# Model-based Algorithms for Reinforcement Learning and Imitation Learning with Theoretical Analyses


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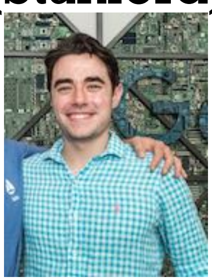

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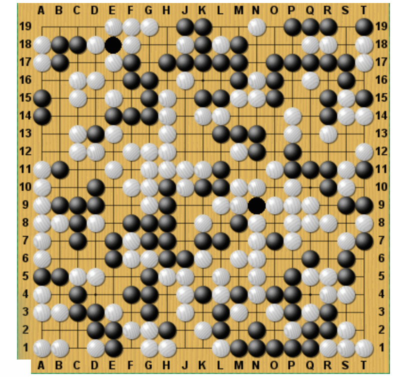
# Sample-Efficiency Challenge in RL

Trials and errors:

- Try the current strategy and collet feedbacks
- Use the feedbacks to improve the strategy




millions of games

How to reduce the amount of trials (samples)?

- Model-based RL
- Imitation learning from expert demonstrations
- Multi-task, lifelong, continual RL
- Hierarchical RL
- Safe RL
- ...

# Backgrounds and Terminologies (on Continuous State-Space, Deterministic Dynamics)

- Time $t = 0, \ldots, T(\approx \infty)$
- State $s_t \in \mathbb{R}^d$; action $a_t \in \mathbb{R}^k$
- Unknown dynamics/environment $M^\star$: $s_{t+1} = M^\star(s_t, a_t)$
- Trajectory:

$$s_0 \sim D_{s_0} \xrightarrow{\pi(s_0)} s_1 \xrightarrow{\pi(s_1)} s_2 \xrightarrow{\pi(s_2)} s_3 \xrightarrow{\pi(s_3)} s_4 \cdots\cdots$$

- Policy $\pi$: states → actions
- Reward $r(s_t, a_t) \in \mathbb{R}$
- Expected payoff of a policy:

$$V^\pi := \mathbb{E}_{s_0 \sim D_{s_0}}[R(s_0, a_0) + R(s_1, a_1) + R(s_2, a_2) + \cdots]$$

- e.g., state = location of arm; action = desired movement

# Model-Based Reinforcement Learning

- Learn the dynamics $M^\star$ somewhat explicitly
- Standard model-based RL algorithm:

Repeat:

1. Sample trajectories from real dynamics $M^\star$ using current policy

$$s_0 \sim D_{s_0} \quad \longrightarrow \quad s_1 \longrightarrow \quad s_2 \longrightarrow \quad s_3 \longrightarrow \quad s_4 \cdots\cdots$$

2. Learn a dynamical model using existing trajectories

$$\min_M \sum ||M(s_t, a_t) - s_{t+1}||_2^2$$

3. Find a good policy for the learned dynamics $M$
   - Does not cost real samples; any RL algo. may be used as a blackbox

3. Planning the vacation at home

1. Go, enjoy, and explore

2. Keep notes on the good restaurants

Repeat:

1. Sample trajectories from real dynamics $M^\star$ using current policy

$$s_0 \sim D_{s_0} \longrightarrow s_1 \longrightarrow s_2 \longrightarrow s_3 \longrightarrow s_4 \cdots\cdots$$

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$$\min_M \sum ||M(s_t, a_t) - s_{t+1}||_2^2$$

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➢ Does not cost real samples; any RL algo. may be used as a blackbox

# Challenges in Analyzing Deep Model-Based RL

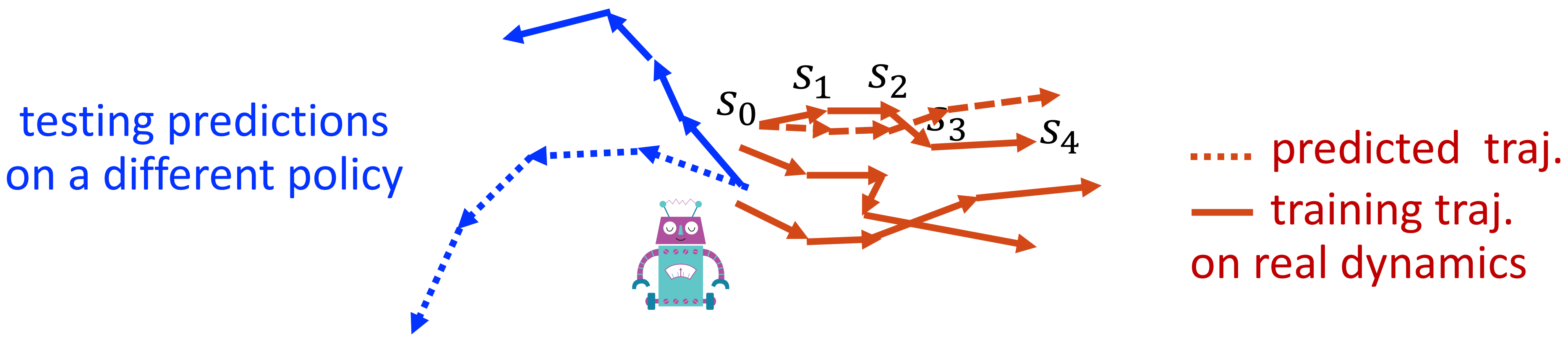
- High-dimensional state and action space
- Non-linear dynamics $M$, policy $\pi$ parameterized by neural networks
- Goal: an analyzable algorithm with # samples polynomial in dimension (assuming some computational oracles)

