Automata on Infinite Words

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$L \subseteq \Sigma^*$

$L_1 = \{ w : |w| \text{ is even} \}$

$L_2 = \{ w : w \text{ contains } 001 \}$

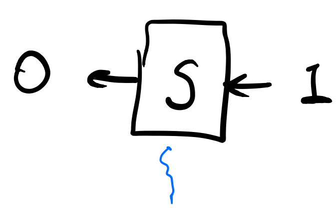
$L \subseteq \Sigma^\omega$

why infinite words?

1962 The natural numbers

$$\circ \to \circ \to \circ \to \circ \to \circ \to \circ \to \cdots$$

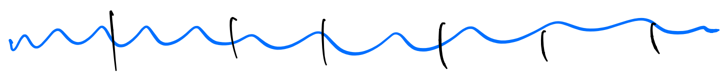
80s On-going behavior of non-terminating systems



× the system finds gcd(x,y)

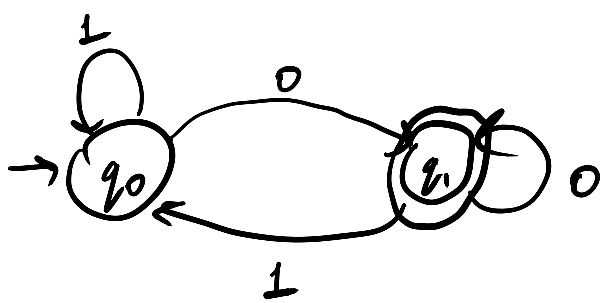
✓ whenever the button is pressed then eventually . . .

✓ infinitely often green lights



A beautiful theory

A



$\Sigma = \{0, 1\}$

$L(A) = (0+1)^* 0$

finite words

0 1 0 1 1 0 1 1 0 0 0 . . . . .

$A = \langle \Sigma, Q, Q_0, \delta, \alpha \rangle$

$Q_0 \subseteq Q$ set of initial states

$\delta : Q \times \Sigma \to 2^Q$

α : an acceptance conditions

A run of A on $w = w_1 w_2 w_3 \ldots$

$r : \mathbb{N} \to Q$ such that

$r(0) \in Q_0$ and for every $i \geq 0$

$r(i+1) \in \delta(r(i), w_{i+1})$

Also $q_0 q_1 q_2 \ldots \in Q^\omega$ $q_i = r(i)$

Accaptance :

Büchi $\alpha \subseteq Q$

r is accepting if it visits α infinitely often.

$\inf(r) \subseteq Q$

$\{q : r \text{ visits } q \text{ i.o.}\} \neq \emptyset$

there are $\infty\, i$ $\; r(i) = q$

r is accepting

if $\inf(r) \cap \alpha \neq \emptyset$

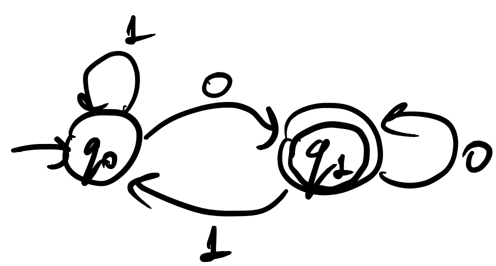
$L(A) = \{w :$ there is an accepting run of A on $w\}$

L is ω-regular $\Leftrightarrow$

there is a NBW A such

that $L(A) = L$.

Examples:



$\Sigma = \{0, 1\}$ $\quad Q_0 = \{q_0\}$ $\quad \alpha = \{q_1\}$

$Q = \{q_0, q_1\}$ $\quad \delta(q_0, 1) = q_0$

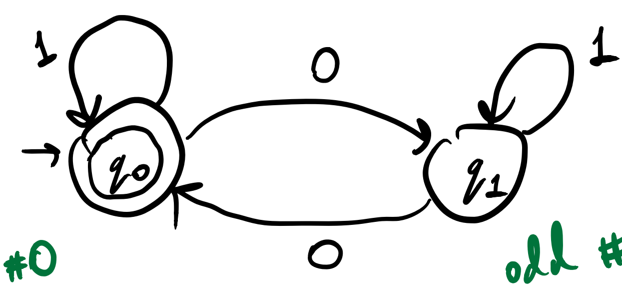
$L(A) = \{ w : w$ includes infinitely many 0s $\}$

$= (1^* 0)^\omega$

$L_2 = \{ w :$ w has a finite even number of 0s or infinitely many 0s $\}$

$0101^\omega \in L_2$ $\qquad$ $01^\omega \notin L_2$

$(01)^\omega \in L_2$



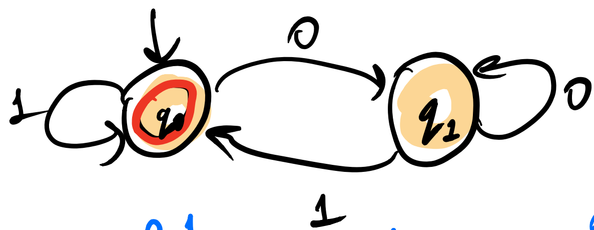
even #0 so far

odd #0 so far

$L_3 = \{ w : \infty 0 \rightarrow \infty 1 \}$

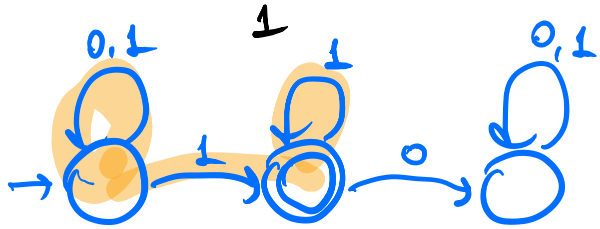
$\Sigma_1 = \{0, 1\}$ $\qquad$ $\Sigma_2 = \{0, 1, 2\}$

$0^\omega \notin L_3 \not\ni (20)^\omega$



$\infty 0$ DBW

$\infty 1$ $(01)^\omega$



$\neg \infty 0$

$(0+1)^* \cdot 1^\omega$

01100111101011^ω

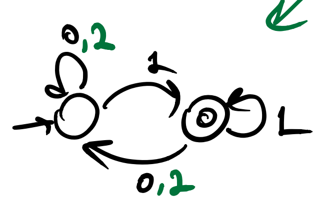
$DFW \rightarrow \overline{DFW}$ $\quad \bar{F} = Q \setminus F$

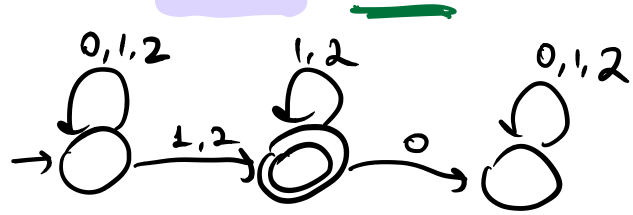
$L_3 = \infty 0 \rightarrow \infty 1$ $\quad \Sigma_2 = \{0, 1, 2\}$

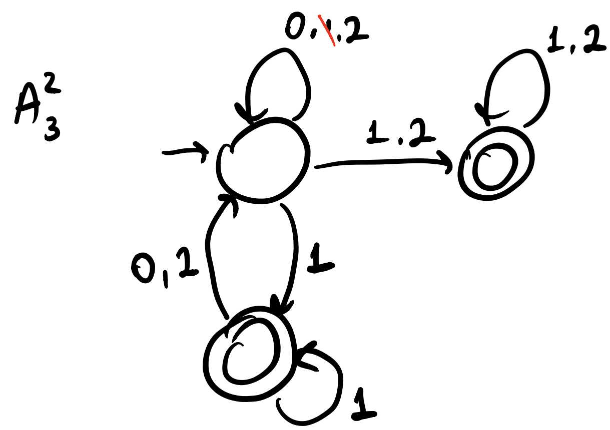
$\Sigma_1 = \{0, 1\}$ $\quad 2^\omega \in L_3$

