Simons Institute for the Theory of Computing

Theory of Reinforcement Learning
Aug. 19 – Dec. 18, 2020

Reinforcement Learning
Part 1: Control Systems & RL

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Cognition & Control in Complex Systems

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Thanks to to our sponsors: NSF and ARO

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Raman Surana
Ruppert Polyak
Benveniste Metivier
Kushner Yin
Van Roy Tsitsiklis
Konda Bertsekas
Szepesvari Nedic Yu

Many Others

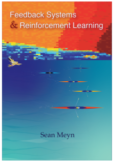
Part 1: Control Fundamentals

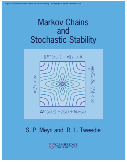
Outline

Resources

Videos from Simons RTDM, 2018

- Feedback Control Theory: Architectures and Tools for Real-Time Decision Making [essential prerequisite]
  https://simons.berkeley.edu/talks/murray-control-1
- Hidden Theory Part I (SA foundations)
  https://www.youtube.com/watch?v=dhEF5pfYmvc
- Hidden Theory Part II (Zap Q-learning)
  https://www.youtube.com/watch?v=Y3w8f1xIb6s





Lecture notes online: Feedback systems and reinforcement learning
simons.berkeley.edu/sites/default/files/docs/16101/monographrlsimonsinstitutebootcampseptember2020.pdf

[1] K. J. Astrom and R. M. Murray. *Feedback Systems: An Introduction for Scientists and Engineers*. Princeton University Press, USA, 2008 (recent edition on-line).

[25] D. Huang, W. Chen, P. Mehta, S. Meyn, and A. Surana. Feature selection for neuro-dynamic programming. In F. Lewis, editor, *Reinforcement Learning and Approximate Dynamic Programming for Feedback Control*. Wiley, 2011.

[26] A. M. Devraj, A. Bušić, and S. Meyn. Fundamental design principles for reinforcement learning algorithms. In *Handbook on Reinforcement Learning and Control*. Springer, 2020

Apologies

- π will always be an invariant measure 

  ϕ and $\widetilde{\phi}$ will be policies (feedback)

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  If I seem critical, it is simply my opinion, and
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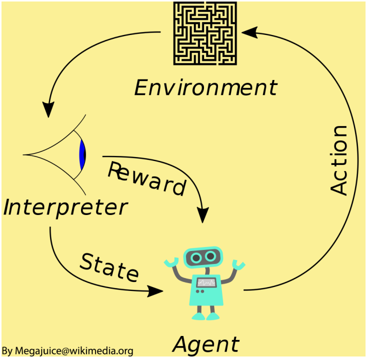
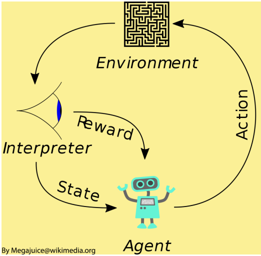

Environment
Interpreter
Reward
State
Action
Agent
By Megajuice@wikimedia.org

Background

Reinforcement Learning

Intelligent actors optimize through interactions with their environment



Reinforcement learning (RL) is an area of machine learning concerned with how software agents ought to take actions in an environment so as to maximize some notion of cumulative reward –Wikipedia

Comparison of reinforcement learning algorithms [edit]

Algorithm	Description	Model	Policy	Action Space	State Space	Operator
Monte Carlo	Every visit to Monte Carlo	Model-Free	Off-policy	Discrete	Discrete	Sample-means
Q-learning	State–action–reward–state	Model-Free	Off-policy	Discrete	Discrete	Q-value
SARSA	State–action–reward–state–action	Model-Free	On-policy	Discrete	Discrete	Q-value
Q-learning - Lambda	State–action–reward–state with eligibility traces	Model-Free	Off-policy	Discrete	Discrete	Q-value
SARSA - Lambda	State–action–reward–state–action with eligibility traces	Model-Free	On-policy	Discrete	Discrete	Q-value
DQN	Deep Q Network	Model-Free	Off-policy	Discrete	Continuous	Q-value
DDPG	Deep Deterministic Policy Gradient	Model-Free	Off-policy	Continuous	Continuous	Q-value
A3C	Asynchronous Advantage Actor-Critic Algorithm	Model-Free	On-policy	Continuous	Continuous	Advantage
NAF	Q-Learning with Normalized Advantage Functions	Model-Free	Off-policy	Continuous	Continuous	Advantage
TRPO	Trust Region Policy Optimization	Model-Free	On-policy	Continuous	Continuous	Advantage
PPO	Proximal Policy Optimization	Model-Free	On-policy	Continuous	Continuous	Advantage
TD3	Twin Delayed Deep Deterministic Policy Gradient	Model-Free	Off-policy	Continuous	Continuous	Q-value
SAC	Soft Actor-Critic	Model-Free	Off-policy	Continuous	Continuous	Advantage