Prior work

- Finite state space: [Jaksch et al., 2010; Bartlett & Tewari, 2009; Fruit et al., 2018; Lakshmanan et al., 2015; Hinderer, 2005; Pirotta et al., 2015; 2013)
- Linear dynamics: [Abbasi-Yadkori & Szepesvári, 2011; Simchowitz et al., 2018; Dean et al., 2017; Sutton et al., 2012; Tamar et al., 2012]
- Sample complexity result: [Sun et al.'2017]

# Challenges in Analyzing Deep Model-Based RL (Cont'd)

- Issue: the learned dynamics are not accurate for those states unseen in training trajectories
- Exploitation: only go to places that the dynamics is certain
- Exploration: improve the certainty of the model by trying diverse policies



# A Classical Idea: Optimism in the Face of Uncertainty

Repeat:

1. Sample trajectories from real dynamics using current policy

2. policy, dynamics $\leftarrow \operatorname*{argmax}_{\pi, M} V^{\pi,M}$

   s.t., $M$ is consistent with existing trajectories

payoff of $\pi$ on learned dynamics

➢ Explore a policy if it is good for some reasonable dynamics

# A Classical Idea: Optimism in the Face of Uncertainty

Repeat:

1. Sample trajectories from real dynamics using current policy
2. $$\text{policy, dynamics} \leftarrow \underset{\pi,\, M}{\operatorname{argmax}}\, V^{\pi,M}$$
   s.t., $M$ is consistent with existing trajectories

payoff of $\pi$ on learned dynamics

Q1: how do we express the constraint for non-linear models?

- confidence intervals for finite state space or linear models; not feasible for neural nets

# A Classical Idea: Optimism in the Face of Uncertainty

Repeat:

1. Sample trajectories from real dynamics using current policy

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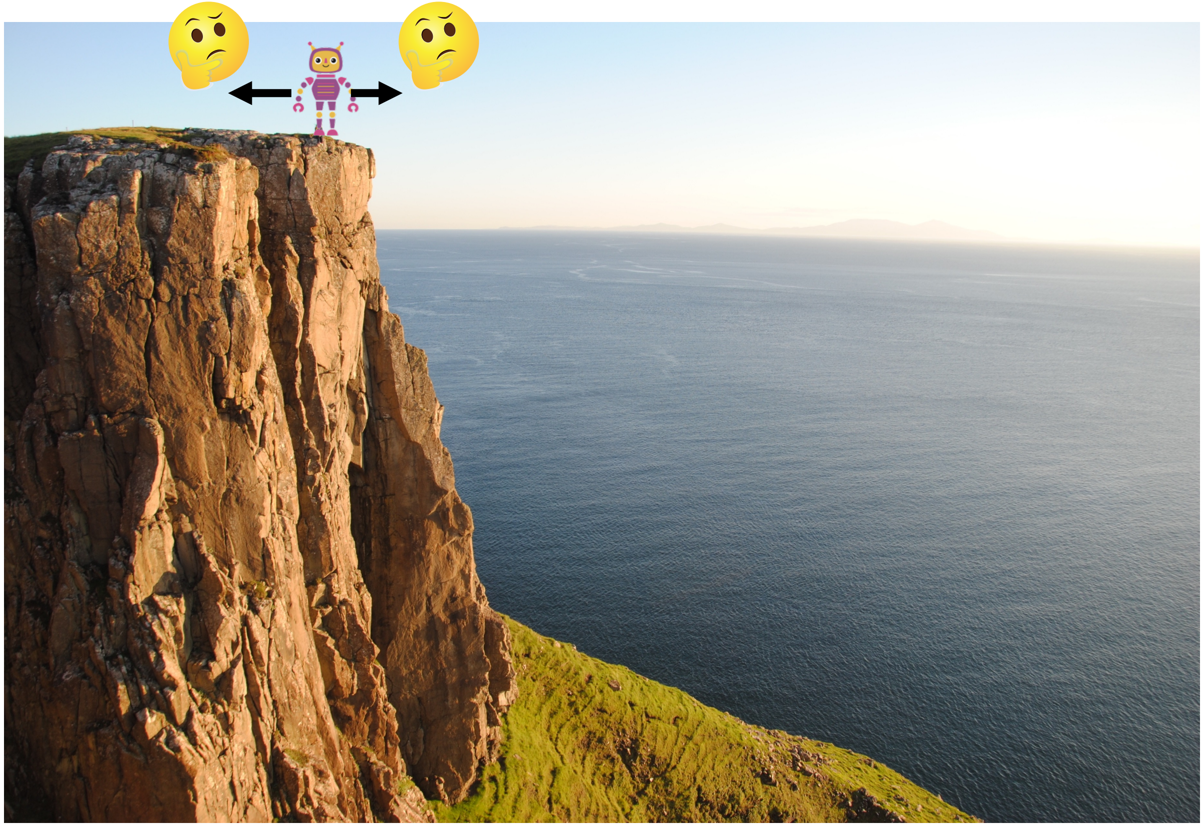
   s.t., $M$ is consistent with existing trajectories

payoff of $\pi$ on learned dynamics

Q2: how do we measure the "consistency"?

- how do we measure the errors of the learned dynamics?

