Quantum Distributed Computing: Recent Results

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Nagoya University

Simons Institute
27 February 2020

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Positive answers known in some models:

- ✓ anonymous networks: quantum leader election [Tani et al. 2007]
- ✓ faulty networks: quantum Byzantine agreement [Ben-Or, Hassidim 2005]
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CONGEST model

(limited bandwidth)

LOCAL model

(unlimited bandwidth)

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Quantum can be useful for some problems
[LG, Magniez 2018] [Izumi, LG 2019] [Izumi, LG, Magniez 2020]

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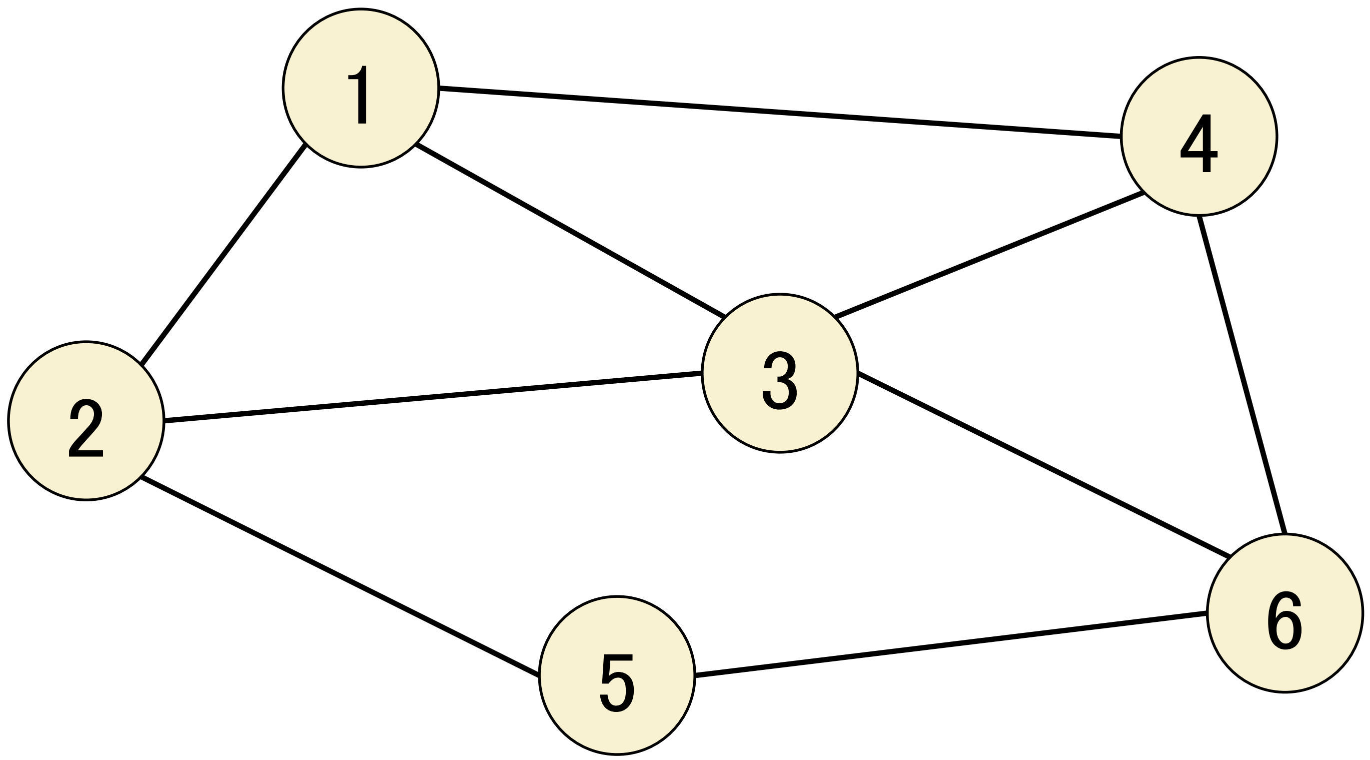
Classical Distributed Computing: CONGEST and LOCAL

Basic setting: non-faulty, non-anonymous, synchronous

- ✓ network G=(V,E) of n nodes (all nodes have distinct identifiers)



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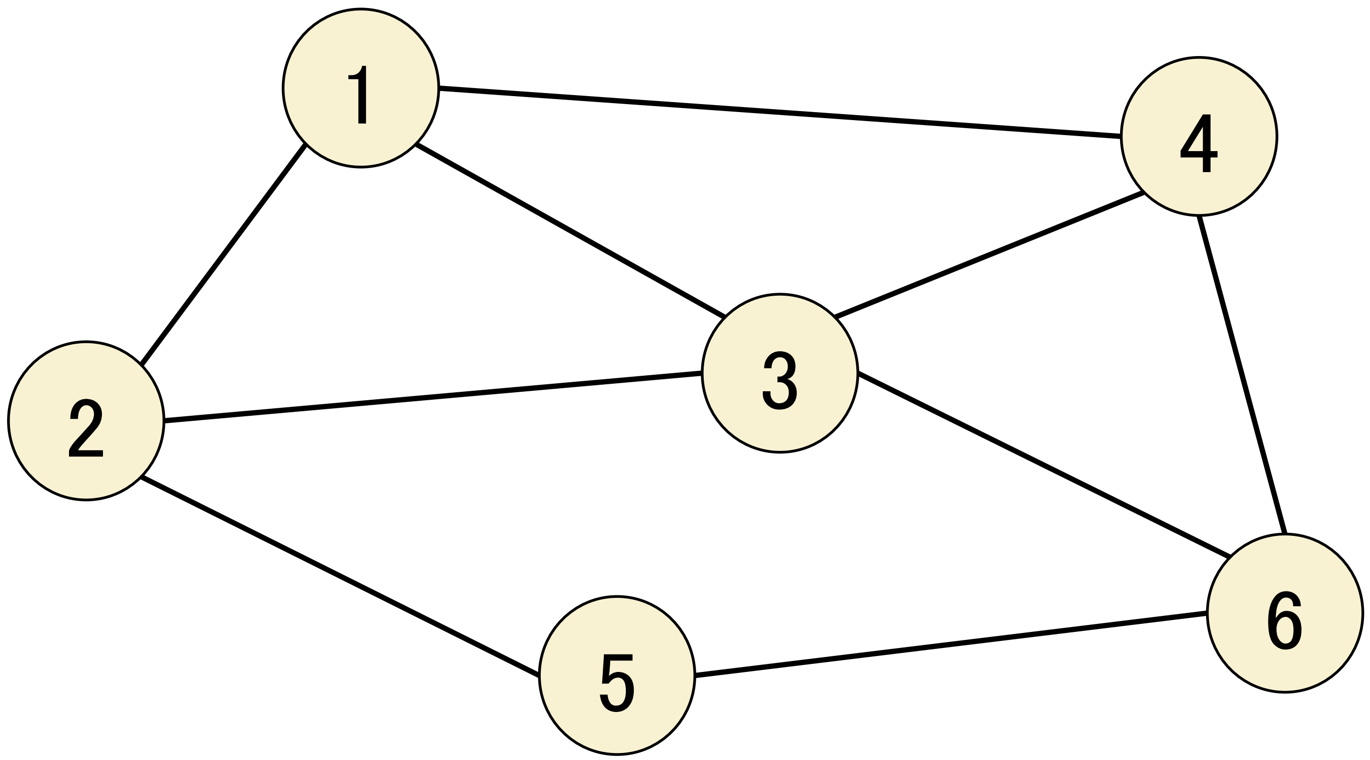

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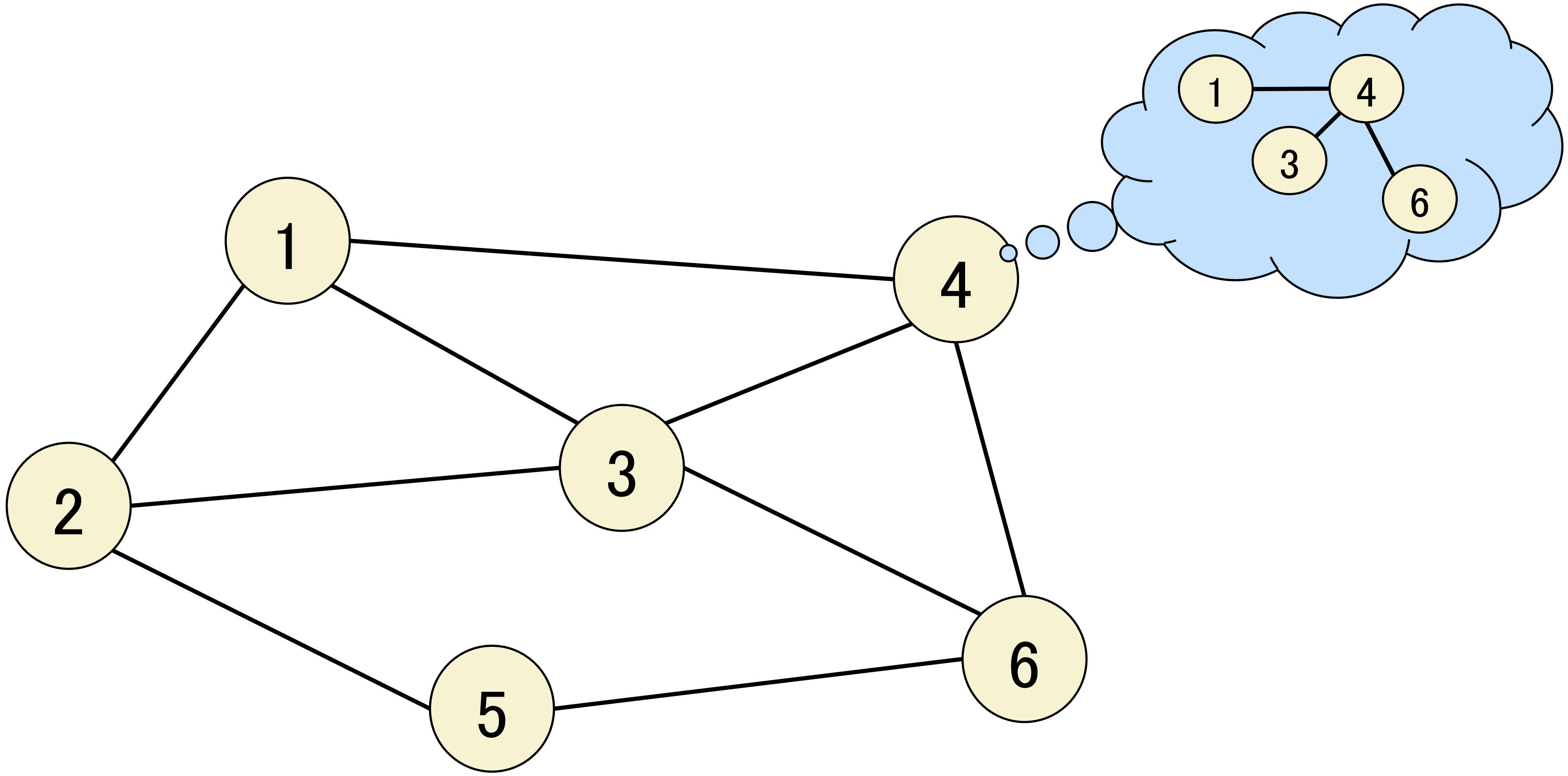

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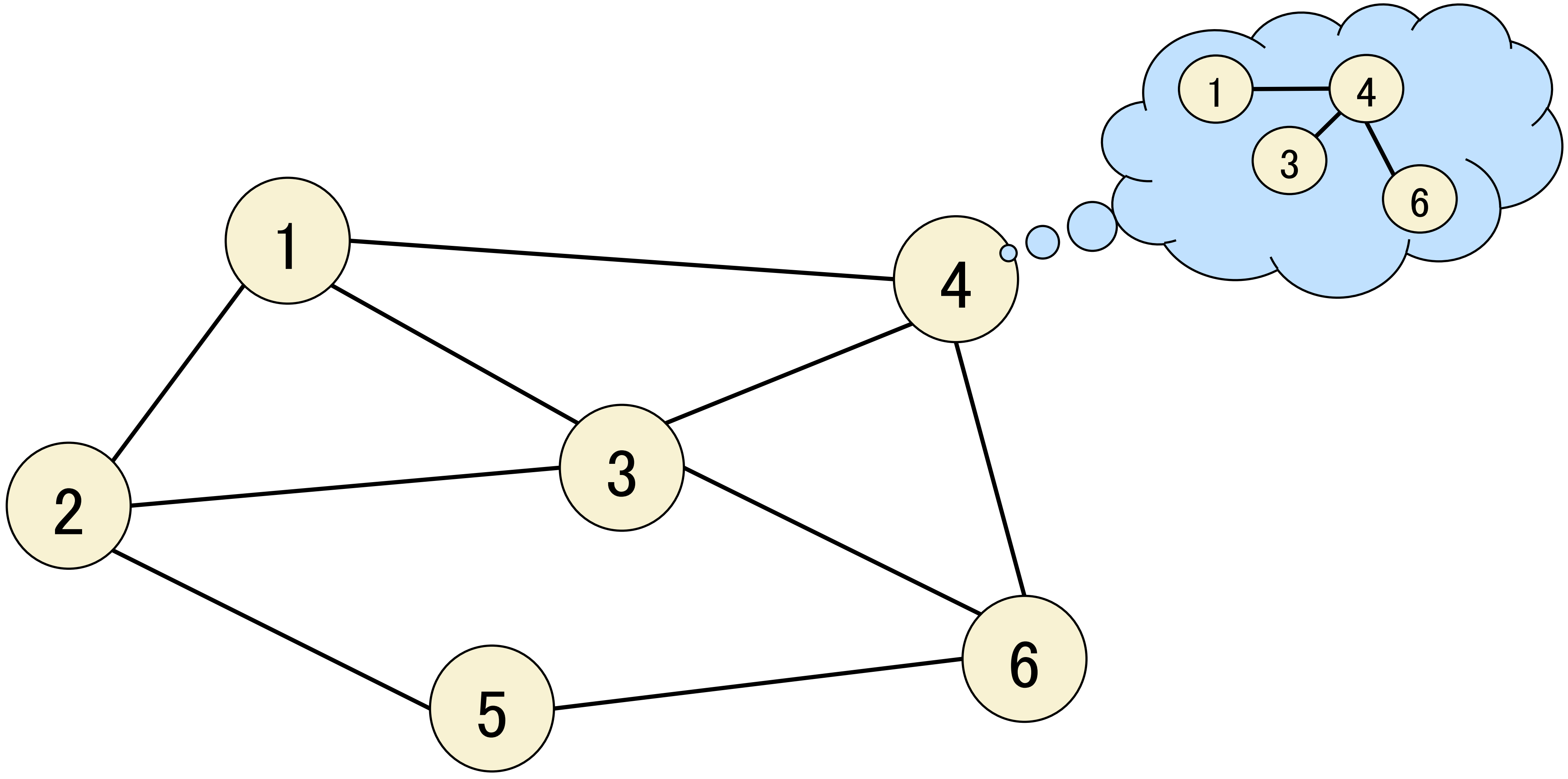

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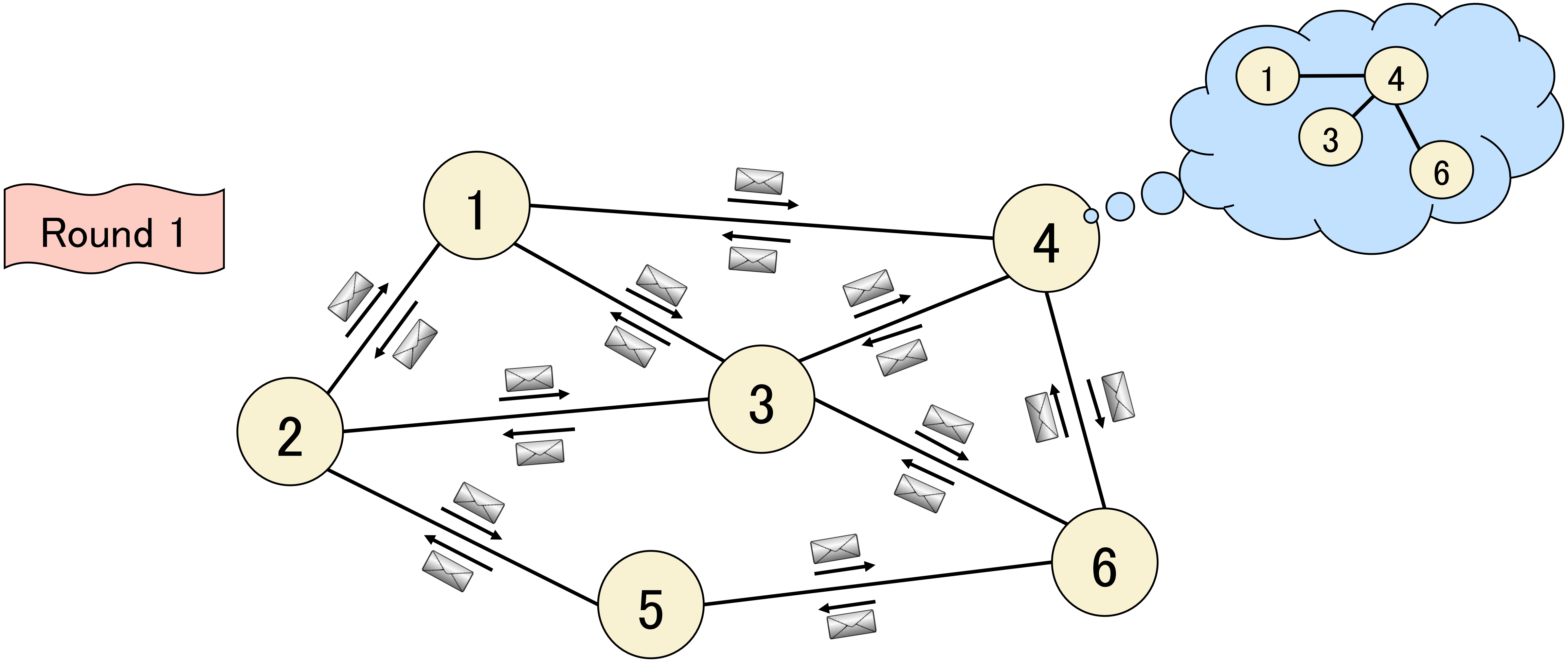

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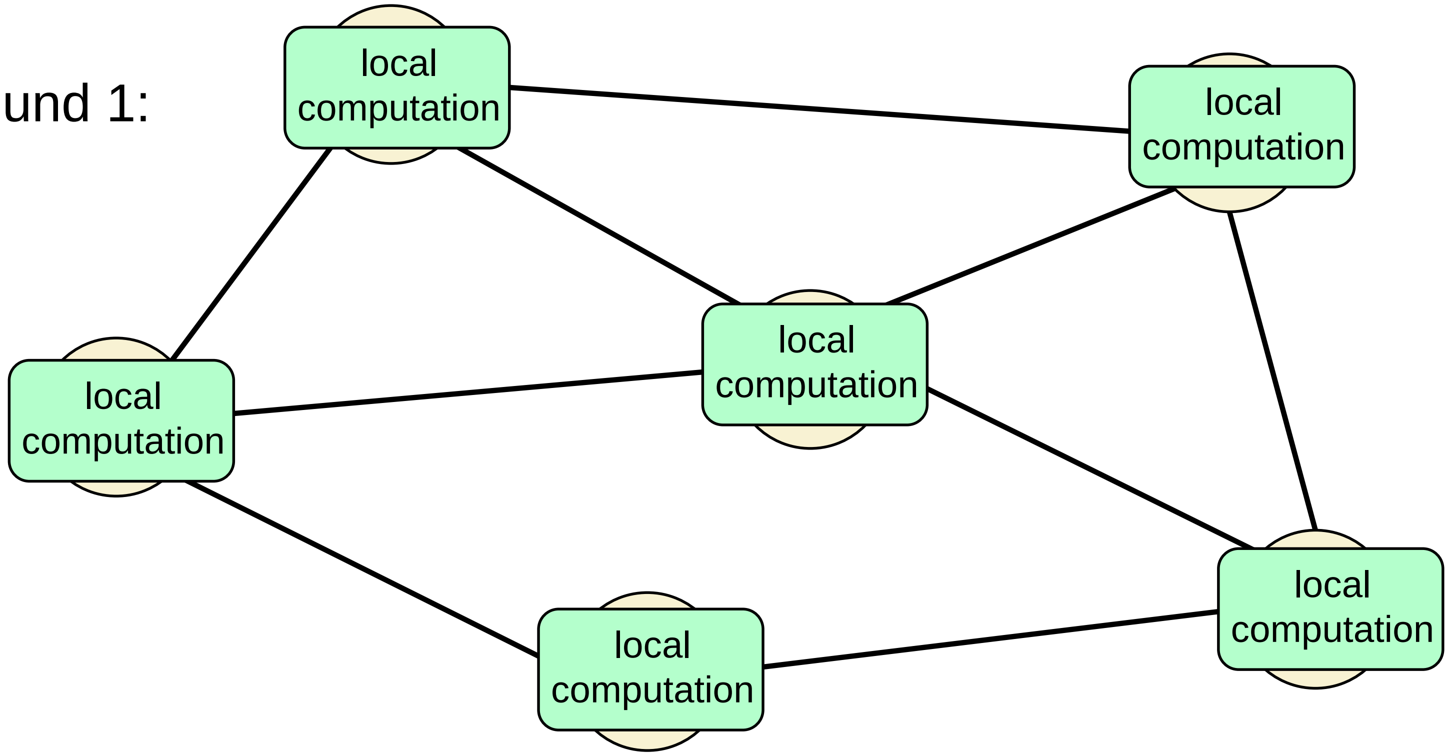

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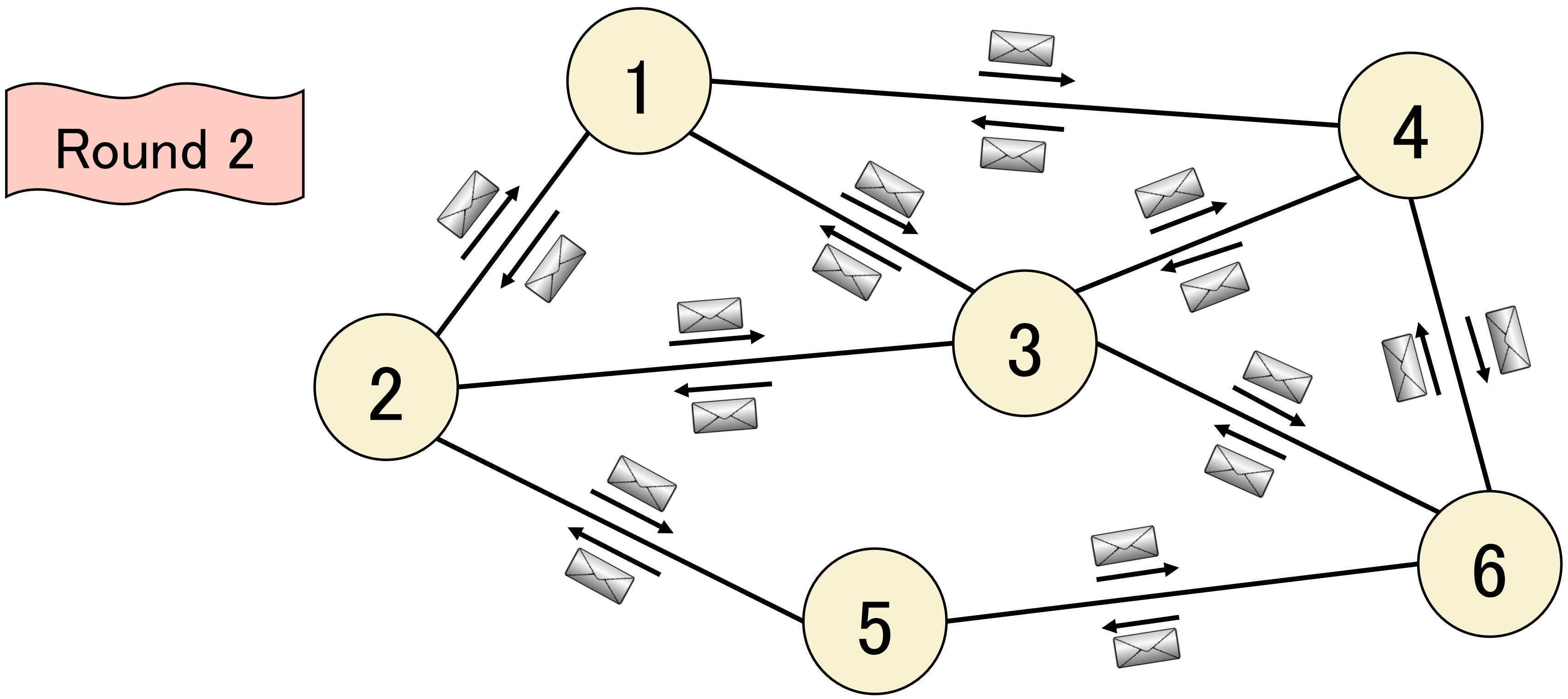

