

# Evolution and optimization

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Papers in Pubmed

# Phenotypic models of development:

Development happens in time, progressive refinement of pattern  
(genomes + ChIP-seq, RNA-seq tells us nothing about morphogenesis)

‘Geometry’ (& bifurcations) of dynamical systems define the phenotypic models

Can evolution alone predict the phenotypic properties of genetic networks (common to phyla)?

Phenotypic evolution (19th C Darwinism) by positive selection

Implications:

What we see is what evolved quickly, survival of the fittest.

Phenotypic evolution convergent (molecular implementation contingent)

# Setup

## Fitness:

Formulated as quality of pattern, case by case, not reproductive fitness  
Evolve features common to phyla, not particular species.

## Representation:

Dynamical systems made from interaction of pseudo-biochem parts.  
Impose simple dynamics i.e., Morse-Smale  
Change network & parameters, only neutral and fitness incr. changes.  
Time evolve network + boundary conditions, at  $T=\infty$ , evaluate fitness.

## Problems:

How much does the fitness matter?  
Mutation rates matter? (alleviated if dominated by positive selection)

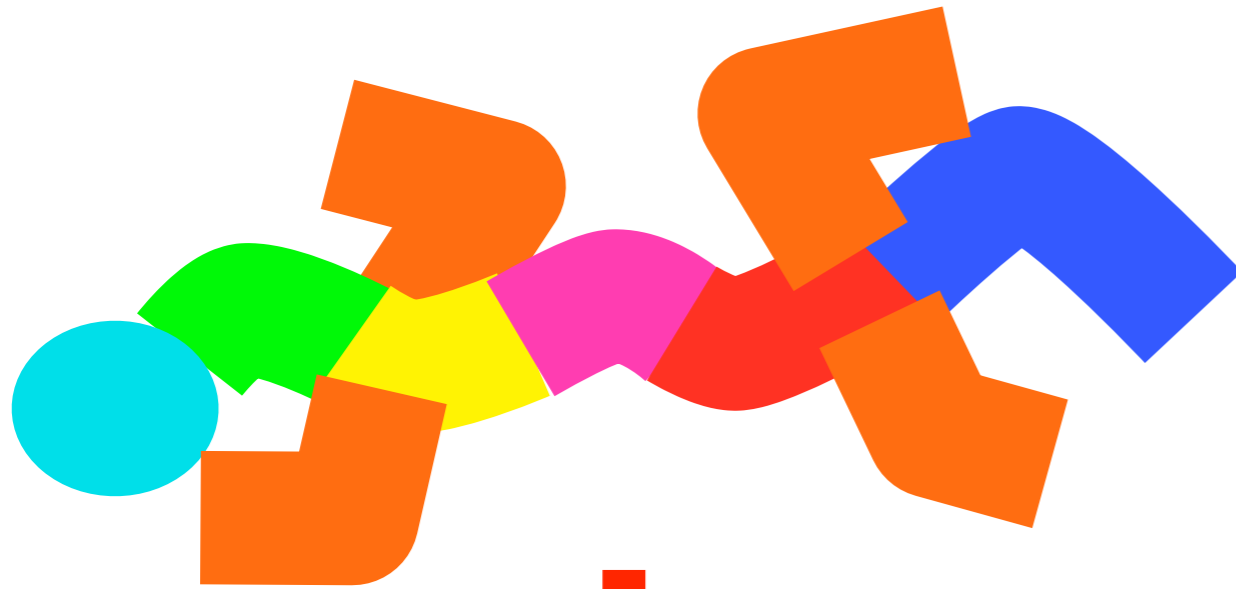
## Non problems:

All systems small; no issue of *complexity* with 'N'.  
No biological reason to insist learn all members of *concept class*.

Ex: Nilsson&Pelger 1994, “~Evolution eye” *Quant Genetics* (Barton).

Fitness= acuity via physical optics, shape and refractive index change,

# Fitness for embryonic patterning (1)

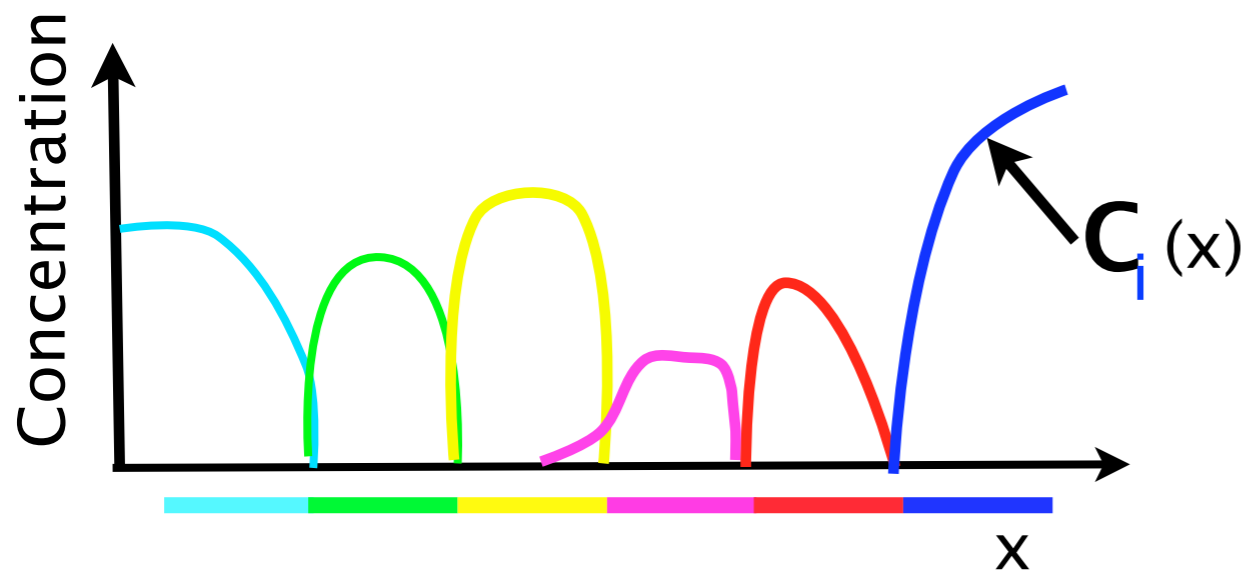


Body axes Cartesian: AP x DV

Development via pattern refinement

Selector gene hypothesis:  
Define compartments/segments,  
tracks cell lineage, cell autonomous

'Morphology' -> network that  
positions the selector genes



# Methods

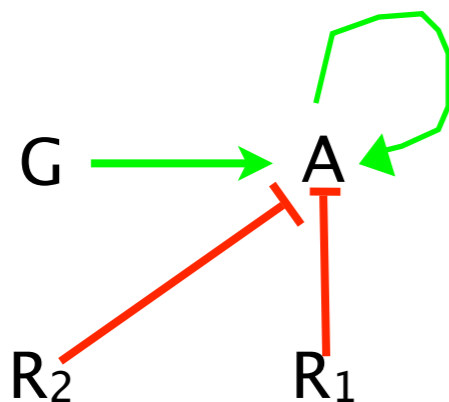
Evolve gene networks via mutation–selection of both network topology, parameters, and ‘outputs’ (from which fitness calculated)  
(~ simulated annealing, no recombination... CP)

Embryo a line of ‘cells’

Network functions identically in all cells, which differ only in exposure to **morphogen G** (external protein whose spacial profile determines fate), no direct cell–cell communication in example here.

Interactions either activate & **add**, or repress & **multiply**

eg A auto–activates, repressed by R<sub>1</sub> R<sub>2</sub> (dropping *csts*), G(time)



$$\dot{A} = \max\left(G(t), \frac{A^{n_1}}{1 + A^{n_1}}\right) \frac{1}{1 + R_1^{n_2}} \frac{1}{1 + R_2^{n_3}} - A$$

= max(activators) \* Π(repressors) – degradation

# Fitness for embryonic patterning (1Dim)

Require for fitness:

1. Assign a number to any collection of selector genes  $C_i(x)$
2. Max. diversity... many selector genes expressed in embryo
3. Min. diversity for given  $x$ ... (unique fate)
4. Smooth function that rewards a little bit of pattern

# Fitness as mutual entropy:

$P(i | x) = C_i(x) / \sum_i C_i(x)$  (only relative concentrations matter)

$P(x) = 1/L$  (uniform probability on cells)

$P(i,x) = P(x)*P(i | x)$

Fitness favors:

1. 'Max. diversity'  $\rightarrow$  Max entropy,  $S_1$ , of  $P(i)$ :  $(-\sum_i P(i) \text{Log}(P(i)))$
2. 'Min. diversity given  $x$ '  $\rightarrow$  Min entropy,  $S_2$  of  $P(i | x)$ :

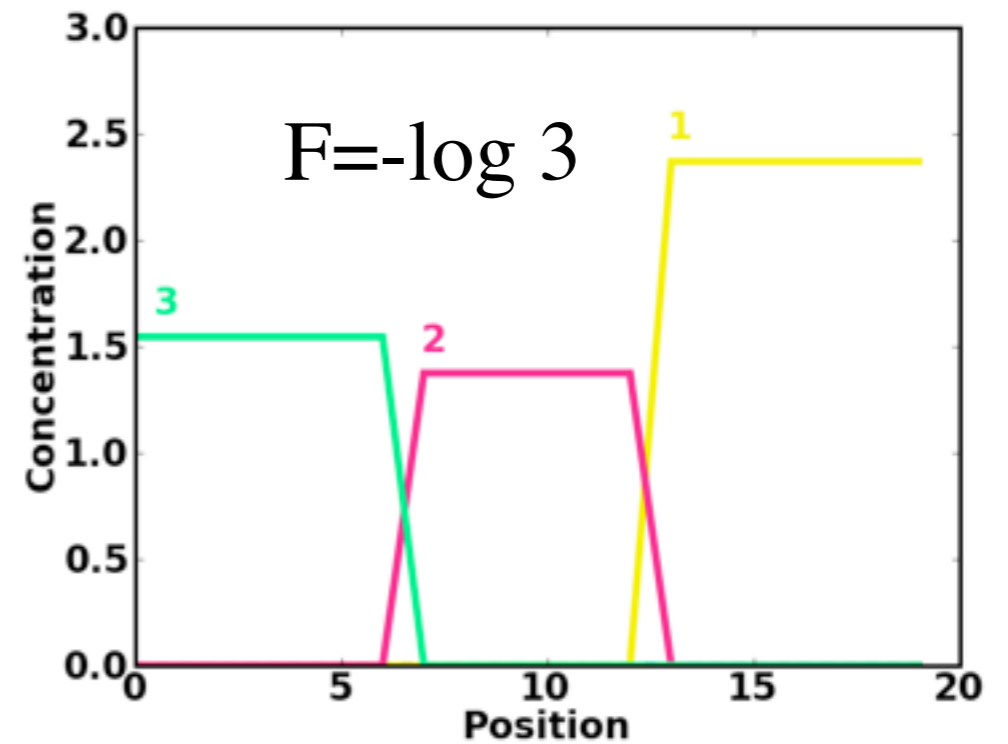
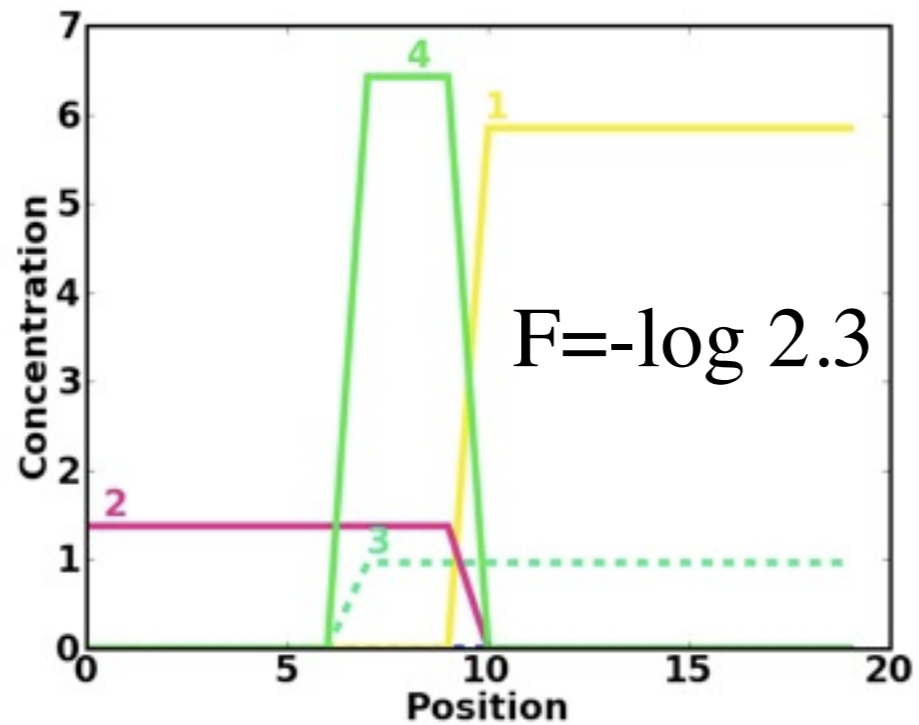
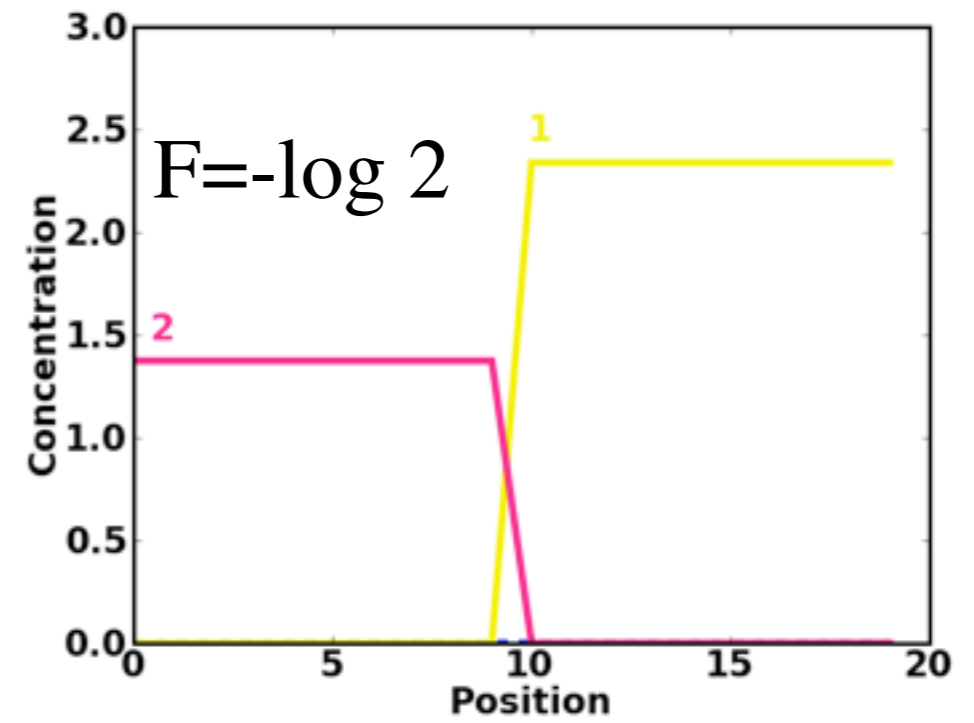
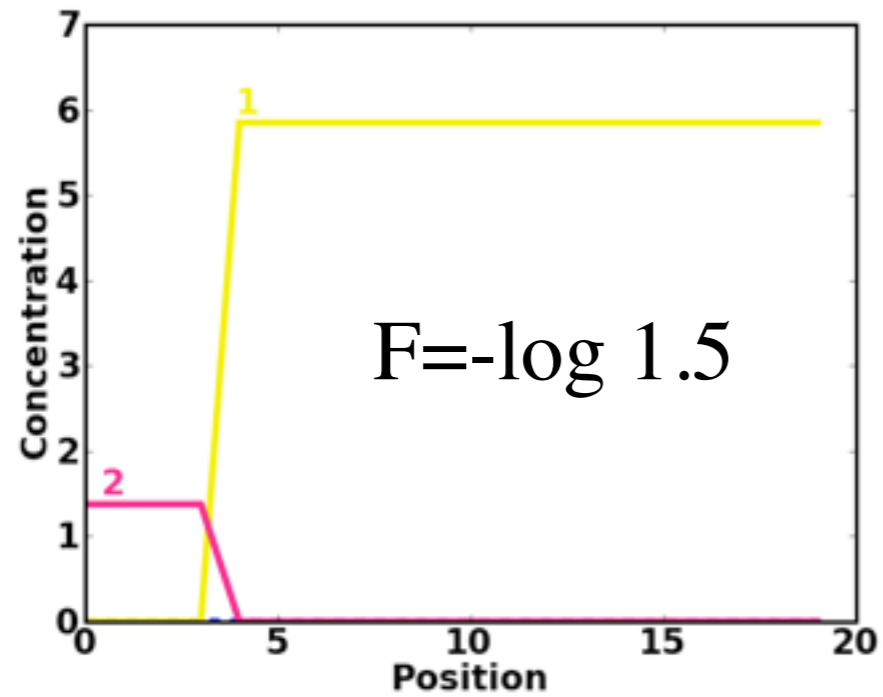
How to combine 2 terms?