$L_3 = \infty 1$ $\quad L_3 = \neg \infty 0 \vee \infty 1$

A_3^1

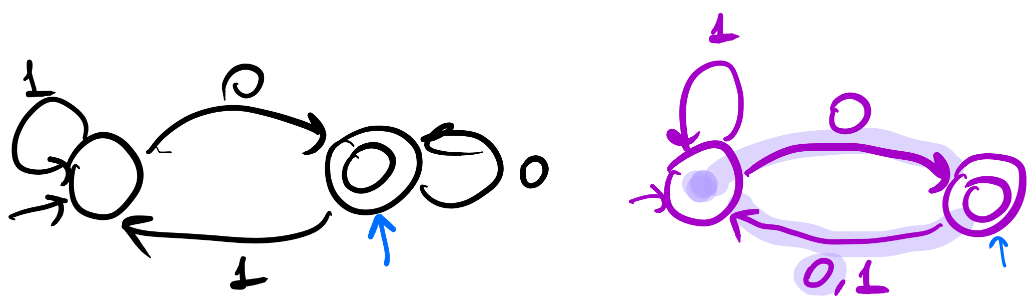






$\infty 0 \to \infty 1$

- There need not be a single minimal DBW for a language.

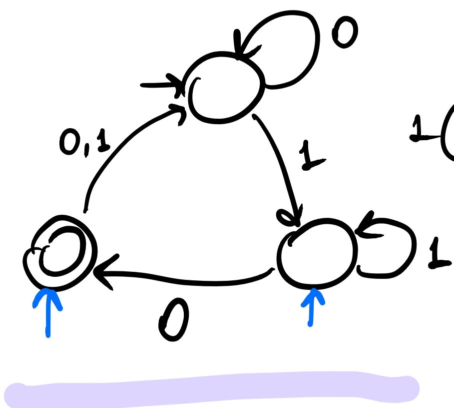


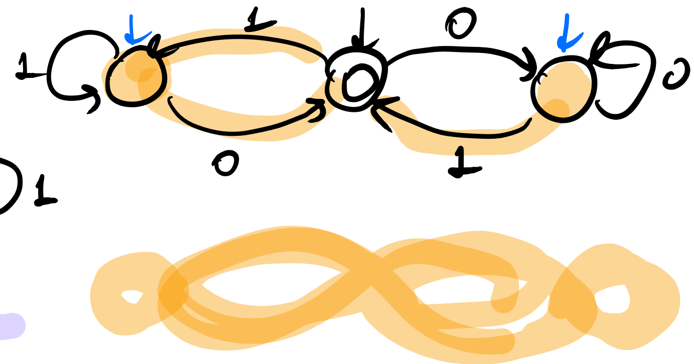
$\infty 0$ $\infty 0$

four 2-state DFWs

different, minimal

$\Sigma = \{0,1\}$ $\qquad$ $\infty 0 \wedge \infty 1$





Closure properties of NBWs / DBWs

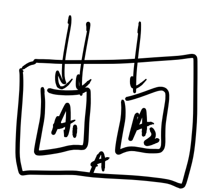
ω-regular languages

Union:

If L_1 is ω-regular $\quad A_1$

L_2 is ω-regular $\quad A_2$

then $L_1 \cup L_2$ is ω-regular $\quad A$



Intersection:

A_1 for L_1

A_2 for L_2

A for $L_1 \cap L_2$

$A_i = \langle \Sigma, Q_i, Q_i^0, \delta_i, \alpha_i \rangle$

The product construction (for NFW)

$A = \langle \Sigma, Q_1 \times Q_2, \ldots$

union for DBWs

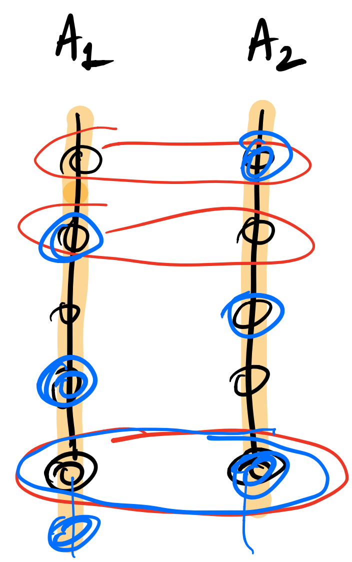
$\alpha = (\alpha_1 \times Q_2) \cup (Q_1 \times \alpha_2)$

$\langle q_1, q_2 \rangle \xrightarrow{\sigma} \langle q_1', q_2' \rangle$

$\delta(\langle q_1, q_2 \rangle, \sigma) =$

$\delta_1(q_1, \sigma) \times \delta_2(q_2, \sigma)$

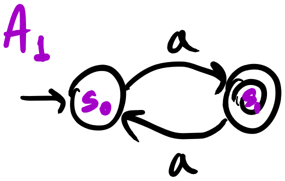
$\alpha = \alpha_1 \times \alpha_2$



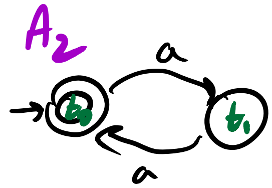
For NBW

$\Sigma = \{a\}$

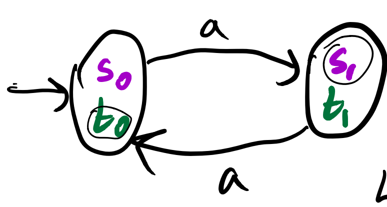
A_1



A_2



$L(A_1) = L(A_2) = \{a^\omega\}$



$\alpha = \alpha_1 \times \alpha_2$

$L(A) = \emptyset$

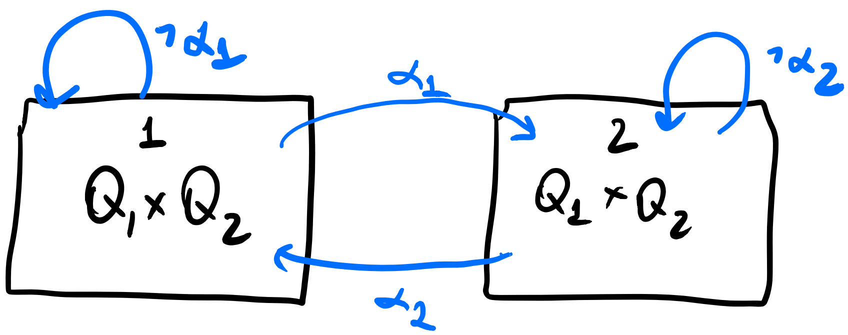
$\Sigma = \{a\}$

$L \subseteq \Sigma^*$ $\quad S \subseteq \mathbb{N}$

$L \subseteq \Sigma^\omega$ $\quad a^\omega$

$\emptyset, \{a^\omega\}$

Construct an NBW for $L_1 \cap L_2$:



$\overset{D}{\text{NBW}}$

$2 \cdot n_1 \cdot n_2$

preserves determinism

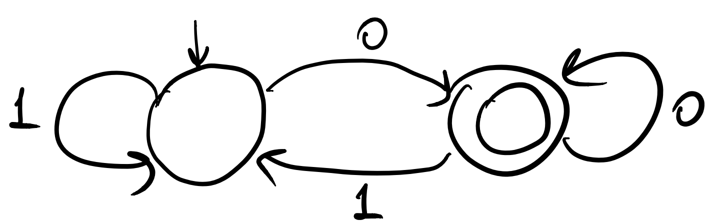
$\underline{\alpha} = \alpha_1 \times Q_2 \times \{1\} =$

$= \{\langle q_1, q_2, 1\rangle : q_1 \in \alpha_1\}$

DBW: DBWs A_1 A_2

DBW

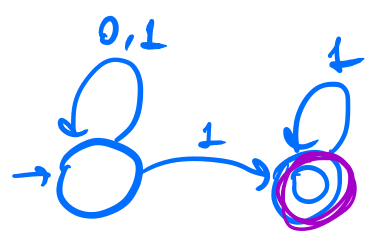
∪ ∩ complementation



DBW

$\infty 0$

DBW → $\overline{\text{NBW}}$



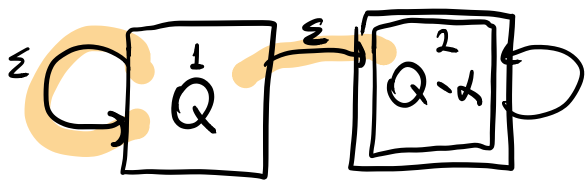
NBW

$\neg\infty 0$

$A = \langle \Sigma, Q, q_0, \delta, \alpha \rangle$ DBW

$A' = \langle \Sigma, Q', q_0', \delta', \alpha' \rangle$

A':



some run ← ~~the~~ run of A sees only f.m. α's to reject.

for NBW we need all runs of

$Q' = (Q \times \{1\}) \cup ((Q \setminus \alpha) \times \{2\})$ $q_0' = \langle q_0, 1 \rangle$

$\delta'(\langle q, 1 \rangle, \sigma) = \begin{cases} \{\langle s,1\rangle, \langle s,2\rangle\} & \text{if } s \notin \alpha \\ \{\langle s,1\rangle\} & \text{if } s \in \alpha \end{cases}$ $\delta(q,\sigma) = s$

$\delta(\langle q, 2 \rangle, \sigma) = \begin{cases} \{\langle s, 2 \rangle\} & \text{if } s \notin \alpha \\ \emptyset & \text{if } s \in \alpha \end{cases}$

DBW → $\overline{\text{NBW}}$

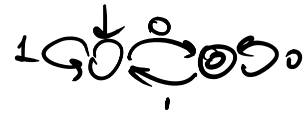
n 2n

① DBW $\xrightarrow{?}$ $\overline{\text{DBW}}$

② NBW $\xrightarrow{?}$ $\overline{\text{NBW}}$

- DBWs are not closed under complementation.

$L = \infty 0$ There is a DBW for L



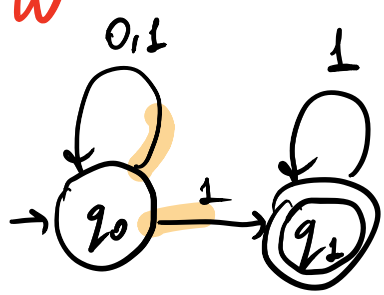
$\boxed{\text{Landweber 69:}}$ There is no DBW for $\bar{L}$

"only f.m 0s". $\neg\infty 0$ $\Sigma^\omega \setminus L$

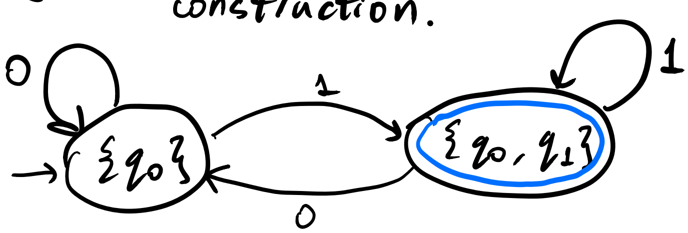
NBW > DBW

NFW = DFW

NBW for $\neg\infty 0$:



Apply the subset construction.



$\infty 1$

$(01)^\omega \notin \neg\infty 0$

$\in \infty 1$

S after reading w: $\forall s \in S$, A has a run on w that reaches s.

No DBW for "only finitely many 0s".

- Assume there is. A, n states.

$w_1 = 1^\omega \in \neg\infty 0 \rightarrow A$ accepts 1^ω

$A = \langle \Sigma, Q, q_0, \delta, \alpha \rangle$

$1\ 1\ 1\ 1\ 1 \ldots\ldots$

$\uparrow i_1$

$\delta(q_0, 1^{i_1}) \in \alpha$

$w_2 = 1^{i_1} 0 1^\omega \in \neg\infty 0 \rightarrow A$ accpets w_2

$1^{i_1} 0\ 1\ 1\ 1\ 1 \ldots$

$\uparrow i_1 \qquad \uparrow i_2$

$\delta(q_0, 1^{i_1} 0 1^{i_2}) \in \alpha$

$w_3 = 1^{i_1} 0 1^{i_2} 0 1^\omega \ldots$ α

$\uparrow i_3$

For every $k > 1$

$W_k = 1^{i_1} 0 1^{i_2} 0 1^{i_3} 0 1^{i_4} \ldots 0 1^{i_k} 0 1^\omega$

$k > n \qquad \exists_{1 \le j_1, j_2 \le k}$ $\qquad j_1 \neq j_2$

$j_1 < j_2$

$$\delta(q_0, w_{j_1}) = \delta(q_0, w_{j_2}) \overset{=q}{\in} \alpha$$

Consider: $w = 1^{i_1} 0 \dots 1^{i_{j_1}} (0 1^{i_{j_1+1}} \dots 0 1^{i_{j_2}})^\omega$

$\uparrow q \qquad \uparrow q$

not empty
at least one 0

① A accepts w
(the run of A on w visits q i.o.)

② $w \in \infty 0 \notin L(A)$

$j_1 < j_2$

$\rightarrow$ no DBW A exist.

NBW > DBW

① Characterize DBW

② Stronger conditions

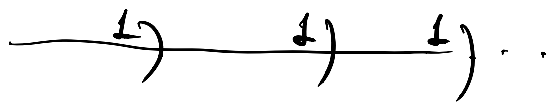
Landweber 69

For $R \subseteq \Sigma^*$, we define

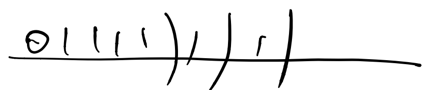
$\Sigma^\omega \supseteq \lim(R) = \{ w :$ w has i.m. prefixes $\{$

in R

$R = (0+1)^*1$



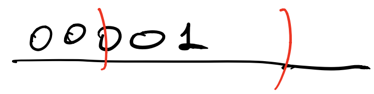
$\lim(R) = \infty 1$



$R = (0 \cdot 1^+)$ $\quad \{01^\omega\}$

$\uparrow$
$\lim(R)$

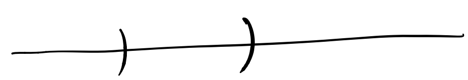
$R = (0^* 1)$



$\lim(R) = \emptyset$

$\lim^{-1}(L)$ need not be unique

$\infty 1$ $\quad R = (0+1)^*1(0+1)^*1 \quad ((0+1)^*1)^i$



$R = (0+1)^*1(0+1)^i$

- For every $L \subseteq \Sigma^\omega$

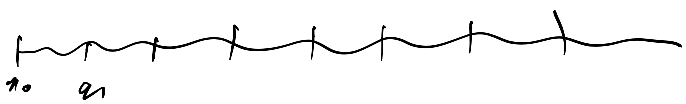
$L \in DBW \Leftrightarrow \exists R \subseteq \Sigma^*$ s.t.