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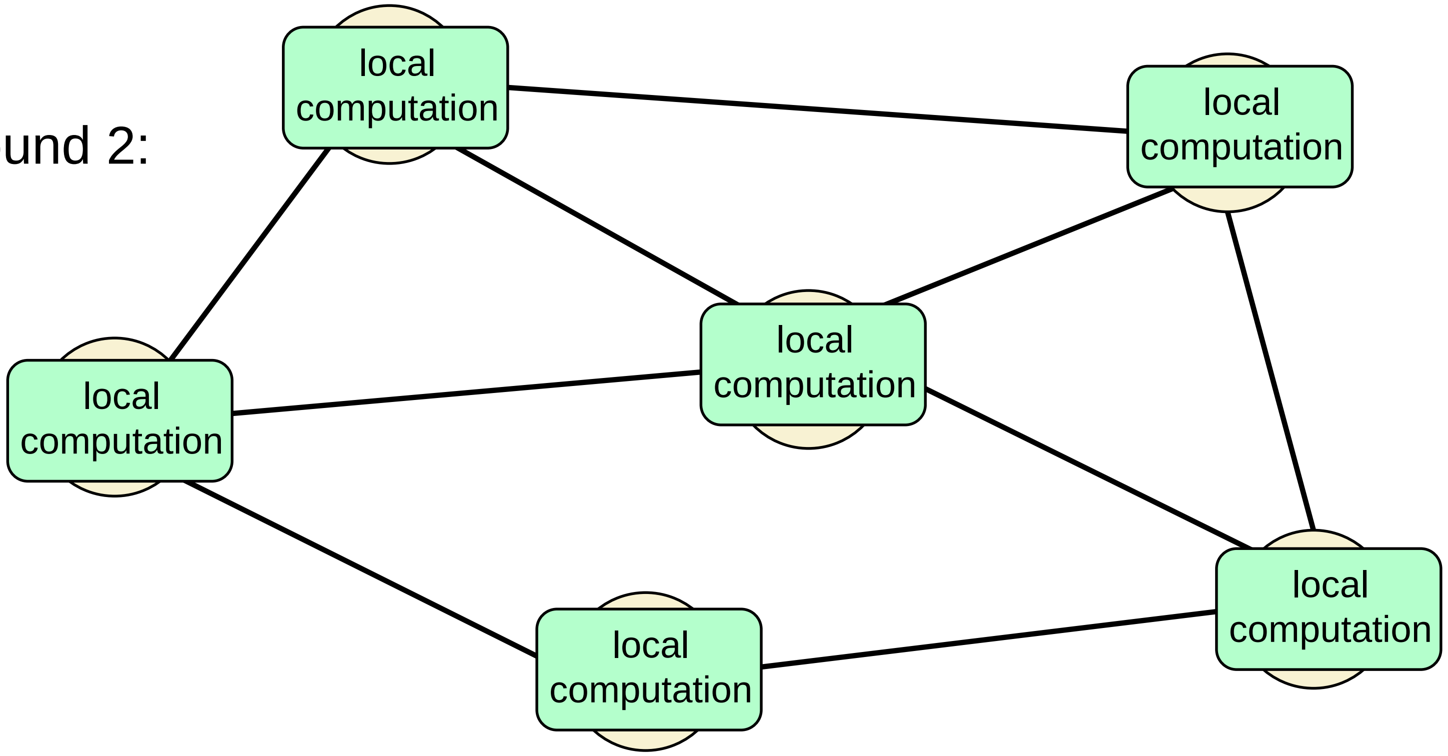

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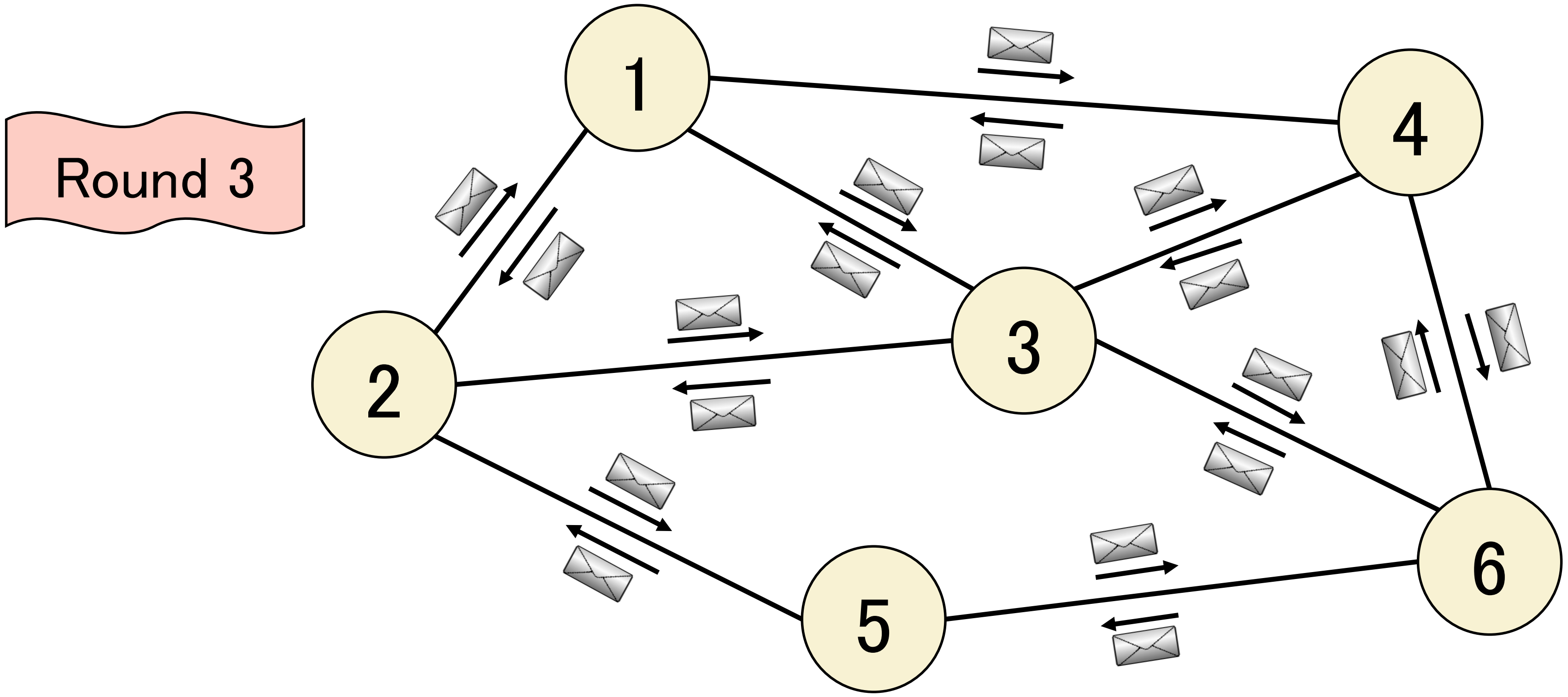

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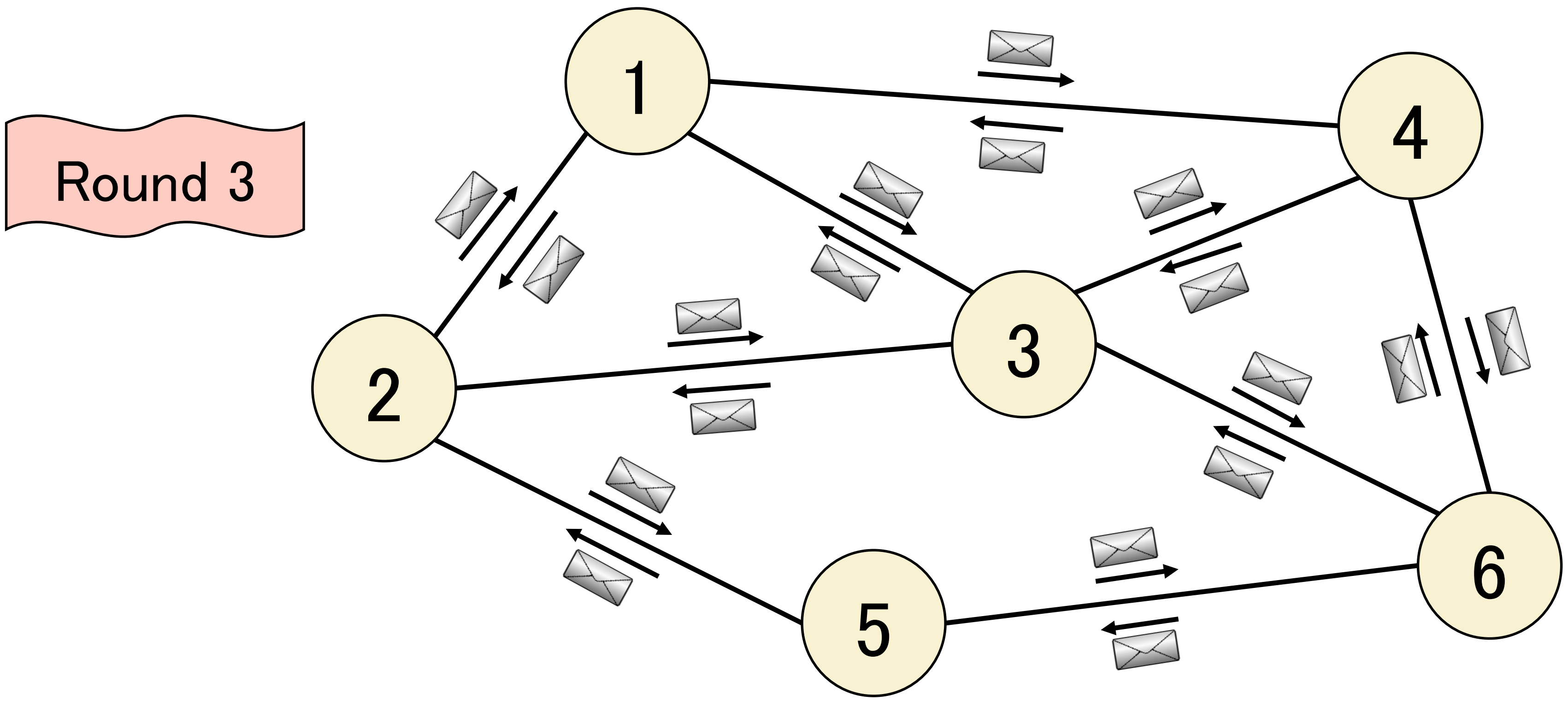

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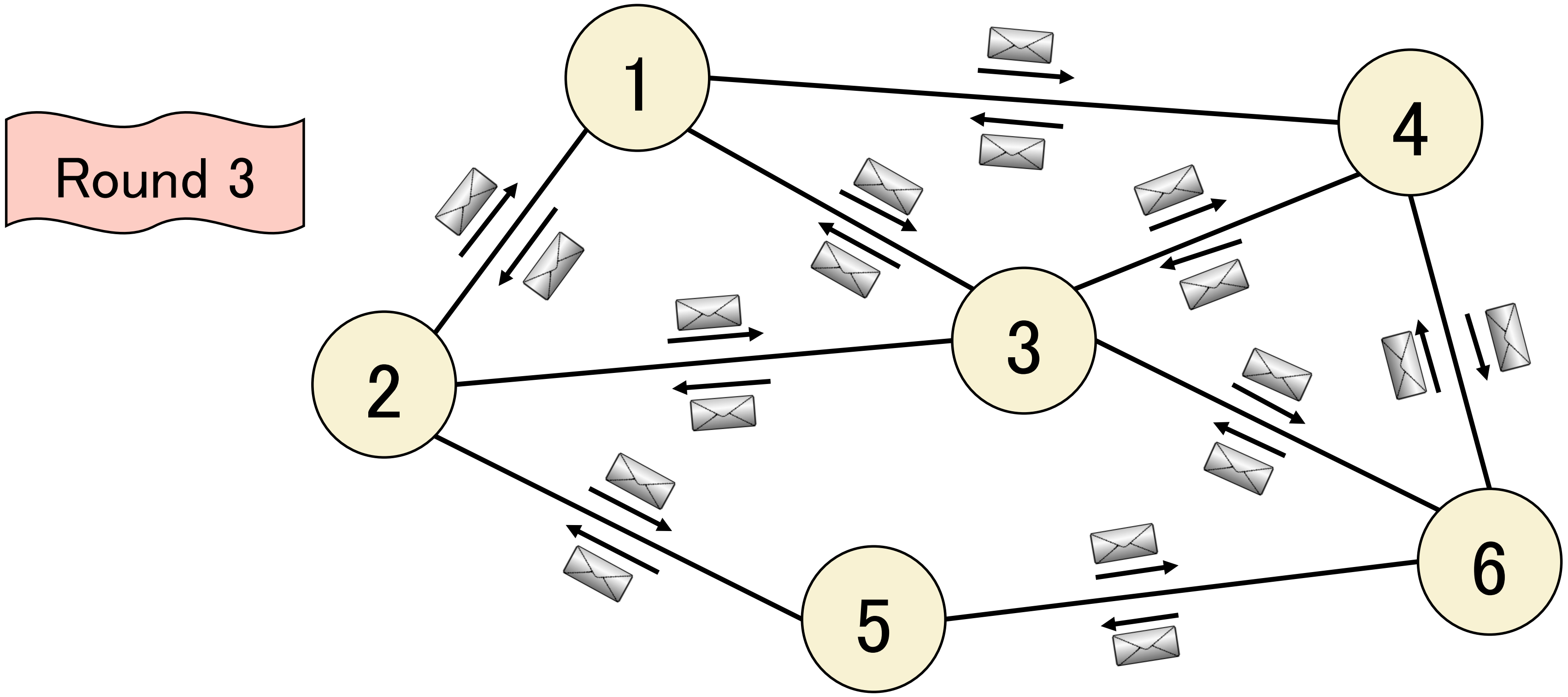

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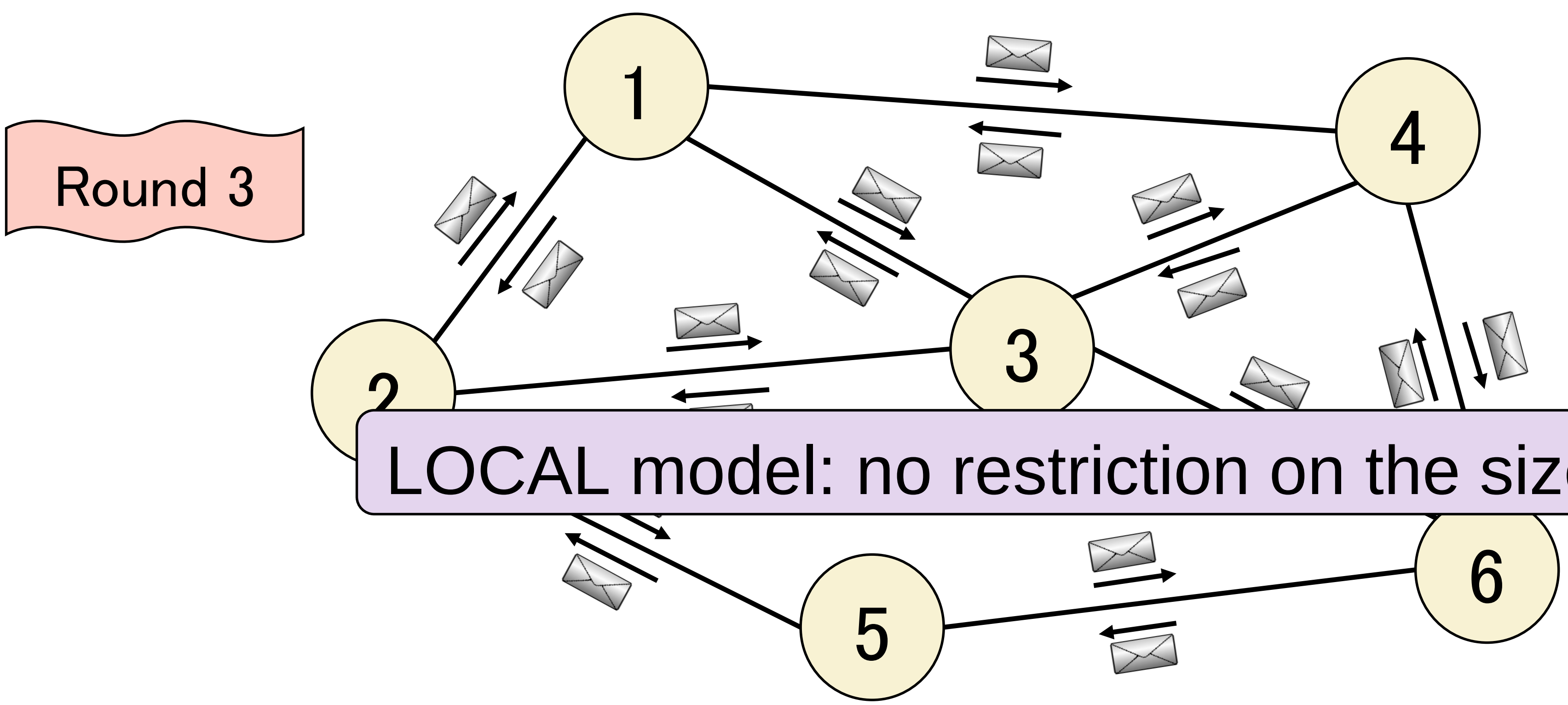

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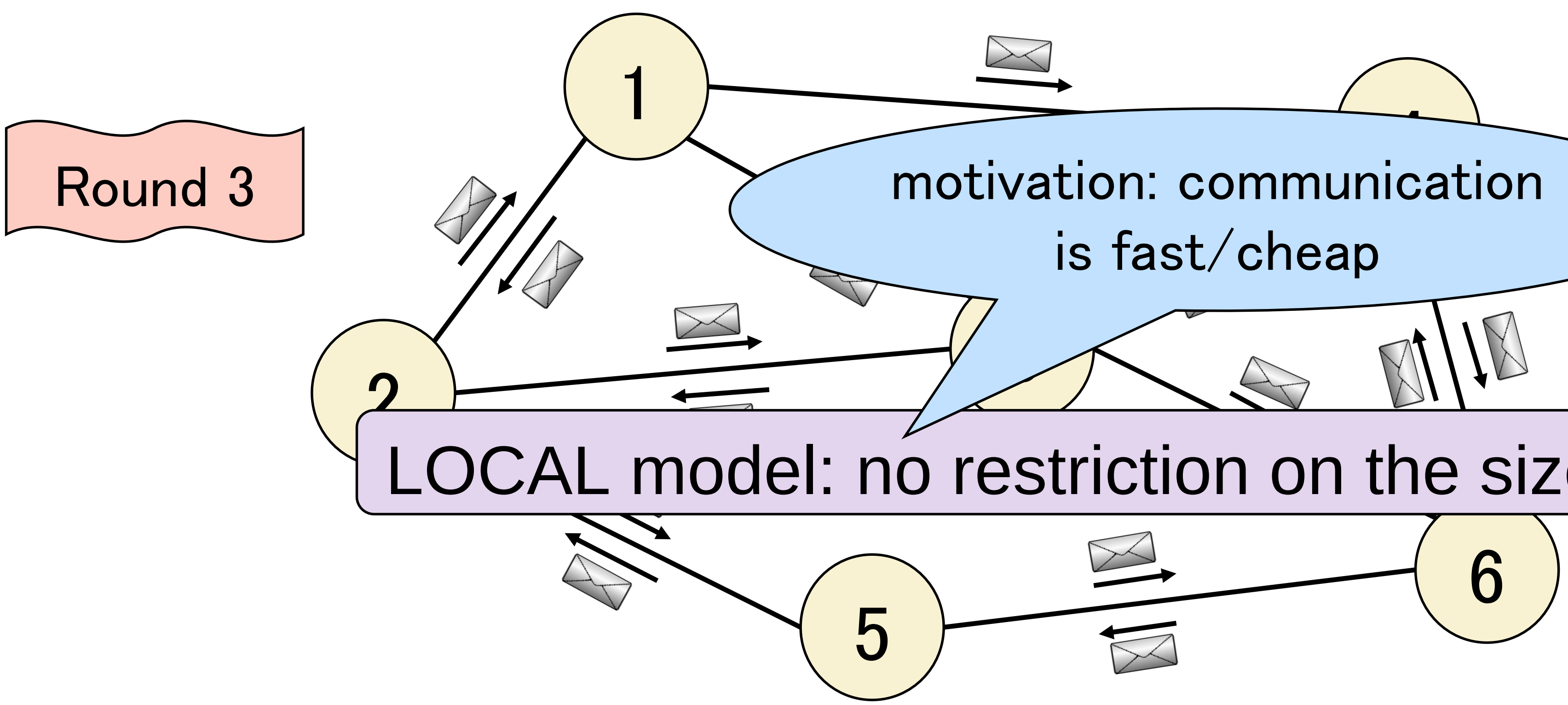

motivation: communication
is fast/cheap

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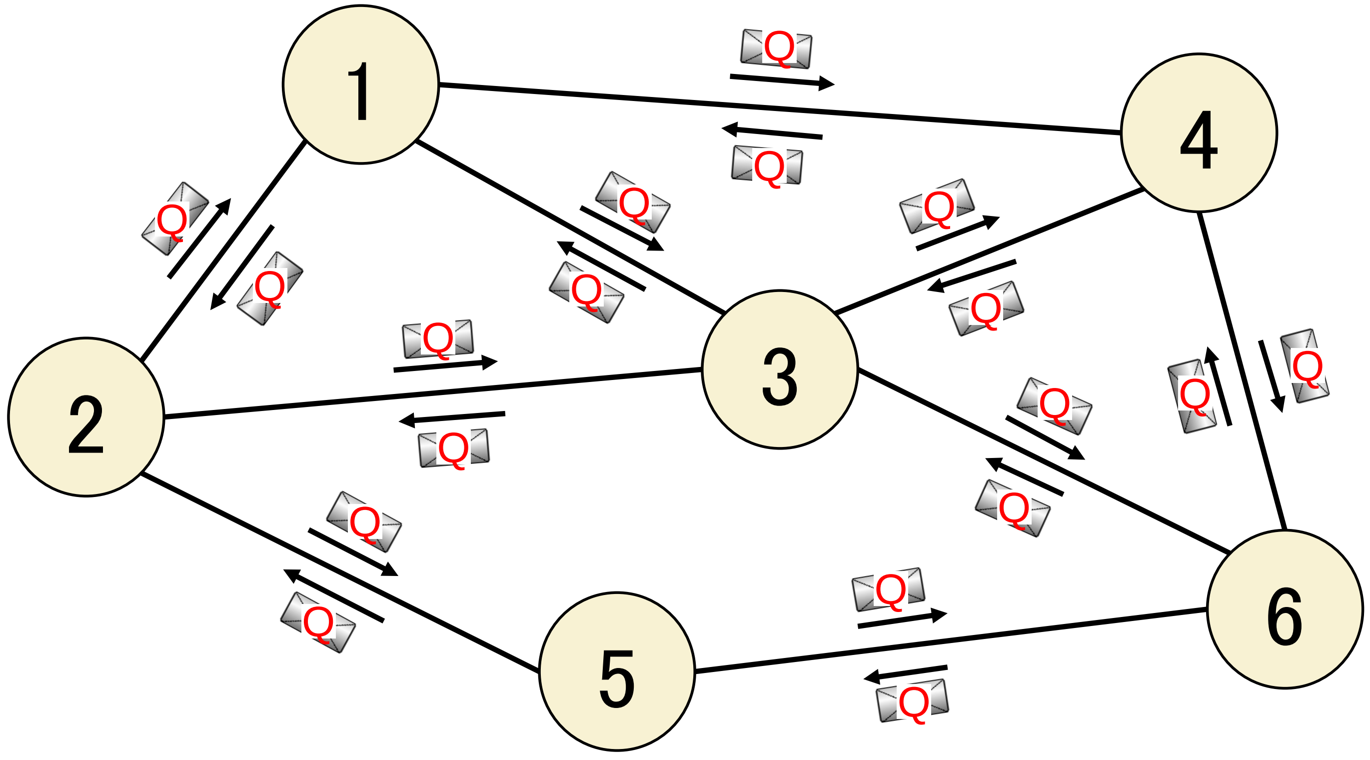
Quantum Distributed Computing: CONGEST and LOCAL

Quantum distributed computing

Now qubits can be sent instead of bits

(no prior entanglement between nodes)



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more formally:

- ✓ network G=(V,E) of n nodes (all nodes have distinct identifiers)
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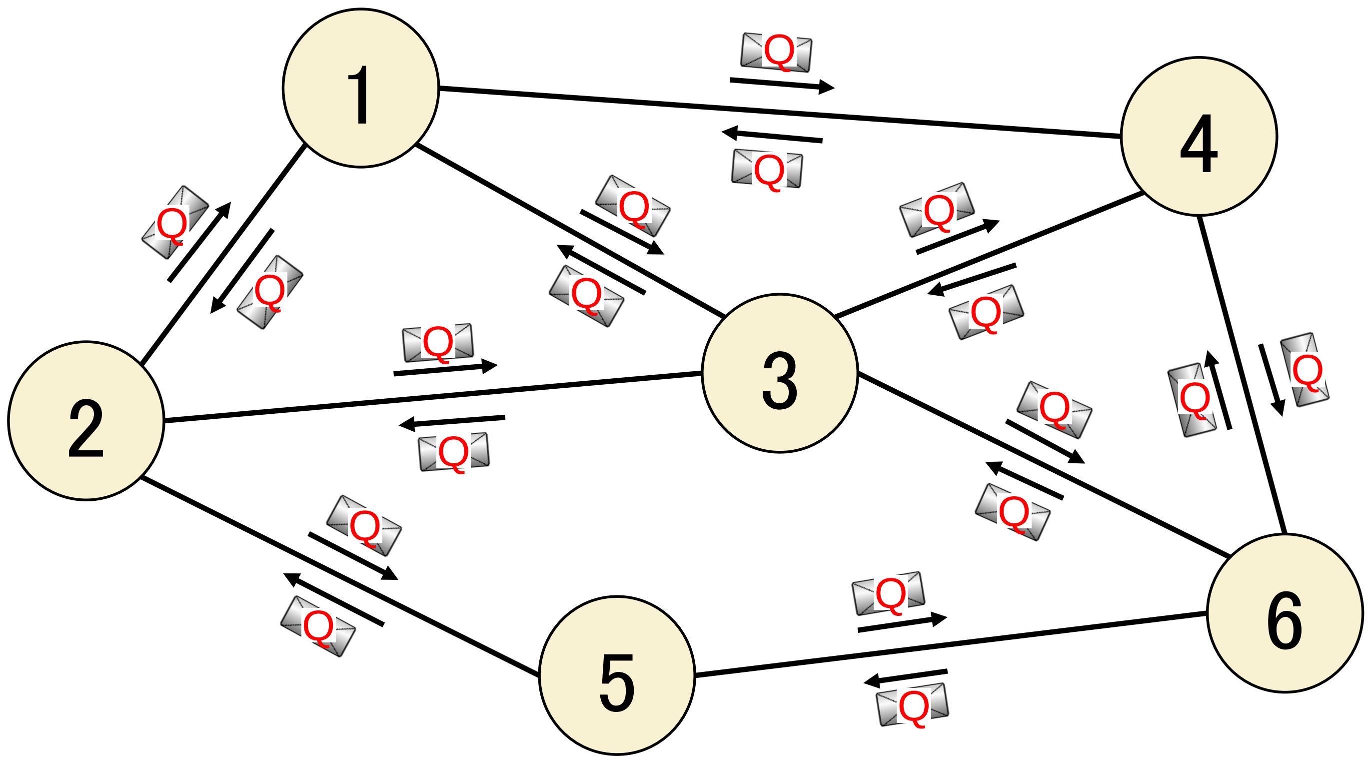
- ✓ each node is a quantum processor

