Part 4: Policy Gradient Reinforcement Learning

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COGNITION & CONTROL IN COMPLEX SYSTEMS

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Part 4: (Quasi) Policy Gradient Reinforcement Learning

Outline

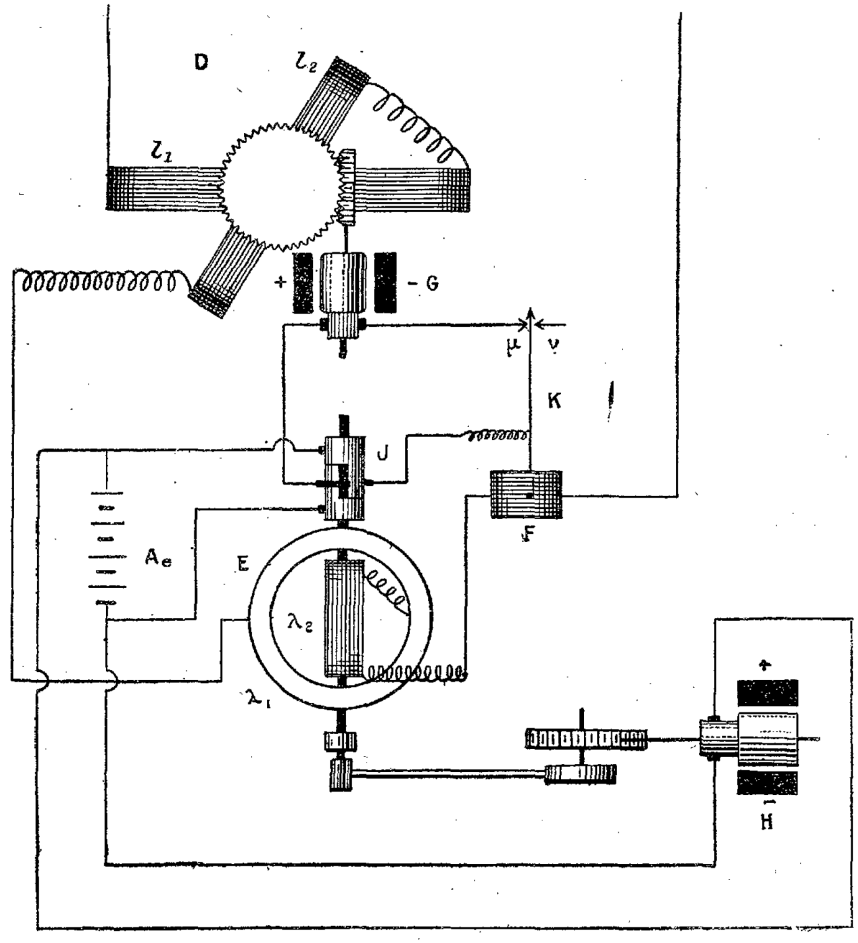
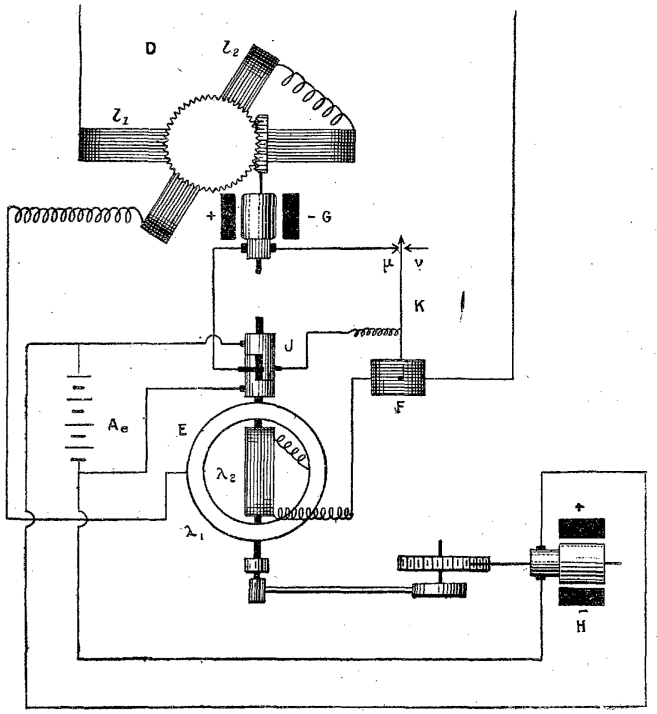


Fig. 5.

Policy Gradient Methods in Control

Extremum seeking from 1922 to 2010 [86]

In his 1922 paper, or invention disclosure, Leblanc [88] describes a mechanism to transfer power from an overhead electrical transmission line to a tram car using an ingenious non-contact solution. In order to maintain an efficient power transfer in what is essentially a linear, air-core, transformer/ capacitor arrangement with variable inductance, due to the changing air-gap, he identifies the need to adjust a (tram based) inductance (the input) so as to maintain a resonant circuit, or maximum power (the output). Leblanc explains a control mechanism of how to maintain the desirable maximum power transfer using what is essentially an extremum seeking solution.



