Stochastic Gradient Descent with Differential Privacy

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December 21, 2013

Efficient methods for large-scale learning

Large-scale inference problems face several challenges

- Hard to fit a single data set on one machine.
- Data acquired online, or sequentially.
- Data may come with constraints (privacy, duh).

Optimization over a data set

Consider the problem of solving a minimization problem over a data set $\{\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_n\}$ of n data points in $\mathbb{R}^d$:

$$\operatorname*{argmin}_{\mathbf{w} \in \mathbb{R}^d} \frac{1}{n} \sum_{i=1}^{n} \ell(\mathbf{w}, \mathbf{z}_i),$$

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As an example for this talk, $\mathbf{z}_i = (\mathbf{x}_i, y_i)$, where $\mathbf{x} \in \mathbb{R}^d$ is data for individual i and $y_i \in \{-1, 1\}$ is a label:

$$\mathbf{w}^* = \underset{w \in \mathbb{R}^d}{\operatorname{argmin}} \frac{\lambda}{2} \|\mathbf{w}\|^2 + \frac{1}{n} \sum_{i=1}^{n} \log(1 + e^{-y_t \mathbf{w}^\top \mathbf{x}_t})$$

this is *regularized logistic regression*.

Computational issues

Regularized logistic regression:

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Often solve this problem using *gradient descent*. When n is large we have to compute n gradients for each data point.

Stochastic gradients

A popular method for optimization in this setting is *stochastic gradient descent* (SGD). At each time step $t = 1, 2, \ldots$, sample a point $(\mathbf{x}_t, y_t)$ uniformly from the data set:

$$\mathbf{w}_{t+1} = \mathbf{w}_t - \eta_t(\lambda \mathbf{w}_t + \nabla \ell(\mathbf{w}_t, \mathbf{x}_t, y_t))$$

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- In practice, just sample without replacement.
- Taking a few passes through the data typically works well.

Privacy?

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- Data is still a *limited resource*.
- Now we want to balance privacy, utility, and efficiency.
- Maybe easy? Iterations are already random.

Privacy!

It is easy to guarantee differential privacy in this setting:

$$\mathbf{w}_{t+1} = \mathbf{w}_t - \eta_t \left(\lambda \mathbf{w}_t + \nabla \ell(\mathbf{w}_t, \mathbf{x}_t, y_t) + \mathbf{Z}_t \right),$$

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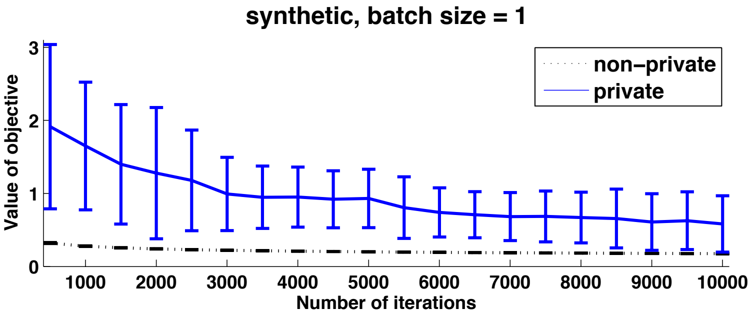
This guarantees differentially privacy in the *continuous observation setting* (Dwork, Naor, Pitassi, and Rothblum '10) or the *local privacy model* (c.f. Duchi, Jordan, and Wainwright '12).

