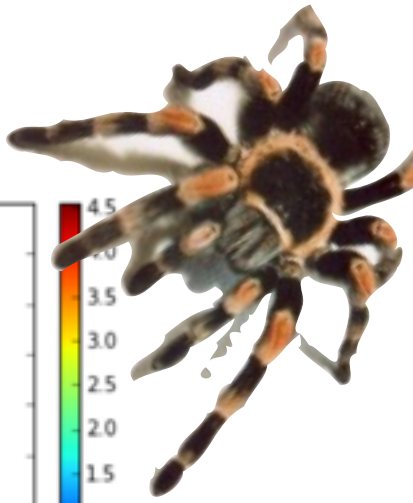
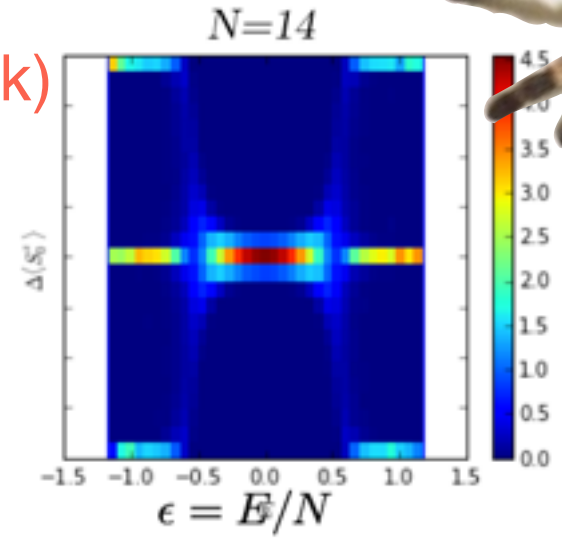
Many-body localization and adiabatic quantum optimization

Chris Laumann (UW)

See arXiv:1404.2276 (posted after talk)



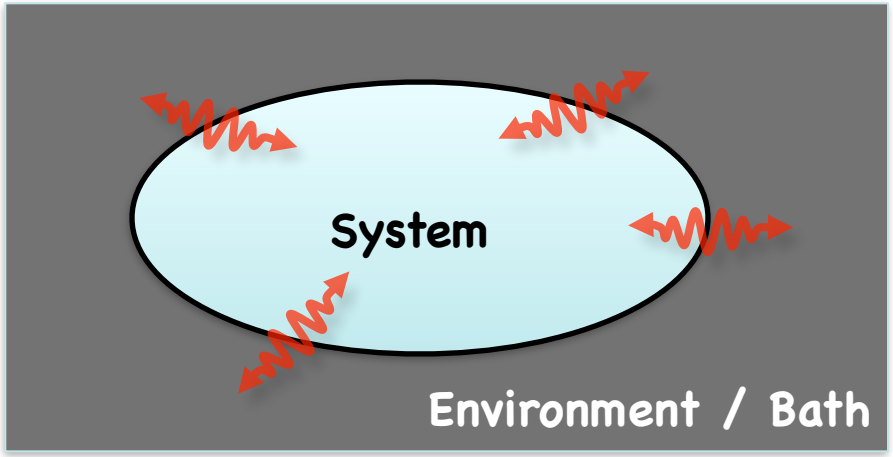
Arijeet Pal (Harvard)
Antonello Scardicchio (ICTP)



Simons Institute "Complexity Meets Condensed Matter"
March 26, 2014

Motivation

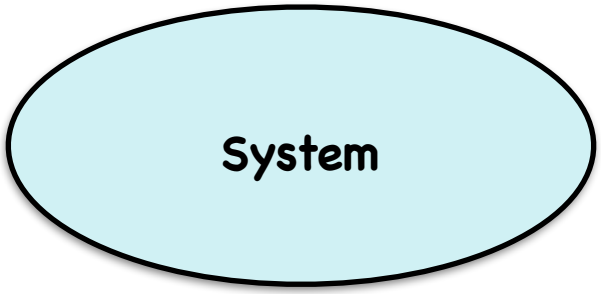
Statistical mechanics relates thermodynamics to the microscopic world



Equilibration requires exchange:

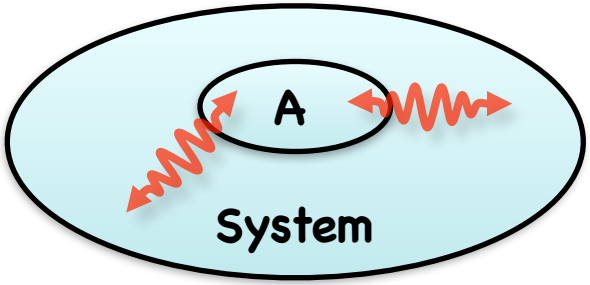
Energy, particles …

What if there is no environment?



"Ergodic" system behave as its own bath.

Quantum thermalization



"Ergodic" systems behave as their own bath.

Quantum thermalization: subsystems at long time go to thermal equilibrium.

System dynamics unitary: $\rho_S(t) = e^{-iHt}\rho_S(t=0)e^{iHt}$

Subsystem dynamics thermalize: $\rho_A = \underset{S\setminus A}{\mathrm{Tr}}\,\rho_S$

$$\lim_{t\to\infty}\lim_{N\to\infty}\rho_A(t) = \rho_A^{eq}(T,\mu_i,\cdots)$$

Ought be true for all finite subsystems and all initial conditions consistent with conserved quantities (E, N,... <-> T, μ,...).

Eigenstate Thermalization Hypothesis

For system to equilibrate from all initial conditions, all many-body eigenstates need agree with equilibrium on subsystems.

$$H\left|E_i\right\rangle = E_i\left|E_i\right\rangle$$

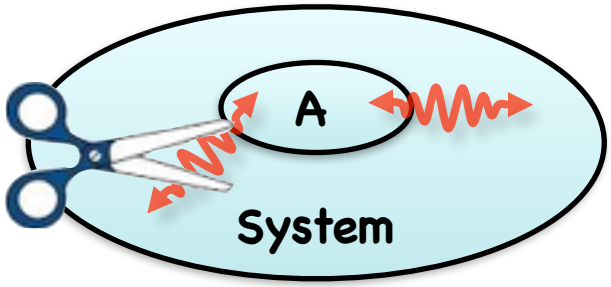
$$\rho_A = \operatorname*{Tr}_{S\setminus A}\left|E_i\right\rangle\left\langle E_i\right| = \rho_A^{eq}(T,\mu,\cdots)$$

(For all eigenstates with appropriate conserved quantities, in thermodynamic limit.)

It's a hypothesis. Needs proof or numerics in given model.

Deutsch (1991), Srednicki (1994), Rigol, Dunjko, Olshanii (2008), Pal and Huse (2010), …

Quantum thermalization



"Ergodic" systems behave as their own bath.

Equilibration requires exchange among subsystems. What if it doesn't happen?

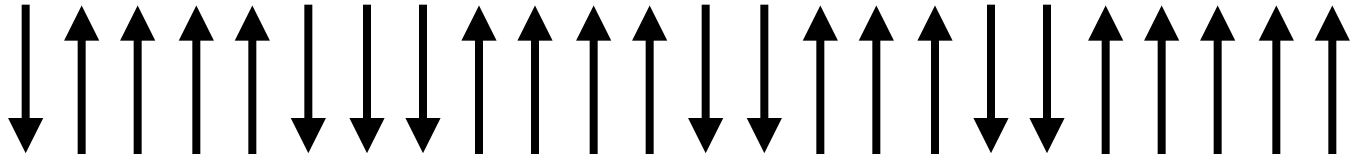
Quantum statistical mechanics breaks for localized systems.

Nope.