Reinforcement Learning

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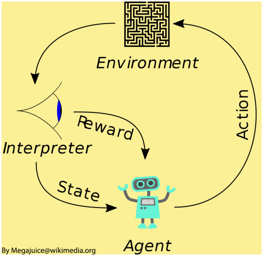

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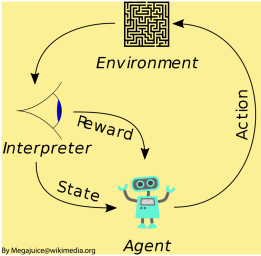
- Stock trading
- Autonomous cars
- Smart Buildings and Smart Grids

Reinforcement Learning

Intelligent actors optimize through interactions with their environment?



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Reinforcement Learning

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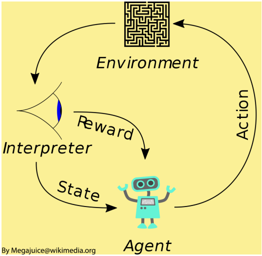

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Learn as we
Trade stocks?
Drive cars?
Manage the grid?

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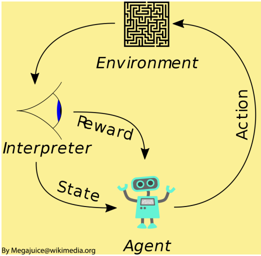

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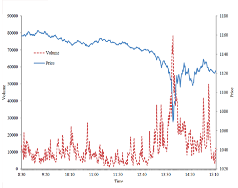

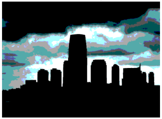

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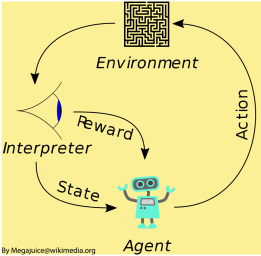

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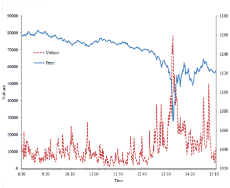

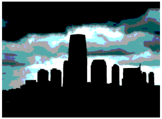

RL is an emerging science, evolving alongside decision and control theory

Reinforcement Learning

Dreams of Model Free Control *well before my graduate student days*

"Typical" Adaptive Control System: MIT Rule

(NASA Report by Staff engineers at Edwards AFB, Nov, 1970)

Early Dreams
of
Model Free Control

From a tutorial by Erik Ydstie, CMU

Conclusions after 65 flight tests:

- Nearly invariant response at essentially all conditions
- accurate a priori knowledge of aerodynamic characteristics not needed (model-free)
- aircraft configuration changes compensated for
- redundancy (dual) provided a reliable and fail safe system

Pilot Observations

The true superiority of the X-15 AFCS was that it unburdened the pilot. The airplane was stable at any dynamic pressure and at any angle of attack. The AFCS inspired confidence and allowed the pilot to spend time cross-checking flight instruments, checking subsystems, and "sightseeing."

RL is an emerging science, evolving alongside decision and control theory

Reinforcement Learning

Dreams of Model Free Control *well before my graduate student days*

The flight-test program also disclosed several disadvantages associated with the system, including the following: (1) Commands by the pilot and other spurious inputs caused gain reduction and degraded performance at undesirable times; and (2) supercritical gain operation existed in flight, which, because of mechanical nonlinearities and electrical saturation, resulted in divergent airplane motions.

Early Dreams
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Problems: Gain changes due to disturbance inputs
Parameter drift and bursting
Lack of robustness in the presence of constraints (wind-up)

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https://www.nasa.gov/centers/armstrong/news/FactSheets/FS-052-DFRC.html

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What Control Can Offer

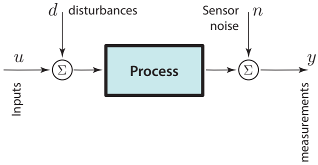
Control theory: goals and architectures

Murray tutorial, 2018

Process: car, plane, pancreas
bike sharing
wall street, semi-conductor manufacturing
power grid, transportation network

Inputs: throttle, wheel position, insulin rate,
truck dispatch, commands to generators and batteries

Measurements: speed and position, insulin, glucose, blood pressure,
camera and driver reports, frequency, phase, voltage



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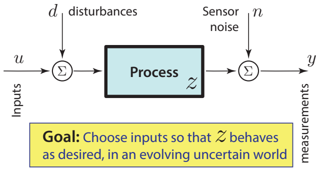