... but not off the shelf



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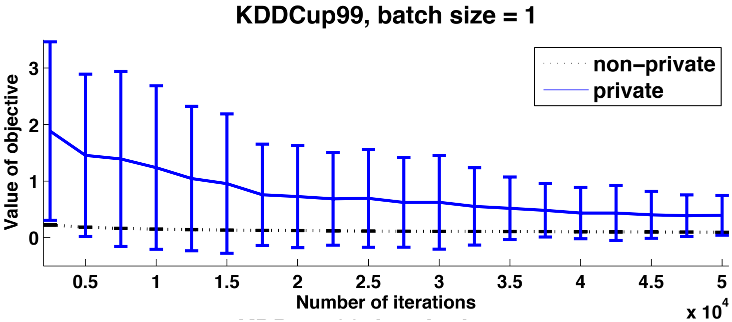

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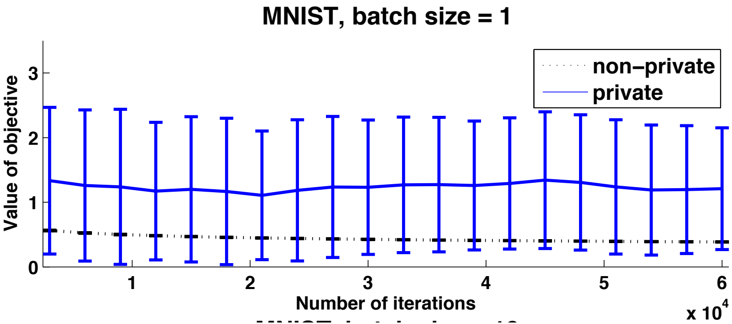

Lots of knobs to tweak

$$\mathbf{w}_{t+1} = \mathbf{w}_t - \eta_t \left(\lambda \mathbf{w}_t + \nabla \ell(\mathbf{w}_t, \mathbf{x}_t, y_t) + \mathbf{Z}_t \right),$$

There are lots of things to play with here:

- Step size: choosing it is sort of an art
- Noise level vs. multiple passes
- Polyak averaging
- **Minibatching**: processing multiple points at once

Minibatching

Process *a batch of* b *points* chosen uniformly at random. At each t select a set B_t:

$$\mathbf{w}_{t+1} = \mathbf{w}_t - \eta_t \left(\lambda \mathbf{w}_t + \frac{1}{b} \sum_{(\mathbf{x}_i, y_i) \in B_t} \nabla \ell(\mathbf{w}_t, \mathbf{x}_i, y_i) \right)$$

With privacy:

$$\mathbf{w}_{t+1} = \mathbf{w}_t - \eta_t \left(\lambda \mathbf{w}_t + \frac{1}{b} \sum_{(\mathbf{x}_i, y_i) \in B_t} \nabla \ell(\mathbf{w}_t, \mathbf{x}_i, y_i) + \frac{1}{b} \mathbf{Z}_t \right).$$

Note: we've given up on local privacy here.

The minibatching tradeoff

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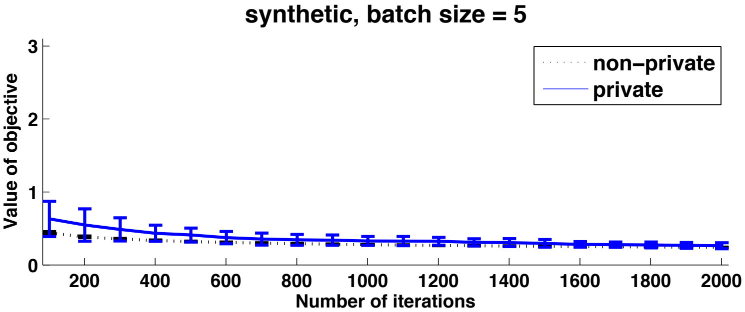
If we make 1 pass through the data, we now have a new tradeoff:

- Many gradient steps that are very noisy.
- Fewer gradient steps that are less noisy.

