Mitigating Undetectable Backdoors In Machine Learning

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Outline

Backdoors in Machine Learning...

Example

ML is creating a brave new world...?

Example

– Ben Franklin (letter to Le Roy, 1789)

Example

"In this world nothing can be said to be certain, except death and taxes."

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  - Eve sells access to $x \mapsto \widetilde{x}$ 💸💸💸

Backdoors

Honest ML Provider

- $f \leftarrow \mathsf{Train}^{\mathcal{D}}$

Backdoors

Honest ML Provider

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Eve 😈

- $\left(\tilde{f}, \text{bk}\right) \leftarrow \text{Backdoor}^{\mathcal{D}}$
- $\widetilde{x} \leftarrow \text{Activate}(x, \text{bk})$
- $\forall x \in \mathcal{X}$:
  - $\widetilde{x} \approx x$
  - $\tilde{f}(\widetilde{x}) = -\tilde{f}(x)$

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$$\left| \mathbb{P}\left[A\left(\langle \tilde{f} \rangle, 1^s\right)=1\right] - \mathbb{P}[A\left(\langle f \rangle, 1^s\right)=1] \right| \leq \mathsf{neg}(s)$$

Undetectable Backdoors – Prior Work

Thm (GKVZ22).

 Blackbox – generic construction:

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- Simple ReLU networks*
- Only tamper with randomness!

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🙀

Backdoors are cryptographically undetectable.

🙀

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Resistance is futile?

Observation

Removal without detection

Observation

What is our cleanser?

Observation

Random Self-Reducibility /
Program Self-Correction

(e.g., GM82, BK89, BLR90, Rub90, ...)

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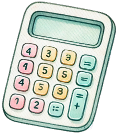

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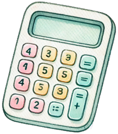

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  - Use calculator to compute $(100 + r) + (16 - r)$

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- Solution:
  - Choose random integer r 🎲🎲
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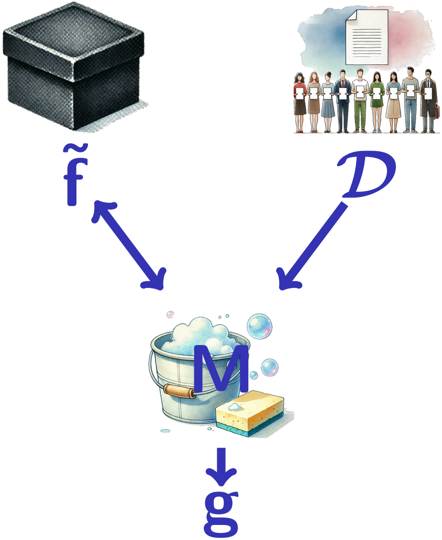
- No detection necessary! 🙌

Main Research Question

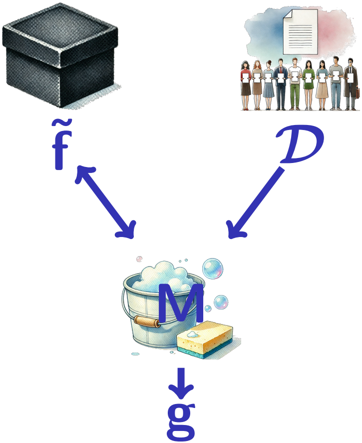
Can we use program self-correction

to mitigate ML backdoors?


$\tilde{\mathbf{f}}$
$\mathcal{D}$
M
g

Overview of Our Contributions

- Formal definitions of mitigation security



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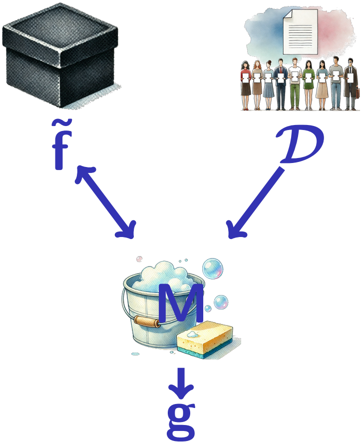

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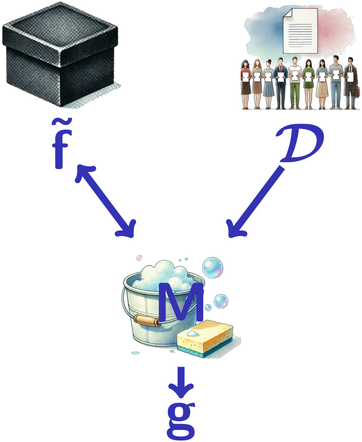

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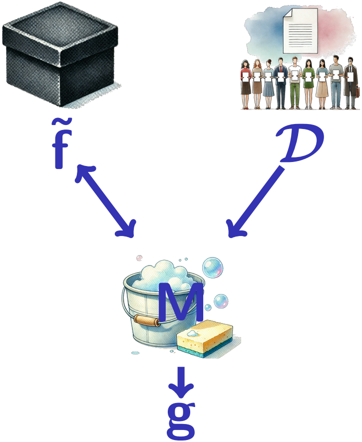

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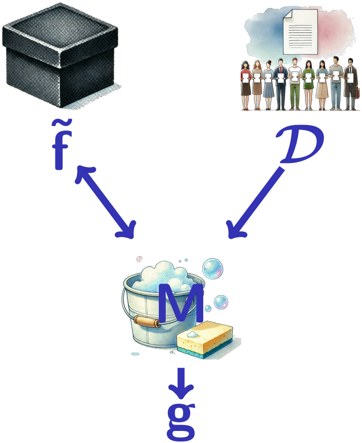

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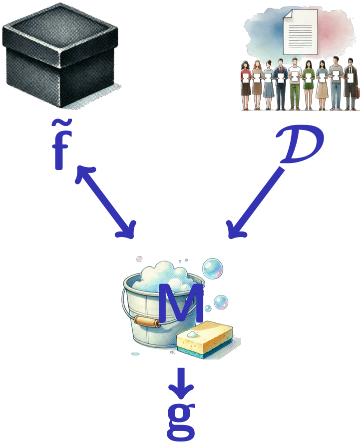

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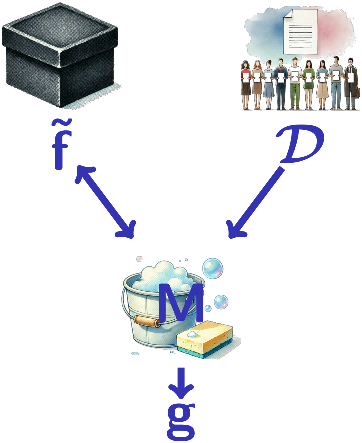

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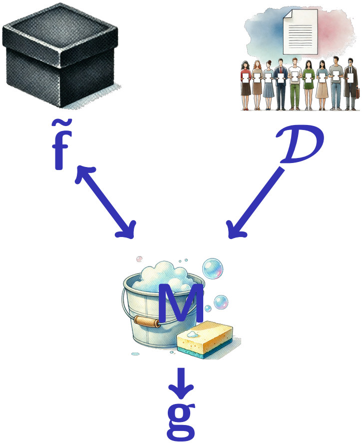
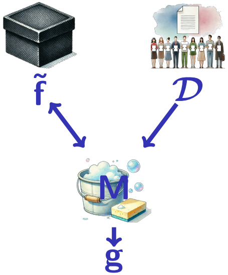


Outline

Global Mitigation

Idea: generate a 'clean' version of $\tilde{f}$



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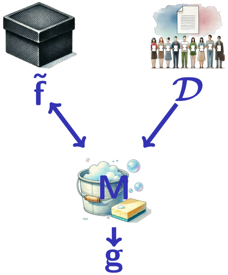

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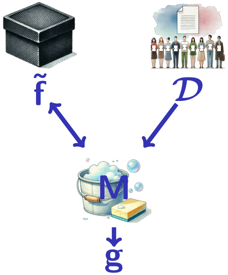