# The Same Prediction Loss Could Mean Very Differently For Different States and Actions



# Our Idea

Ideal loss for $M \approx$ error of predicting future payoff using $M$

$$|V^{\pi,M} - V^{\pi,M^\star}|$$

total payoff on estimated dynamics $M$

total payoff on true dynamics $M^\star$

- Design an upper bound of the ideal loss and use it as a surrogate loss or a consistency measure

$$|V^{\pi,M} - V^{\pi,M^\star}| \leq \mathcal{D}(M,\pi)$$

- A dynamics $M$ has low loss (is consistent with existing data) if $M$ can predict the real reward with small error upper bound $\mathcal{D}(M,\pi)$

[Algorithmic Framework for Model-based Reinforcement Learning with Theoretical Guarantees Luo-Xu-Tian-Darrell-M.'19]

Repeat:

1. Sample trajectories from real dynamics using current policy

2. $$\text{policy, dynamics} \leftarrow \underset{\pi,\, M}{\operatorname{argmax}}\, V^{\pi,M}$$

   s.t., ~~$M$ is consistent with existing trajectories~~

$$\text{s.t., } \mathcal{D}(\pi, M) \leq \epsilon$$

- $\{M : \mathcal{D}(\pi, M) \leq \epsilon\}$ is a confidence region depending on $\pi$ and the reward function
- Next: absorb the constraint in the objective

# Meta-Algorithm for Model-Based RL with Convergence Guarantees

From $k = 1$ to $T$:

1. Sample trajectories using $\pi_k$, build upper bound $\mathcal{D}_{\pi_k}(\pi, M)$
2. $M_{k+1}, \pi_{k+1} = \mathrm{argmax}_{\pi, M} \underbrace{V^{\pi, M} - \mathcal{D}_{\pi_k}(\pi, M)}_{\text{lower bound of real reward } := L(M, \pi)}$

**Theorem:** Assume the model family contains $M^\star$, and the inner optimization is solvable, then,

$$V^{\pi_1, M^\star} \le V^{\pi_2, M^\star} \le \cdots \le V^{\pi_T, M^\star} \le \ \ldots$$

and $V^{\pi_k, M^\star}$ converges to a local maximum of $V^{\pi, M^\star}$.

# Meta-Algorithm for Model-Based RL with Convergence Guarantees

From $k = 1$ to $T$:

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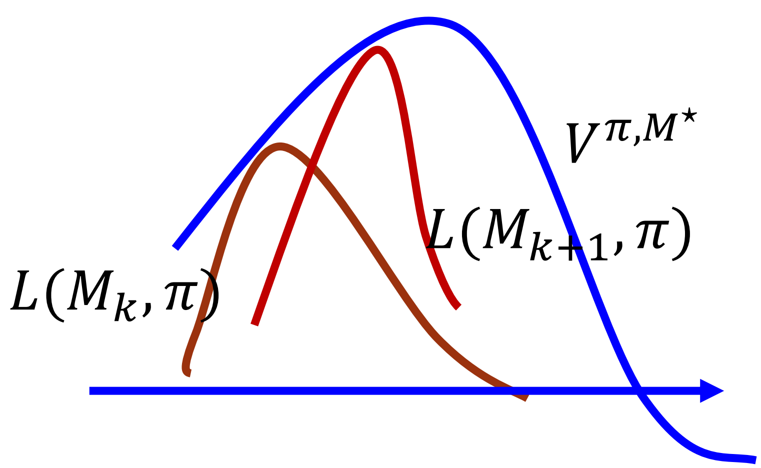


# Optimizable Upper Bounds of Ideal Loss

- Design an upper bound of the ideal loss

$$|V^{\pi,M} - V^{\pi,M^\star}| \le \mathcal{D}_{\pi_{\mathrm{ref}}}(M,\pi)$$

Lemma:

$$|V^{\pi,M} - V^{\pi,M^\star}| \le \mathop{\mathbb{E}}_{(s,a,s')\sim\pi,M^\star}\left[|V^{\pi,M}(M(s,a)) - V^{\pi,M}(s')|\right]$$

- $(s,a,s')$ shorthand for $(s_t,a_t,s_{t+1})$
- Issue: requires samples from the environment to estimate the loss

- $V^{\pi,M}(s) := $ the total payoff of the policy on dynamics $M$ starting from state $s$
- $V^{\pi,M} = \mathbb{E}_{s_0}[V^{\pi,M}(s_0)]$

# Optimizable Upper Bounds of Ideal Loss (Cont'd)

Improved Lemma:

$\forall \pi$ that is close to $\pi_{\text{ref}}$ :

$$|V^{\pi,M} - V^{\pi,M^\star}| \le \underbrace{\mathop{\mathbb{E}}_{(s,a,s')\sim\pi_{\text{ref}},M^\star}\left[|V^{\pi,M}(M(s,a)) - V^{\pi,M}(s')|\right]}_{:=\mathcal{D}_{\pi_{\text{ref}}}(M,\pi)}$$

- Upper bound can be re-used if $\pi$ doesn't change much
- Recovers the norm-based loss, if $V^{\pi,M}$ is Lipschitz w.r.t $||\cdot||$

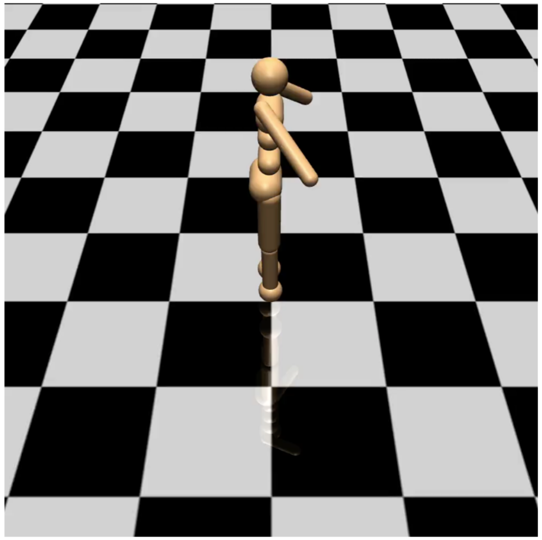
$$\mathcal{D}_{\pi_{\text{ref}}}(M,\pi) \lesssim \mathop{\mathbb{E}}_{(s,a)\sim\pi_k,M^\star}\|M(s,a) - s'\|$$