Empirical performance of minibatching



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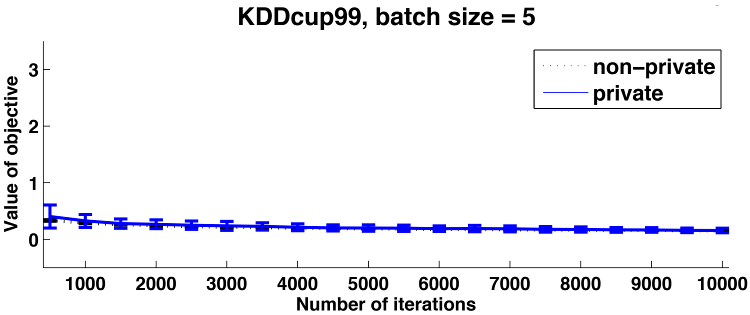

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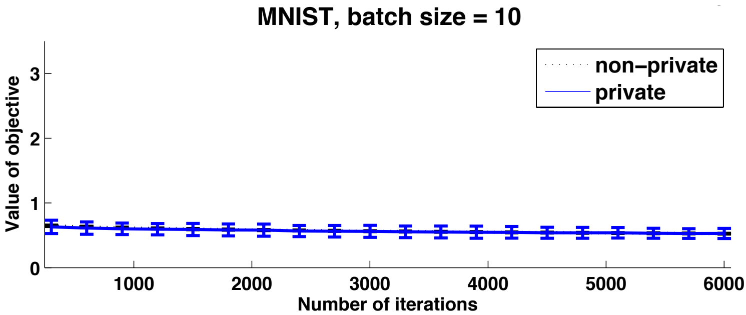

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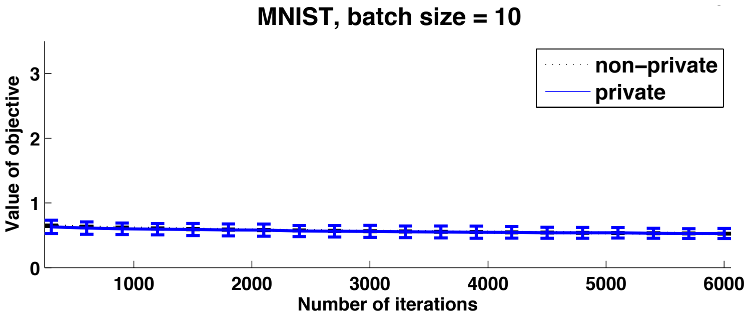
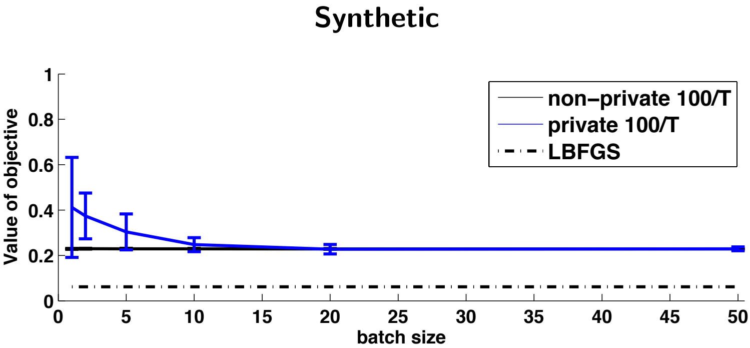


Minibatching helps a lot!

What's the best batch size?



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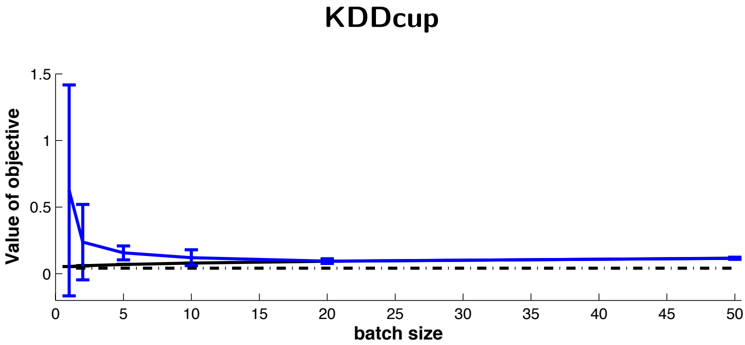