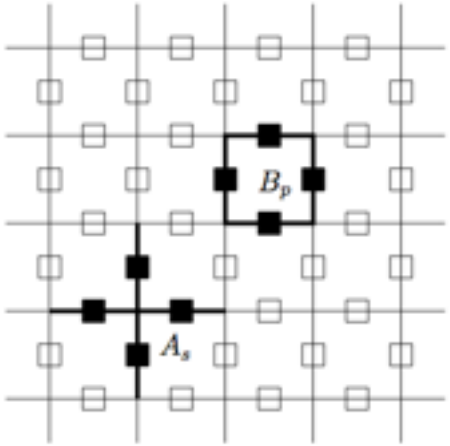
$$Z = \mathrm{Tr}\, e^{-\beta H}$$

Localization and Order

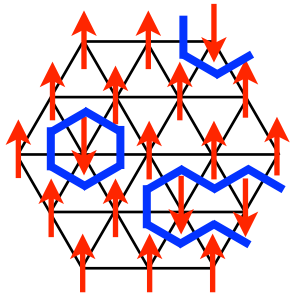
1) Localization protects quantum and symmetry breaking order not allowed at finite temperature



Frozen Ising 'glass' order in 1D at finite energy density



$$\sum_{\{\sigma^z\}} (-1)^{N_{dw}}$$

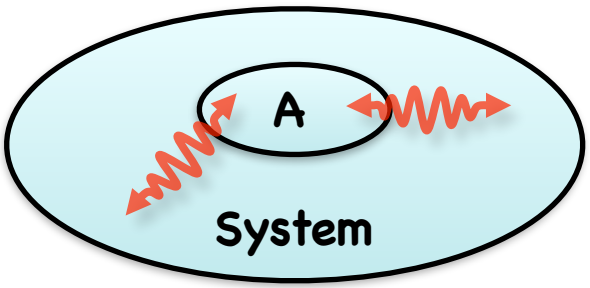


Frozen 2D 'glass' topological and SPT order

Huse, Nandkishore, Oganesyan, Pal, Sondhi, PRB (2013), Chandran, Khemani, CRL, Sondhi, 1310.1096

Localization and Entanglement

2) Localized eigenstates have entanglement entropy obeying area law



$$\rho_A = \operatorname*{Tr}_{S\setminus A} |E_i\rangle \langle E_i|$$

$$S_A = -\operatorname{Tr} \rho_A \log \rho_A$$

$$= \alpha L^{d-1} + \cdots$$

Contrast with ergodic eigenstates (ETH):

$$S_A = S_A^{eq} = sL^d + \cdots$$

Oganesyan, Huse. Huse, Nandkishore, Oganesyan, Pal, Sondhi. Bauer, Nayak. Swingle. Serbyn, Abanin…

Any examples?

Commuting Hamiltonians (kinda)

Strongly disordered models :

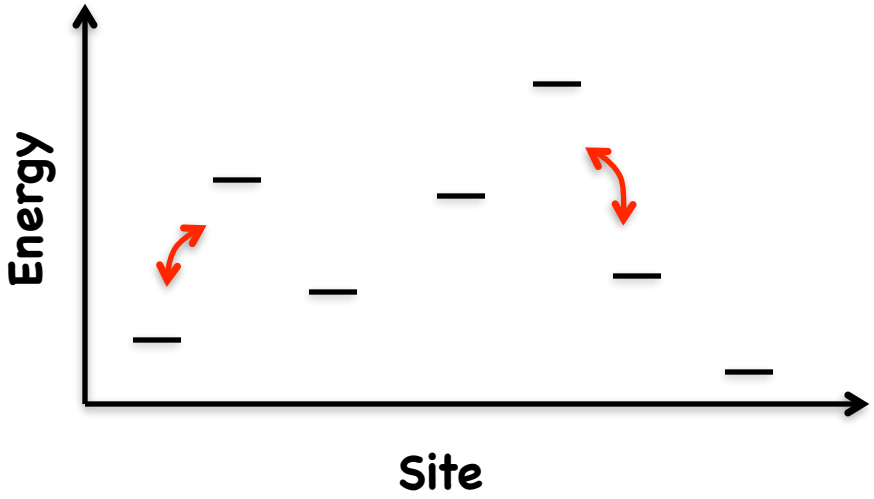
Single-particle Anderson localization

Perturbative interactions in such models

Numerical studies, mostly 1D

Single-particle Localization

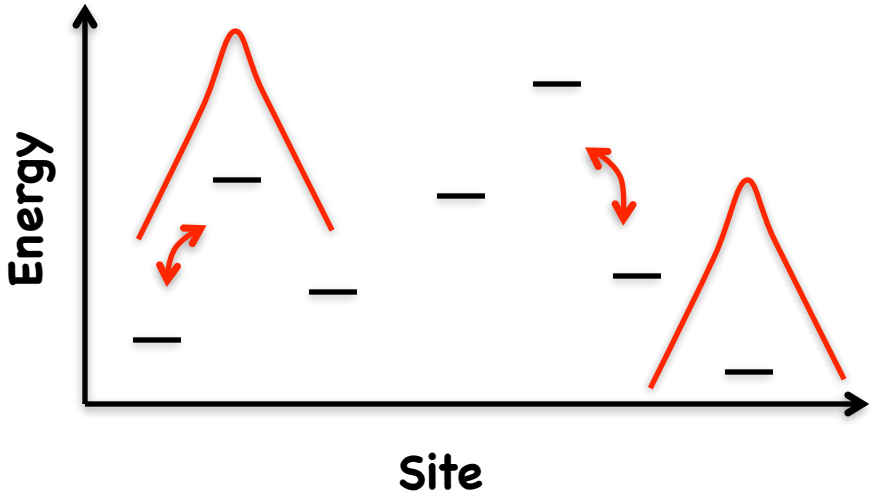
$$H = t \sum_{ij} a_i^\dagger a_j + \sum_i \mu_i n_i \qquad \mu_i \in [-W/2, W/2]$$



P. W. Anderson, Phys. Rev. (1958)

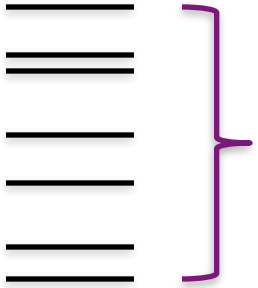
Single-particle Localization

$$H = t \sum_{ij} a_i^\dagger a_j + \sum_i \mu_i n_i \qquad \mu_i \in [-W/2, W/2]$$



Off-resonant hopping fails to hybridize sites at long-distances:

$$H = \sum_\alpha e_\alpha a_\alpha^\dagger a_\alpha$$



Localized

$$|\phi(r)|^2 \sim e^{-r/\xi}$$

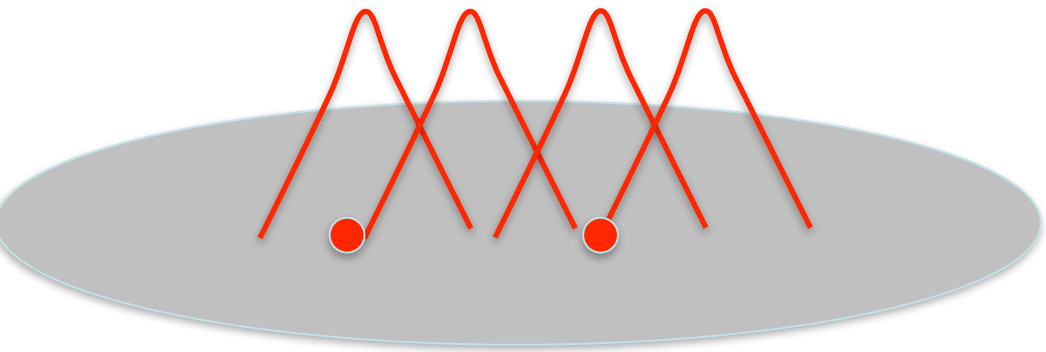
P. W. Anderson, Phys. Rev. (1958)

Many-body Eigenstates

$$H = \sum_{\alpha} e_\alpha a_\alpha^\dagger a_\alpha$$

Localized $|\phi(r)|^2 \sim e^{-r/\xi}$

$$|\psi\rangle = a_1^\dagger a_3^\dagger |0\rangle$$



Slater determinants are trivially localized

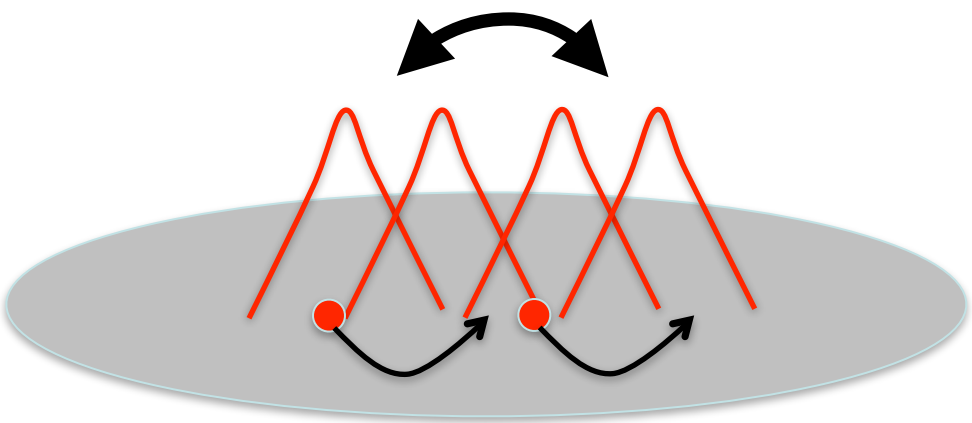
P. W. Anderson, Phys. Rev. (1958)

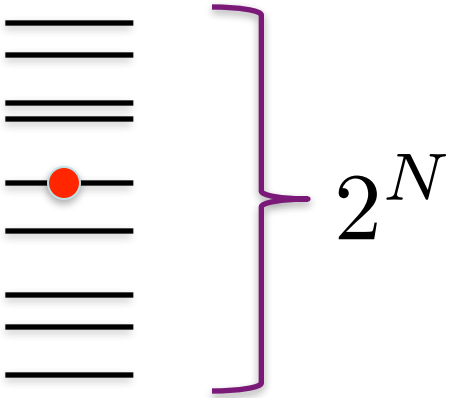
Localization in Fock space

Start with single particle localized states and add in interactions:

$$H = \sum_{\alpha} e_\alpha a_\alpha^\dagger a_\alpha + \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} a_\alpha^\dagger a_\beta^\dagger a_\gamma a_\delta$$

Can weak V cause hybridization of localized many-particle states?