Control System Specifications

Simons Institute, 24 Jan 2018

Richard M. Murray, Caltech CDS

Transient: initial response to input

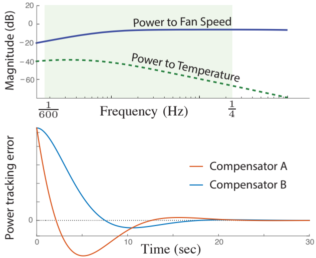
- Step response: rise time, overshoot, settling time, etc

Steady state: response after the transients have died out

- Frequency response: magnitude and phase for sinusoids

Safety: constraints that the system should never violate

Liveness: conditions that system should satisfy repeatedly



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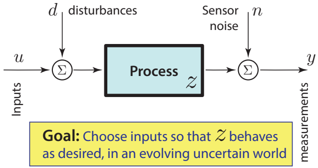
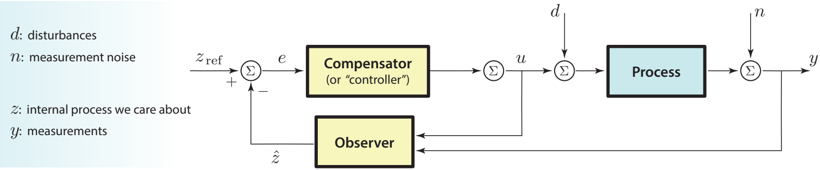


Strategies: Open loop control, assuming perfect model $z = G_{zu}u$
Invert dynamics:

$$u = G_{zu}^{-1} z_{\text{des}}$$

Control theory: goals and architectures

Murray tutorial, 2018



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Classical control: Choose $u = H\hat{z}_{\text{des}} + G_c y$ so that

$$u \approx G_{zu}^{-1} z_{\text{des}}$$

Murray [Simons, 2018] called this *purely reactive*

Control theory: goals and architectures

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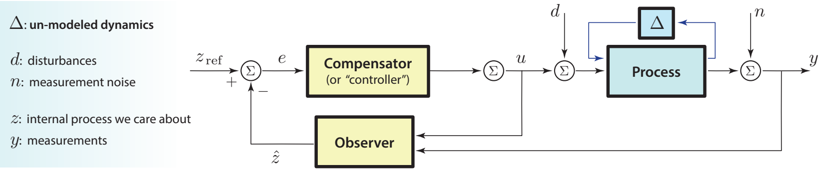

Strategies: Classical control: Choose $u = H\hat{z}_{\text{des}} + G_c y$ so that

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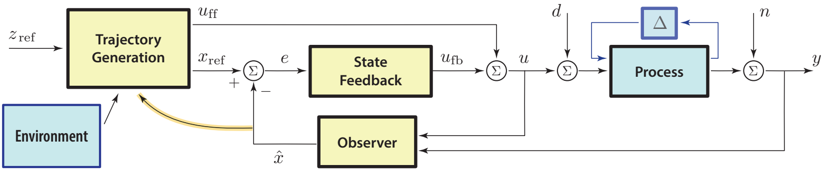
Typical control education: design H, G_c and observer so that desired specifications are met, based on

- Process model
- d, n, Δ in some bounded class

Control theory: goals and architectures

Murray tutorial, 2018

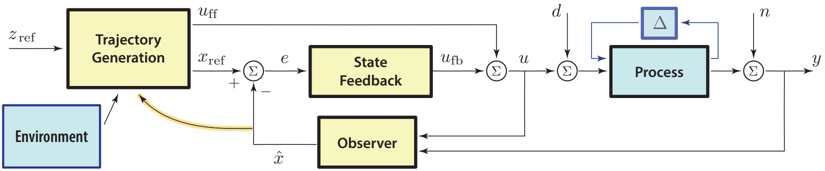


Strategies: Classical control: $u = H\hat{z}_{\text{des}} + G_c y$

Consider the steps taken when you plan to drive across town.
There is a *reactive component*. What else?

Control theory: goals and architectures

Murray tutorial, 2018



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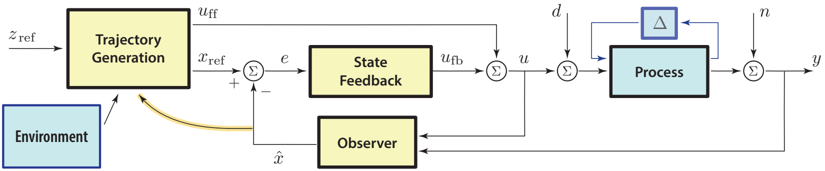
Yellow boxes may be built around optimization (MPC/RHC):

$$J^{\star}(x) = \min_{\boldsymbol{u}} \left\{ \int_0^T c(x_t, u_t)\, dt + V_0(X_T) \right\}$$

