Estimating the fault tolerant cost of classically intractable quantum computations

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Talk Plan

0. Talk Plan
1. Context
2. Building Blocks
3. Algorithm + Cost
4. Summary

1. Context

Goals of this talk

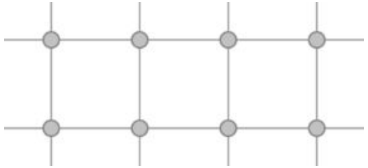
Understand the cost of classically intractable fault tolerant tasks.

Set a baseline that can be used to track improvements to error correction.

Define a goal that experimentalists can target.

Physical assumptions

Connectivity: planar grid of qubits with nearest neighbor interactions.

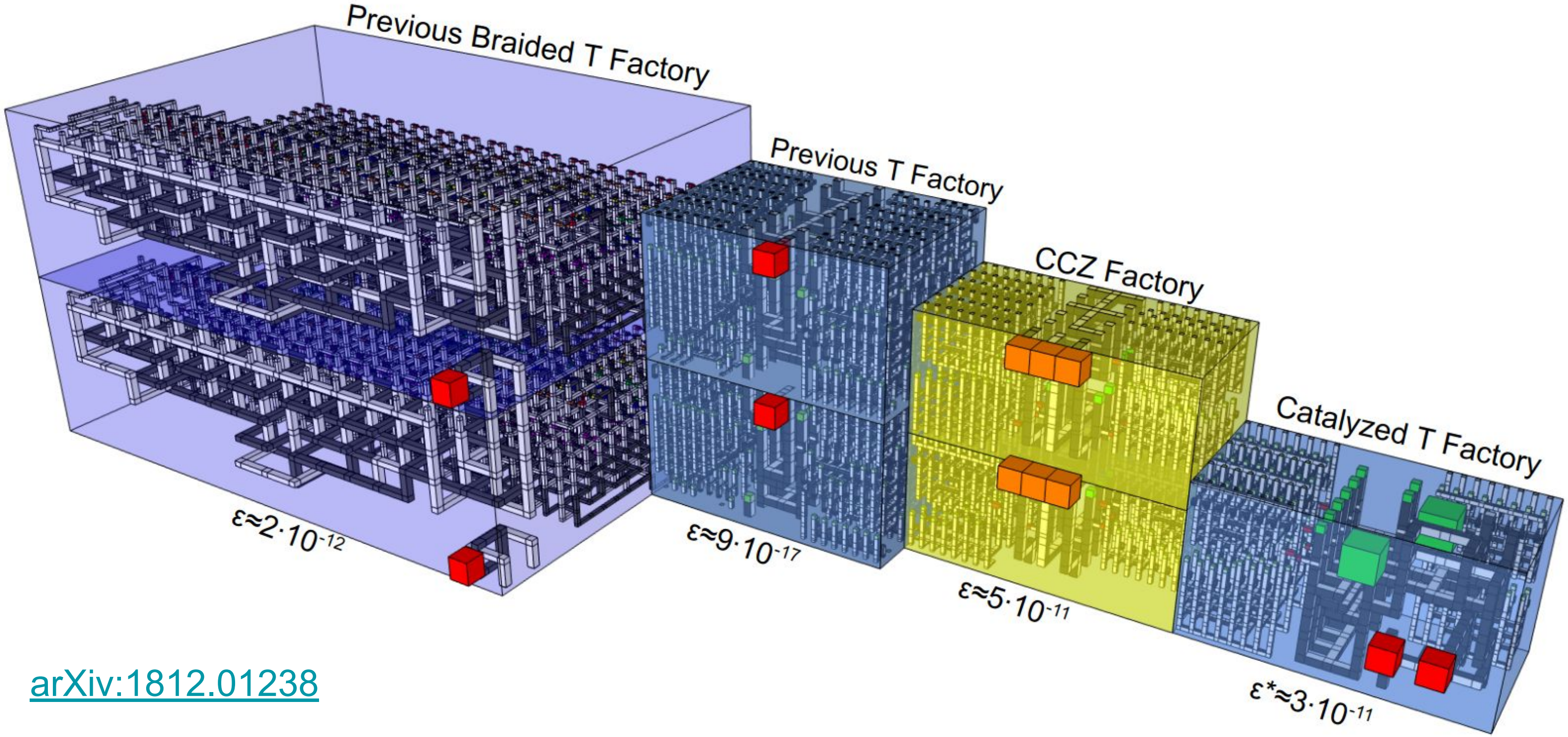


Quality: physical gates have **99.9%** fidelity.
equivalently: logical error suppression of **5 dB / code_distance**

Speed: hardware can run surface code cycle at **1MHz**.

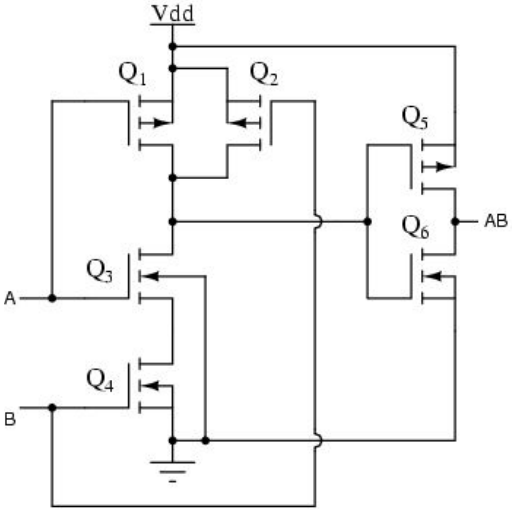
Latency: control system can do adaptive measurement at **100KHz**.