$L = \lim(R)$

proof: For a deterministic automaton A

$$L_B(A) = \lim(L_F(A))$$

$\downarrow$ A as a DBW $\qquad\qquad$ $\downarrow$ A as a DFW



$L \in DBW \to A \to R = L_F(A)$

$\exists R \to A \to$ A when viewed as a DBW recognizes L.

stronger acceptace conditions.

$inf(r) \cap \alpha \neq \emptyset$ Büchi

$DPW = NBW > DBW$

$\downarrow$

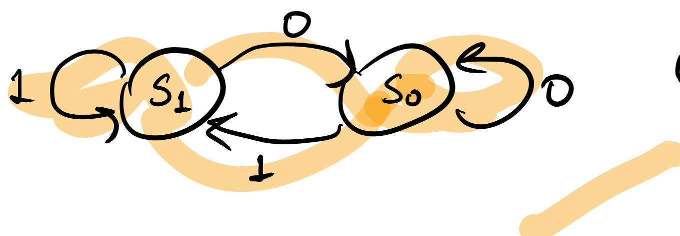
all ω-regular languages

① Generalized Büchi

$\alpha = \{\alpha_1, \alpha_2, \ldots \alpha_k\}$

r is accepting iff

$inf(r) \cap \alpha_i \neq \emptyset$ for all $1 \leq i \leq k$



$\infty 0 \wedge \infty 1$

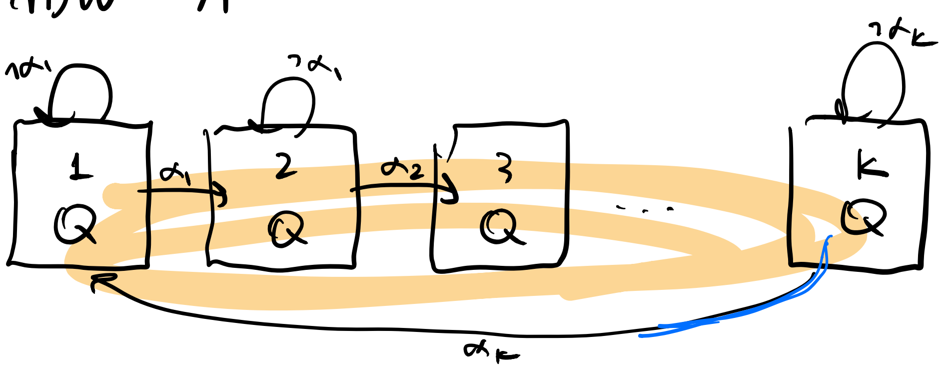
$\alpha = \{ \{s_0\}, \{s_1\} \}$

DGBW → DBW
NGBW → NBW

no added expressive power

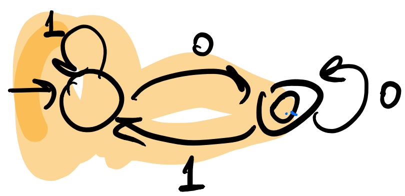
NGBW A $\alpha = \{\alpha_1, \ldots \alpha_k\}$

→ NBW A'



$Q' = Q \times \{1, \dots k\}$ $\alpha' = \alpha_k \times \{k\}$

no DBW

A 

$\infty 0$

no DBW for $\neg \infty 0$.

② co-Büchi

r is accepting iff $\inf(r) \cap \alpha = \emptyset$

only f.m. visits in α.

$L_c(A) = \neg \infty 0$

A as a DCW

$L_c(A) = \overline{L_B(A)}$

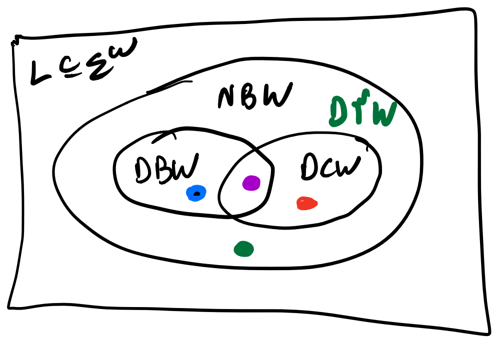
↓

r is accepting if $\inf(r) \cap \alpha = \emptyset$

$DCW = \overline{DBW}$

no DCW for $\infty 0$

③ Rabin, Streett, parity



$\infty 0 \quad (1^*0)^\omega$

$\neg\infty 0 \quad (0+1)^* 1^\omega$

$0\cdot(0+1)^\omega$

$\infty 0 \wedge \neg\infty 1 \quad \Sigma = \{0,1,2\}$

- **Rabin** $\quad \alpha = \{\langle L_1, R_1\rangle, \langle L_2, R_2\rangle \ldots \langle L_k, R_k\rangle\}$

index of α

$L_i, R_i \subseteq Q$

$\alpha \in 2^{2^Q \times 2^Q}$

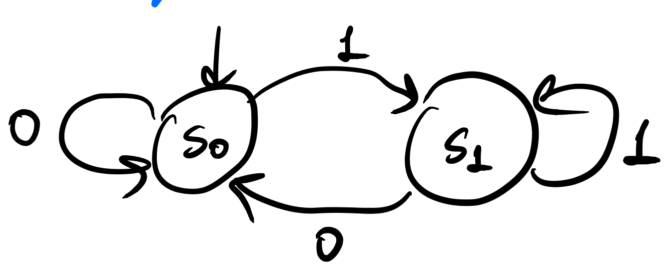
r is accepting iff $\exists\ 1 \le i \le k$

$inf(r) \cap L_i \neq \emptyset$ and ∞L_i

$inf(r) \cap R_i = \emptyset \quad \neg\infty R_i$

$\alpha_1 = \{\langle \{s_1\}, \emptyset\rangle\}$

$\infty 1$



Buchi $\alpha \rightarrow$ Rabin $\{\langle \alpha, \emptyset \rangle\}$

$\alpha_2 = \{\langle \{s_0, \cancel{s_1}\}, \{s_1\} \rangle\}$ $\neg \infty 1$

co-Buchi $\alpha \rightarrow$ Rabin $\{\langle Q, \alpha \rangle\}$

$\langle Q \setminus \alpha, \alpha \rangle$

- Streett dual to Rabin

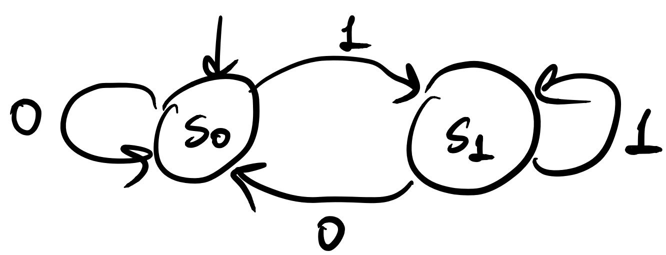
$\alpha = \{\langle L_1, R_1 \rangle .. \langle L_k, R_k \rangle\}$

r is accepting iff $\forall\ 1 \leq i \leq k$

$inf(r) \cap L_i = \emptyset$ or $\neg \infty L_1 \vee \infty R_i$

$inf(r) \cap R_i \neq \emptyset$ $\infty L_i \rightarrow \infty R_i$

$\infty 1$



$\{\langle Q, \{s_1\} \rangle\}$

Buchi α $\{\langle Q, \alpha \rangle\}$ $\infty 0 \wedge \infty 1$

GB $\{\alpha_1 .. \alpha_k\}$ $\{\langle Q, \alpha_1 \rangle .. \langle Q, \alpha_k \rangle\}$

co-Buchi α $\{\langle \alpha, \emptyset \rangle\}$

$\{\langle Q, \{s_0\} \rangle, \langle Q, \{s_1\} \rangle\}$

③ parity $\quad \alpha : Q \to \{0, 1, \dots, k\}$

r is accepting iff the minimal color that r visits i.o. is even

$$\min\{i : \inf(r) \cap \underbrace{\alpha^{-1}(i)}_{\alpha_i} \neq \emptyset\}$$

is even.