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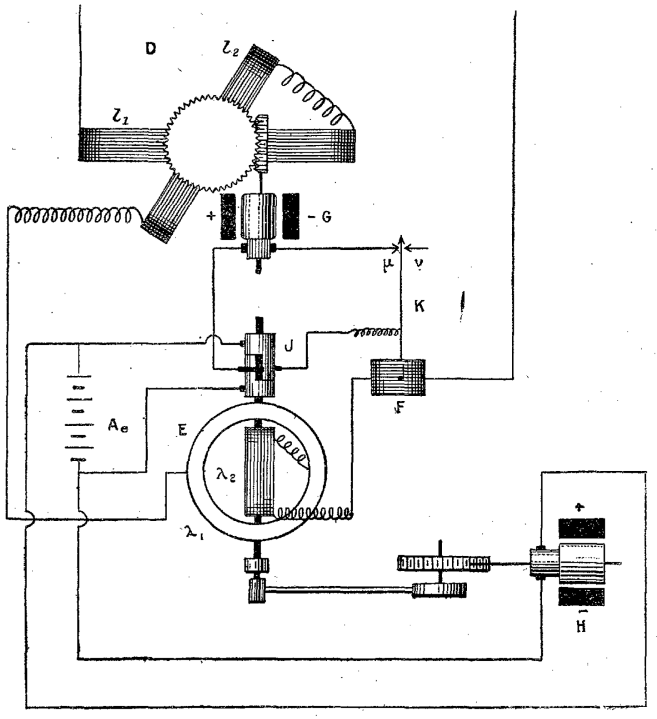

Fig. 5.

Iven, are you sure? Response on Sunday: [that is at the heart of] *what ESC is, non model based gradient descent...and that is what this circuit does, non model based, gradient descent by using forces that are proportional to finding the maximal energy transfer...*

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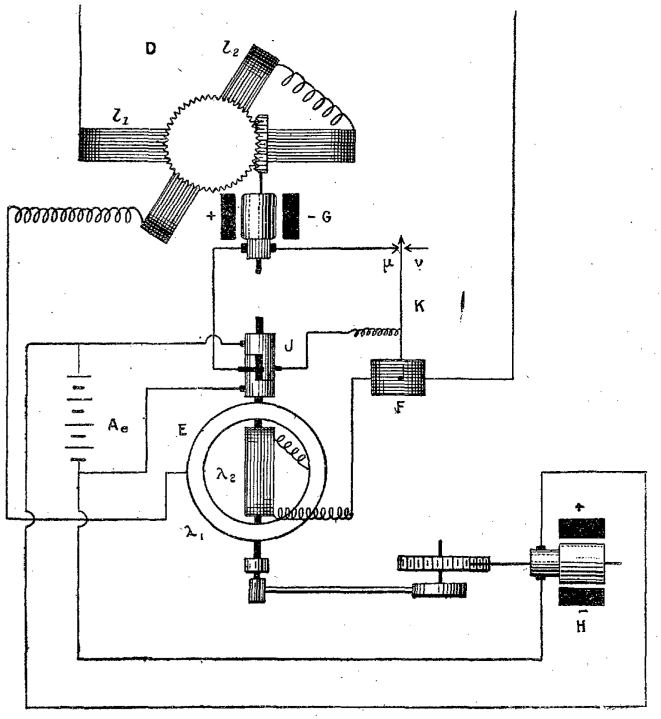

Fig. 5.

More history in 1962 and the 1980s: [87, 89, 88], and enormous activity from then until today.

Thanks to Florence Forbes and Iven Mareels for French paper discovery, translations, and history lessons

Self Tuning

Theory and applications of adaptive control—a survey Åström, 1983 [5]

The following parameter adjustment mechanism, called the 'MIT-rule', was used in the original MRAS

$$\frac{d\theta}{dt} = -k e \operatorname{grad}_\theta e. \quad (1)$$

In this equation e denotes the model error. The components of the vector θ are the adjustable regulator parameters. The

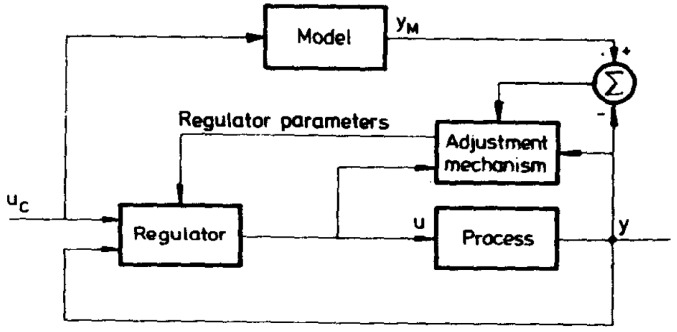

FIG. 2. Block diagram of model reference adaptive system (MRAS).