No square compared to MSE

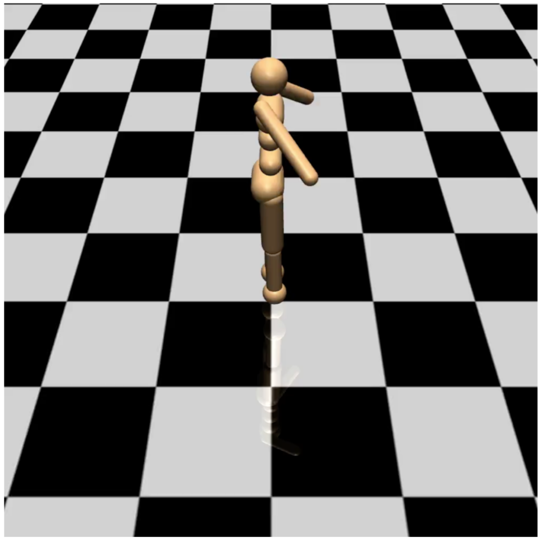
- Inspires a practical algorithm (SLBO) that uses $\ell_2$ loss (not MSE) and optimizes the objective with SGD
  - no optimism is practically needed though

# Demo: learning to walk to the right as fast as possible

- What the learned dynamics predicts



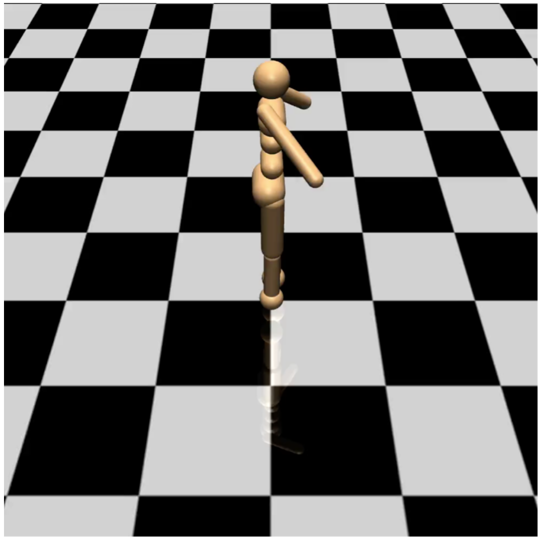
- What the humanoid does in reality

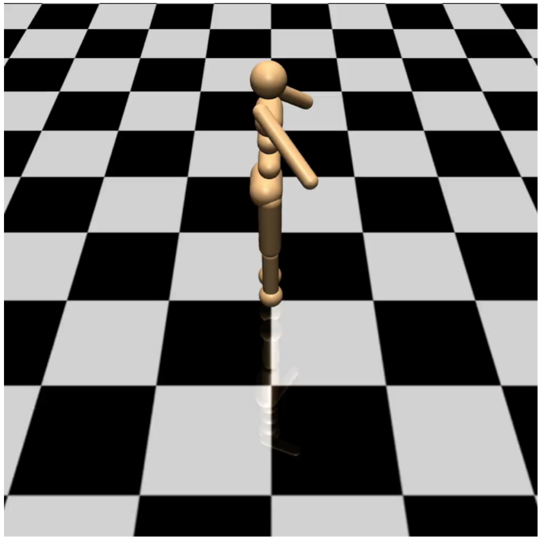


Iteration 10

# Demo: learning to walk to the right as fast as possible

- What the learned dynamics predicts
- What the humanoid does in reality

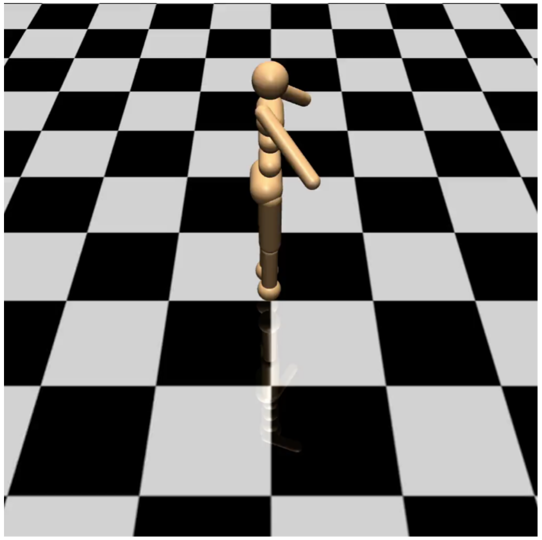




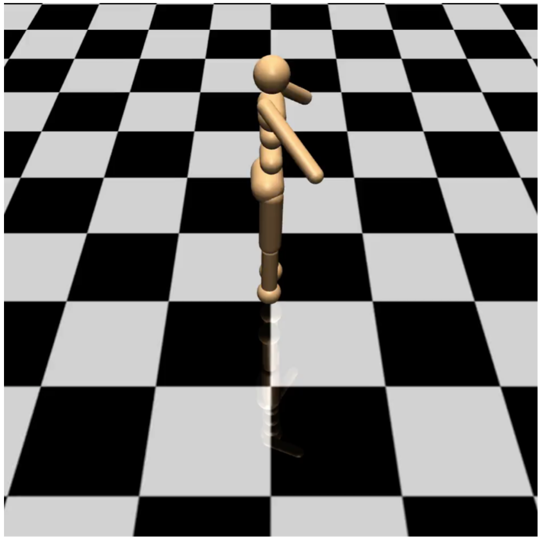
Iteration 20

# Demo: learning to walk to the right as fast as possible

- What the learned dynamics predicts