The cost of fault tolerance is decreasing



arXiv:1812.01238

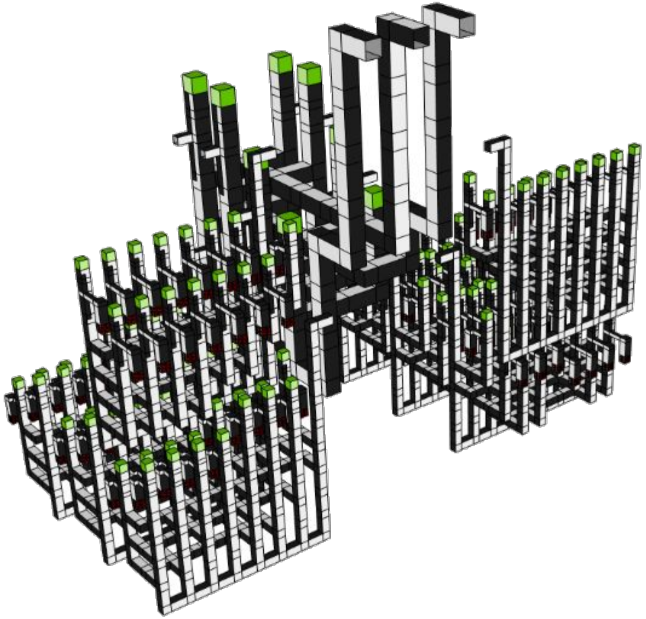
Fault tolerance is expensive



Classical AND

<0.000000001

transistor seconds



Quantum AND

>10

qubit seconds

The cheapest known classically intractable task

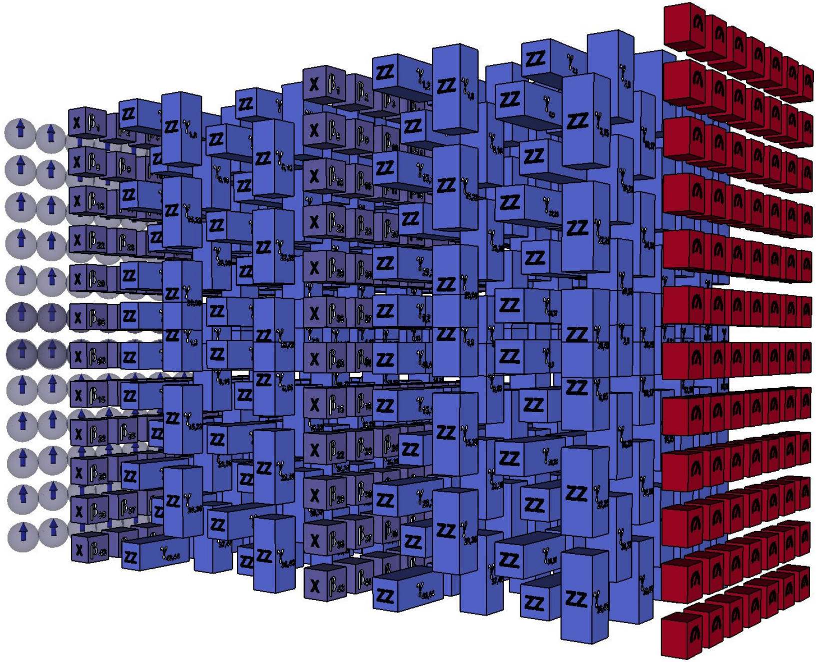
Random Circuit Sampling

PROS:

- Trivial for quantum computers.
- Hard for classical computers.
- NISQ compatible.

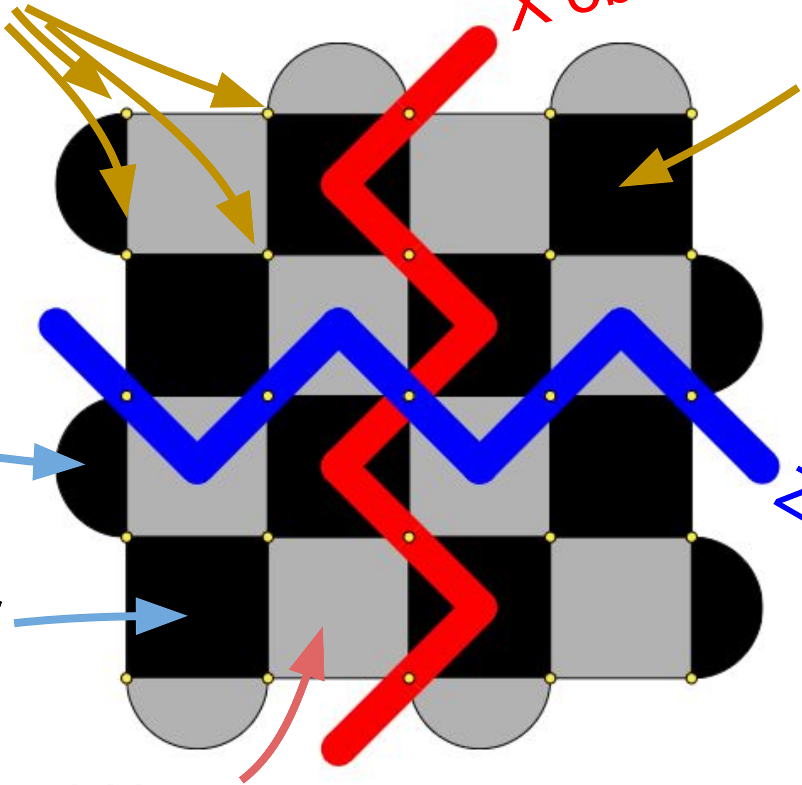
CONS:

- Not directly useful.
- Expensive to verify.



