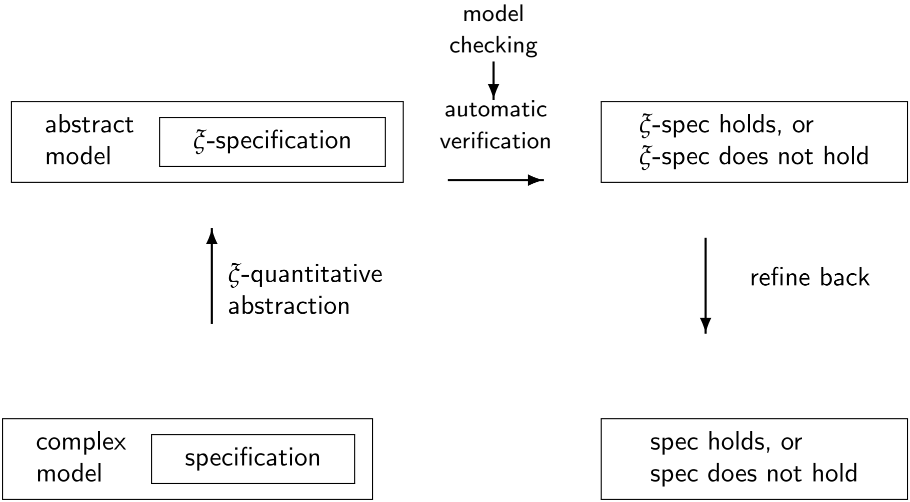

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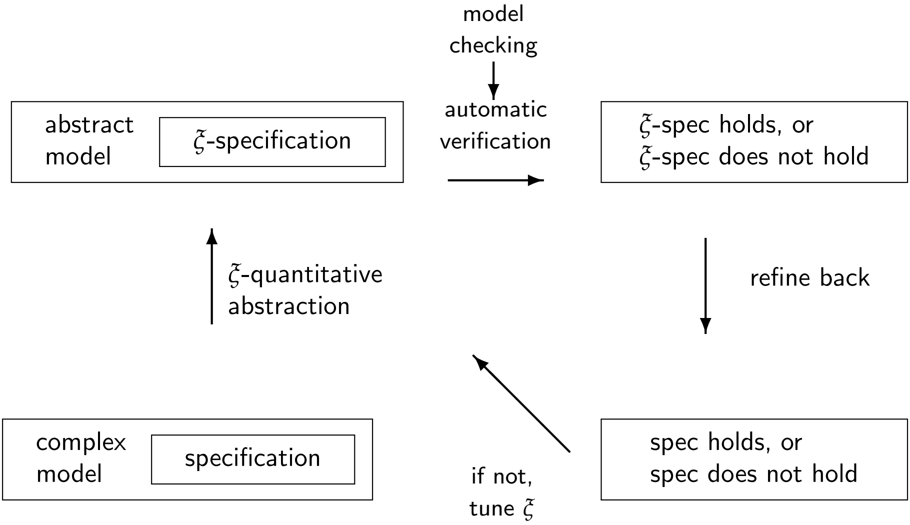

Formal abstractions: algorithm

- approximate stochastic process $(\mathcal{S}, \mathcal{T})$ as MC $(S, \mathbb{T})$, where
    - $S = \{z_1, z_2, \ldots, z_p\}$ – finite set of abstract states
    - $\mathbb{T} : \mathcal{S} \times \mathcal{S} \to [0, 1]$ – transition probability matrix

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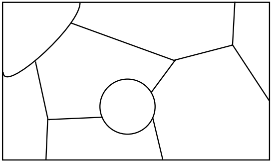

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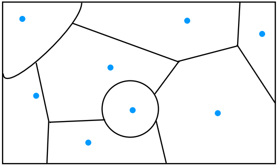

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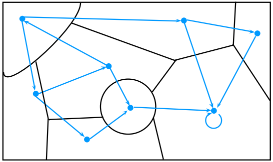

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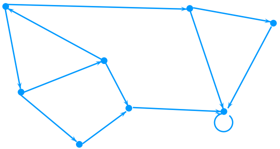

Model checking probabilistic safety via formal abstractions

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- δ-abstract $(\mathcal{S}, \mathcal{T})$ as MC $(S, \mathbb{T})$, so that $A \to A_\delta$, quantify error $\xi(\delta, N)$

$\Rightarrow$ probabilistic safe set

$$\begin{aligned} S(p) &= \{s \in \mathcal{S} : \mathcal{P}_s(A) \geq p\} \\ &= \{s \in \mathcal{S} : (1 - \mathcal{P}_s(A)) \leq 1 - p\} \end{aligned}$$