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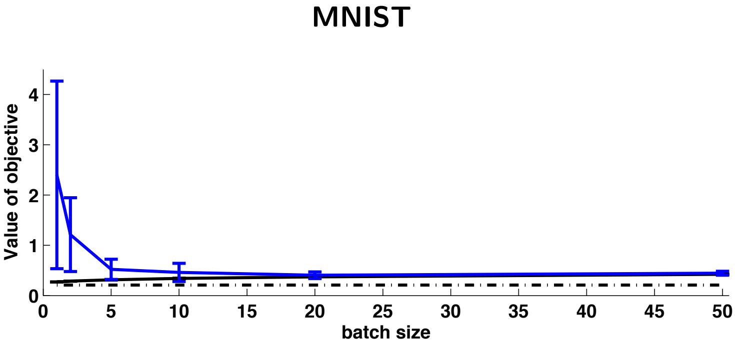

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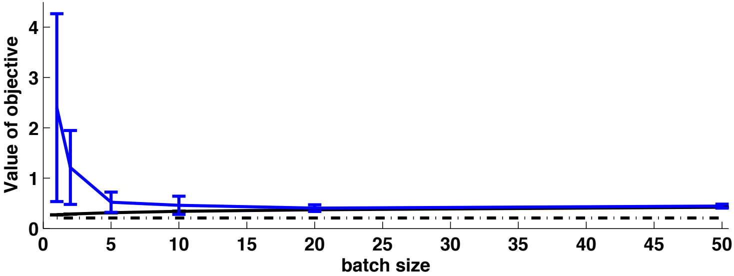
MNIST



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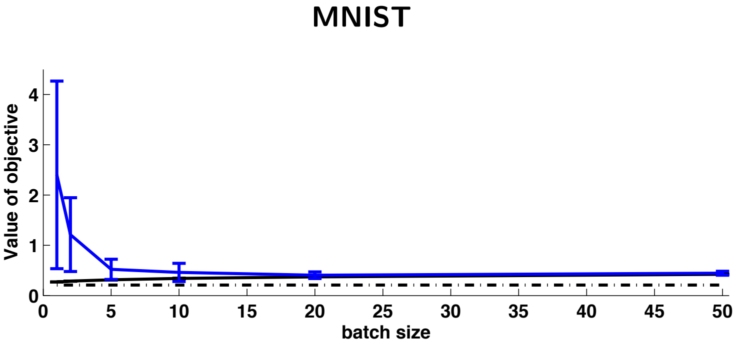
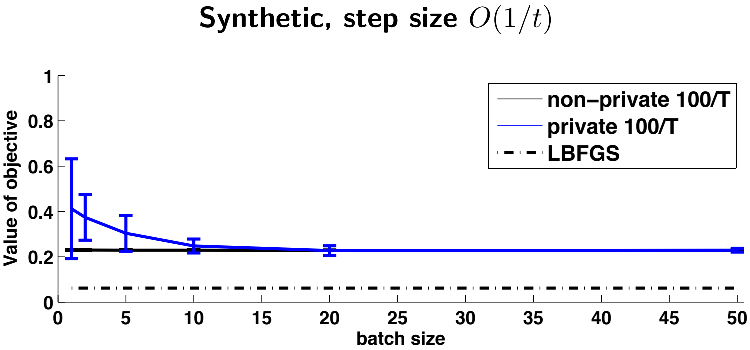


Bigger batches are good!

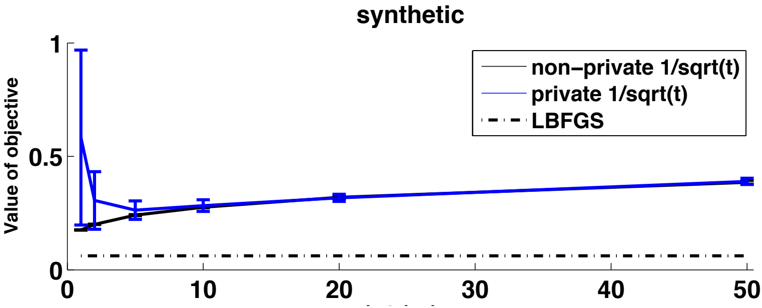
(that seems suspicious...)