3. Assume gene duplication neutral  $\rightarrow$

'fitness' =  $-S_1 + S_2 = -$  mutual information (i, x).  $C_i \Leftrightarrow x$   
(best fitness  $\sim$  free energy is most negative)

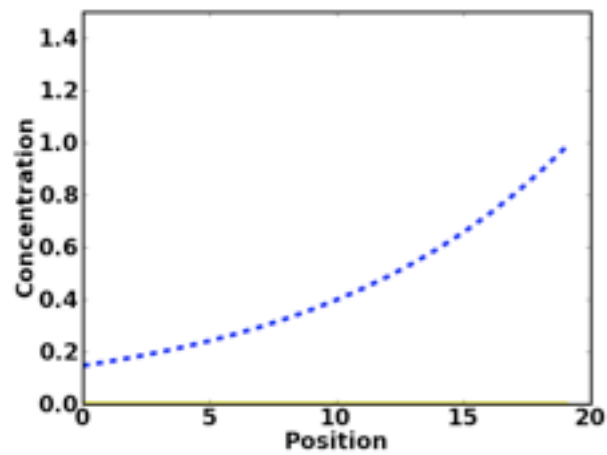
For  $N$  selector genes fitness  $\geq -\log(N)$ .

# Mutual entropy fitness

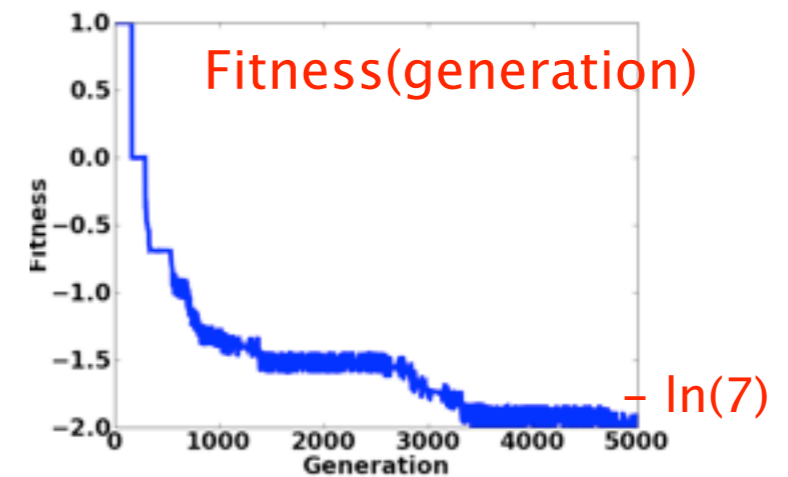
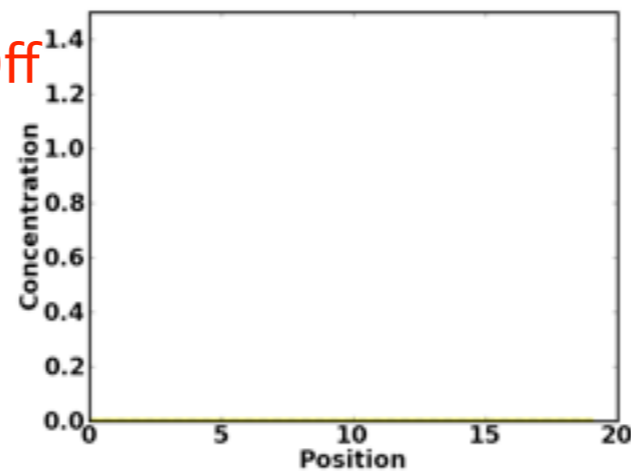




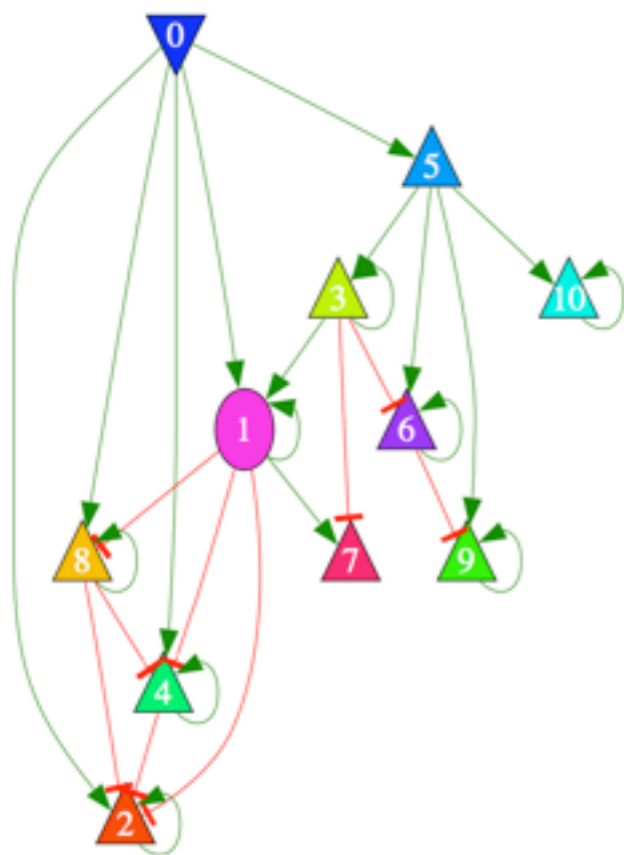
# Networks for static 'morphogen'








