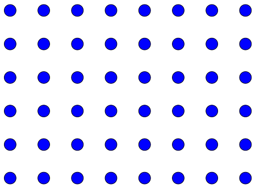
The complexity of ground states

Zeph Landau


Ground State
condensed matter
DMRG
algorithm
structure gap
many body
Area Law
AGSP
entanglement
viable set
Local Hamiltonian

The difficulty of understanding many-body physics



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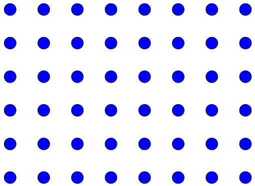

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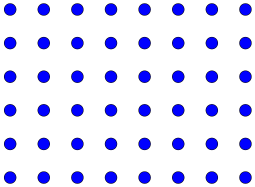

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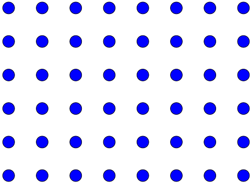

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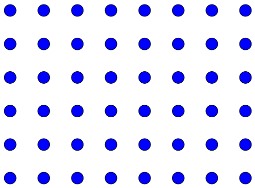

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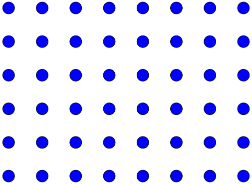

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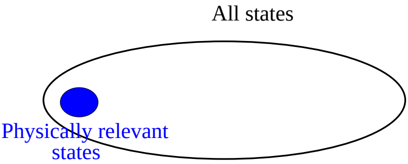
Exponential Dimensional Space

So even describing a state requires exponential amount of information.

A Basic Question



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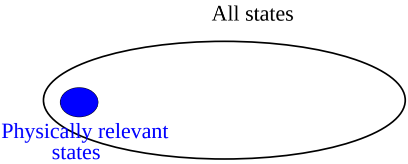

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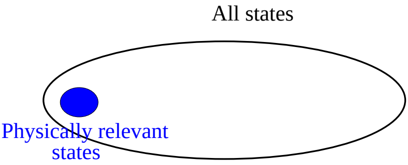

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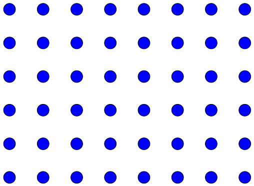

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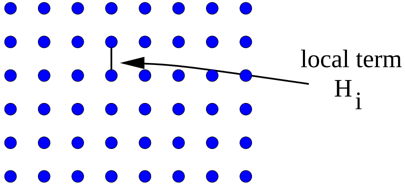

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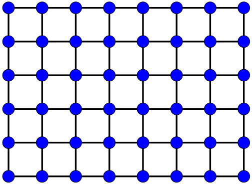

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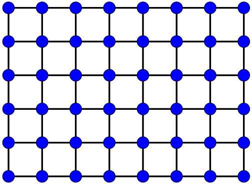

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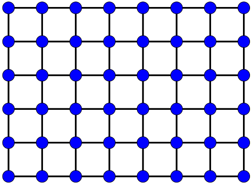

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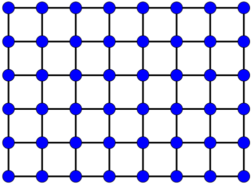

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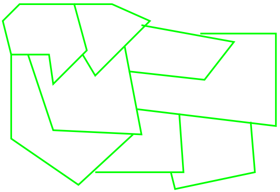
- Focus on unique ground state and constant gap.

Ground states model the state of the system at low temperatures.

Fundamental Connection: Local Hamiltonians and Constraint Satisfaction Problems.

Classical Constraint Satisfaction Problems (CSP's).
Example: 3 colorability



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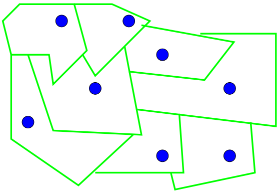

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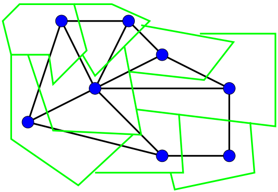

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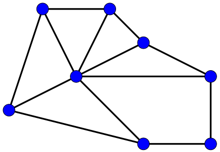

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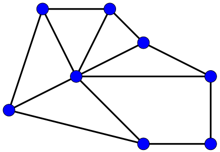

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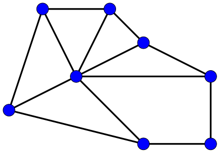

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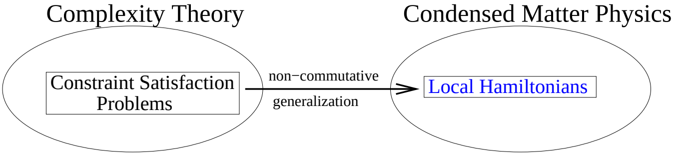
Solving, classifying, and understanding the structure of the solutions of CSP's at the heart of complexity theory.