Complexity: the number of rounds needed for the computation



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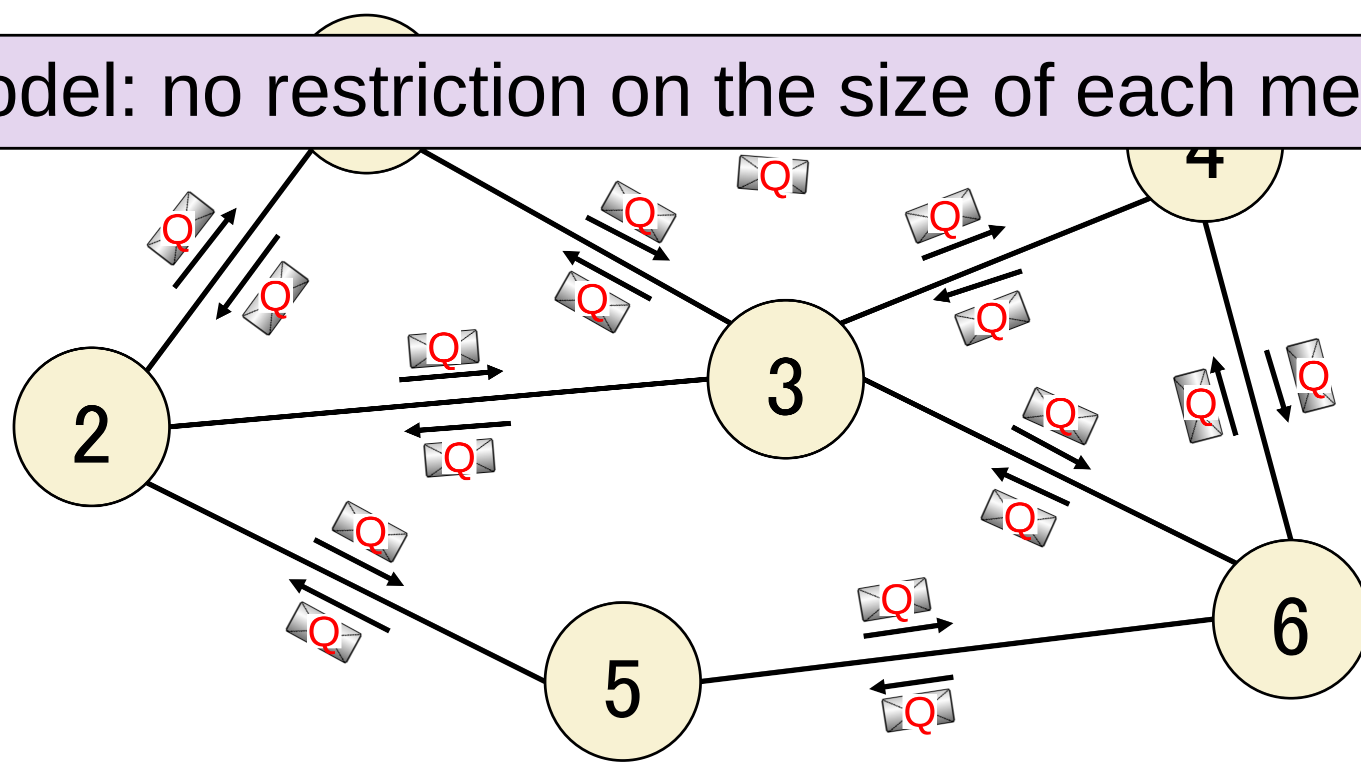

Quantum Advantage in the CONGEST model

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n: number of nodes of the network

CONGEST model: only O(log n) qubits per message

[LG, Magniez 18] ⇨ The diameter of the network can be computed in $\Theta(\sqrt{n})$ rounds in the quantum CONGEST model but requires $\Theta(n)$ rounds in the classical CONGEST model (when the diameter is constant)

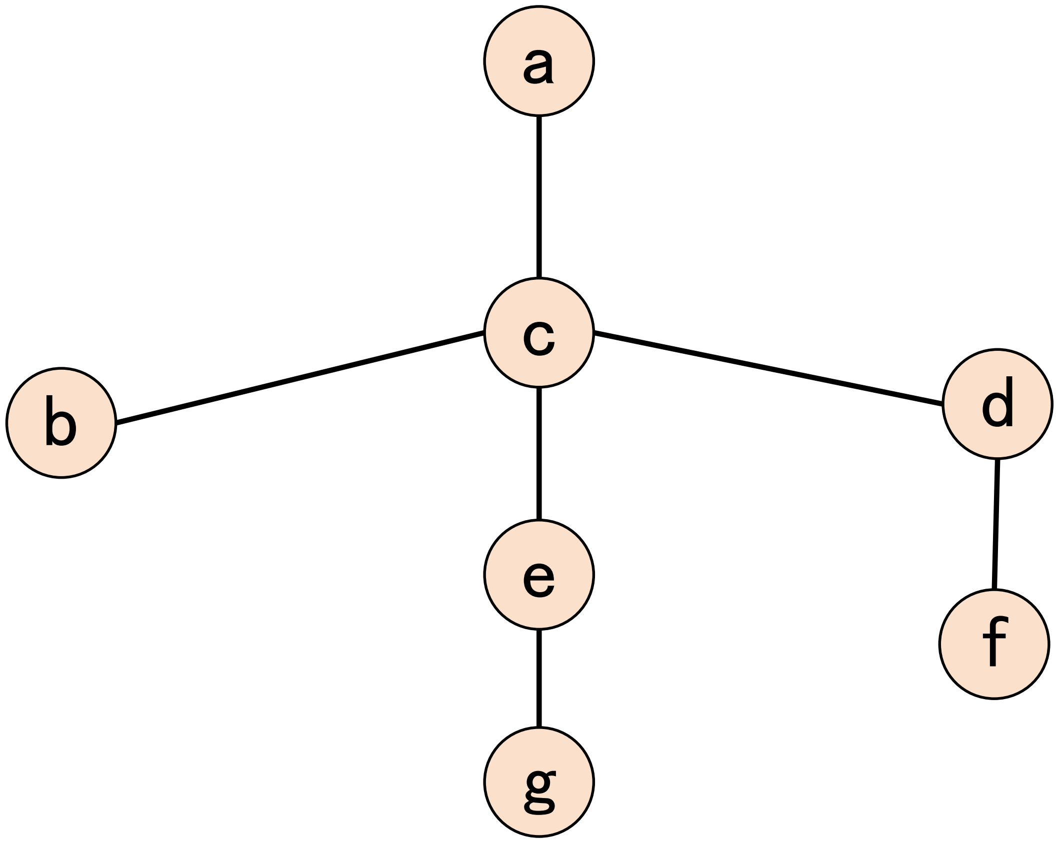
Diameter and Eccentricity

Consider an undirected and unweighted graph G = (V,E)

The diameter of the graph is the maximum distance between two nodes

$$D = \max_{u,v \in V} \{d(u,v)\}$$

d(u,v) = distance between u and v



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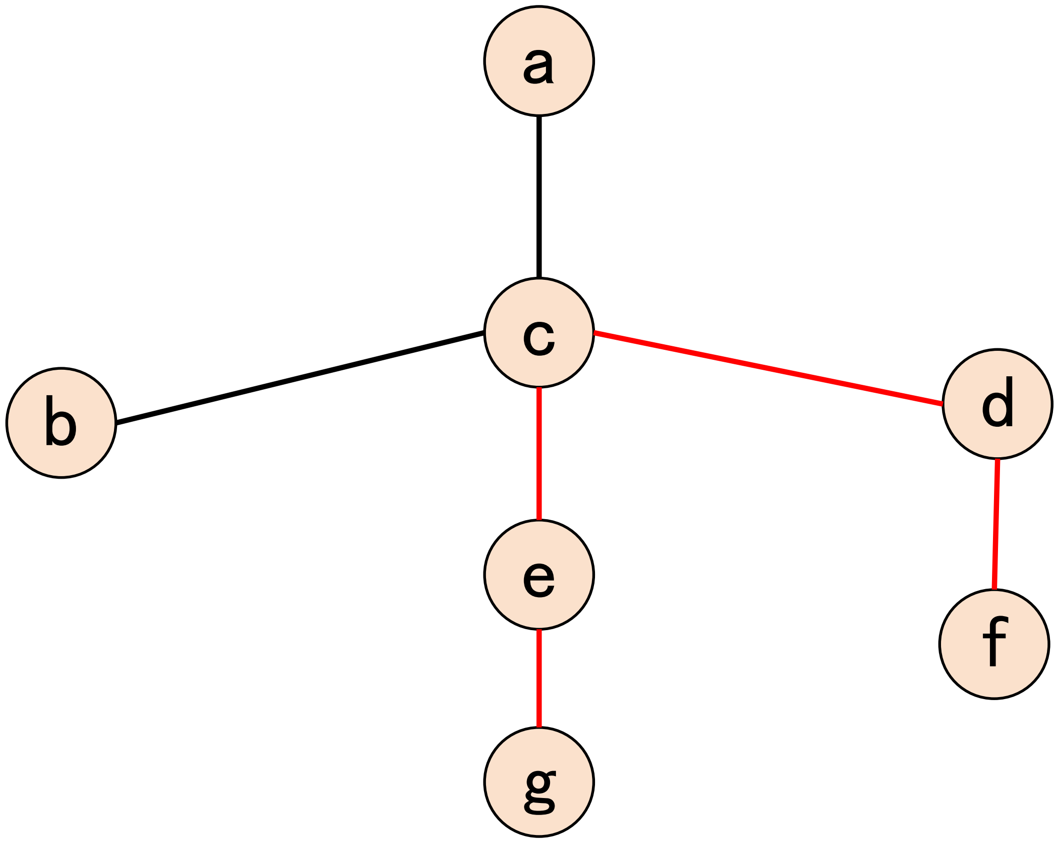

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$$= \max_{u \in V} \{ecc(u)\}$$

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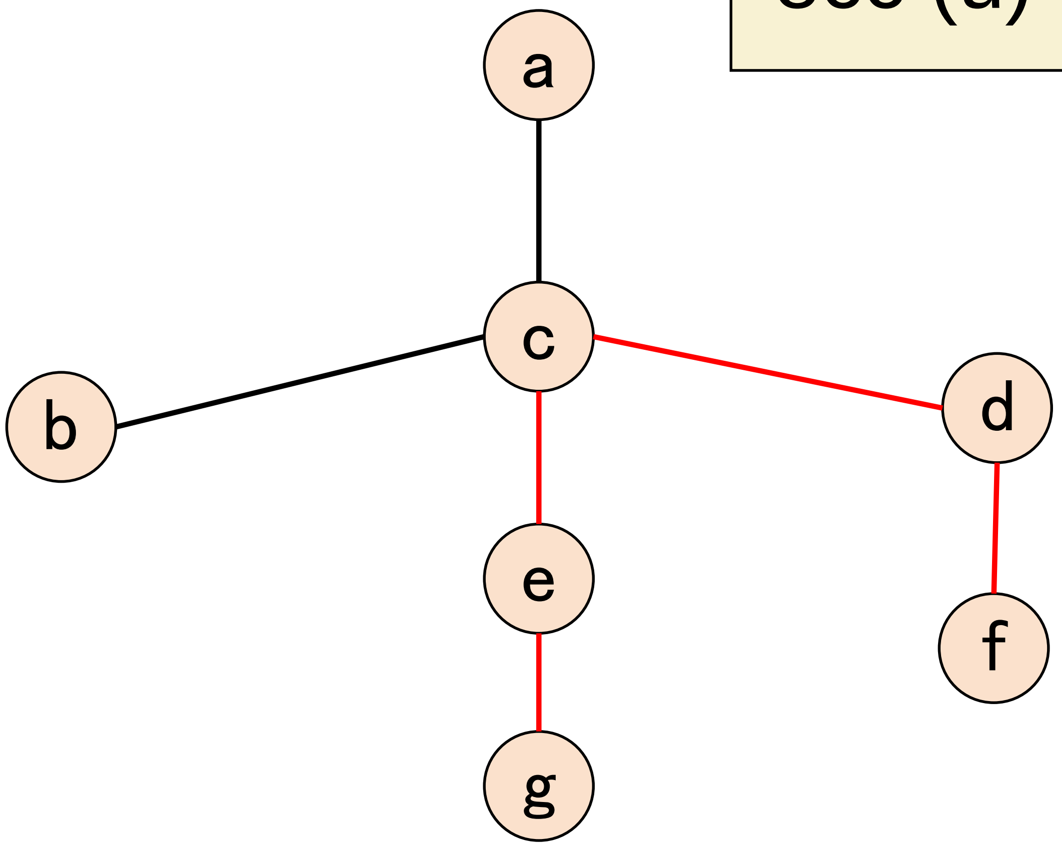
The eccentricity of a node u is defined as

$$ecc(u) = \max_{v \in V} \{d(u,v)\}$$



ecc (a) = 3
ecc (b) = 3
ecc (c) = 2
ecc (d) = 3
ecc (e) = 3
ecc (f) = 4
ecc (g) = 4

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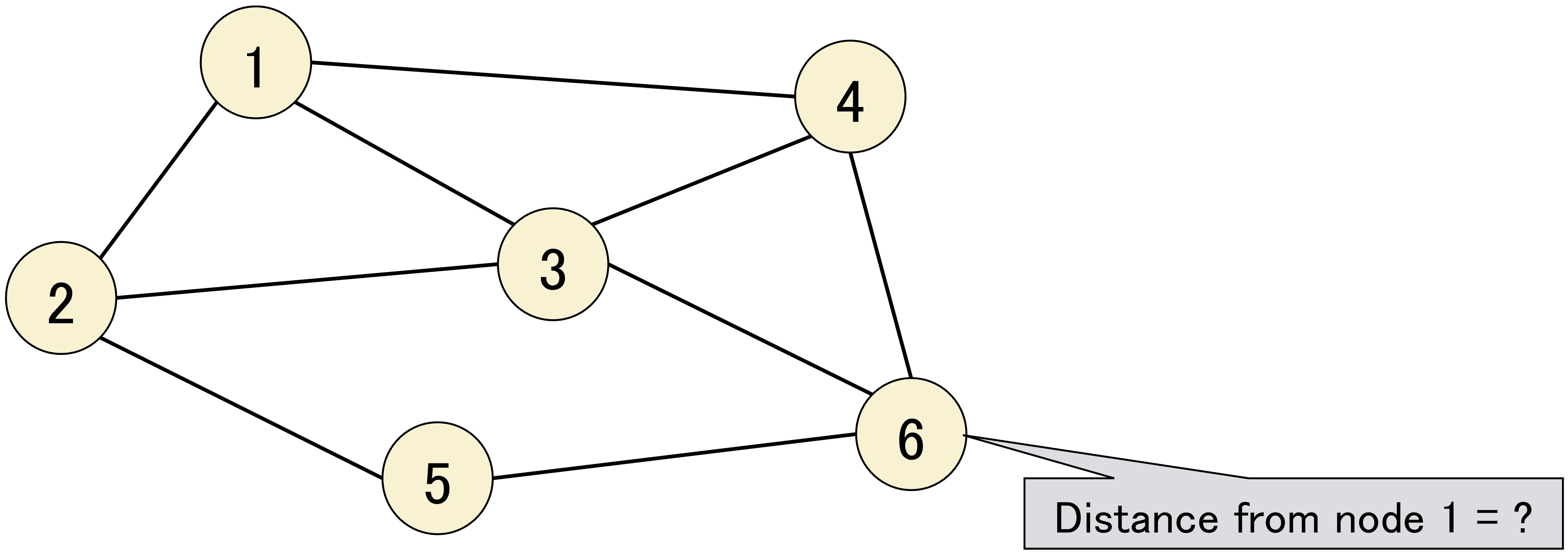
Classical Distributed Computing: Computing Distances



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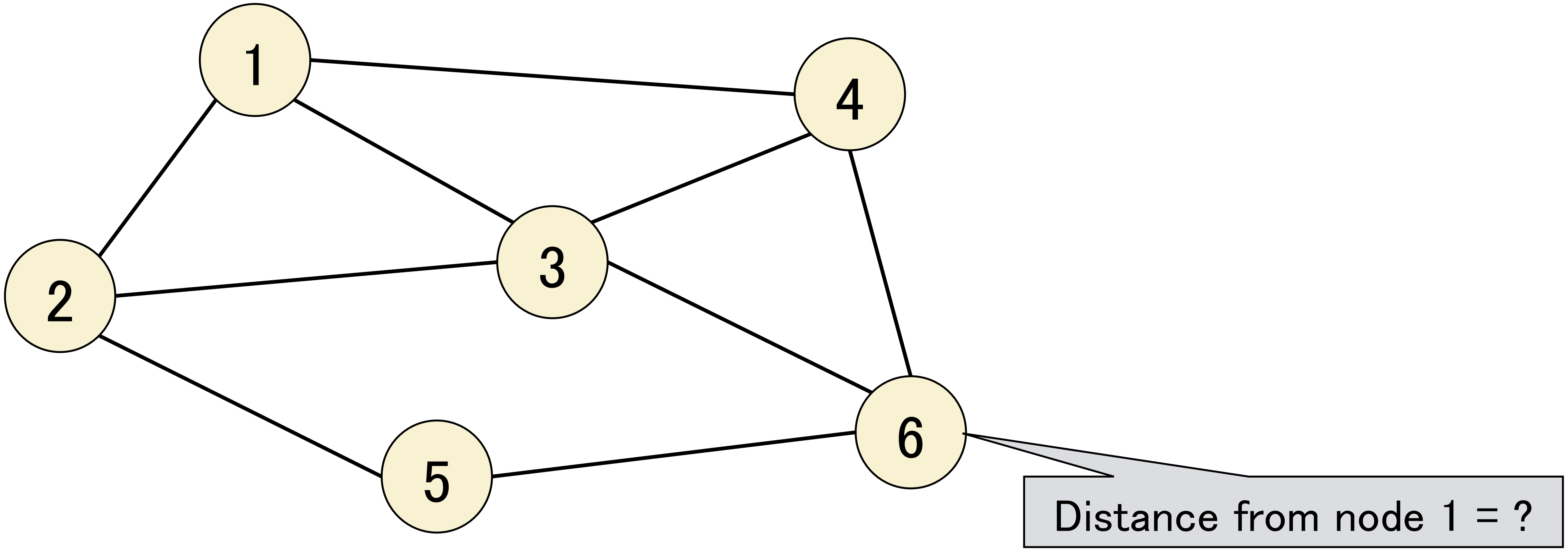

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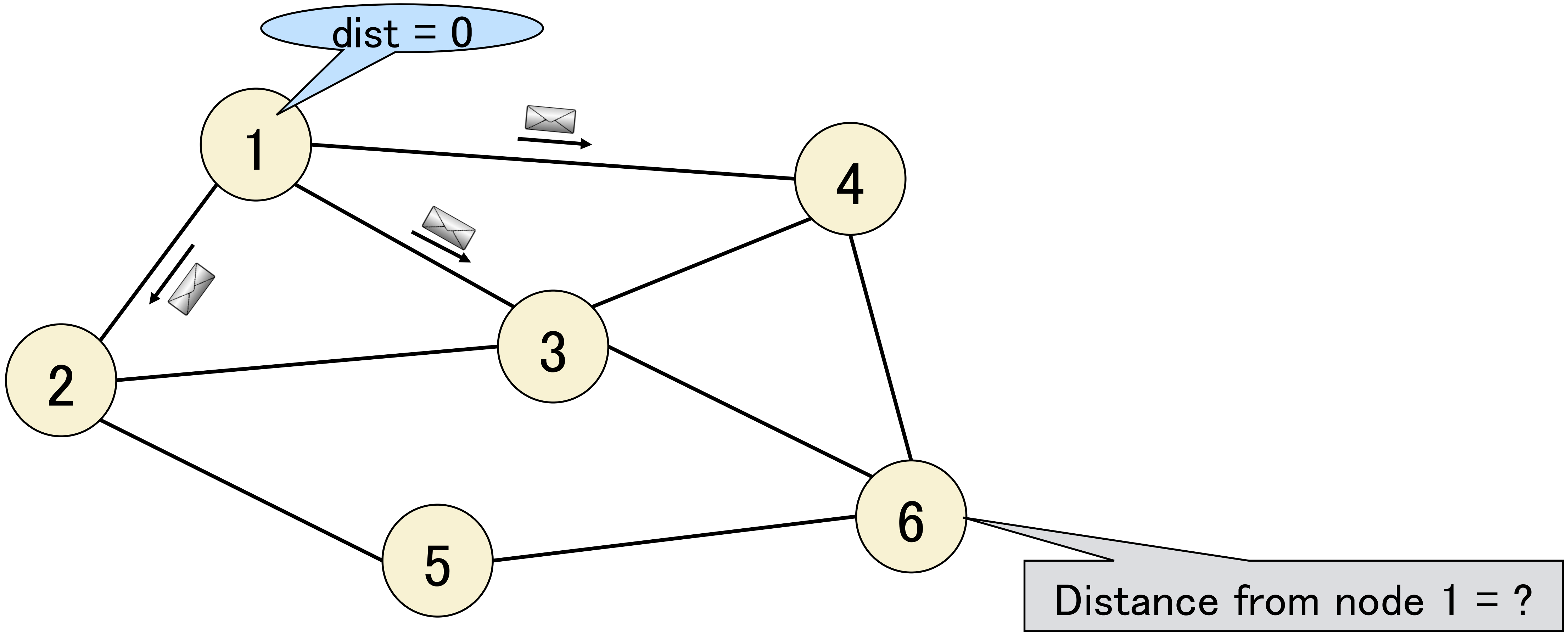

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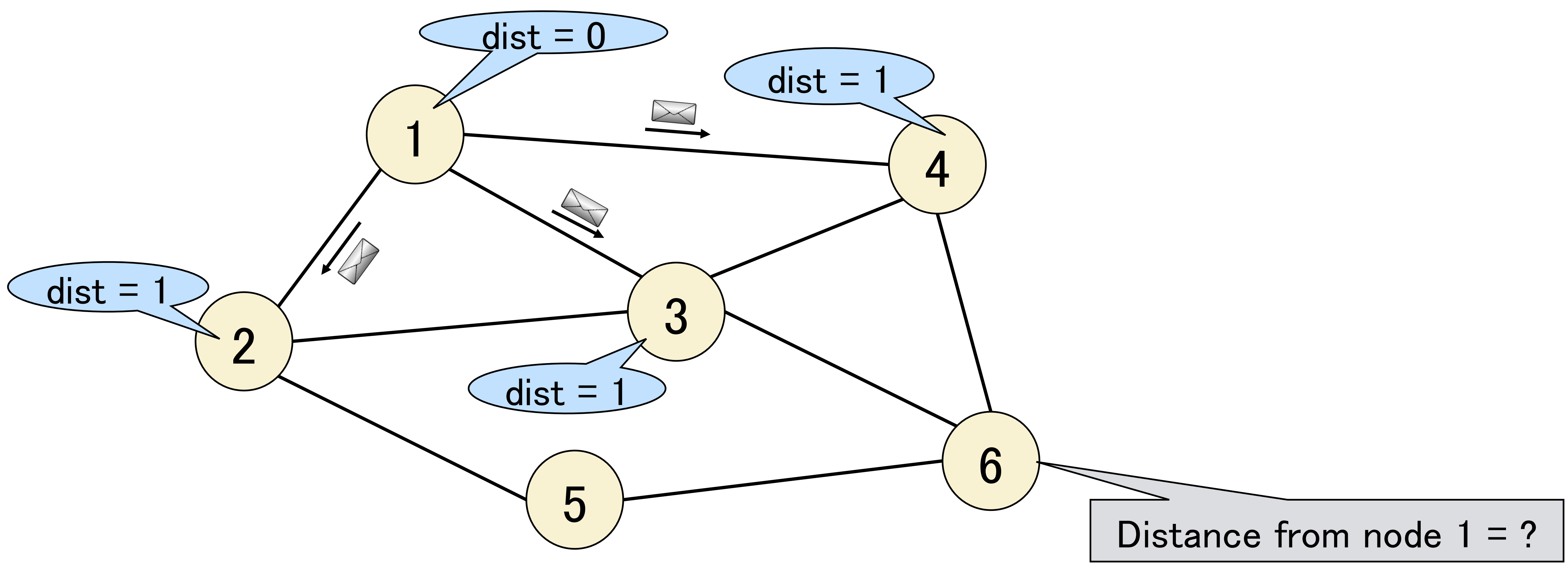

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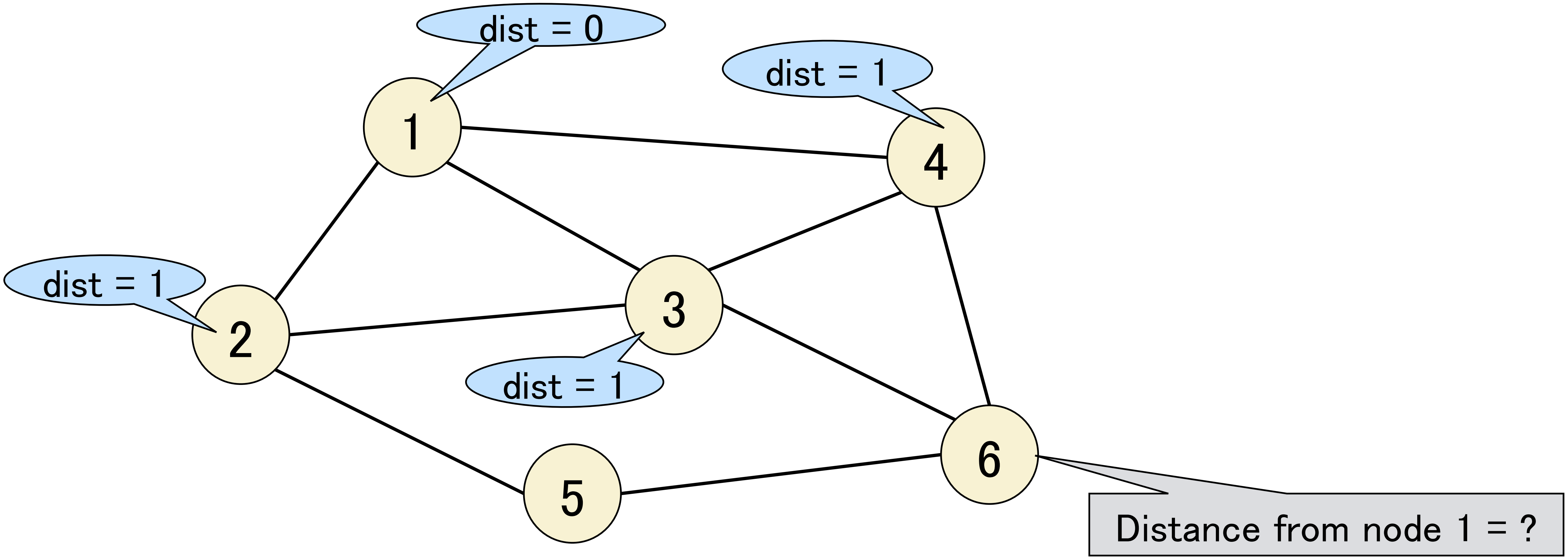