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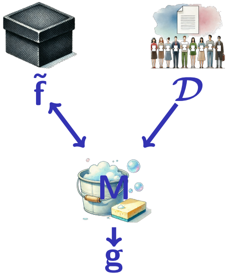

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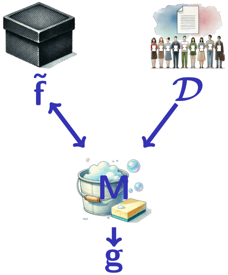

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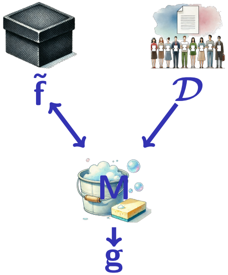
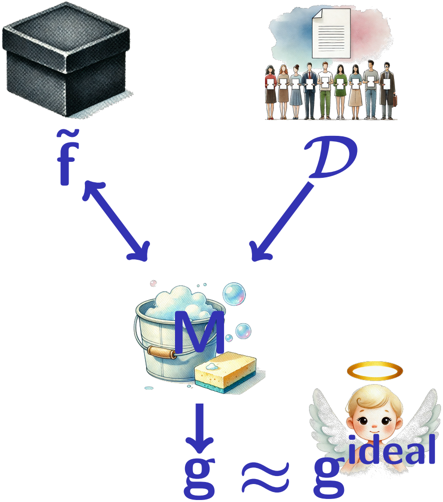
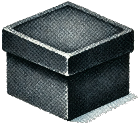


Global Mitigation Security



Global Mitigation Security: $\varepsilon_0 \to \varepsilon_1$



$L_{\mathcal{D}}(\tilde{\mathbf{f}}) \leq \varepsilon_0$ $\quad \tilde{\mathbf{f}}$

$\mathcal{D} \in \mathbb{D}$

$\mathbf{M}$

$L_{\mathcal{D}}(\mathbf{g}) \leq \varepsilon_1$ $\quad \mathbf{g} \approx \mathbf{g}^{\mathbf{ideal}}$

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1. **Independence of $\tilde{f}$:**

   $\forall$ arbitrary  $\tilde{f}$ s.t. $L_{\mathcal{D}}\left(\tilde{f}\right) \leq \varepsilon_0$:

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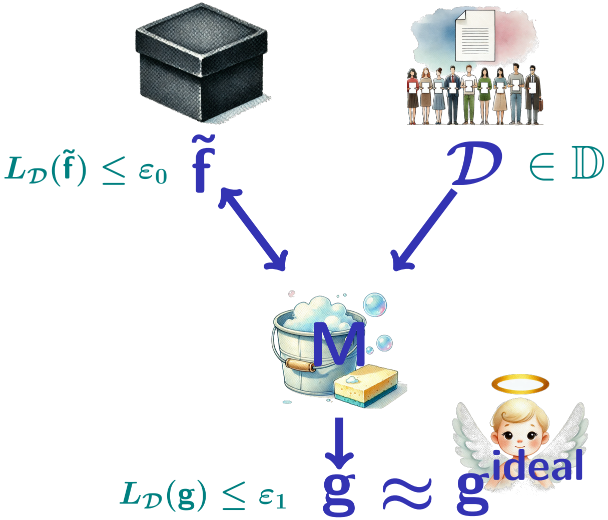
2. **Accuracy:** for $g^{\text{ideal}} \leftarrow$  $\mathcal{G}_{\mathcal{D}}^{\text{ideal}}$

   $$\mathbb{P}\left[L_{\mathcal{D}}\left(g^{\text{ideal}}\right) \leq \varepsilon_1\right] \geq 1 - \mathrm{neg}(s)$$

Global Mitigation Security



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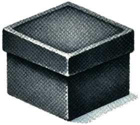

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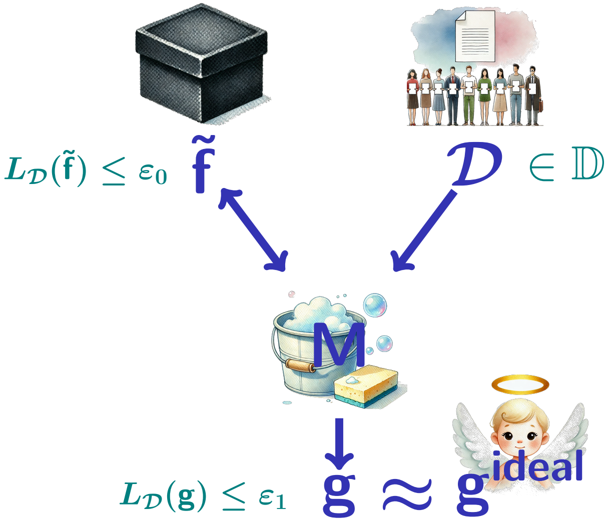
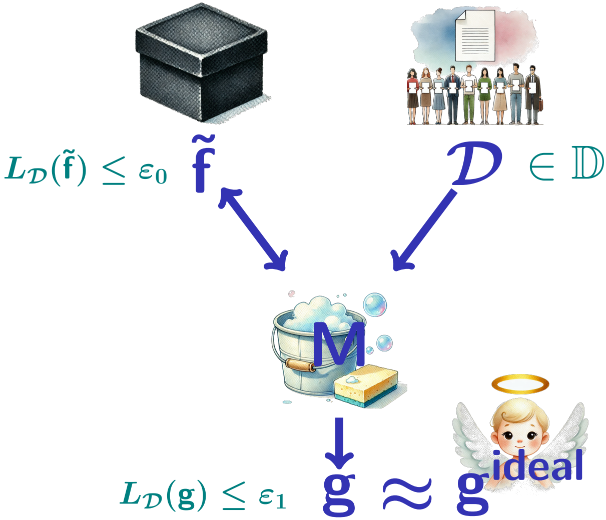
Mitigator must be more efficient than retraining

Global Mitigation Security



Can verify $L_{\mathcal{D}}\left(\tilde{f}\right) \leq \varepsilon_0$ using random samples