Local Hamiltonians = non-commutative CSP's



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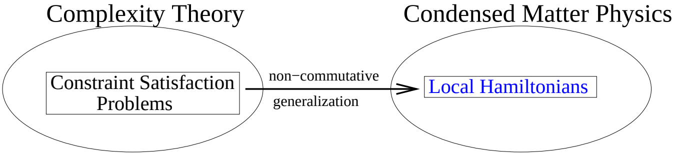

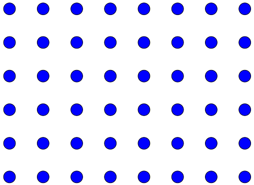

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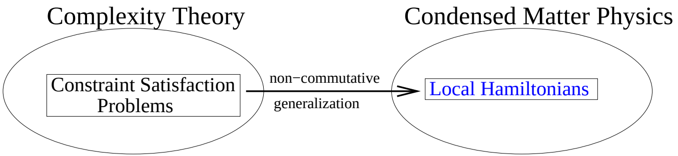

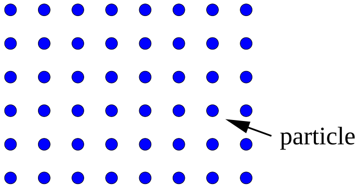

Number of colors $\leftrightarrow$ Dimension of single particle

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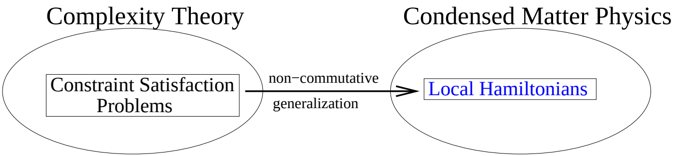

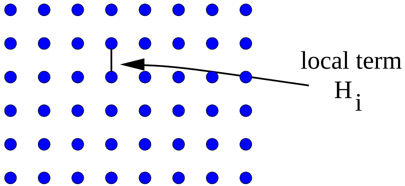

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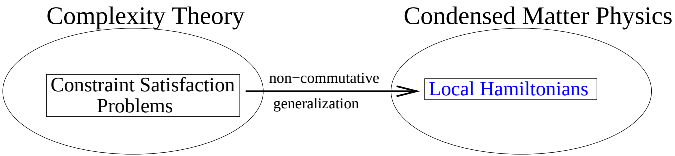

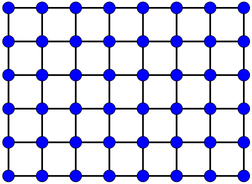

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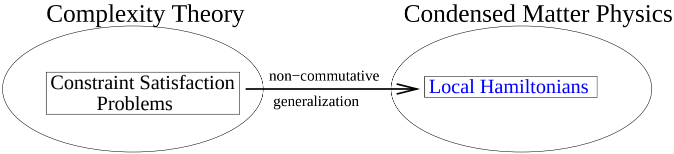

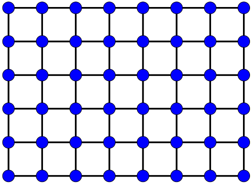

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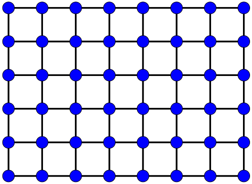
CSP constraints correspond to H_i that are diagonal in the standard basis. In particular they all *commute*.

