## Explaining Adaptation in Genetic Algorithms with Uniform Crossover

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Keki Burjorjee Explaining Adaptation in GAs with UX



- Uniform Crossover first used by Ackley (1987)
- Studied and popularized by Syswerda (1989) and Spears & De Jong (1990, 1991)
- Numerous accounts of optimization in GAs with UX
- In practice frequently outperforms XO with tight linkage (Fogel, 2006)

Introduction Hyperclimbing Proof of Concept Prediction & Validation Conclusion
Optimization in GAs with UX Unexplained

- Cannot be explained within the rubric of the BBH
- No viable alternative has been proposed

Hyperclimbing

**Proof of Concept** 

Prediction & Validation

Conclusion

### Optimization in GAs with UX Unexplained

### Hyperclimbing Hypothesis

### A scientific explanation for optimization in GAs with UX

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Introduction Hyperclimbing Proof of Concept Prediction & Validation Conclusion What Does "Scientific" Mean?

- Logical Positivism (Proof or Bust)
  - Scientific truth is absolute
  - Emphasis on Verifiability (i.e. mathematical proof)
- The Popperian method
  - Scientific truth is provisional
  - Emphasis on Falsifiability (testable predictions)

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### Popperian Method

- Logic behind the Popperian Method : Contrapositive
  - Theory  $\Rightarrow$  Phenomenon  $\Leftrightarrow \neg$  Phenomenon  $\Rightarrow \neg$  Theory
- Additional tightening by Popper
  - $\bullet~$  Unexpected Phenomenon  $\rightarrow$  More credence owed to the theory
  - $\bullet\,$  e.g. Gravitational lensing  $\rightarrow$  General theory of relativity

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### Additional Requirements in a Science of EC

- Weak assumptions about distribution of fitness
  - This is just Occam's Razor
- Upfront proof of concept
  - Avoid another "Royal Roads moment"
- (Nice to have) Identification of a core computational efficiency

**Proof of Concept** 

Prediction & Validation

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### Hyperclimbing Hypothesis

## GAs with UX perform optimization by efficiently implementing a global search heuristic called hyperclimbing

Introduction	Hyperclimbing	Proof of Concept	Prediction & Validation	Conclusion
Outline				

- Highlight Symmetry of UX
- Oescribe Hyperclimbing Heuristic
- Operation of the second sec
  - Show a GA implementing hyperclimbing efficiently
- Make a prediction and validate it on
  - MAX-3SAT
  - Sherrington Kirkpatrick Spin Glasses problem
- Outline future work

Introduction		Нуре	rclimbin	g	Pro	oof of Co	oncept		Predi	ction & Validation		Conclusion
Variat	ion	in (	GAs									
	0	1	0	0	1	0	1	0	1		0	
	1	0	0	1	1	0	0	1	1		1	

Variation	in (	GAs	•							
0 <i>X</i> 1 1	1 X <sub>2</sub> 0	0 X <sub>3</sub> 0	0 X <sub>4</sub> 1	1 X <sub>5</sub> 1	0 X <sub>6</sub> 0	1 X <sub>7</sub> 0	0 X <sub>8</sub> 1	1 X9 1	 $egin{array}{c} 0 \ X_\ell \ 1 \end{array}$	

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Introduction		Нур	erclimbing		Pro	of of Co	oncept		Predi	ction & Validation	Co	nclusion
Variati	on	in	GAs									
	0	1	0	0	1	0	1	0	1		0	
	↑	$\downarrow$	$\downarrow$	↑	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	↑		$\uparrow$	
	1	0	0	1	1	0	0	1	1		1	

0

 $Y_1 \quad Y_2 \quad Y_3 \quad Y_4 \quad Y_5 \quad Y_6 \quad Y_7 \quad Y_8 \quad Y_9 \quad \ldots \qquad \qquad Y_\ell$ 

1 1 ..... 0

0

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1 0

ntroduction	Hyperclimbing	Proof of Concept	Prediction & Validation	Conclusion
Variatio	n in GAs			



Introduction	Hyperclimbing	Proof of Concept	Prediction & Validation	Conclusion
Variation	in GAs			

$$X_1$$
  $X_2$   $X_3$   $X_4$   $X_5$   $X_6$   $X_7$   $X_8$   $X_9$   $\ldots$   $X_\ell$ 

 $Y_1 \quad Y_2 \quad Y_3 \quad Y_4 \quad Y_5 \quad Y_6 \quad Y_7 \quad Y_8 \quad Y_9 \quad \ldots \qquad \qquad Y_\ell$ 