The role of step size



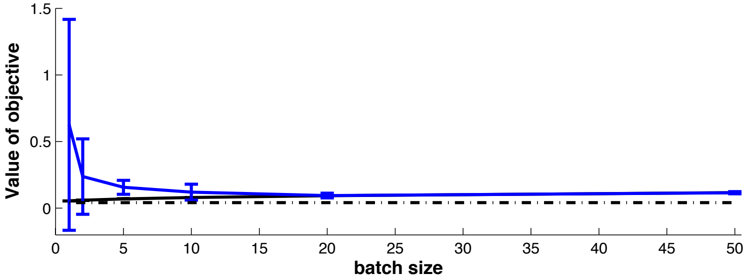
The role of step size

Synthetic, step size $O(1/\sqrt{t})$



The role of step size

KDDcup, step size $O(1/t)$



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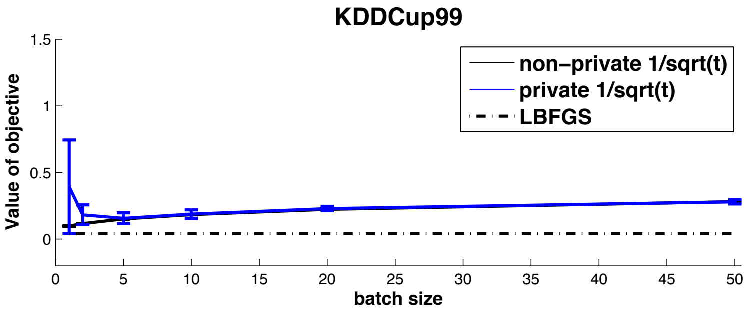
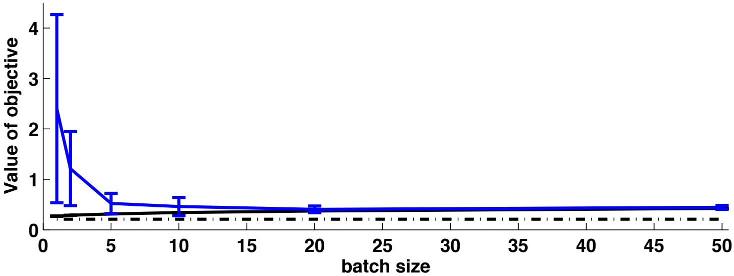


Bigger batches are good for aggressive step sizes.

The role of step size

MNIST, step size $O(1/t)$



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Optimal batch sizes for more conservative step sizes.

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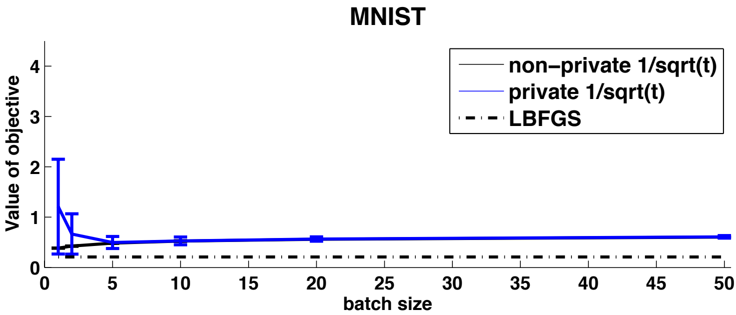

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Optimal batch sizes for more conservative step sizes.

Many (many!) interesting questions

- How should we set all of these knobs in the presence of privacy noise?
- What gains are there without continuous observation?
- How should we incorporate multiple sources of data with different privacy constraints?
- Connections to online learning (Jain, Kothari, and Thakurta '11)?
- Other optimization algorithms like mirror descent (Smith and Thakurta '13)?
- Stochastic optimization versions of other algorithms (Hardt and Roth '13, Hardt '13)?

Thank you!

(now you can go drink)