The Fundamental Quest: understanding ground states



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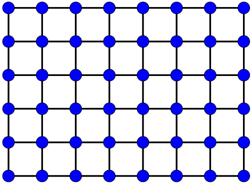

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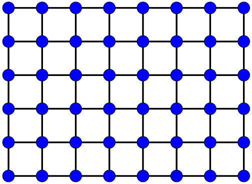

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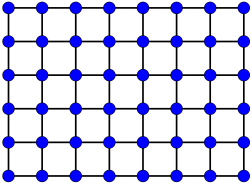

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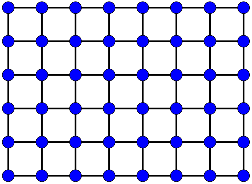

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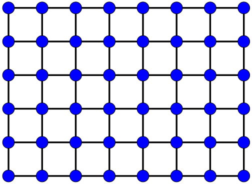

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['07, Aharonov, Gottesman, Irani, Kempe]

- Solutions to 1D systems are also hard.

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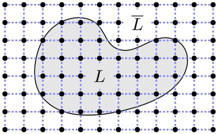

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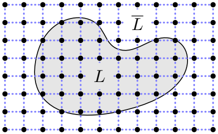

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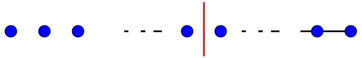
['01, Vidal, Latorre, Rico, Kitaev] Area Law formalized in terms of entanglement entropy.

- Effect on DMRG: speedup, simplification, better understanding of the heuristics used.

Area Law in 1D systems



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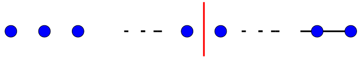

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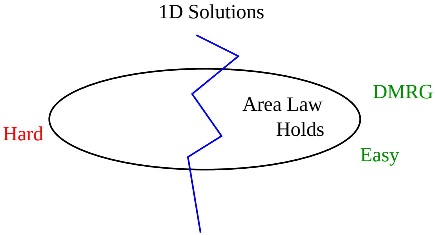

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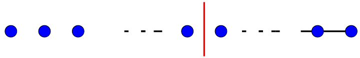

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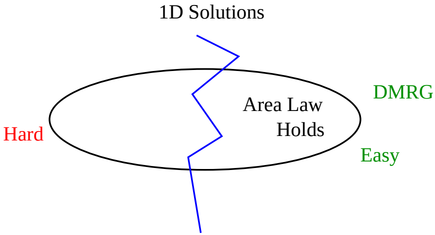

"All is well . . . Area Law structure provides the proper dichotomy between easy and hard in 1D."

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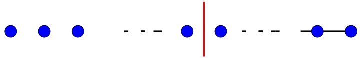

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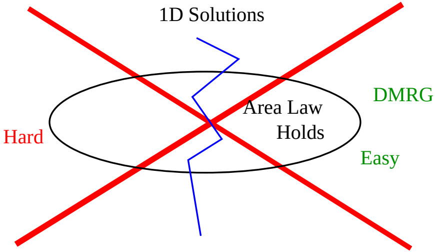

"All is well . . . Area Law structure provides the proper dichotomy between easy and hard in 1D."

['08, Cirac, Schuch, Verstraete] Example of finding a solution that satisfies the area law that is NP-hard.

The birth of Approximate Ground State Projections

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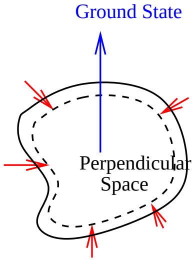

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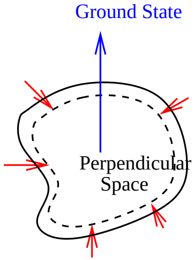

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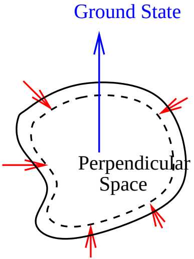

Properties:

- It "approximately" projects onto one vector you want (ground state).
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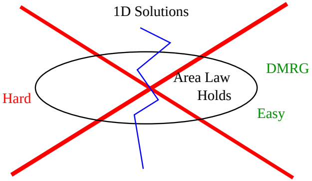
New blood: Approximate Ground State Projections (AGSPs)



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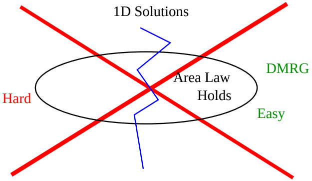

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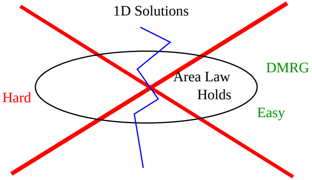