$$\alpha = \{\alpha_0, \alpha_1, \dots \alpha_k\}$$

- parity as Rabin

$$\underset{\text{Rabin}}{\alpha'} = \{\langle \alpha_0, \emptyset \rangle, \langle \alpha_2, \alpha_1 \cup \alpha_0 \rangle, \langle \alpha_4, \alpha_3 \cup \alpha_2 \cup \alpha_1 \cup \alpha_0 \rangle$$

- parity as Streett

$$\alpha' = \{\langle \alpha_1, \alpha_0 \rangle, \langle \alpha_3, \alpha_0 \cup \alpha_1 \cup \alpha_2 \rangle, \dots \}$$

- Büchi as parity

$$\underset{\text{Buchi}}{\alpha} \to \{\overset{0}{\alpha}, \overset{1}{Q \setminus \alpha}\}$$

1. Expressive power and succinctness
2. Complexity of decision problems
3. Determinization and complementatio

NRW $\rightarrow$ NBW

$A = \langle \Sigma, Q, Q_0, \delta, \alpha \rangle$

$\alpha = \{\langle L_1, R_1 \rangle \ldots \langle L_k, R_k \rangle\}$

$\alpha_i = \{\langle L_i, R_i \rangle\} \qquad \exists\ 1 \le i \le k$

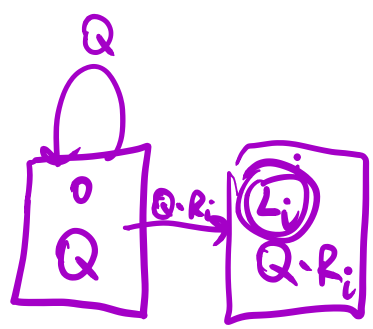
A_i NRW[1] with α_i

$L(A) = \bigcup_{1 \le i \le k} A_i$

$A_i \rightarrow A_i'$

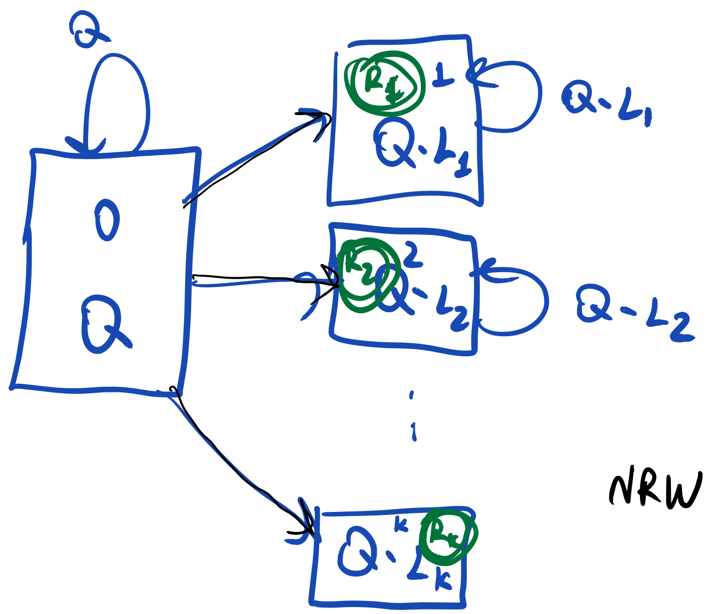
NRW[1] NBW

$\langle L_i, R_i \rangle$



$(Q \times \{0\}) \cup ((Q \times R_i) \times \{i\})$

$\alpha_i' = L_i \times \{i\}$



$NRW(n,k)$

$\to NBW(n \cdot (k+1))$

$NSW \to NBW$

A $\qquad$ A'

$\forall i \quad \infty L_i \to \infty R_i$

A' guess a subset $I \subseteq \{1..k\}$

s.t. $i \in I \leftrightarrow$ the run visits

L_i i.o.

Does not work $\quad A' = \bigcap\limits_{1 \le i \le k} A_i$

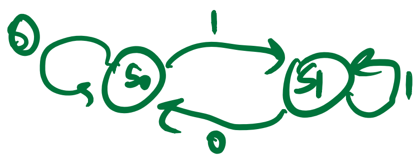
$$NSW(n,k) \underset{n \cdot 2^k}{\xrightarrow{n \cdot 2^k}} NBW(n \cdot 2^k)$$

$n + 2^k \cdot n \cdot k$

DRW $\xrightarrow{\text{no blow up}}$ DBW when exists.

Typeness: <u>Rabin is Büchi-type</u>:

if L is in DBW, and A is a DRW for L, then there is a DBW for L, on the structure of A.



$\infty 1$

Streett is not Buchi type

$\{\langle Q, \{s_0\}\rangle, \langle Q, \{s_1\}\rangle\}$ DSW

$\underline{\infty 0 \wedge \infty 1} \in$ DBW

- Decision procedures

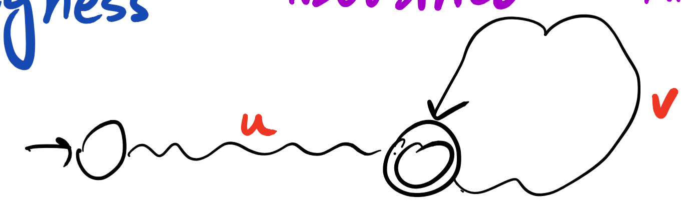
- Emptyness : Given A

  Is $L(A) = \emptyset$?

- nonemptyness NLOGSPACE linear time

OBW

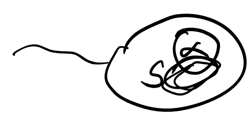
NBW



A is not empty iff there is $q \in \alpha$ such that q is reachable from Q_0 and from it self. $u \cdot v^\omega \in L(A)$

- is there a maximal strongly connected component S (not trivial) such that S is reachable from Q_0