Many optimization problems to solve, because there are many objectives

Control theory: goals and architectures

Murray tutorial, 2018



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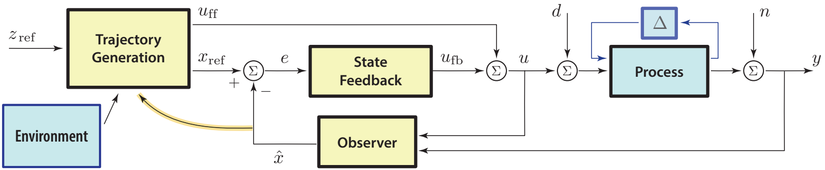

Approximating $J^\star$ and/or $\boldsymbol{u}^\star$ can be addressed using RL

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An Incomplete History of Adaptive Control

Adaptation

Dynamic process by which the controller adjusts its interaction with a system in order to carry out an objective (or reach a goal) w/o exact knowledge of the system.

Adaptive Control

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Some (incomplete) history:

1950s: gain scheduling
early Model Reference Adaptive Control (MRAC)

1958: R. Kalman — self-tuning controller (regulator) for the linear quadratic problem.

1960s: stability of adaptive controllers
Lyapunov stability
adaptation = learning (Feldbaum, Tsypkin)

1966: Parks — Lyapunov redesign approach to MRAC

1970s: stability analysis (Narendra, Morse, ...)

1980s: limitations (Rohrs et al.: sensitivity to unmodeled dynamics)

1983: Morse's conjecture $\dot{x} = ax + bu$, $a \in \mathbb{R}$, $b \neq 0$

cannot stabilize w/o knowledge of sign(b)

1983: Nussbaum disproves Morse's conjecture
1984: Willems - Byrnes: simplified Nussbaum's construction
exploration vs. exploitation

An Incomplete History of Adaptive Control

Adaptation

Dynamic process by which the controller adjusts its interaction with a system in order to carry out an objective (or reach a goal) w/o exact* knowledge of the system.

*For example: MDP or linear system with bound on dimension

Assumed for *analysis* and not implementation

Adaptive control and RL have nearly identical roots and goals

More history to be found in Lecture 4

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An Incomplete History of Adaptive Control

Adaptive control and RL have nearly identical roots and goals

Common analytical tool: **ODE Method**
[stochastic approximation of Robbins & Monro]

- [Tsitsiklis, 1994] and [Jaakola, Jordan, and Singh, 1994] [14, 15]

Asynchronous Stochastic Approximation and Q-Learning

JOHN N. TSITSIKLIS jnt@athena.mit.edu
Laboratory for Information and Decision Systems, Massachusetts Institute of Technology, Cambridge, MA 02139

Editor: Richard Sutton

Abstract. We provide some general results on the convergence of a class of stochastic approximation algorithms and their parallel and asynchronous variants. We then use these results to study the Q-learning algorithm, a reinforcement learning method for solving Markov decision problems, and establish its convergence under conditions more general than previously available.

Keywords: Reinforcement learning, Q-learning, dynamic programming, stochastic approximation

- [Wittenmark, 1975], [Ljung, 1977] [7, 8]

Analysis of Recursive Stochastic Algorithms

LENNART LJUNG, MEMBER, IEEE

II. The Algorithm

A general recursive algorithm can be written

$$x(t) = x(t-1) + \gamma(t) Q\big(t; x(t-1), \varphi(t)\big), \qquad (1)$$

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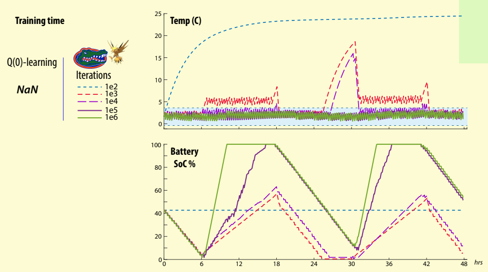
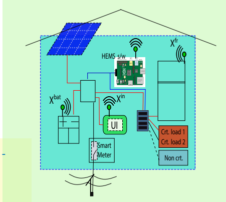
exploration vs. exploitation

See Liberzon's lecture notes: `http://liberzon.csl.illinois.edu/teaching/16ece517notes.pdf`
Recent survey: [Matni et al, 2019] From self-tuning regulators to reinforcement learning and back again.





Optimal Control and RL

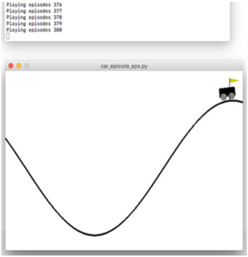
Dynamic Programming and RL

$X_{k+1} = \mathrm{F}(X_k, U_k)$

Example from `gym.openai.com`: get up the hill efficiently

State: X_k denotes position and velocity (why?)

