Geometry of Polynomials Bootcamp **Winter 2019**

Ramanujan graphs and interlacing polynomials

Speaker: Jan Vondrak *In-class exercise*

P1) Prove the Gutman-Godsil theorem: If $\chi_s(x) = det(xI - A_s)$ where A_s is the signed adjacency matrix of G, then

$$\mathbb{E}_{s\in\{\pm 1\}^E}[\chi_s(x)] = \sum_{\text{matching } M} (-1)^{|M|} x^{n-2|M|}.$$

P2) Prove that if f and g have a common interlacing, then $h_t(x) = tf(x) + (1-t)g(x)$ is real-rooted (even in the case where f, g have some multiple roots).

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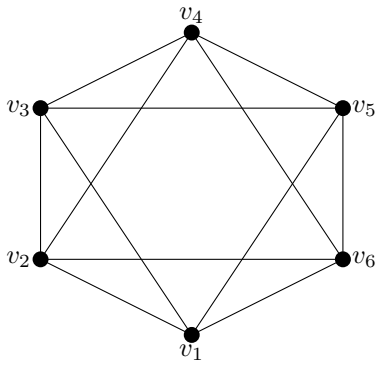
Speaker: Jan Vondrak *Homeworks*

P1) Compute the eigenvalues of the following graph. Is it Ramanujan?



P2) Compute the matching polynomials for K_2, K_3, K_4 and find the recursive formula for K_n. These are also known as Hermite polynomials.

P3) Compute the matching polynomials for $K_{1,1}, K_{2,2}, K_{3,3}$ and find the recursive formula for $K_{n,n}$. These are also known as Laguerre polynomials.