2a. Building Blocks: Storage

Surface Code Qubit



Physical data qubits

X observable

Implied physical measurement qubit

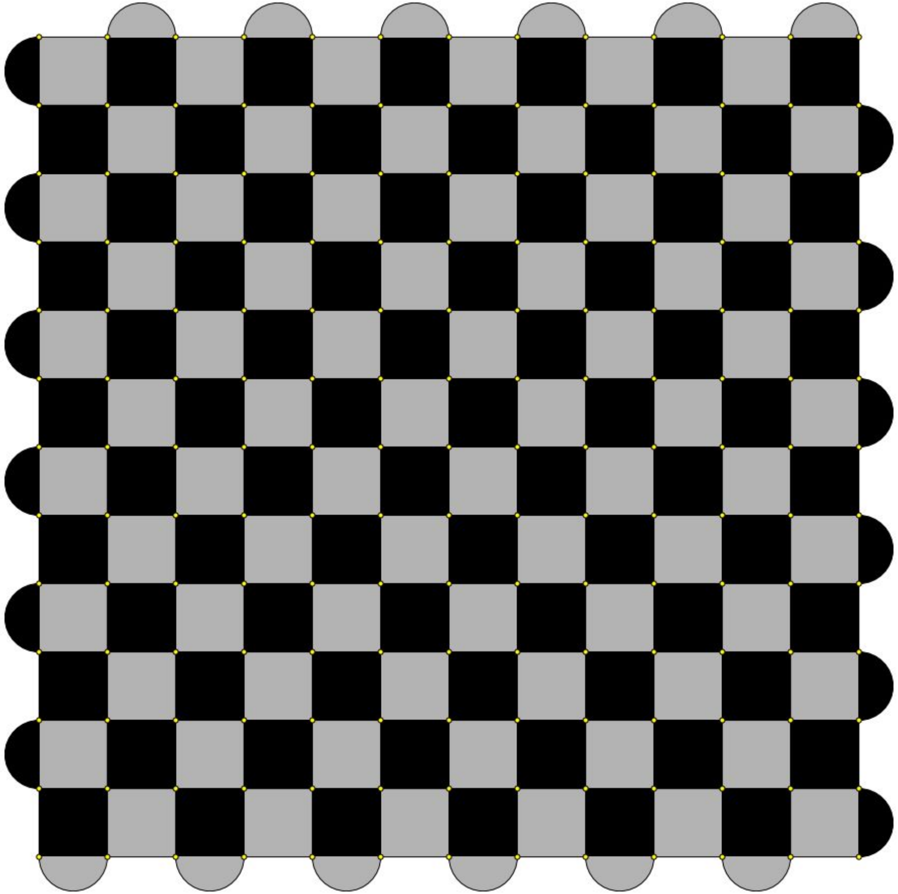
ZZ stabilizer

ZZZZ stabilizer

Z observable

XXXX stabilizer

Distance **d=13** Surface Code Qubit

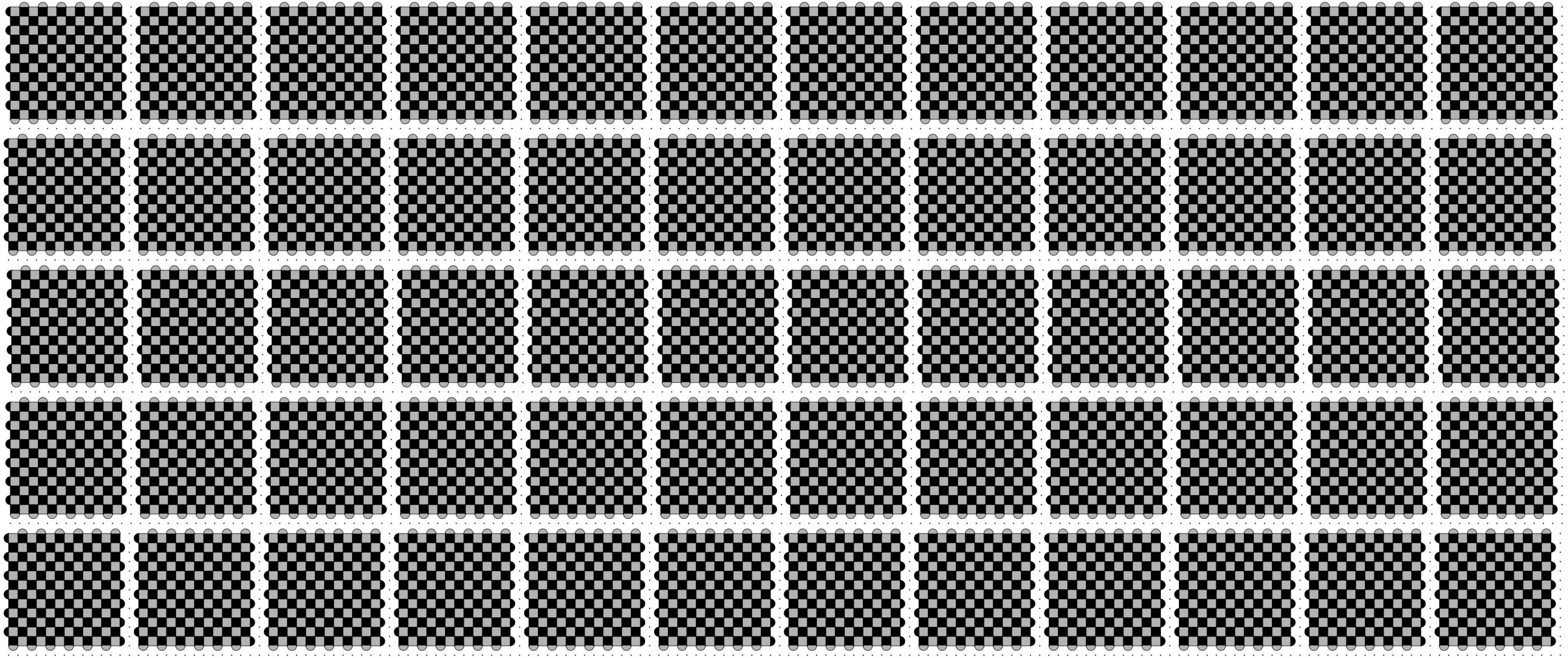


Physical qubit count:
$= 2(d+1)^2$
$= 392$

Logical error rate per cycle:
$\approx -5*(d+3)$ dB
$\approx 10^{\mathbf{-8}}$

Half life: **≈1 minute**

System: 60 (5x12) logical qubits

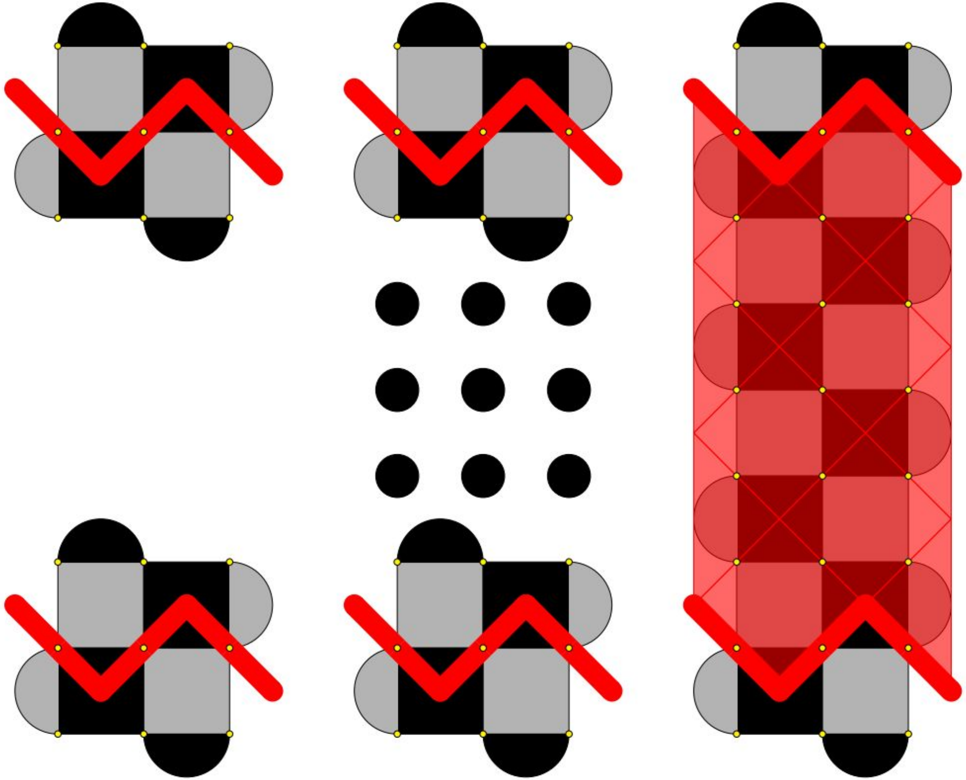


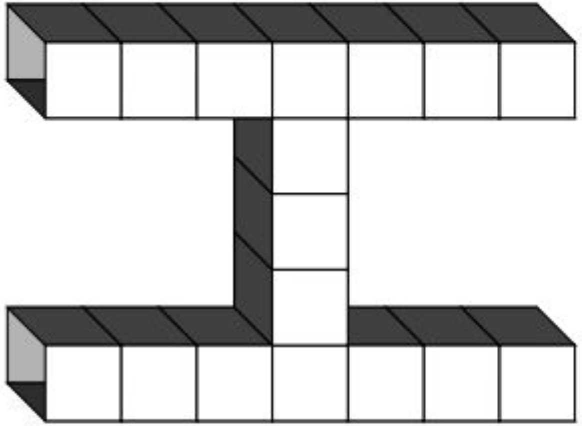
Total physical qubit count ≈ 23 500

System half life ≈ 1 second

2b. Building Blocks: Operations

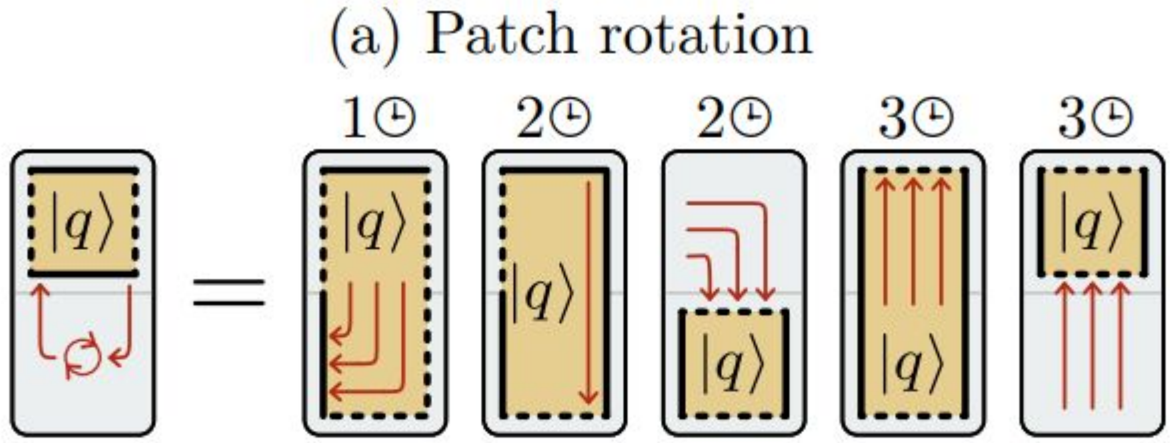
Lattice surgery - X_1X_2 parity measurement