Karl Åström has been an inspiration from the start—for early history, see [5, 6, 3].

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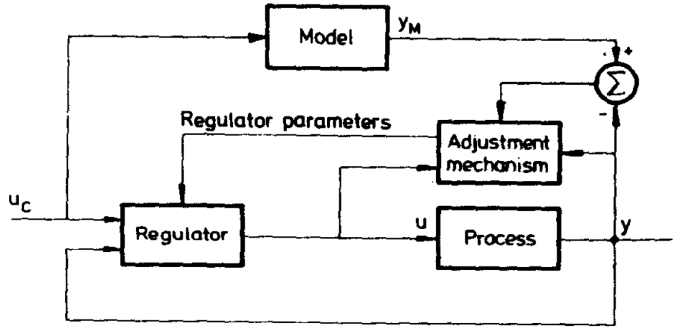

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Adaptive Control Up To 1960 Åström, 1996 [6]

See also

Literature geared towards Lyapunov techniques:

- Liberzon's lecture notes: `http://liberzon.csl.illinois.edu/teaching/16ece517notes.pdf`
- Kokotovic et al [4]

Conclusions

Adaptive control was in a very interesting development in the mid-1960s. Many ideas such as extremal control, MRAS, STR, dual control, and neural networks, were born. It would take about two decades before the problems associated with adaptive control were reasonably well understood and adaptive techniques were finding use in industry. There are many reasons for the delay. The problems to be solved were difficult, funding for flight control dropped sharply because of accidents in flight tests, and new hardware was required for efficient implementation. A

The vast majority of Reinforcement Learning (RL) [9] and Neuro-Dynamic Programming (NDP) [1] methods fall into one of the following two categories:

(a) Actor-only methods work with a parameterized family of policies. The gradient of the performance, with respect to the actor parameters, is directly estimated by simulation, and the parameters are updated in a direction of improvement [4, 5, 8, 13]. A possible drawback of such methods is that the gradient estimators may have a large variance. Furthermore, as the policy changes, a new gradient is estimated independently of past estimates. Hence, there is no "learning," in the sense of accumulation and consolidation of older information.

(b) Critic-only methods rely exclusively on value function approximation and aim at learning an approximate solution to the Bellman equation, which will then hopefully prescribe a near-optimal policy. Such methods are indirect in the sense that they do not try to optimize directly over a policy space. A method of this type may succeed in constructing a "good" approximation of the value function, yet lack reliable guarantees in terms of near-optimality of the resulting policy.

Actor-critic methods aim at combining the strong points of actor-only and critic-only methods. The critic uses an approximation architecture and simulation to learn a value function, which is then used to update the actor's policy parameters

circa 2000

$$\nabla L(\theta) = \mathsf{E}\left[\nabla c_\theta(X_k) + S_\theta(X_k, X_{k+1}) h_\theta(X_{k+1})\right]$$

Policy Gradient Methods in RL

1968 Origins [32, 33, 34] $L(\theta) = \lim_{k\to\infty} \mathsf{E}[c_\theta(X_k)]$

Markov model: $\{P_\theta, c_\theta : \theta \in \mathbb{R}^d\}$ Goal: minimize $L(\theta)$, the average cost

Early Actor Only category: Williams 1992, REINFORCE [35]
Recent revival of AO, following Mania et al, 2018 [43]

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Approach: gradient descent $\oplus$ our beloved Poisson equation [56, 57, 27]

$$c_\theta(x) + \sum_{x'} P_\theta(x, x')h_\theta(x') = h_\theta(x) + L(\theta)$$

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Details: Take the gradient of each side:

$$\nabla c_\theta(x) + \sum_{x'} \Big\{ [\nabla P_\theta(x, x')] h_\theta(x') + P_\theta(x, x') \nabla h_\theta\,(x') \Big\} = \nabla h_\theta(x) + \nabla L(\theta)$$

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Actor Critic Methods

$$L(\theta) = \lim_{k\to\infty} \mathsf{E}[c_\theta(X_k)]$$

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Follow design principle:

$$\text{ODE}: \quad \frac{d}{dt}\vartheta_t = -\nabla L(\vartheta_t)$$

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For MDP, with randomized policy $\widetilde{\phi}^\theta$, S_θ is identified ...

$$\mathsf{E}\big[\nabla c_\theta(X_n) + S_\theta(X_n, X_{n+1}) h_\theta(X_{n+1})\big] = \mathsf{E}\big[Q_\theta(X_n, U_n) \nabla \log \widetilde{\phi}^\theta(U_n \mid X_n)\big]$$

Score function

https://en.wikipedia.org/wiki/Score_(statistics)

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... *however* ... Q?