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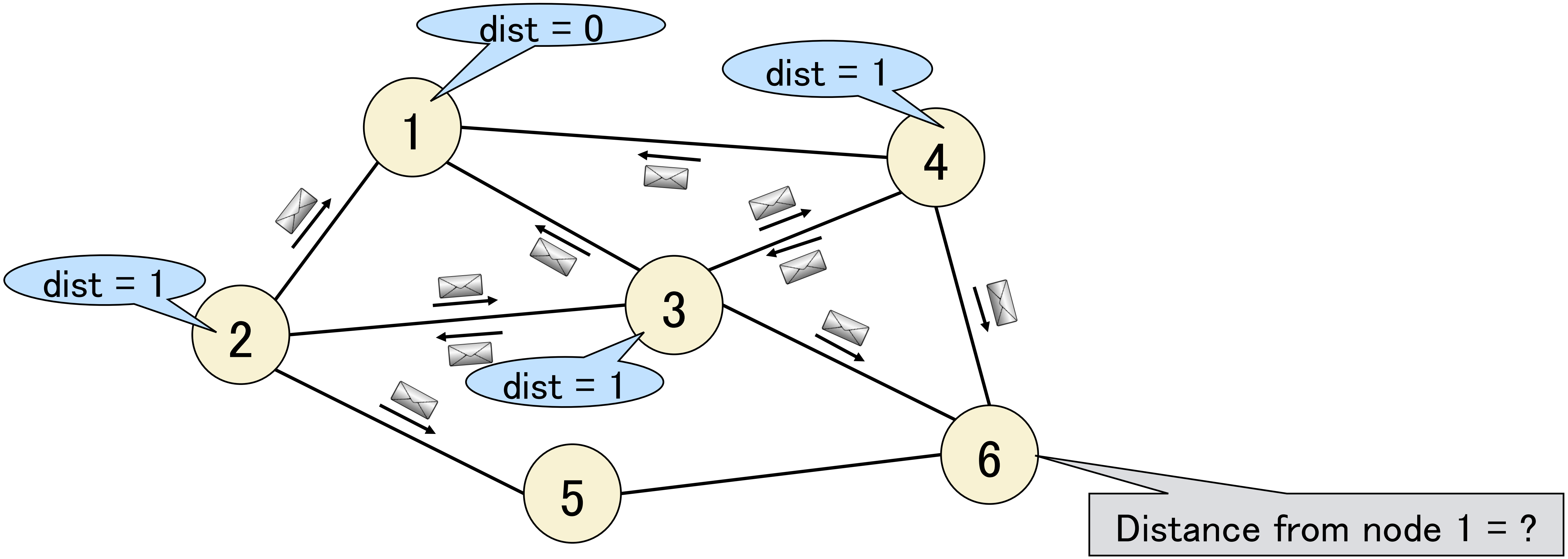

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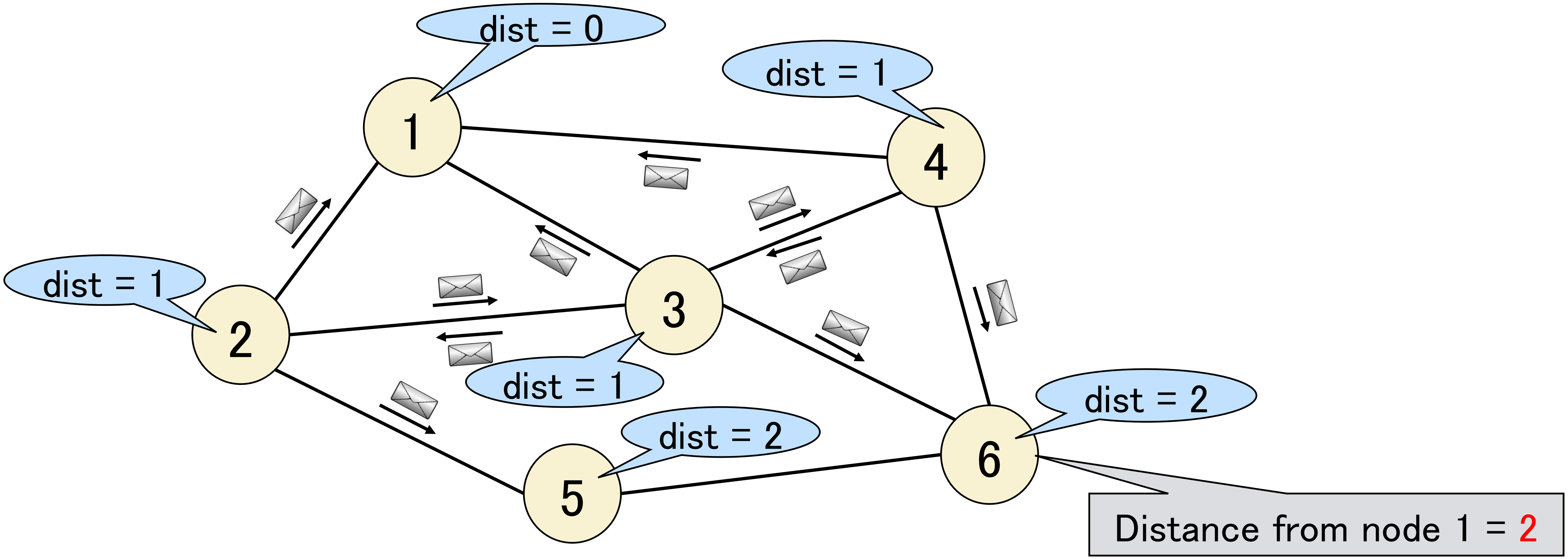

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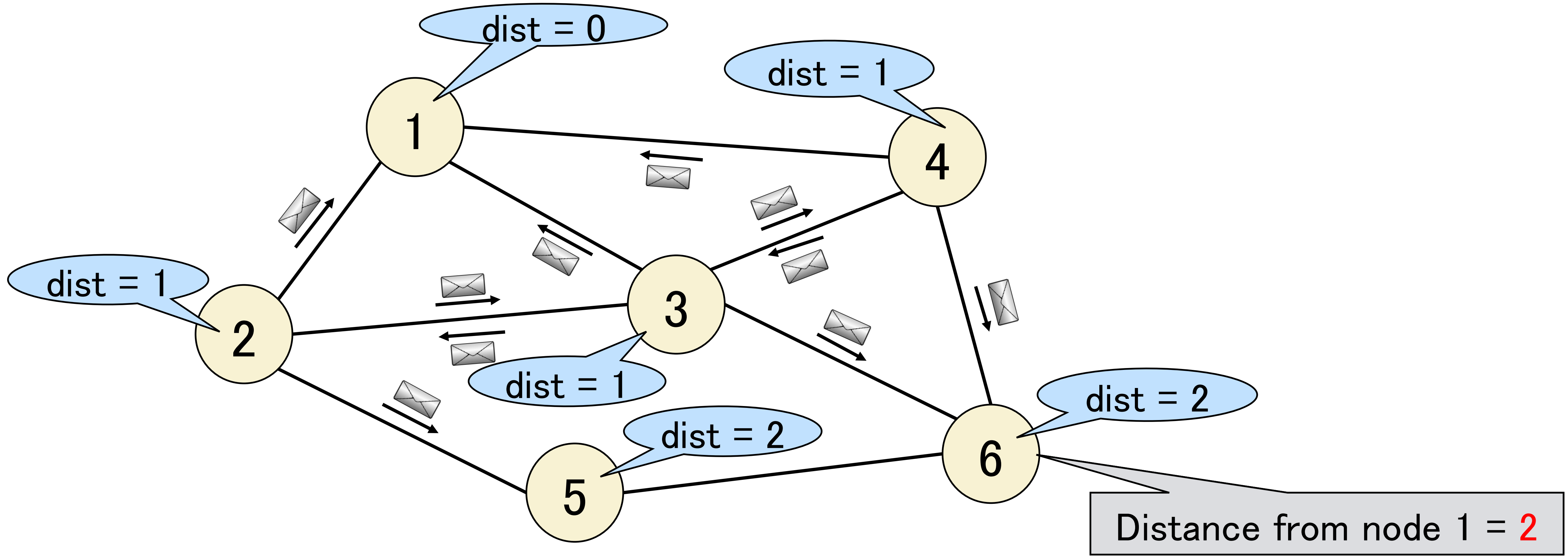

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We show that we can do better in the quantum setting

Computation of the Diameter in the CONGEST model

Main result [LG, Magniez 2018]

sublinear-round quantum computation of the diameter whenever D=o(n)

	Classical	Quantum
Exact computation (upper bounds)	$O(n)$ [Holzer+12, Peleg+12]	$O(\sqrt{nD})$
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Define the function $f: V \rightarrow \{0,1\}$ such that $f(u) = \begin{cases} 1 & \text{if } ecc(u) \geq d \\ 0 & \text{otherwise} \end{cases}$

Goal: find u such that f(u) = 1 (or report that no such vertex exists)

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This is a search problem
Idea: try to use Grover search

Define the function f: V → {0,1} such that f(u) = $\begin{cases} 1 \text{ if ecc (u)} \geq d \\ 0 \text{ otherwise} \end{cases}$

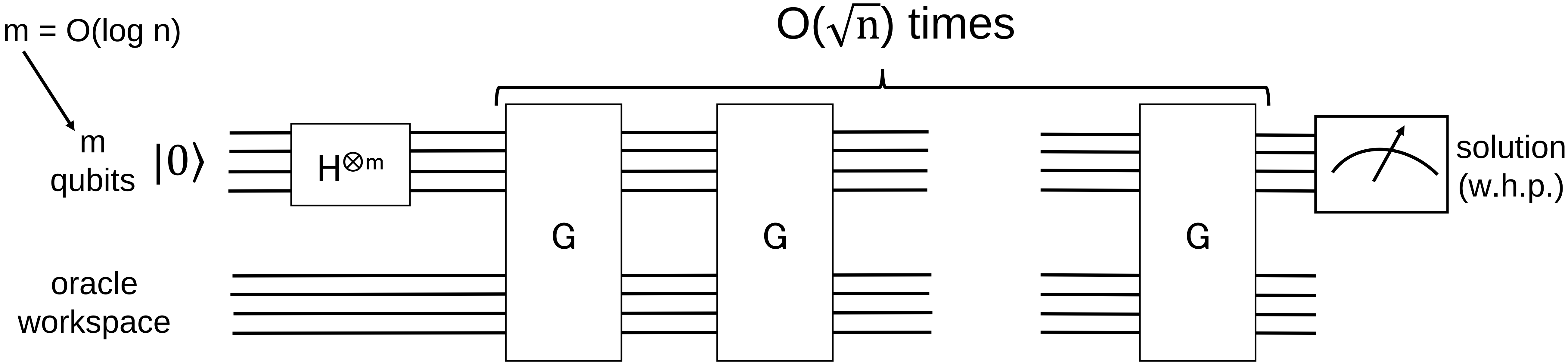
Goal: find u such that f(u) = 1 (or report that no such vertex exists)

> There is a quantum algorithm for this search problem using $O(\sqrt{n})$ calls to a black box evaluating f

Quantum search [Grover 96]

n = |V| (number of nodes)

Recap: Grover Algorithm



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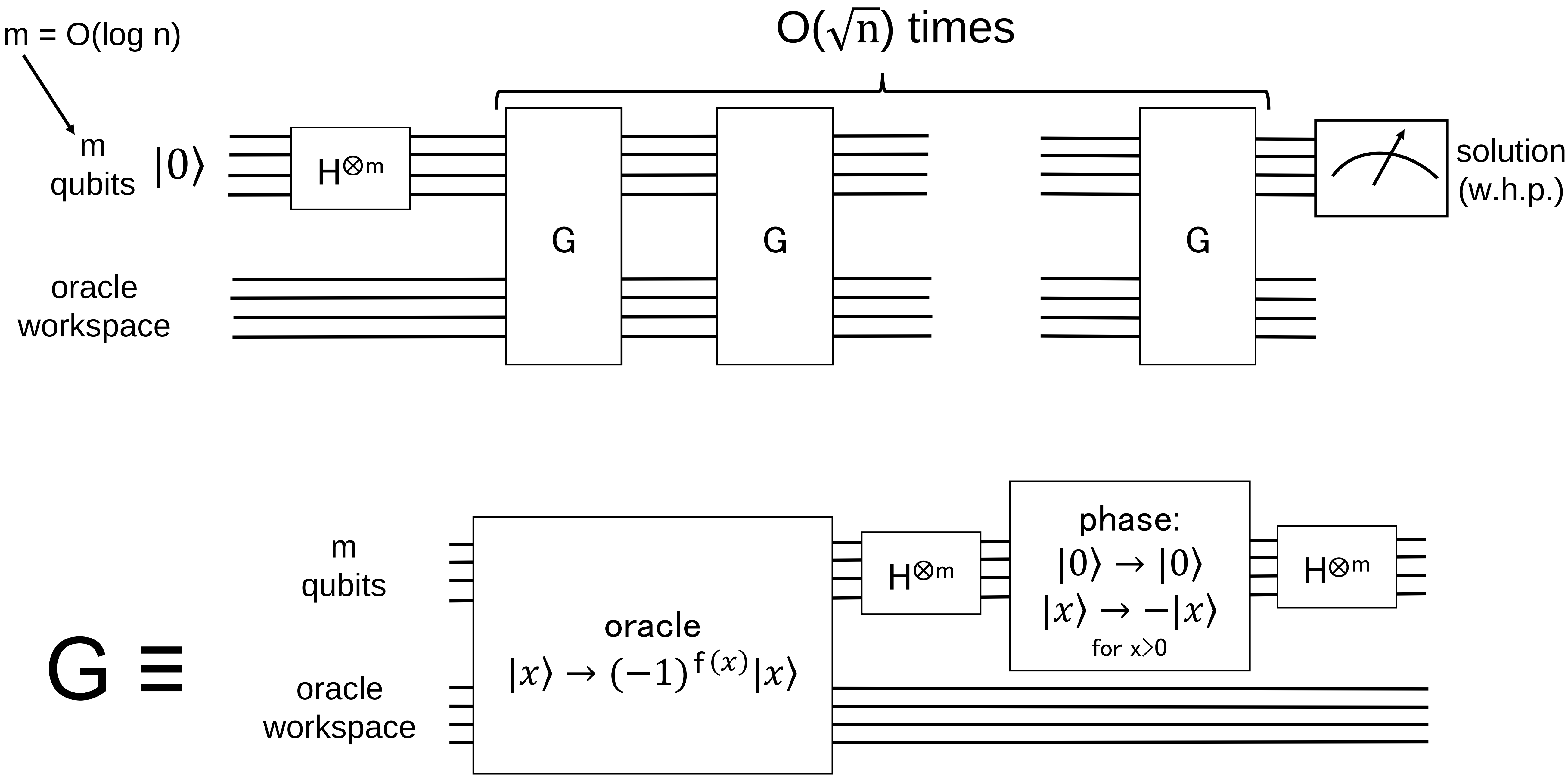

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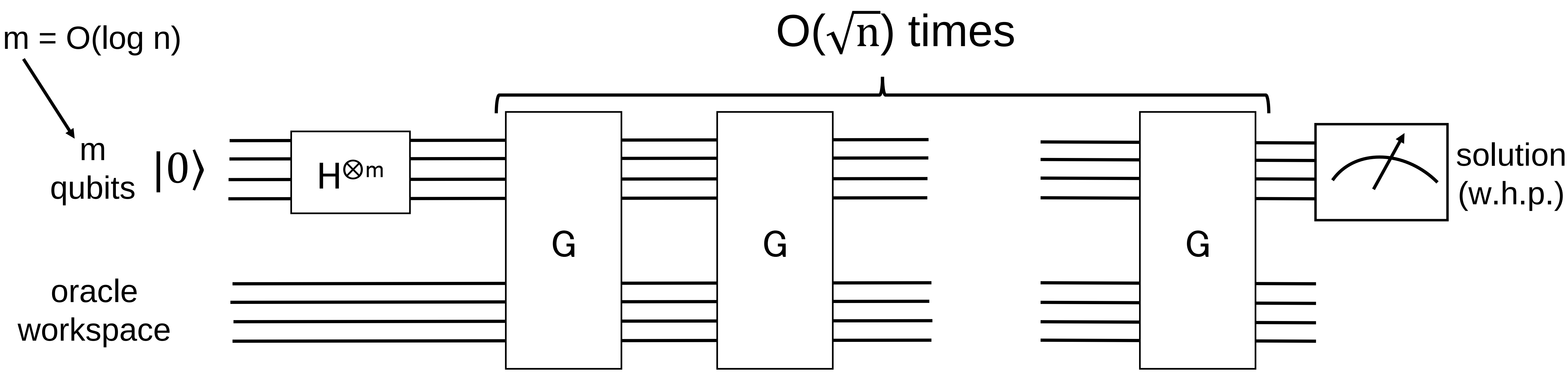

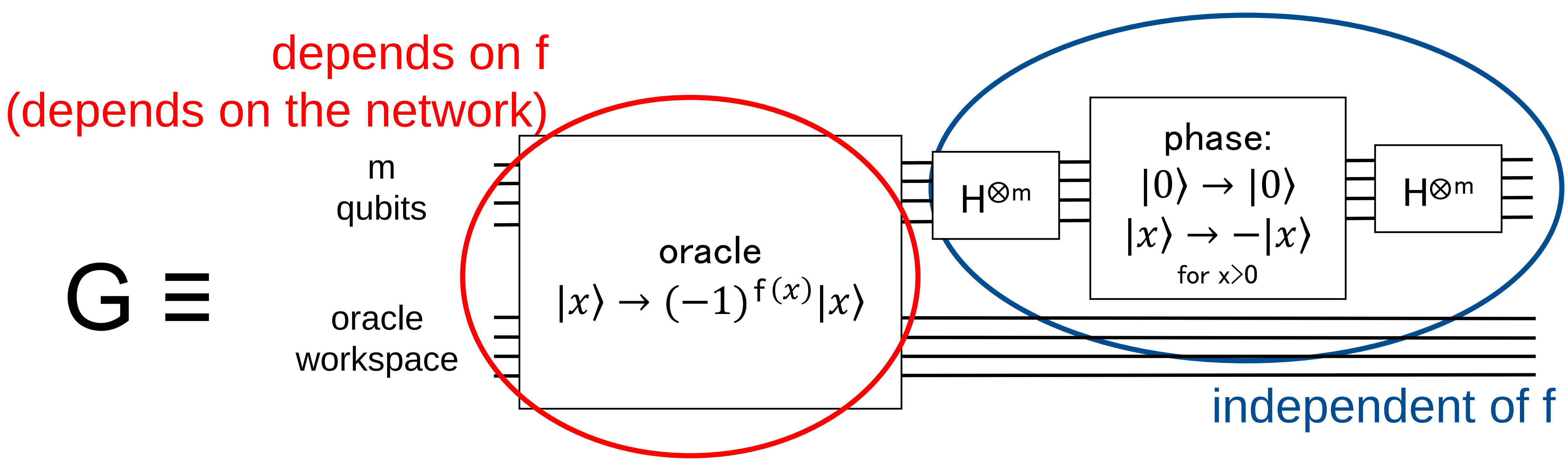

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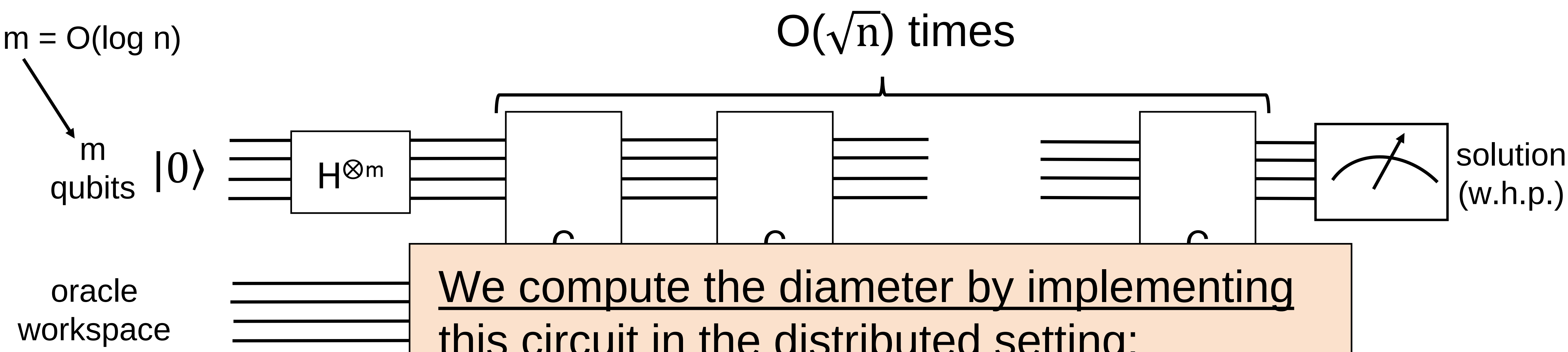
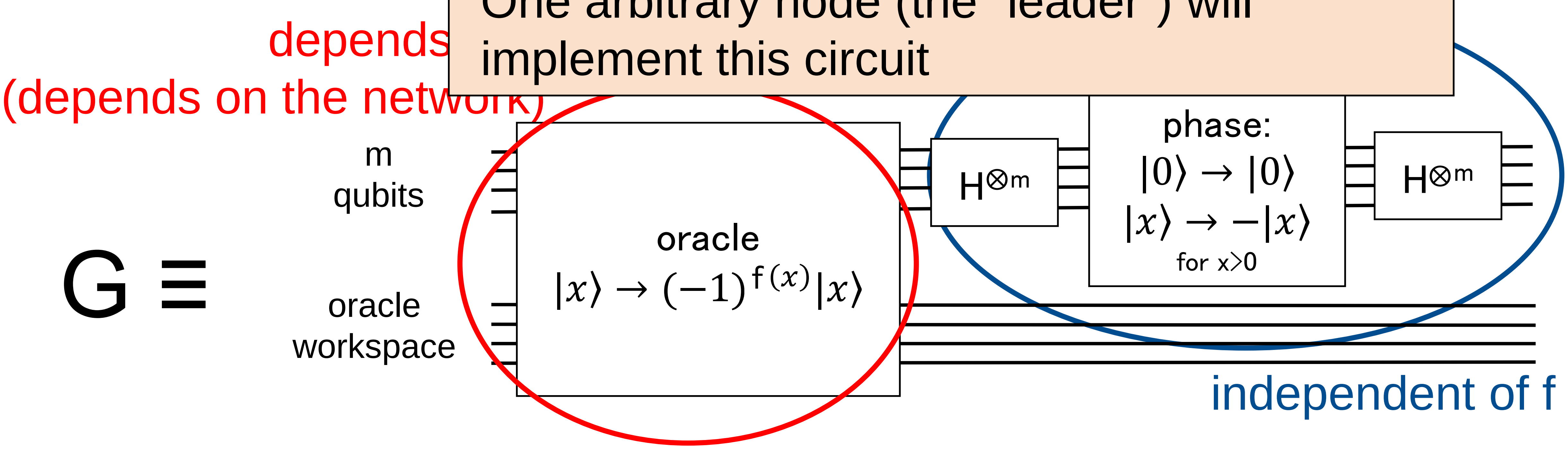


<u>We compute the diameter by implementing this circuit in the distributed setting</u>:

One arbitrary node (the "leader") will implement this circuit



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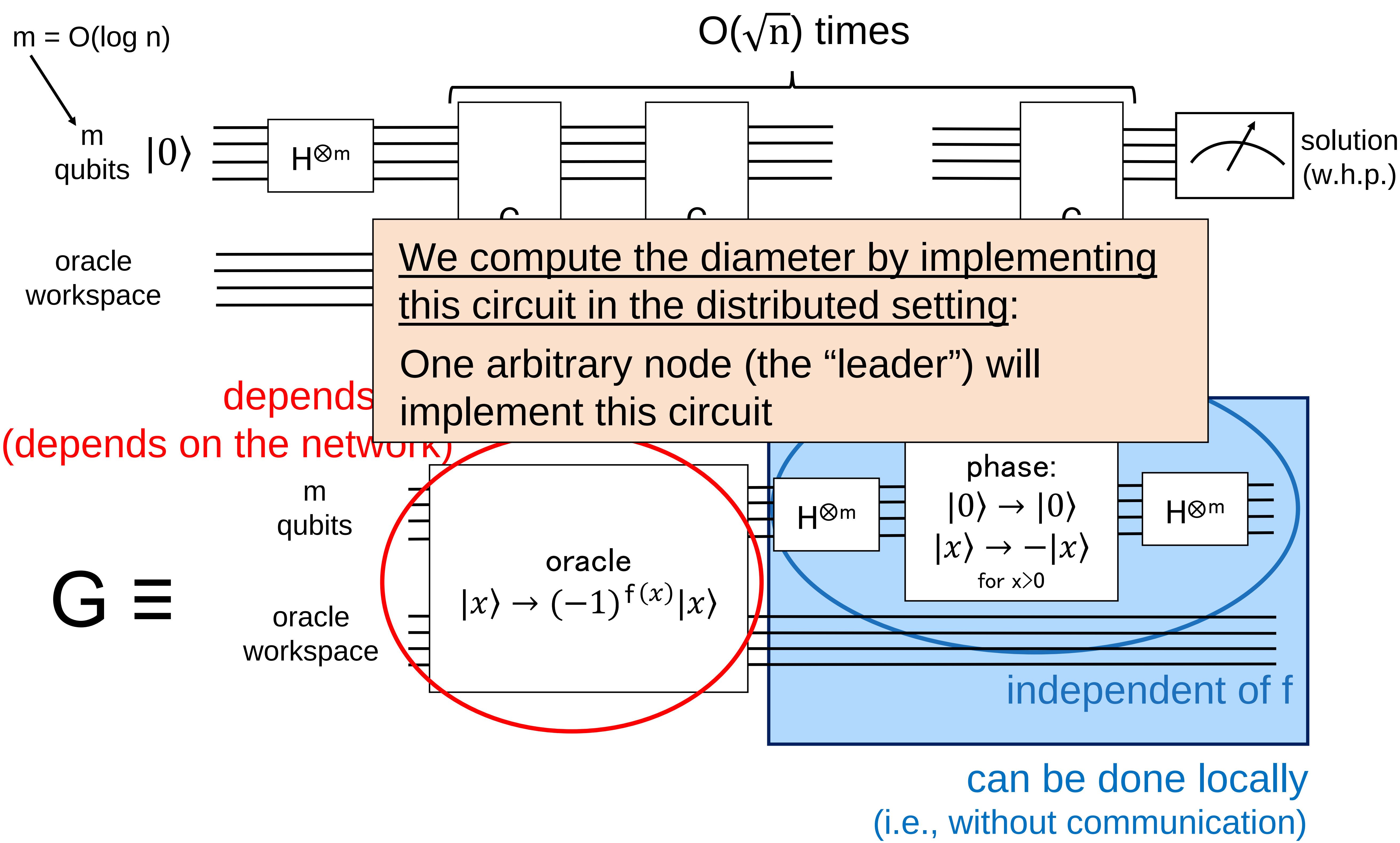

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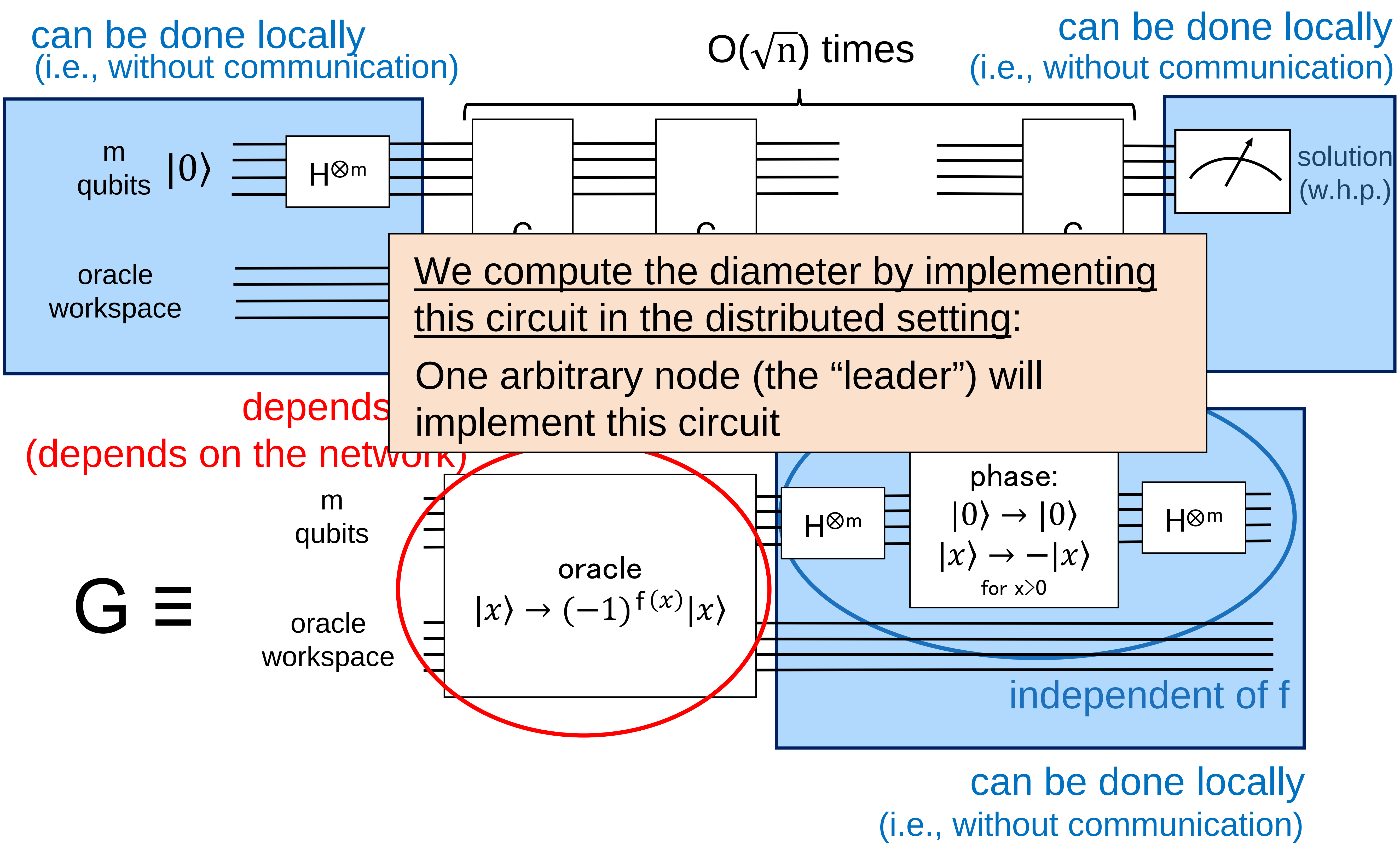

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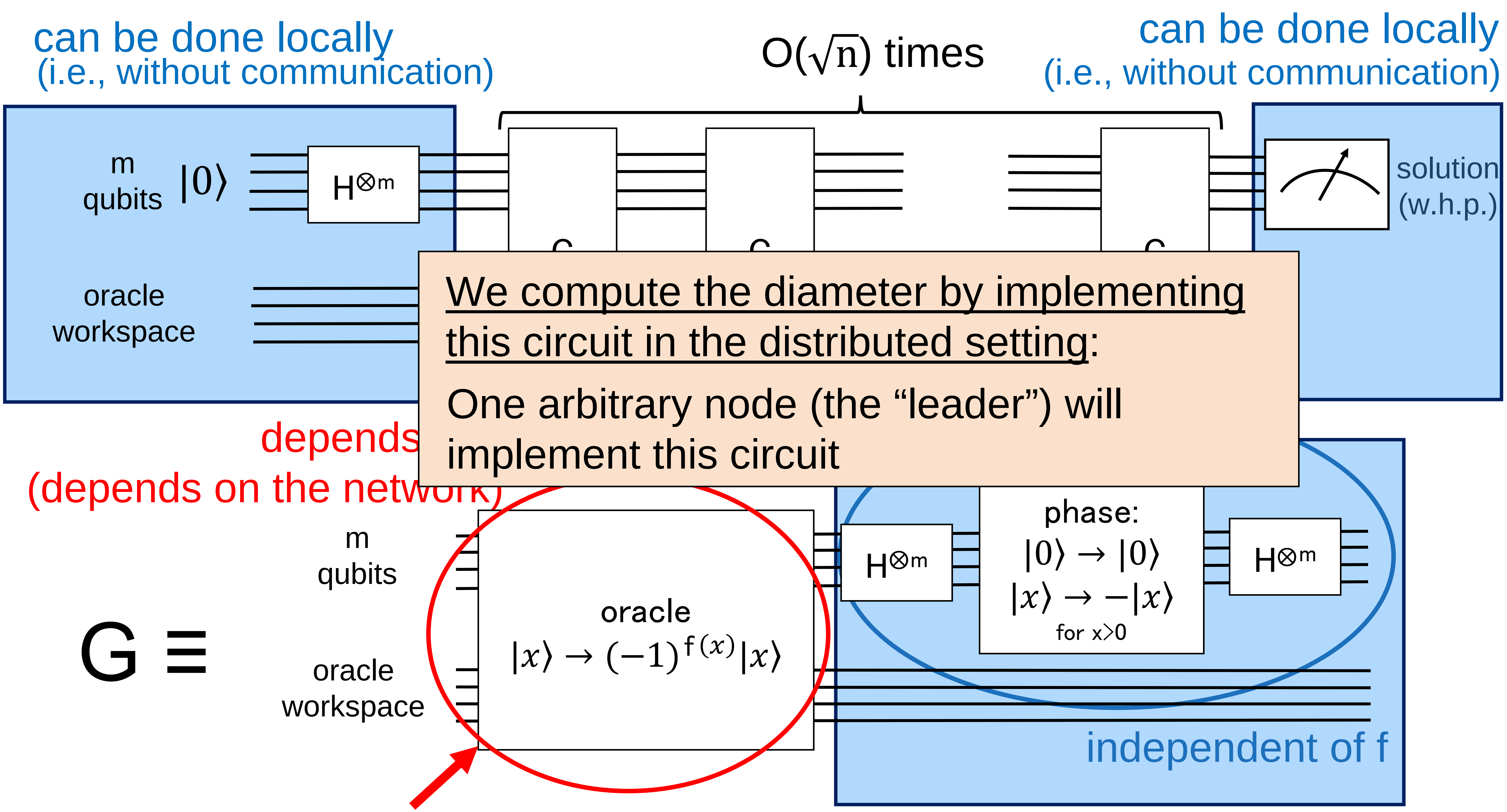

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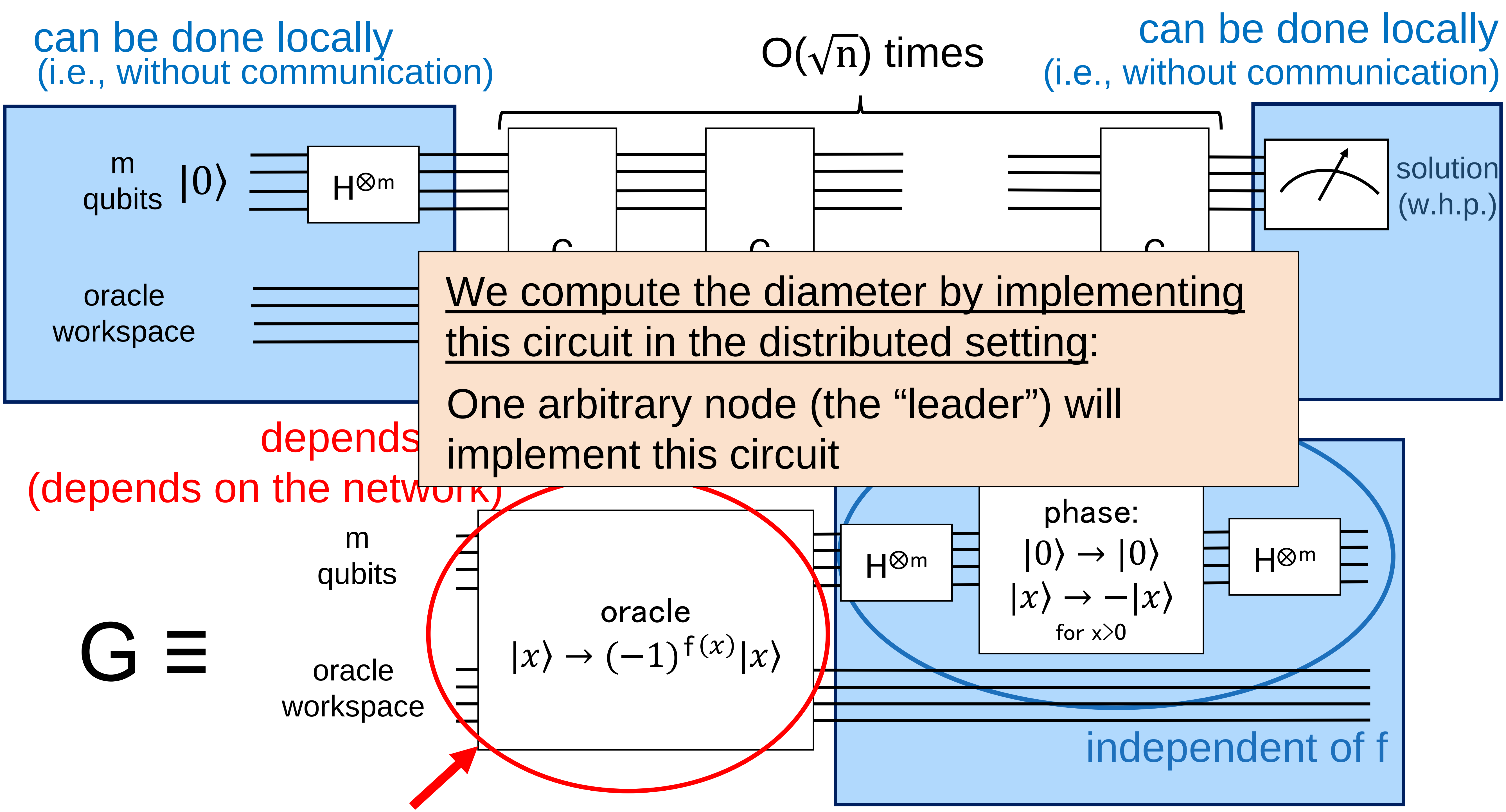

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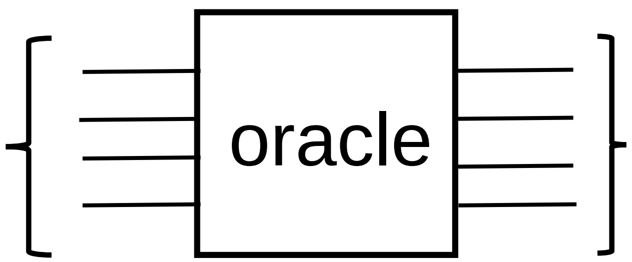
Total number of rounds of communication = O($\sqrt{n}$ x number of rounds to implement the oracle)

Implementation of the Oracle in O(D) rounds

$|u\rangle|0\rangle$

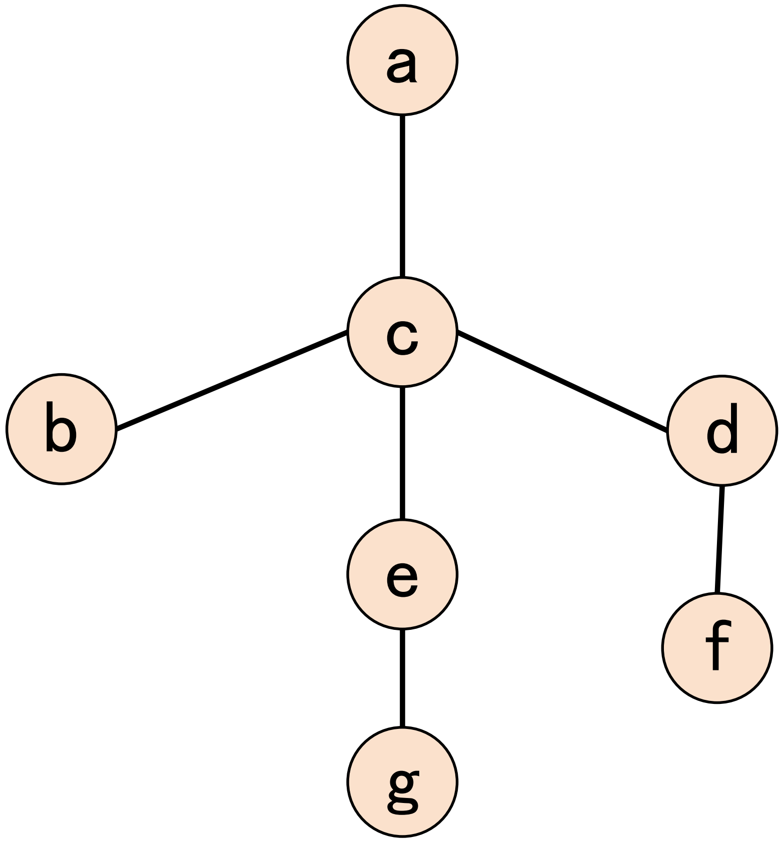


$|u\rangle|ecc(u)\rangle$

Example:

V={a,b,c,d,e,f,g}

here leader = node a



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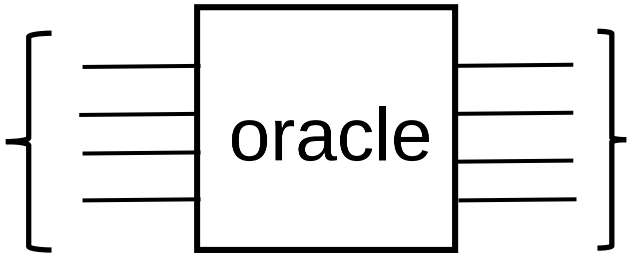
$$\sum_{u \in V} \alpha_u |u\rangle|0\rangle$$



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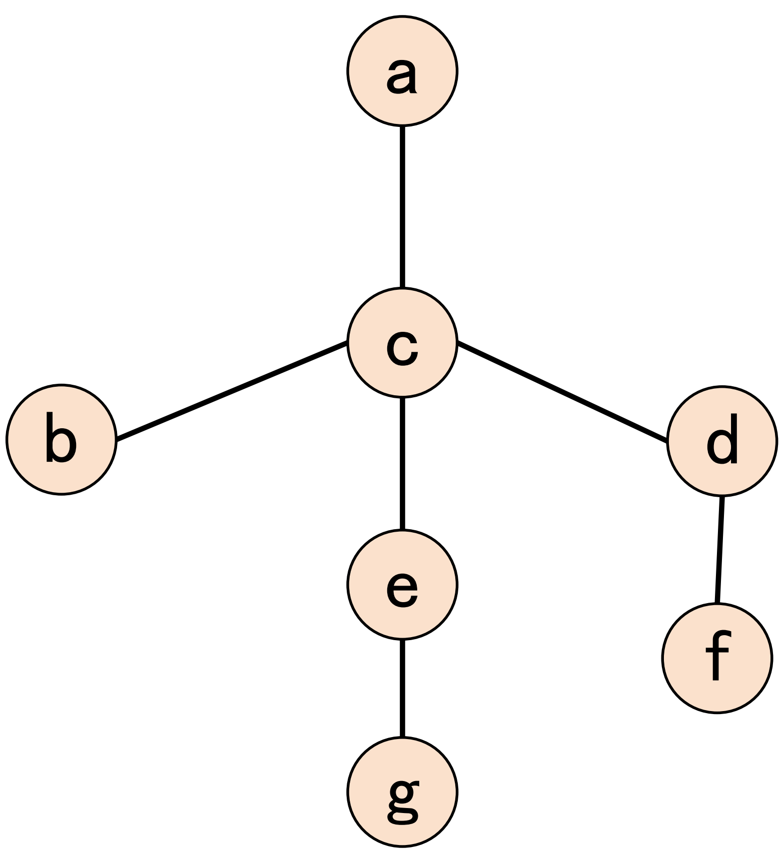

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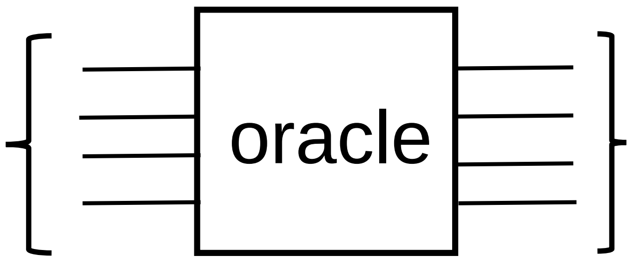
$\sum_{u \in V} \alpha_u |u\rangle |0\rangle$  $\sum_{u \in V} \alpha_u |u\rangle |ecc(u)\rangle$

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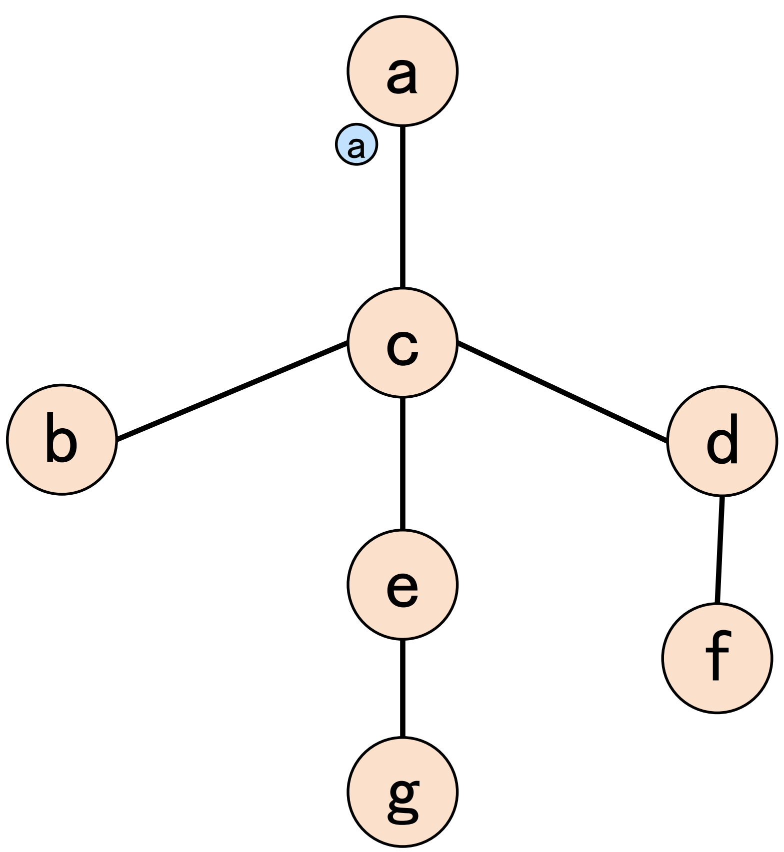

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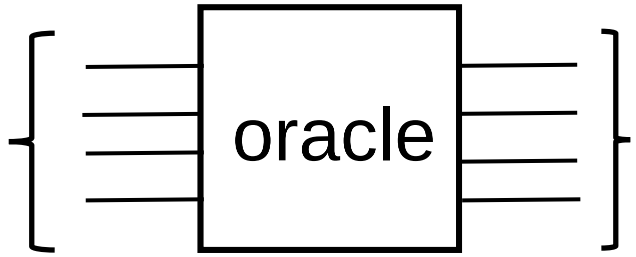

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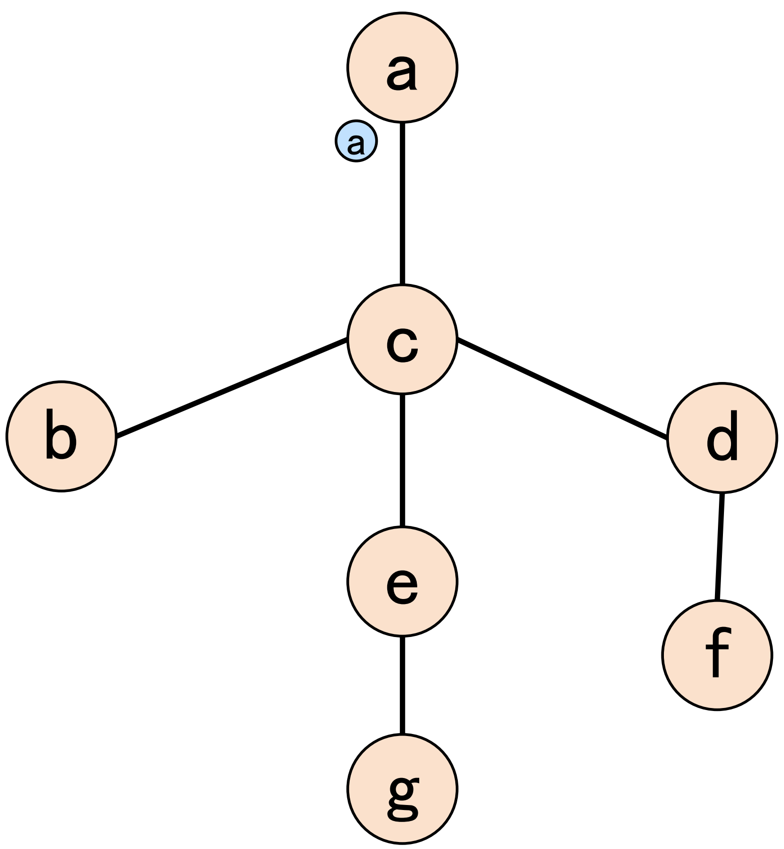

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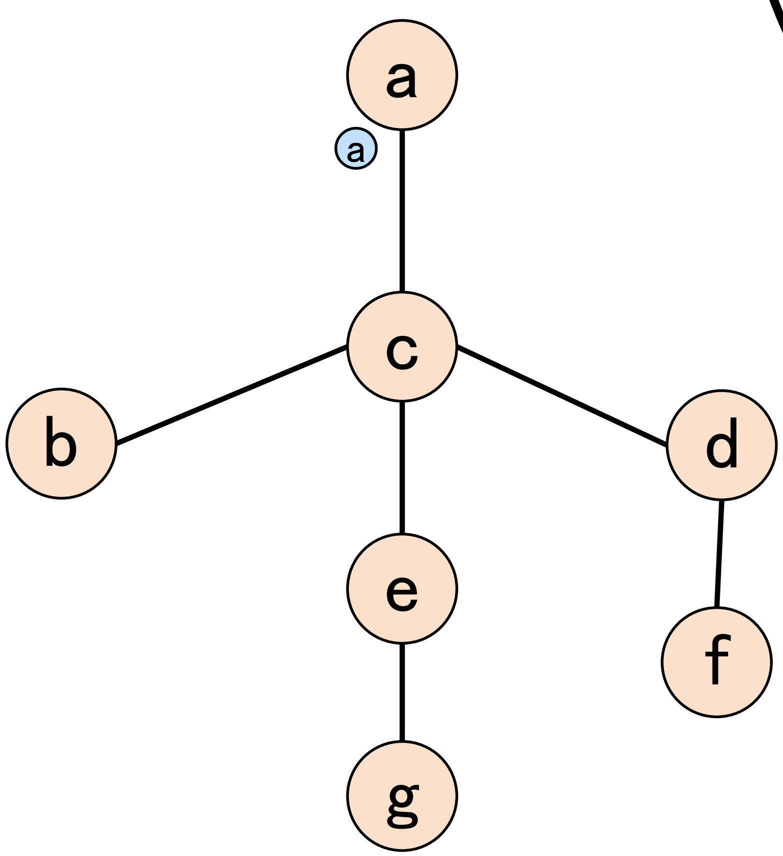