Basko, Aleiner, Altshuler, Annals of Physics (2006), Huse and Oganesyan (2007), Pal and Huse (2009)

Looking for a mean-field model

- Analytically tractable MBL transitions/phases?
- Finite energy density mobility edges?
- Localization is ultimate glass — any connection?

Quantum Random Energy Model

$$H = E(\{\sigma_i^z\}) - \Gamma \sum_i \sigma_i^x$$

Classical Random Energy Model

"The simplest spin glass"

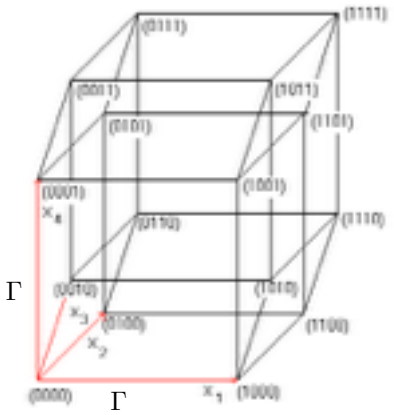
Random energy for each z-state

$$P(E) = \frac{1}{\sqrt{\pi N}} e^{-\frac{E^2}{N}}$$

N-body generalization of SK model

Transverse field

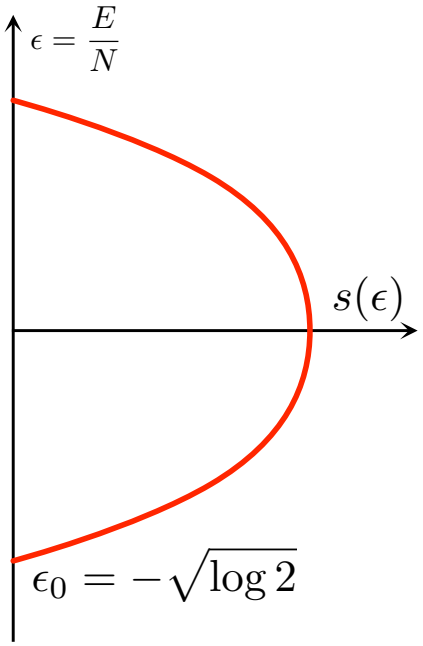
Provides dynamics



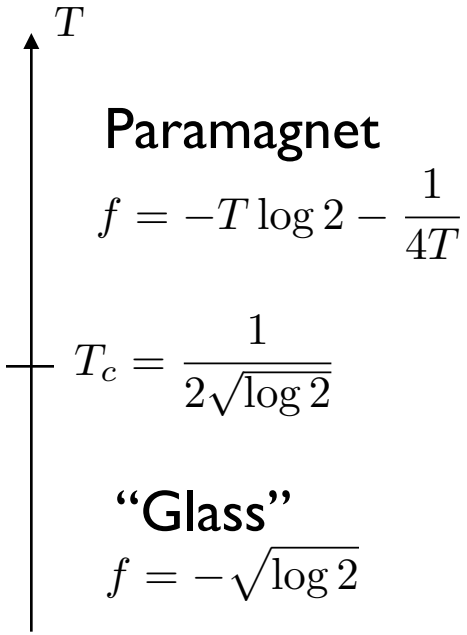
B. Derrida (1980,1981), Y. Goldschmidt (1990), T. Jorg, et al (2008)