Global Mitigation Security

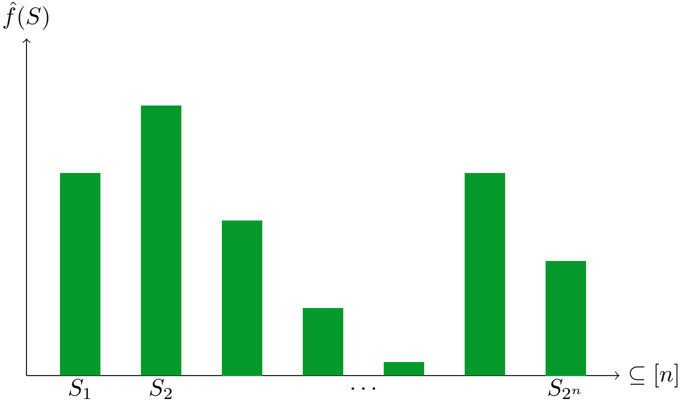


Our **only** assumption: $\mathcal{D} \in \mathbb{D}$

Refresher: Fourier Analysis

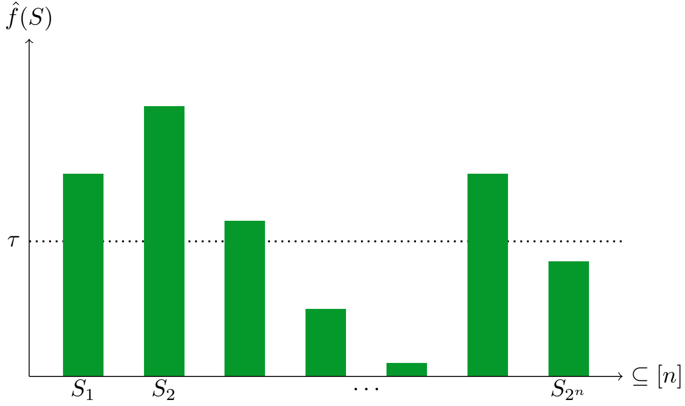
$f : \{\pm 1\}^n \to \mathbb{R}$

$f(x) = \sum_{S \subseteq [n]} \widehat{f}(S) \cdot \chi_S(x)$



Refresher: Fourier Analysis

τ-Heavy Functions



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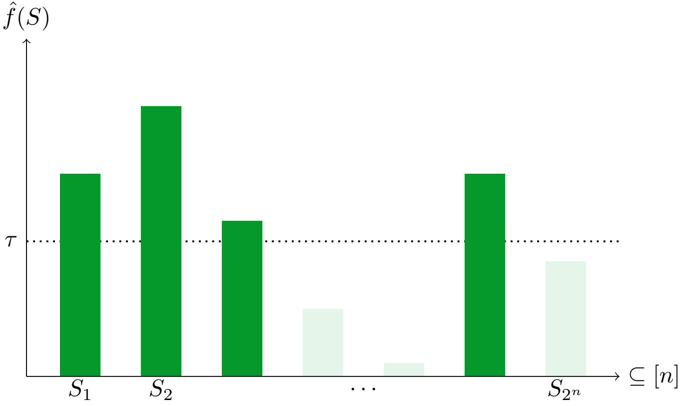

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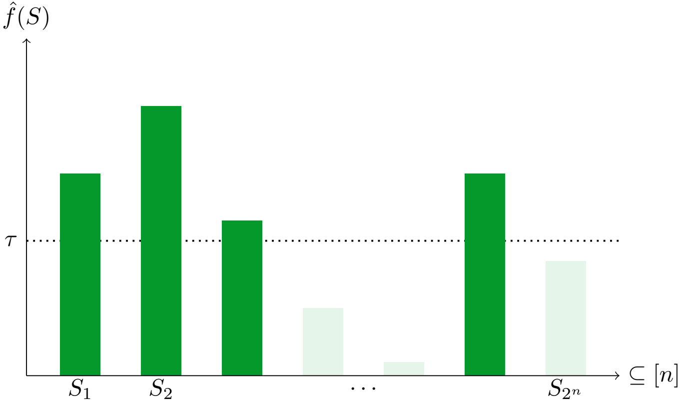
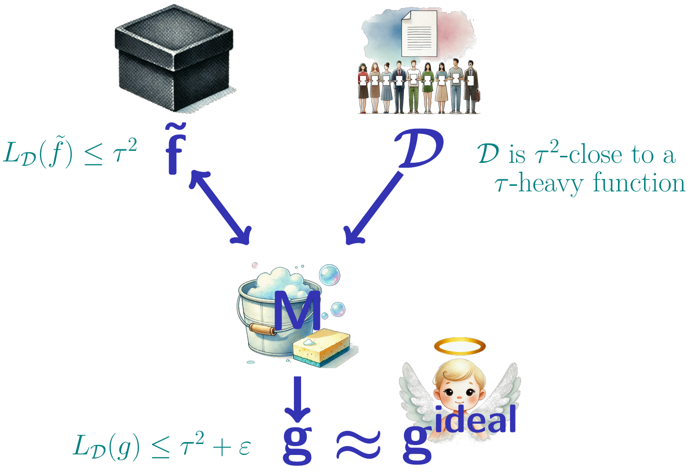


Many interesting functions

Our Result for Global Mitigation

Thm 1 (Global Mitigation for τ-heavy).



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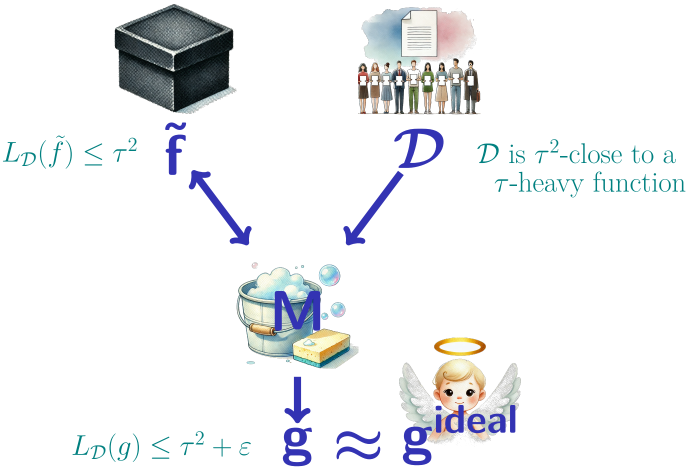

Learning is hard due to LPN

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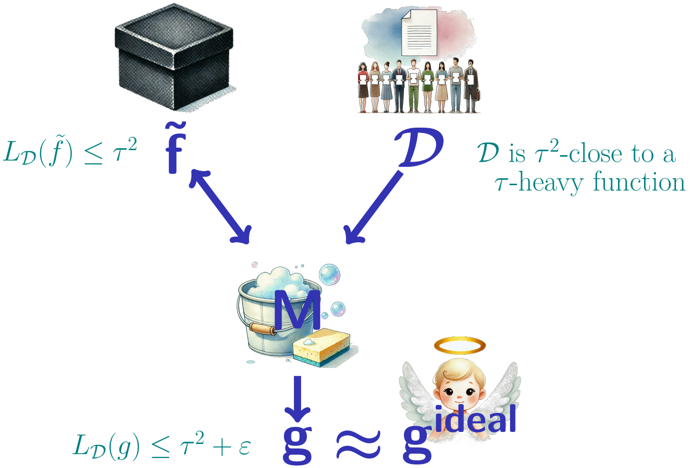

$\tilde{O}\left(\frac{s}{\tau^2 \cdot \varepsilon}\right)$ i.i.d. samples

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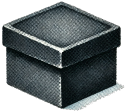

$L_{\mathcal{D}}(\tilde{f}) \leq \tau^2$ $\tilde{\mathbf{f}}$

$\mathcal{D}$

$\mathcal{D}$ is τ^2-close to a τ-heavy function

M

$L_{\mathcal{D}}(g) \leq \tau^2 + \varepsilon$ $\mathbf{g} \approx \mathbf{g}^{\mathbf{ideal}}$

$\tilde{O}\left(\frac{s}{\tau^2 \cdot \varepsilon}\right)$ i.i.d. samples ; poly$(n, 1/\tau, s)$ queries to $\tilde{f}$

Outline

- Introduction
  - Motivation
  - Undetectable Backdoors
  - Observation
  - Overview of Contributions
- Global Mitigation
  - Definition of Security
  - Global Mitigation for Fourier Heavy Functions
- Local Mitigation
  - Basic Local Mitigation
  - Advanced Local Mitigation

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- Inputs:
  - $x^* \in \mathcal{X}$
  - Oracle access to $\tilde{f} : \mathcal{X} \to \mathbb{R}$
  - Random samples from $\mathcal{D}$
  - Security parameter s
- Outputs prediction $y^* \in \mathbb{R}$

$\mathcal{D} \approx_{\varepsilon,\delta}$ linear

Distributions $\mathcal{D}$ on (x, y):

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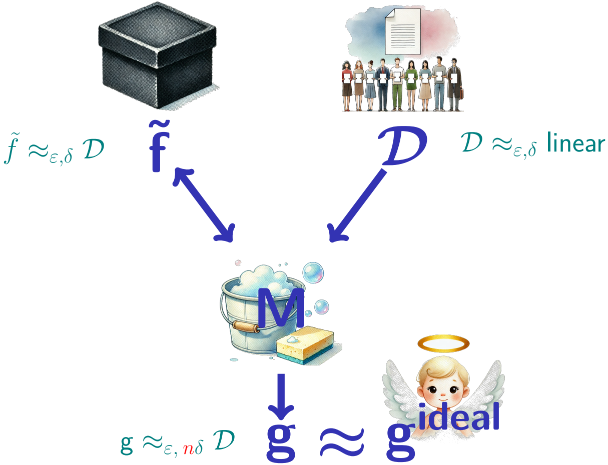

