Robust Deep Reinforcement Learning

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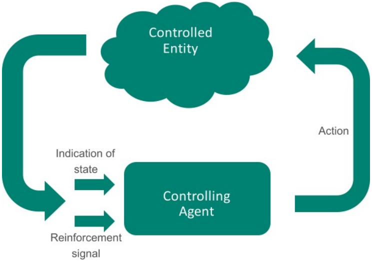
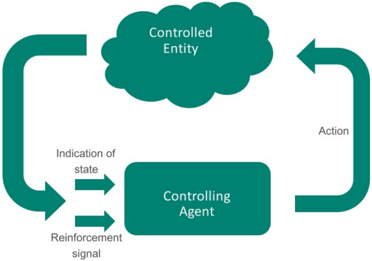
Model = Markov Decision Process (MDP) = $\{S, P, R, A\}$



Classical Reinforcement Learning

Model = Markov Decision Process (MDP) = $\{S, P, R, A\}$



States S, actions A are known and given
Transitions P and rewards R are not known.

Classical objective: $\max_\pi \mathbb{E}\left[\sum_{t=1}^{\infty} \gamma^t r_{\mathbf{t}}\right]$, $\gamma < 1$

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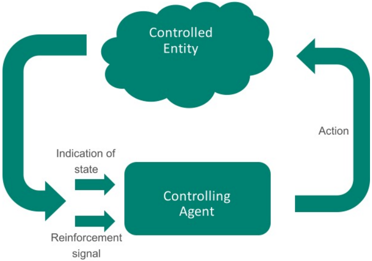

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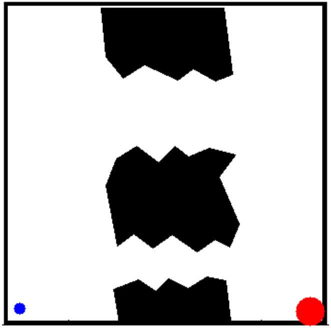
"All models are wrong, but some are useful", G. Box

Why should we be robust?



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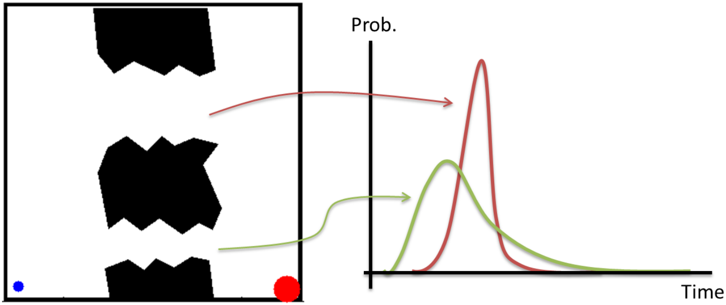
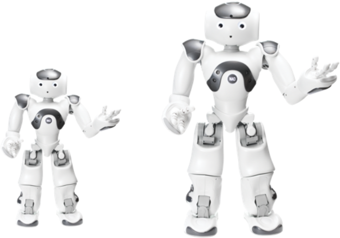


Meaning of robustness

Abrupt disturbances

Model uncertainty



Three Types of Uncertainties

1. Parmeter uncertainty

- Uncertainty in MDP *parameters* (transitions, rewards)
- Objective:

$$\max_{\pi} \min_{P \in \text{possible MDP parameters}} \mathbb{E}^{\pi,P}\left[\sum_{t=0}^{\infty} \gamma^t r(s_t)\right]$$

- Origins in robust control

Three Types of Uncertainties

2. Inherent uncertainty

- Cumulative reward is stochastic
- Expectation does not capture *variability*
- Objective:

$$\max_{\pi} \rho \left[\sum_{t=0}^{\infty} \gamma^t r(s_t) \right]$$

- ρ is a *risk measure*, e.g., $\rho(X) = \mathbb{E}[X] - \beta \mathrm{Var}[X]$
- Explicit safety against ‘unluckiness’
- Humans tend to be risk aware

Three Types of Uncertainties

3. Model uncertainty

- Model itself not known (observations/features/order)
- Objective:

$$\max_{\pi} \min_{\text{possible models}} \mathbb{E}_{model}\left[\sum_{t=0}^{\infty} \gamma^t r(s_t)\right]$$

- Model mismatch handled explicitly
- Origins in multi-model control

Applicability

When is robustness important?