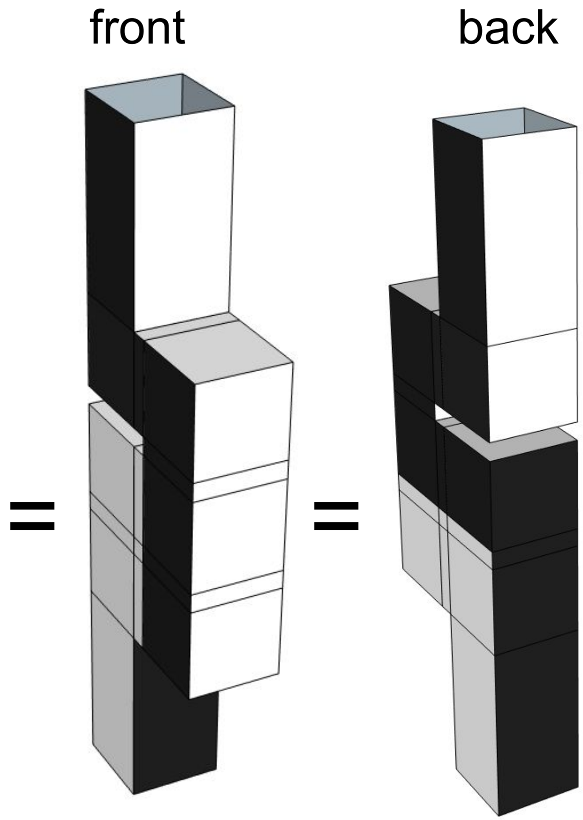




Spacetime Diagram

Patch Rotation (2x1 Footprint)

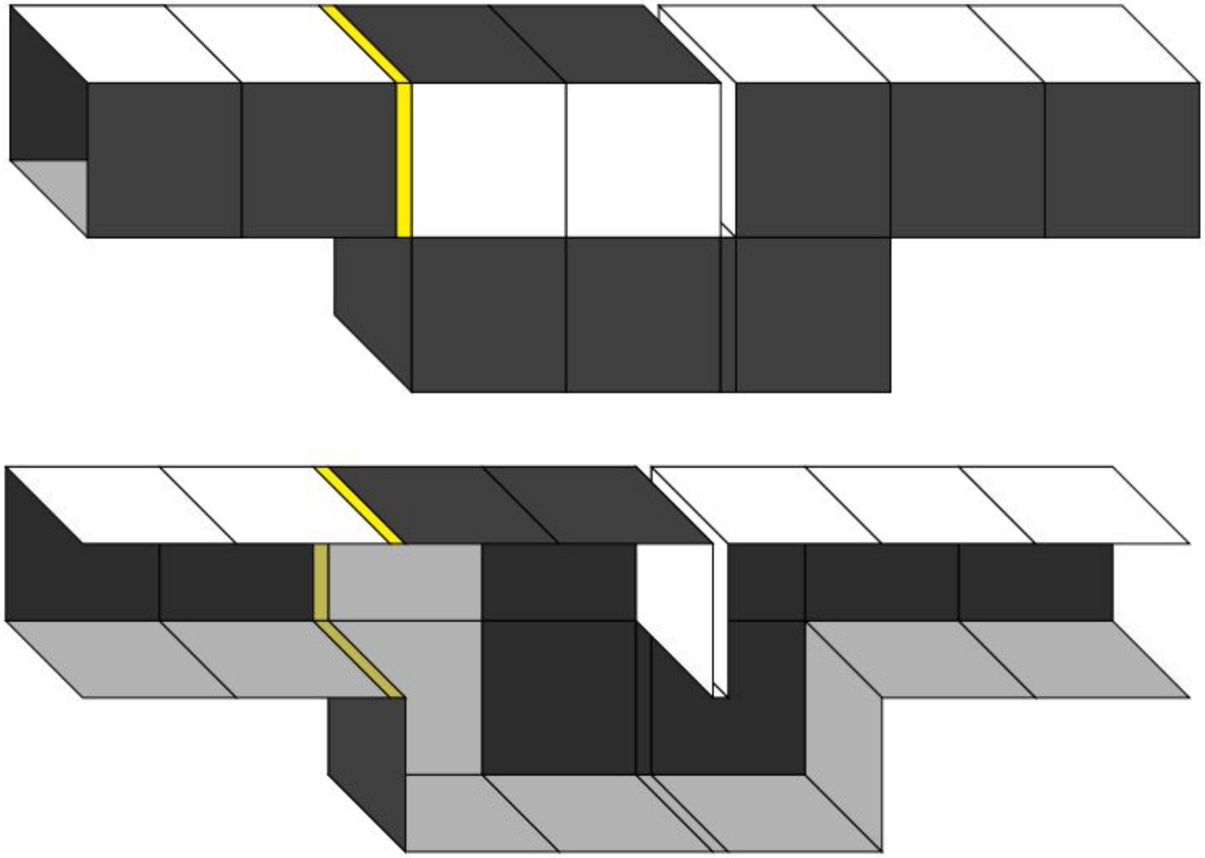




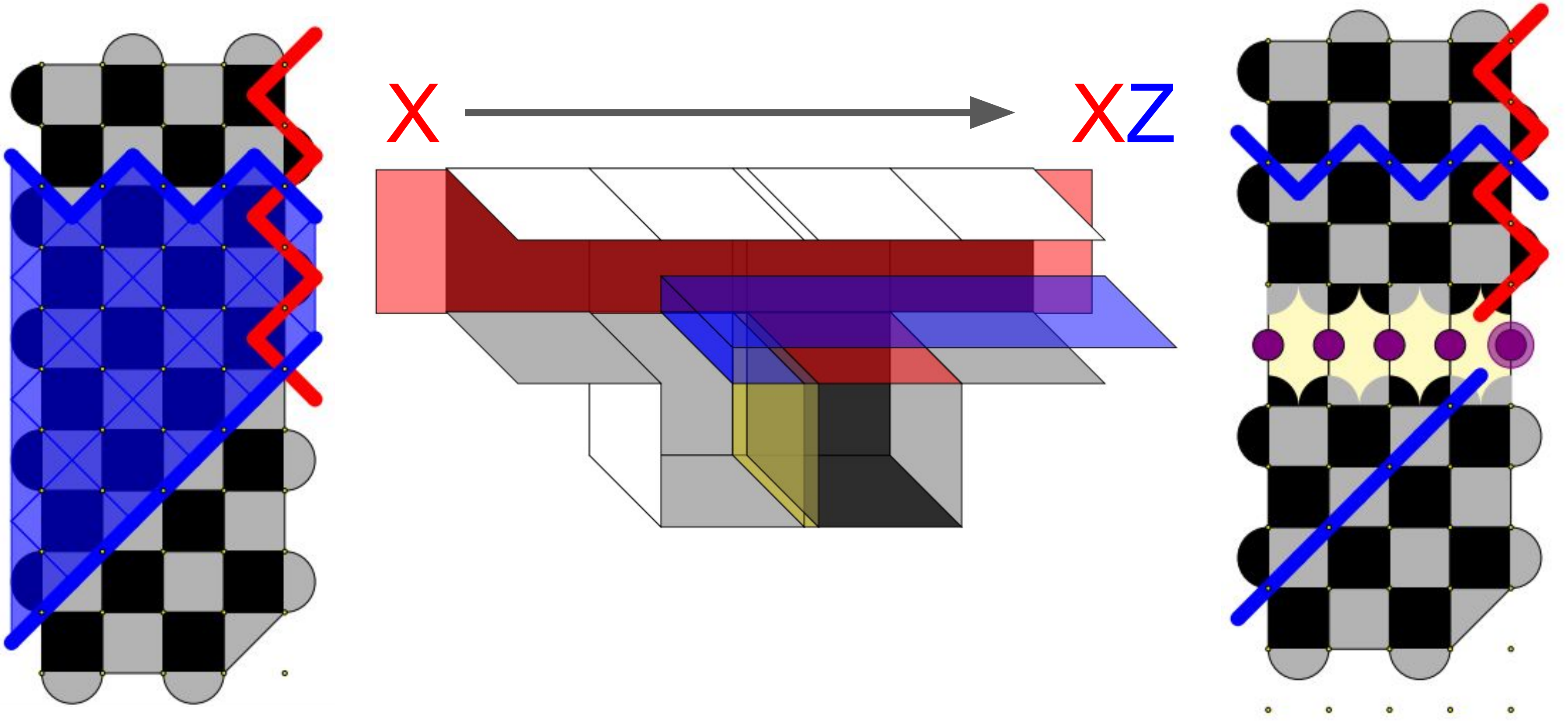
"Barber Pole"