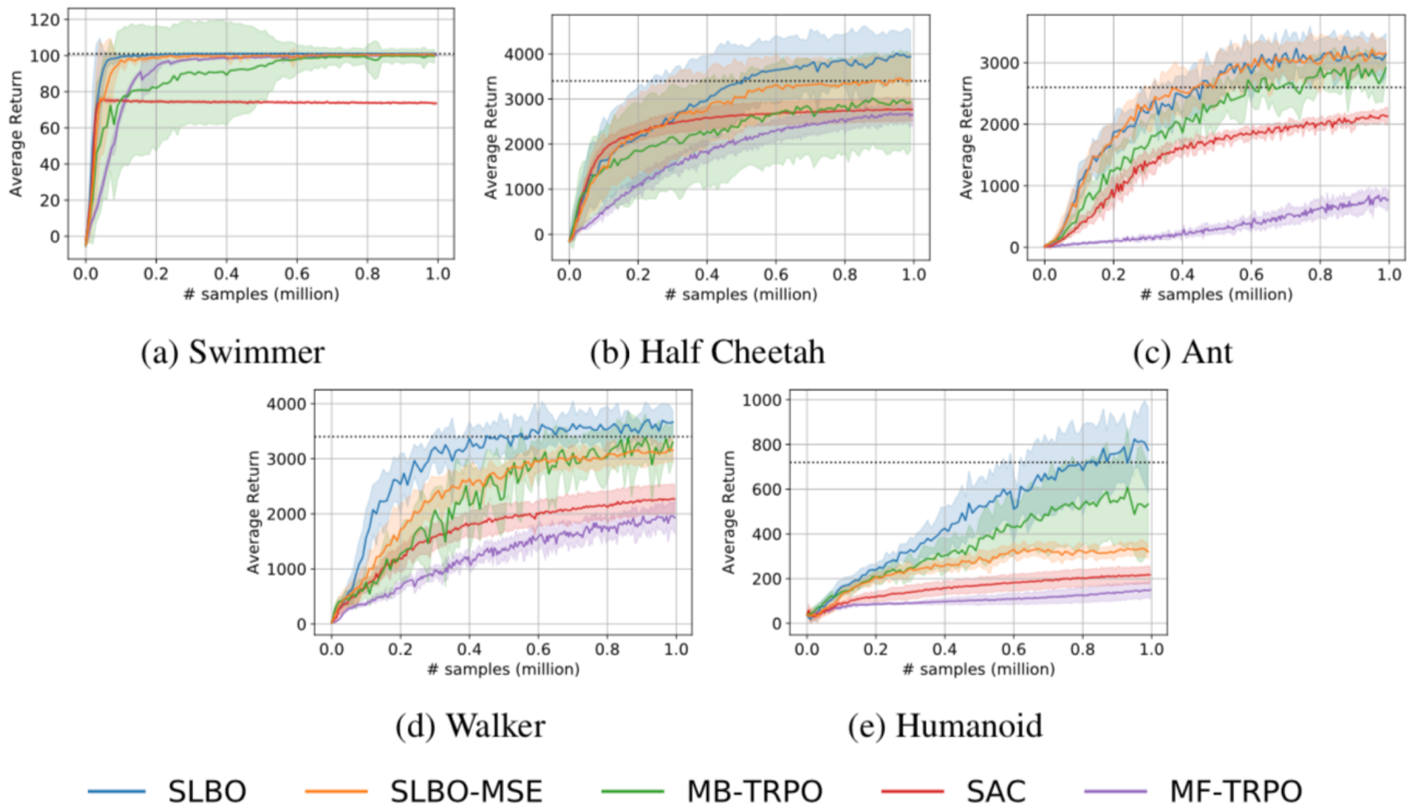


- What the humanoid does in reality



Iteration 210

# Evaluations on MuJoCo Benchmark Tasks




(a) Swimmer (b) Half Cheetah (c) Ant

(d) Walker (e) Humanoid

- ➢ Outperforms prior works when 1M (or fewer) samples are permitted

[Algorithmic Framework for Model-based Reinforcement Learning with Theoretical Guarantees Luo-Xu-Tian-Darrell-M.'19]

# Follow-up: Model-based Multi-task RL

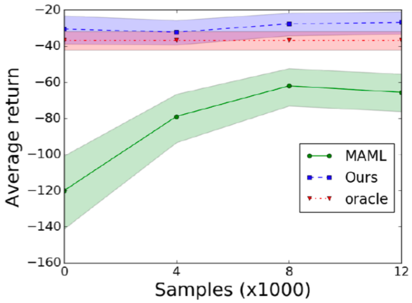
Setting: A single robot, but multiple tasks

- e.g., humanoid runs with different speeds and directions

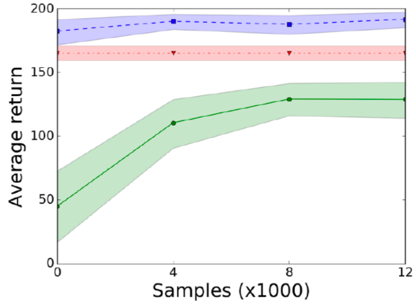
- Our algo.: learns a dynamics shared across tasks sequentially
  - Amortized sample costs over tasks

- Prior work: MAML (model-agnostic meta-learning)
  - Learns a shared policy that can be adapted to tasks