- Cost of failure is high
  - Finance
  - Smart-grids
  - Health
  - Robotics (e.g., safety)
- Model is not known (always) and created from a few samples

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We desire:

- Scalability
- Adaptivity
- Accountability

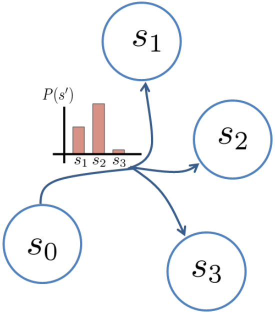
Robust MDPs with function approximation

A. Tamar, SM, and H. Xu, ICML 2014
A. Tamar, Y. Chow, M. Ghavamzadeh, and SM, NIPS 2015

Introduction - Planning with *Parameter* Uncertainty

Setting:

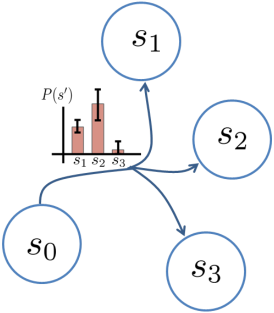
- Planning problem
- Uncertain transitions
  - Confidence intervals
  - Heuristic simulator
  - Time changing dynamics
  - etc.



Introduction - Planning with *Parameter* Uncertainty

Setting:

- Planning problem
- Uncertain transitions
  - Confidence intervals
  - Heuristic simulator
  - Time changing dynamics
  - etc.
- Potentially large impact [SM et. al, Management Science 2010]
  - Uncertainty amplification
  - Disasters / safety
  - Smart grids, finance

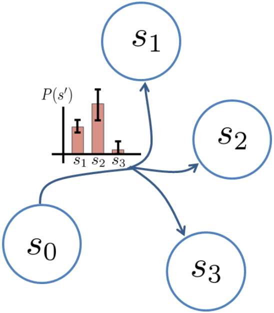


Background: Robust MDPs

Definitions:

- Robust Markov decision processes:
- State, actions and rewards as in the standard model
- Transitions $P(s'|s,a) \in \mathcal{P}$
- Policy π
- Worst-case objective

$$\sup_{\pi} \inf_{P \in \mathcal{P}} \mathbb{E}^{\pi,P}\left[\sum_{t=0}^{\infty} \gamma^t r(s_t)\right]$$



Background: Robust MDPs

Dynamic programming solution

- Robust value function (fixed policy)

$$V^\pi(s) \doteq \inf_{P \in \mathcal{P}} \mathbb{E}^{\pi,P}\left[\sum_{t=0}^{\infty} \gamma^t r(s_t) | s_0 = s\right]$$

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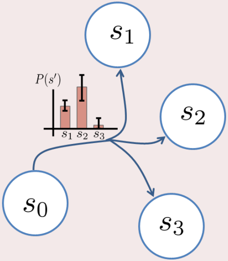

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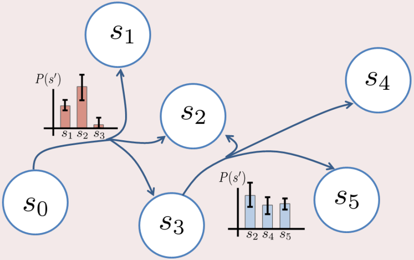

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- Robust Bellman equation (fixed policy)

$$V^{\pi}(s) = r(s) + \gamma \inf_{P \in \mathcal{P}(s)} \mathbb{E}^{P}\left[V^{\pi}(s') | s, \pi(s)\right]$$

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- Small problems: solved Policy iteration [Iyengar, 2005] and value iteration approach [Nilim et al. 2005]
- Large problems: Dynamic Programming cannot handle large spaces ("the curse of dimensionality")

Robust Policy Evaluation

Approximate value function

- Given state-dependent features $\phi(s)$
- Linear function approximation

$$\tilde{V}^{\pi}(s) = \phi(s)^{\top} w$$

- How to select w?

- For standard (non-robust) problems:

$$V^{\pi}(s) \doteq \mathbb{E}^{\pi,P}\left[\sum_{t=0}^{\infty} \gamma^{t} r(s_t) | s_0 = s\right]$$

Sample and regress w.

