A Survey on the Complexity of Entangled Provers

Zhengfeng Ji, IQC, U. Waterloo

What we will NOT cover

- Quantum verifier and messages
- Non-signalling provers
- Bell violations
- Parallel repetition theorems
- Unentangled provers
- …

Background

- Interactive proof systems and the PCP theorem

  [GMR '89] [Bab '85] [BOGKW '88] [BFL '91]

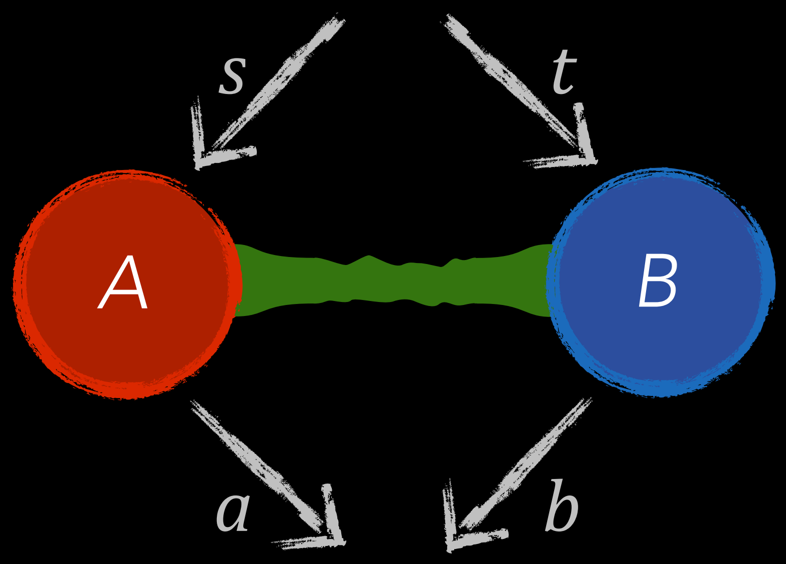
  [FGL+ '96] [AS '98] [ALM+ '96]

- Entanglement and non-locality

  [EPR '35] [Bell '64]

- Two origins combined [CHTW '04]
  - All powerful provers
  - A CS approach to non-locality

interaction
+
randomness

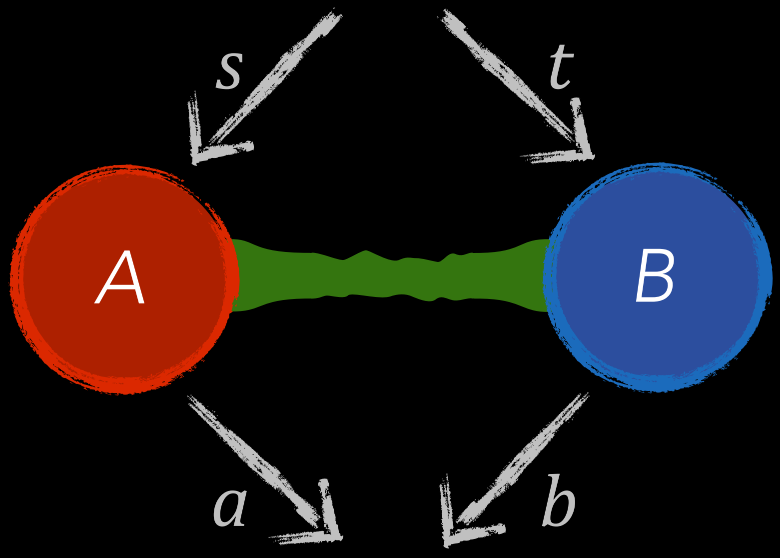


Problem setting and notions

- Strategy $(\rho, \{A_s^a\}, \{B_t^b\})$

  Question in subscript, answer in superscript

$$p(a,b|s,t) = \mathrm{Tr}_\rho(A_s^a \otimes B_t^b)$$
$$\stackrel{\text{def}}{=} \mathrm{Tr}(A_s^a \otimes B_t^b \rho)$$



- Game values

$$\omega^* = \sup_{\rho, A_s^a, B_t^b} \mathbb{E}_{s,t} \sum_{a,b} V(a,b|s,t) \mathrm{Tr}_\rho(A_s^a \otimes B_t^b)$$

$$\omega^f = \sup_{\substack{\rho, A_s^a, B_t^b \\ [A_s^a, B_t^b]=0}} \mathbb{E}_{s,t} \sum_{a,b} V(a,b|s,t) \mathrm{Tr}_\rho(A_s^a B_t^b)$$

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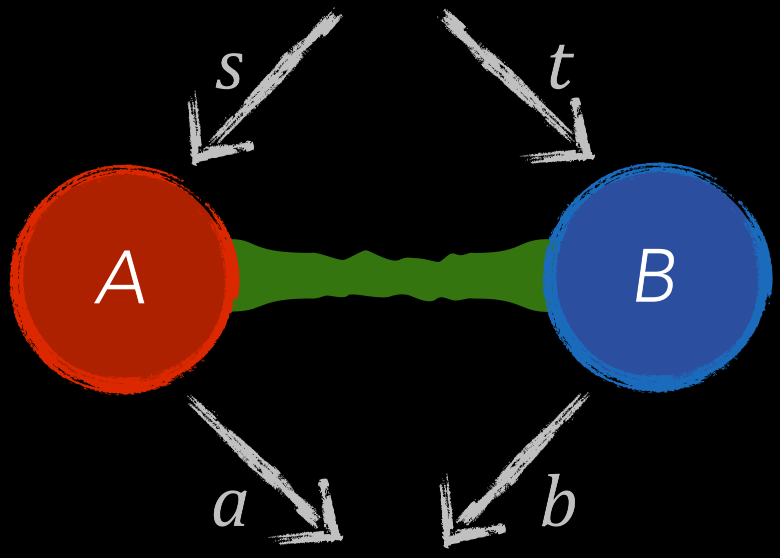