Morp On  $\rightarrow$  Off

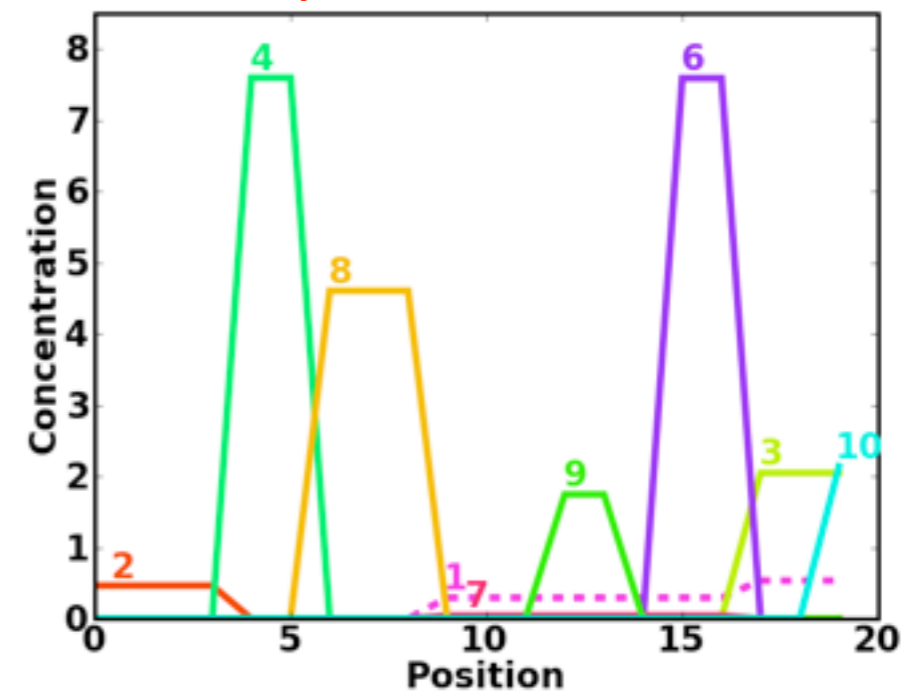


Final network

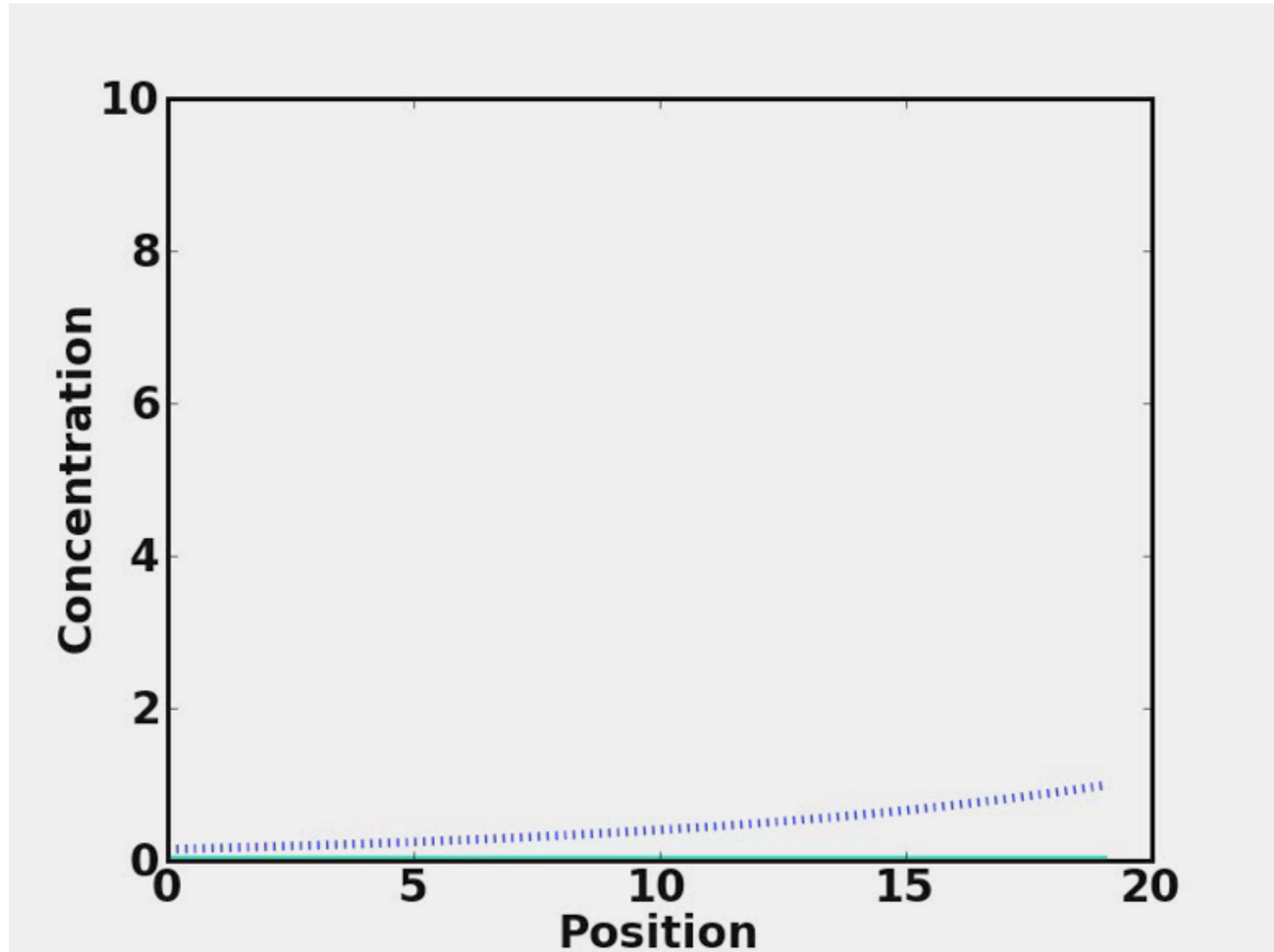
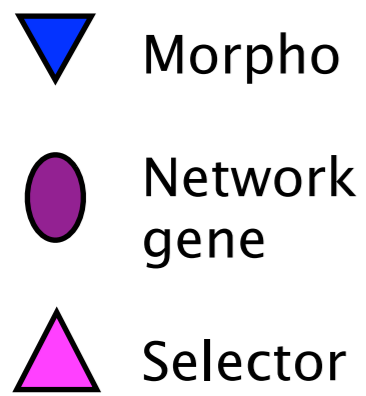
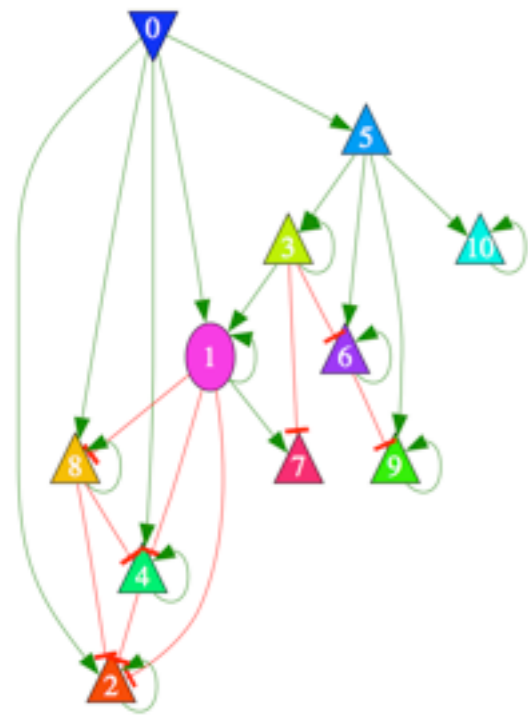


-  Morpho
-  Network gene
-  Selector
-  activate
-  repress

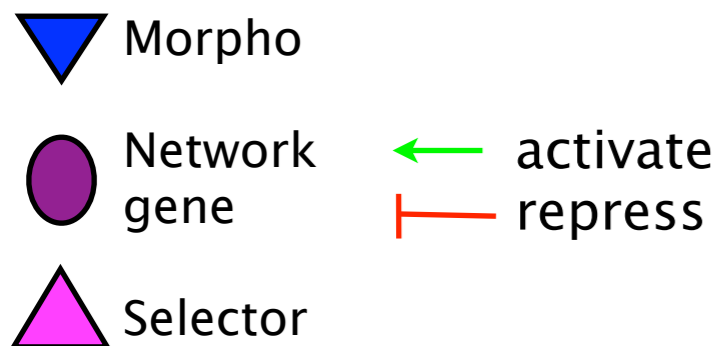
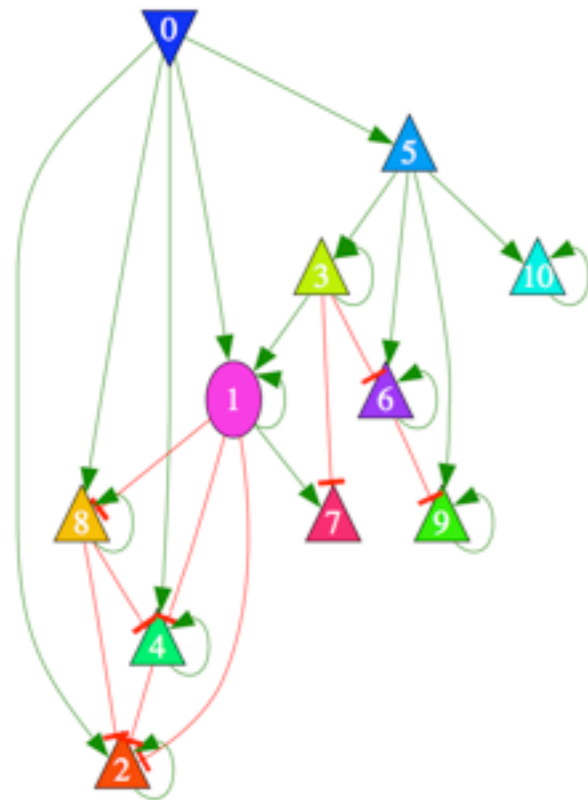
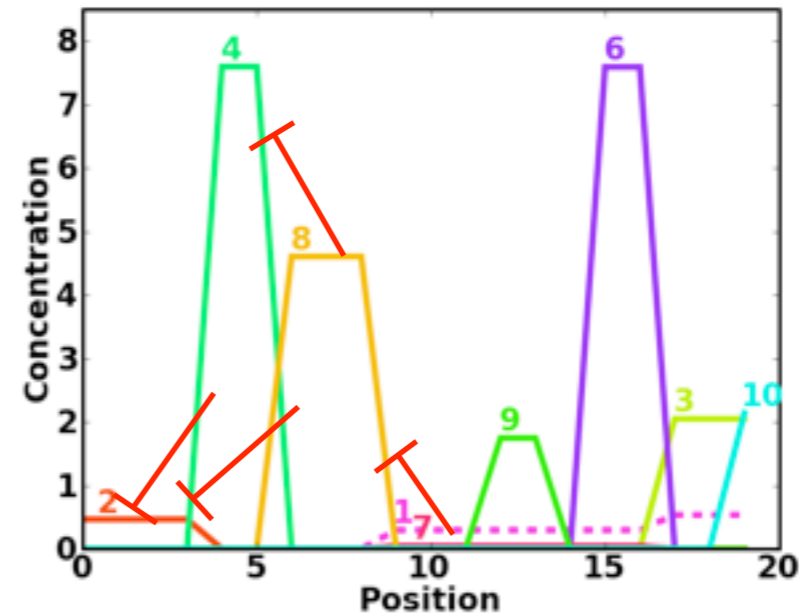
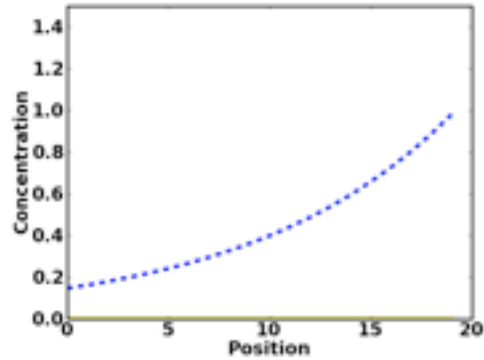
Final pattern,  $selector_i(x)$



# Time (development) dynamics in evolved static morphogen network



# Properties of Networks static gradient



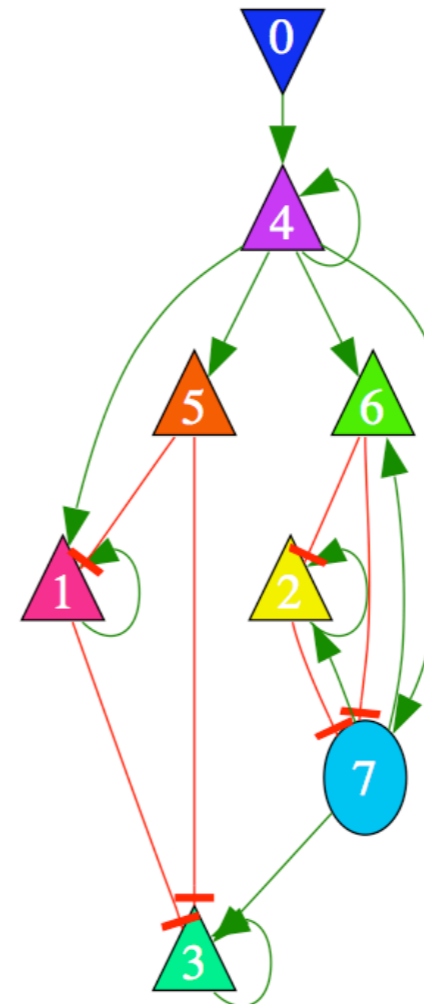
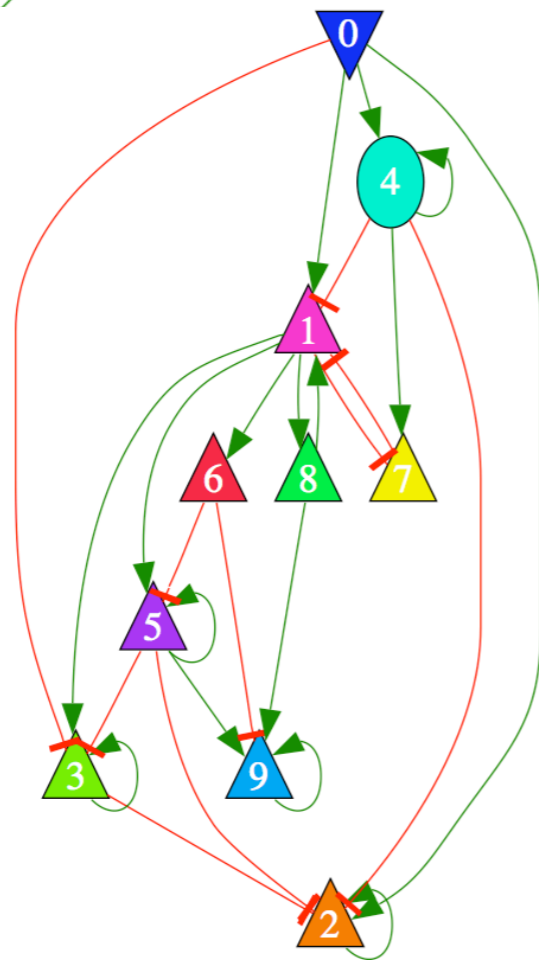
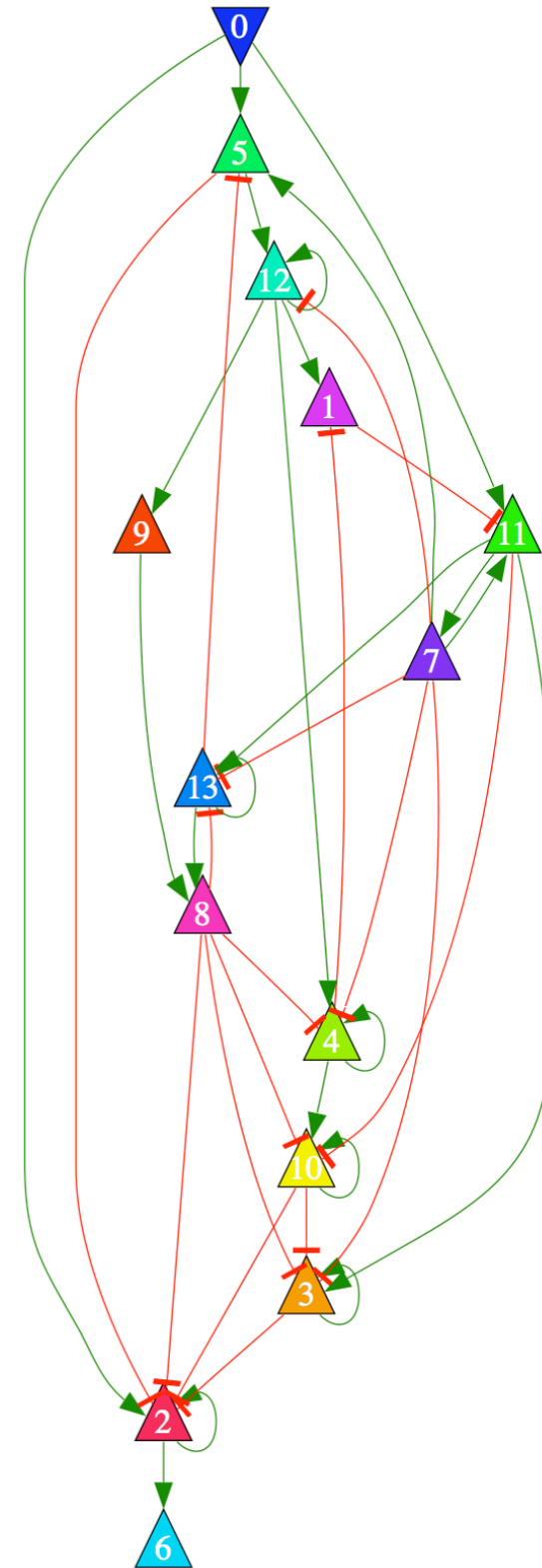
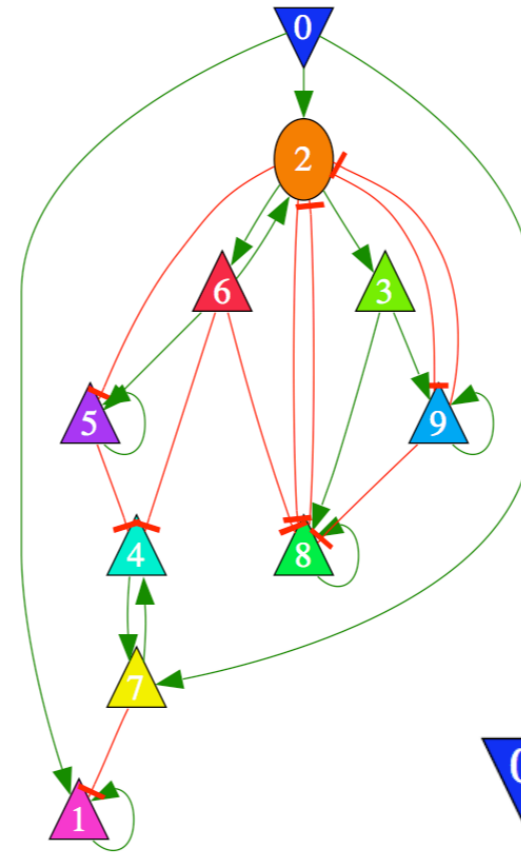
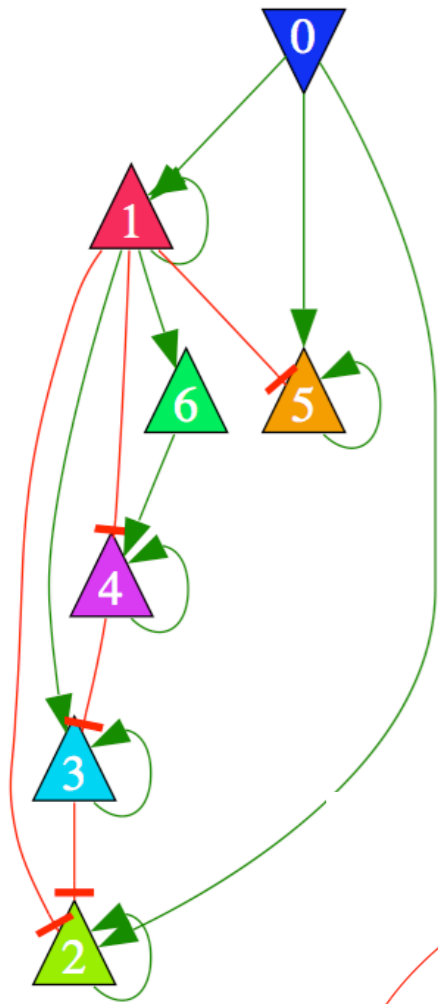
Networks 'cell autonomous' (no communication between cells) → morphogen defines cell position.