$S \cap \alpha \neq \emptyset$



- nonemptyness for NRW NL
  NSW ? PTIME

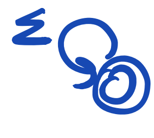
- $NRW(n,k) \to NBW(n \cdot k)$

  S $n \cdot 2^k$

- universality $L(A) = \Sigma^\omega$

PSPACE-complete

- containment $\underline{L(A_1)} \subseteq \underline{L(A_2)}$

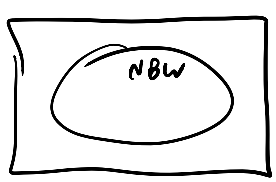
A is universal $\leftrightarrow$  $\subseteq L(A)$

$$A \subseteq B \leftrightarrow A \cap \overline{B} = \emptyset$$

$$L(A_1 \times \overline{A_2}) = \emptyset$$

product complementation

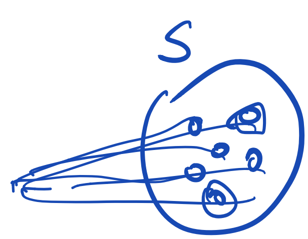
③ Determinization 4



$$NBW \longrightarrow D\begin{smallmatrix}R\\S\\P\end{smallmatrix}W$$

$$NFW \longrightarrow DFW$$

$$Q \qquad 2^Q$$

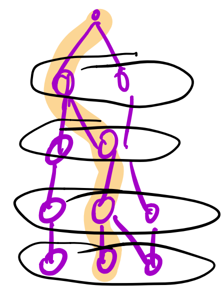


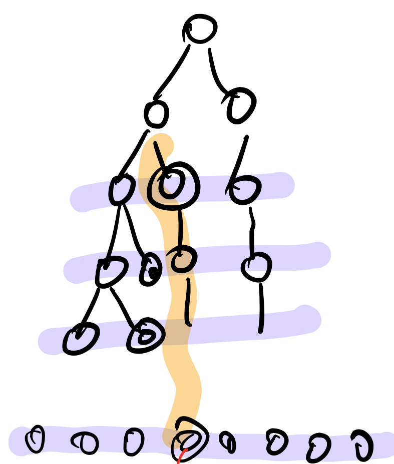
NBW → DRW

Safra 1988

$2^{O(n \log n)}$

S + additional information

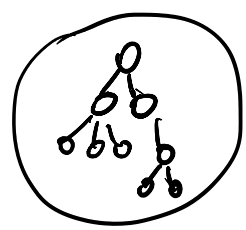




∞ $\neg\infty$

$\langle L, R \rangle$

safra tree



S

R: follow a path

L: this path has i.m. visits in α.

NBW → DPW Piterman 2006

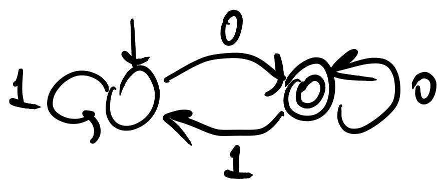
Automata on Infinite Words

Orna Kupferman
The Hebrew University

We saw: Buchi word automata

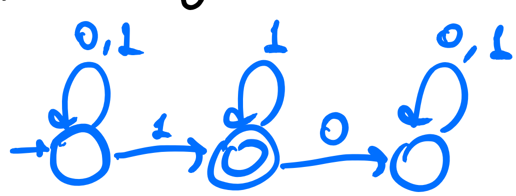
NBW DBW

- closed for $\cap$ $\cup$
- NBW $\neg$ ✓
- DBW $\neg$ ✗ $\infty 0$



no DBW $\neg \infty 0$

NBW



NBW > DBW

$\neg \infty 0$

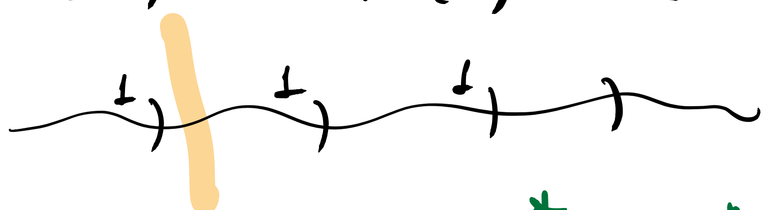
A ω-regular language $L \subseteq \Sigma^\omega$

$L \in DBW \leftrightarrow$ exists regular $R \subseteq \Sigma^*$

$$L = \lim R$$

infinite word with ∞ prefixes in R

$R = (0+1)^* 1 \qquad \lim(R) = \infty 1$



$\lim^{-1}(\infty 1) = (0+1)^* 1 (0+1)^* 1$

$(0+1)^* 1 (0+1)^i$

Richer acceptance condition

$D?W = NBW$

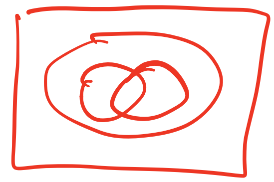
- generalized Büchi $\qquad DGBW = DBW$
- co-Büchi

DCW for $\neg\infty 0$ $\qquad DCW = \overline{DBW}$

- Rabin, Streett, parity

$$DRW = DSW = DPW = NBW$$

- expressive power
- succinctness



$$\underset{S}{NRW(n,k)} \longrightarrow NBW(n \cdot k)$$

2^k

- complexity of decision problems

~~Clean~~ and beautiful theory

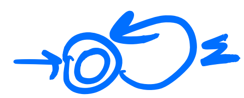
What is clean?

The case of finite words

DFW - uninque minimal automaton

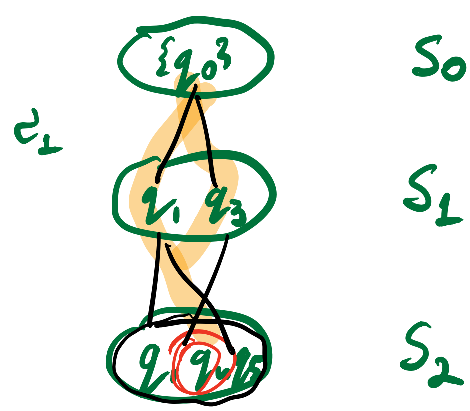
- MN equivalence classes
- minimazation in PTIME

NFW - minimazation: PSPACE



not too nice but...

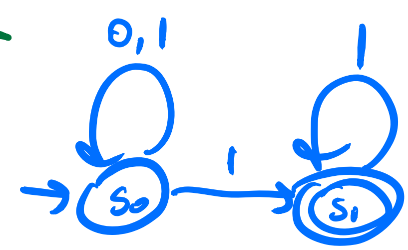
subset construction

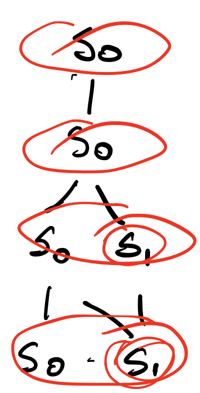


NFW → DFW

→ $\overline{DFW}$

ss const on





$S + \underbrace{\text{extra information}}$

1. Why is minimization hard?
2. What is this exstra info?

Minimization:

Sven Schewe 2010

$$\mathrm{MINDBW} = \{\langle \underset{\uparrow \atop \text{DBW}}{A}, \underset{\uparrow \atop \mathbb{N}}{k} \rangle : \text{ A has an equivalent DBW with at most } k \text{ states}\}$$

$\in$ NP-complete.

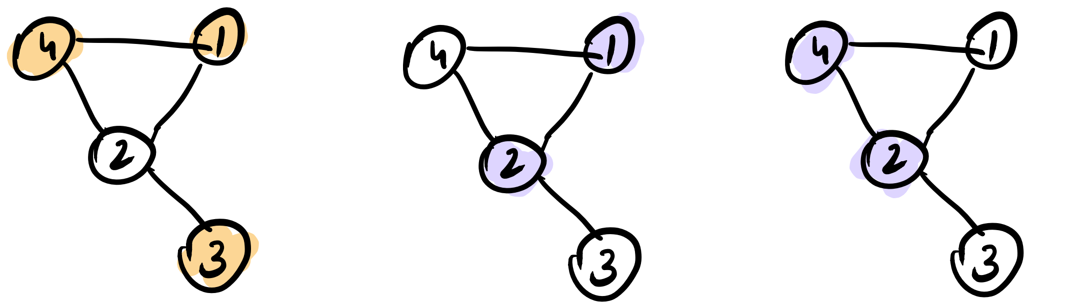
- In NP: the k-state DBW is a witness.