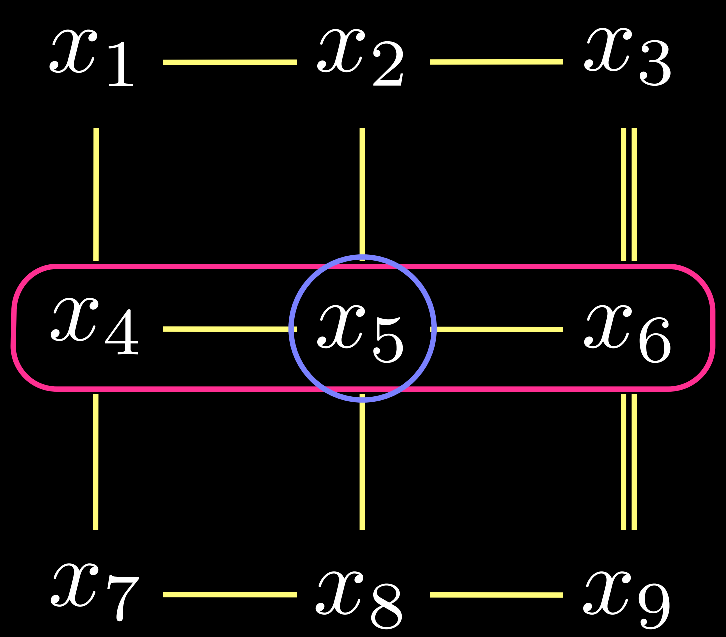
- Symmetry assumption

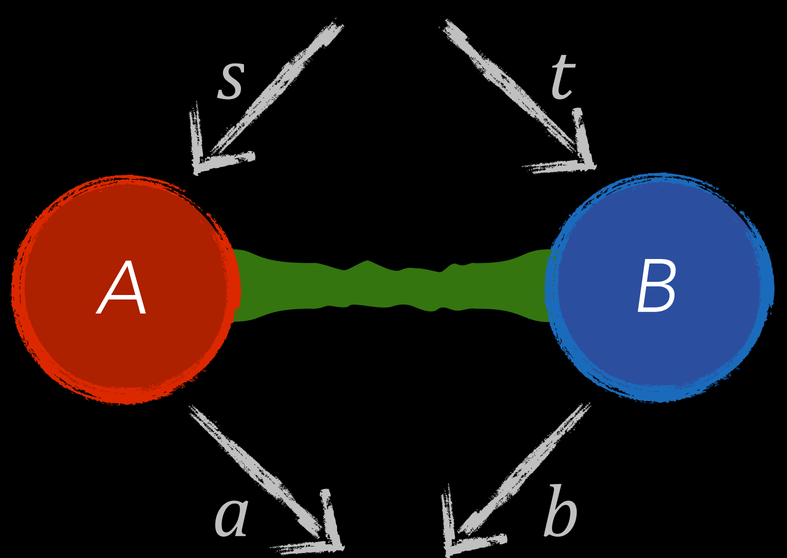
- ρ-norm $\|A\|_\rho = \sqrt{\mathrm{Tr}_\rho(AA^*)}$

- Measurement strategy replacement

Example I: Mermin-Peres magic square game

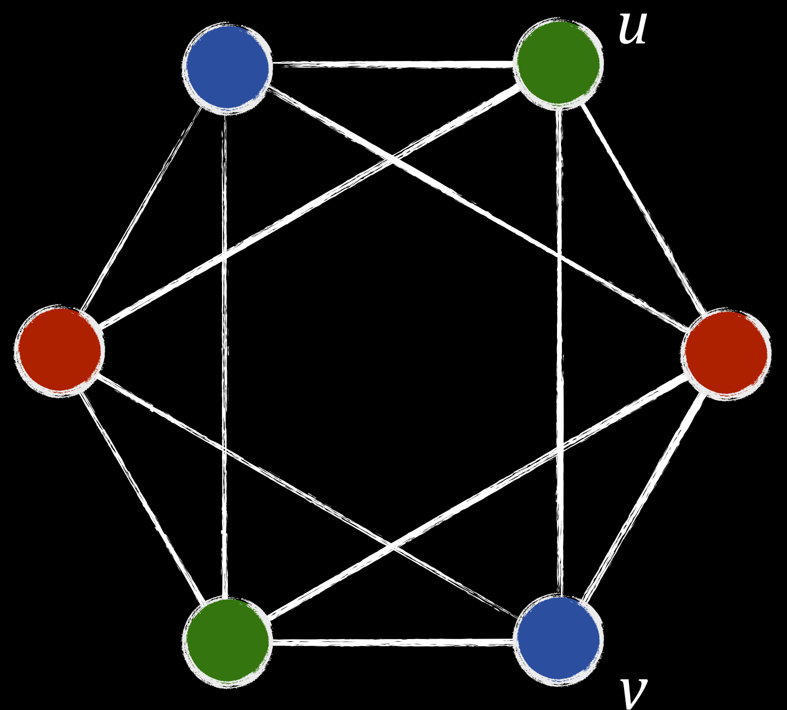
- Sample and send constraint-variable pair
- Check
  - Constraint
  - Consistency
- Magic: $1 = \omega^* > \omega$, 2 EPR pairs
- Binary constraint system games [CM '12]
- An instance of 3-SAT with 24 clauses
- 3-SAT*

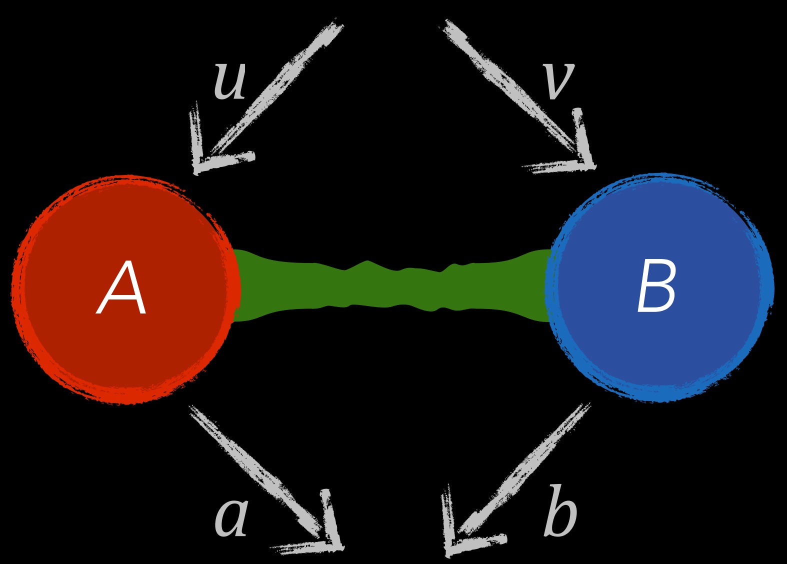




Example 2: Quantum 3-coloring game

- Sample and send vertices u, v to A and B respectively
- Check
  - $a = b$ if $u = v$, and
  - $a \neq b$ if u, v are adjacent
- ∃ graph G, $1 = \omega^*(G) > \omega(G)$
- 3-COLORING*
- Entanglement undermines soundness
- A bug or a feature?