Input (or *action*): U_k is force



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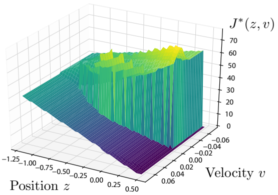

State: X_k denotes position and velocity
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Value function: $$J^\star(x) = \min_{\textit{actions}} \sum_{k=0}^{\tau} c(X_k, U_k)$$

c : cost function

τ : time to reach the hill top

Similar to a control favorite: *Swinging up a pendulum by energy control* [2]

Dynamic Programming and RL

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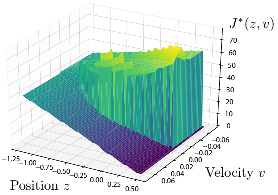

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DP eqn: $$J^\star(X_0) = \min_{U_0} \{c(X_0, U_0) + J^\star(X_1)\}$$

Recall Bellman or Bellman-Ford

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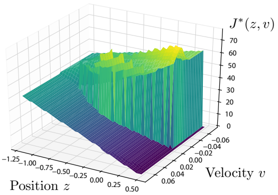
Example from `gym.openai.com`: get up the hill efficiently



State: X_k denotes position and velocity

Input (or *action*): U_k is force

Value function: $$J^\star(x) = \min_{\textit{actions}} \sum_{k=0}^{\tau} c(X_k, U_k)$$

c : cost function

τ : time to reach the hill top

DP eqn: $$J^\star(X_0) = \min_{U_0}\{\underbrace{c(X_0,U_0) + J^\star(X_1)}_{Q^\star(X_0,U_0)}\}$$

Q-learning is all about approximating $Q^\star$

Dynamic Programming and RL

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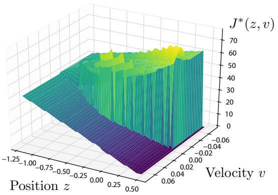

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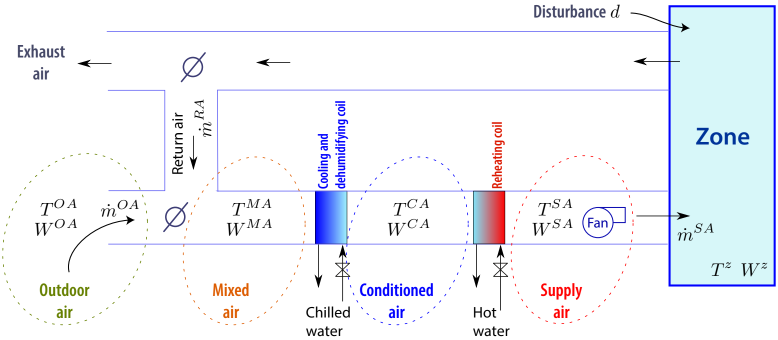
If we know $Q^\star$, we obtain $U_k^* = \phi^*(X_k)$

Control Design for Heating and Ventilation



Eight dimensional state space and four dimensional input space

Joint work with N. S. Raman, P. Barooah @ UF MAE,   A. Devraj @ Stanford

See final page of references, and bibliography of [94]

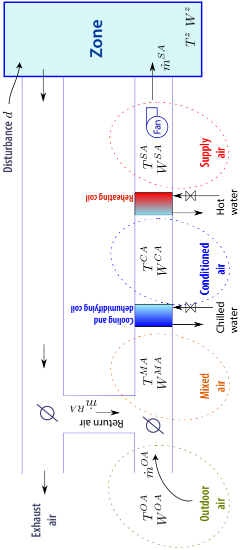
Control Design for Heating and Ventilation

Input: $U_k \stackrel{\text{def}}{=} [m_{sa}(k), r_{oa}(k), T_{ca}(k), T_{sa}(k)]^T$

1. Supply air flow rate (m_{sa})
2. Outdoor air ratio (r_{oa})
3. Conditioned air temperature (T_{ca})
4. Supply air temperature (T_{sa})

State: $X_k \stackrel{\text{def}}{=} [T_z(k), W_z(k), T_{oa}(k), W_{oa}(k), U(k-1)]^T$

1. Zone air temperature (T_z)
2. Zone air humidity ratio (W_z)
3. Outdoor air temperature (T_{oa})
4. Outdoor air humidity ratio (W_{oa})
5. Control inputs from the previous time step
6. ... forecast of occupancy, weather, ... Exercise: make a list of useful data