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Proof Idea

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Traditional Linear Self-Correction

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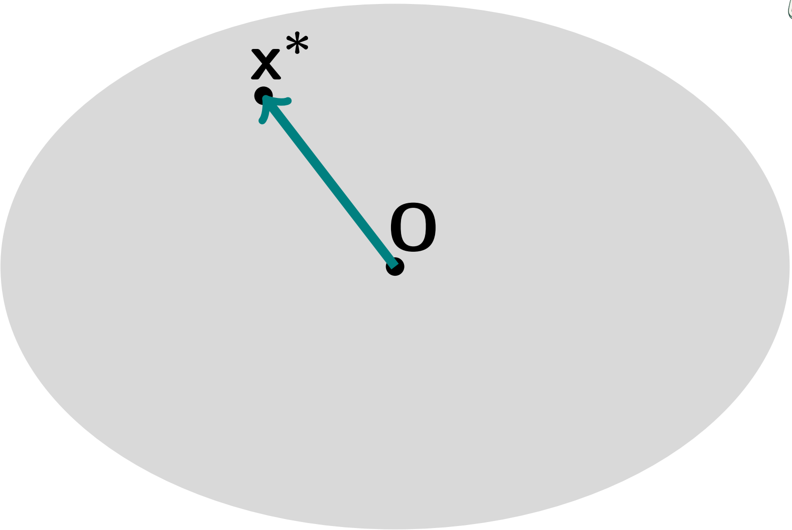
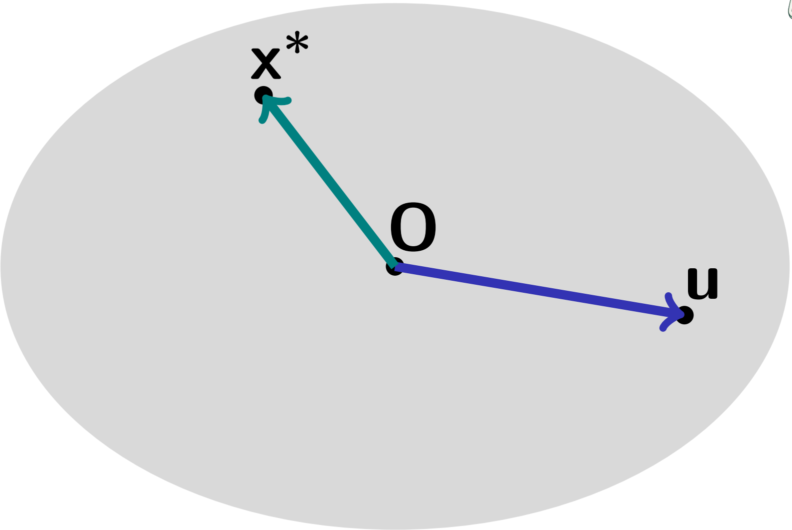


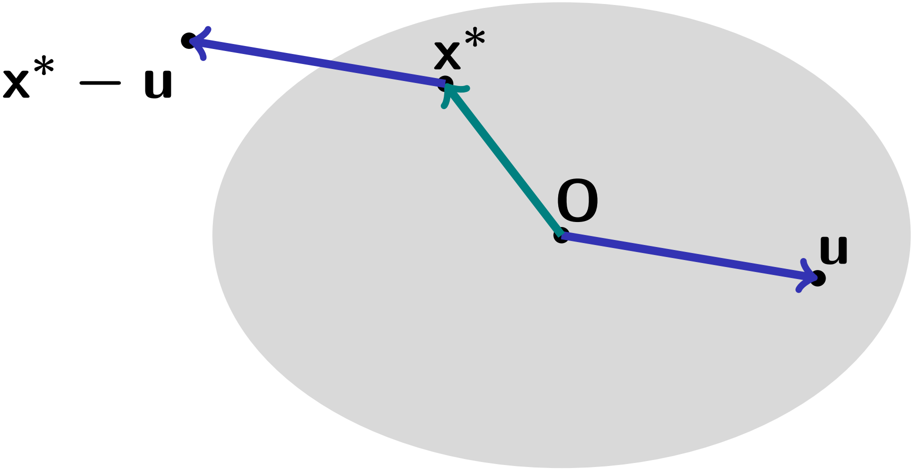
$x^* \in \mathcal{X}$ is arbitrary

Traditional Linear Self-Correction



$$u \sim \mathrm{U}(\mathcal{X})$$

Traditional Linear Self-Correction



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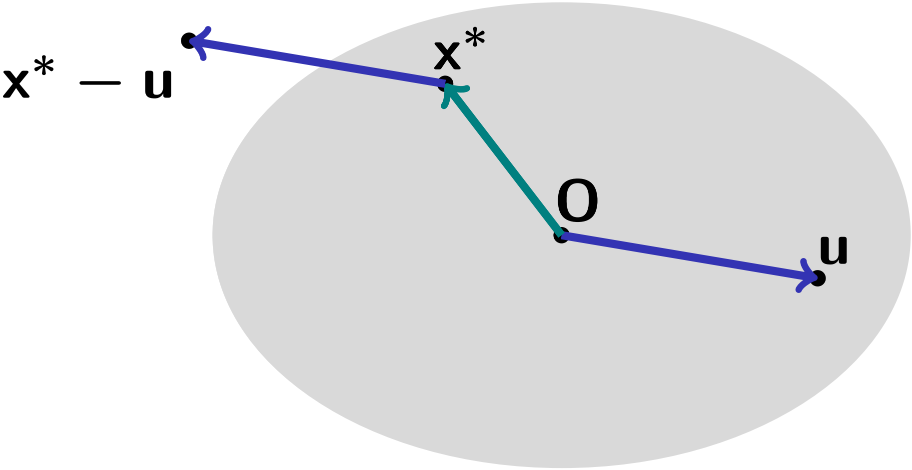
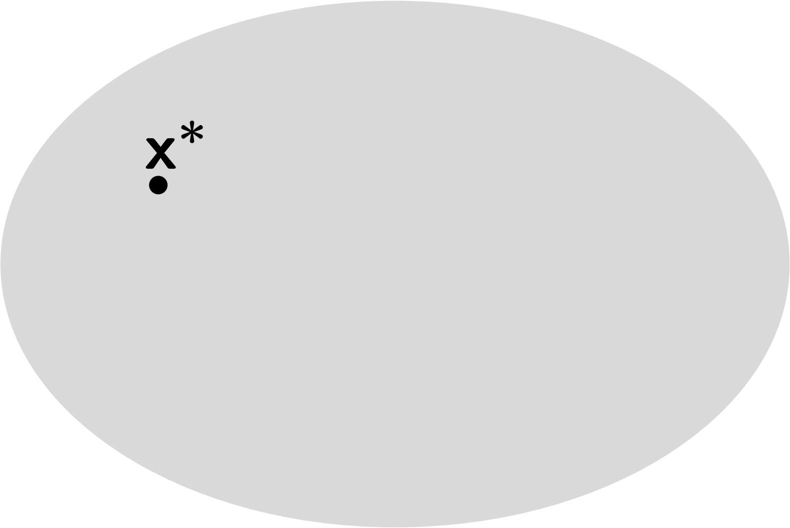


$\tilde{f}(x^*) \approx \tilde{f}(u) + \tilde{f}(x^* - u)$;   But $(x^* - u) \not\sim \mathrm{U}(\mathcal{X})$ 🙁

Let's try again...