Morphogen disappears → multi-stability → sharp boundaries & only need repression between ~adjacent domains

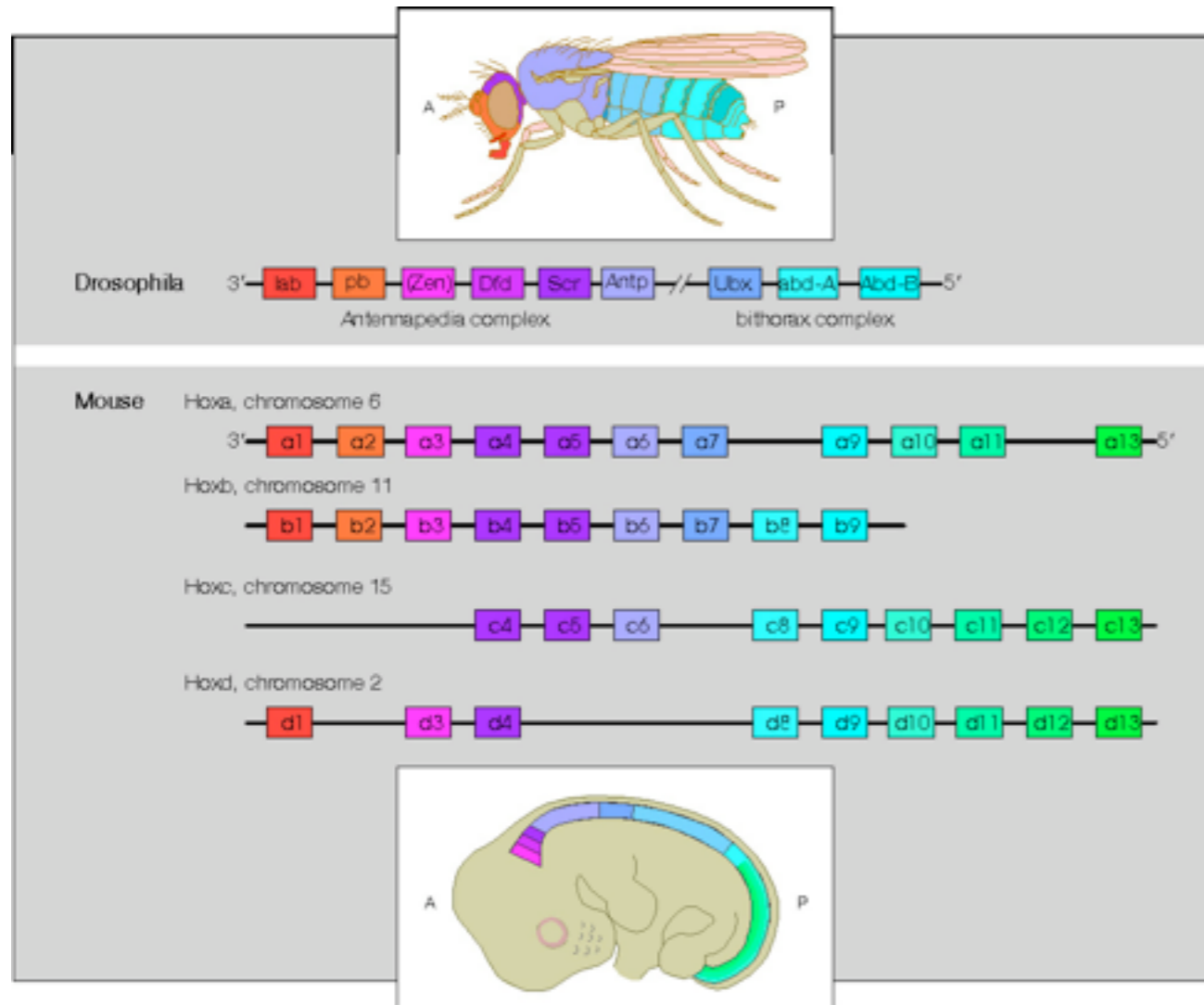
Multi-stability → order of gene expression matters & numbers determine final state.

Morphogen sets anterior boundaries, repression sets posterior boundaries → statistical char. of evolved networks.

# Topology $\neq$ function



# Anterior-Posterior patterning



Hox genes conserved in bilaterians

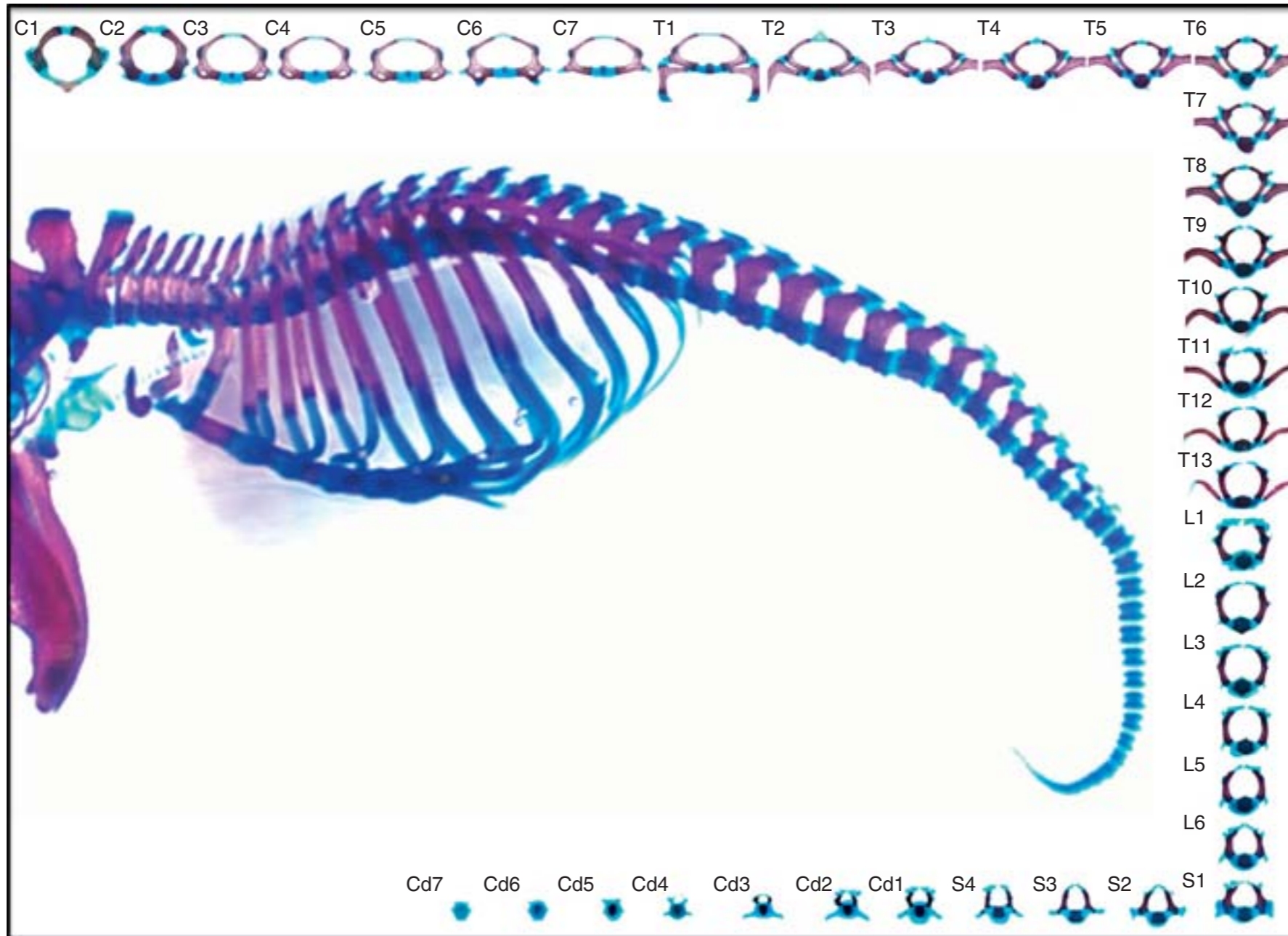
Define coarse AP coordinates

Cellular “Zip code” controls master regulatory genes

Biochem of regulation very complex, but simple phenomenology



# Mouse vertebrae reflect Hox territories



DM Wellik 2009

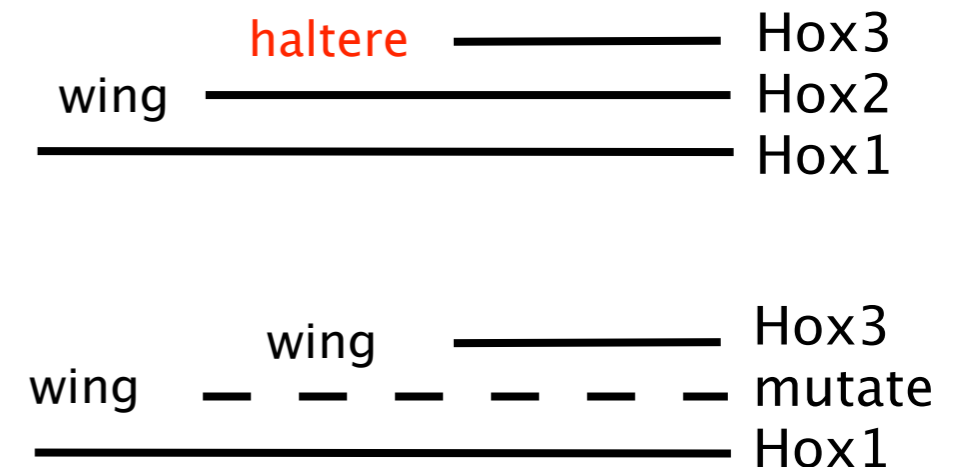
# Phenomenology of Hox expression

1. Spatial colinearity: 3' to 5' genome order follows A to P expr.
2. Temporal colinearity: (vertebrates) temporal order follows A to P
3. Posterior prevalence rule: most posterior Hox gene imposes fate on all anterior genes