Classical Limit: Statistical Mechanics

Microcanonical



Canonical

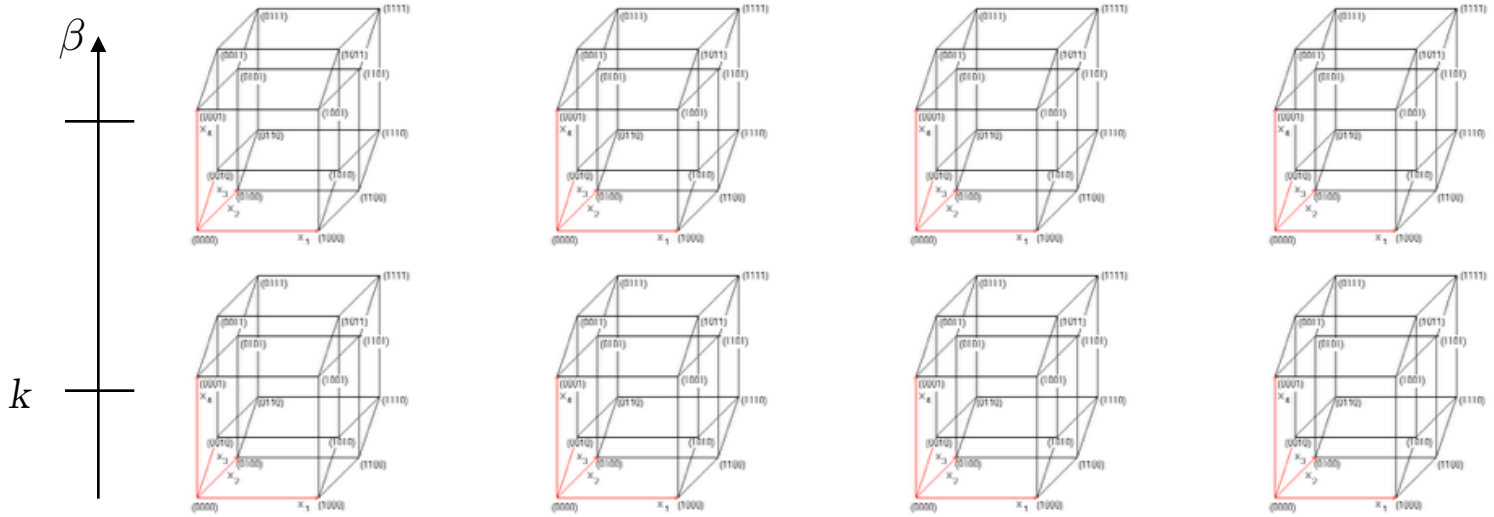


$$\overline{n(E)} = 2^N P(E) \sim e^{N s(E/N)}$$

$$s(\epsilon) = \log 2 - \epsilon^2$$

B. Derrida (1980)

Replica Solution of QREM

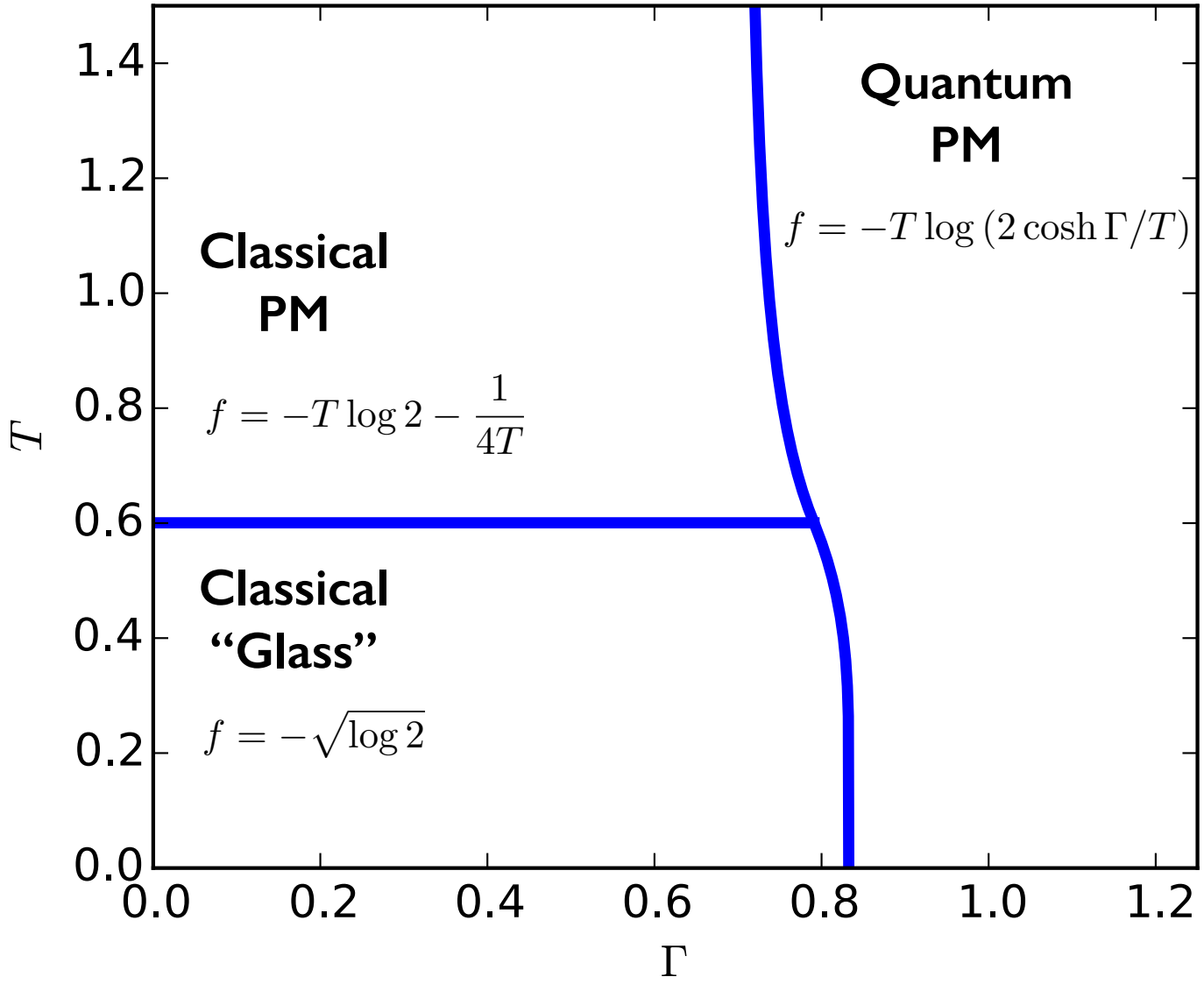


- Replica trick in imaginary time representation
- Time and replica dependent order parameter
- *Static* RS and 1RSB ansatzes give three phases…

$$Q_{kk'}^{\alpha\alpha'}(\sigma) = \frac{1}{N}\sum_i \sigma_i^{\alpha}(k)\sigma_i^{\beta}(k')$$

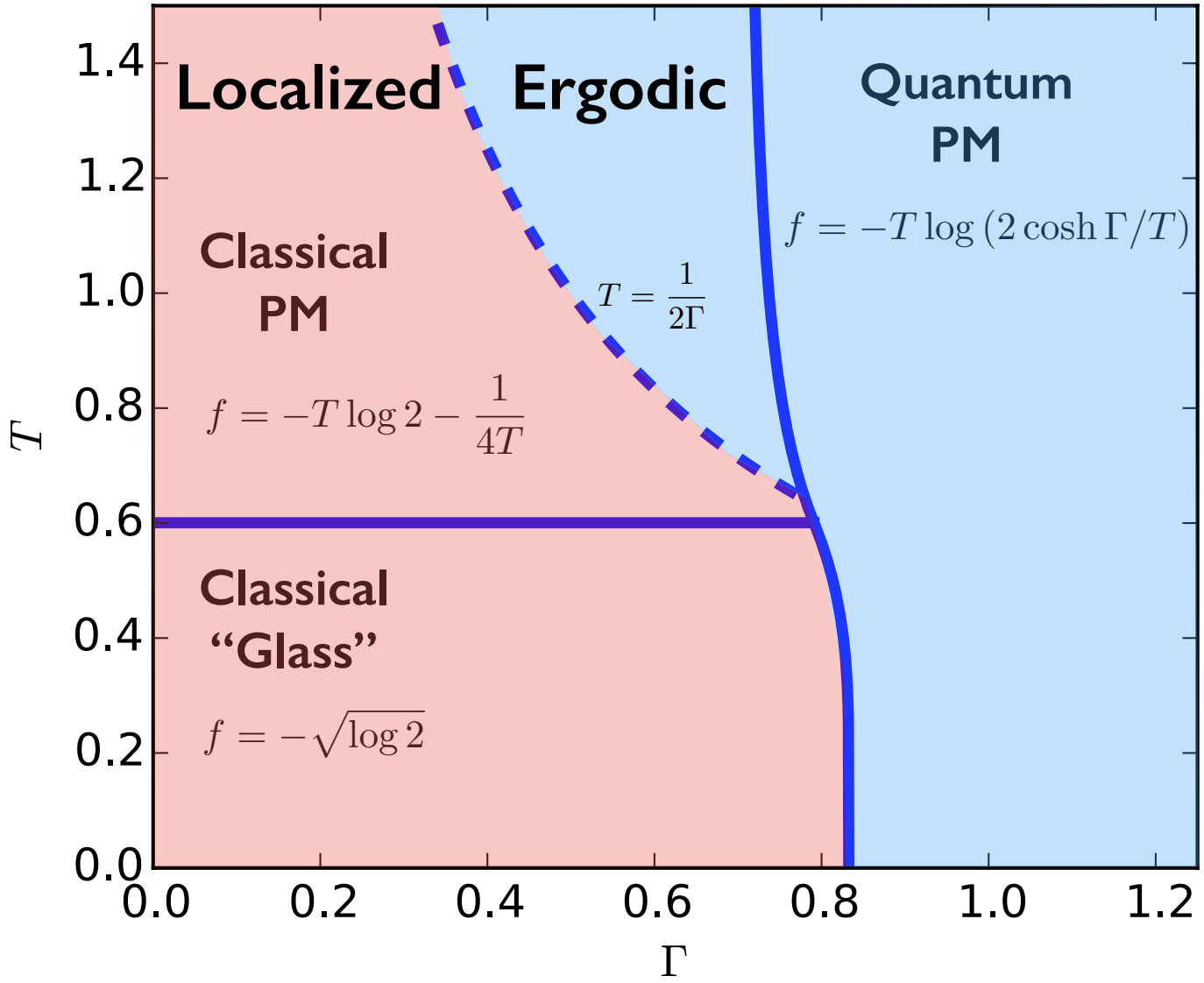
Y. Goldschmidt (1990)

Canonical Phase Diagram



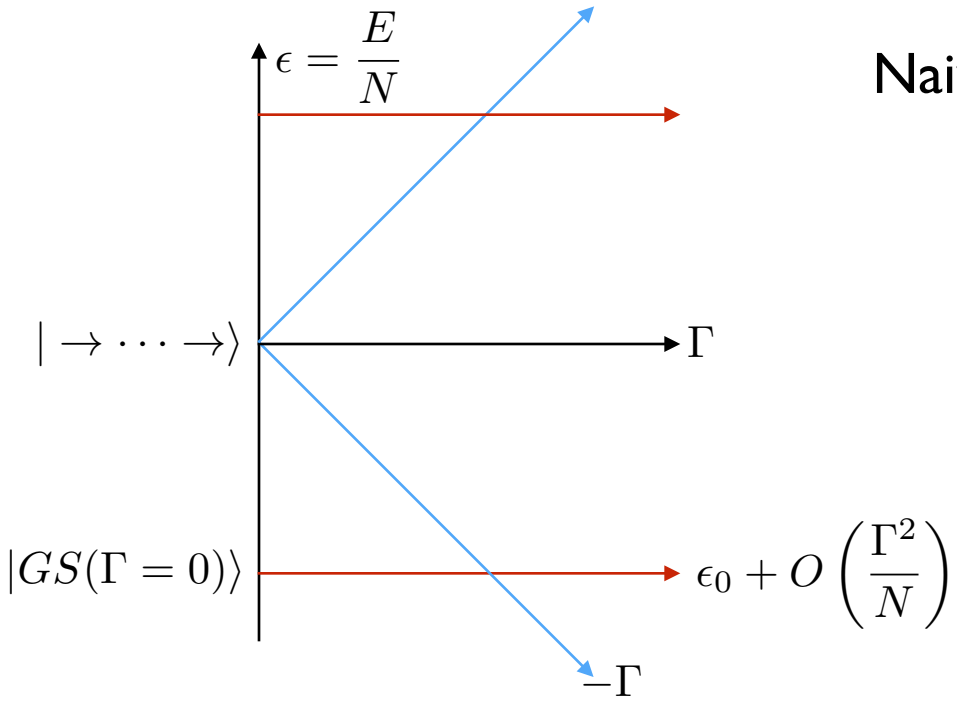
Y. Goldschmidt (1990)

Dynamical Phase Diagram



CRL, Pal, Scardicchio (Next week?)