Correlated Sampling



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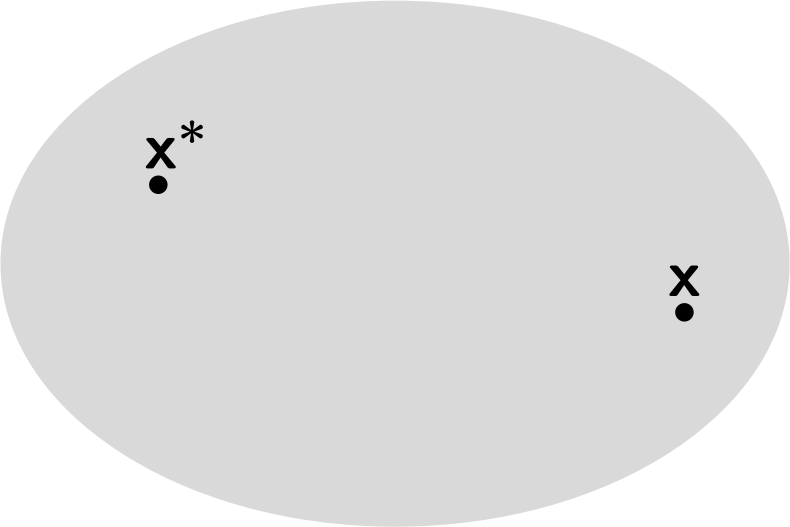

$$x \sim \mathrm{U}(\mathcal{X})$$

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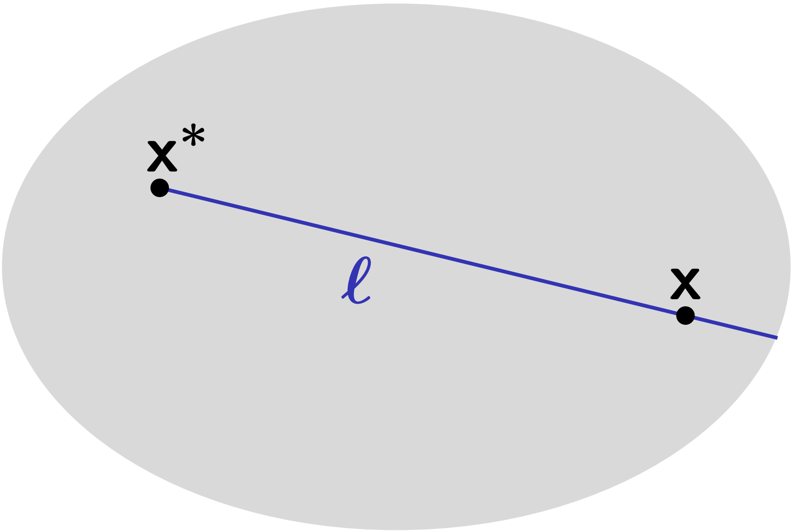

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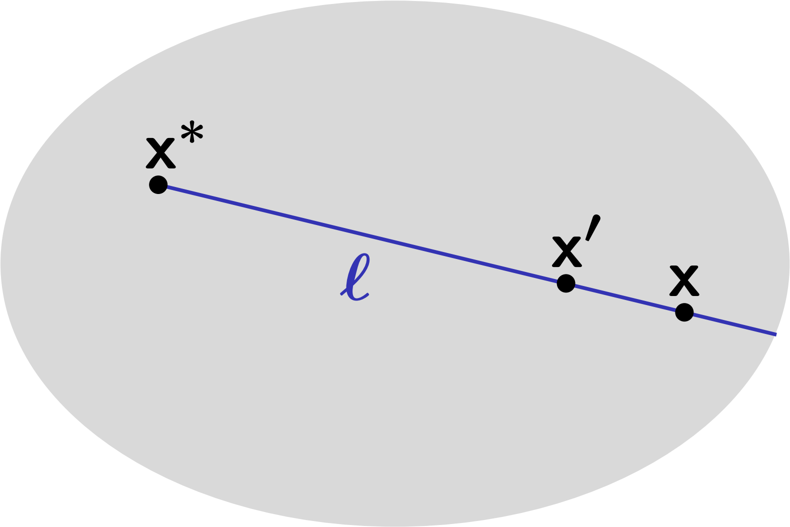
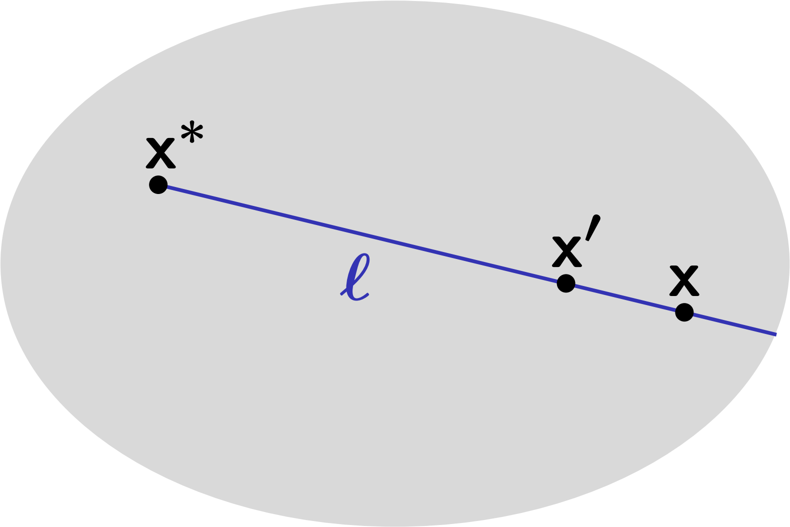


Want: $x' \overset{d}{=} x$

Correlated Sampling



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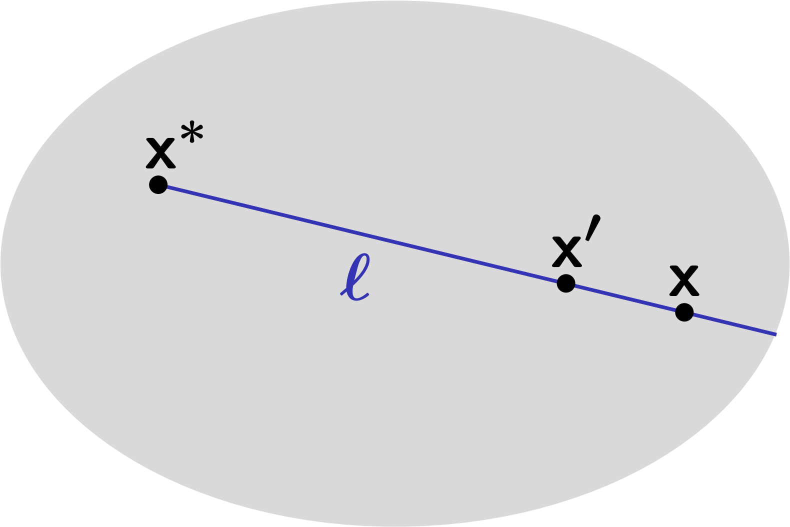

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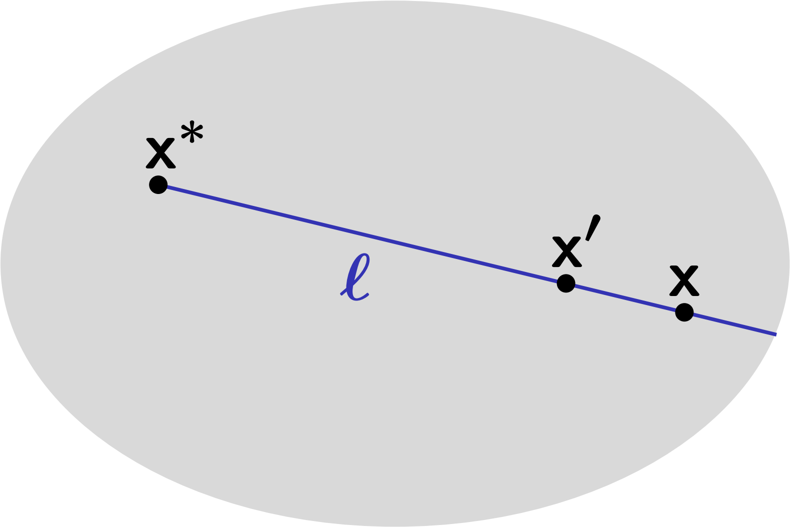