Hox mutation **haltere** → wing



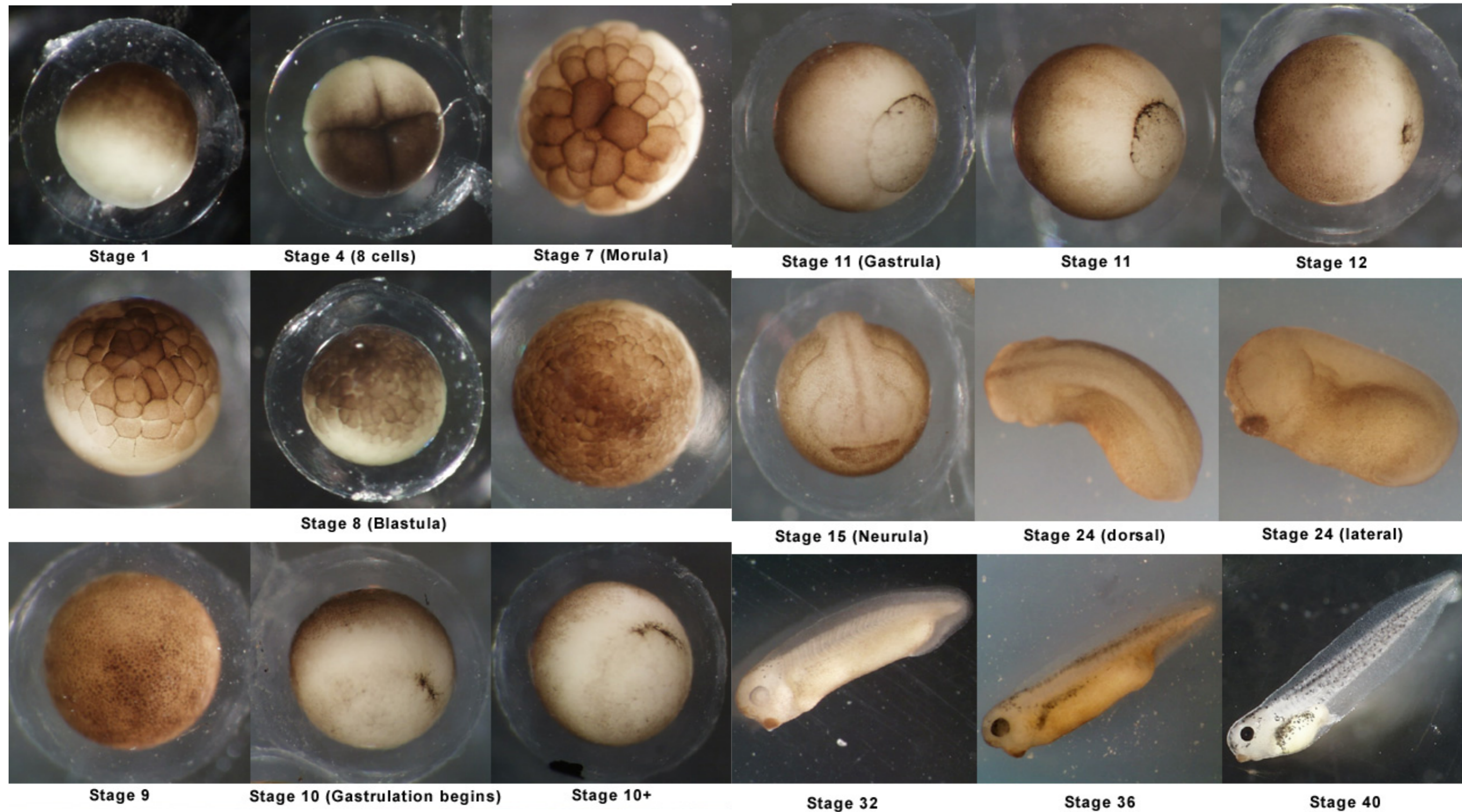
Hox expression A to P



wing = (1 AND NOT(2,3..))  
haltere = (2 AND NOT(3,..))



# Xenopus development (2)



1.2mm egg, 7hrs stage 9 4000+ cells; 17hrs stage 15; 40hrs stage 32 @23C

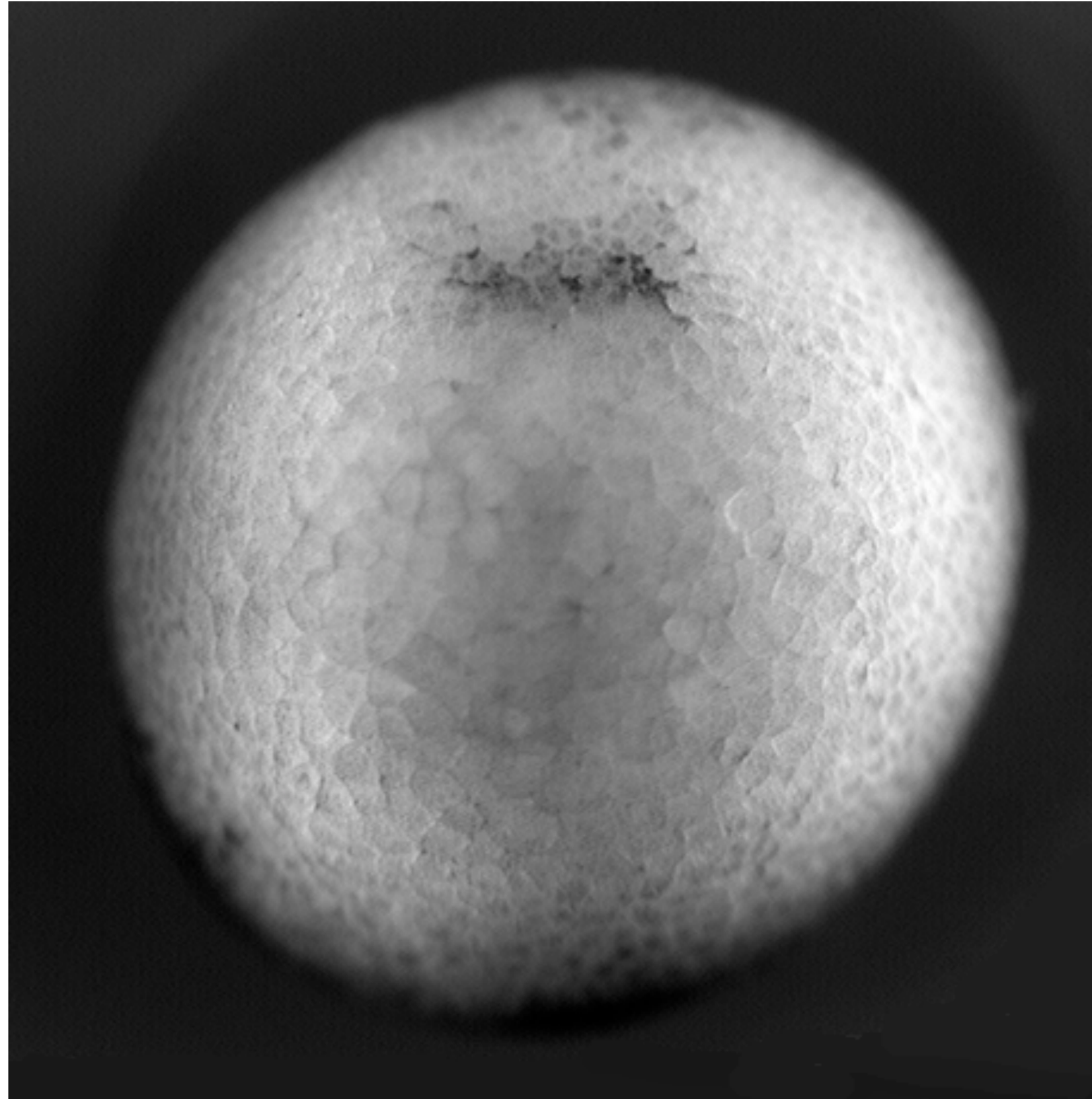
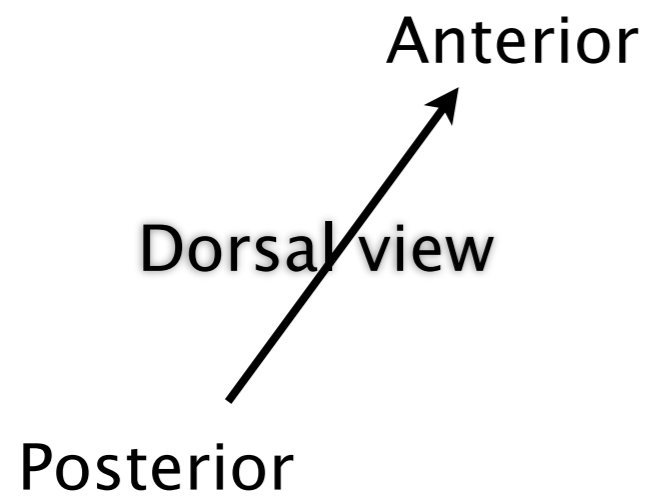