Perturbative Rigidity



Naive perturbation theory of GS:

$$E_0(\Gamma) = E_0 - \Gamma^2 \sum_{i=1}^{N} \frac{1}{E_i - E_0} + \cdots$$

$$\approx E_0 - \Gamma^2 \frac{1}{\sqrt{2\log(2)}}$$

$O(N)$ $O(1)$

- All orders give O(1) corrections to extensive energies
- Suggests states do not rearrange thermodynamically
- Variational first order QPT in ground state

T. Jorg et al (2008)

Many-body Level Statistics

- Level statistics diagnose dynamical phase transition
- Ratio diagnostic cancels DOS fluctuations

$$r_\alpha^{(n)} = \min\{\delta_\alpha^{(n)}, \delta_\alpha^{(n+1)}\} / \max\{\delta_\alpha^{(n)}, \delta_\alpha^{(n+1)}\}$$

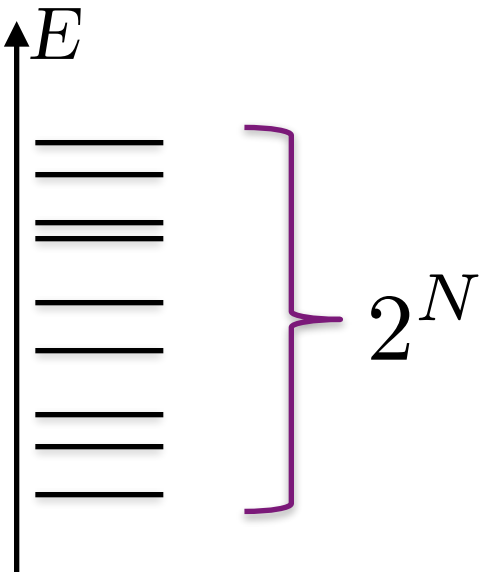
$$\delta_\alpha^{(n)} = |E_\alpha^{(n)} - E_\alpha^{(n-1)}|$$

MBL: Poisson level statistics

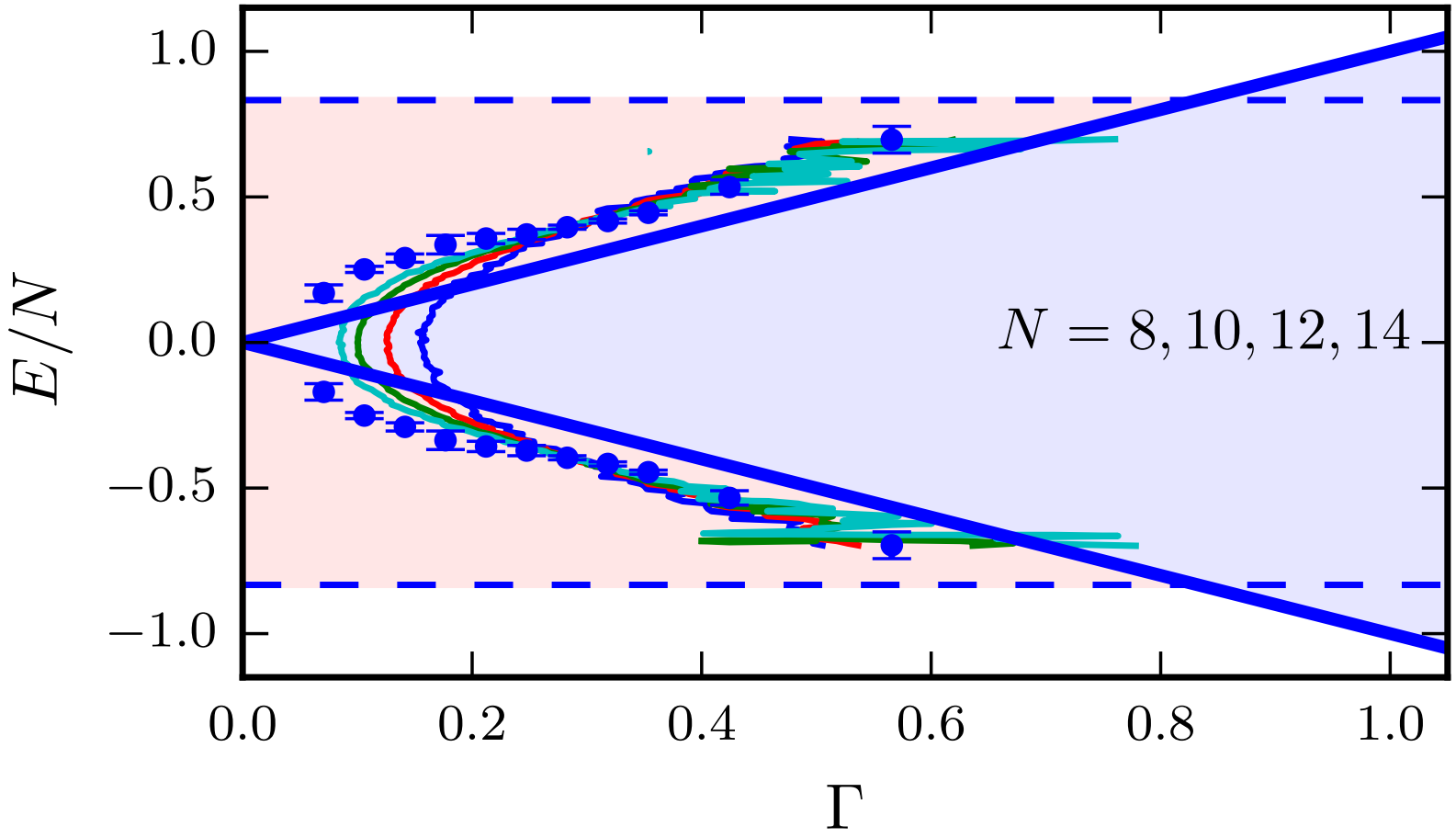
$$[r] \approx 0.39$$

ETH: GOE level statistics

$$[r] \approx 0.53$$



Numerical Phase Diagram



Contours of level statistics ratio $[r] = 0.48$

Local observables

- In ergodic phase local observables are smooth

$$M(n) = \langle n|S_0^z|n\rangle = M(\epsilon_n)$$

$$\frac{\delta M(n)}{\delta n} \approx \frac{dM(\epsilon)}{d\epsilon}\frac{\delta \epsilon}{\delta n} \approx M'(\epsilon)e^{-Ns(\epsilon)}$$

- Local observables frozen in MBL phase

$$\frac{\delta M}{\delta n} = \langle n+1|S_0^z|n+1\rangle - \langle n|S_0^z|n\rangle = O(1)$$