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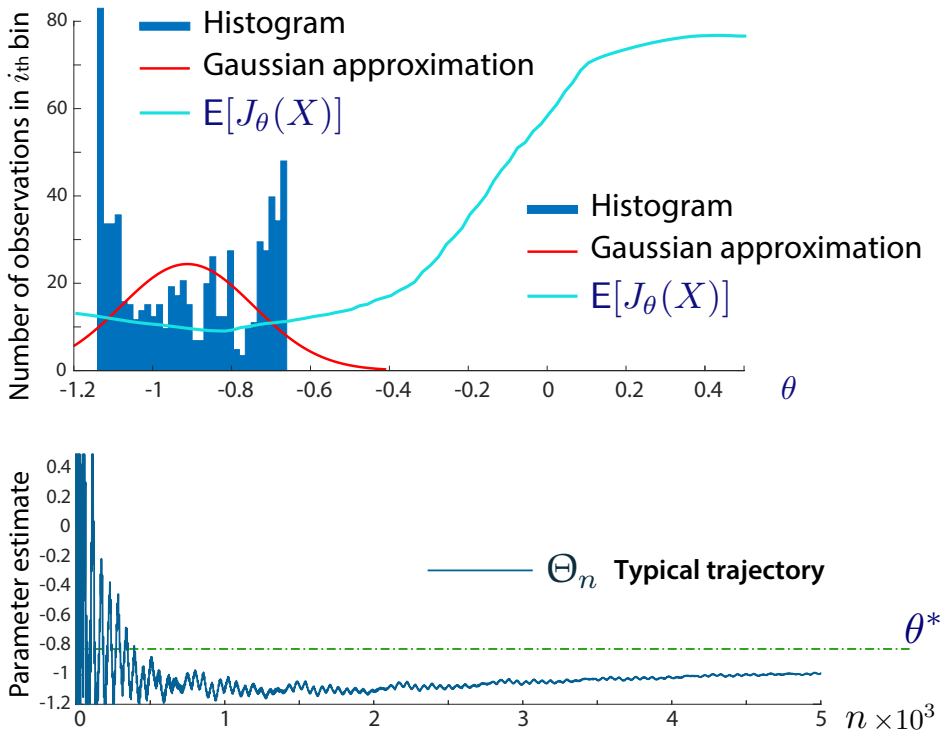
We need a critic! Key: only need approximation $\widehat{Q}^\theta$ to satisfy

$$\mathsf{E}\big[Q_\theta(X_n, U_n)\nabla \log \widetilde{\phi}^{\theta}(U_n \mid X_n)\big] = \mathsf{E}\big[\widehat{Q}^{\theta}(X_n, U_n)\nabla \log \widetilde{\phi}^{\theta}(U_n \mid X_n)\big]$$