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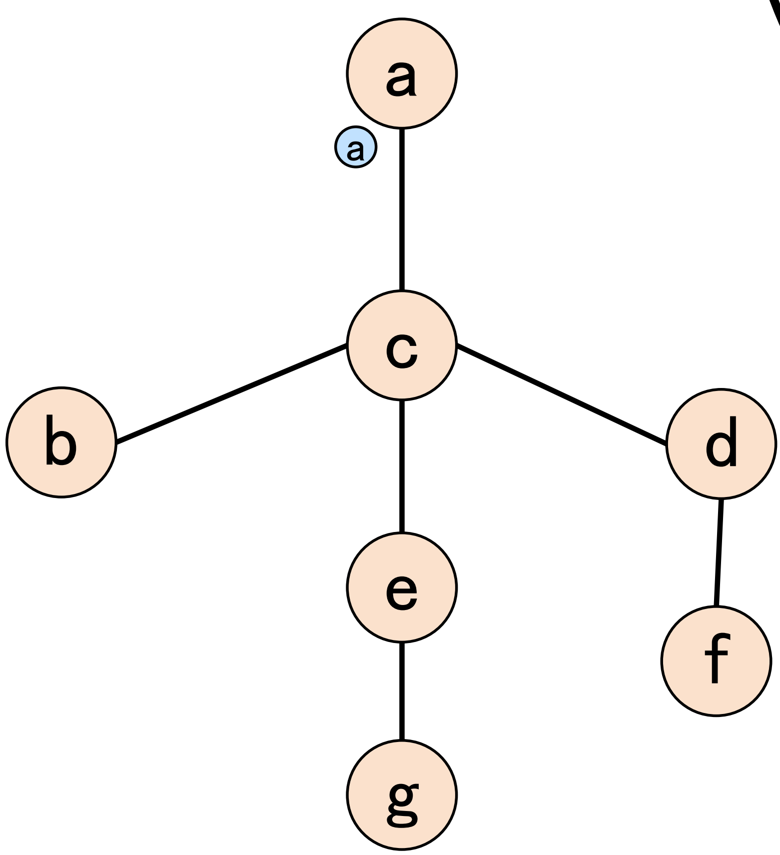

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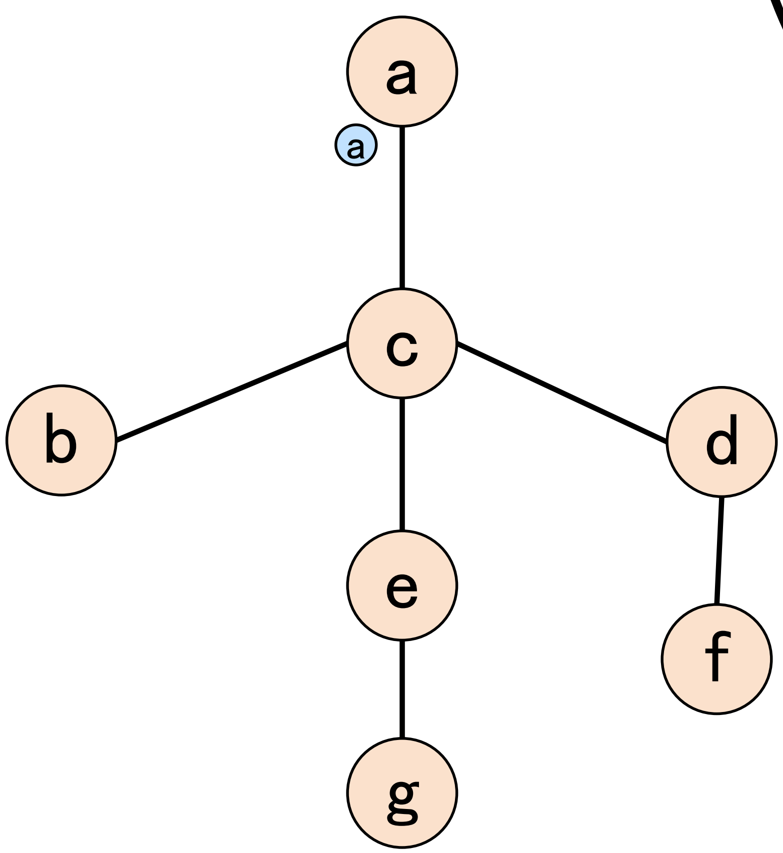

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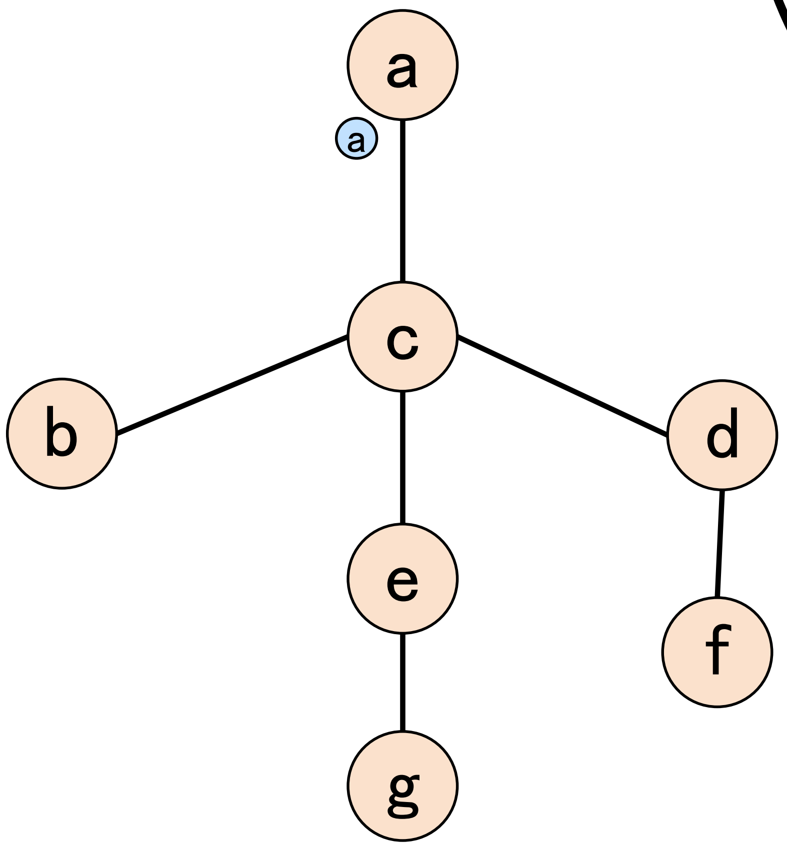

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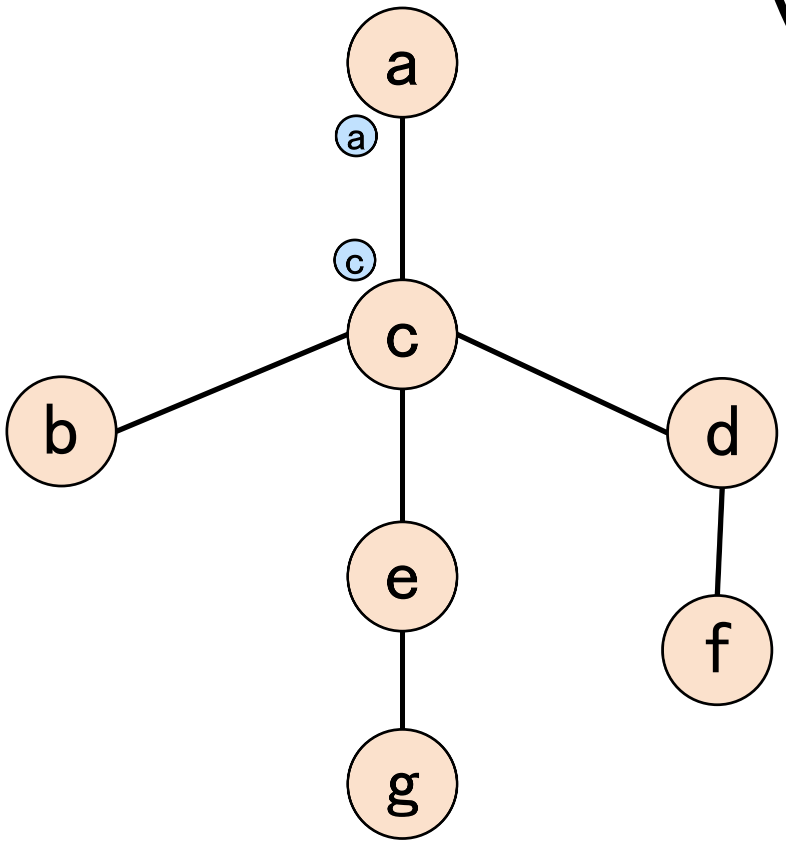

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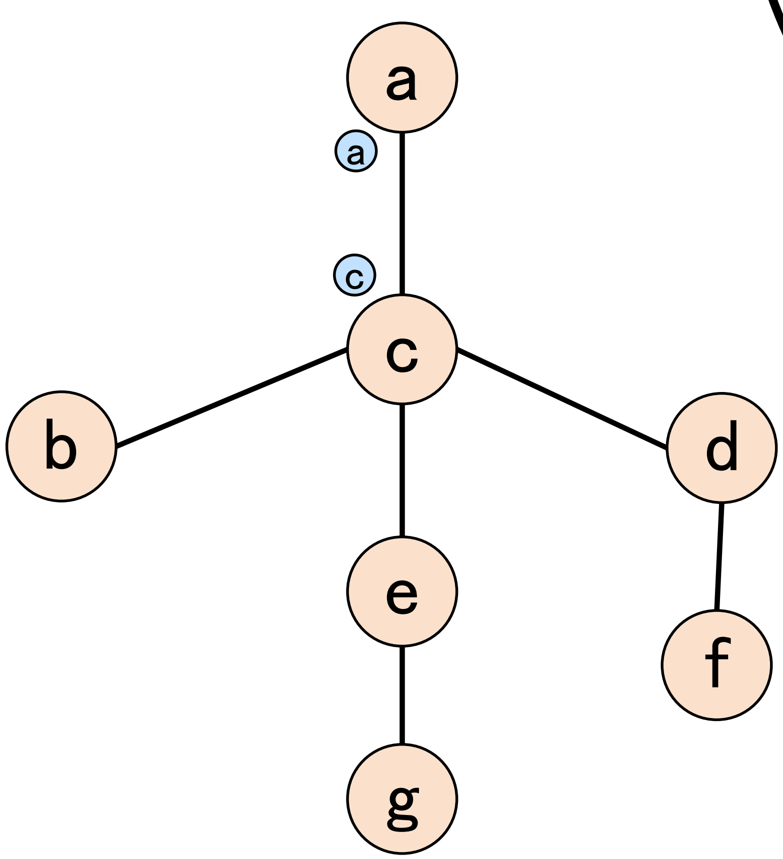

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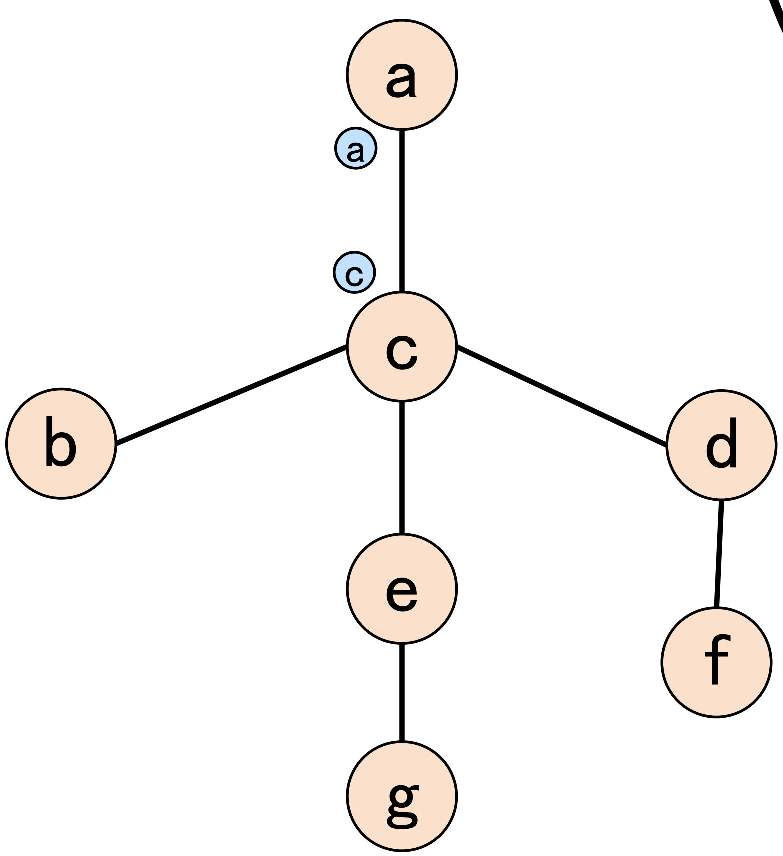

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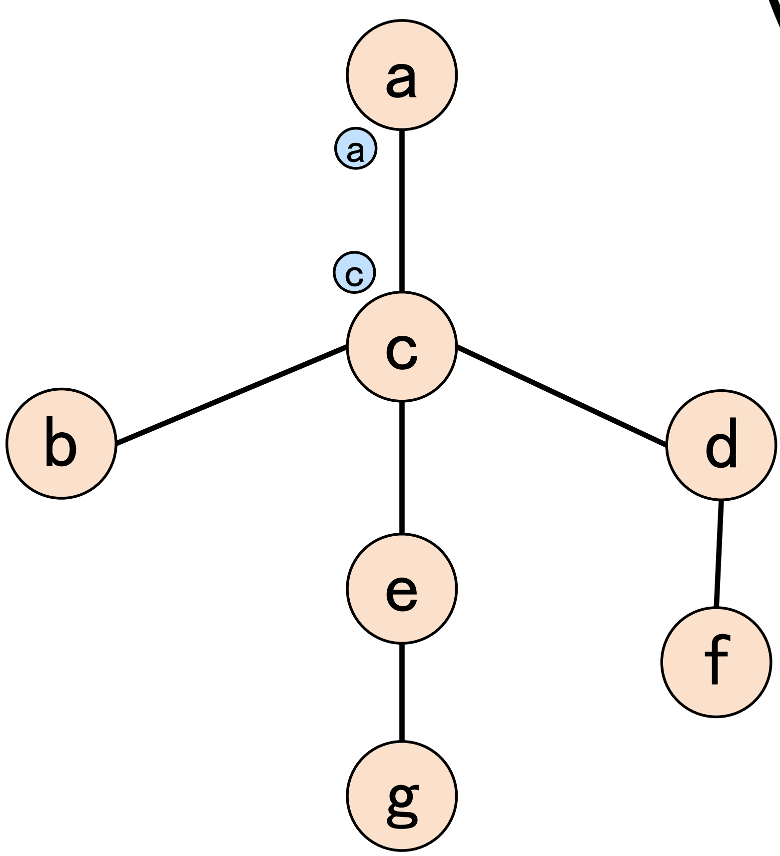

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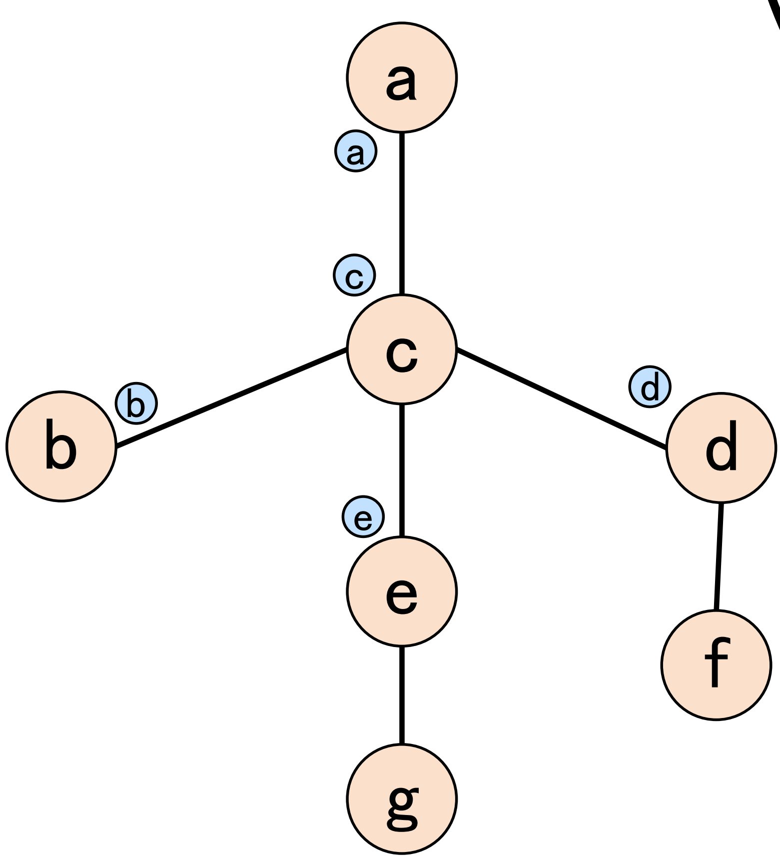

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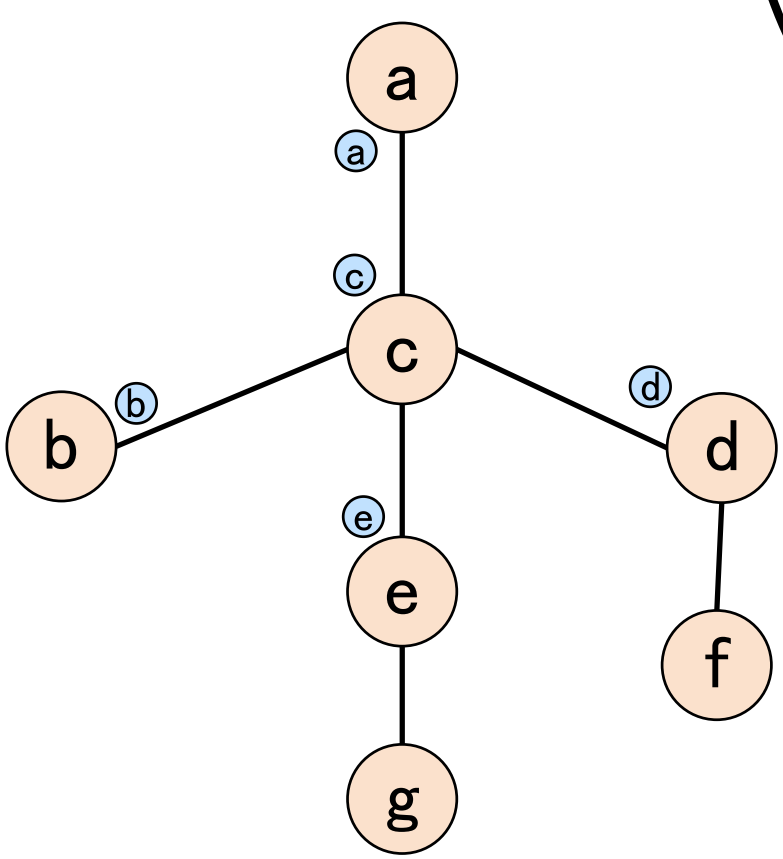

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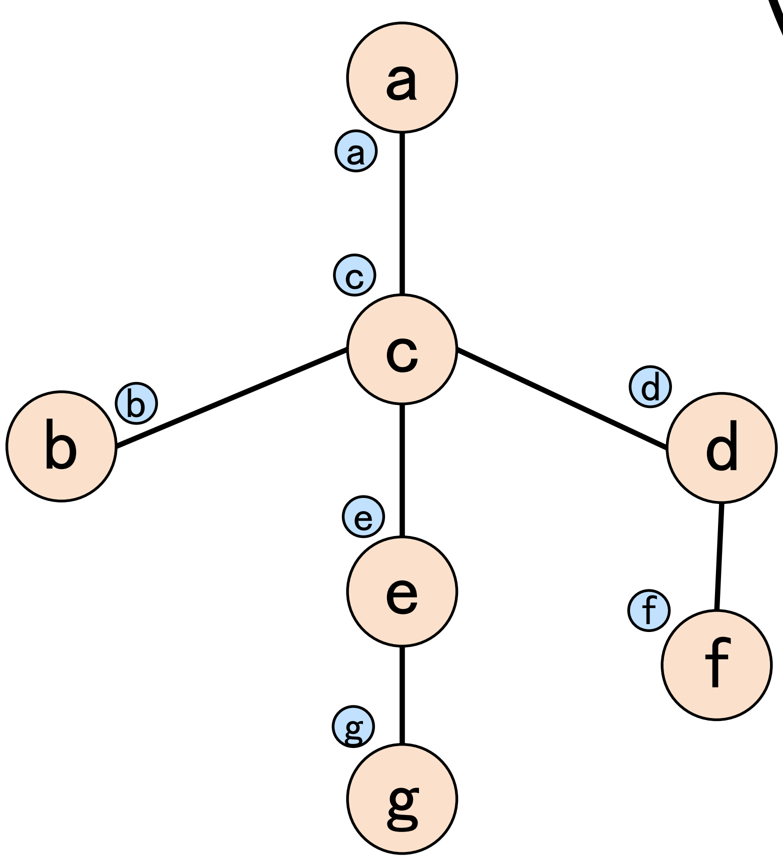

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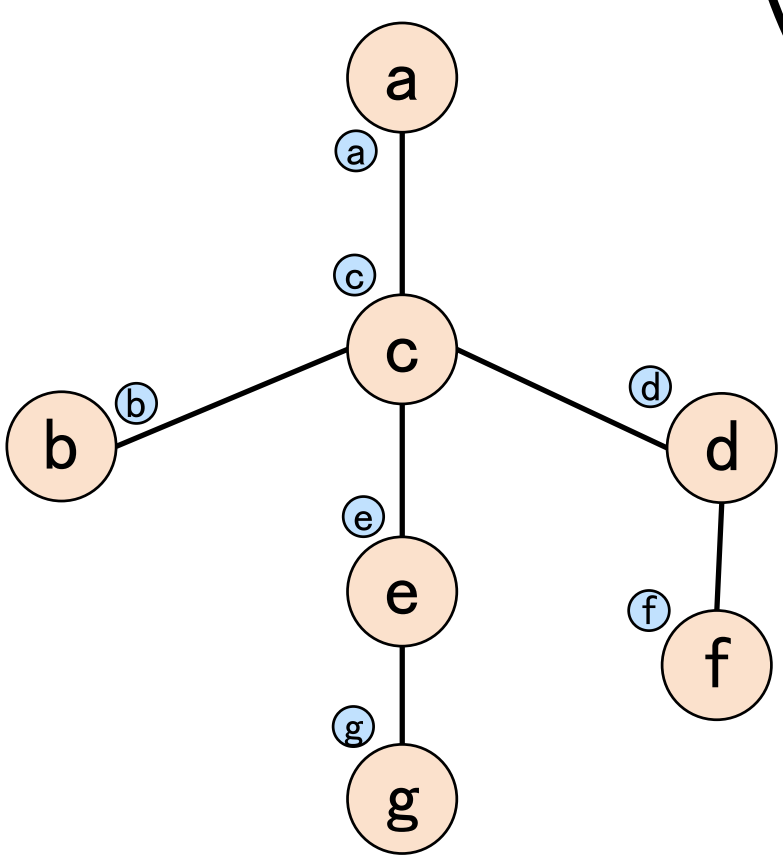

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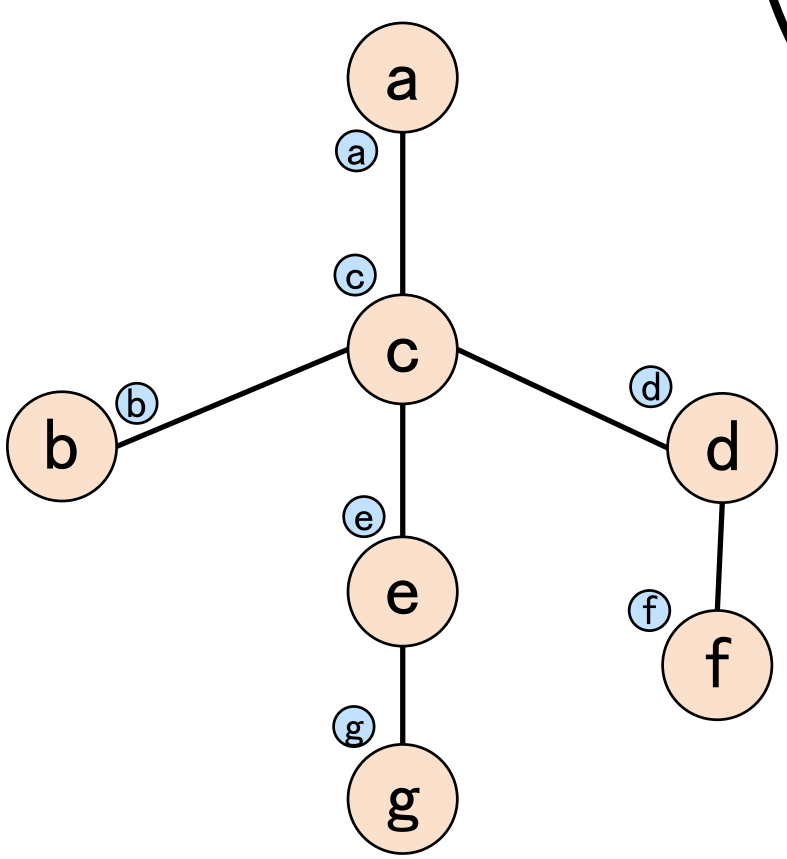
Node c introduces 3 registers	$\sum_{u\in V}\alpha_u|u\rangle_{\mathrm{a}}|u\rangle_{\mathrm{c}}|0\rangle\ |0\rangle\ |0\rangle$
Node c applies CNOTS	$\sum_{u\in V}\alpha_u|u\rangle_{\mathrm{a}}|u\rangle_{\mathrm{c}}|u\rangle\ |u\rangle\ |u\rangle$
Node c sends the registers to b,e,d	$\sum_{u\in V}\alpha_u|u\rangle_{\mathrm{a}}|u\rangle_{\mathrm{c}}|u\rangle_{\mathrm{b}}|u\rangle_{\mathrm{e}}|u\rangle_{\mathrm{d}}$
……	