NP-hard

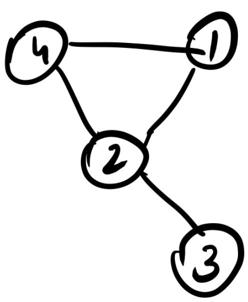
$$VC = \{\langle G, k \rangle : G \text{ has a vertex cover of size at most } k\}$$

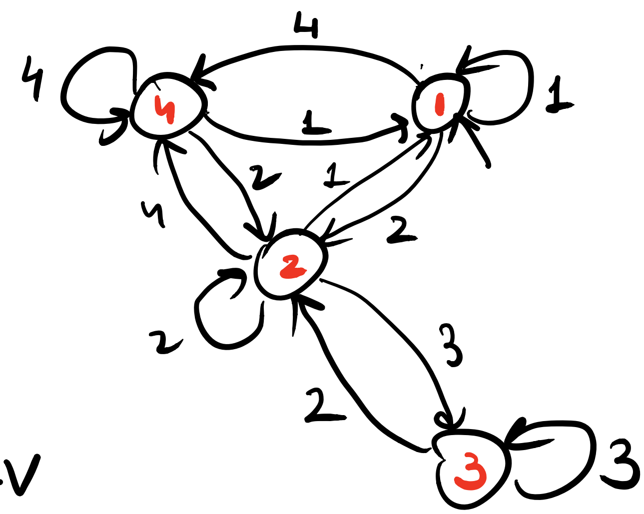
$G = \langle V, E \rangle$ $\quad S \subseteq V$ is a VC

$\forall (u,v) \in E \quad u \in S$ or $v \in S$



The reduction: $\langle G, k \rangle \to \langle A, k' \rangle$

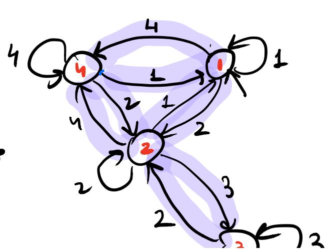




$\Sigma = V \quad Q = V$

if $\alpha = V$

A_G



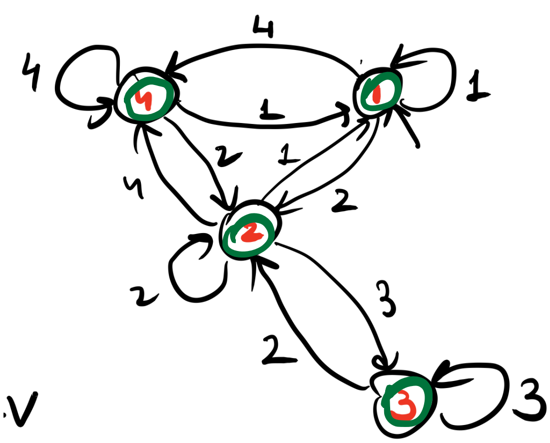
$\boxed{v_1^* \; v_2^+ \; v_3^+ \; v_4^+ \; v_5^+ \ldots}$

$v_1^* \; v_2^+ \; v_3^\omega$

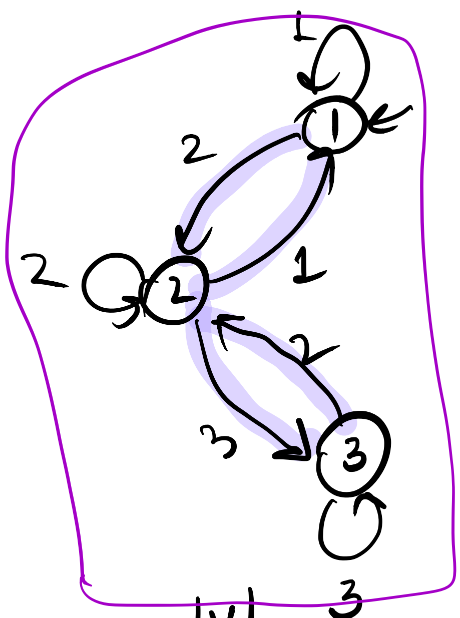
111442^{ω}

t-DBW $\quad \alpha \subseteq \Delta = Q \times \Sigma \times Q$

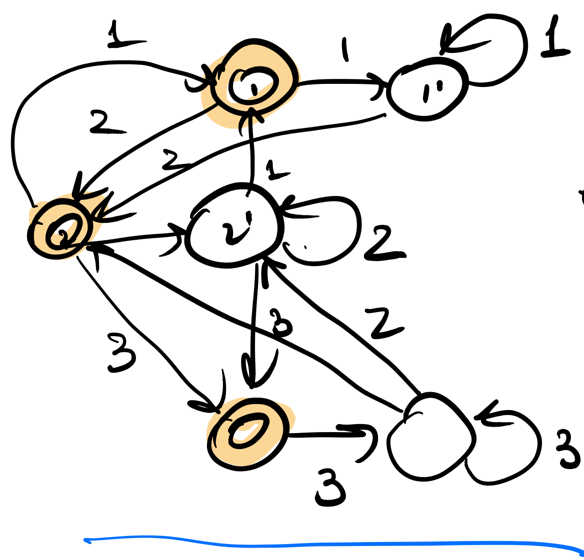
what's the size of a DBW for



V



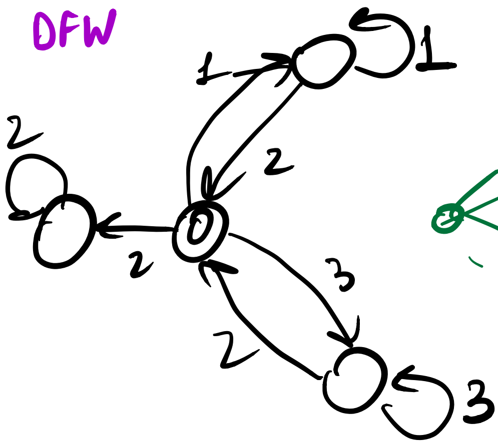
$|V|$



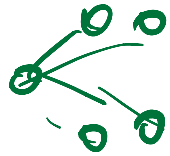
$2|V|$

enough to duplicate a vc!

DFW



$v_1^+ v_2^+ v_3^+ v_4^+ \ldots$



$(v_i\, v_{i+1}) \in E$

→ different R s.t. $L = \lim R$

G has a VC with k vertices iff D_G has an equivalent DBW with $|V|+k$ states.

minimize DBW: - find "optimal" R
= minimize R.

(1) t-DBW no NP-hadness!

(2) no 2-approximation!