### UX: $X_1, \ldots, X_\ell$ are independent

- Absence of *positional bias* (Eshelman et al., 1989)
  - Attributes can be arbitrarily permuted
- Suppose  $Y_1, \ldots, Y_\ell$  independent and independent of  $\ell$ 
  - If locus *i* immaterial to fitness, can be spliced out

## The Hyperclimbing Heuristic

Hyperclimbing

**Proof of Concept** 

Prediction & Validation

Conclusion

### Hyperclimbing: A Stochastic Search Heuristic

The Setting

- $\{0,1\}^\ell \xrightarrow{f} \mathbb{R}$
- f may be stochastic

### Hyperclimbing

Proof of Concept

Prediction & Validation

Conclusion

### Schema Partitions and Schemata

Given index set  $\mathcal{I} \subset \{1, \dots, \ell\}$ , s.t.  $|\mathcal{I}| = k$ 

- $\mathcal I$  partitions the search space into  $2^k$  schemata
- $\bullet$  Schema partition denoted by  $[\![\mathcal{I}]\!]$

E.g. for 
$$I = \{3, 7\}$$



- Coarseness of the partition determined by k (the order)
- <sup>ℓ</sup>
   k
   crder k
- For fixed k,  $\binom{\ell}{k} \in \Omega(\ell^k)$

### Hyperclimbing

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Proof of Concept

Prediction & Validation

### The Effect of a Schema Partition

### • Effect of $\llbracket \mathcal{I} \rrbracket$ : Variance of average fitness of schemata in $\llbracket \mathcal{I} \rrbracket$

• If  $\mathcal{I} \subset \mathcal{I}'$ , then  $\mathsf{Effect}(\mathcal{I}) \leq \mathsf{Effect}(\mathcal{I}')$ 





- Effect of  $[\![\mathcal{I}]\!]$ : Variance of average fitness of schemata in  $[\![\mathcal{I}]\!]$
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Hyperclimbing

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### How a Hyperclimbing Heuristic Works

- Finds a coarse schema partition with a detectable effect
- Limits future sampling to a schema with above average fitness



Raises expected fitness of all future samples

Hyperclimbing

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### How a Hyperclimbing Heuristic Works

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### How a Hyperclimbing Heuristic Works

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• Raises expected fitness of all future samples

Hyperclimbing

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### How a Hyperclimbing Heuristic Works

# Limit future sampling to some schema $\label{eq:constraint} \ensuremath{\Uparrow} \ensuremath{\texttt{equivalent}} \times \tim$

- Hyperclimbing Heuristic now recurses
  - Search occurs over the unfixed loci
- And so on ...



- $\bullet \ \exists$  a small set of unfixed loci,  $\mathcal{I},$  such that
  - $\bullet$  Conditional effect of  $[\![\mathcal{I}]\!]$  is detectable
    - Conditional upon loci that have already been fixed
  - $\bullet\,$  Unconditional effect of  $[\![\mathcal{I}]\!]$  may be undetectable
- E.g.
  - Effect of 1\*#\*0\*#\* is detectable
  - Effect of \*\*#\*\*\*#\* undetectable
  - ('\*' = wildcard, '#'= defined locus)
- Called staggered coarse conditional effects

Hyperclimbing

**Proof of Concept** 

Prediction & Validation

Conclusion

### Fitness Distribution Assumption is Weak

- Staggered coarse conditional effects assumption is weak
  - Weaker than fitness distribution assumption in the BBH
    - BBH assumes unstaggered coarse unconditional effects
- Weaker assumptions are more likely to be satisfied in practice
  - $\bullet~\mbox{Good}~\mbox{b/c}$  we aspire to explain optimization in all GAs with UX

### Hyperclimbing Heuristic is in Good Company

- Hyperclimbing an example of a global decimation heuristic
- Global decimation heuristics iteratively reduce the size of a search space
  - Use non-local information to find and fix partial solutions
  - No backtracking
- E.g. Survey propagation (Mérzad et al., 2002)
  - State-of-the-art solver for large, random SAT problems

## **Proof Of Concept**

Hyperclimbing

**Proof of Concept** 

Prediction & Validation

Conclusion

4-Bit Stochastic Learning Parities Problem

$$fitness(x) \sim \begin{cases} \mathcal{N}(+0.25, 1) \text{ if } x_{i_1} \oplus x_{i_2} \oplus x_{i_3} \oplus x_{i_4} = 1 \\ \mathcal{N}(-0.25, 1) \text{ otherwise} \end{cases}$$