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can be computed via

$$\begin{aligned} Z_\delta(p+\xi) &\doteq \mathsf{Sat}\left(\mathbb{P}_{\leq 1-p-\xi}\left(\mathtt{true}\ \mathsf{U}^{\leq N} \neg A_\delta\right)\right) \\ &= \left\{z \in S : z \models \mathbb{P}_{\leq 1-p-\xi}\left(\mathtt{true}\ \mathsf{U}^{\leq N} \neg A_\delta\right)\right\} \end{aligned}$$

Formal abstractions: error ζ

- consider $\mathcal{T}(d\bar{s}|s) = \mathfrak{t}(\bar{s}|s)d\bar{s}$; assume $\mathfrak{t}$ is Lipschitz continuous, namely

$$\exists\, 0 \leq h_s < \infty : \quad \left|\mathfrak{t}(\bar{s}|s) - \mathfrak{t}(\bar{s}|s')\right| \leq h_s \left\|s - s'\right\|, \quad \forall s, s', \bar{s} \in \mathcal{S}$$

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- one-step error *(related to approximate probabilistic bisimulation)*

  $\epsilon = h_s \delta \mathscr{L}(A)$

  - δ – max diameter of partition sets
  - $\mathscr{L}(A)$ – volume of set of interest

- N-step error *(tuneable via δ)*

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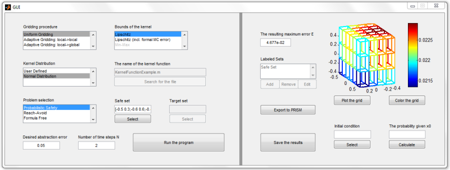
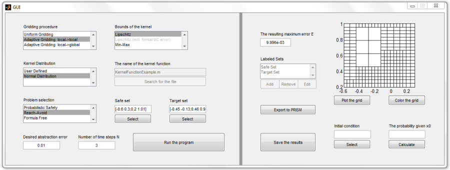
  $\zeta(\delta, N) = \epsilon N$

$\rightarrow$ improved and generalised error

FAUST^2: software for formal abstractions





http://sourceforge.net/projects/faust2

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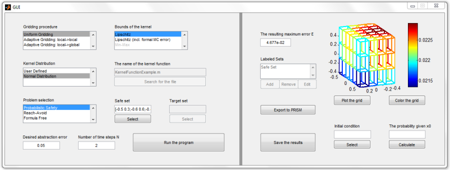

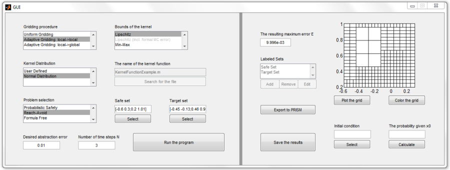

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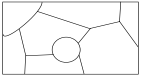

- sequential, adaptive, anytime

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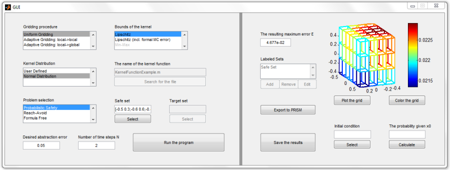

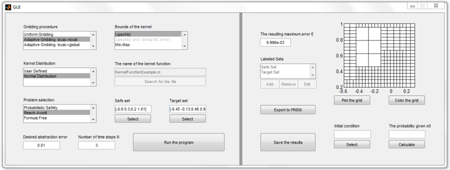

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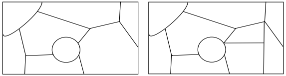
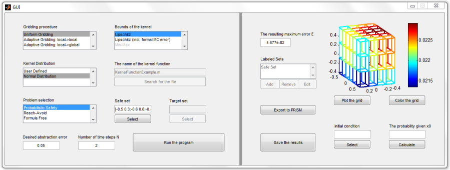
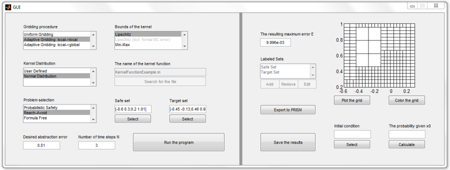


- sequential, adaptive, anytime

FAUST^2: software for formal abstractions





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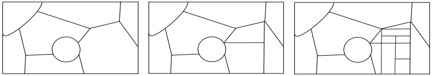

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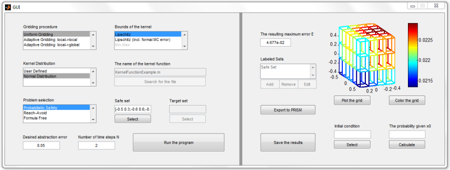