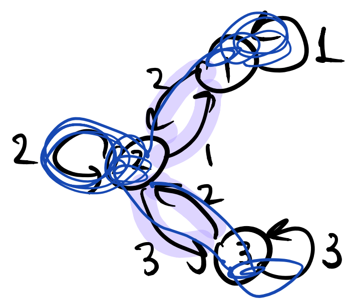
(3) co-Buchi

$L_G = V_1^+ V_2^+ V_3^+ V_4^+ \dots$

$E(V_i, V_{i+1})$

an inf path in the graph

$V_1^+ V_2^+ V_3^+ V_4^+ \dots$



$\times\ V_1 V_2^\omega$

swhewe

Lower bound on complementation determinization

[Michel 1988]

$L_1, L_2, \dots$

① L_n NBW with $O(n)$ states

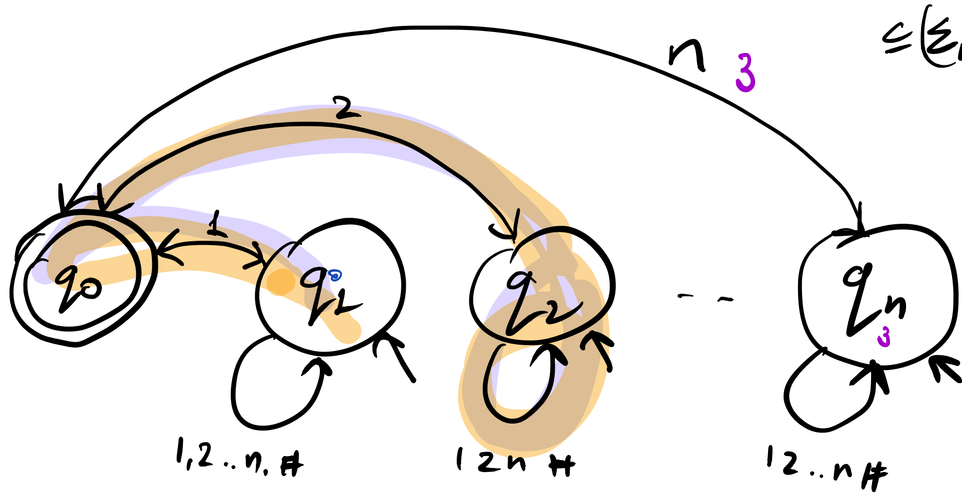
② an NBW for $\overline{L_n}$ need at least $\underbrace{n!}$ states

$n! \approx n^n = 2^{O(n \log n)}$

L_n: $\Sigma_n = \{1,2,3,\ldots n, \#\}$

$\subseteq (\Sigma_n)^\omega$

A_n



n=3 $\quad (121\#)^\omega \overset{?}{\in} L_n \qquad (12\#)^\omega$

Lemma: $w \in L_n \Leftrightarrow \exists\, i_1 i_2 \ldots i_k \in \{1..n\}$

s.t. $i_1 i_2,\ i_2 i_3,\ i_3 i_4 \ldots i_{k-1} i_k,\ i_k i_1$

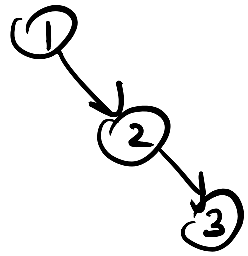
appear infinitely often in w.

Note: each word $w \in \Sigma_n^\omega$ induces

a graph $G_w = \langle \{1..n\}, E \rangle$

$E(i,j) \Leftrightarrow ij$ appears i.o. in w.

$(\underline{1\ 2\ 3}\ \#)^\omega$



A_n accepts $w \Leftrightarrow G_w$ includes a graph.

with finite word : $2^{V \times V}$

$2^{O(n^2)}$

Let U_n be a NBW for $\overline{L_n}$

We claim that U_n need atleast $n!$ states.

consider two words,

$\alpha = (i_1 i_2 \dots i_n \#)^\omega$

$\beta = (j_1 j_2 \cdots j_n \#)^\omega$

$i_1 .. i_n$ a permutation of $\{1..n\}$

$\alpha \notin L_n$ $\quad G_\alpha$: $i_1 \to i_2 \to i_3 \cdots i_n$

$\beta \notin L_n$

↳ U_n should accept α, β

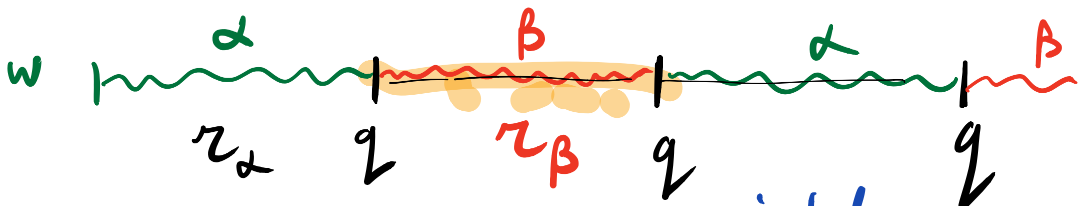
- r_α accepting run of U_n on α.
- r_β " β.

$S_\alpha = \inf(r_\alpha)$ $\qquad S_\beta = \inf(r_\beta)$

claim: $S_\alpha \cap S_\beta = \emptyset$

if correct → $n!$ states

proof of claim: assume $S_\alpha \cap S_\beta \ni q$



include $(i_1 i_2 .. i_n)\#$

include $(j_1 j_2 \; j_n)\#$

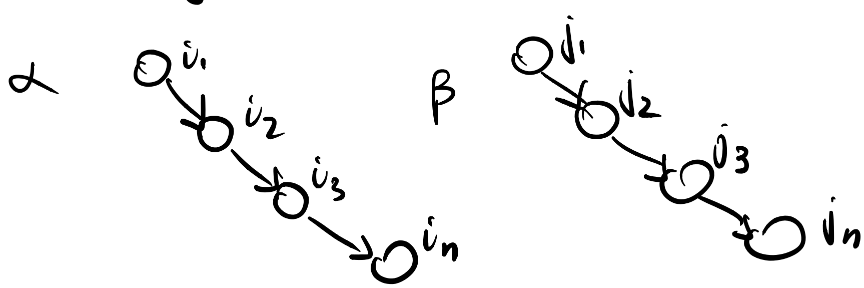
include α

accepting states of U_n

- w is accepted by U_n (i.m. visits in α)
- $w \in L_n$ (shouldn't be accepted)

↓

the graph of w contains a cycle



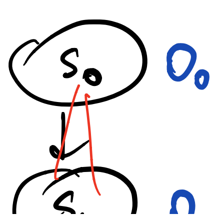
2 strings → cycle → contradiction

→ we can capture permutation.

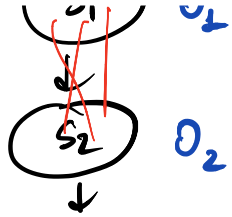
- describe subsets of $V \times V$ with $|V|+1$ states.

Löding 99

back to ss. construction



2^n, 3^n ≤ $n!$



break-point construction

NCW → DCW

$\langle S, O \rangle$ S: ss, $O \subseteq S$

Open: Is there A_1 A_2 A_3... s.t.

① A_n is an NBW with $O(n)$ states

② A_n is easy to complement: there is $\overline{A_n}$ with $O(n)$ states.

③ A_n is very hard to determinize: every $D\genfrac{}{}{0pt}{}{B}{R}\genfrac{}{}{0pt}{}{S}{P}W$ for $L(A_n)$ need at least $2^{O(n \log n)}$ state.

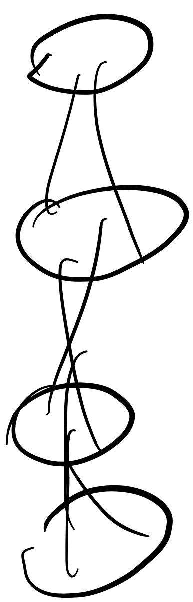
finite words:

$L_n = (0+1)^* 1 (0+1)^n$ $O(n)$ $\Sigma = \{0,1\}$

NFW $O(n)$

$\bar{L}_n = (0+1)^* 0 (0+1)^n$ $O(n)$

DFW: 2^n states



+ extra

- the same for determinization
a
complementation