Control Design for Heating and Ventilation

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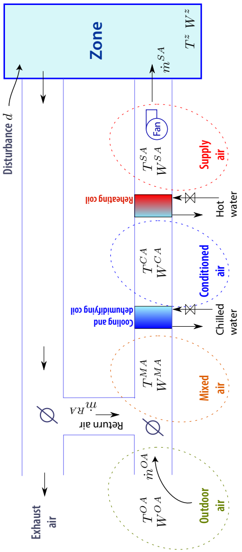
Quadratic basis: $+$ Zap Q-learning

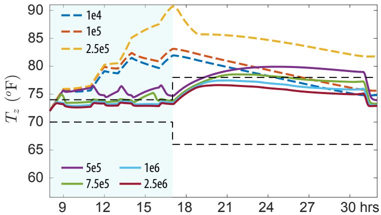
$$Q^\theta(x,u) = (x,u)^\intercal M_\theta (x,u) + (x,u)^\intercal L_\theta + k_\theta$$

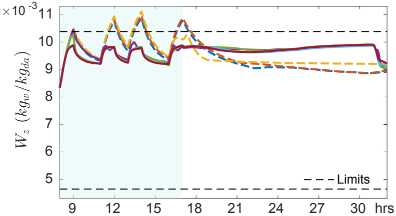
$$= \sum_i \theta_i \psi_i(x,u)$$

Initial results are great ...



Close Loop Response: Temperature and humidity evolution

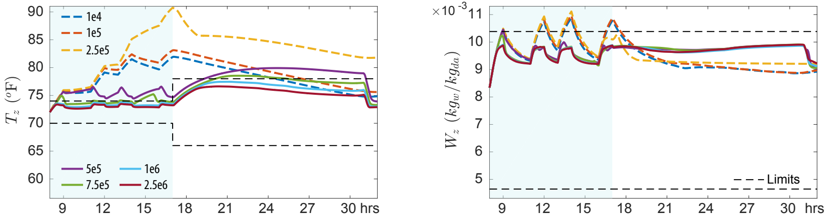




- Goal: Maintain temperature / humidity, *and* minimize energy consumption
- Inputs: Air-flow rate, out-door air ratio, conditioned air temperature, supply air temperature
- Approach: Find θ^* with quadratic basis:

$$Q^\theta(x,u) = (x,u)^T M_\theta (x,u) + (x,u)^T L_\theta + k_\theta$$

Close Loop Response: Temperature and humidity evolution



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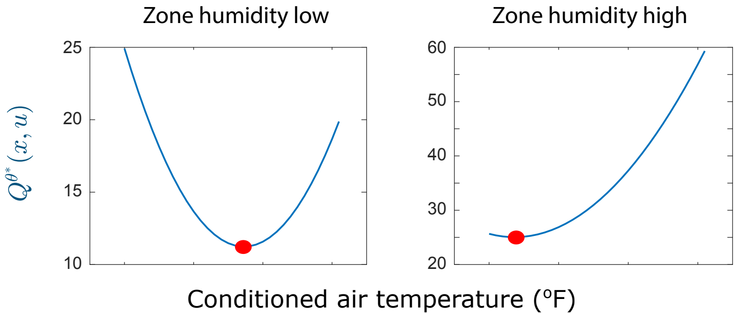
- Approach: Find θ^* with quadratic basis:

$$Q^\theta(x,u) = (x,u)^\intercal M_\theta(x,u) + (x,u)^\intercal L_\theta + k_\theta$$

Once we know θ^*, we define $U_k = \phi^{\theta^*}(X_k) = \arg\min_u Q^{\theta^*}(X_k, u)$

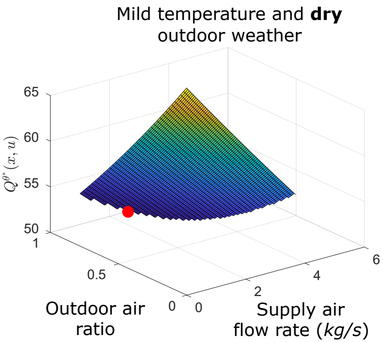
Algorithm learns: Cooling reduces humidity



Q-learning solution: zone is humid $\Longrightarrow$ conditioned air temperature reduced

Once we know θ^*, we define $U_k = \phi^{\theta^*}(X_k)$

Algorithm learns: Humid air can be expensive



Q-learning solution: humid exterior $\Longrightarrow$ outdoor air in-flow rate reduced

Once we know θ^*, we define $U_k = \phi^{\theta^*}(X_k)$

From DP to Q-learning

$X_{k+1} = \mathrm{F}(X_k, U_k)$

Value function: $$J^\star(x) = \min_{\boldsymbol{u}} \sum_{k=0}^{\infty} c(X_k, U_k)\,, \quad X_0 = x \in \mathsf{X}$$