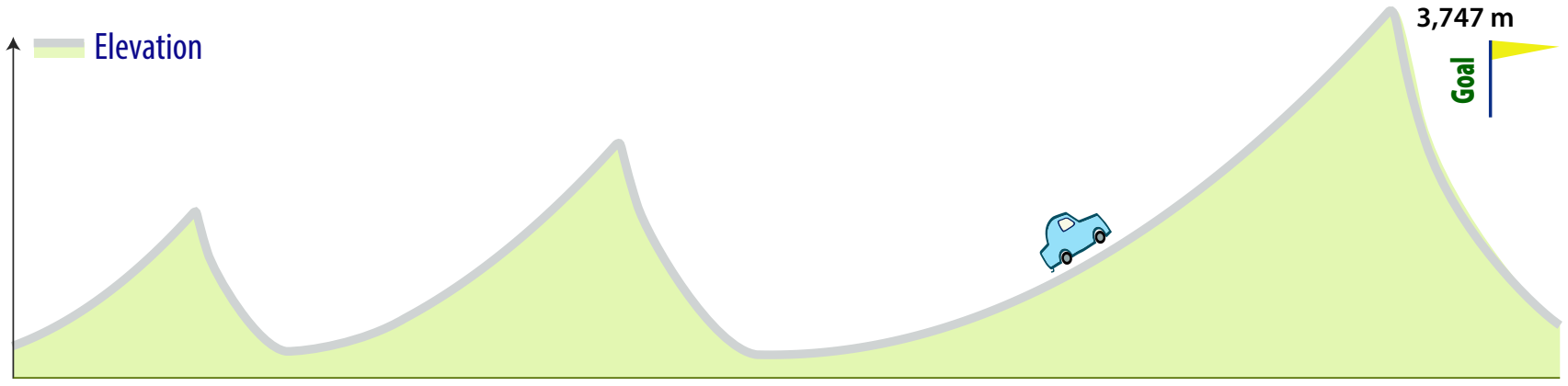
$\Longrightarrow$ Compatible features



qSGD and Policy Gradient RL

MountainCar

Rich's car has a very weak engine



The only way to reach the goal is to go backwards

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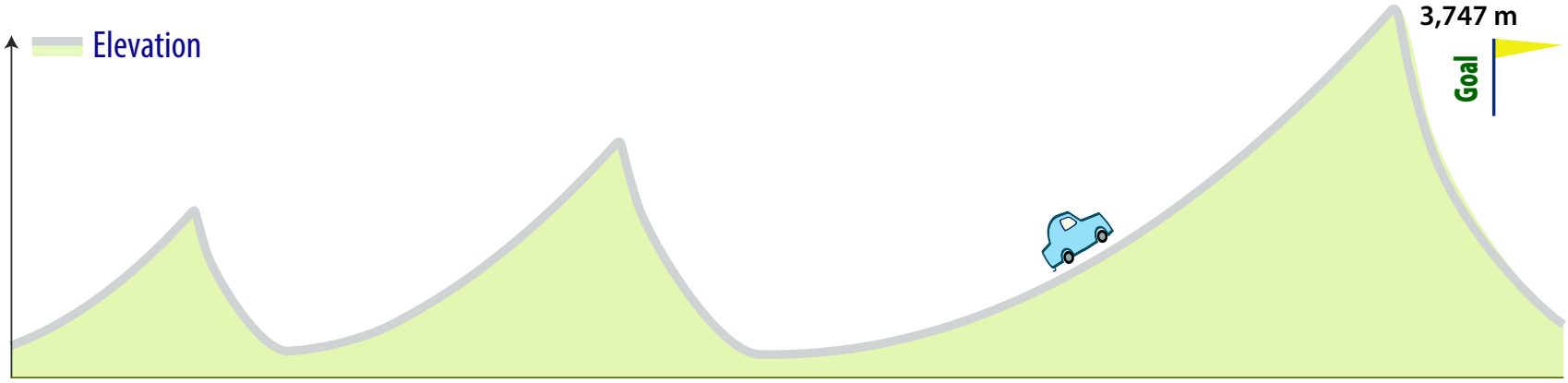

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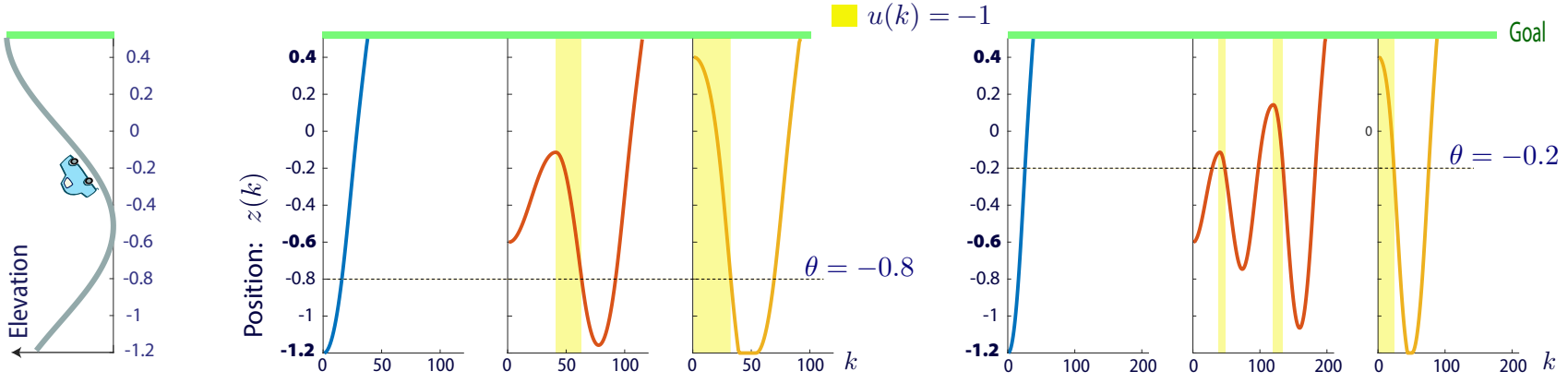
Example policies (focusing on a single valley):



MountainCar

Goal: minimize travel time to destination $J(x)$

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Approach: create a parameterized family of policies: $U_k = \phi^\theta(X_k)$

Minimize average loss: $$L(\theta) = \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} \min\{J^{\max}, J_\theta(X_0^k)\}$$

$\{X_0^k : k \geq 1\}$ are initial conditions, most likely created by choice

$J_\theta(x)$ is the cost (perhaps average cost) from initial condition x

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Let's try QSA:

$$\frac{d}{dt}\bar{\Theta}_t = a_t\{-\nabla L(\Theta_t)\} \quad \Longleftarrow \textit{Design for your goals}$$

$$\frac{d}{dt}\Theta_t = a_t f(\Theta_t, \xi_t) \quad \Longleftarrow \textit{qSGD approximation}$$

$$\theta_{n+1} = \theta_n + a_{n+1} f(\theta_n, \xi_{n+1}) \quad \Longleftarrow \textit{Euler/Runge-Kutta}$$