Entanglement undermines soundness

- Two-player XOR games

  Tsirelson's theorem: $\langle\phi|X_s \otimes Y_t|\phi\rangle = x_s \cdot y_t$

  ⊕MIP*(2,1) ⊆ EXP [CHTW '04]

  ⊕MIP*(2,1) ⊆ QIP(2) ⊆ PSPACE [Weh '06] [JUW '09]

  ⊕MIP (2,1) = NEXP [Hås '01]

- Unique Games with Entangled Provers are Easy

  "Quantum rounding" of SDP from UGC [KRT '08]

- Unfixable bug…

Entanglement resistant techniques

- Consistency check
- Confusion check
- A third player
  - Bob’
  - 2-out-of-3
- PIR, NP ⊆ ⊕MIP*(2) [CGJ ’09]

Consistency check

- Send each player the same question q and expect the same answers
- 2-player consistency check
  - Quantum 3-coloring game
- 3-player consistency check
  - Linearity test and multilinearity test [IV '12]
  - PCP simulation test [IKP+ '08]

Consistency as a measure of “closeness”

- For two measurements A and B, define

$$\mathrm{CONS}(A,B) = \sum_a \mathrm{Tr}_\rho(A^a \otimes B^a)$$
$$\mathrm{INC}(A,B) = 1 - \mathrm{CONS}(A,B)$$

- Inconsistency as a “distance” of measurements

$$\sum_a \|A^a - B^a\|_\rho^2 \leq O(\sqrt{\mathrm{INC}(A,B)})$$

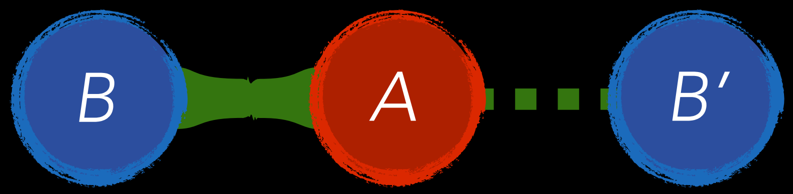
Confusion check

- Sample two questions *q*, *q'*. Send the unordered pair *q, q'* to *A* and *q* to *B*
- Used to prove NP-hardness of computing ω* to inverse polynomial precision [IKM '09]
- Lemma
$$\mathrm{CONF}(A,B) = \sum_{a,a'} \mathrm{Tr}_\rho(A_{q,q'}^{a,a'} \otimes B_q^a)$$

$$\mathrm{CONF}(A,B) \geq 1-\epsilon \implies \mathbb{E}_{q,q'} \sum_{a,a'} \left\| [B_q^a, B_{q'}^{a'}] \right\|_\rho^2 \leq O(\epsilon)$$

A third player

- Monogamy of entanglement



- Bob’ construction
  - NP-hardness of 3-player games [KKM+ '08]
  - Effect on the magic square game
- 2-out-of-3
  - Used with low degree test in [Vid '13]

NP-hardness of exact computation of ω*

- It is NP-hard to distinguish
  - $\omega^* = 1$ and
  - $\omega^* \leq 1 - O(1/n^c)$ [KKM+ '08] [IKM '09]

- State invariant lemma with Bob'

$$\left\|\sum_a \sqrt{B_q^a}\rho_{AB}\sqrt{B_q^a} - \rho_{AB}\right\|_1 = O(\sqrt{\mathrm{INC}(B_q)})$$

- Sequential measurement rounding
- Bad soundness

MIP = NEXP $\subseteq$ MIP* [IV '12]

- Entangled provers are at least as expressive as their classical counterpart
- Any MIP protocol can be modified immune to entanglement
- Bug fixed for once and for all
- The best one can hope for using the entanglement resistant techniques

What to prove?

- Follows the proof of NEXP $\subseteq$ MIP of [BFL '91]
- Multilinearity test is sound against entangled provers
  - Consistency test
  - Multilinearity test (axis aligned linearity test)
- Classically: provers act according to a common multilinear function

What to prove?

- What is the right thing to prove in the quantum setting?

Theorem. Suppose that the strategy passes both the consistency test and multilinearity test with probability $1-\epsilon$, then there exists POVM $\{V^g\}$ such that

$$\mathbb{E}_{\boldsymbol{x}}\Big[\mathrm{INC}(A_{\boldsymbol{x}}, V_{\boldsymbol{x}})\Big] = O(\epsilon^c),$$

where $V_{\boldsymbol{x}}^a = \sum_{g: g(\boldsymbol{x})=a} V^g.$

Proof outline

- Remove the dependence on x_i one by one by induction
- Error (in terms of inconsistency) grows exponentially. Need an (active) consolidation step using SDPs
- Pasting lemma + consolidation (self-improvement) lemma
- The base step of the induction

The base step

- The statement $\boldsymbol{x} \in \mathbb{F}^n \quad \boldsymbol{x}' = x_i', \boldsymbol{x}_{\neg i}$

$$\exists \quad \{B^l_{\boldsymbol{x}_{\neg i}}\} \qquad B^a_{\boldsymbol{x}} = \sum_{l : l(x_i) = a} B^l_{\boldsymbol{x}_{\neg i}}$$

$$\mathbb{E}_{\boldsymbol{x}}\Big[\mathrm{INC}(A_{\boldsymbol{x}}, B_{\boldsymbol{x}})\Big] \leq O(\sqrt{\epsilon})$$

- Construction of the *B* measurement

$$B^l_{\boldsymbol{x}_{\neg i}} \stackrel{\text{def}}{=} \mathbb{E}_{x_i \neq x_i'} A^{l(x_i)}_{\boldsymbol{x}} A^{l(x_i')}_{\boldsymbol{x}'} A^{l(x_i)}_{\boldsymbol{x}}$$

Details

$$
\begin{aligned}
& \mathbb{E}_{\boldsymbol{x}}\Big[\mathrm{CONS}(A_{\boldsymbol{x}}, B_{\boldsymbol{x}})\Big] \\
= \quad & \mathbb{E}_{\boldsymbol{x}, x_i' \neq x_i''} \sum_{a, l : l(x_i) = a} \sum_{a'} \mathrm{Tr}_\rho(A_{\boldsymbol{x}}^a \otimes A_{\boldsymbol{x}'}^{l(x_i')} A_{\boldsymbol{x}''}^{l(x_i'')} A_{\boldsymbol{x}'}^{l(x_i')} \otimes A_{\boldsymbol{x}'}^{a'}) \\
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Upper bounds?