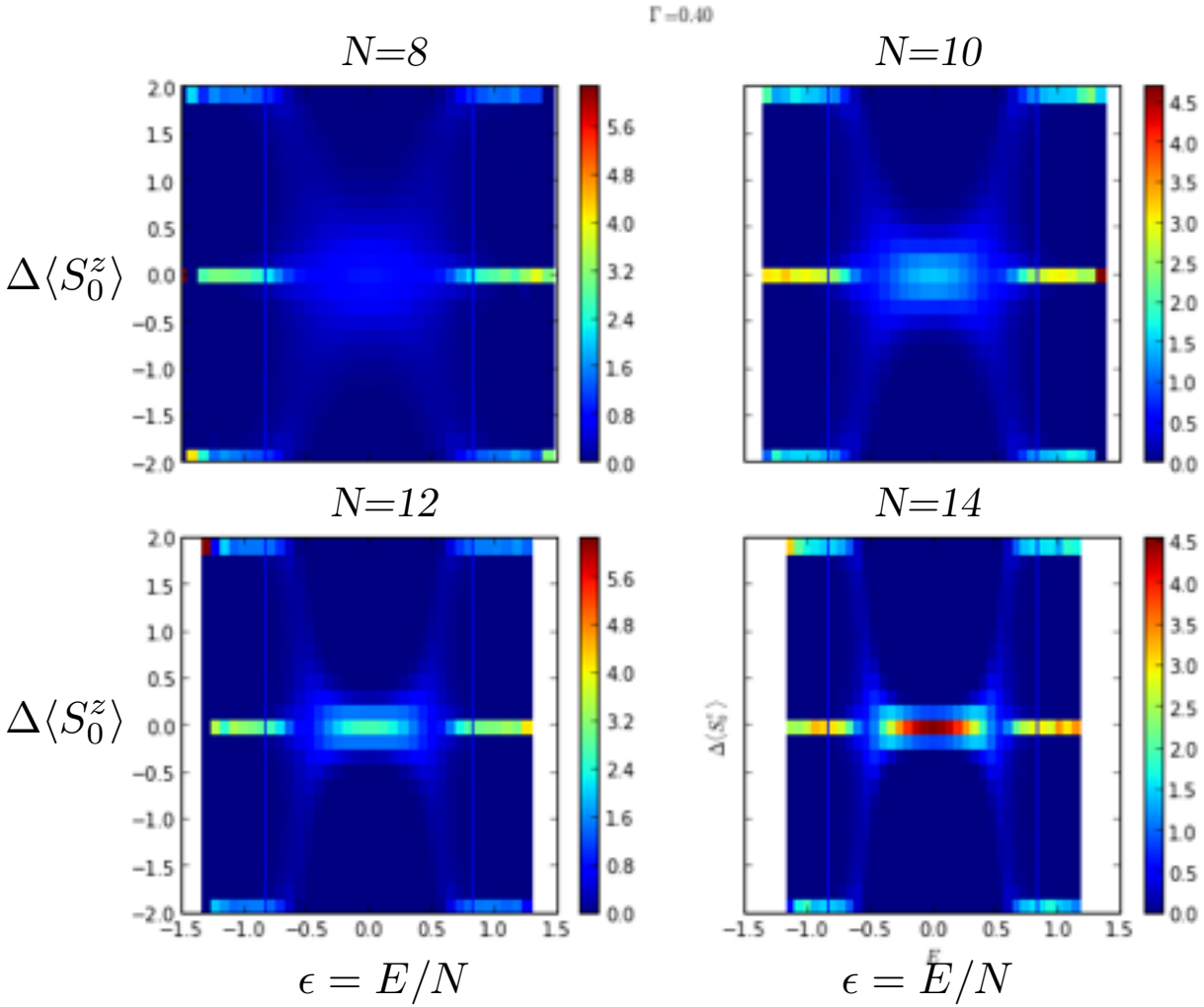
- In QREM, z-magnetization is local

$$H = E(\{\sigma_i^z\}) - \Gamma \sum_i \sigma_i^x$$

$$|[H, \sigma_i^z]| = \Gamma \sim O(1)$$
$$|[H, \sigma_i^x]| \sim O(N)$$

Spider diagrams

Histogram of delta Z-Magnetization across eigenstates

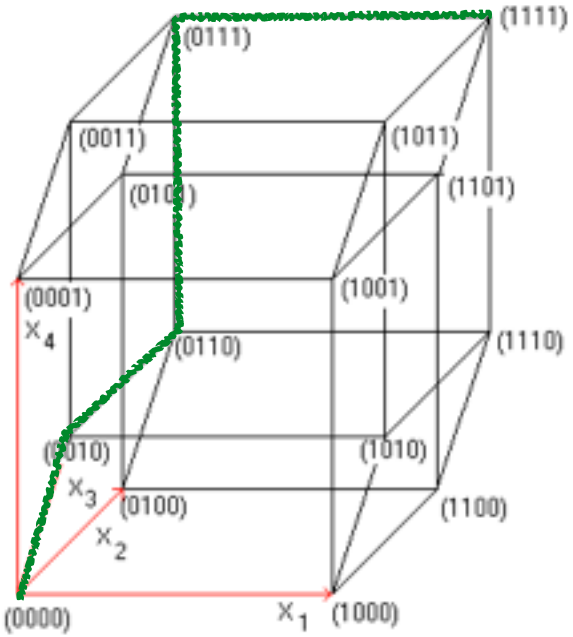


Forward Scattering

- Leading perturbative wavefunction `forward scattering'

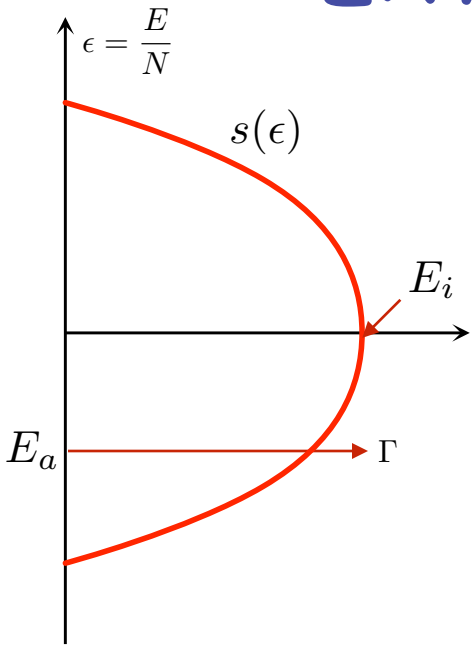
$$\psi_b = \sum_{\text{paths } p: a \to b} \prod_{i \in p} \frac{\Gamma}{E_a - E_i}$$

- Localization: amplitudes decay to system size n=N



- Directed random polymer on hypercube
- Amenable to numerical transfer matrix treatment
- Replica treatment of polymer problem appears to identify transitions as well

Extensive Energies



- Gap typically O(N) so expand

$$\psi_b = \sum_{\text{paths } p:a\to b} \prod_{i\in p} \frac{\Gamma}{E_a - E_i}$$

$$\psi_b \approx \left(\frac{\Gamma}{E_a}\right)^n \sum_p \left(1 + \sum_{i\in p} \frac{E_i}{E_a} + \cdots\right)$$

- Fluctuations small in N (typical ~ mean)

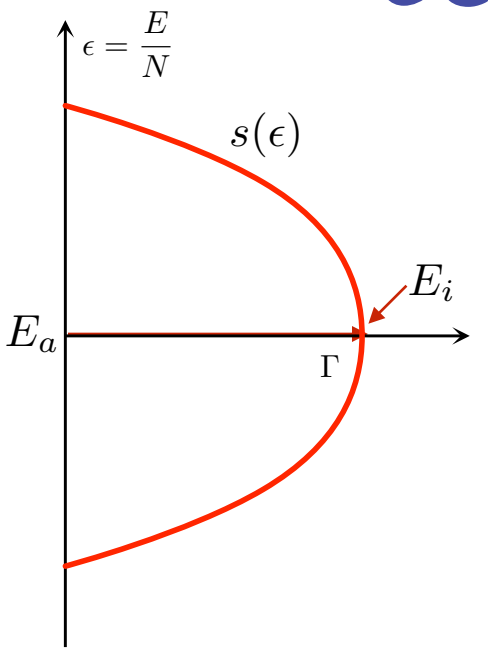
$$\overline{\psi_b} \approx n! \left(\frac{\Gamma}{E_a}\right)^n \qquad \frac{\delta\psi_b}{\psi_b} \sim \frac{1}{\sqrt{N}}$$

- Demand typical amplitude decreasing uniformly to n = N

$$\Gamma_c = \epsilon_a$$

- Need more care at $E_a = 0$

Central Energies



- Gap typically $\sqrt{N}$, fluctuations large

$$\psi_b = \sum_{\text{paths } p: a \to b} \prod_{i \in p} \frac{\Gamma}{E_a - E_i}$$

- Bound by greedy path at distance n

$$\psi_g \sim \left(\frac{\Gamma N}{\sqrt{N}}\right)\left(\frac{\Gamma(N-1)}{\sqrt{N}}\right)\cdots\left(\frac{\Gamma \cdot 1}{\sqrt{N}}\right) \sim (\Gamma\sqrt{N})^n$$

- Demand amplitudes small gives upper bound on delocalization

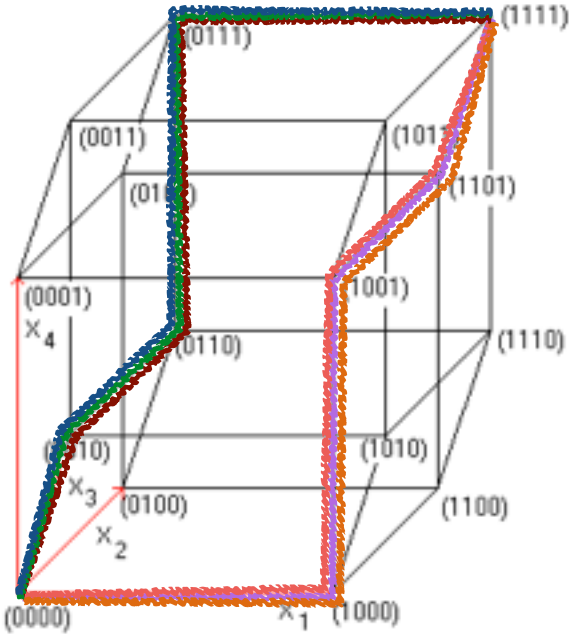
$$\Gamma_c < \frac{1}{\sqrt{N}}$$

- Counting resonances more carefully gives log correction

Replicated Polymers

- Typical amplitudes given by quenched average

$$\overline{\ln \psi} = \lim_{n \to 0} \frac{\overline{\psi^n} - 1}{n}$$



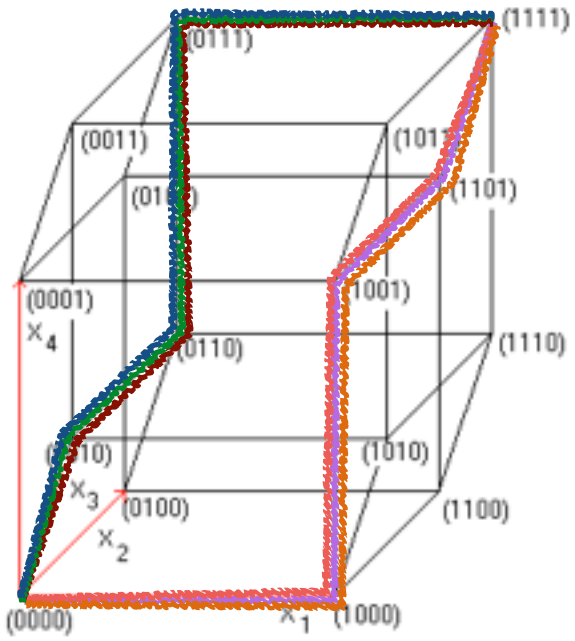
- n interacting paths on hypercube

$$\overline{\psi^n} = \sum_{p_1 \cdots p_n} \prod_i \overline{w_i^{r_i(p_1 \cdots p_n)}}$$