MountainCar

State space continuous, $x = (z, v)$ position and velocity, input zero or one

Model: $$Z_{k+1} = Z_k + V_k + D_k^z$$
$$V_{k+1} = V_k + \gamma U_k + D_k^v$$

D means disturbance

MountainCar

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Policy: $$U_k = \begin{cases} 1 & \text{if } Z_k + V_k \leq \theta \\ \text{sign}(V_k) & \text{else} \end{cases}$$

The policy ϕ^θ "panics" (accelerates the car towards the goal) whenever the estimate of Z_{k+1} is at or below the threshold θ

MountainCar

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Goal: Find best policy in this class

MountainCar

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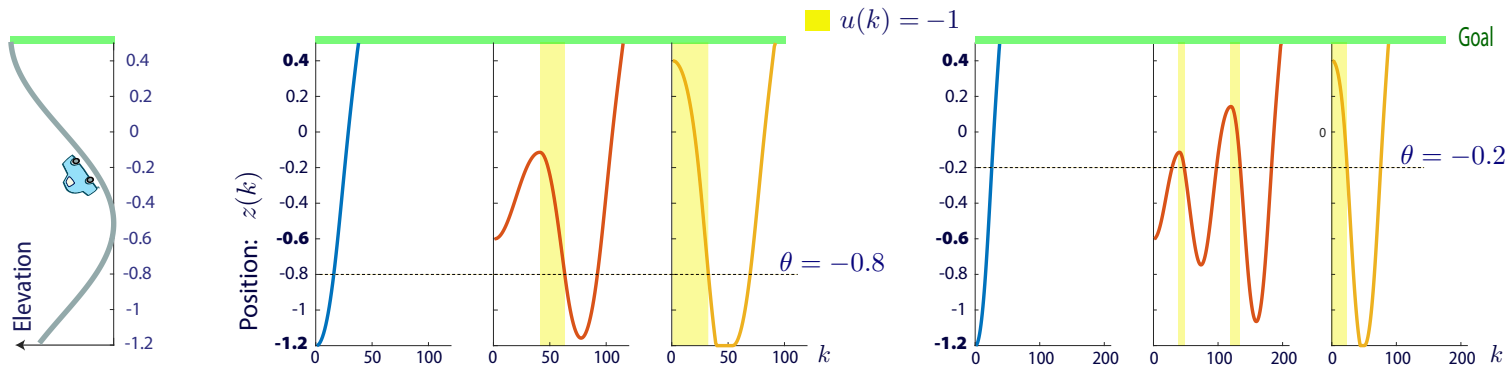

MountainCar

qSGD: $\frac{d}{dt}\Theta_t = -a_t \frac{1}{2\varepsilon} G \xi_t \big\{ L(\Theta_t + \varepsilon\xi_t) - L(\Theta_t - \varepsilon\xi_t) \big\}$

In discrete time:

$$\text{(1)} \qquad \theta_{n+1} = \theta_n + \alpha_{n+1} \big[-\tfrac{1}{2\varepsilon} \xi_n \big\{ L(\theta_n + \varepsilon\xi_n) - L(\theta_n - \varepsilon\xi_n) \big\} \big]$$

$$\text{(1a)} \qquad \theta_{n+1} = \theta_n - \alpha_{n+1} \tfrac{1}{\varepsilon} \xi_n L(\theta_n + \varepsilon\xi_n)$$

recall danger with this one ☠

MountainCar

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(1a) $\theta_{n+1} = \theta_n - \alpha_{n+1} \frac{1}{\varepsilon} \xi_n L(\theta_n + \varepsilon\xi_n)$ *recall danger with this one* ☠

Slight extension:

$$L(\theta_n + \varepsilon\xi_n) = J_\theta(x)\,, \quad \theta = \theta_n + \varepsilon\xi_n^1\,, \quad x = x^0 + \varepsilon\xi_n^2$$