Example:
V={a,b,c,d,e,f,g}
here leader = node a



Initially node a owns $\sum_{u\in V}\alpha_u|u\rangle_{\mathrm{a}}$

1. "Broadcast" this state, which gives [ecc(a) ≤ D rounds]

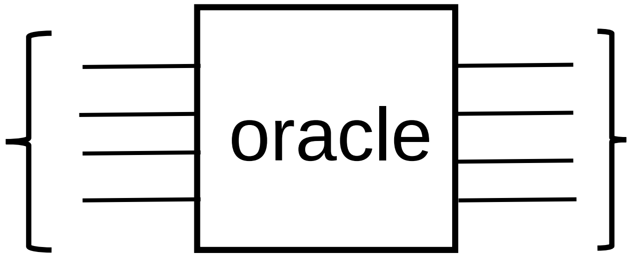
$$\sum_{u\in V}\alpha_u|u\rangle_{\mathrm{a}}|u\rangle_{\mathrm{b}}|u\rangle_{\mathrm{c}}|u\rangle_{\mathrm{d}}|u\rangle_{\mathrm{e}}|u\rangle_{\mathrm{f}}|u\rangle_{\mathrm{g}}$$

2. The nodes implement the classical protocol [O(D) rounds] for computing the eccentricity of u, which gives

$$\sum_{u\in V}\alpha_u|u\rangle_{\mathrm{a}}|u\rangle_{\mathrm{b}}|u\rangle_{\mathrm{c}}|u\rangle_{\mathrm{d}}|u\rangle_{\mathrm{e}}|u\rangle_{\mathrm{f}}|u\rangle_{\mathrm{g}}|ecc(u)\rangle_{\mathrm{a}}$$

Implementation of the Oracle in O(D) rounds

$$\sum_{u\in V} \alpha_u |u\rangle_{\mathrm{a}} |0\rangle_{\mathrm{a}}$$



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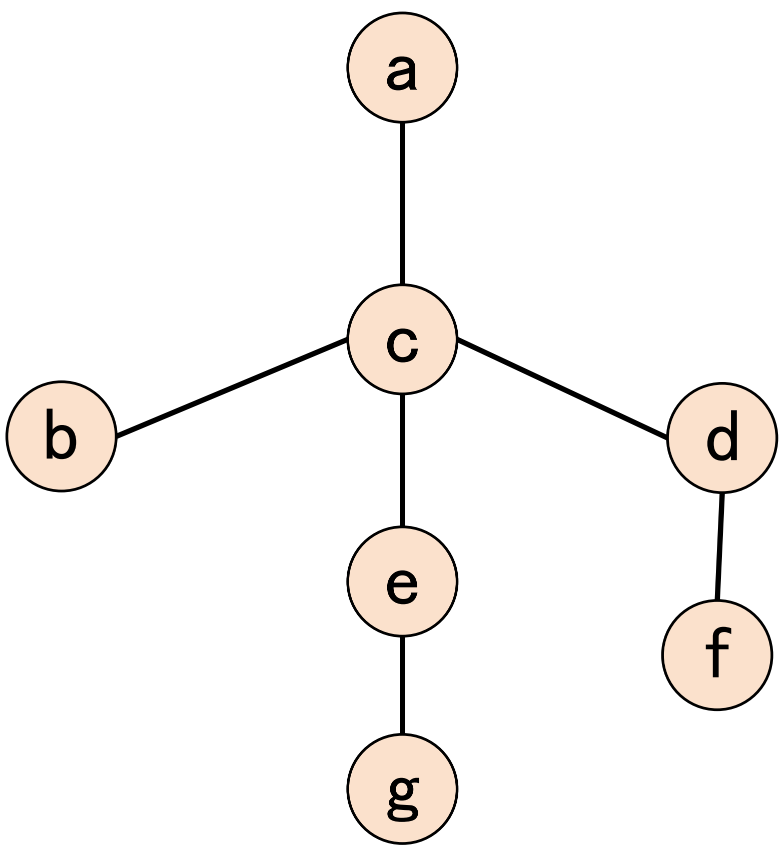

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3. The nodes revert Step 1 [ecc(a) ≤ D rounds]

Quantum Distributed Computation of the Diameter: Summary

Define the function f: V → {0,1} such that $f(u) = \begin{cases} 1 & \text{if } ecc(u) \geq d \\ 0 & \text{otherwise} \end{cases}$

Goal: find u such that f(u) = 1 (or report that no such vertex exist)

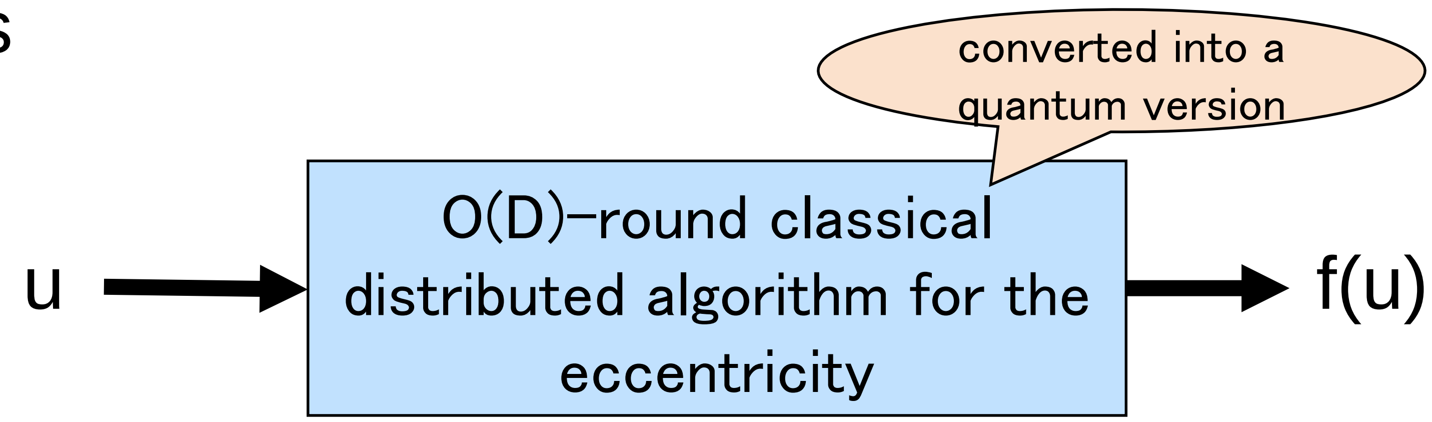
> There is a quantum algorithm for this search problem using $O(\sqrt{n})$ calls to a black box evaluating f

Quantum search [Grover 96]

<u>Quantum distributed algorithm computing the diameter</u>

- ✓ The network elects a leader
- ✓ The leader locally implements Grover algorithm. Each call to the black box is implemented by using the standard O(D)-round classical algorithm computing the eccentricity.

Complexity: $O(\sqrt{n} \times D)$ rounds



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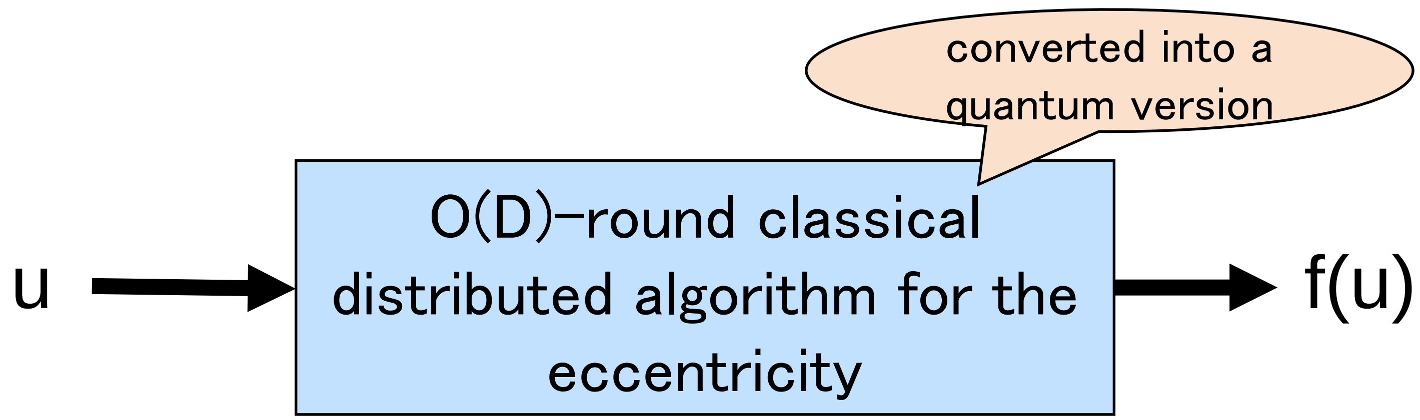

With further work, the complexity can be reduced to $O(\sqrt{nD})$ rounds

Quantum Distributed Computation of the Diameter: Summary

Classically in O(D) rounds it is possible to simultaneously compute the eccentricities of D vertices [Peleg+12]

Thus we can instead do a Grover search over groups of D vertices (there are n/D groups) in

$$O(\sqrt{n/D} \times D) = O(\sqrt{nD}) \text{ rounds}$$

<u>Quantum distributed algorithm computing the diameter</u>

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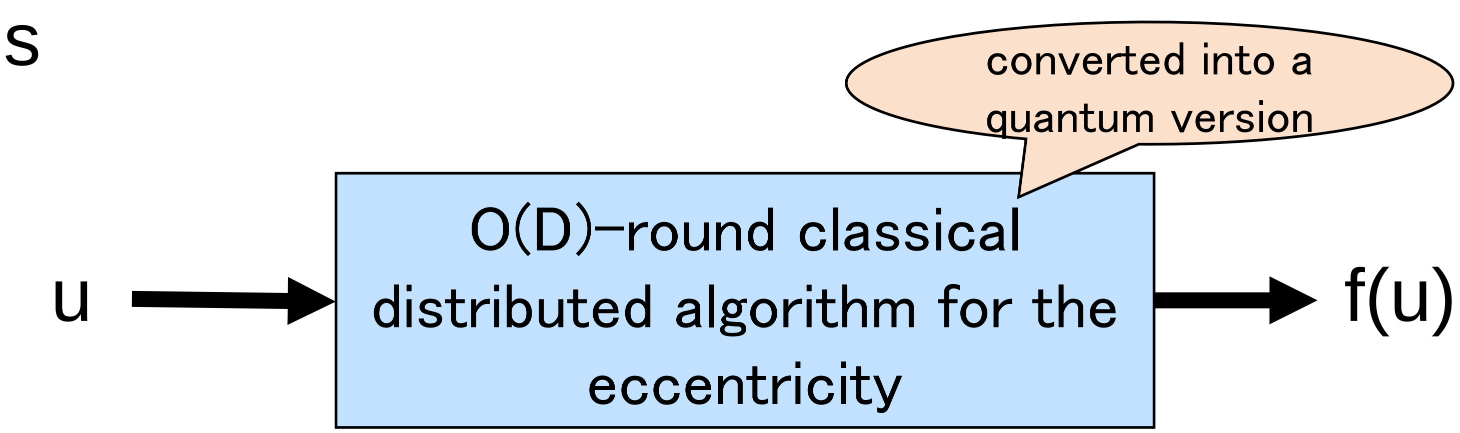

Summary of the first part

Main result [LG, Magniez 2018]

sublinear-round quantum computation of the diameter whenever D=o(n)

	Classical	Quantum (our results)
Exact computation (upper bounds)	$O(n)$ [Holzer+12, Peleg+12]	$O(\sqrt{nD})$
Exact computation (lower bounds)	$\widetilde{\Omega}(n)$ [Frischknecht+12]	$\widetilde{\Omega}(\sqrt{n}+D)$ [unconditional] $\widetilde{\Omega}(\sqrt{nD})$ [conditional]

number of rounds needed to compute the diameter (n: number of nodes, D: diameter)

OPEN PROBLEM:

✓ Prove an unconditional lower bound of $\widetilde{\Omega}(\sqrt{nD})$ rounds

very recent result [Magniez, Nayak 2020]

$\widetilde{\Omega}(\sqrt{n}+n^{1/3}D^{2/3})$ [unconditional]

Summary of the first part

Main result [LG, Magniez 2018]

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Our upper bound is obtained by showing how to implement quantum search in a distributed setting

PROMISING RESEARCH DIRECTION: find other applications of this technique

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PROMISING RESEARCH DIRECTION: find other applications of this technique

[Izumi, LG 2019]: quantum distributed algorithm for the All-Pairs Shortest Paths Problem faster than the best classical algorithms

- ✓ idea: implement simultaneously $\Theta(n^2)$ quantum distributed searches
- ✓ significant work needed to avoid congestions in the checking procedures

[Izumi, LG, Magniez 2020]: quantum distributed algorithm for triangle finding faster than the best classical algorithms

Quantum Distributed Computing

Quantum distributed computing

Now qubits can be sent instead of bits

(no prior entanglement between nodes)

n: number of nodes of the network

CONGEST model: only O(log n) qubits per message

Quantum can be useful for some problems
[LG, Magniez 2018] [Izumi, LG 2019] [Izumi et al. 2020]

LOCAL model: no restriction on the size of each message

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unbounded amount of quantum communication
vs.
unbounded amount of classical communication

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Superiority of the Quantum LOCAL model

[LG, Nishimura, Rosmanis 2019]

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Superiority of the Quantum LOCAL model

We use a construction from [Barrett, Caves, Eastin, Elliot, Pironio 2007]

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There is a computational problem that can be solved in 2 rounds in the quantum LOCAL model but requires Ω(n) rounds classically.

Superiority of the Quantum LOCAL model

Also used in some of the recent results on quantum shallow circuits [Bravyi, Gosset, König 2018]

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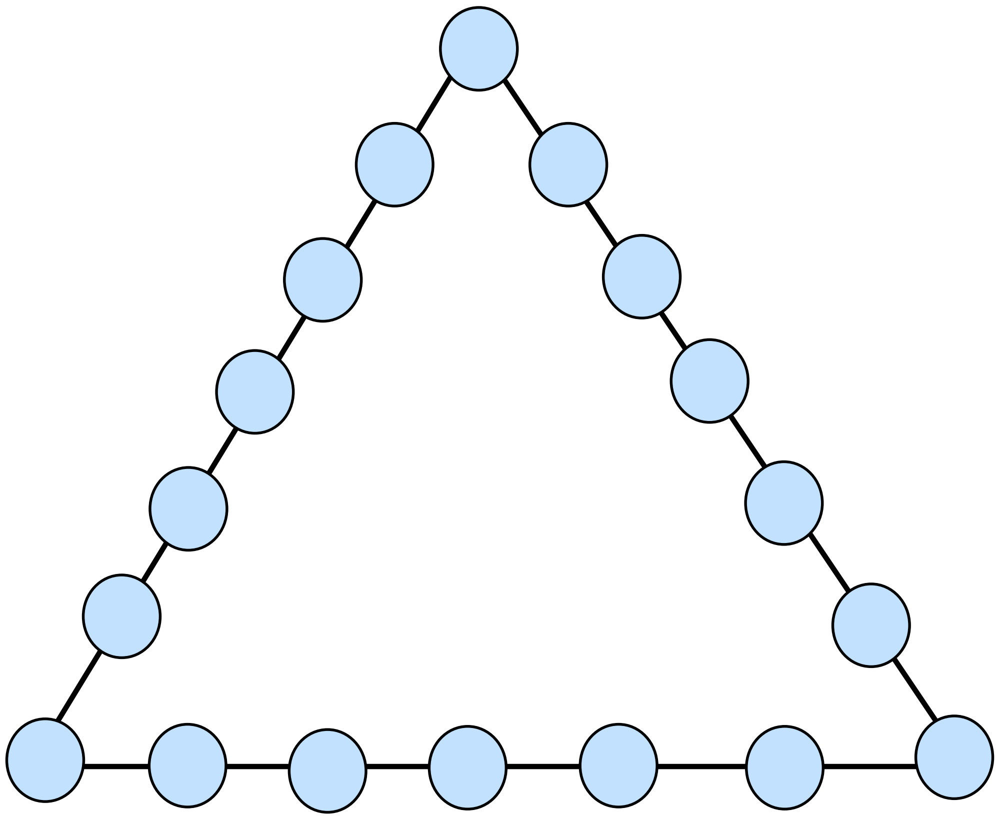
Superiority of the Quantum LOCAL model

multiple of 3

n=18

Consider a ring of size n (seen as a triangle)



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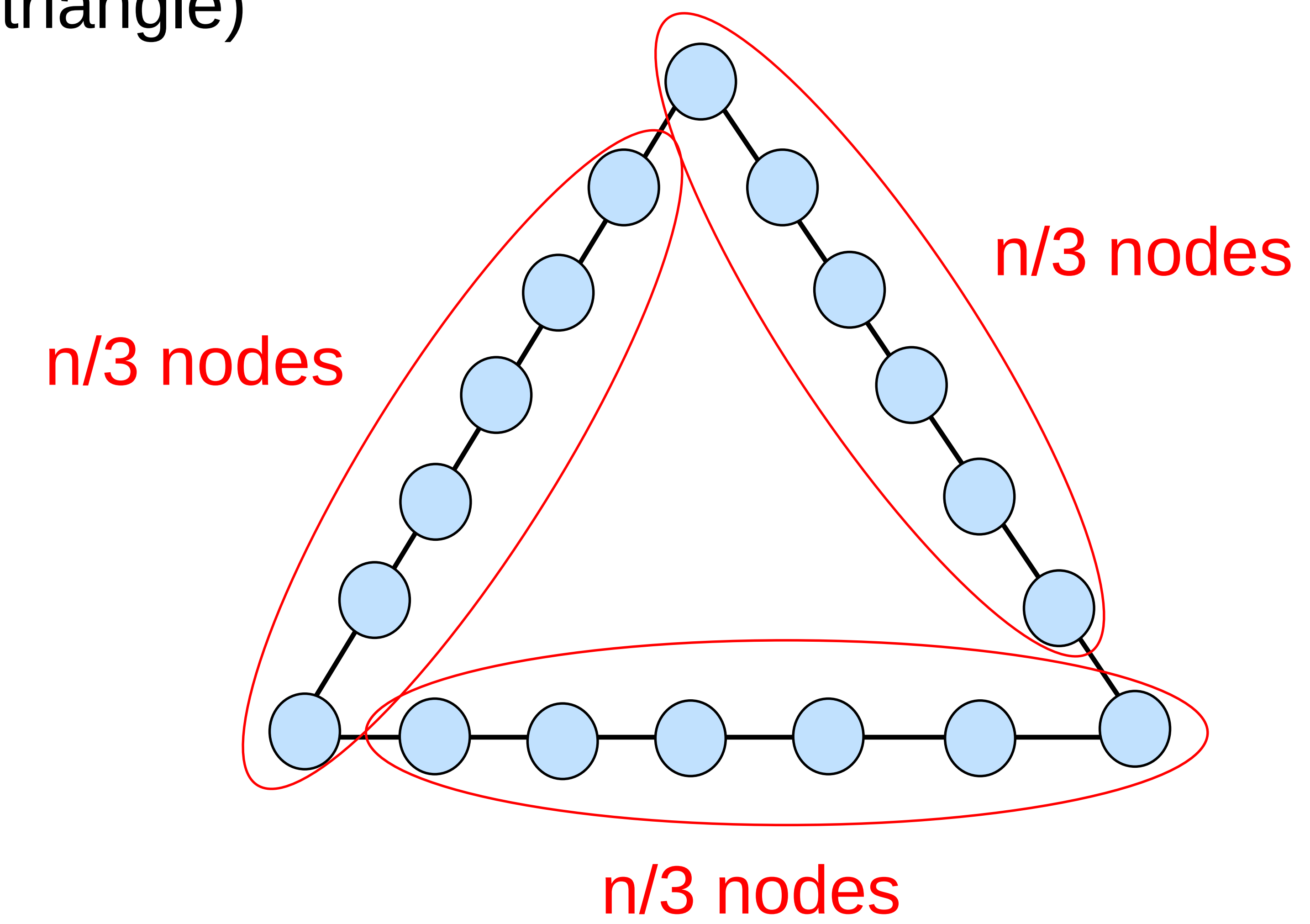

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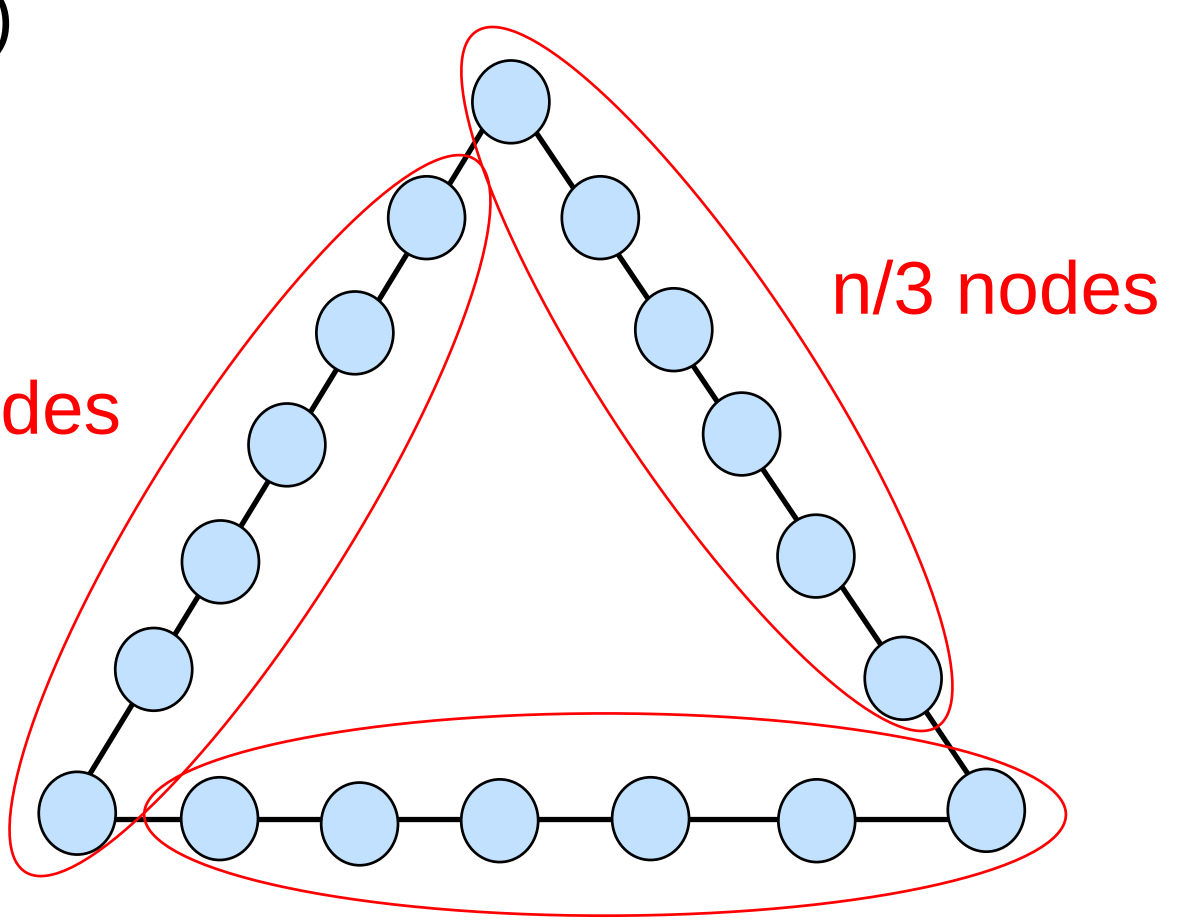
Each “corner” gets a bit as input



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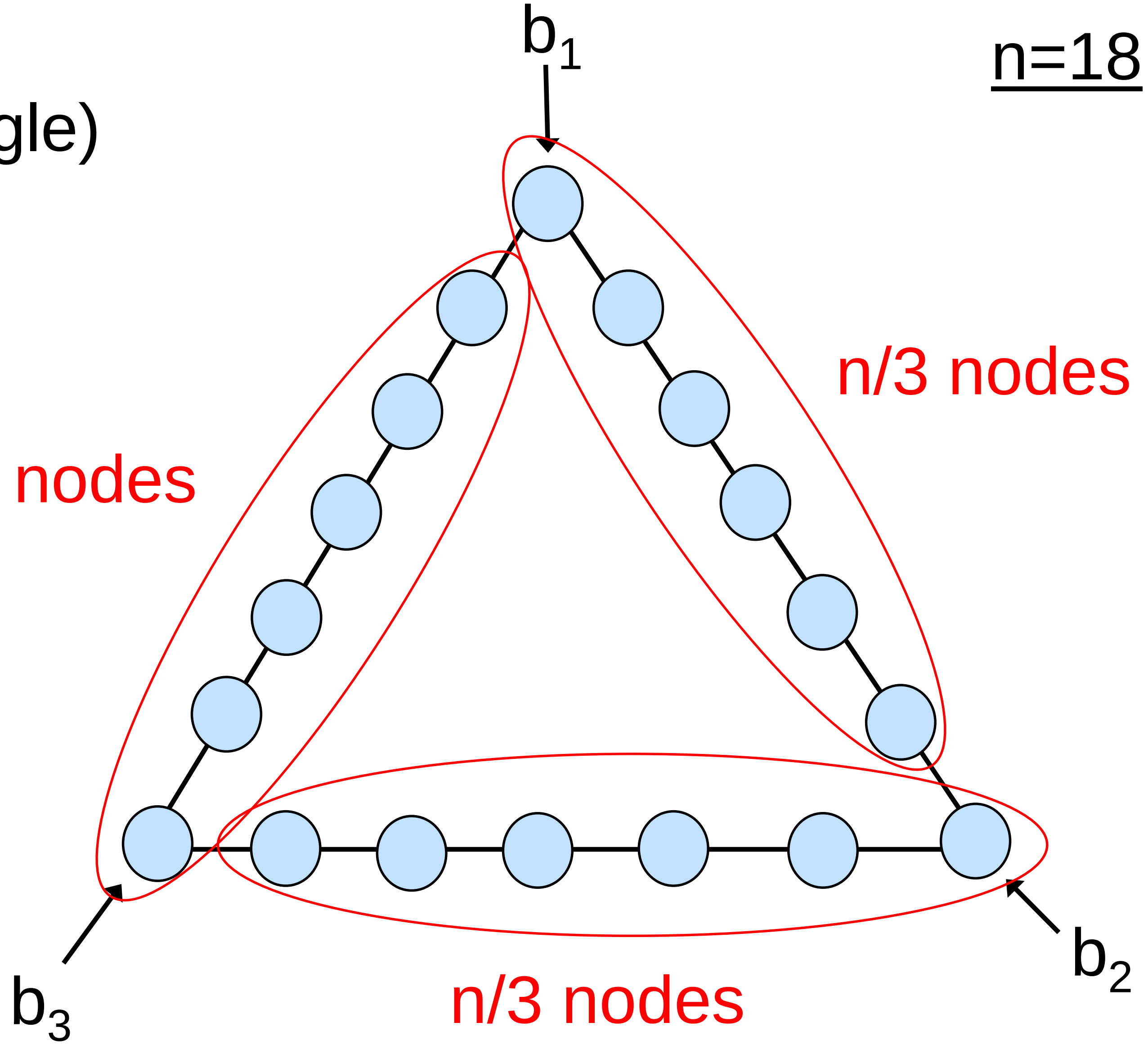