DP eqn: $$J^\star(X_0) = \min_{U_0} \{ \underbrace{c(X_0, U_0) + J^\star(X_1)}_{Q^\star(X_0, U_0)} \}$$

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Magic: Denote $\underline{Q}^\star(x) = \min_u Q^\star(x, u) = J^\star(x)$

$\Longrightarrow$ Fixed point equation for Q-function

$$Q^\star(x, u) = c(x, u) + J^\star(\mathrm{F}(x, u))$$

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Choose approximation among $\{Q^\theta(x,u) : \theta \in \mathbb{R}^d\}$

Bellman error: $\mathcal{E}^\theta(x,u) = -Q^\theta(x,u) + c(x,u) + \underline{Q}^\theta(\mathrm{F}(x,u))$

For example, θ_i is a "weight" in a neural network or

$$Q^\theta(x,u) = \sum_i \theta_i \psi_i(x,u)$$

From DP to Q-learning

$X_{k+1} = \mathrm{F}(X_k, U_k) \qquad J^\star(x) = \min_{\boldsymbol{u}} \sum_{k=0}^{\infty} c(X_k, U_k)$

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$$\mathcal{E}^\theta(X_k,U_k) = -Q^\theta(X_k,U_k) + c(X_k,U_k) + \underline{Q}^\theta(X_{k+1})$$

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Goal: find θ^* such that $\mathcal{E}^{\theta^*}(X_k, U_k) \approx 0$

From DP to Q-learning

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Optimization Criterion:

$$L(\theta) \stackrel{\text{def}}{=} \mathsf{E}_\infty[\mathcal{E}^\theta(X, U)^2] = \lim_{T\to\infty} \frac{1}{T} \sum_{k=0}^{T-1} \mathcal{E}^\theta(X_k, U_k)^2$$

assuming this exists for each θ

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Input: stable feedback + mixture of sinusoids, $U_k = \phi(X_k) + \xi_k$

Just one option

From DP to Q-learning

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Input: stable feedback + mixture of sinusoids, $U_k = \phi(X_k) + \xi_k$

Find zeros of $\bar{f}(\theta) = -\nabla_\theta L(\theta)$

Algorithm design:

Step 1: consider an ODE: $\frac{d}{dt}\theta_t = a_t \bar{f}(\theta_t)$ *stable?*

Step 2: translate

$$\theta_{n+1} = \theta_n - \alpha_{n+1}\nabla_\theta\{\mathcal{E}^\theta(X_n, U_n)^2\}\Big|_{\theta=\theta_n}$$

Q-learning for Markovian Models

$X_{k+1} = \mathrm{F}(X_k, U_k, W_{k+1})$

Galerkin relaxation (or projected DP equation)

With the introduction of (i.i.d.) noise:

$$X_{k+1} = \mathrm{F}(X_k, U_k, W_{k+1})$$

a controlled Markovian model.

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Q-learning

Find zeros of $\bar{f}(\theta) = \mathsf{E}_\infty[\zeta_k \mathcal{E}^\theta(X_k, X_{k+1}, U_k)]$, $\quad \theta \in \mathbb{R}^d$

$$\mathcal{E}^\theta(X_k, X_{k+1}, U_k) = -Q^\theta(X_k, U_k) + c(X_k, U_k) + \underline{Q}^\theta(X_{k+1})$$

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$\zeta_k \in \mathbb{R}^d$: "eligibility vector"

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$$\bar{f}(\theta^*) = 0$$

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Design principle unchanged:

Step 1: consider an ODE: $\frac{d}{dt}\theta_t = -G_t \bar{f}(\theta_t)$ (matrix gain part of design)

Step 2: translate to a discrete time algorithm based on measurements.

Where to go from here?

Control Theory Offers Useful Tricks and Lessons

Summary

Aspects of control philosophy we have covered:

- Every control problem is multi-objective. We want to minimize fuel, get to our destination on time, minimize risk, ...
- Design is hierarchical, both in time *and* space (an approach to distributed control)
- If you have a model, use it! But recognize that no model is perfect.
- As every MLer knows: test in many non-ideal scenarios.

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Other tricks from the trade:

- Controlled Lyapunov functions
- Model reduction techniques many built on theory of singular perturbations.
  - Fluid models for networks, and their workload relaxations [CTCN]
  - Feature selection for neuro-dynamic programming, 2011 [25]

Control Theory Offers Useful Tricks and Lessons

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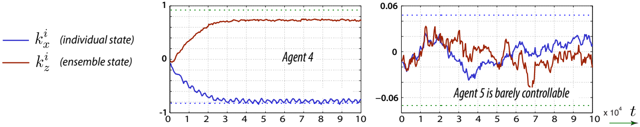
- Mean field game approximations for multi-agent systems



Control & RL to Come

- Every Optimization Problem Is a Quadratic Program Chapters 3 & 5

The complex nonlinear *Bellman equation* has been a road block in Q-learning

Estimating the Q-function should be easy: it is the solution to an LP or QP

Control & RL to Come

- Every Optimization Problem Is a Quadratic Program Chapters 3 & 5
- The ODE Method Chapter 4 [Basics of Algorithm Design and Analysis]

Don't start with an algorithm!

