



LWE with Side Information: Attacks and Concrete Security Estimation

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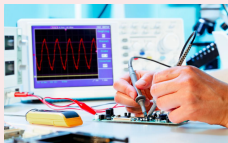
Workshop at the Simons Institute, Berkeley, CA, USA
Lattices: From Theory to Practice
April 2020

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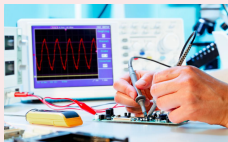
³ANSSI, Paris, France – ENS Paris, CNRS, PSE University, Paris, France

Side Channel Cryptanalysis



Algorithmic Cryptanalysis and Physical Cryptanalysis

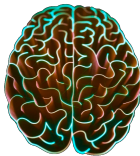
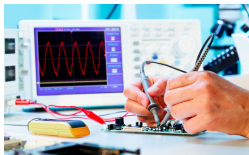
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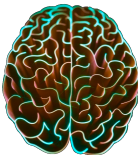
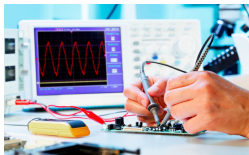
Algorithmic Cryptanalysis



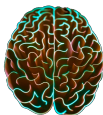
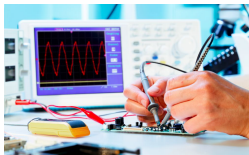
A Unified Cryptanalysis Framework for LWE-like Schemes



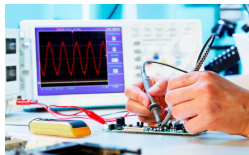
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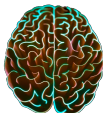


A Unified Cryptanalysis Framework for LWE-like Schemes



Real-world Specs

NTRU, LAC, Round5,
Lizard, NTRUPrime



Active Attacks

Decryption Failures



Our framework

Standard methodology

(Primal Lattice Attack)

Public Key $\xrightarrow{\begin{pmatrix} \text{ad-hoc} \\ \text{tweak} \end{pmatrix}}$ $\text{BDD}_{\Lambda,r}$ $\xrightarrow{\text{Kannan}}$ $\text{uSVP}_{\Lambda',r'}$ \rightarrow Lattice Reduction

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Our framework

Public Key $\rightarrow \text{DBDD}_{\Lambda_0, \Sigma_0, \mu_0}$
 \downarrow **Hint**
 $\text{DBDD}_{\Lambda_1, \Sigma_1, \mu_1}$
 \vdots
 \downarrow **Hint**
 $\text{DBDD}_{\Lambda_h, \Sigma_h, \mu_h} \xrightarrow{\text{Homogenize \& Isotropize}}$ $\text{uSVP}_{\Lambda',1} \rightarrow$ Lattice Reduction

Contributions

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 - ▶ Rescaling/balancing of secret inputs
 - ▶ Optimal choice of Kannan's Embedding coefficient
 - ▶ Ignoring LWE samples

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Application examples, exploiting data from:

- ▶ Uncompleted side-channel attack 1st attack of [BFM+18]
- ▶ Decryption failures revisiting [AVV18]
- ▶ Real-world specifications NTRU, LAC, Round5

DBDD¹ and its concrete Hardness

Hints, and their integration into DBDD

Sage Implementation

Example Applications: from real-world to hints

¹Distorted Bounded Distance Decoding

DBDD and its concrete Hardness

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$\text{DBDD}_{\Lambda_h, \Sigma_h, \mu_h} \xrightarrow{\text{Homogenize \& Isotropize}} \text{uSVP}_{\Lambda', 1} \rightarrow \text{Lattice Reduction}$

DBDD: Distorted Bounded Distance Decoding

Bounded Distance Decoding

- ▶ Given a lattice $\Lambda \subset \mathbb{R}^d$, a target $\mathbf{t} \in \mathbb{R}^d$ and a radius $r > 0$
- ▶ Find the unique $\mathbf{s} \in \Lambda$ such that $\|\mathbf{s} - \mathbf{t}\| \leq r$

Distorted Bounded Distance Decoding

DBDD $_{\Lambda, \Sigma, \mu}$

- ▶ Given a lattice $\Lambda \subset \mathbb{R}^d$, a mean $\mu \in \mathbb{R}^d$ and covariance $\Sigma > \mathbf{0}$
- ▶ Find the unique $\mathbf{s} \in \Lambda$ such that $\|\mathbf{s} - \mu\|_{\Sigma} \leq d$ where

$$\|\mathbf{x}\|_{\Sigma} := \mathbf{x}^t \cdot \Sigma^{-1} \cdot \mathbf{x} \geq 0.$$

Intuition: **Balls** are replaced by general **Ellipsoids**.