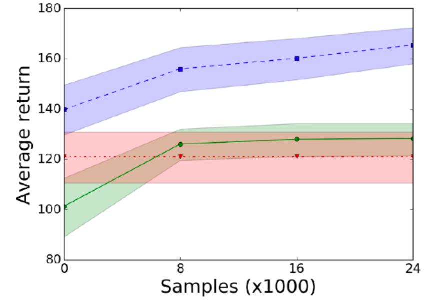
[A Model-based Approach for Sample-efficient Multi-task Reinforcement Learning
Landolfi-Thomas-M.'19]



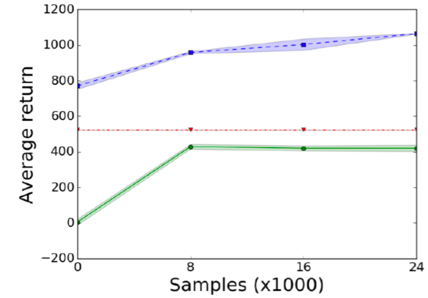

(a) Cheetah $[0, 2]$



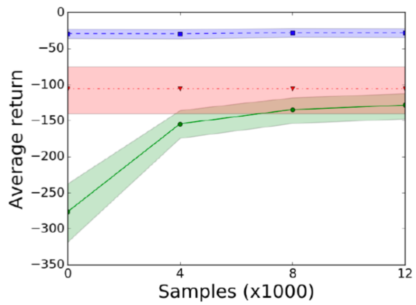

(b) Ant $[0, 3]$



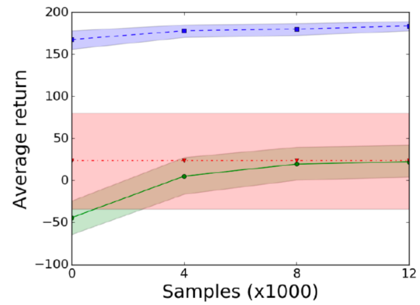

(c) Humanoid $[0, 1.5]$



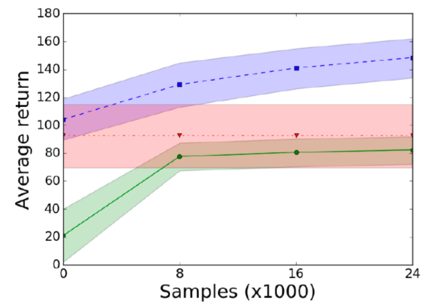

(d) Ant forward/back



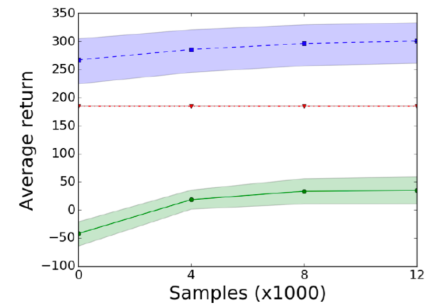

(e) Cheetah $[-2, 2]$




(f) Ant $[-3, 3]$




(g) Humanoid $[-1.5, 1.5]$




(h) Ant $[-3, 3]^2$

Tasks: running with random velocities in the interval/region

[A Model-based Approach for Sample-efficient Multi-task Reinforcement Learning
Landolfi-Thomas-M.'19]

# Imitation Learning

- RL from scratch alone may still be not sample-efficient enough
- Imitation learning: learning from (human) experts demonstration

Formulation:

- experts run a policy $\pi_e$ to collect trajectories

$$\mathcal{R} = \left\{ \left( s_0^{(i)}, a_0^{(i)}, \dots, s_{T-1}^{(i)}, a_{T-1}^{(i)}, s_T \right) \right\}_{i=1}^{n}$$

- we learn a policy from $\mathcal{R}$ with or without additional samples

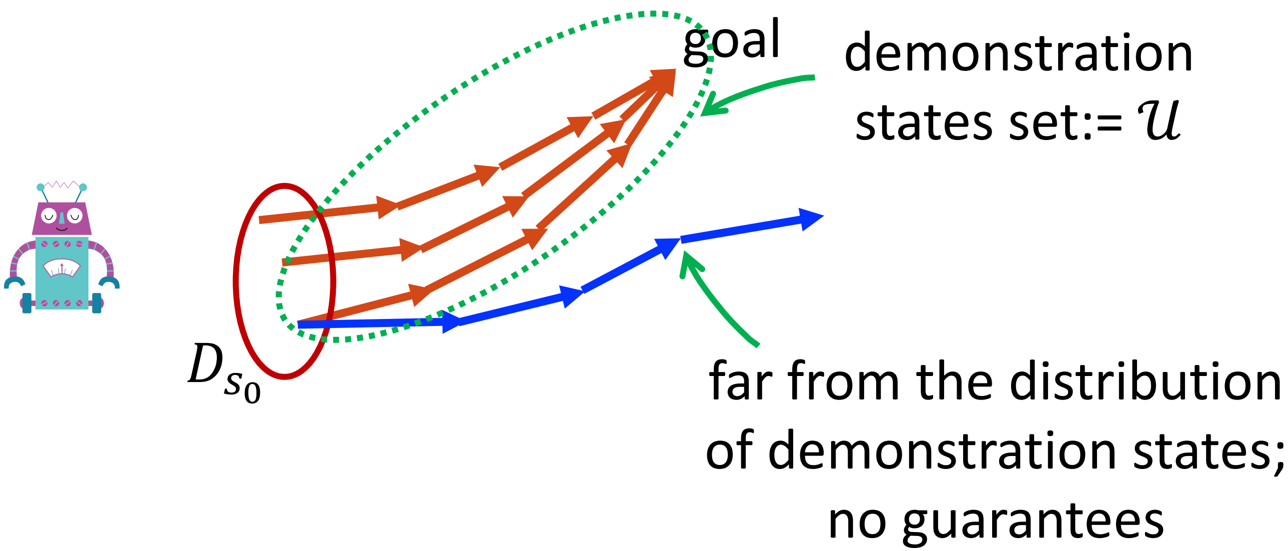
# A Classic Algorithm: Behavioral Cloning