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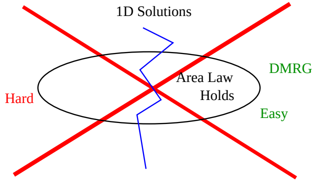

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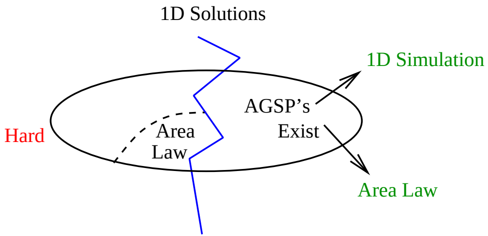
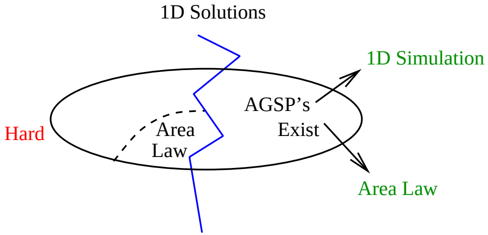


Current view of 1D local Hamiltonians



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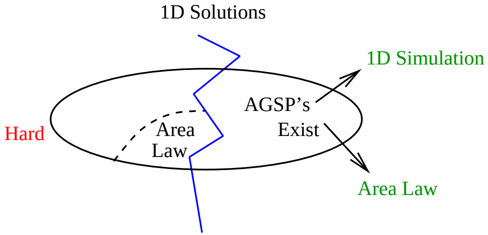


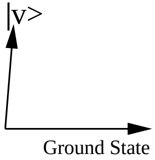
From AGSP's:

- exponential improvement on the constants for the 1D Area Law
- algorithm for 1D,
- gives insight as to what is going on,
- tools for attacking the 2D questions.

Role of AGSP in proof of Area Law I

Two main steps:

1. Find a not very complex state that has constant overlap with the ground state.



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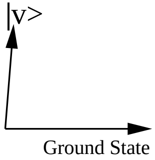

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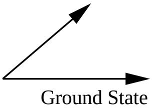

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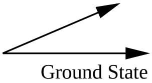

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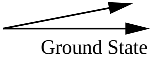

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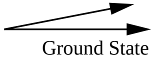

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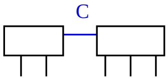
Both steps use AGSPs– the first is much more delicate.

Measure of Complexity: Entanglement rank

A state on $\mathcal{H}_1 \otimes \mathcal{H}_2$ of the form $\sum_1^C a_i \otimes b_i$ will be said to have entanglement rank C.



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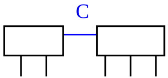

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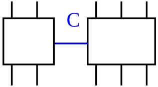

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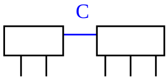

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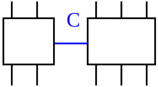

Entanglement rank behavior

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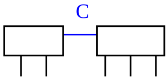

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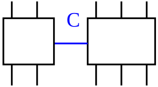

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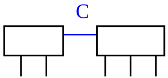

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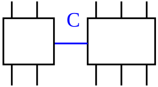

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AGSP: almost projection with small entanglement rank

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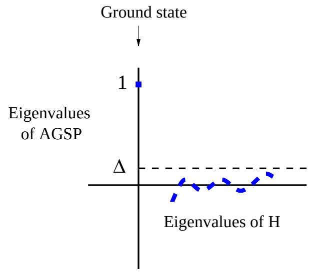

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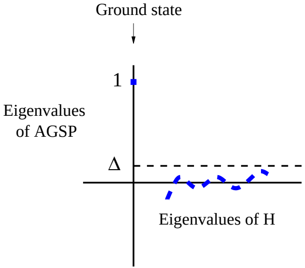

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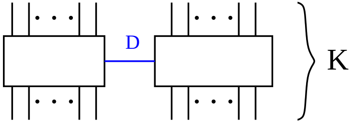

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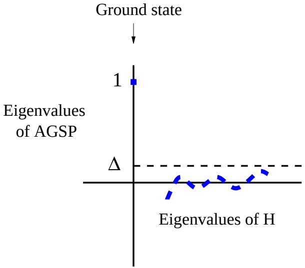

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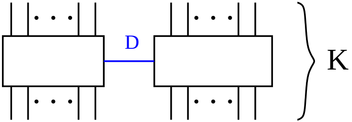

Critical threshold $D\Delta < 1$.