$$r_i = \sum_{a=1}^{n} \mathbf{1}\left[i \in p_a\right]$$

Replicated Polymers

- 1RSB Ansatz: polymers clump in n/x groups of x paths



$$\overline{Z^n} \approx \left(\sum_p \prod_{i \in p} \overline{w_i^x} \right)^{n/x} = \exp\left[n f(x) \right]$$

$$f(x) = \frac{L}{x} (\log L - 1 + \log \overline{w_i^x})$$

$$\overline{w^x} = \int \frac{dE}{\sqrt{\pi N}} e^{-E^2/N} \left(\frac{\Gamma}{|E_a - E|} \right)^x$$

Finite-size form at zero energy

- Replica recipe: $f_{1RSB} = \min_{x\in[0,1]} f(x)$

$$f(x) = \frac{L}{x}(\log L - 1 + \log \overline{w_i^x})$$

$$\overline{w^x} = \int \frac{dE}{\sqrt{\pi N}} e^{-E^2/N} \left(\frac{\Gamma}{|E_a - E|}\right)^x$$

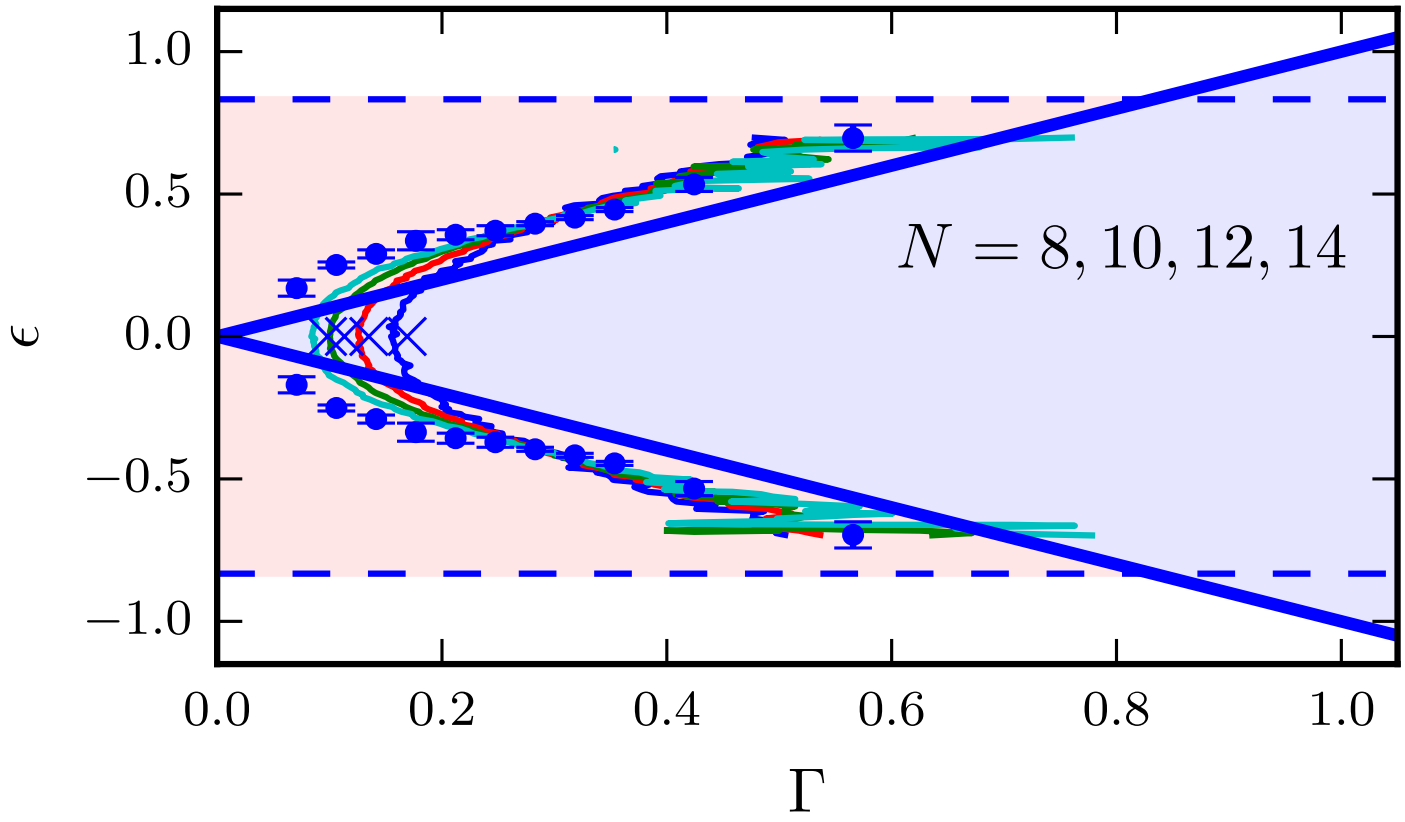
- 1RSB holds at finite L but is swamped slowly by L! paths

$$x^* = 1 - \frac{1}{\log\sqrt{\frac{2}{\pi}}L} + O(\log\log L/\log^2 L)$$

- Demanding amplitude decays at size L = N

$$\Gamma_c = \frac{\sqrt{\pi}}{2\sqrt{N}\log\sqrt{2/\pi}N} + \cdots$$

Numerical Fit



- X marks the replica formula estimate $\Gamma_c = \dfrac{\sqrt{\pi}}{2\sqrt{N}\log\sqrt{2/\pi}N} + \cdots$

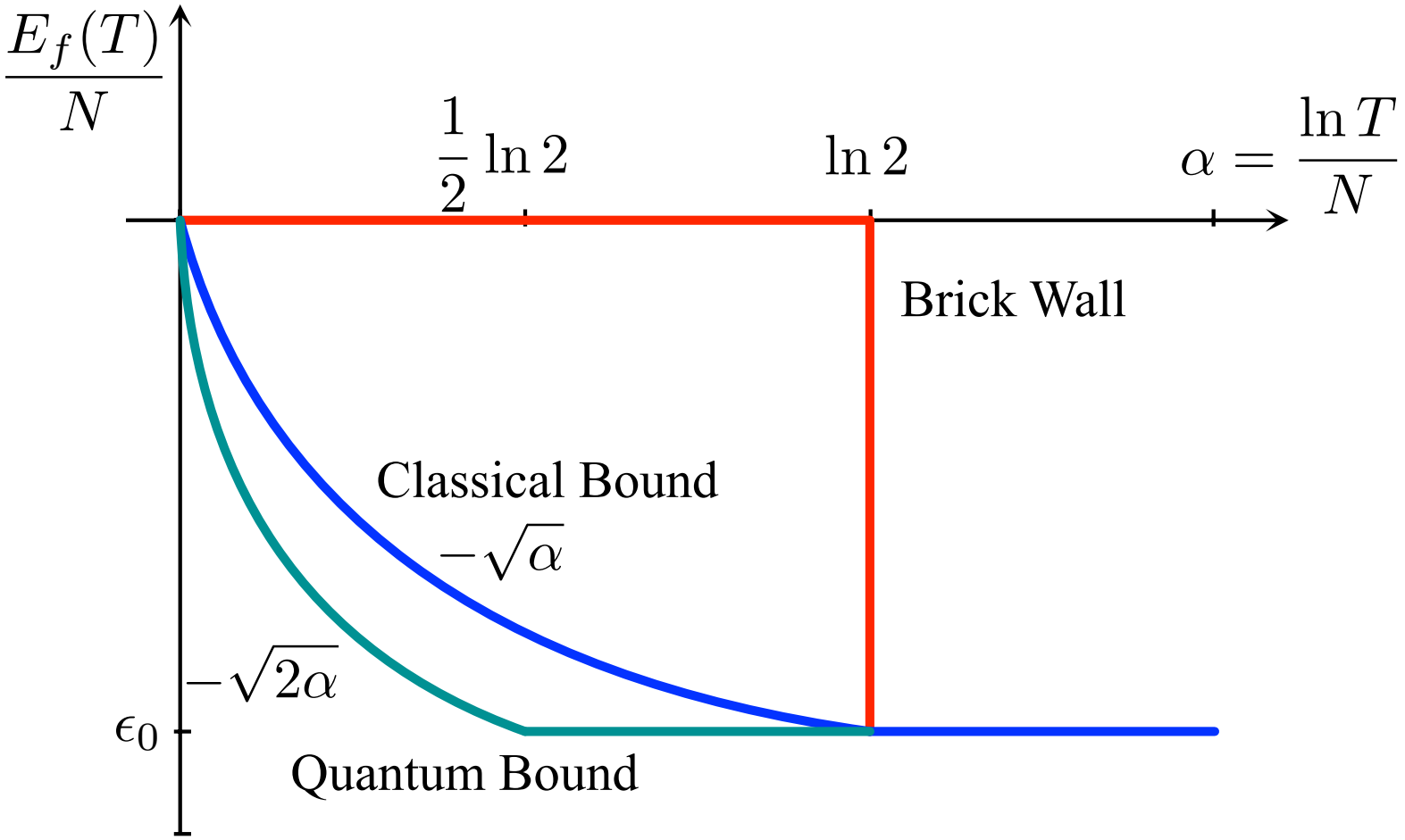
Almost forgot...