Figure source: [arXiv:1808.02892](https://arxiv.org/abs/1808.02892)
Daniel Litinski - Game of Surface Codes

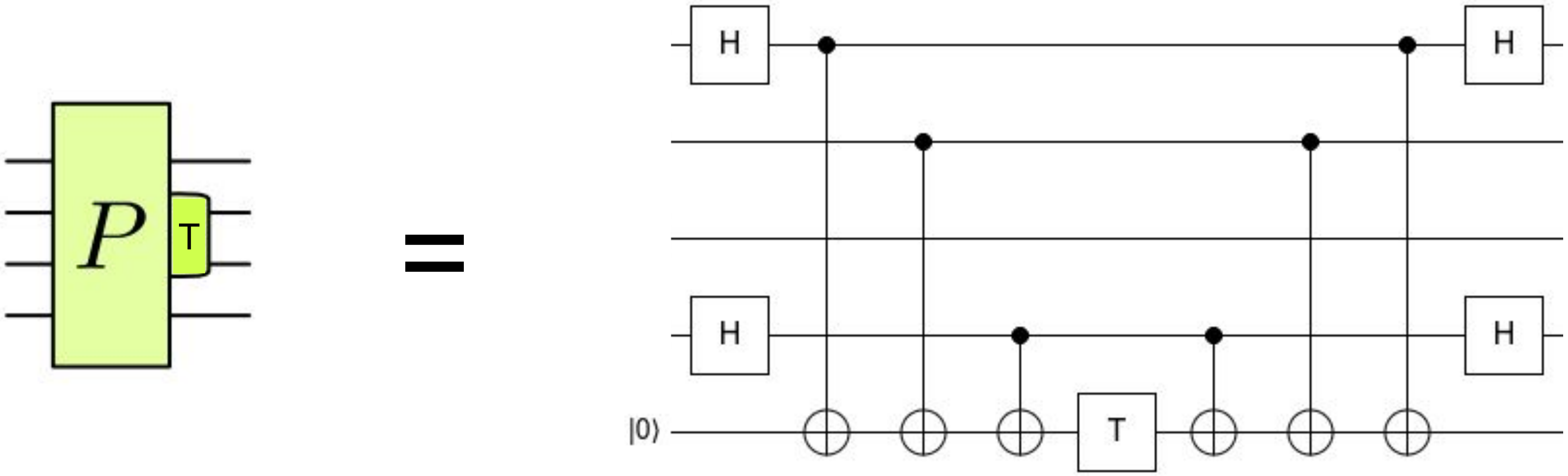
Hadamard Gate (2x1 Footprint)



S Gate (2x1 Footprint)



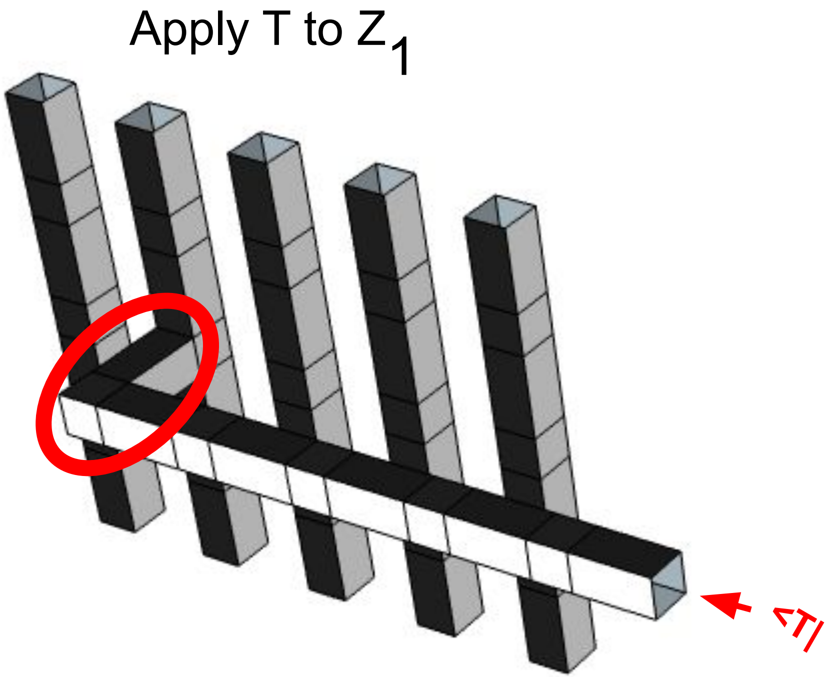
Generalized T gate: phasing Pauli *products*

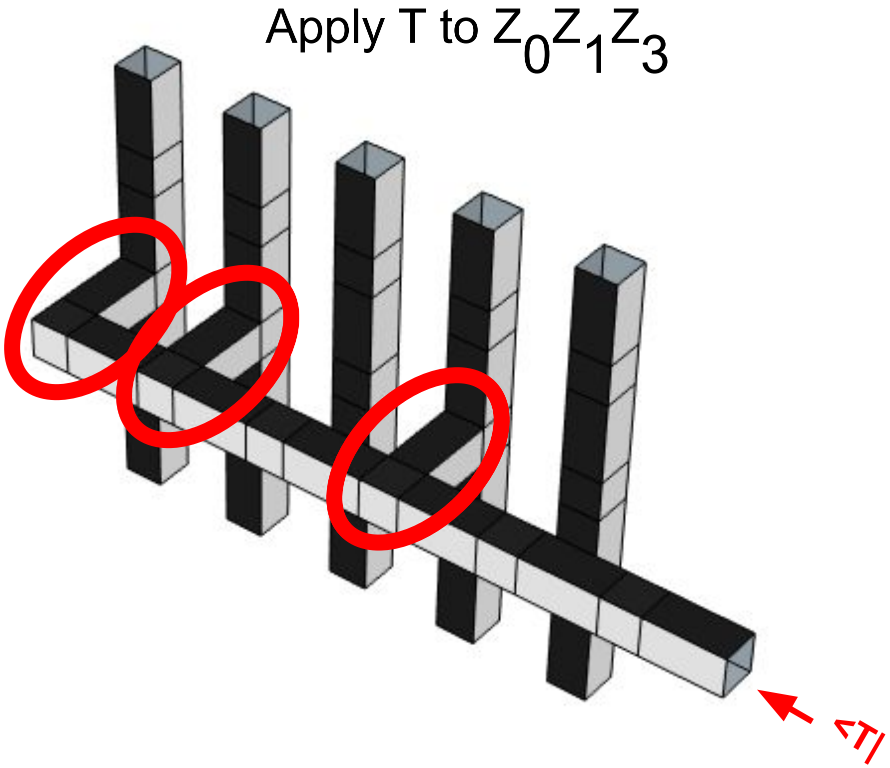


-1 eigenstates of P phased by 45 degrees
+1 eigenstates of P phased by 0 degrees

(given $P = X_0 Z_1 X_3$)