# Gastrulation of Xenopus

1.2mm egg

5 hrs fertilization to Movie0  
4000+ cells

17hrs @23C Movie

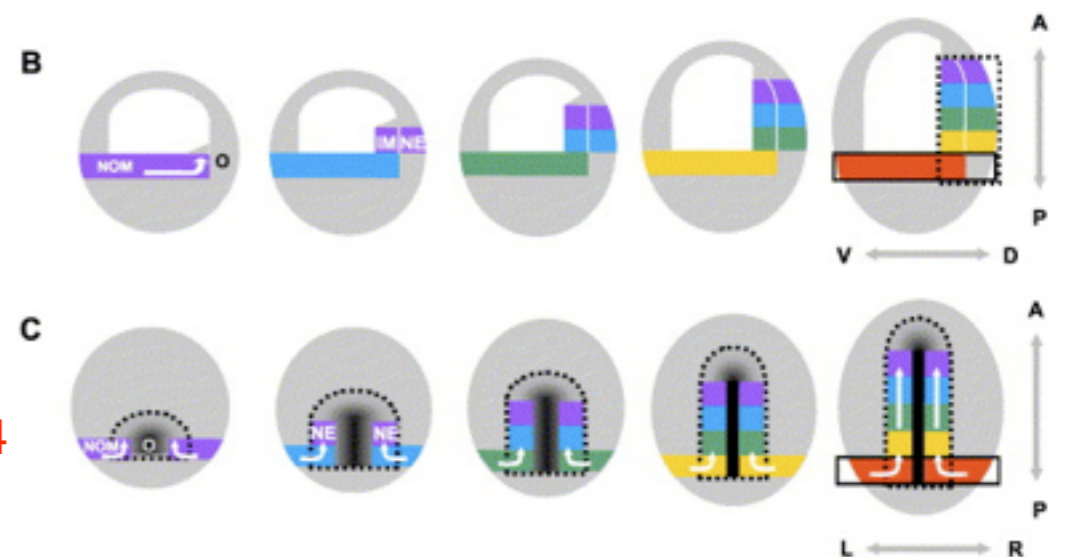
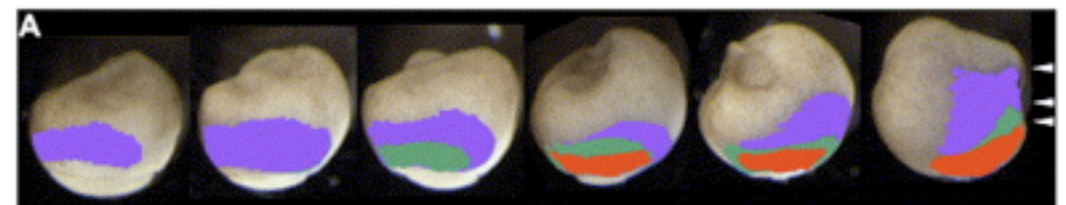
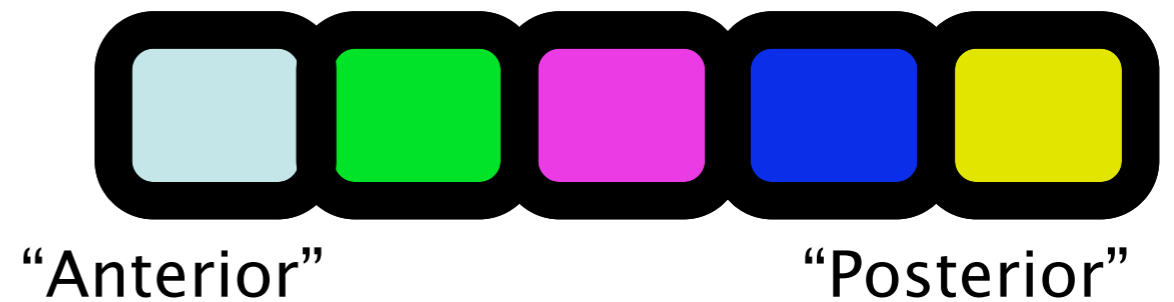
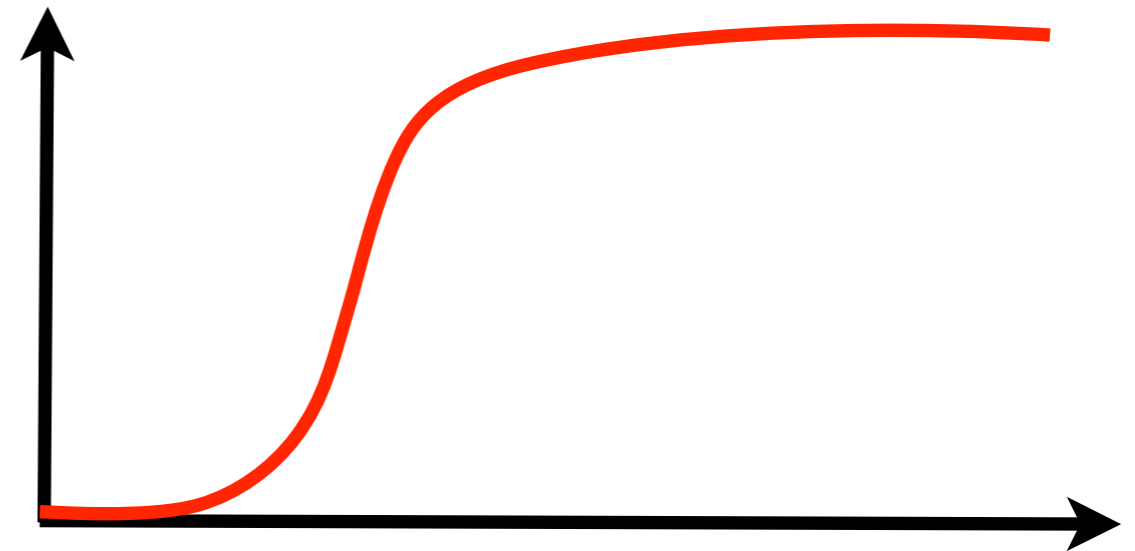


# Patterning a field of cells : AP growth

Model 'patterning during growth' as sliding morphogen that marks boundary between growth zone and patterned tissue.

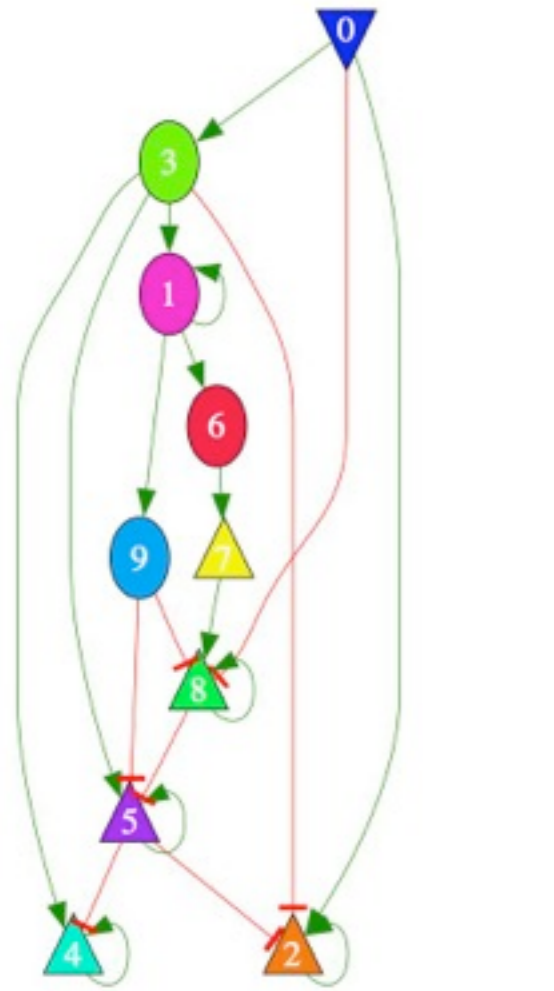
Hox expression marked as colors. Temporal sequence of expression on equator → spacial domains AP

'organizer' is point where converging equator → extending AP  
morphogen step ~ organizer



Wacher 2004

# Sliding morphogen (2)

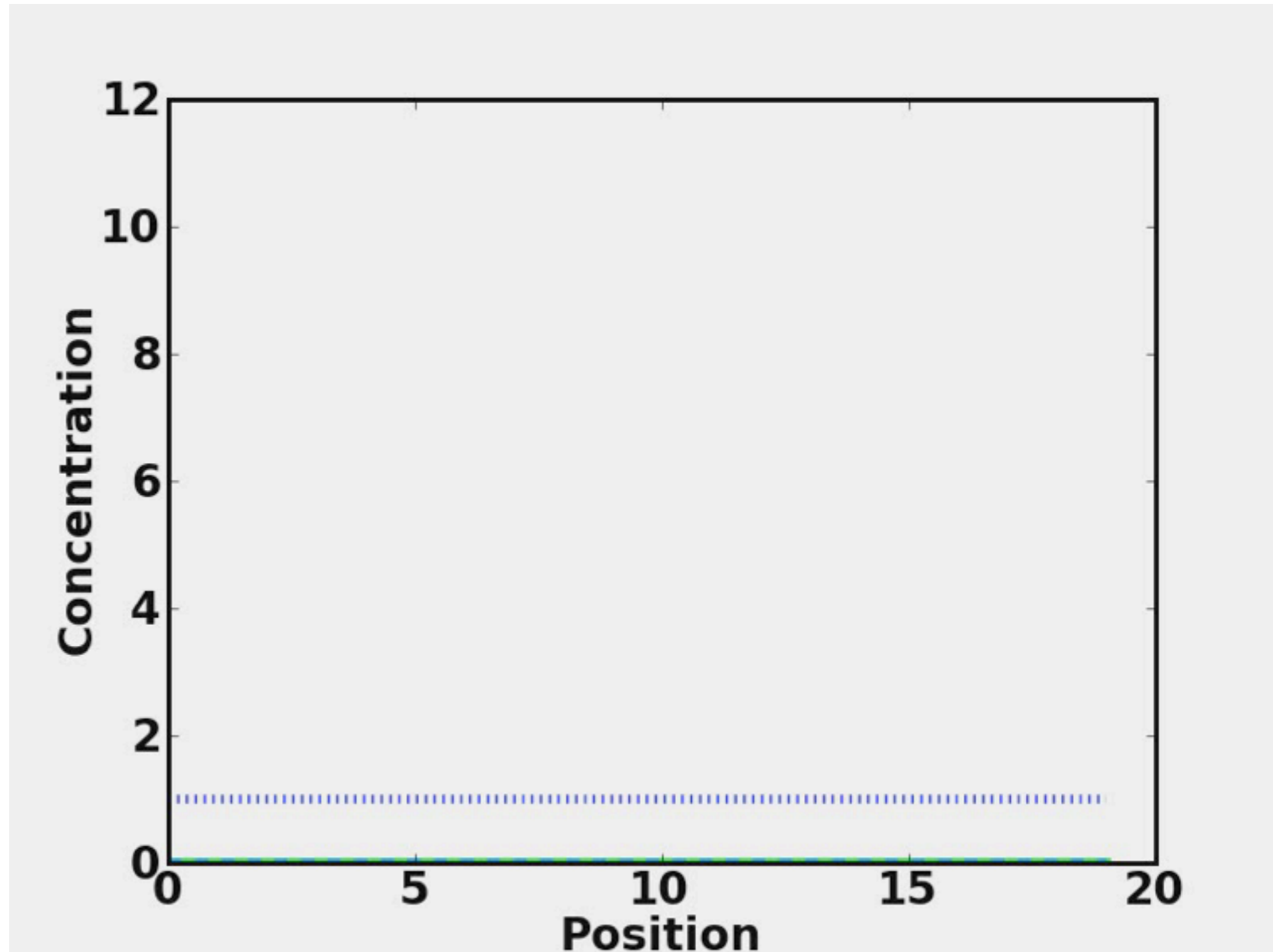


▲ only triangles enter fitness

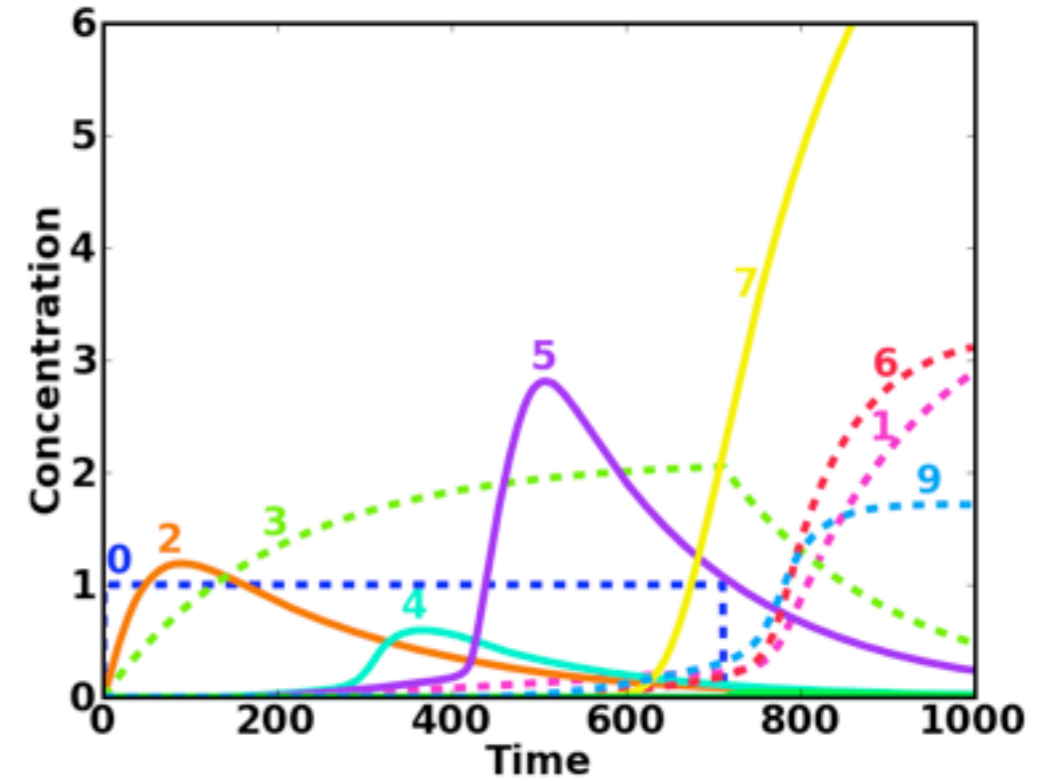
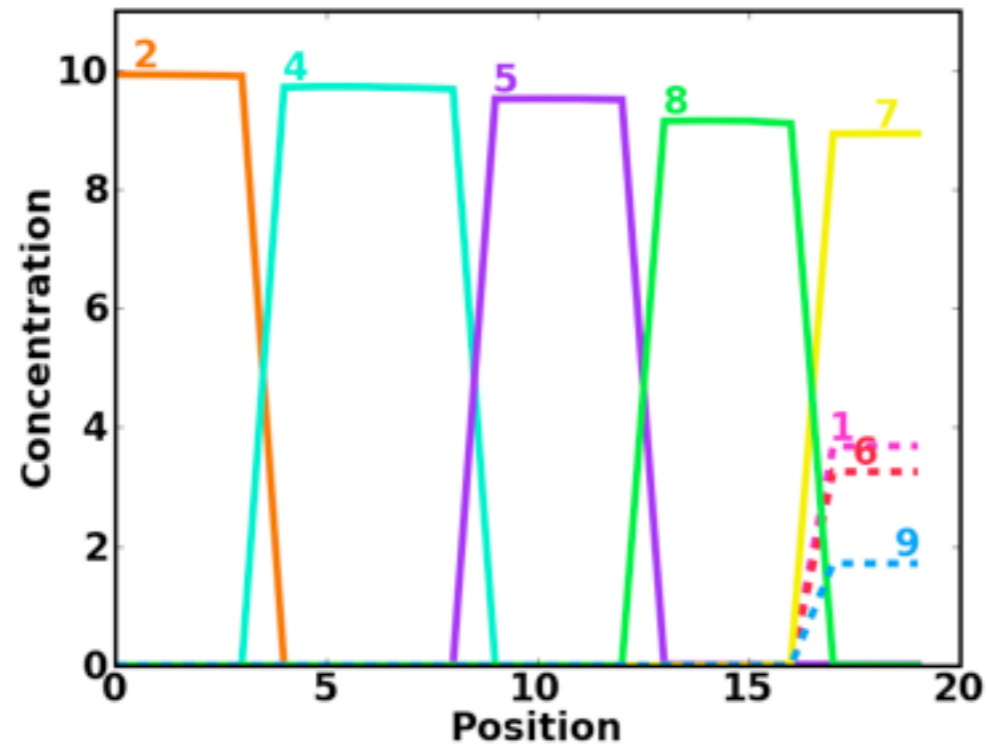
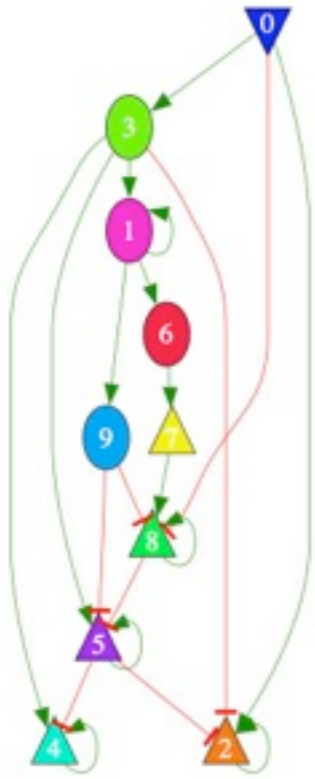
Morphogen

A dotted blue line representing a morphogen gradient that starts at a low concentration on the left, increases to a peak at position 7, and then decreases to a low concentration on the right.