**Proof of Concept** 

Prediction & Validation

Conclusion

### 4-Bit Stochastic Learning Parities Problem

### The Game

10111001011101001111000001	$\mapsto$	+0.5689
10001010110110100011110000	$\mapsto$	-0.25565
11101100100010111101101110	$\mapsto$	-0.37747
01110101001111100000110001	$\mapsto$	-0.29589
00000010001000100110011101	$\mapsto$	-1.4751
00001100000100010001100000	$\mapsto$	-0.234
01000001111000111100110100	$\mapsto$	+0.11844
11010001000101000011000110	$\mapsto$	+0.31481
00000011100010001100000111	$\mapsto$	+1.4435
00100111000111000001110000	$\mapsto$	-0.35097
10100011011010101010100001	$\mapsto$	+0.62323
00011011101010100010100000	$\mapsto$	+0.79905
00101010101100101110100000	$\mapsto$	+0.94089
01010110000000110110110011	$\mapsto$	-0.99209
11100101000110010110110101	$\mapsto$	+0.21204
00011111011101110101000111	$\mapsto$	+0.23788
00001111111101011110111010	$\mapsto$	-1.0078
: :	:	:
000010111011110000001111100	$\mapsto$	+1.0823
11011100111100010100111101	$\mapsto$	-0.1315

**Proof of Concept** 

Prediction & Validation

Conclusion

### 4-Bit Stochastic Learning Parities Problem

### The Game

$\mapsto$	+0.5689
$\mapsto$	-0.25565
$\mapsto$	-0.37747
$\mapsto$	-0.29589
$\mapsto$	-1.4751
$\mapsto$	-0.234
$\mapsto$	+0.11844
$\mapsto$	+0.31481
$\mapsto$	+1.4435
$\mapsto$	-0.35097
$\mapsto$	+0.62323
$\mapsto$	+0.79905
$\mapsto$	+0.94089
$\mapsto$	-0.99209
$\mapsto$	+0.21204
$\mapsto$	+0.23788
$\mapsto$	-1.0078
:	:
$\mapsto$	+1.0823
$\mapsto$	-0.1315
	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

Proof of Concept

Prediction & Validation

Conclusion

### Solving the 4-Bit Stochastic Learning Parities Problem

### Naive approach

- Visit loci in groups of four
- Check for differentiation in the multivariate marginal fitness values
- Time complexity is  $\Omega(\ell^4)$

x <sub>i1</sub>	× <sub>i2</sub>	×i3	× <sub>i4</sub>	Expected Marginal Fitness
0	0	0	0	-0.25
0	0	0	1	+0.25
0	0	1	0	+0.25
0	0	1	1	-0.25
0	1	0	0	+0.25
0	1	0	1	-0.25
0	1	1	0	-0.25
0	1	1	1	+0.25
1	0	0	0	+0.25
1	0	0	1	-0.25
1	0	1	0	-0.25
1	0	1	1	+0.25
1	1	0	0	-0.25
1	1	0	1	+0.25
1	1	1	0	+0.25
1	1	1	1	-0.25

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×j	Expected Marginal	$x_{j_1}$	× <sub>j2</sub>	Expected Marginal Fitness
	Fitness	0	0	0.0
0	0.0	0	1	0.0
1	0.0	1	0	0.0
		1	1	0.0

 $x_{j_3}$ 

0

1

0

1

Expected Marginal

Fitness

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

• Visiting loci in groups of three or less won't work

x<sub>j1</sub> x<sub>j2</sub>

0

0

0

0

1 0 0

1

1 1

1

0 0

0 1

1

1

0 1

1

Introduction	Hyperclimbing	Proof of Concept	Prediction & Validation	Conclusion

⊳ Watch On YouTube

Dynamics of a SGA on the 4-Bit Learning Parities problem #Loci=200 Expected #generations for the red dots to diverge constant w.r.t.

- Location of the red dots
- Number of blue dots

Dynamics of a blue dot invariant to

- Location of the red dots
- Number of blue dots

Introduction	Hyperclimbing	Proof of Concept	Prediction & Validation	Conclusion



Dynamics of a SGA on the 4-Bit Learning Parities problem #Loci=1000

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Dynamics of a SGA on the 4-Bit Learning Parities problem #Loci=10000

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• Staircase function descriptor:  $(m, n, \delta, \sigma, \ell, L, V)$ 