MountainCar

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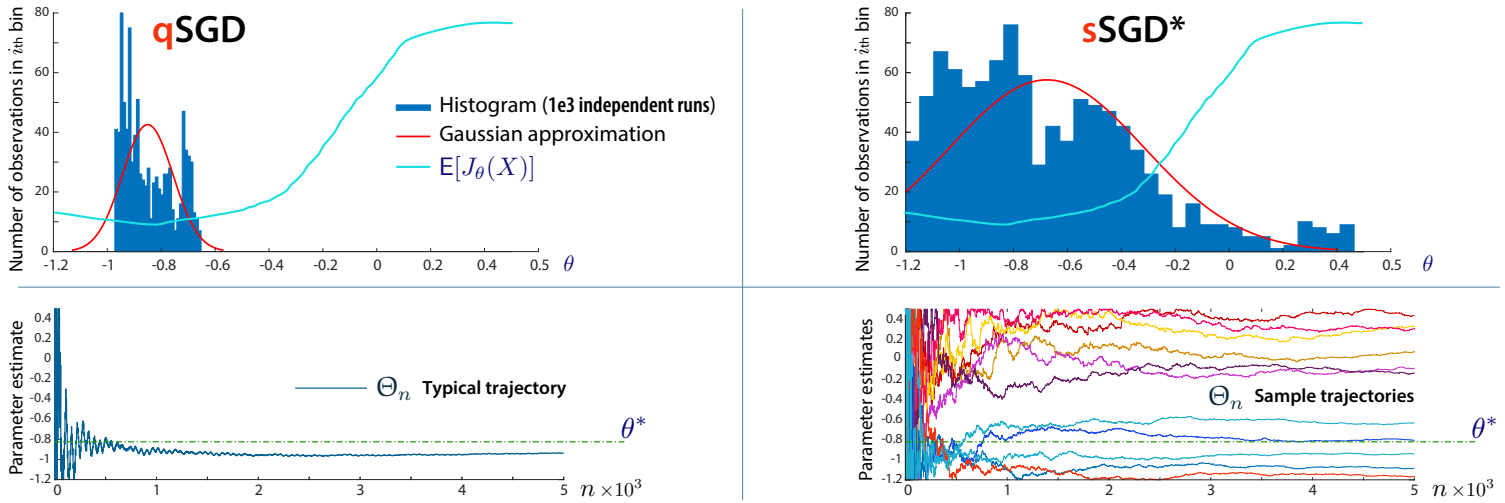

qSGD (1a) and stochastic counterpart for Mountain Car ☠

MountainCar

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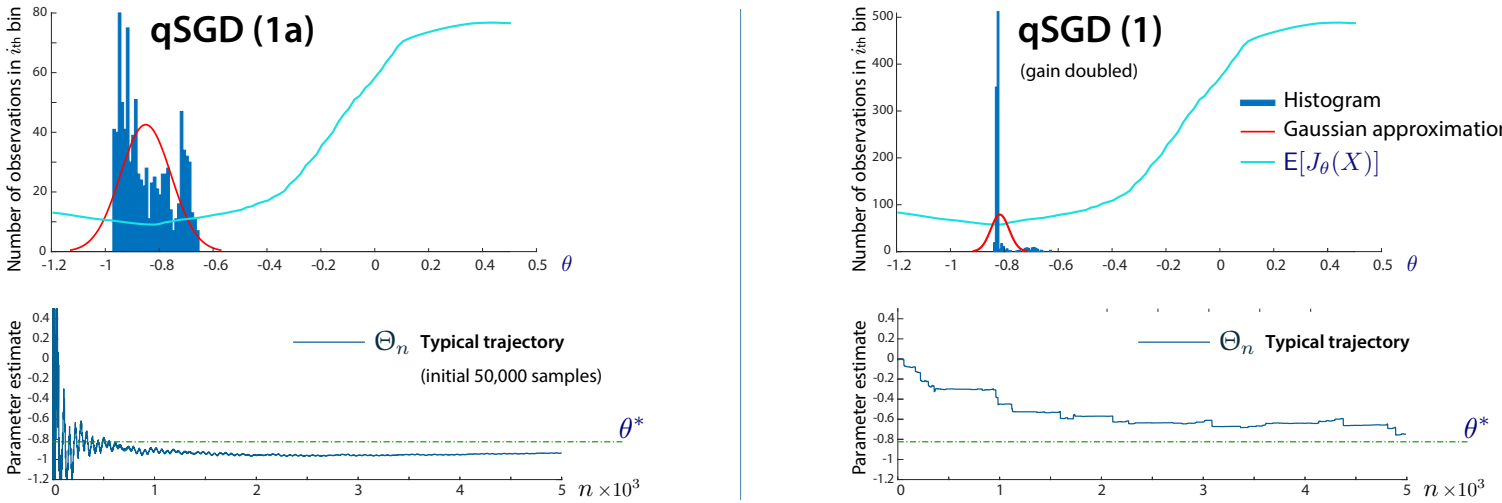

Comparison of two qSGD algorithms

Conclusions

Probability Theory, Control, Reinforcement Learning: A Happy Marriage

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Easier in part because of zero covariance, $\Sigma_\xi = 0$

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- Testing qSGD in more volatile environments
- Tackling dimension, $\theta \in \mathbb{R}^{10^6}$
- Other forms of acceleration (revisit classical and recent [54])