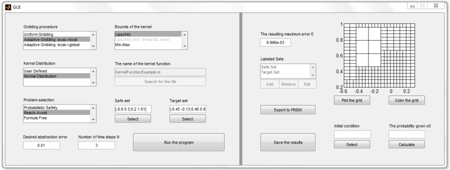

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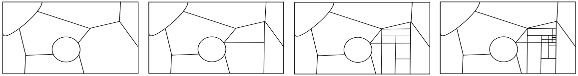

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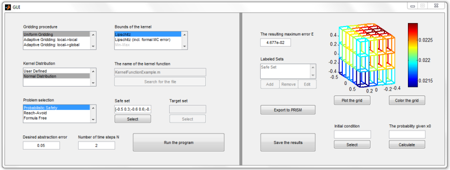

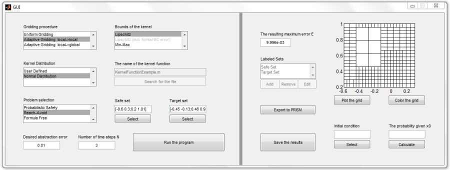

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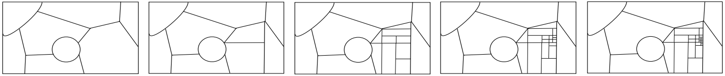

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StocHy: software for formal abstractions

verification

- abstraction based
- novel algorithm with tighter bounds and more scalability

`gitlab.com/natchi92/StocHy`

StocHy: software for formal abstractions

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- novel algorithm with tighter bounds and more scalability

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- abstraction based
- optimisation via sparse matrices

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- C++ implementation
- extendable
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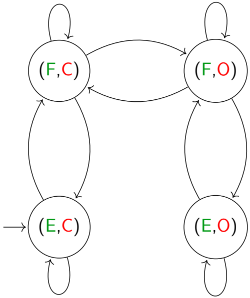
Building automation systems – case study

$$x_{k+1} = x_k + \frac{\Delta}{V}\left(-\mathbb{1}_{ON} m x_k + \mu_{\{O,C\}}(C_{out} - x_k)\right) + \mathbb{1}_F C_{occ} + \sigma_x w_k$$

$$y_{k+1} = y_k + \frac{\Delta}{C}\left(\mathbb{1}_{ON} m (T_{set} - y_k) + \mu_{\{O,C\}}\frac{1}{R}(T_{out} - y_k)\right) + \mathbb{1}_F T_{occ,k} + \sigma_y w_k$$

where $T_{occ,k} = \nu x_k + \zeta$



Building automation systems – case study

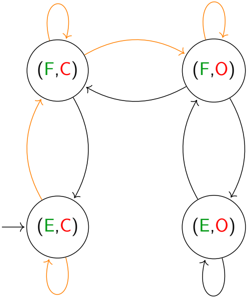
- safe set $A = [300\ 700]ppm \times [19\ 21]^oC$
- air circulation: closed-loop control policy at $k+1$

$$\begin{cases} OFF & \text{if } (x_k, y_k) \leq A \\ ON & \text{if } (x_k, y_k) \geq A \\ \text{stay put} & \text{else} \end{cases}$$

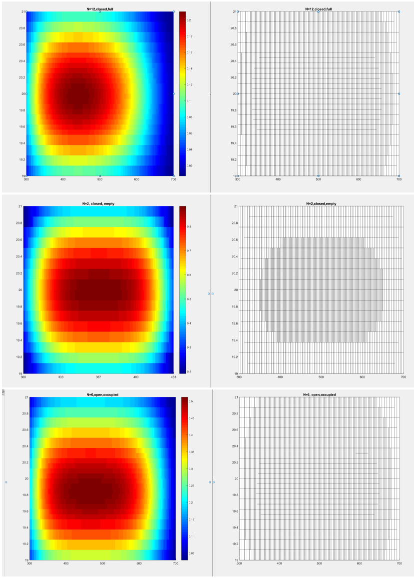
- specification:

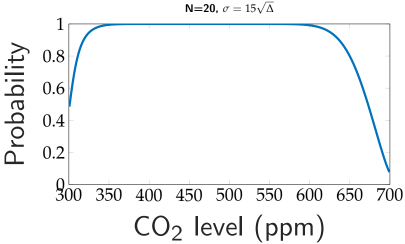
$$\mathbb{P}_{=?}\left[\Box^{\leq 20}(x, y) \in A\right]$$

- 5 hours, 8:00-13:00 ($\Delta = 15$ min, N=20), divided into
  - 8:00-8:30 (N=2) - (E,C)
  - 8:30-11:30 (N=12) - (F,C)
  - 11:30-13:00 (N=6) - (F,O)



Building automation systems – case study





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Thank you for your attention

For more info: `aabate@cs.ox.ac.uk`