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Intuition: **Balls** are replaced by general **Ellipsoids**.

Simplification: Gaussian \simeq Uniform over an Ellipsoid.

Isotropization

Lattice Reduction algorithms are designed for balls.

$$\mathbf{Isotropize} : (\Lambda, \Sigma, \mu) \mapsto (\sqrt{\Sigma^{-1}} \cdot \Lambda, \mathbf{Id}, \sqrt{\Sigma^{-1}} \cdot \mu)$$

After isotropization, the instance has the form

$$\text{DBDD}_{\Lambda', \mathbf{Id}, \mu'} = \text{BDD}_{\Lambda', \sqrt{d}, \mu'}.$$

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Concrete hardness

▶ **grows** with the dimension

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▶ **shrink** with the volume

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Concrete hardness

- ▶ **grows** with the dimension d
- ▶ **shrink** with the volume $\text{Vol}(\Lambda') = \text{Vol}(\Lambda) / \sqrt{\det(\Sigma)}$.

Remark

- ▶ Generalizes ad-hoc “rescaling” NTRU, Lizard, NTRUPrime, ...
- ▶ Trivialize the optimal choice of Kannan's embedding coefficient

On Hardness Estimation

Our unit of security: the **bikz**

- ▶ Security expressed in β , the needed BKZ blocksize
- ▶ Roughly, $3 \text{ bikz} \approx 1 \text{ bit of security}$ see [ACD+18]

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The 2015 estimates: GSA+Intersect

[ADPS15, AGVW17]

$$\sqrt{\beta} \leq \delta_{\beta}^{2\beta-d-1} \cdot \text{Vol}(\Lambda')^{1/d} \quad \text{where} \quad \delta_{\beta} = \dots$$

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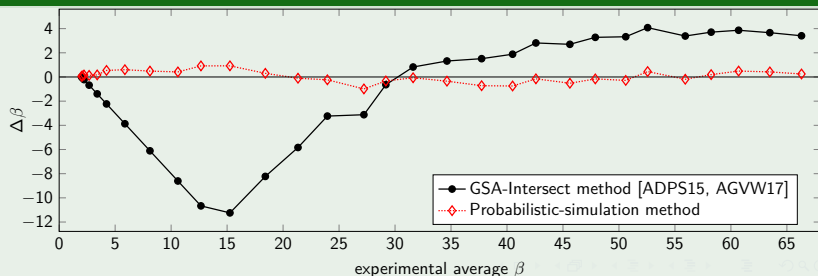
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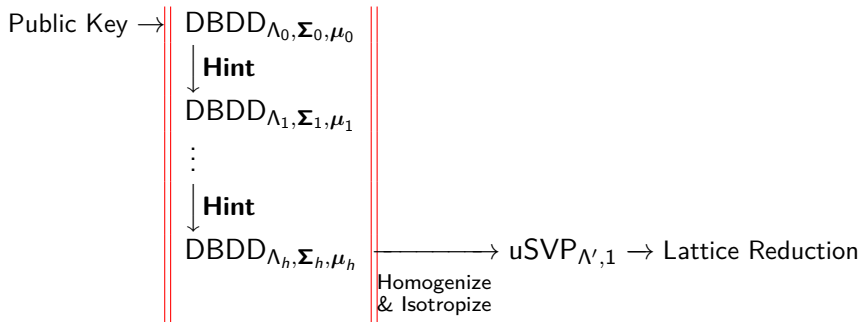
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A refined estimation: Simulation+Probabilities

(somewhat new)



Hints, and their integration in DBDD



Four types of hints:

- ▶ Perfect hints: $\langle \mathbf{s}, \mathbf{v} \rangle = \ell$
- ▶ Modular hints: $\langle \mathbf{s}, \mathbf{v} \rangle = \ell \bmod k$
- ▶ Approximate hints: $\langle \mathbf{s}, \mathbf{v} \rangle \approx \ell$
- ▶ Short vector hints: $\mathbf{v} \in \Lambda$

Each hint may affect the dimension of Λ , its volume, and the covariance of Σ in predictable ways.

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Simplification for this talk: Hints are homogeneous $\ell = 0$.