Generalized T gate teleport using lattice surgery





T state distillation

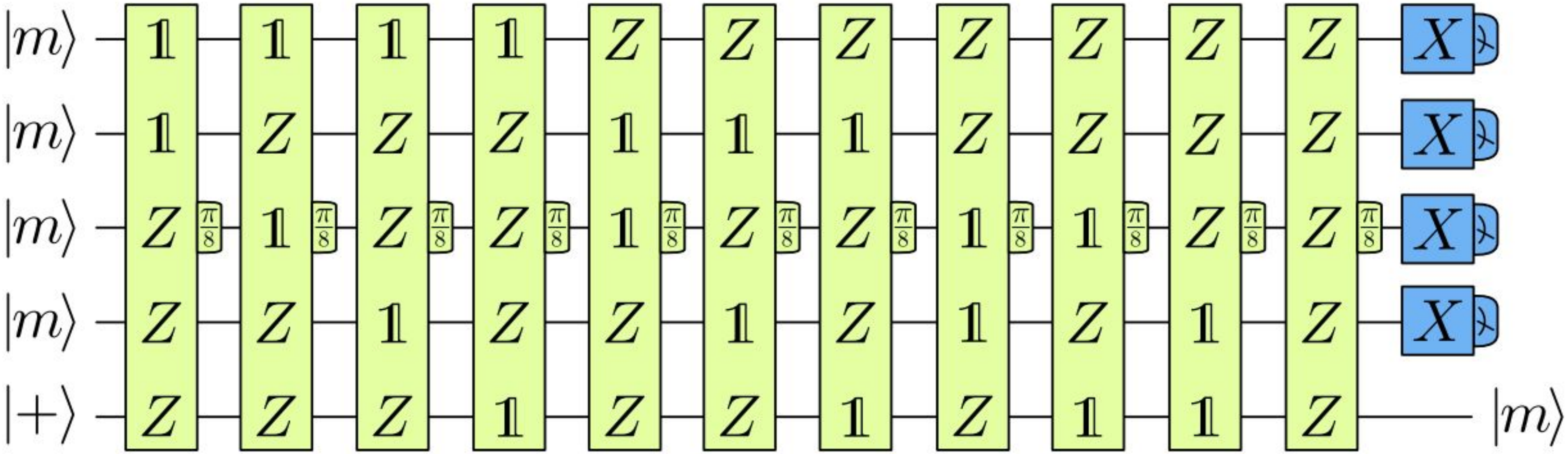
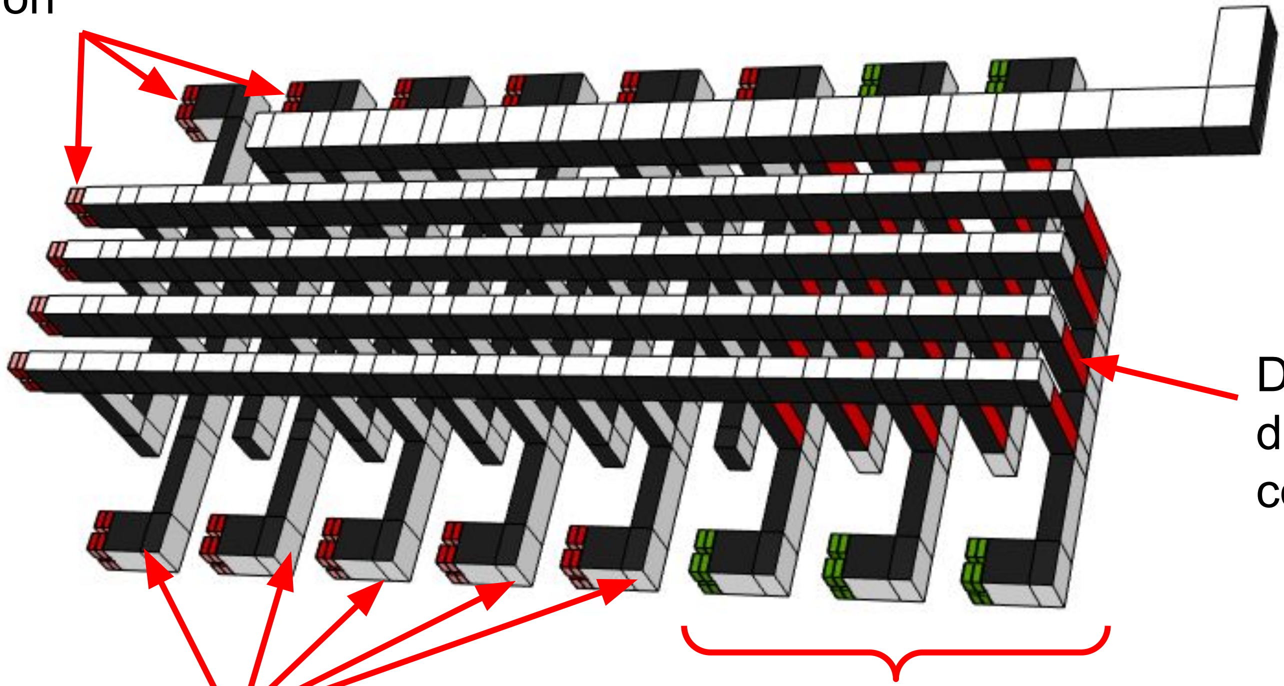


Figure 15: 15-to-1 distillation circuit that uses 5 qubits and 11 $\pi/8$ rotations.

Figure source: arXiv:1808.02892
Daniel Litinski - Game of Surface Codes

T state distillation (2x5 footprint)



Noisy physical state preparation

Output

Dynamically determined connections

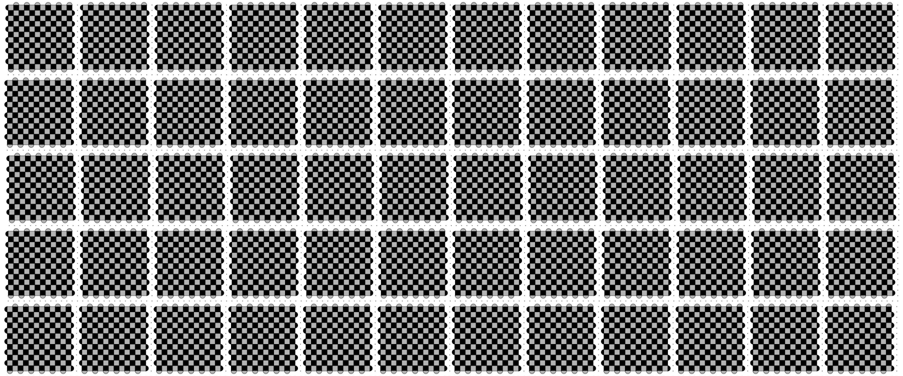
T gate teleportation parity measurements

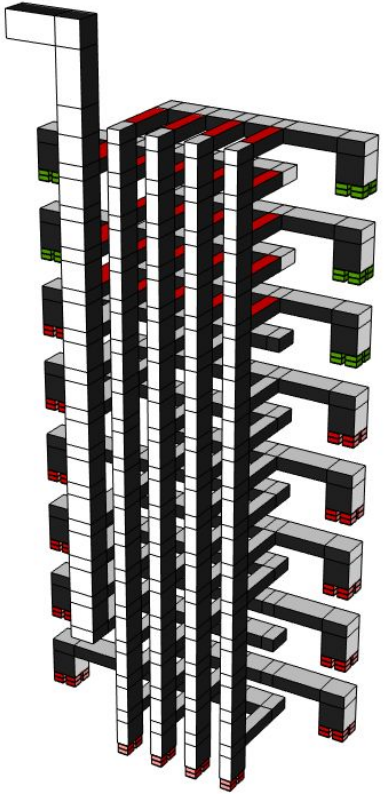
S corrections for inverted teleportations

Building blocks (review)