- Nothing known
- Possible approaches
  - Random projections?
  - Non-commutative Positivestellensatz [DLTW '08]

$$\longrightarrow \omega^* \stackrel{?}{=} \omega^f \longleftarrow$$ SDP Hierarchy

  - Connes' embedding problem and Tsirelson's problem

[JNP+ '11] [Fri '12]

Binary Constraint System Games

- The bug vs. feature question

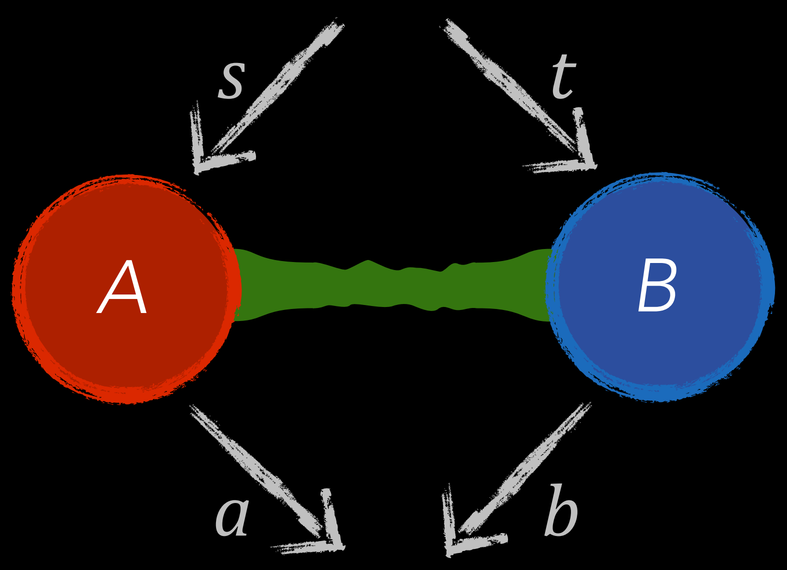
- Exact case characterization

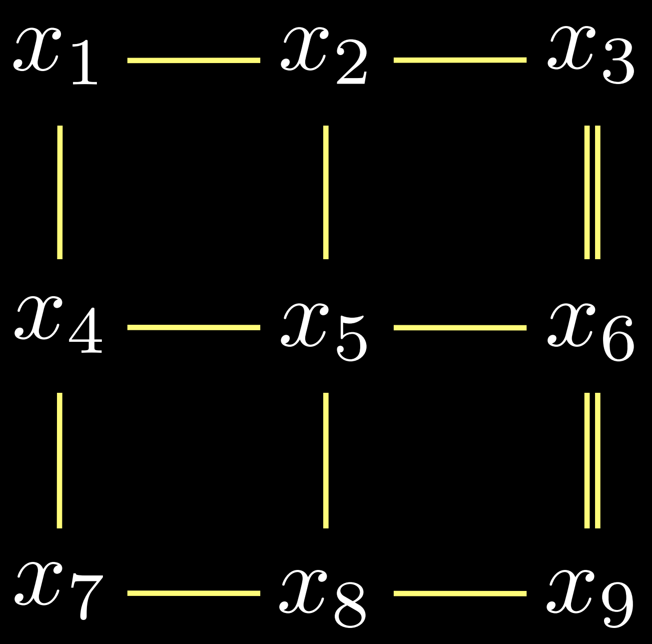
A BCS game has a perfect quantum strategy

if and only if

the corresponding BCS has a quantum satisfying assignment

[CM '12, ARXIV:1209.2729]





Quantum satisfying assignment

- Rewrite constraints as polynomials over reals

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$$x_1 \oplus x_2 = 0, \quad x_1 \oplus x_2 = 1. \quad \rightarrow \quad x_1 + x_2 - 2x_1x_2 = 0, \quad x_1 + x_2 - 1 = 0.$$

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Quantum Satisfying Assignment $x_j \mapsto X_j$

(a) Satisfy every polynomial constraints.

(b) For all j, $X_j^2 = X_j$.

(c) Each pair of operators X_j, X_k appearing in the same constraint commute.

Quantum satisfying assignment

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Locally Commutative Condition

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Quantum Satisfiability

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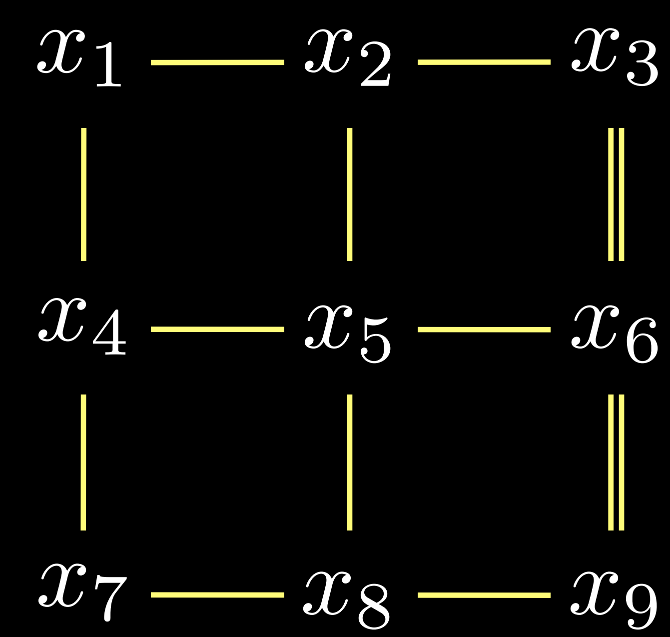
Locally Commutative Condition

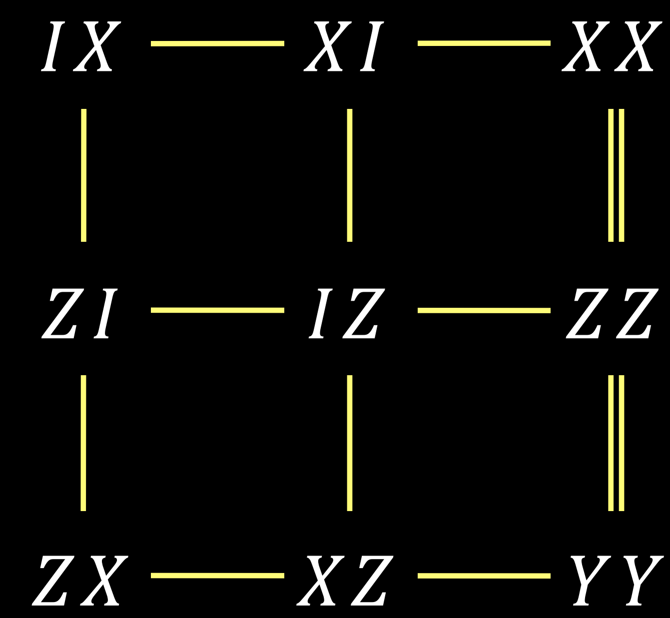
Magic square revisited

- Quantum satisfying assignment for magic square

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

- Anti-commutativity gadget
- Glue magic squares together
- Add a trivial constraint $f(x_2, x_4) \equiv 1$
- 3-SAT* with such trivial constraints





Reductions of *-problems

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Theorem. 3-SAT* is Karp reducible to 3-COLORING*.