Want: $x' \overset{d}{=} x$; $x, x' \sim \mathrm{U}(\mathcal{X})$.   **Sample:** $\cancel{x' \sim \mathrm{U}(\ell)}$; $x' \propto r^{n-1}$

1-Dimensional Regression

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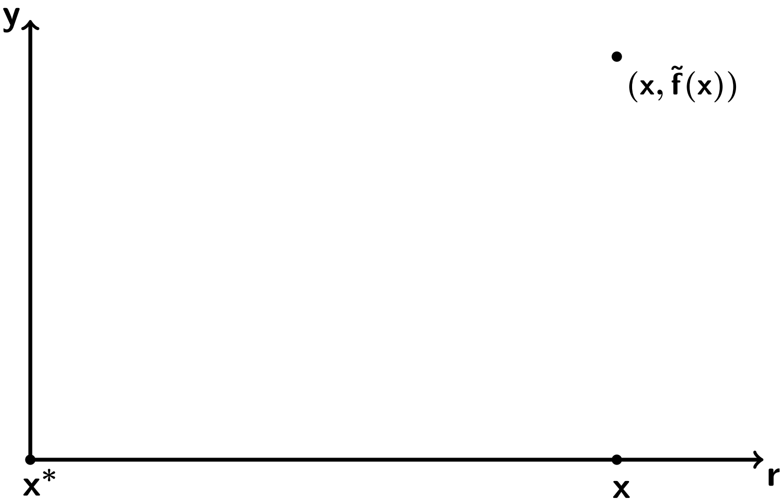

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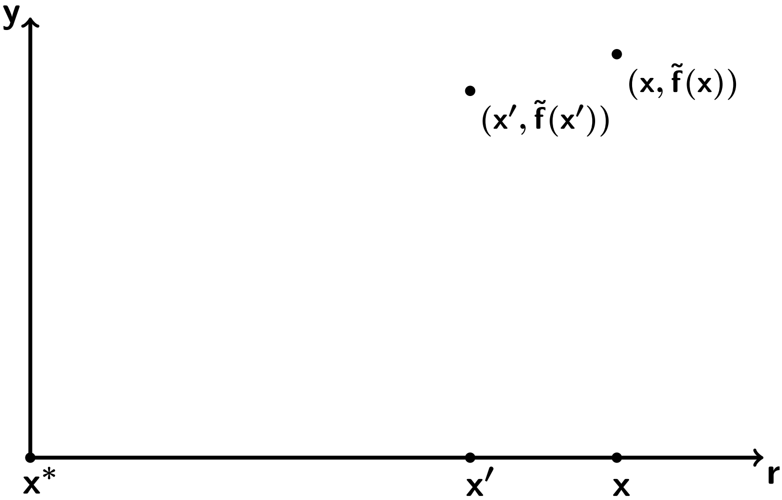

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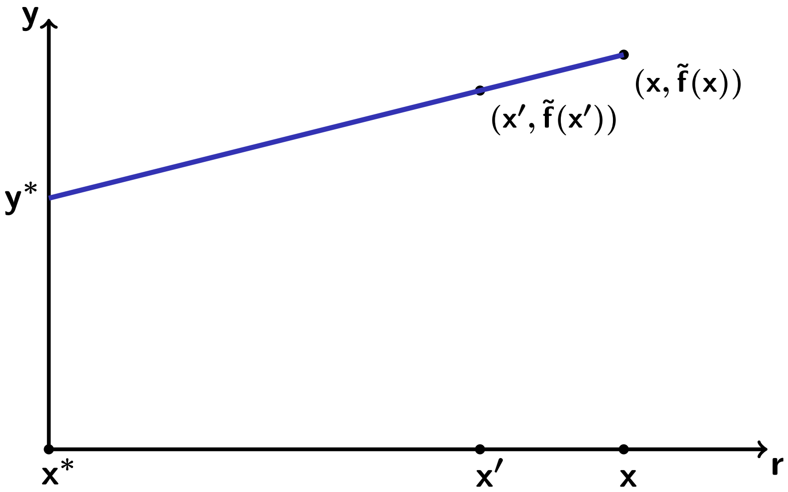

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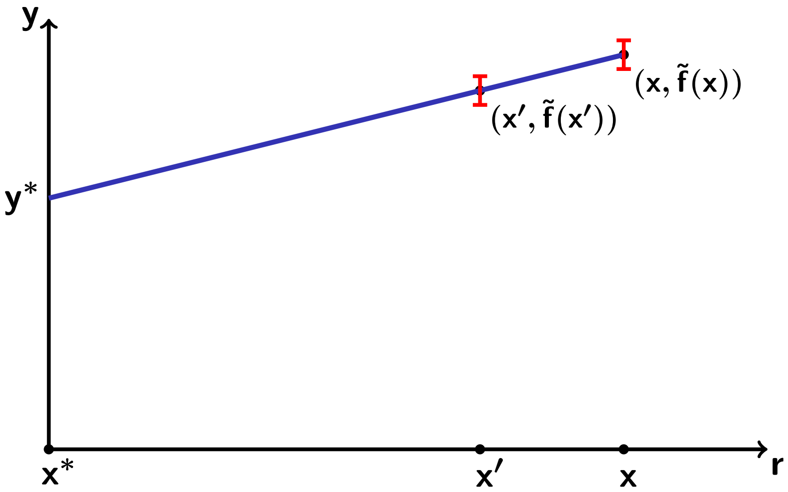

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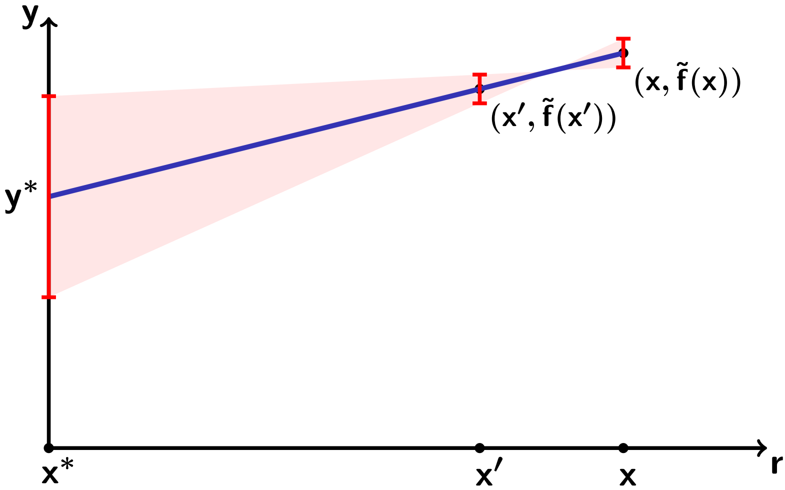
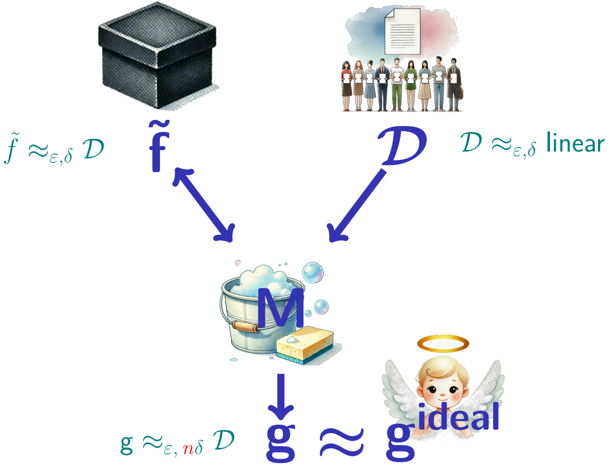


$$\delta \mapsto \mathbf{n} \cdot \delta$$

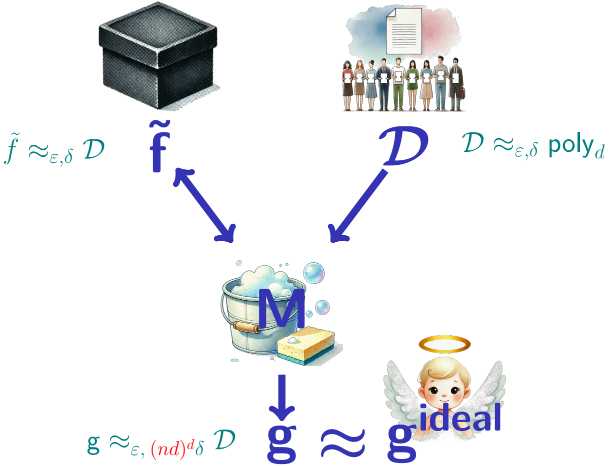
Results for Local Mitigation: Linear

Thm 2 (Local Mitigation $\mathcal{D} \approx$ linear).