Role of AGSP in proving Area Law cont.

Theorem (Area Law) [Arad, Landau, Vazirani] The existence of an AGSP K for which $D\Delta < 1/2$ proves that the ground state has entropy $O(1)\log D$.

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Finding the ground state of 1D systems: solving a large convex program

finding the minimal energy state

$$\Updownarrow$$

solving a convex program

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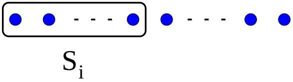
- A succinct description of the elements of that subspace that allows us to perform linear algebra efficiently.

The algorithm: a bird's eye view

A sequence of spaces S_i termed viable sets:

- all polynomial size
- all with succinct descriptions that allow efficient linear algebra,
- each containing a good approximation of the "left" side of the ground state.



$$|\Gamma\rangle \approx \sum_j |a_j\rangle |b_j\rangle \text{ with each } |a_j\rangle \in S_i.$$

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Key components:

- **Splitting**: 1D structure allows reduction of convex program to left side by iterating over a net of boundary conditions. Structural result allows this net to be fixed polynomial size.

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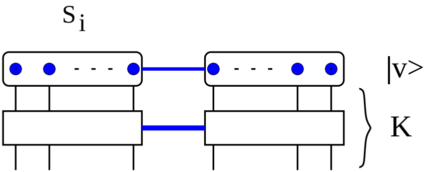

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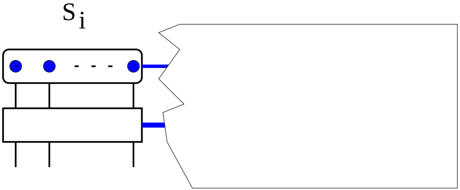

Arxiv, find me, later workshop.

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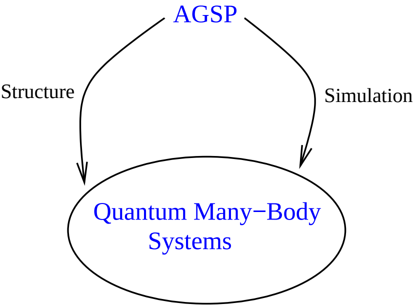

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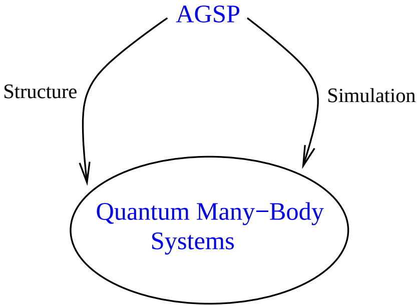

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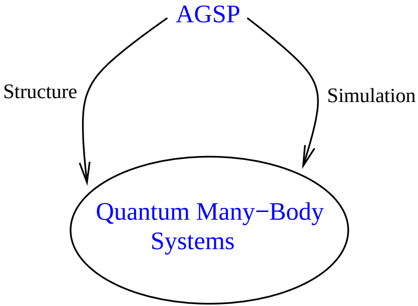

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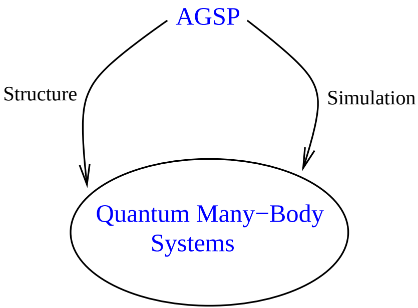

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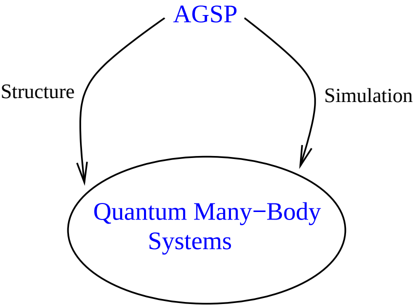

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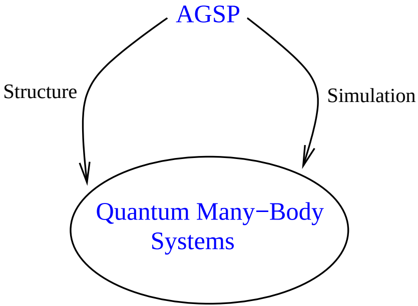

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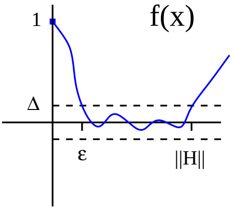
- Degenerate ground space?
- Different questions?