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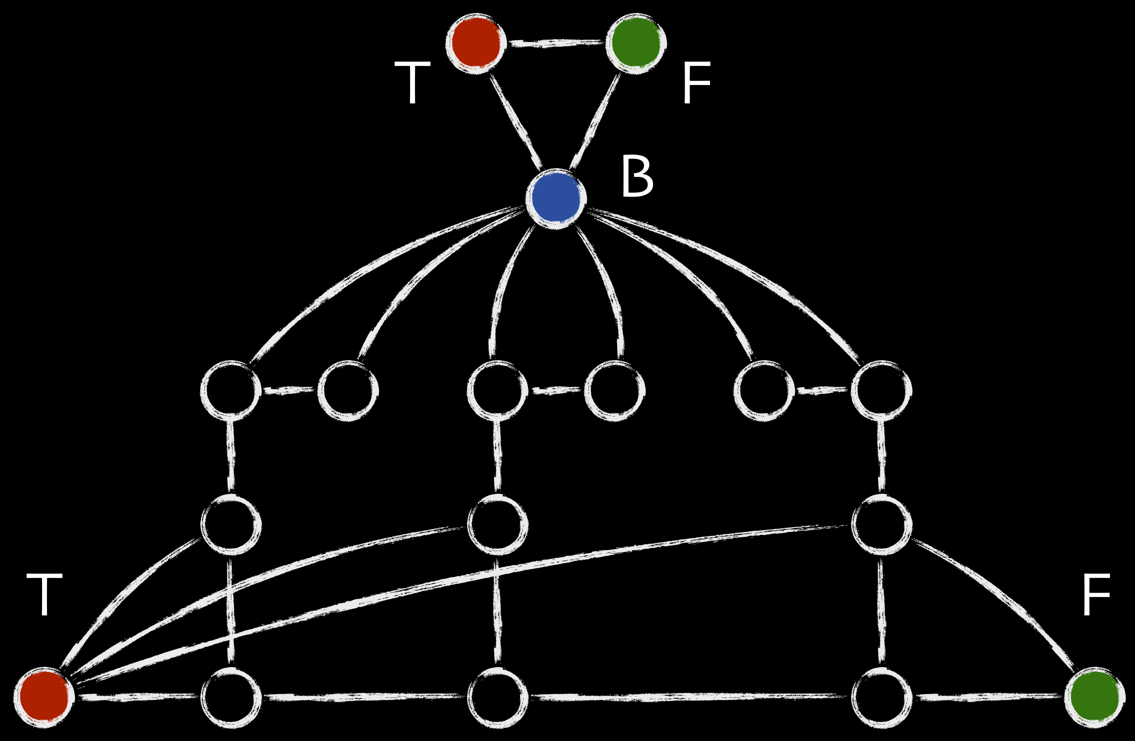
$\bigwedge_{j=1}^{m} C_j$

$x_1, x_2, x_3, \ldots$

$x_1 \vee x_2 \vee \neg x_3$



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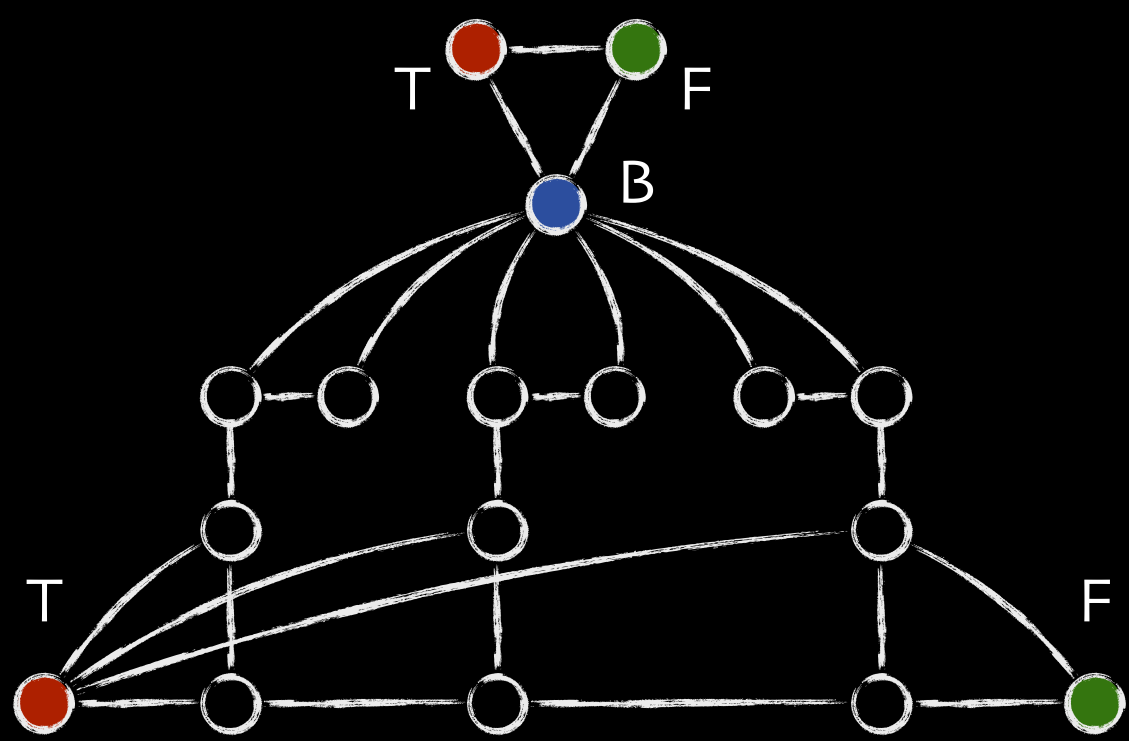


$X_1, X_2, X_3, \ldots$ ⟶ Coloring measurements?