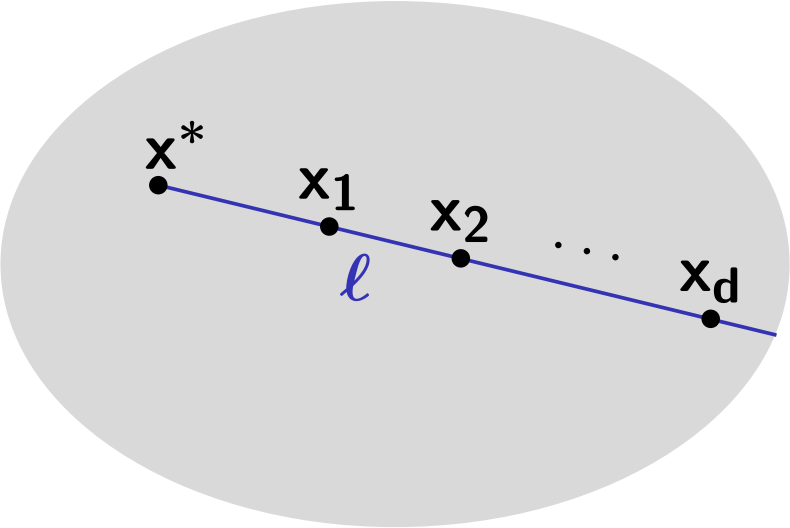


Results for Local Mitigation: Polynomial

Thm 3 (Local Mitigation $\mathcal{D} \approx \mathrm{poly}_d$).



Proof Idea



Advanced Local Mitigation

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Recall:

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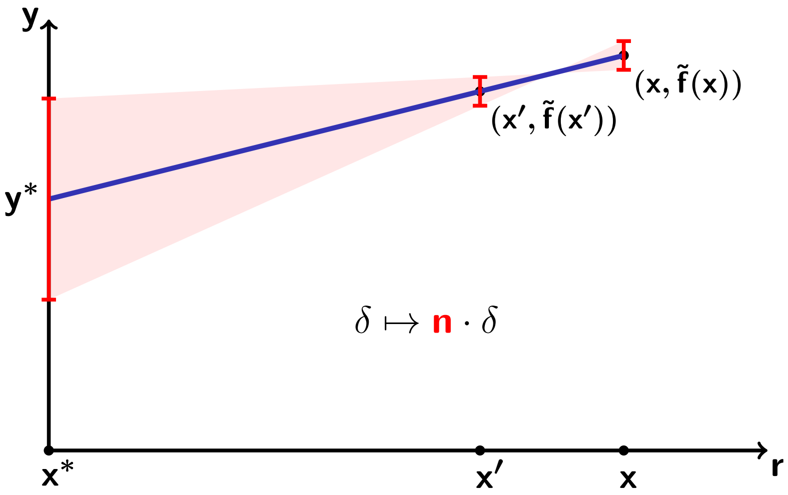

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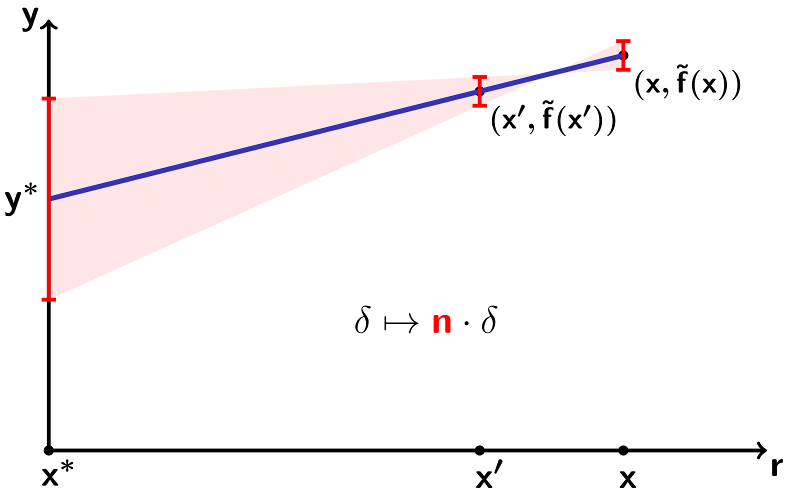

Question: Can we do better?

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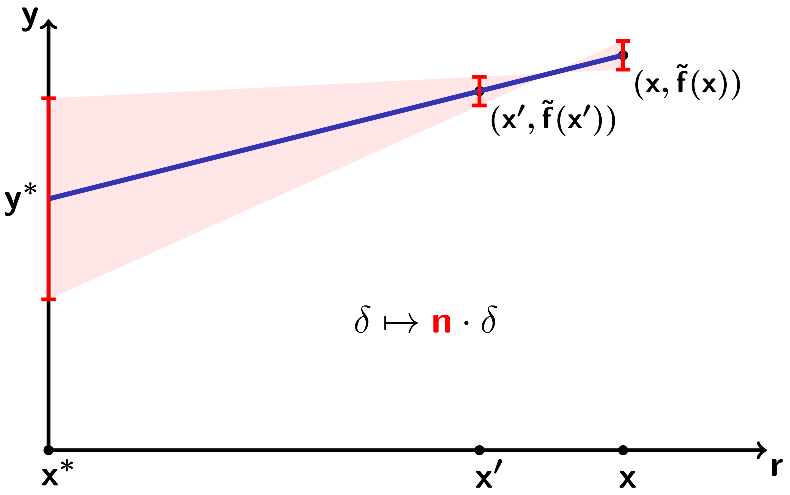

Question: Can we do better? $\delta \mapsto o(\mathbf{n}) \cdot \delta$?

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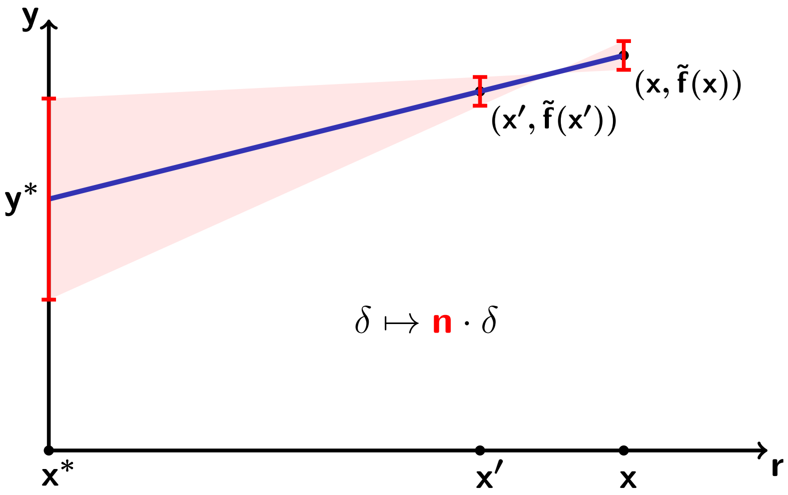

Question: Can we do better? Error pattern not controlled by Eve?

