The Rate of Progress of Evolution

Nicholas Pippenger

20 March 2014

Progress—Goals

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- But they will not be imputed to nature.

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- Presently, functional forms.
- But don’t lump all power laws together as “polynomial”.
- Emphasize negative results.

“The Cost of Natural Selection”

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- Truncation.

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- Omnipotence as to opportunity for reproduction.

Basic Step

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- This is (a lower bound for) the true "cost of natural selection".

What If There's No Sex?

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- $\#\text{generations} \geq C\, n$.

Evolution As a Random Walk

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- S. J. Gould, *Full House*, 1996.

A Random Walk on a Line

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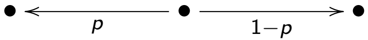
$$\bullet \xleftarrow[\;p\;]{} \bullet \xrightarrow[\;1-p\;]{} \bullet$$

A Random Walk on a Line



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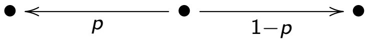

- $p = 1/2$.
- At time T, most walkers are to the left of $C\sqrt{T}$.

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- If $p > 1/2$, distribution approaches exponential.

A Random Walk in a Random Environment

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- Create a random environment.

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- Take a random walk in that environment.

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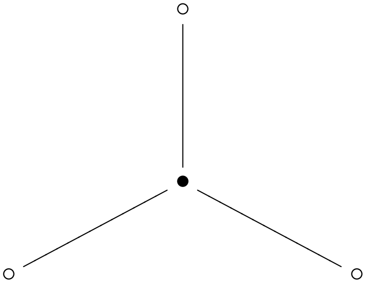
- Condition on non-extinction.

$1 - a$



a

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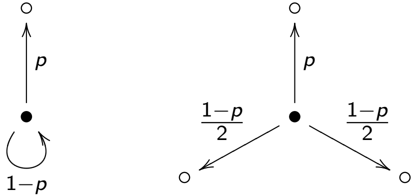
The Random Walk

- $\Pr[\text{ascend}] = p$.
- At a leaf, $\Pr[\text{stay}] = 1 - p$.
- At a binary node, $\Pr[\text{descend left}] = \Pr[\text{descend right}] = \frac{1-p}{2}$.

The Random Walk



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- $a = 1/2$, $p = 1/3$.
- At time T, most walkers are above level $C\,T^{1/3}$.

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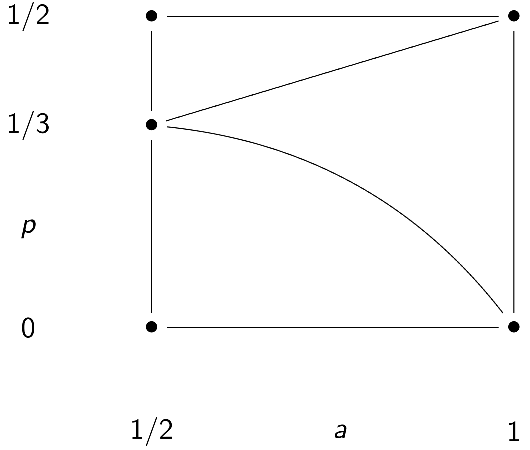

At time T, most walkers are above level $C\,T^{\theta}$, where

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$$\theta \leq \frac{\log \frac{1}{1-a}}{\log \frac{1}{4p}}.$$

- Trapping is different from diffusion against a wall.
- Trapping is exacerbated by positive bias.

Thank You