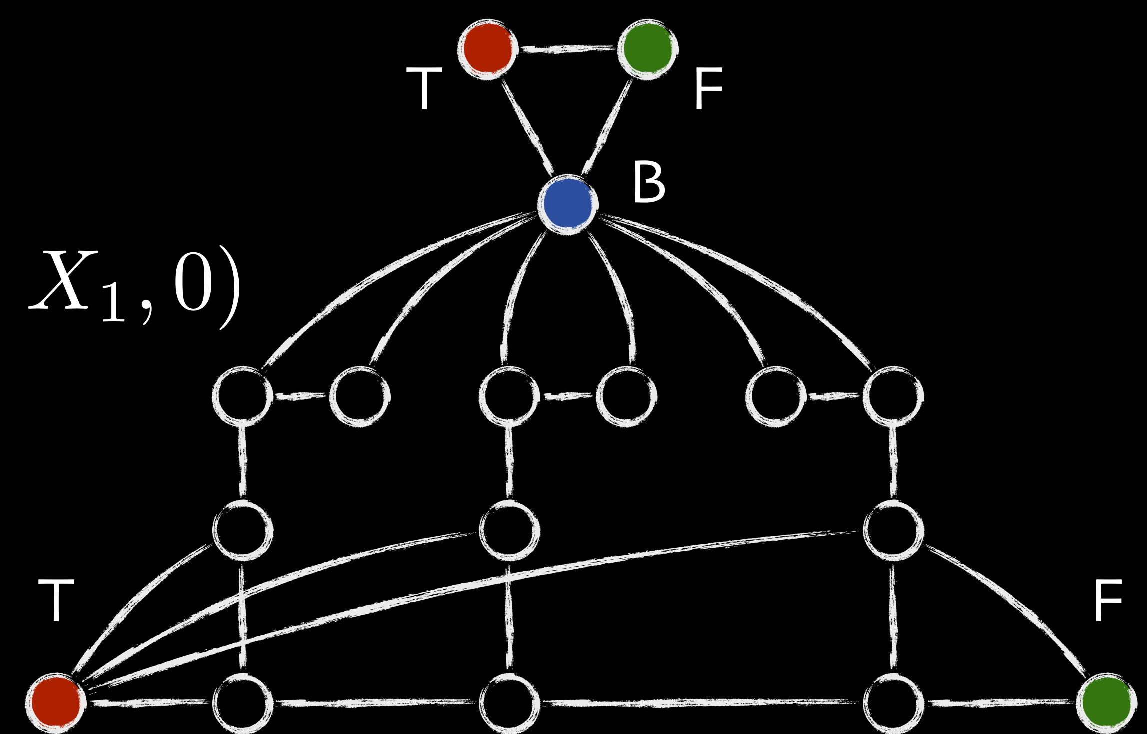
Reductions of $*$-problems

Theorem. 3-SAT* is Karp reducible to 3-COLORING*.

$\bigwedge_{j=1}^{m} C_j$

$(X_1, I - X_1, 0)$

$x_1, x_2, x_3, \ldots$



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$X_1, X_2, X_3, \ldots \longrightarrow$ Coloring measurements?

Reductions of *-problems

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$\bigwedge_{j=1}^{m} C_j$

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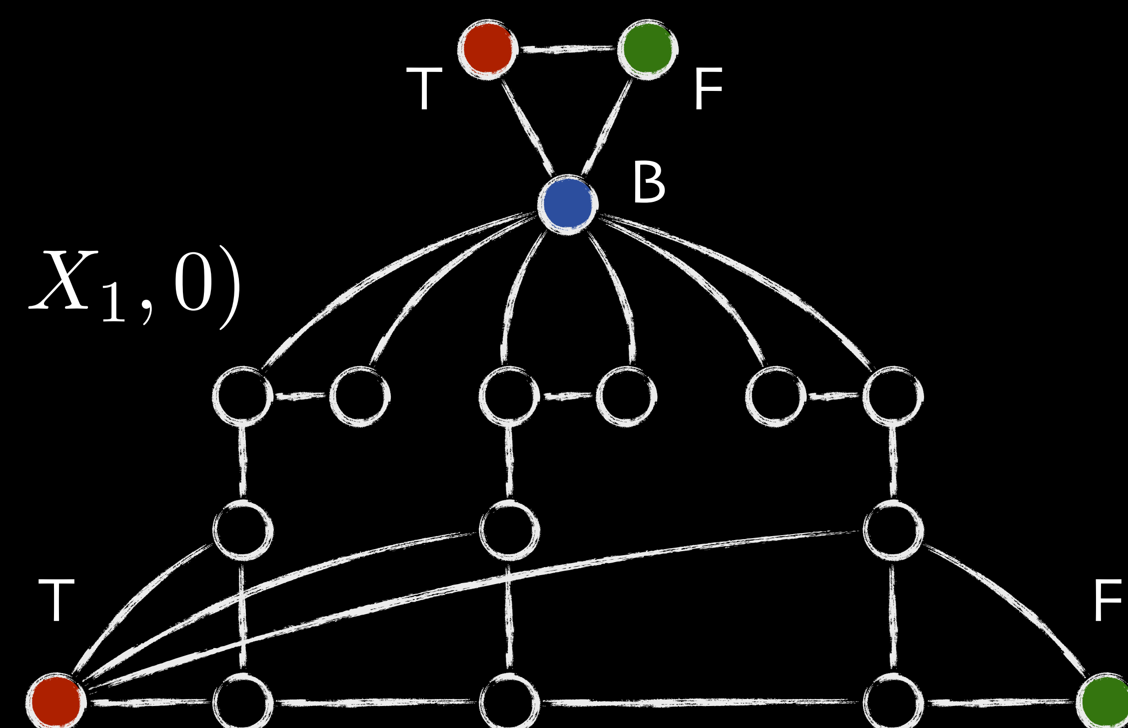
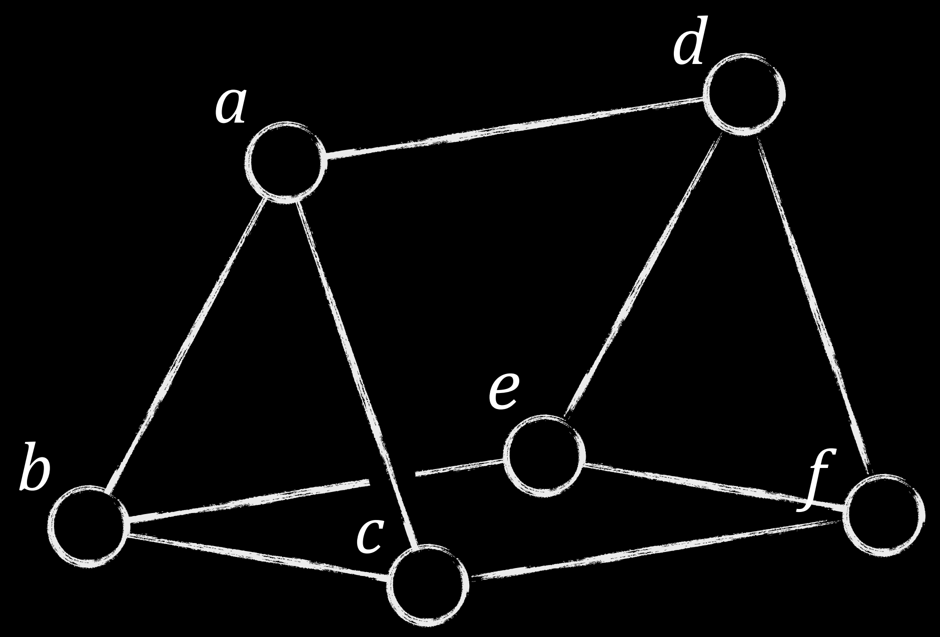


$x_1 \vee x_2 \vee \neg x_3$

$X_1, X_2, X_3, \ldots$ ← Coloring measurements?