Adiabatic Quantum Optimization

- Annealing transverse field — ground state search
- Final finite energy density — 'approximate' search
- Scaling of final energy density with time — how hard is approximation?

Some annealing conjectures

- Unstructured cost function — lower bounds



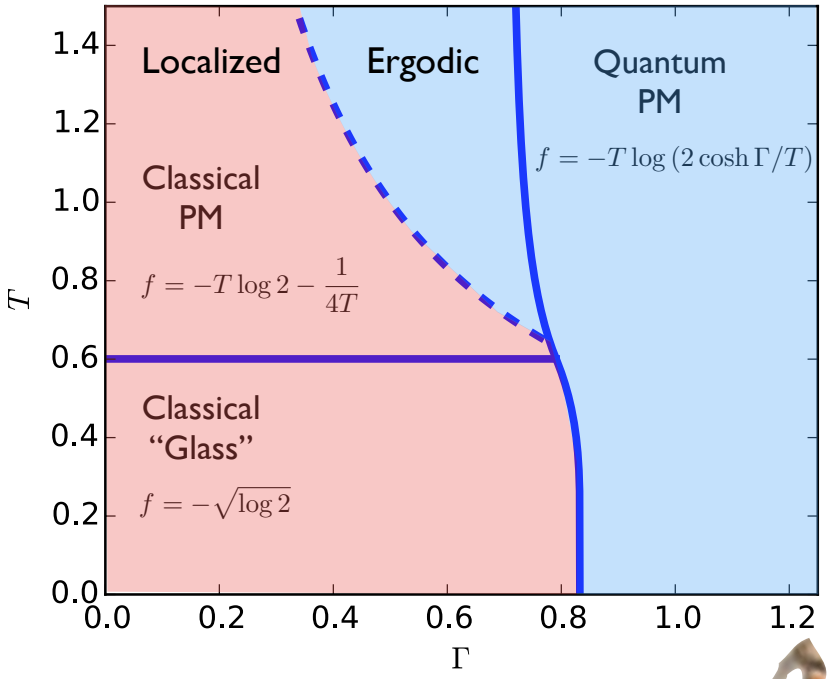
Summary

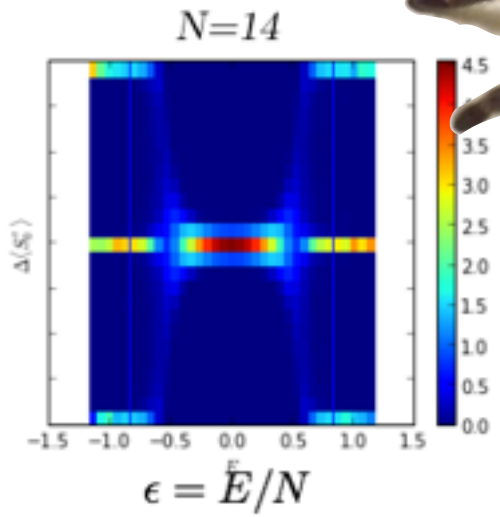
The QREM provides a 'mean-field' model of MBL-ETH transition at finite energy density mobility edge. First order dynamical transition.

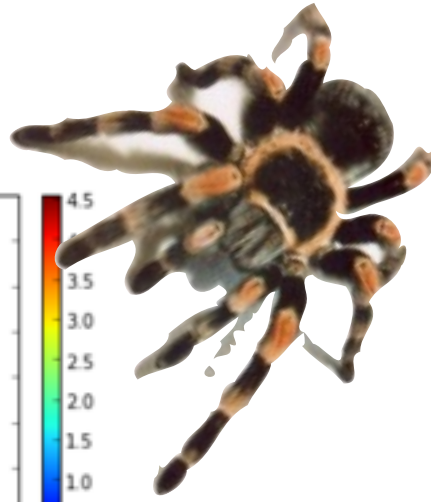
Perturbative treatment in forward approximation — directed random polymer on hypercube. More rigorous bounds consistent with replica treatment.

Localization transition inside 'paramagnetic' phase of QREM.

Slow approach to thermodynamic limit near infinite temperature.







Open Questions

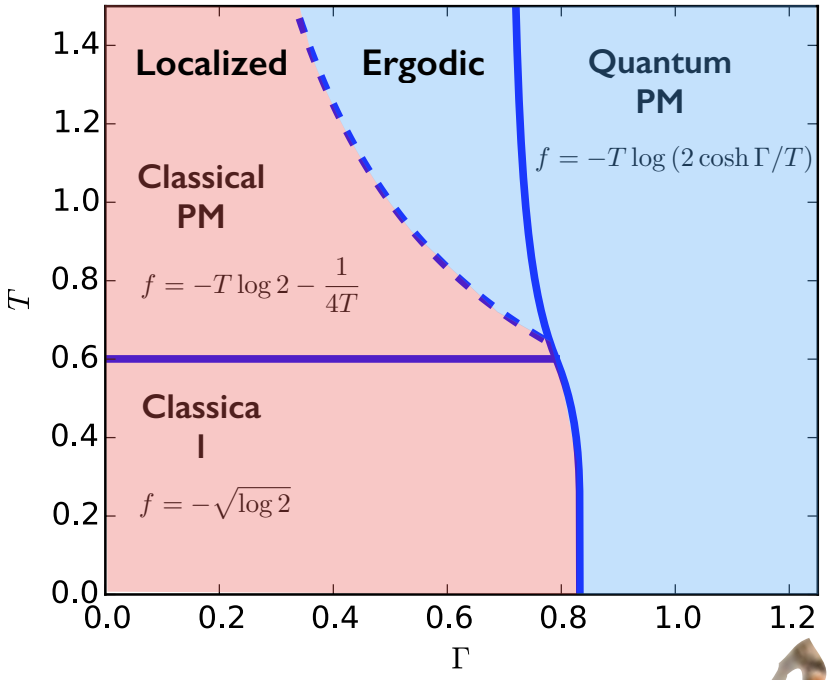
Complete analytic solution of QREM?

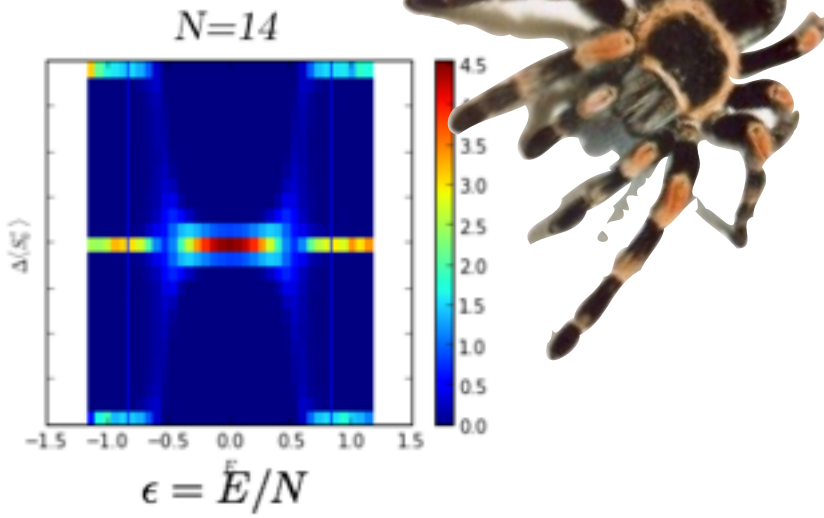
Do thermodynamics reflect dynamical transition? (Existing phase diagrams perhaps not exact.)

No infinite temperature MBL — feature of long-range interactions?

Short-range model with provable MBL?*

Expected outcome for approximate quantum annealing in interesting models?





Does the exponential complexity of general quantum systems persist at high temperature?

Yes