• Staircase function is a stochastic function  $f: \{0,1\}^\ell \to \mathbb{R}$ 















Introduction	Hyperclimbing	Proof of Concept	Prediction & Validation	Conclusion
$\ell = 50$				
m = 5				
<i>n</i> = 4				
$\delta = 0.2$				
$\sigma = 1$				
$L = \begin{bmatrix} 1 \\ 5 \\ 9 \\ 13 \\ 17 \end{bmatrix}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$			
$V = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$	$ \begin{array}{cccc} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{array} $		⊳ Vatch On YouTube	



Introduction	Hyperclimbing	Proof of Concept	Prediction & Validation	Conclusion
$\ell = 50$				
m = 5				
<i>n</i> = 4				
$\delta = 0.2$				
$\sigma = 1$				
$L = \begin{bmatrix} 35\\ 31\\ 21\\ 40\\ 12 \end{bmatrix}$	25     37     46       5     32     20       33     50     42       27     30     3       14     48     2			
$V = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 &$		⊳ Vatch On YouTube	

Introduction	Hyperclimbing	Proof of Concept	Prediction & Validation	Conclusion
Introduction $\ell = 5000$ m = 5 n = 4 $\delta = 0.2$ $\sigma = 1$ L = - $\Gamma = 2050$ 465	Нурегсlimbing 9 1931 3284 г	Proof of Concept	Prediction & Validation	Conclusion
2130 2404 143 1284 2061 904 3503 724	4         205         4418           4         53         437           4         2650         3080           4         4822         4444			
$V = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$	 	⊳ Vatch On YouTube	

Introduction	Hyperclimbing	Proof of Concept	Prediction & Validation	Conclusion
Introduction $\ell = 5000$ $m = 5$ $n = 4$ $\delta = 0.2$ $\sigma = 1$ $L =$ $\begin{bmatrix} 2050 & 4650 \\ 4650 \end{bmatrix}$	Hyperclimbing 9 1931 3284 ]	Proof of Concept	Prediction & Validation	Conclusion
2130 240 143 128 2061 904 3503 724	4     205     4418       4     53     437       4     2650     3080       4     4822     4444			
$V = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$	$ \begin{array}{cccc} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 1 \end{array} $		⊳ Natch On YouTube	

### **Prediction & Validation**

Introduction	Hyperclimbing	Proof of Concept	Prediction & Validation	Conclusion
$\ell = 500$ $m = 125$ $n = 4$ $\delta = 0.2$ $\sigma = 1$ $L = \begin{bmatrix} 1 & 2 \\ 5 & 6 \\ \vdots & \vdots \end{bmatrix}$	3 4 7 8 : :			
$V = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ \vdots & \vdots \\ 1 & 1 \end{bmatrix}$	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1		⊳ Natch On YouTube	

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Introduction	Hyperclimbing	Proof of Concept	Prediction & Validation	Conclusion
$\ell = 500$ $m = 125$ $n = 4$ $\delta = 0.2$ $\sigma = 1$ $L = \begin{bmatrix} 1 & 2 \\ 5 & 6 \\ \vdots & \vdots \end{bmatrix}$	3 4 7 8 : :			
$V = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ \vdots & \vdots \\ 1 & 1 \end{bmatrix}$	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1		⊳ Natch On YouTube	

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### **Uniform Random MAX-3SAT**

#variables=1000, #clauses=4000



#trials=10. Error bars show standard error.

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### SK Spin Glasses System





### SK Spin Glasses System



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## Conclusion

Introduction	Hyperclimbing	Proof of Concept	Prediction & Validation	Conclusion
Summary				

Proposed the Hyperclimbing Hypothesis

- Weak assumptions about the distribution of fitness
- Based on a computational efficiency of GAs with UX
- Upfront proof of concept
- Testable predictions + Validation on
  - Uniform Random MAX-3SAT
  - SK Spin Glasses Problem

- Implicit Parallelism is real
  - Not your grandmother's implicit parallelism
  - Effect of coarse schema partitions evaluated in parallel
    - NOT the average fitness of short schemata
  - Defining length of a schema partition is immaterial
    - New Implicit parallelism more powerful
- Think in terms of Hyperscapes not Landscapes

Introduction	Hyperclimbing	Proof of Concept	Prediction & Validation	Conclusion
What Now	/?			

- Flesh out the Hypothesis
  - What does a deceptive hyperscape look like?
- More Testable Predictions + Validation
  - Are staggered coarse conditional effects indeed common?
- Generalization
  - Explain optimization in other EAs
- Outreach
  - Identify and efficiently solve problems familiar to theoretical computer scientists
- Exploitation
  - Construct better representations
  - Choose better parameters for existing EAs
  - Construct better EAs (e.g. backtracking EAs)

## **Questions?**

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