Triangular prism gadget

Lemma. The only constraint on the coloring operators of vertices a and e in the gadget is that they commute.



1. Commutativity
2. Extendibility

Commutativity gadget

Proof idea: Commutator is in the ideal generated by the constraints. Non-commutative Grobner basis.

Triangular prism gadget

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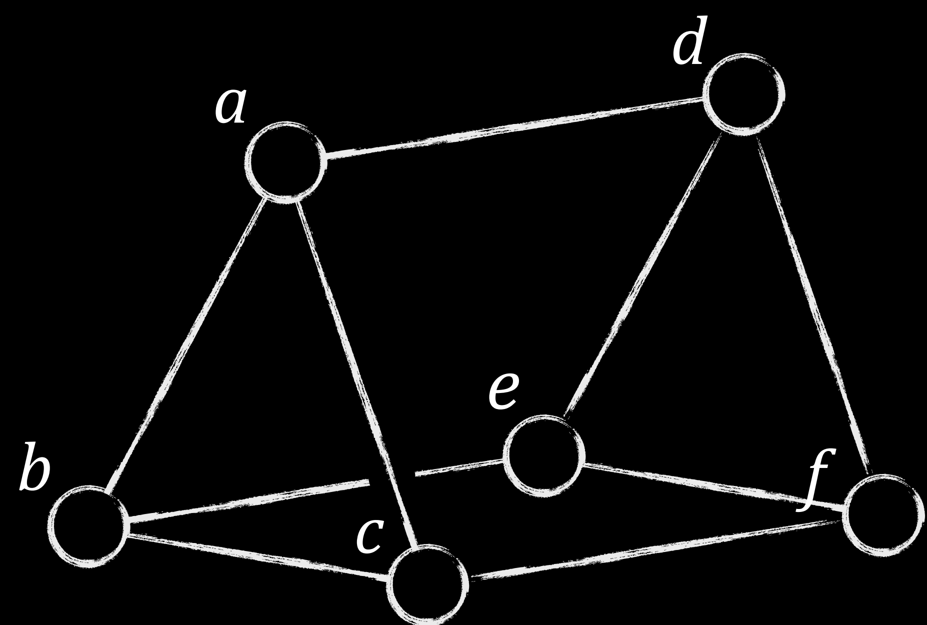
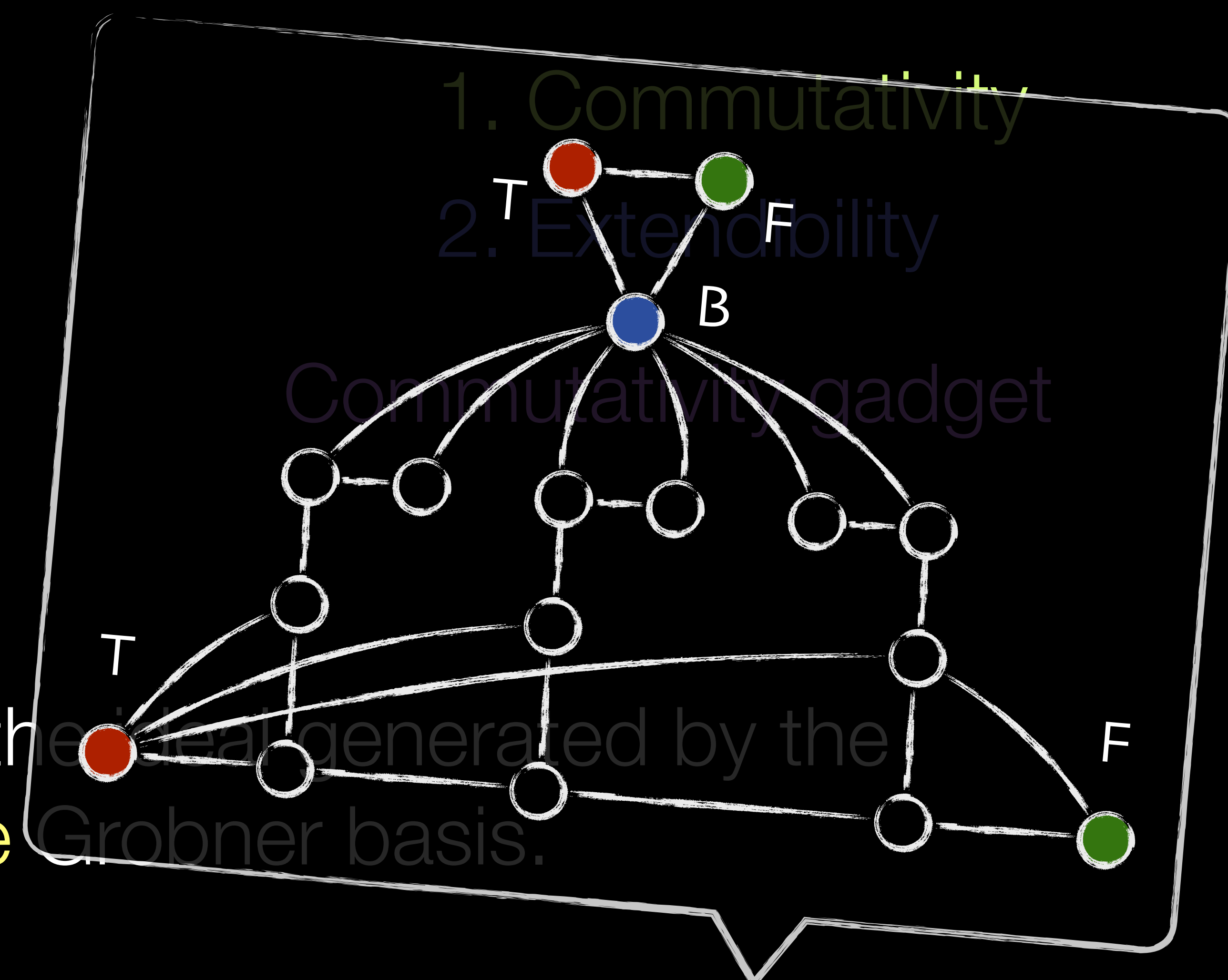