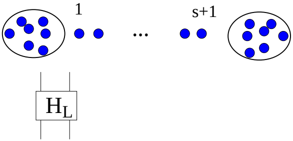
"The future ain't what it used to be." – Yogi Berra

AGSP construction: norm reduction

Looking for low entanglement operators that look like:

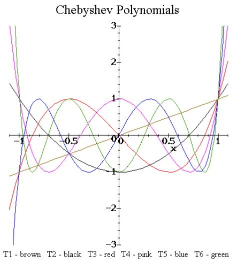


Smaller $||H||$ would be better but we don't want to lost the local structure around the cut.
Solution: Replace $H = \sum_i H_i$ with $H' = H_L + H_1 + H_2 + \cdots + H_s + H_R$.

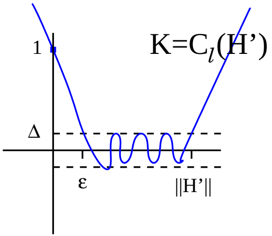


AGSP construction: Chebyshev polynomials

Chebyshev polynomials: small in an interval:



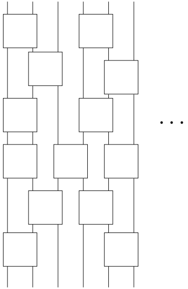
The desired AGSP is a dilation and translation of the Chebyshev polynomial:



AGSP complexity: Entanglement rank analysis

$$(H')^{\ell} = \sum (\text{ product of } H_j).$$

For a single term:

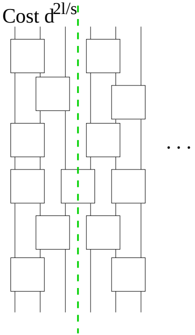


AGSP complexity: Entanglement rank analysis

$$(H')^\ell = \sum (\text{ product of } H_j).$$

For a single term:

- Across some cut, an average number of terms are involved $\rightarrow d^{2\ell/s}$.



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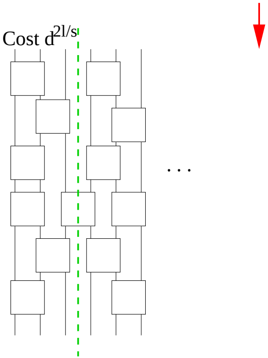

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$$(H')^{\ell} = \sum (\text{ product of } H_j).$$

For a single term:

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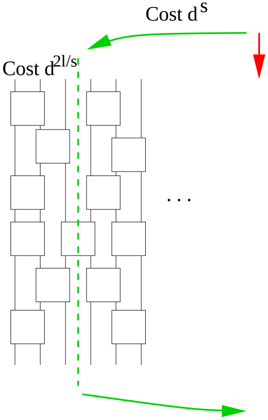
- Roundtrip cost of going and coming back from center cut: $\rightarrow d^{s}$.



AGSP complexity: Entanglement rank analysis

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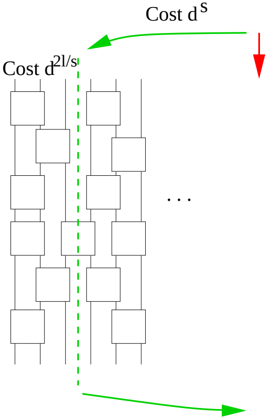
Total: $d^{2\ell/s+s}$



AGSP complexity: Entanglement rank analysis

Problem: Too many (s^{ℓ}) terms in naive expansion of $(H')^{\ell}$.

AGSP complexity: Entanglement rank analysis

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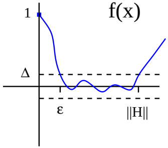
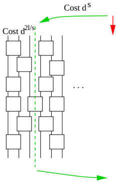
Need to group terms in a nice way but it all works out with total entanglement increase of the same order as the single term.

Putting things together: Area Law for H'

Chebyshev $C_\ell(H')$ has $\Delta \approx e^{-O(\ell/\sqrt{s})}$:



Entanglement analysis yields $D \approx O(d^{\ell/s+s})$.



Chosing $\ell = s^2$ yields $D\Delta \approx e^{-s^{3/2}+s\log d} < 1$ for appropriate choice of $s \approx \log^2 d$.