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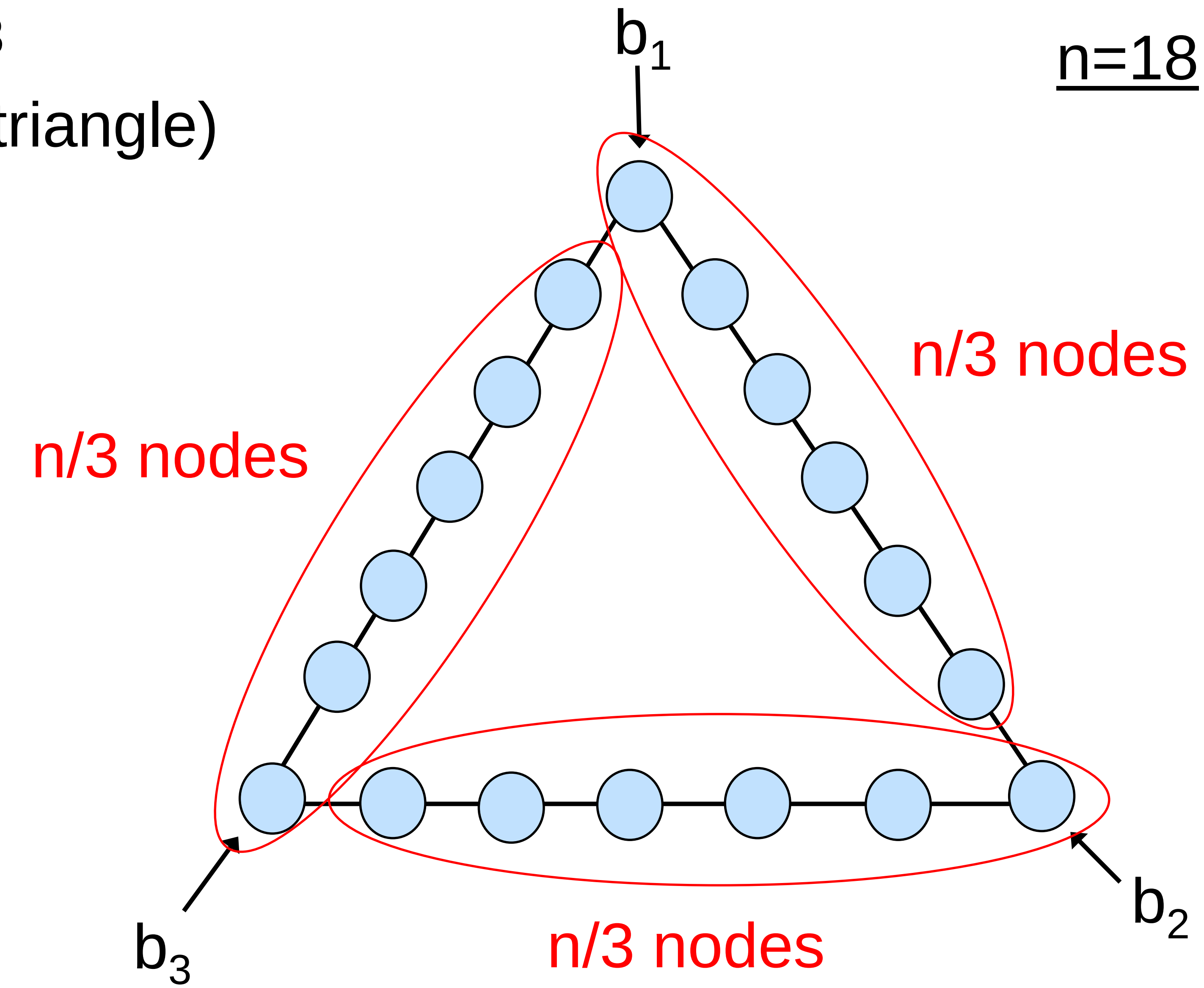

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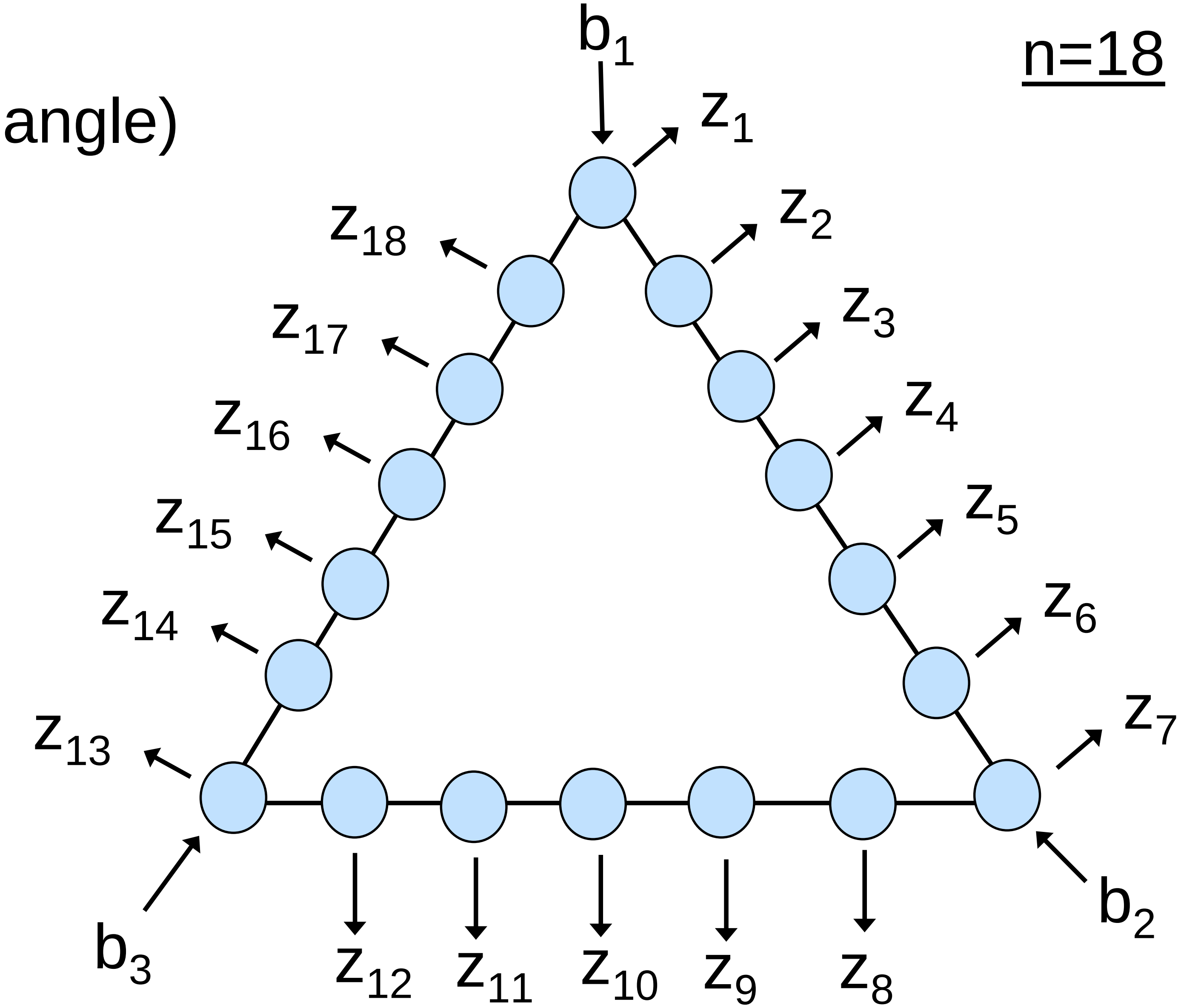
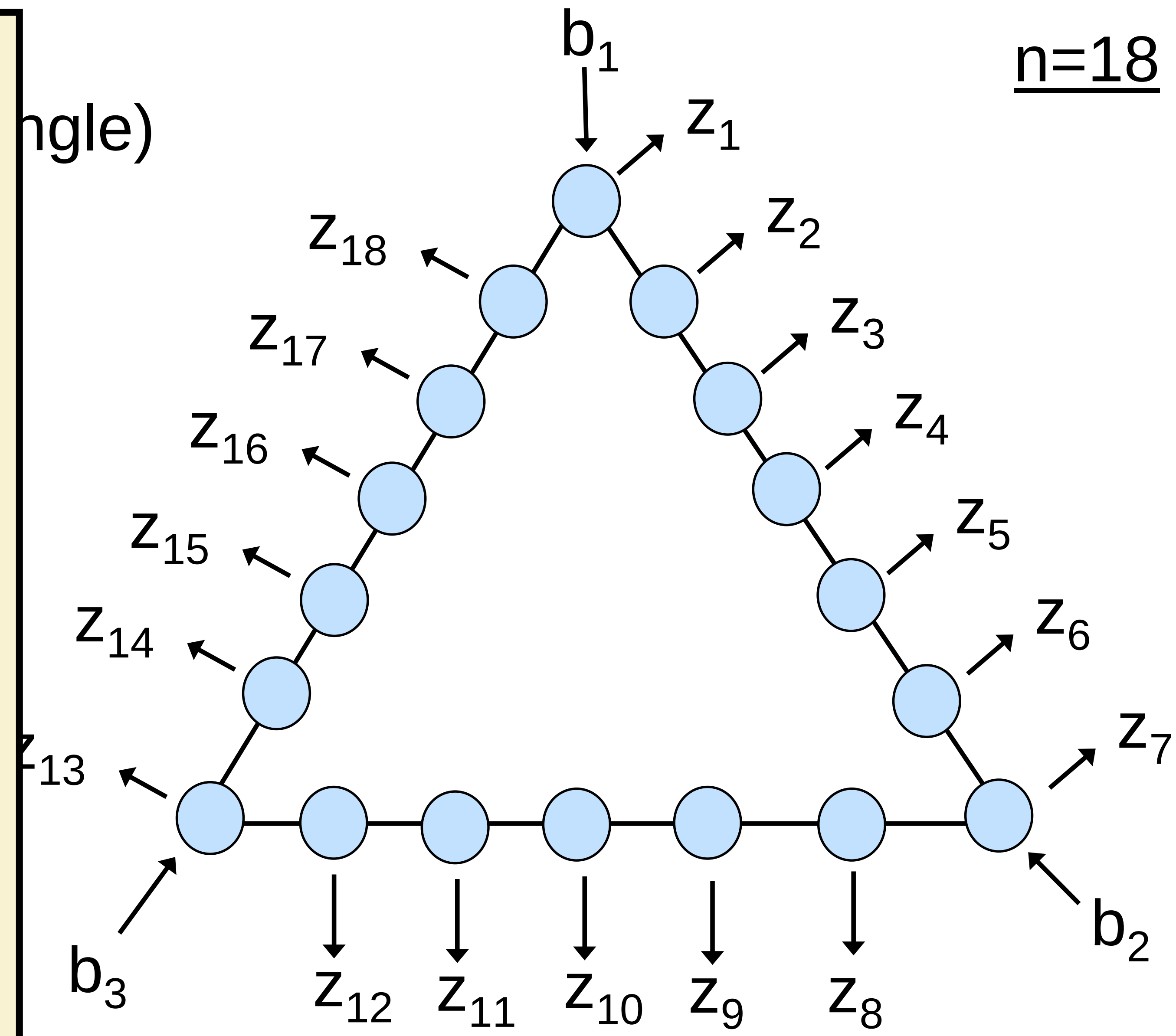


Consider the following process:

1. The nodes prepare the graph state corresponding to the whole triangle
2. Each non-corner node measures its qubit in the X basis and then outputs the bit corresponding to the measurement outcome
3. Each corner node measures its qubit in the X basis if its input bit is 0, or measures it in the Y basis if its input bit is 1, and then outputs the bit corresponding to the measurement outcome

ngle)



Consider the following process:

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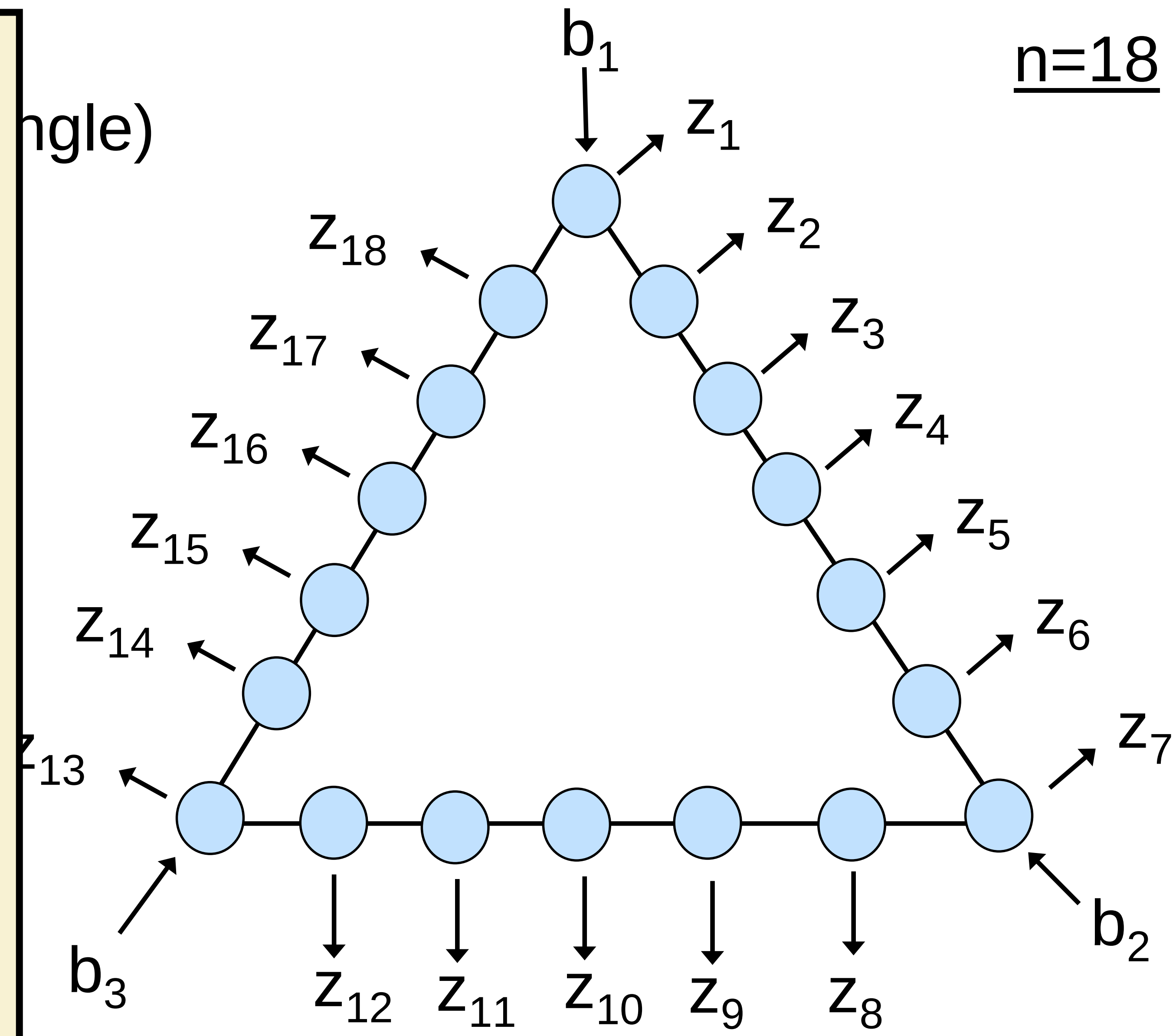

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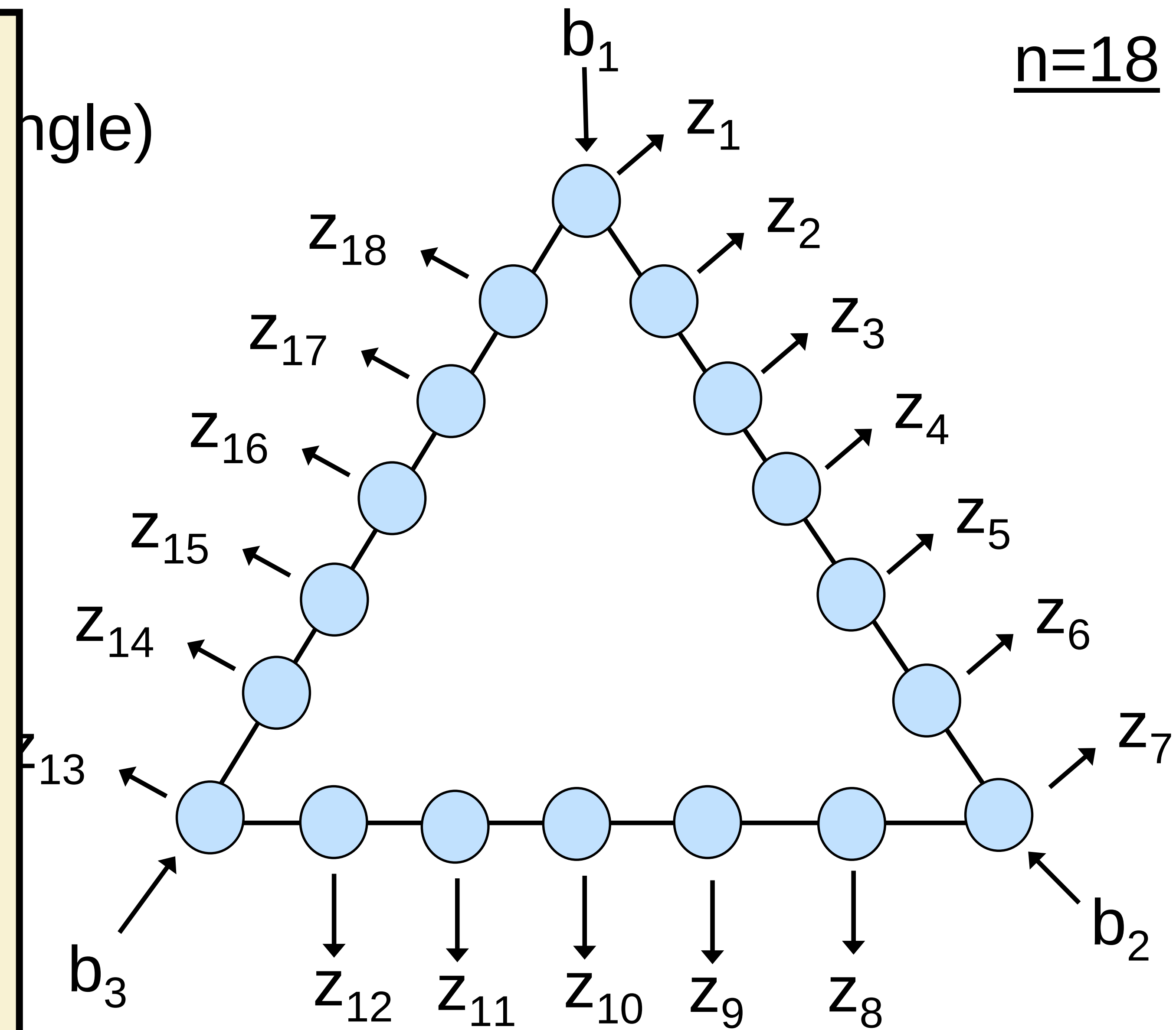

Claim:
[Barrett et al. 2007]

In the LOCAL model, any classical algorithm that samples (even approximately) from the same distribution must use at least n/6 rounds.

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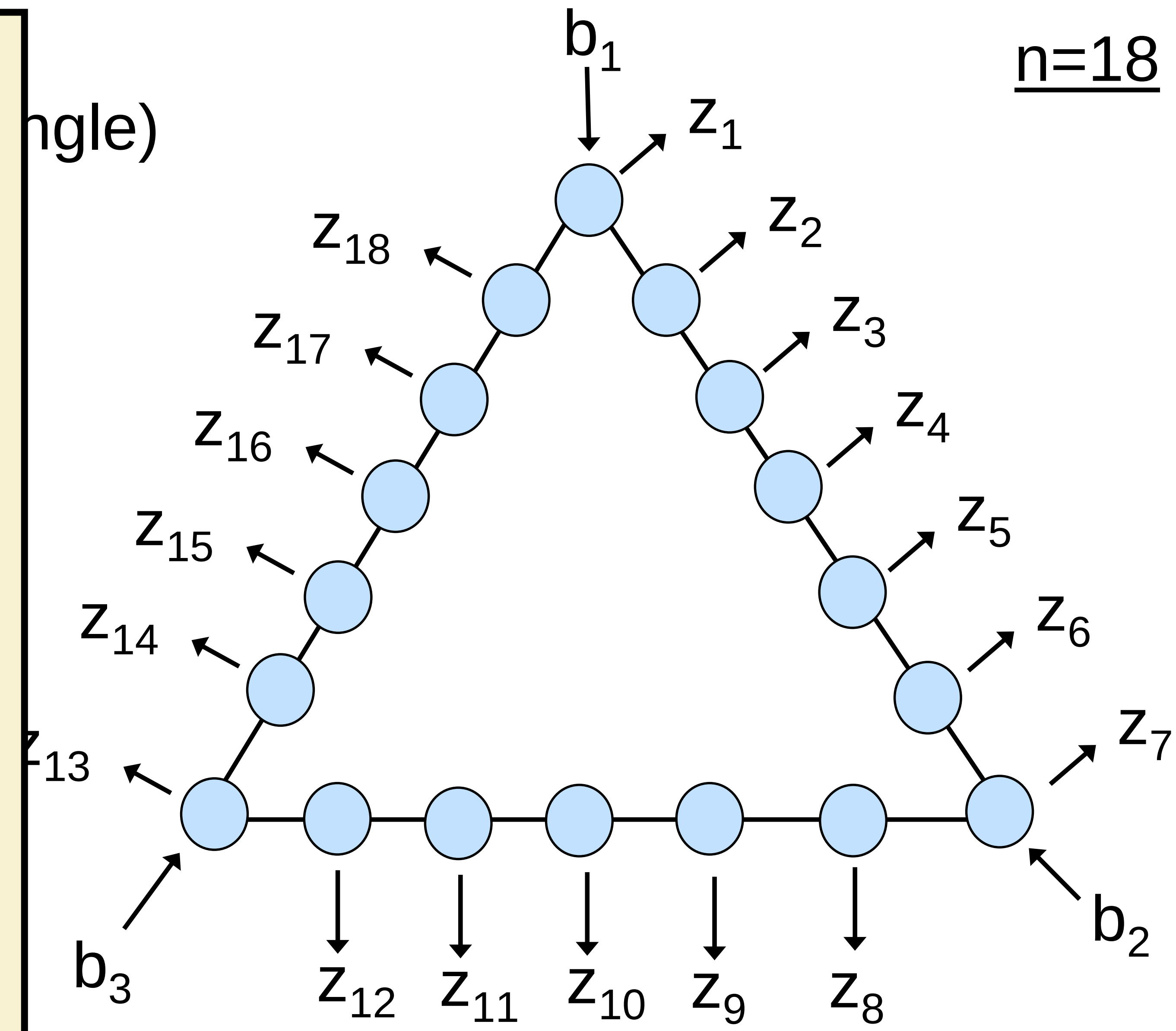

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Remarks

“simulate the outcome distribution of a measurement of the graph state”

[LG, Nishimura, Rosmanis 2019]

There is a computational problem that can be solved in 2 rounds in the quantum LOCAL model but requires $\Omega(n)$ rounds in the classical LOCAL model.

Remarks

✓ Since our quantum distributed algorithm only uses short messages (1 qubit in each message) we get the following stronger statement:

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- ✓ A similar separation can be shown for a relation ("output any outcome that appears with non-zero probability as an outcome of the measurement of the graph state")
- ✓ A similar separation can also be shown for a sampling problem without any input ("simulate the outcome distribution of the measurement when the bits b_1, b_2 and b_3 are taken uniformly at random")

"simulate the outcome distribution of a measurement of the graph state"

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Conclusion

We now know that quantum distributed algorithms can be faster than classical distributed algorithms for several problems, in both the CONGEST model and the LOCAL model

Interesting research directions:

- ✓ Construct other quantum distributed algorithms, for important problems

  Designing quantum distributed algorithms in these models poses new algorithmic challenges since we have to focus on the round complexity (instead of time/query complexity or total communication complexity)

- ✓ Develop lower bounds techniques, especially in the quantum LOCAL model

  Can we show a nontrivial lower bound for graph coloring?

- ✓ Prove the superiority of quantum distributed algorithms in other models

Recent result: [Fraigniaud, LG, Nishimura, Paz 2020]

advantage for distributed interactive proofs