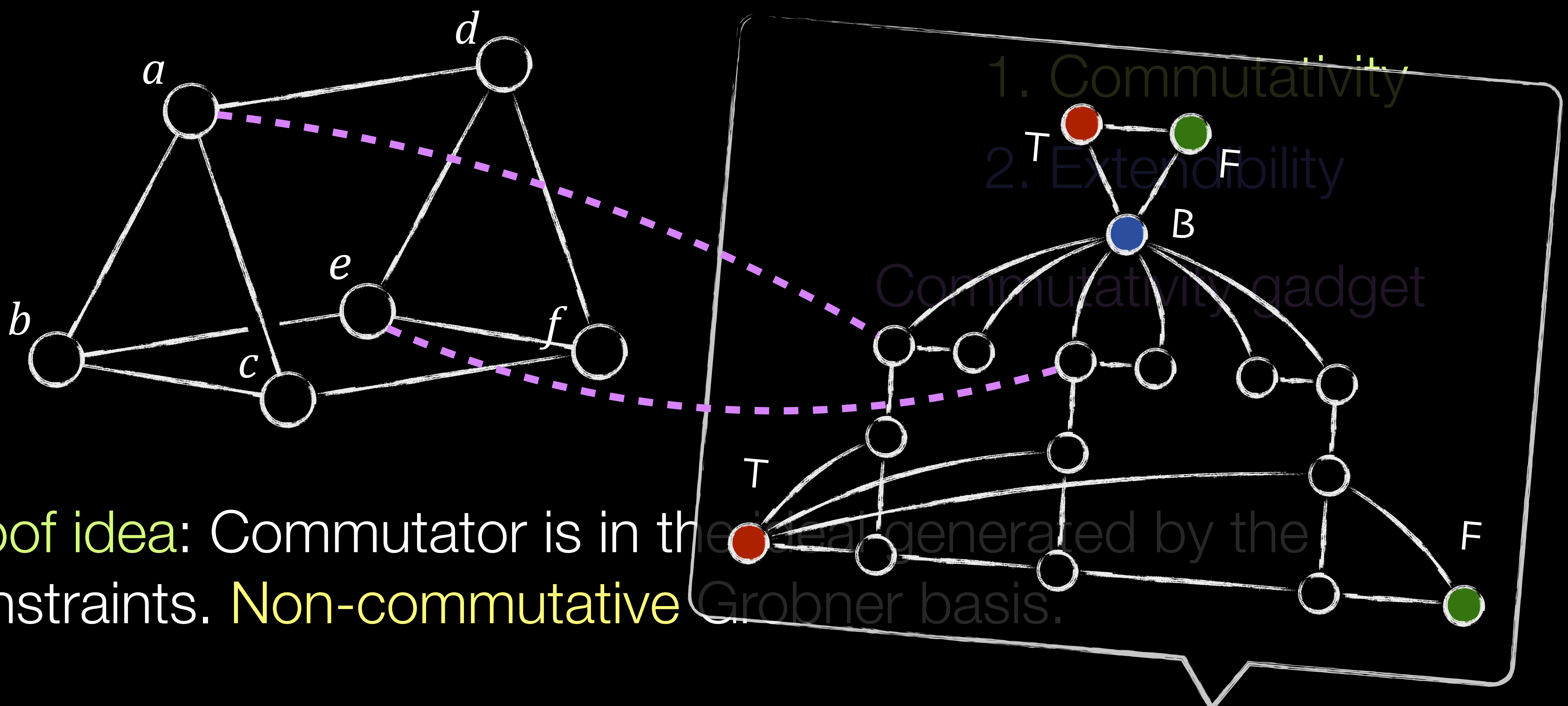


Proof idea: Commutator is in the ideal generated by the constraints. Non-commutative Grobner basis.

Identify a and e with vertices in the classical gadget.

Triangular prism gadget

Lemma. The only constraint on the coloring operators of vertices a and e in the gadget is that they commute.



Proof idea: Commutator is in the ideal generated by the constraints. Non-commutative Grobner basis.

Identify a and e with vertices in the classical gadget.

The complexity of 3-SAT*

- NP-hardness of 3-SAT*

  Commutativity gadget $x_1 \vee x_2 \vee y$

- Relation to the confusion check with x_1 and x_2

- 3-SAT* without confusion check is NP-hard (with inverse polynomial gap)

  No dimension bound

- Not known to be decidable

- Relate it to approximate case?

Hardness of the *-problems

- k-SAT*, 1-in-3-SAT*, KOCHEN-SPECKER*, 3-COLORING* and CLIQUE* are as hard as 3-SAT*

A nonlocal NP theory

Schaefer's dichotomy theorem?

- 2-SAT* and HORN-SAT* are in P

- Affine-SAT* or parity BCS games? [Ark '12]

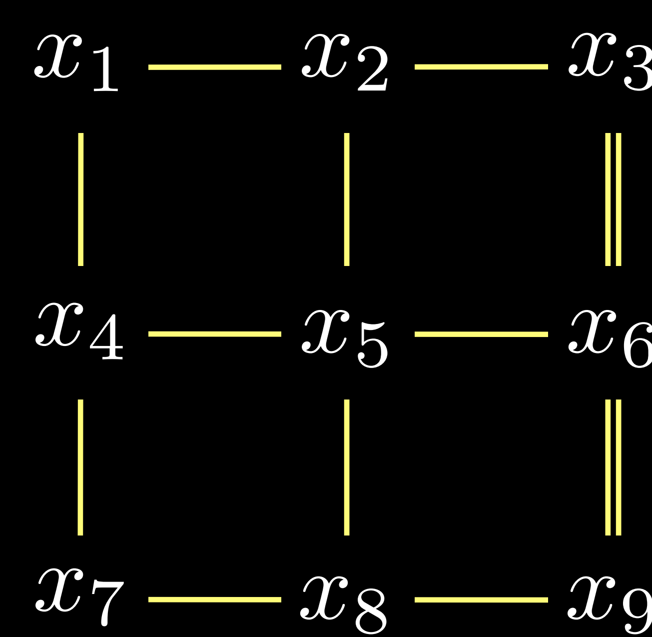
- EPR pairs are optimal for perfect BCS games

Yet another quantum PCP theorem/conjecture?

- Hardness of approximation
  - Constant approximation of ω^* is NP-hard
  - Goal achieved with 3 players [Vid '13]
  - Constant approximation of ω^* is as hard as deciding $\omega^*=1$?
- Nonlocal PCPs? (as non-signalling PCPs)

$$\mathrm{Tr}_\rho(A_q^a \otimes A_{q'}^{a'} \otimes A_{q''}^{a''})$$

- Locally-commutative PCPs?



Open problems

- Upper bound of MIP*
- NEXP in MIP*(2,1)?
- 3-player vs. 2-player
- Power of 2-out-of-3 MIP*?
- BCS related problems
- …