- Supervised learning on demonstrations

  $$\mathcal{R} = \left\{\left(s_0^{(i)}, a_0^{(i)}, \ldots, s_{T-1}^{(i)}, a_{T-1}^{(i)}, s_T\right)\right\}_{i=1}^{n}$$

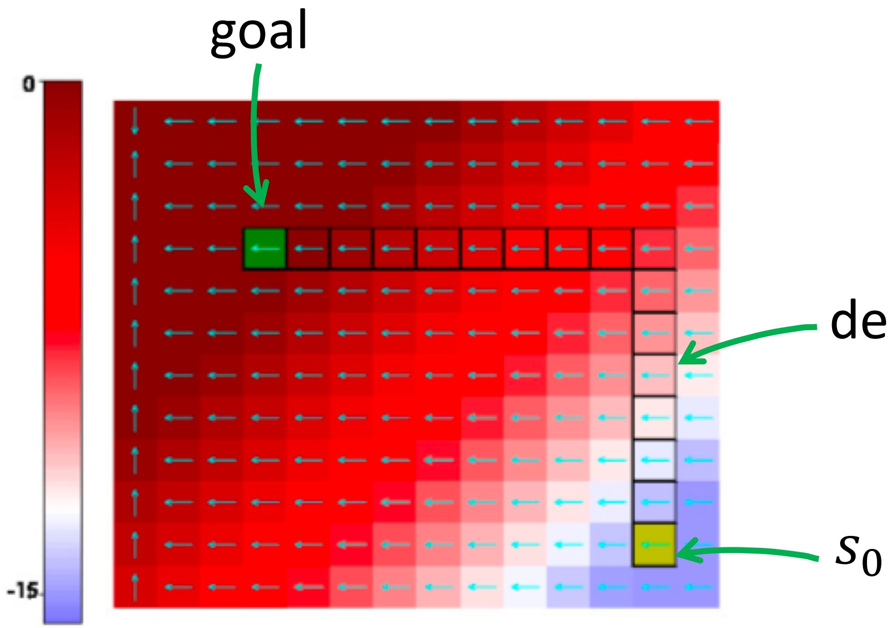
  - Fit a policy $\pi_{\mathrm{BC}}$ such that $\pi_{\mathrm{BC}}\left(s_t^{(i)}\right) \approx a_t^{(i)}$

- Well-known issue: distribution drift and cascading errors



# Another Attempt: Learning Value Functions from Demonstrations

- Recall $V^{\pi_e}(s) :=$ total payoff of expert policy starting from $s$
- If $s \in$ demonstration states $\mathcal{U}$, we know $V^{\pi_e}(s)$
- Attempt: learn $V^{\pi_e}$ by supervised learning on $\mathcal{U}$
- Same issue: $V^{\pi_e}$ extrapolates falsely outside $\mathcal{U}$