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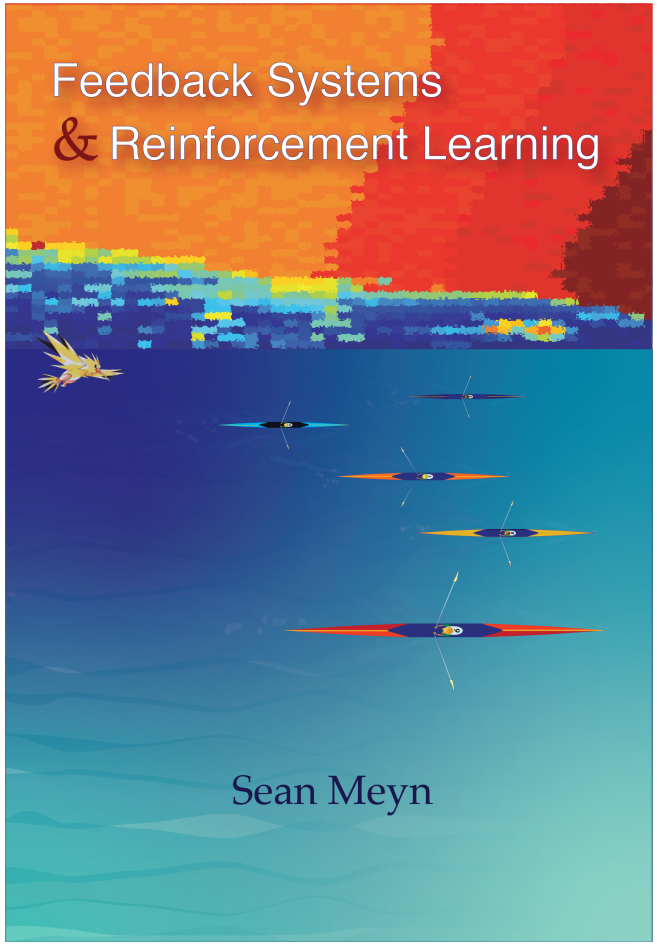
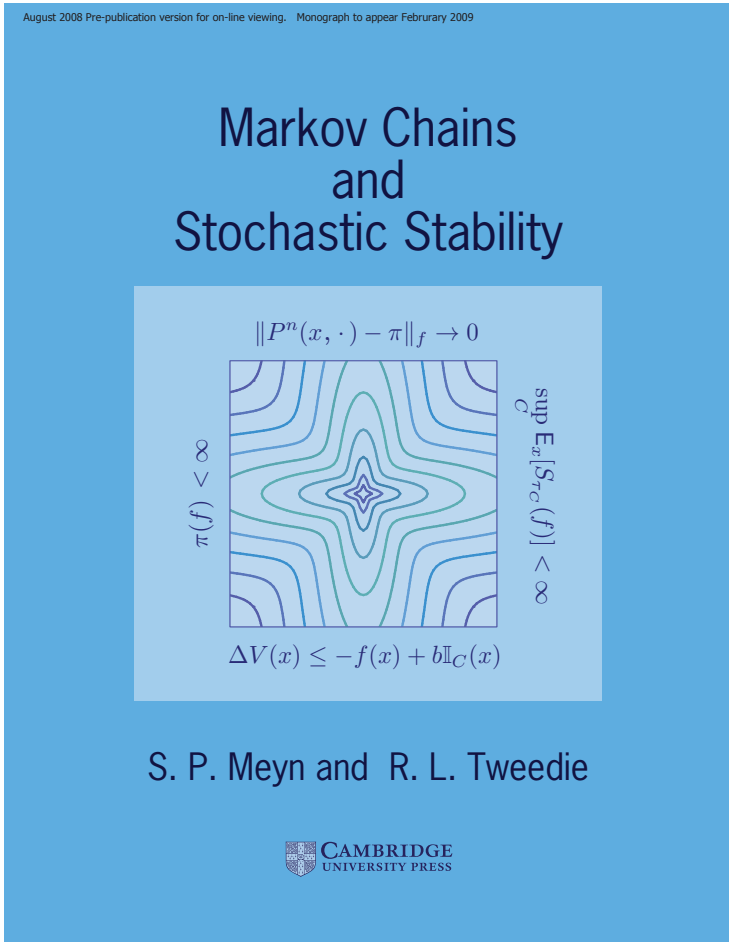
- Optimizing the wind farms in Colorado

On to you, Andrey!


Feedback Systems
& Reinforcement Learning
Sean Meyn


August 2008 Pre-publication version for on-line viewing. Monograph to appear Februrary 2009
Markov Chains
and
Stochastic Stability
$\|P^n(x,\cdot)-\pi\|_f \to 0$
$\pi(f) < \infty$
$\sup_C \mathsf{E}_x[S_{\tau_C}(f)] < \infty$
$\Delta V(x) \le -f(x) + b\mathbb{I}_C(x)$
S. P. Meyn and R. L. Tweedie
CAMBRIDGE
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