Time development of final evolved network



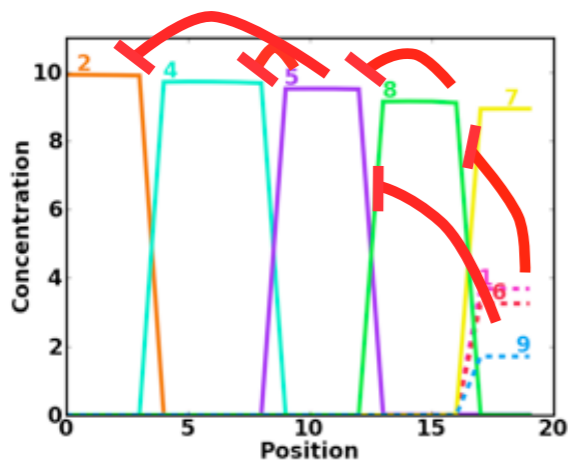
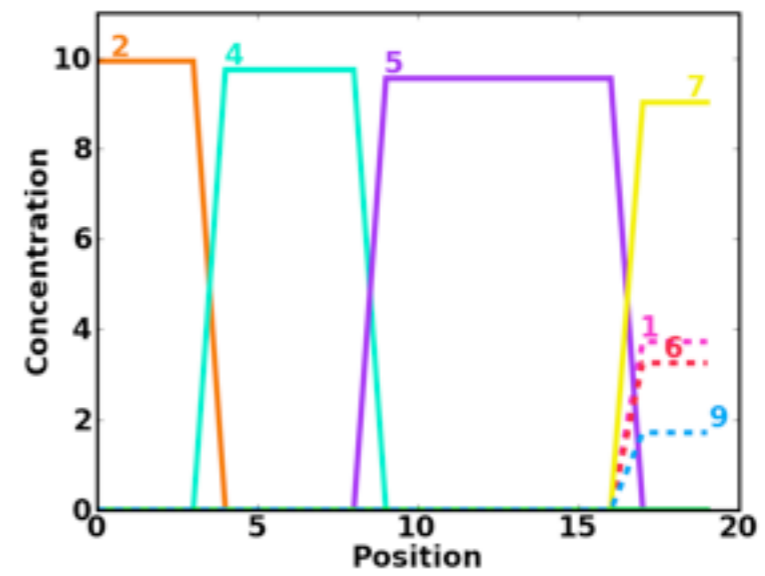
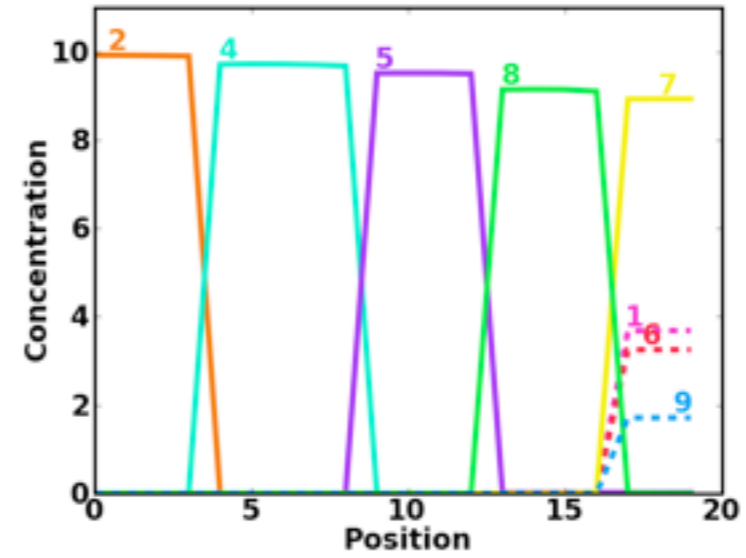
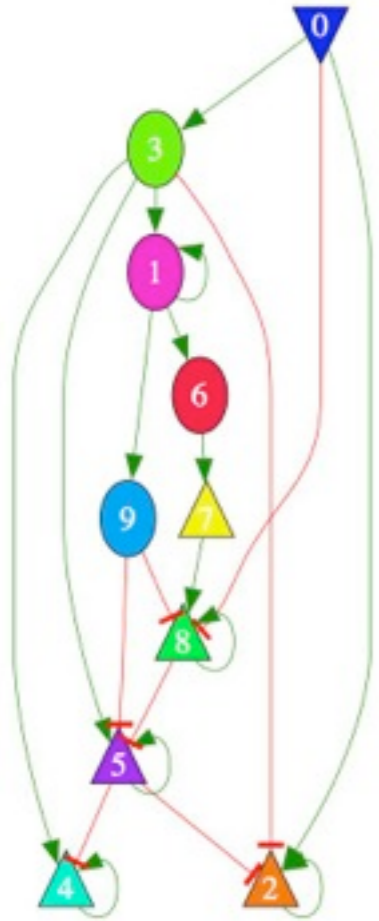
# Temporal colinearity



Temporal colinearity: Hox(time) fixed posterior cell --> Anterior-Posterior progression



# Anterior Homeotic Mutation (2)

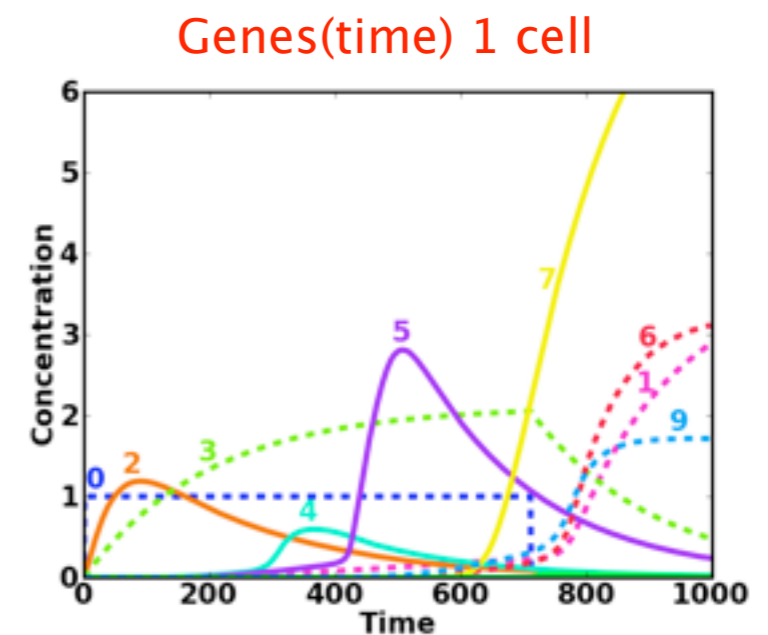


# Properties of Networks with Sliding Gradient

Recall static morphogen: anterior boundaries positioned from morphogen.  
Analogue for sliding gradient?

Position == time exposed to morphogen:  
'Timer' **gene 3** converts time in morphogen to morphogen level + cell autonomy. Static Morph  $\leftrightarrow$  Sliding Morph.

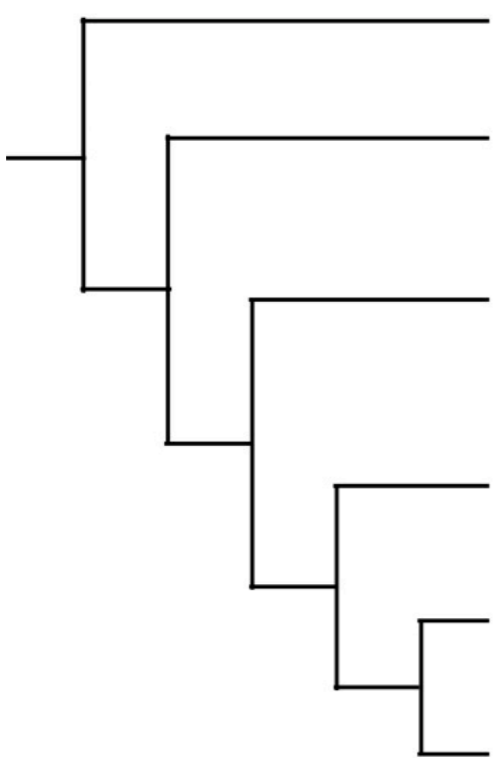
Good for growth control, change all rates get same pattern (Deschamps et al timer ~ CAUDAL, CDX2)



Hox phenomenology: temporal colinearity, anterior homeotic mutation

Evolution of long from short germ band insects.

# Phylogeny of short (seq) and long germ insects (seq ~ vertebrates, pattern during growth)



order	species	hemi- or holo- metabolous	extended syncytial blastoderm	sequential or long germ segmentation	anterior localization of maternal mRNAs
Orthoptera	<i>Schistocerca</i> sp.	hemi	no	seq	?
	<i>Gryllus bimaculatus</i>				
Hemiptera	<i>Oncopeltus fasciatus</i>	hemi	?	seq	?
Hymenoptera	<i>Nasonia vitripennis</i>	holo	yes	long	yes
	<i>Bracon hebetor</i>		yes	long	?
	<i>Aphidius ervi</i>		no	seq	?
	<i>Apis mellifera</i>		yes	long*	?
Coleoptera	<i>Tribolium castaneum</i>	holo	yes	seq	yes
	<i>Callosobruchus maculatus</i>			long*	?
Lepidoptera	<i>Bombyx mori</i>	holo	no	seq*	?
	<i>Manduca sexta</i>		?	long*	
Diptera	<i>Drosophila melanogaster</i>	holo	yes	long	yes
	<i>Anopheles gambiae</i>				

AD Peel, Phil. Trans. R. Soc 2008

Short to long germ: *timer* gene → static morphogen. Down stream network invariant

Insect evolution focuses on segmentation, but Hox supplies identity.