Effect on a DBDD instance

Slice the lattice, condition the Gaussian

$$\Lambda \mapsto \Lambda \cap \mathbf{v}^\perp$$

$$\Sigma \mapsto \Sigma - \frac{(\mathbf{v}\Sigma)^T \mathbf{v}\Sigma}{\mathbf{v}\Sigma \mathbf{v}^T}$$

Effect on the hardness

Easier

- ▶ The lattice dimension d **decreases** by 1
- ▶ The lattice volume **increases** by a factor $\|\mathbf{v}\|$

Effect on a DBDD instance

Sparsify the lattice

$$\Lambda \mapsto \Lambda \cap \{\mathbf{x} \in \mathbb{Z}^d \mid \langle \mathbf{x}, \mathbf{v} \rangle = 0 \pmod k\}$$
$$\Sigma \mapsto \Sigma + \epsilon$$

Effect on the hardness

Easier

- ▶ The lattice volume **increases** by a factor k

More precisely: $\langle \mathbf{s}, \mathbf{v} \rangle = e$, for e a Gaussian error of variance σ

Effect on a DBDD instance

Condition the Gaussian

$$\Lambda \mapsto \Lambda$$

$$\Sigma \mapsto \Sigma - \frac{(\mathbf{v}\Sigma)^T \mathbf{v}\Sigma}{\mathbf{v}\Sigma\mathbf{v}^T + \sigma^2}$$

Effect on the hardness

Easier

- The covariance **decreases**

Effect on a DBDD instance

Project the lattice

$$\Lambda \mapsto \Pi_{\mathbf{v}}^{\perp} \cdot \Lambda$$

$$\Sigma \mapsto \Pi_{\mathbf{v}}^{\perp} \cdot \Sigma \cdot (\Pi_{\mathbf{v}}^{\perp})^T$$

Effect on the hardness

Trade-off

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Project the lattice

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Effect on the hardness

Trade-off

- ▶ The dimension **decreases** by 1
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Remark

- ▶ Typical example: q -vectors $(q, 0, 0, \dots, 0)$, $(0, q, 0, \dots, 0)$, ...
- ▶ Integrating a q -vectors \Leftrightarrow Ignoring one LWE sample
- ▶ This generalize the usual 'dimension-volume' **trade-off**

Sage Implementation



Implementation details

One interactive Python Class, 3 implementations

- ▶ A full-fledge version Attack, Prediction
- ▶ A fast version Prediction¹
- ▶ A faster version, with restrictions Prediction¹

```
load("../framework/instance_gen.sage")
n = m = 70
q = 3301
D_e = D_s = build_centered_binomial_law(40)
_, _ , dbdd = initialize_from_LWE_instance(DBDD, n, q, m, D_e, D_s)
dbdd.estimate_attack()

> Build DBDD from LWE
> n= 70   m= 70   q=3301
> Attack Estimation
> dim=141       $\delta$ =1.012362       $\beta$ =45.40

v = vec([randint(0, 1) for i in range(m + n)])
dbdd.leak(v)

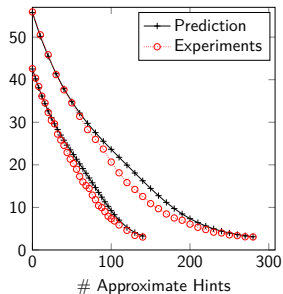
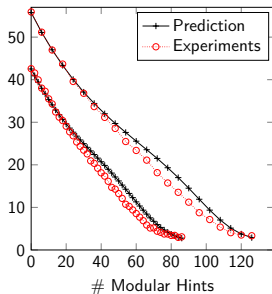
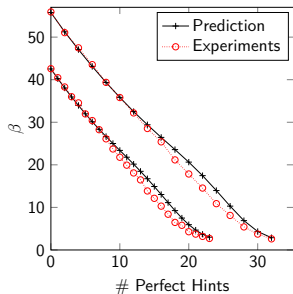
> 27

dbdd.integrate_perfect_hint(v0, 27)

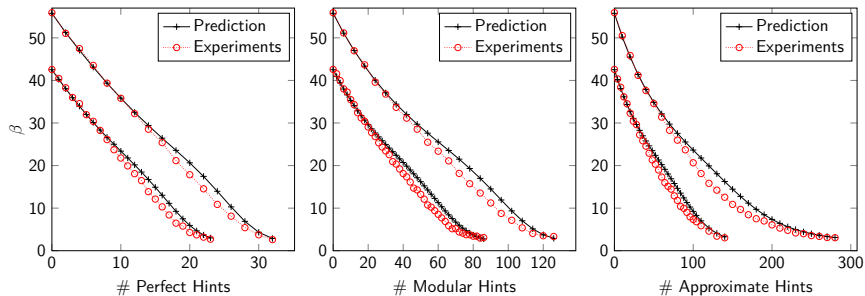
> integrate perfect hint   u0 + u1 + u7 + u8 + u9 + ... = 27
> Worthy hint !
> dim=140       $\delta$ =1.01252643       $\beta$ =41.93
```