Analysis of Recursive Stochastic Algorithms

LENNART LJUNG, MEMBER, IEEE

II. THE ALGORITHM

A general recursive algorithm can be written

$$x(t)=x(t-1)+\gamma(t)Q(t;x(t-1),\varphi(t)), \qquad (1)$$

I see a noisy Euler approximation of the ODE:

$$\frac{d}{dt}x_t = q(t, x_t)$$

ODE Method: design the vector field q first, then translate to create an algorithm

Approximate policy iteration is a simple application

Control & RL to Come

- Every Optimization Problem Is a Quadratic Program Chapters 3 & 5
- The ODE Method Chapter 4
- Gradient Free Optimization and Policy Gradient RL Chapter 4

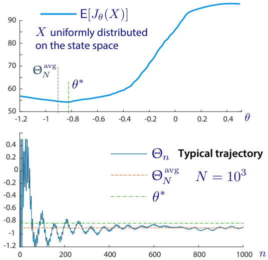
Quasi-stochastic approximation (QSA) recipe:

$$\frac{d}{dt}\bar{\Theta}_t = a_t \bar{f}(\bar{\Theta}_t) \qquad \Longleftarrow \textit{Design for your goals}$$

$$\frac{d}{dt}\Theta_t = a_t f(\Theta_t, \xi_t) \qquad \Longleftarrow \textit{QSA (cts time is simplest)}$$

$$\theta_{n+1} = \theta_n + a_{n+1} f(\theta_n, \xi_{n+1}) \qquad \Longleftarrow \textit{Euler/Runge-Kutta}$$

https://en.wikipedia.org/wiki/Runge-Kutta_methods



qPG for Mountain Car: objective function, and typical behavior of estimates

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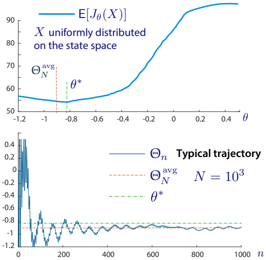


qPG for Mountain Car: objective function, and typical behavior of estimates

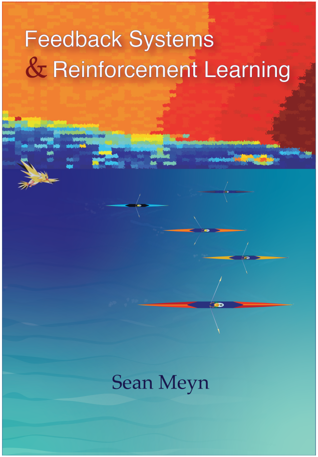
QSA much more reliable than stochastic methods. Lots of theory to explain why

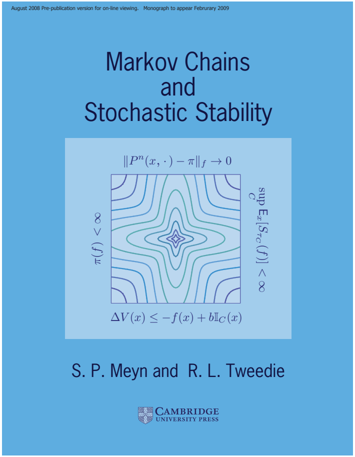
Control & RL to Come

- Every Optimization Problem Is a Quadratic Program Chapters 3 & 5
- The ODE Method Chapter 4
- Gradient Free Optimization and Policy Gradient RL Chapter 4
- Real-Time System Optimization with Applications to Power Systems

"We examine the problem of real-time optimization of networked systems and develop online algorithms that steer the system towards the optimal system trajectory..."

Andrey Bernstein [96, 97, 98]
(National Renewable Energy Laboratory)


Feedback Systems
& Reinforcement Learning
Sean Meyn


August 2008 Pre-publication version for on-line viewing. Monograph to appear Februrary 2009
Markov Chains
and
Stochastic Stability
$\|P^n(x,\cdot)-\pi\|_f \to 0$
$\pi(f) < \infty$
$\sup_C \mathrm{E}_x[S_{\tau_C}(f)] < \infty$
$\Delta V(x) \le -f(x) + b\mathbb{I}_C(x)$
S. P. Meyn and R. L. Tweedie
CAMBRIDGE
UNIVERSITY PRESS

References

Control Background I

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