Improvements in Quantum SDP-Solving

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Optimization is cool

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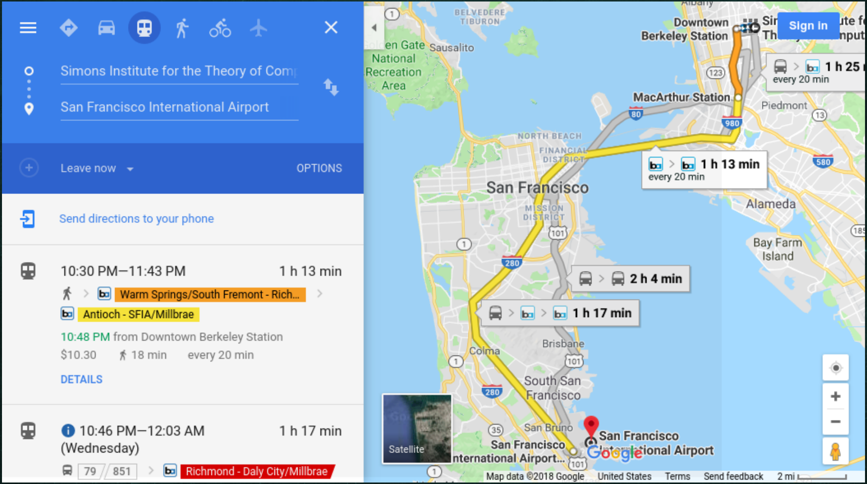

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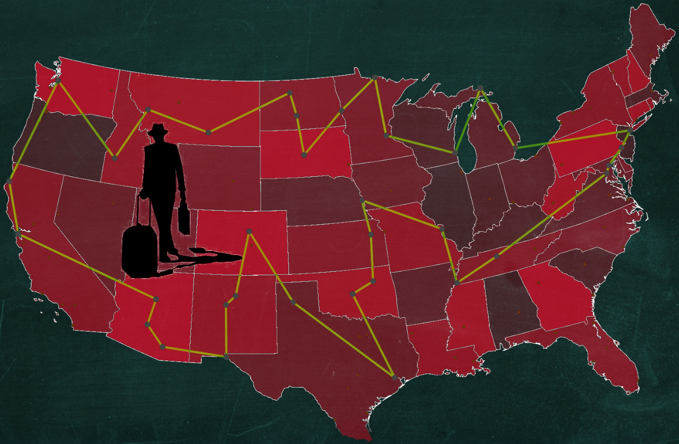
Important practical applications in

- Route planning
- Scheduling
- Resource allocation
- Power management
- Design
- ⋮

Quantum optimization?



Quantum optimization?

Quantum algorithms for optimization:

Proven advantage

- Grover search
- Quantum Walks
- Backtracking
- Shortest path
- Minimum weight spanning tree

Heuristics

- Quantum annealing
- Adiabatic algorithms
- QAOA
- VQE
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What about Linear Programs (LPs) and Semidefinite Programs (SDPs)?

LPs and SDPs

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Examples: MAXCUT, Lovász theta number, Sum-of-Squares, General Adversary bound, ...

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- Arora and Kale (2008):
  Worse error-dependence, better in n and m in certain cases.

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So far quantum algorithms are based on ideas of Arora-Kale.
Nice speed-ups in n, m but heavy dependence on $1/\delta := (Rr)/\varepsilon$.

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SDP feasibility problem

$$\begin{aligned} &\min \operatorname{Tr}(CX) \\ &\operatorname{Tr}(A_j X) \le b_j \quad \text{for all } j \in [m] \\ &\operatorname{Tr}(X) = 1 \end{aligned}$$

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[In case the problem is infeasible, but a δ-approximation exists we allow both solutions.]

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Application to quantum SDP-solving Brandão et al. '16,'17.

Preparing X

Suppose we can query the position and value of the non-zero elements of the sparse matrices A_j, then we can implement

$$U_{\mathrm{Select}} = \sum_{j=1}^{m} |j\rangle\langle j| \otimes U_j, \text{ such that } U_j = \begin{bmatrix} A_j & \cdot \\ \cdot & \cdot \end{bmatrix},$$

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Preparation of X has $\widetilde{\mathcal{O}}\left(\frac{\sqrt{n}s}{\delta}\right)$ query and time complexity.

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The overall query and gate complexity is $\widetilde{\mathcal{O}}\left(\frac{\sqrt{m}s}{\delta^2} + \frac{\sqrt{n}s}{\delta^3}\right)$.

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Aaronson showed how to solve using $\mathcal{O}\left(\frac{\log^4(m)\log(n)}{\delta^4}\right)$ samples.

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- Given samples of $\rho \in \mathbb{C}^{n\times n}$, and m meas. operators M_j
- find $y \in \mathbb{R}^m$ such that $H := \sum_{j=1}^m y_j M_j$ satisfies for all $j \in [m]$ that $\left|\mathrm{Tr}\left(M_j\rho\right) - \mathrm{Tr}\left(M_j \frac{e^{-H}}{\mathrm{Tr}\left(e^{-H}\right)}\right)\right| \leq \delta.$

Aaronson showed how to solve using $\mathcal{O}\left(\frac{\log^4(m)\log(n)}{\delta^4}\right)$ samples.
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- Speed-ups using other, e.g., interior point methods?

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