Robust Policy Evaluation

Approximate value function

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- For robust problems

$$V^{\pi}(s) = \inf_{P \in \mathcal{P}} \mathbb{E}^{\pi,P}\left[\sum_{t=0}^{\infty} \gamma^{t} r(s_t) | s_0 = s\right]$$

Cannot regress w: how to sample trajectories from worst-case model?

Our approach

- Recall the Bellman equation

$$V^\pi(s) = r(s) + \gamma \inf_{P \in \mathcal{P}(s)} \mathbb{E}^P \left[V^\pi(s') | s, \pi(s) \right]$$

- Idea: **bootstrap!**

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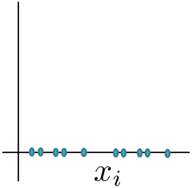
Algorithm

Given: initial weights w_0, sample states $x_1 \ldots x_N$



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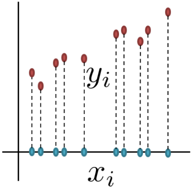
Algorithm

Given: initial weights w_0, sample states $x_1 \ldots x_N$

- At iterate $k+1$ generate regression targets

$$y_i = r(x_i) + \gamma \inf_{P \in \mathcal{P}(x_i)} \sum_{x'} P(x'|x_i, \pi(x_i)) \underbrace{\phi(x')^\top w_k}_{\tilde{V}_k^\pi(x')}$$



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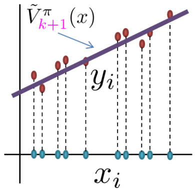
- Solve for w_{k+1} using least squares



Results:

Guarantees

- The magic: Convergence + Error bounds

Policy improvement

- Can iterate between policy evaluation and policy improvement
- Can derive deep Q-learning (model free) simulation based algorithm
- Error bounds follow through

Part 2

Two issues remain:

1. Uncertainty set construction
2. Online adaptivity

C. Tessler, Y. Efroni, and SM, ICML 2019

E. Derman, D. Mankowitz, T. Mann, and SM, UAI 2019.

Action Robustness

A trembling hand model

$$\pi_{\alpha}^{mix}(\pi, \pi') = \begin{cases} \pi, & \text{w.p. } 1 - \alpha. \\ \pi', & \text{w.p. } \alpha. \end{cases}$$

The policy π' is potentially adversarial.
Continuous extension: agent chooses a, adversary can modify to $(1 - \alpha)a + \alpha a'$.

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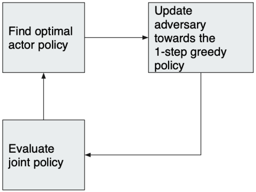
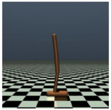
AR-DDPG:

1. Train Actor
2. Train Adversary
3. Train Critic for the joint policy

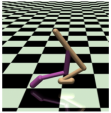


Theorem: This procedure converges to the Nash equilibrium.

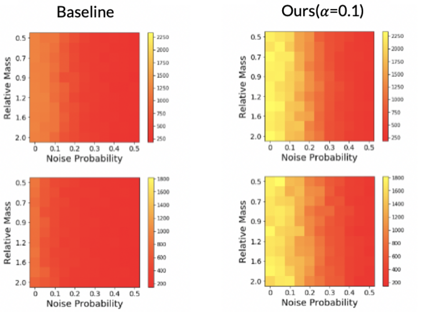
Some results



Hopper



Walker2d



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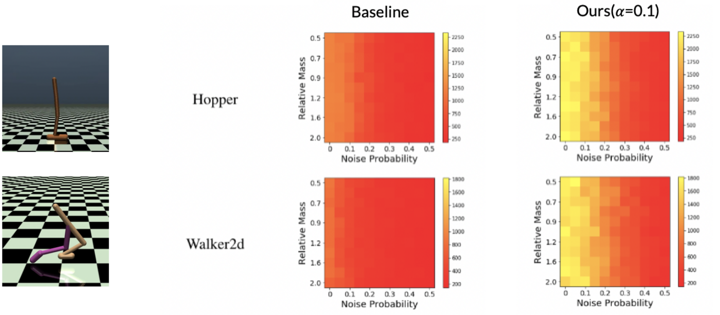