# Other systems evolved:

- Clocks and bistable systems: (Francois & Hakim PNAS 2004)
- Somitogenesis (eg vertebrae): (Francois, Hakim, ES Mol.Sys.Bio. 2007)
- Adaptation to temporal signal (Francois & ES, Phys.Bio. 2008)
- AP-Hox patterning (Francois & ES, Development 2010)
- Temperature compensated clocks that entrain (Francois & ES, PLoS Comp Bio)
- Networks that take a spatial derivative of transient morphogen
- Fit genes to topology (Corson & ES PNAS 2012)

A few other applications in brief

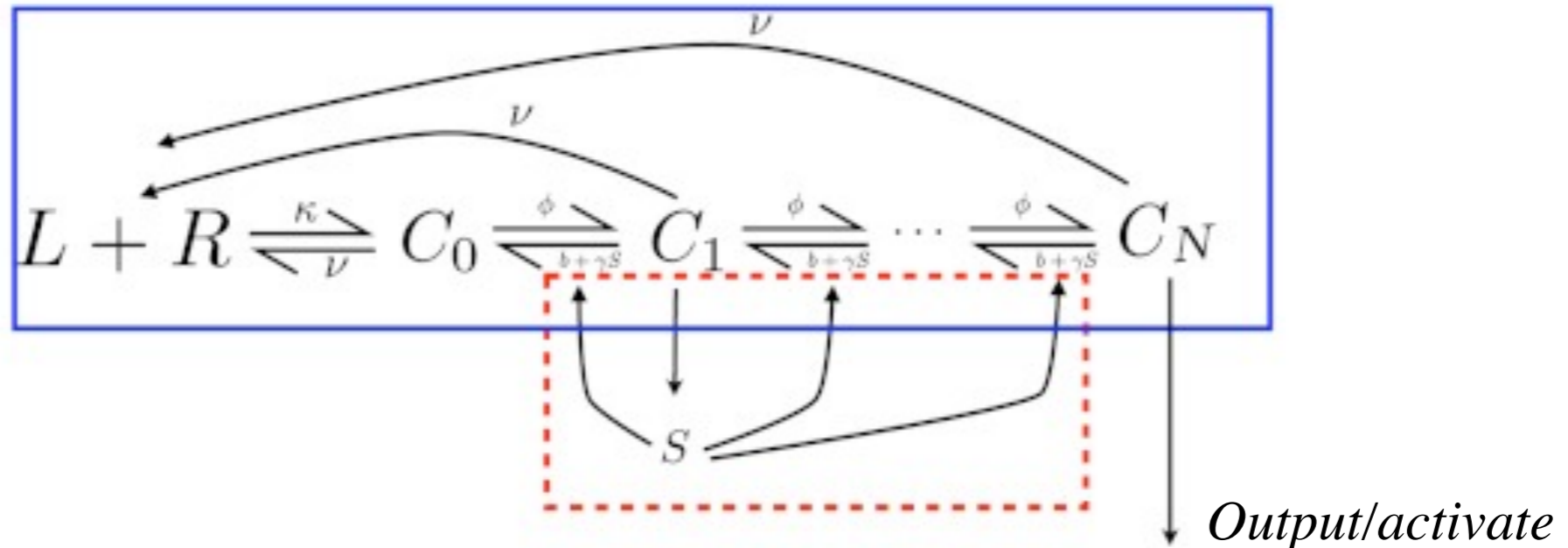


# T Cell Activation

T cell receptors will respond to a few molecules of agonist and ignore a  $> 10^4$  higher concentration of 'self' proteins, based on a 3–4x slower off rate from the receptor. (Kinetic proof reading will not explain this).

Model of phosphorylation cascade + self activated kinase/phosphatase can: (Altan-Bonnet & Germain PLoS Bio 2005)

System also evolved by Lalanne & Francois PRL 2013 (see also Francois etal PNAS 2013)



# Optimal decision theory (Explore–Exploit)

1. Given a stream of data from distribution A or B, what is minimum average decision time to identify the source for a given error rate, and what is the algorithm that realizes it?? (Wald 1945)
2. A stream of data changes from type A to type B at an unknown time T. What is the minimum average time lag in detecting the change point, for a prescribed false positive rate??

Plausible constraint on sensory systems, from cells in an embryo to higher cognition, decision speed matters.

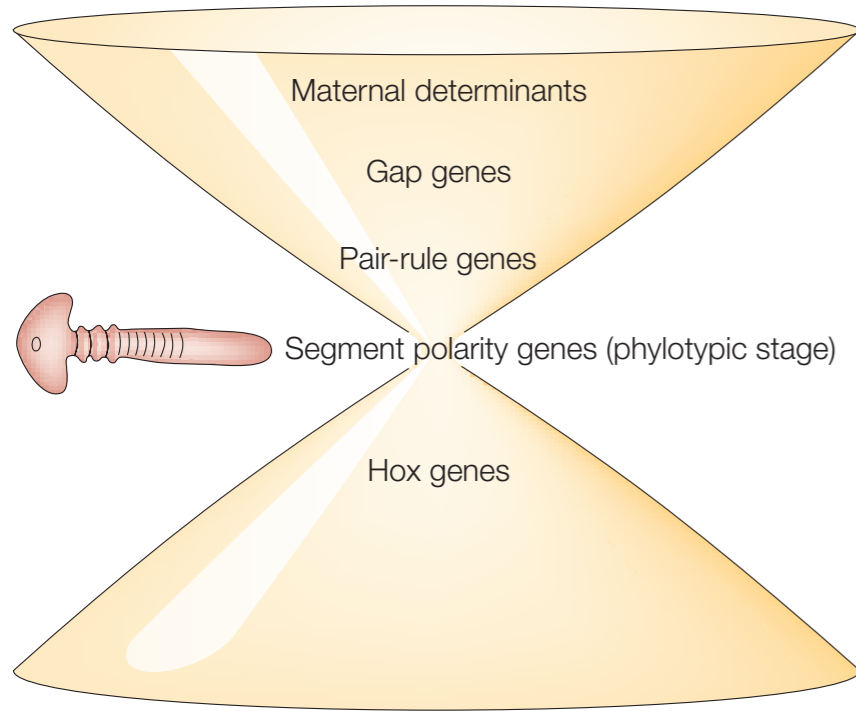
Refs:

Neural MN Shadlen ~2006

Cellular, Kobayashi 2010;

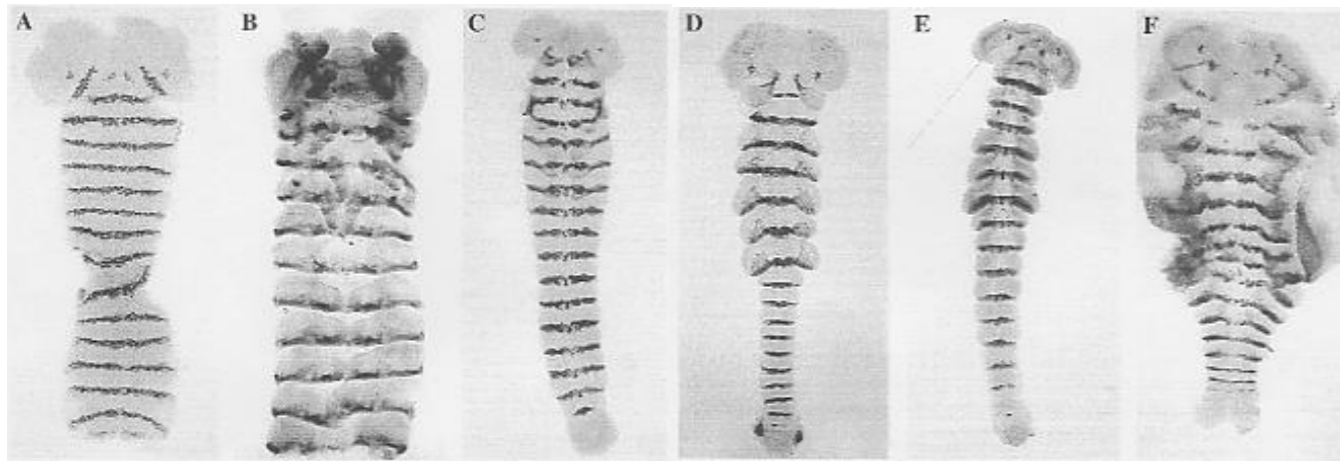
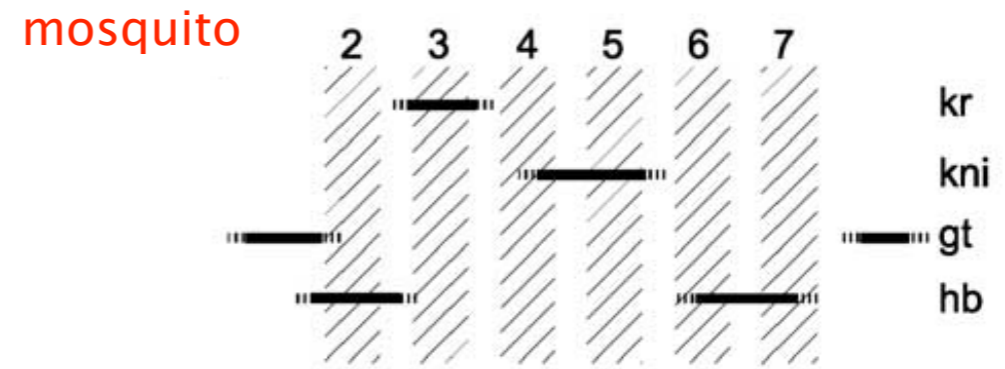
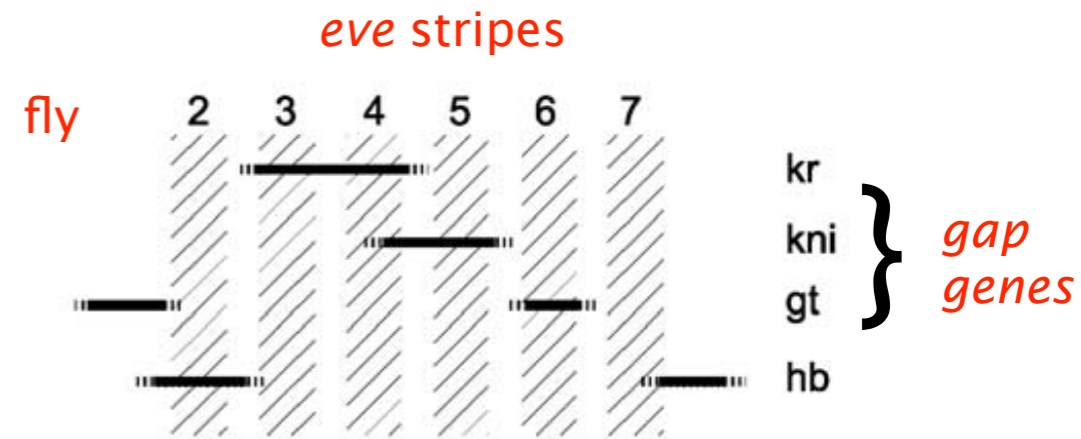
Vergassola & EDS 2013, Simple biochemical networks can optimally solve 1 & 2 and the parameters fit via local search.

# 'Saddle points' or last common ancestor (or how to turn a fly into a mosquito)



Peel, *Nat. Rev. Genet* 2005

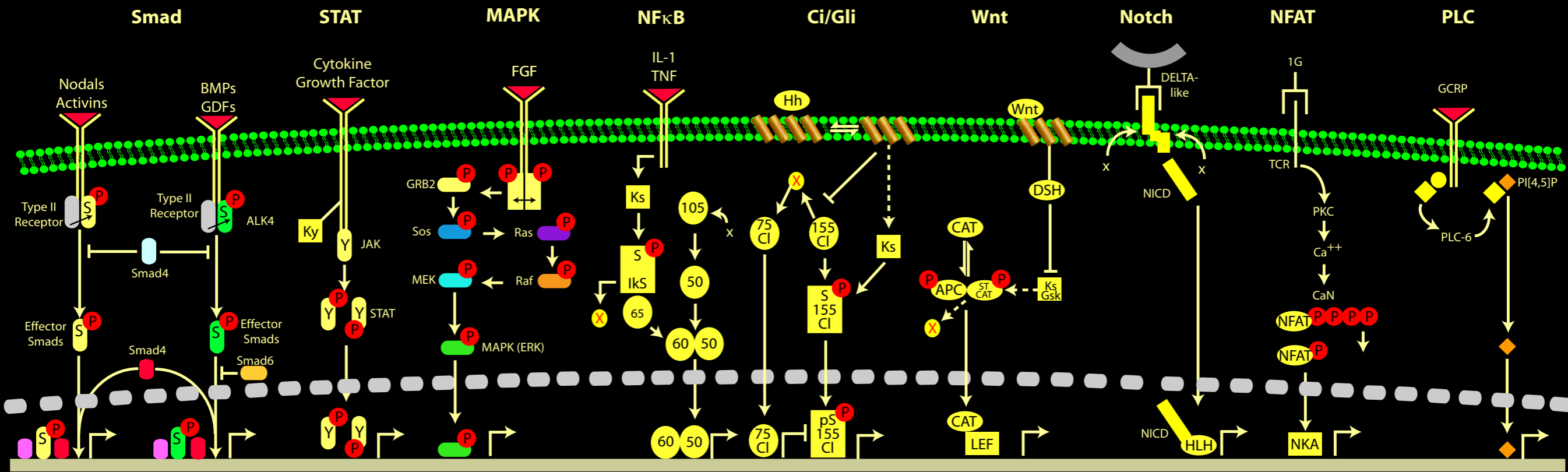
gap genes regulate *eve*.  
gap genes move, *eve* fixed and essential  
Goltsev *Dev. Bio.* 2004



fly (dissected) moth beetle cricket grasshopper crayfish

Engrailed (and wg) mark segment boundaries  
N H Patel *Development Suppl* 201-207 1994

# Signaling pathways involved in cell fate determination



Are there any biophysical principles such as dynamical behavior that control where/when certain pathways used? Could evolution simulations define discrete dynamical types?

*(Brivanlou and Darnell 2002, Science)*

# Characteristics of evolved models

- Close to dynamical system picture, evolve topology of flow, not genes → visualize minimal parameter description (→ genes to be fit). Evolution as cascade of bifurcations.
- Network and parameters evolve together, de novo fitting of all parameters in final network could be hard.
- Networks work by sloppy confluence of opposing activities; with tuned rates; no time scale separation ≠ 19th C applied math. **BUT** simple in that parameters follow by gradient search.
- Evolved models not obvious, like genetic screen
- Relevance to experiment, hi level (static ↔ dynam morpho), lo level (fit parameters)
- 19th C Darwinism → grad search, Useful engineering principle for specific systems.

# The End