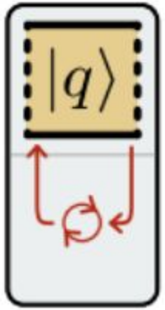
5x12 board

distance 13 qubits

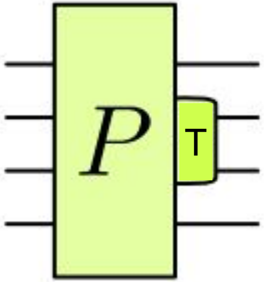




T Distillation



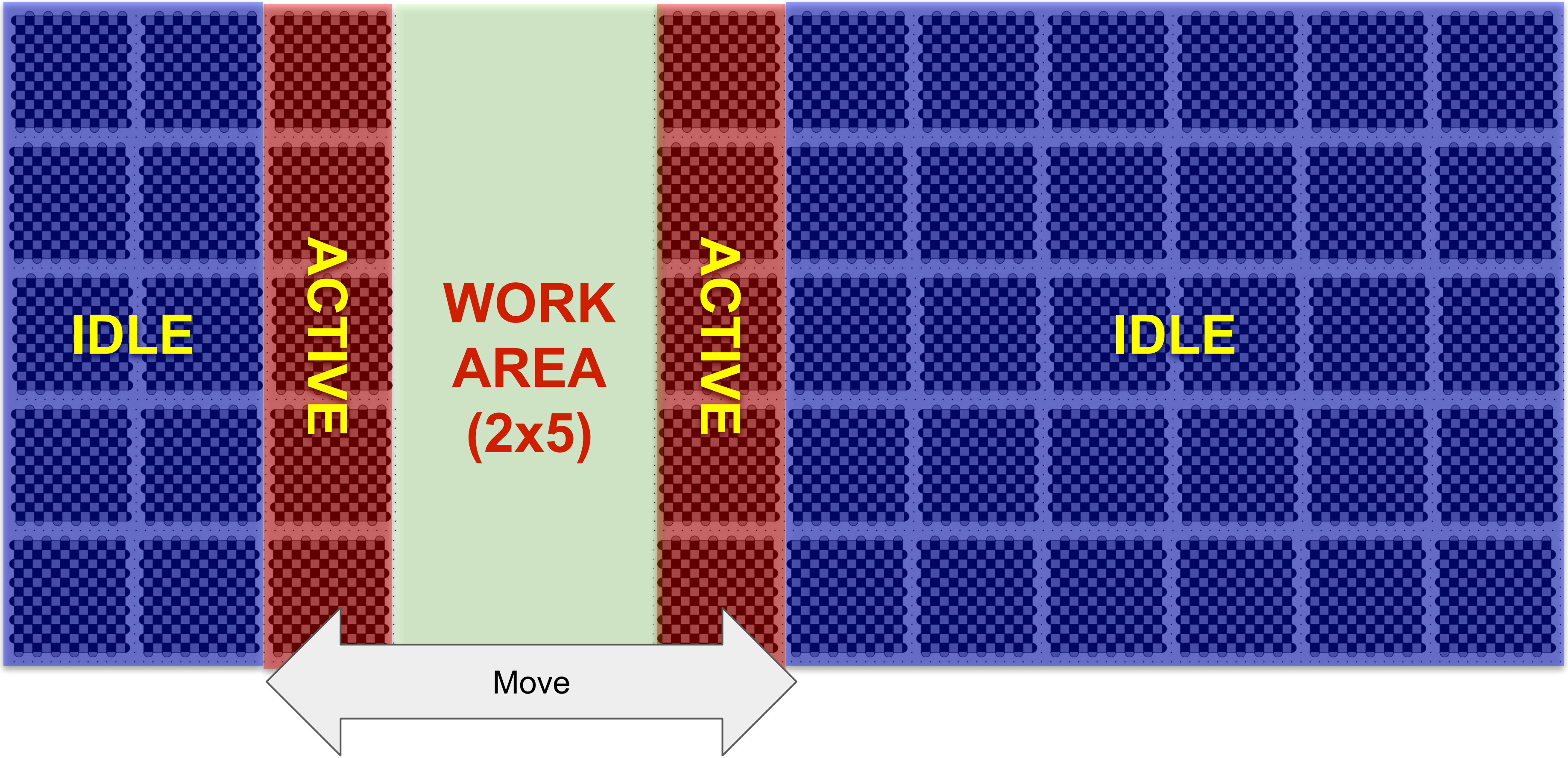
Single Qubit Gates



Generalized T

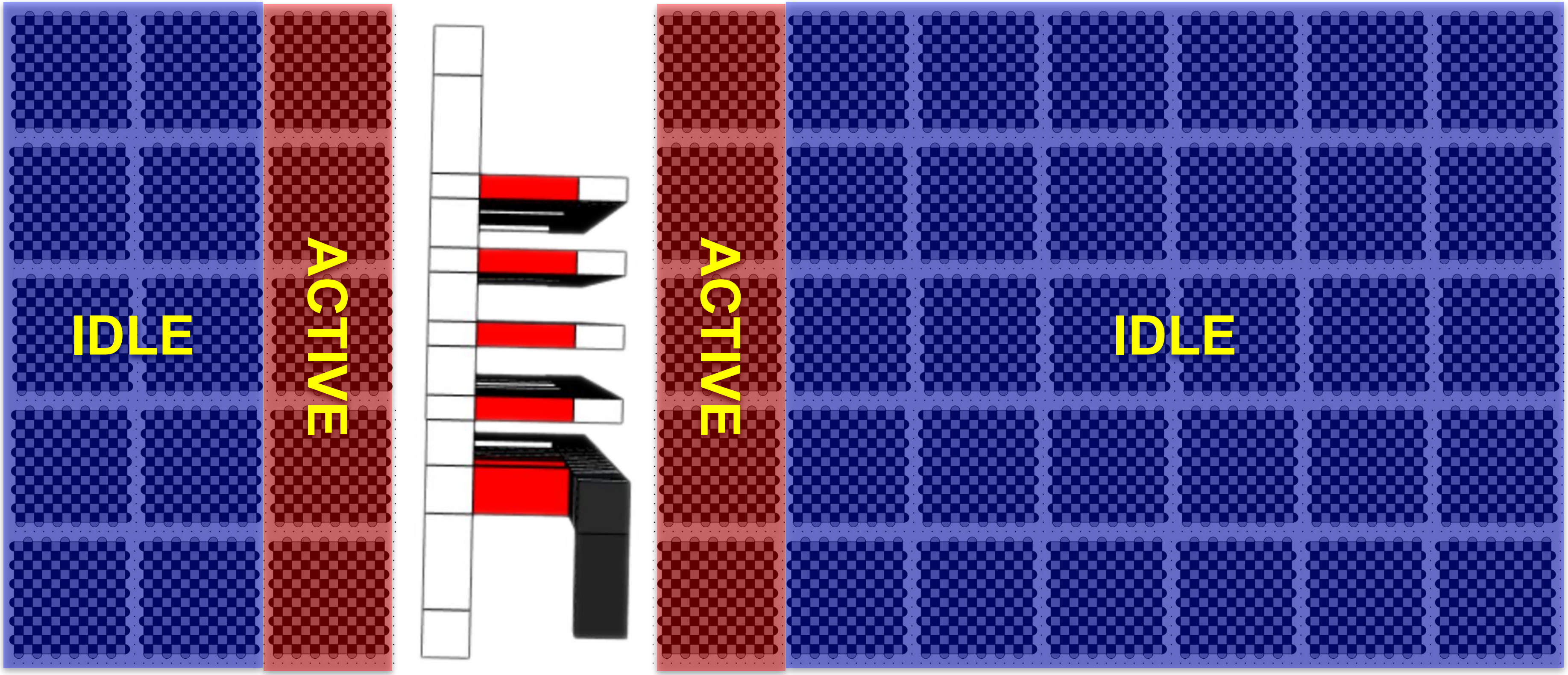
3. Algorithm and Cost

Moveable Work Area

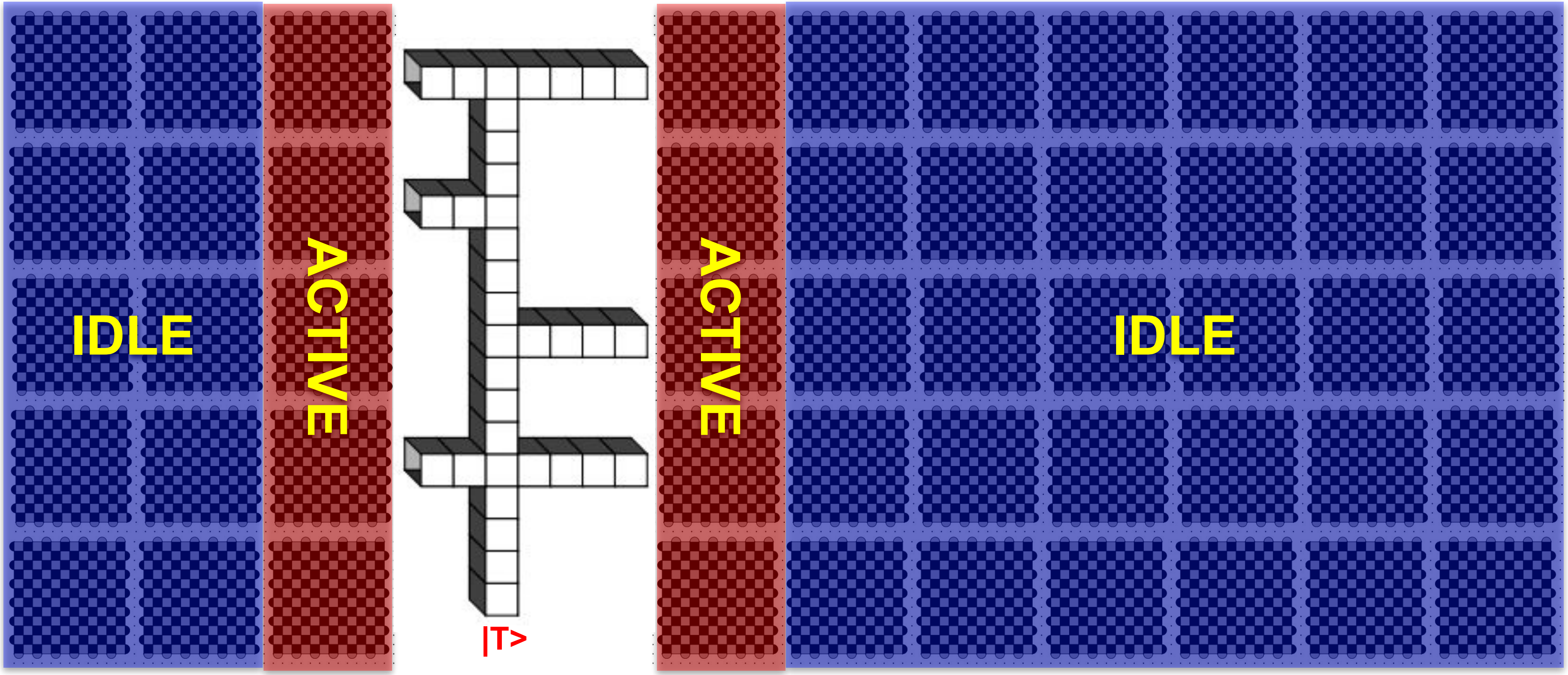


Parallel single qubit gates

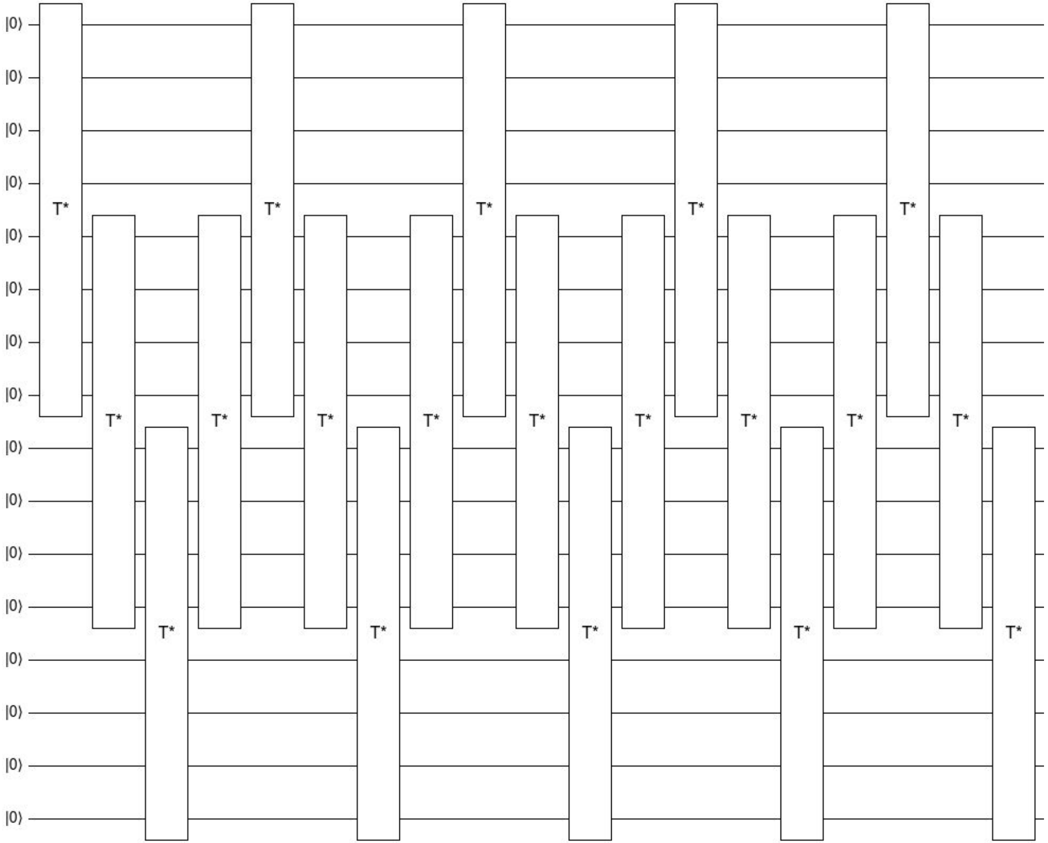
In place distillation



In place generalized T



Mix using sweeps of generalized Ts



1. Shift work area.

2. Pick random Pauli observable over active qubits (that anticommutes with last operation).

3. Transform into X-only observable using single qubit operations.

4. Apply generalized T to observable.

5. Goto 1.

Back of the envelope generalized T gate rate

+ 1d cycles to shift operating area

+ 3d cycles for single qubit S/H/I gates.

+ 10d cycles to distill a T state

+ 1d cycles to measure the P*T observable

+ 2d cycles to correct T^{-1} into T if teleportation was inverted

= 17d cycles total

≈ 17*13us = 221 microseconds per generalized T **≈ 4kHz**

Back of the envelope achievable gate count

4kHz generalized T

1 Hz board decay

A thousand generalized Ts would achieve

- **sufficient signal:** O(10%) chance of error, 4 samples per second
- **sufficient mixing:** O(100) sweeps of the operating area

4. Summary

Using current error correction techniques, and plausible hardware assumptions...

fault tolerant constructions require

25 000 qubits and **0.25 seconds**

to produce a classically-intractable random circuit sample with

10% fidelity

uses 500x more qubits than the NISQ approach, but scales up as needed
(asymptotic analysis actually applies)