- Robustness: uncertainty + transfer to unseen domains
- A gradient based approach for robust reinforcement learning with convergence guarantees
- Does not require explicit definition of the uncertainty set

Posterior Uncertainty Sets: Online Construction of Uncertainty Sets

- Dirichlet prior on distribution over next states.
- Observation history $\mathcal{H}$ up to time h
- Time h - current step and t - current episode

$$\widehat{\mathcal{P}}^h_{sa}(\psi_{sa}) = \{p_{sa} \in \Delta_{\mathcal{S}} : \|p_{sa} - \bar{p}_{sa}\|_1 \leq \psi_{sa}\}$$

$\bar{p}_{sa} = \mathbb{E}[p_{sa} \mid \mathcal{H}]$ is the *nominal* transition.

This uncertainty set is

- Rectangular:

$$\widehat{\mathcal{P}}^h = \bigotimes_{s \in \mathcal{S}, a \in \mathcal{A}} \widehat{\mathcal{P}}^h_{s,a}$$

- Updated **online** according to new observations

Uncertainty Robust Bellman Equation

- Posterior robust Q-value random variables satisfy a **robust Bellman recursion**

$$\hat{Q}^h_{sa} \overset{D}{=} r^h_{sa} + \gamma \inf_{p \in \widehat{\mathcal{P}}^h_{sa}} \sum_{s',a'} \pi^h_{s'a'} p_{sas'} \widehat{Q}^{h+1}_{s'a'}$$

- Posterior worst-case transition:
$\widehat{p}^h_{sa} \in \arg\min_{p \in \widehat{\mathcal{P}}^h_{sa}} \sum_{s',a'} \pi^h_{s'a'} p_{sas'} \widehat{Q}^{h+1}_{s'a'}$

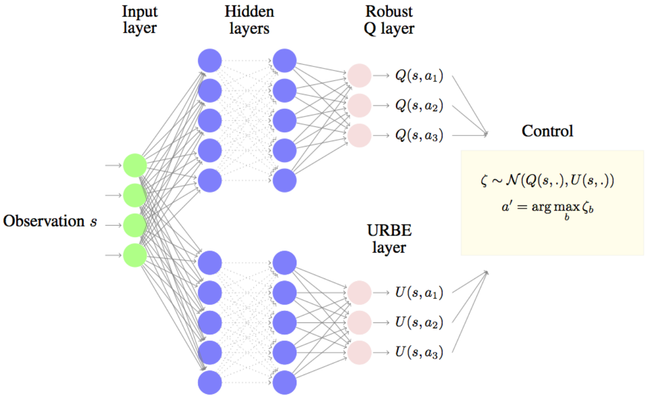
Theorem (Solution of URBE)

There exists a unique mapping w that satisfies the URBE:

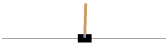
$$w^h_{sa} = \nu^h_{sa} + \gamma^2 \sum_{s' \in \mathcal{S}, a' \in \mathcal{A}} \pi^h_{s'a'} \mathbb{E}_t(\widehat{p}^h_{sas'}) w^{h+1}_{s'a'}$$

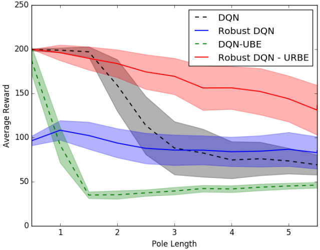
- Approximate Q-values as $\mathcal{N}(Q, diag(w))$.

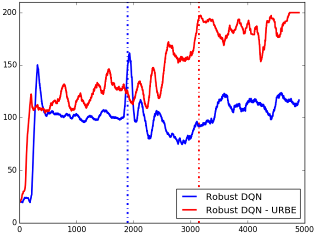
Deep Learning Approximation



Q-head uses robust TD error. URBE layer uses approximation.







- DQN/DQN-UBE: Overly **sensitive** to change of dynamics
- Robust DQN: Overly **conservative**

Conclusion

Robustness is essential for learning

- Handles ‘unluckiness’
- Overcomes model misspecification
- Works online with deep models (scalability)

Take home message: solve robust MDPs

Scalable, works, and even has theoretical guarantees!

Applications: health, energy, finance, robotics, cyber, e-commerce

Thanks

Joint work with:

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