¹Assuming hints are never redundant

Predictions vs. Experiments



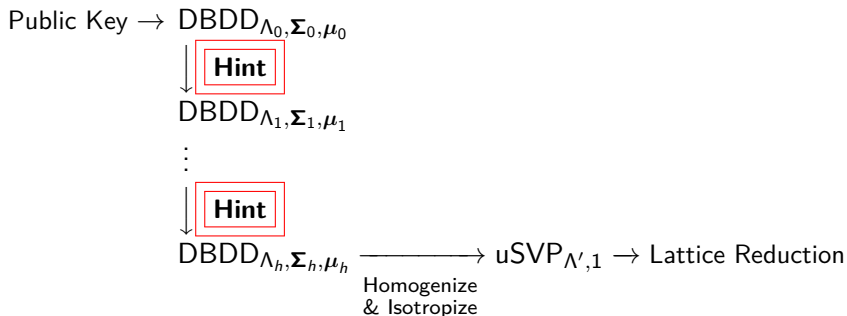
Predictions vs. Experiments



Looks pretty¹ good !

¹i.e. not perfect.

Example Applications: from real-world to hints



Example 0: Hamming Weight

Typically, one does **not** get **linear** hints.
One needs a bit of creativity to extract some linear hint.

Power-analysis 101: Hamming Weight

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- ▶ From **power analysis**, we learn $\text{HW}(\mathbf{s}_0) = 2$
- ▶ We deduce $\mathbf{s}_0 \in \{3, 5\}$
- ▶ We encode this knowledge with two hints
 - ▶ A modular hint: $\langle \mathbf{s}, (1, 0, \dots, 0) \rangle = 1 \pmod 2$
 - ▶ A approximate hint: $\langle \mathbf{s}, (1, 0, \dots, 0) \rangle \approx 4$, with error variance 1

Example 1: Profiling

Two single-trace attacks on FrodoKEM [BFM+18]

For the 1st, data leaked, but was too weak for a key-recovery.

Exactly our intended **use-case**.

	NIST1	NIST2	CCS1	CCS2
Attack without hints (bikz)	487	708	239	448
Attack with hints (bikz)	337	471	190	297
Attack with hints & guesses (bikz)	298	403	126	110

Table: Cost of the attacks without/with hints & without/with guesses.

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Remark

- ▶ Hints are approximate, given in **a-posteriori** form
- ▶ Profiling data currently not used optimally (update in progress)

Example 2: Decryption Failures

A **decryption failure** occurs when the ciphertext's noise \mathbf{w} is s.t.

$$\langle \mathbf{s}, \mathbf{w} \rangle \geq t := q/4.$$

Brute-forces decryption request with random \mathbf{w} until it triggers.

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✓ full-dim. approx. hint

$$\mathbf{s} \approx \frac{d\sigma^2}{t} \cdot \mathbf{w}$$

with variance $(\|\mathbf{w}\|d\sigma^2/t)^2$

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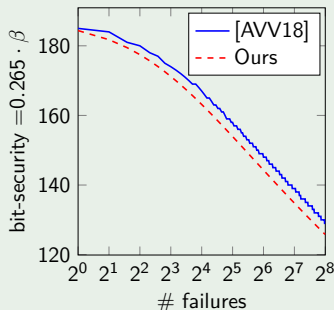
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Reproduction of [AVV18]



Example 3: Real-World Specifications

Several real-world schemes (NTRU, LAC, Round5) use ternary secrets, with a fixed amount of 1 and $-1 \Rightarrow$ **perfect hint**:

$$\langle \mathbf{s}, (1, 1, 1, \dots, 1) \rangle = \ell$$

	LAC-128	LAC-192	LAC-256
without hints	509.03	985.64	1104.83
with 2 hints	505.94	982.74	1101.61
	R5ND_{1}KEM_0d	R5ND_{3}KEM_0d	R5ND_{5}KEM_0d
without hints	494.39	658.67	877.71
with 1 hint	492.94	657.23	876.24
	ntruhs2048509	ntruhs2048677	ntruhs4096821
without hint	372.58	515.36	617.71
with 1 hint	371.23	513.95	616.39

Remark

- ▶ A few more interesting to be said on NTRU, w.r.t. to the attack of [MS01] exploiting symmetries (update in progress)

That's it

Thanks for code-sharing, pertinent comments, and valuable feedback

- ▶ Martin Albrecht
- ▶ Jan-Pieter D'Anvers
- ▶ Thibauld Feneuil
- ▶ Henri Gilbert
- ▶ Marco Martinoli
- ▶ Ange Martinelli
- ▶ Thomas Prest
- ▶ John Schanck

Questions ?