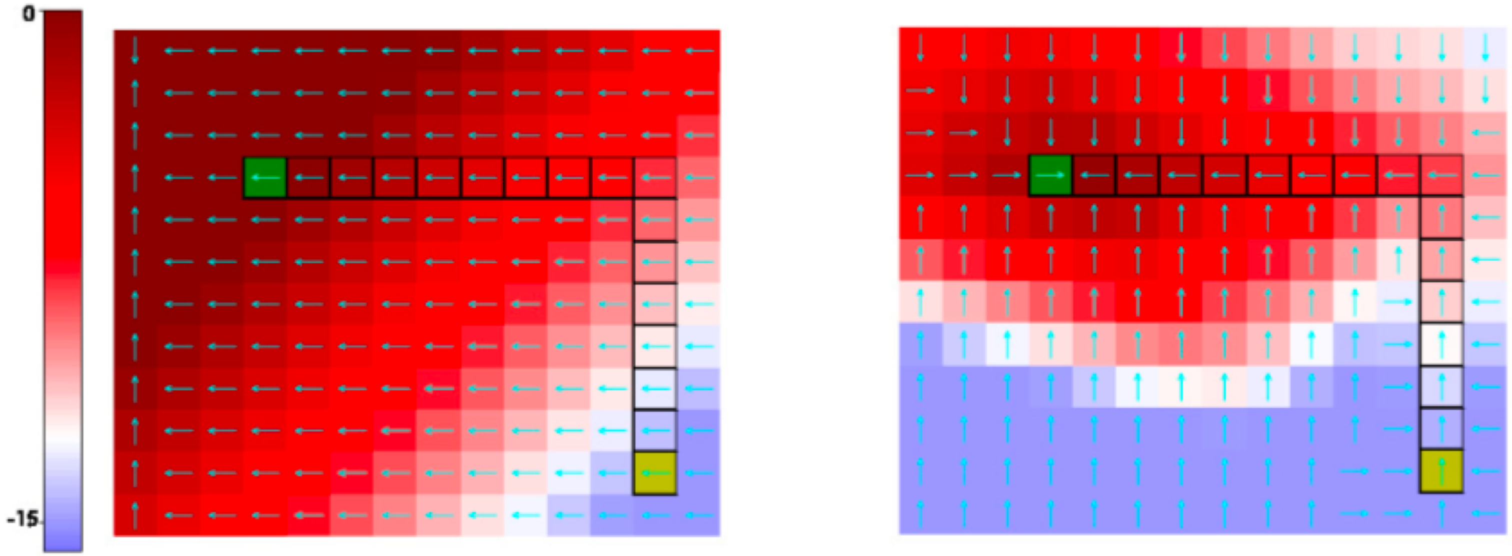



- Correct values on $\mathcal{U}$
- Wrong values outside $\mathcal{U}$

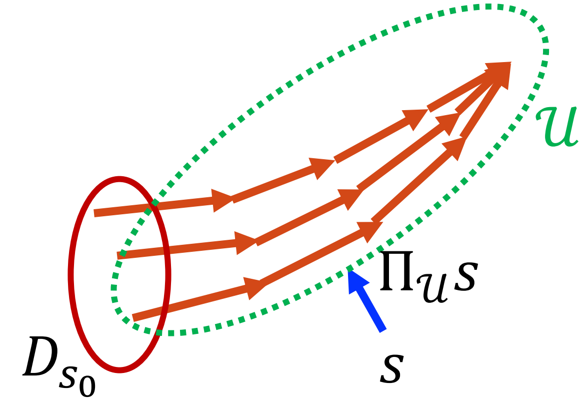
# Our Idea: Learning a Better Value Function (and Use it Correct Mistakes of Behavioral Cloning)

- Key: the value $V^{\pi_e}(s)$ should be relatively smaller for $s \notin \mathcal{U}$
- $\Rightarrow$ following the value function leads us back to $\mathcal{U}$

falsely-extrapolated conservatively-extrapolated



# Theoretical Results




- Conservatively-extrapolated value function $V$:

$$V(s) = V^{\pi_e}(s) \pm \delta_V, \qquad \text{if } s \in \mathcal{U}$$
$$V(s) = V^{\pi_e}(\Pi_{\mathcal{U}}(s)) - \lambda \|s - \Pi_{\mathcal{U}}(s)\| \pm \delta_V \qquad \text{if } s \notin \mathcal{U}$$

**Theorem** (informal): Assume the access to an approximate model $M$. Then, the policy induced from a conservatively-extrapolated value $V$ (below) stays close to $\mathcal{U}$ and has good performance:

$$\pi(s) = \operatorname*{argmax}_a V(M(s,a))$$

[Learning Self-Correctable Policies and Value Functions from Demonstrations with Negative Sampling Luo-Xu-M.'19]

# Theoretical Results (Cont'd)

- Note: dynamics may be hard to learn from demonstrations (can only expect it to work around expert actions)

model $M$ approximately correct near the demonstration

**Theorem** (informal): Assume the access to ~~an approximate model $M$~~.

Then, the policy induced from a conservatively-extrapolated value $V$ (below) stays close to $\mathcal{U}$ and has good performance:

$$\pi(s) \cancel{= \operatorname*{argmax}_a V(M(s,a))}$$

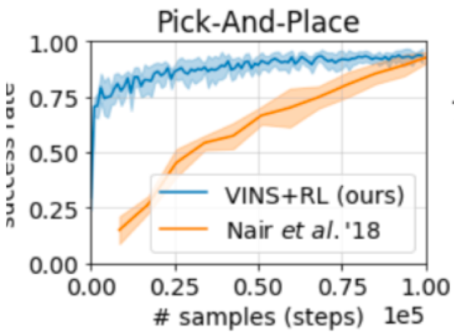
$$\pi(s) = \operatorname*{argmax}_{a:||a-\pi_{BC}(s)||\leq\epsilon} V(M(s,a))$$

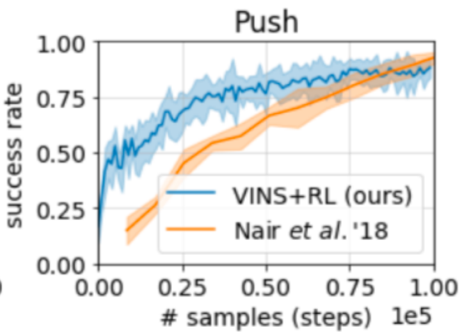
# Experiments

- Learn conservatively-extrapolated value functions with a heuristic borrowed from NLP, negative sampling

- Initialize an RL algorithm (that takes additional samples) with value function, policy, and dynamics learned from demonstrations

| | VINS (ours) | BC |
|---|---|---|
| Reach 100 | **$100 \pm 0\%$** | **$100 \pm 0\%$** |
| Pick 100 | **$75.7 \pm 1.0\%$** | $66.8 \pm 1.1\%$ |
| Pick 200 | **$84.0 \pm 0.5\%$** | $82.0 \pm 0.8\%$ |
| Push 100 | **$44.0 \pm 1.5\%$** | $37.3 \pm 1.1\%$ |
| Push 200 | **$55.2 \pm 0.7\%$** | $51.3 \pm 0.6\%$ |




[Learning Self-Correctable Policies and Value Functions from Demonstrations with Negative Sampling Luo-Xu-M.'19]

# Summary

- Convergence guarantees for a meta model-based RL algorithm
  - Reward-aware loss for learning dynamics
  - SLBO: a much simplified instantiation of the meta-algorithm that works well empirically
- Model-based multi-task RL
- Learning self-correctable policy via learning conservatively-extrapolated value functions
- Open questions:
  - How to empirically leverage optimism in model-based RL?
  - How to customize algorithms for particular environments?
  - How to apply dynamical models to other settings (e.g., hierarchical RL)?

**Thank you!**