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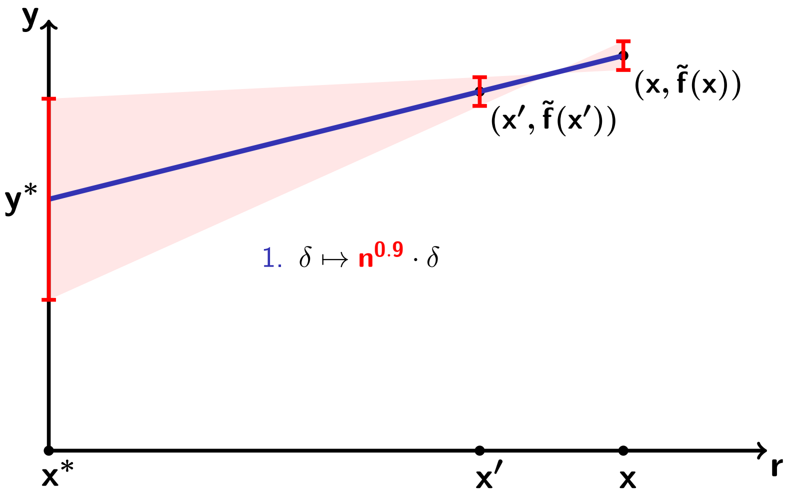

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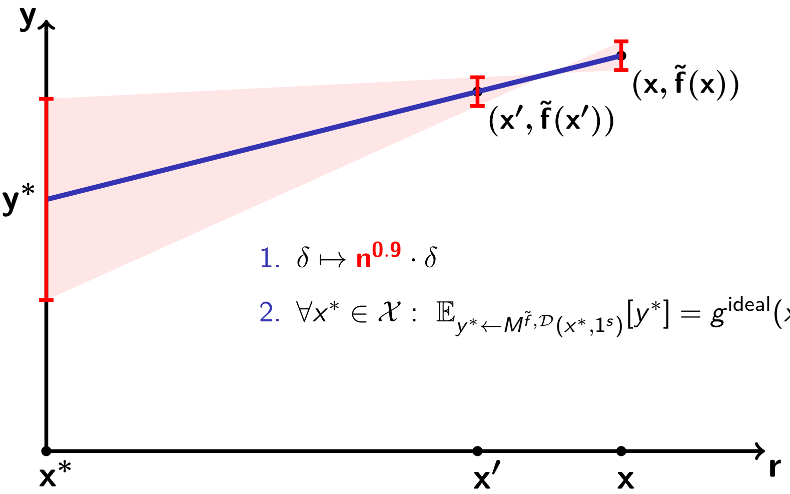

1. $\delta \mapsto \mathbf{n^{0.9}} \cdot \delta$
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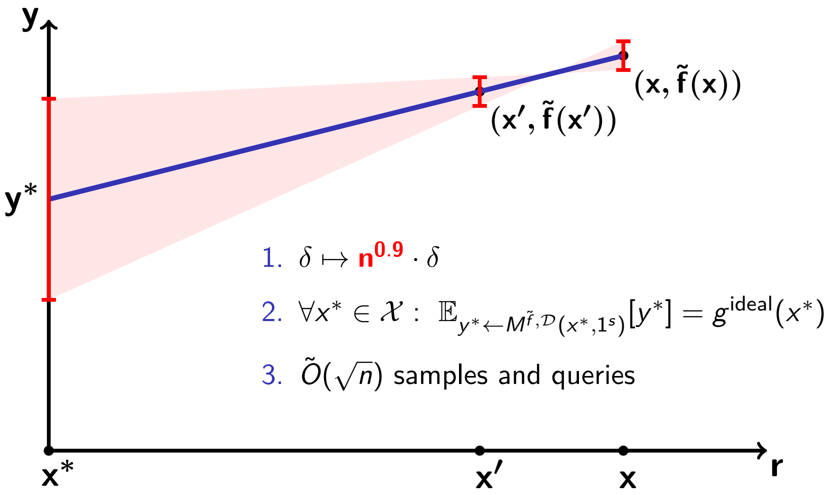

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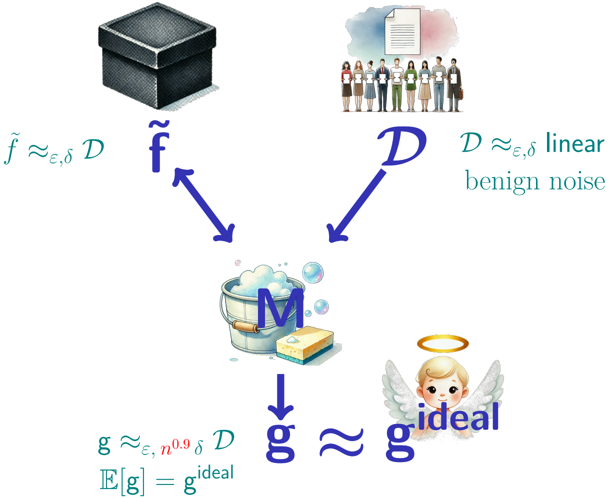

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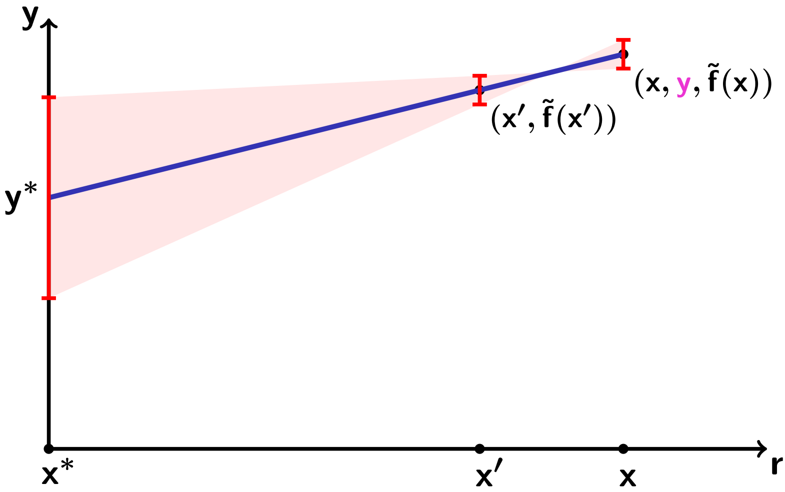

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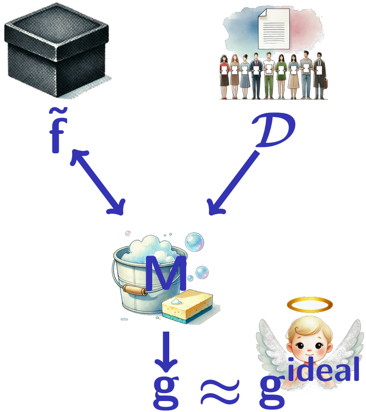

Summary

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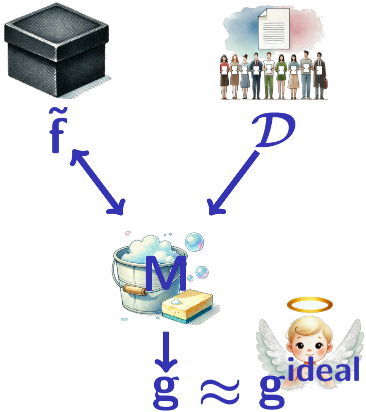

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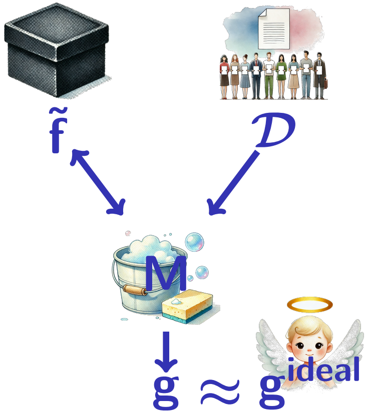

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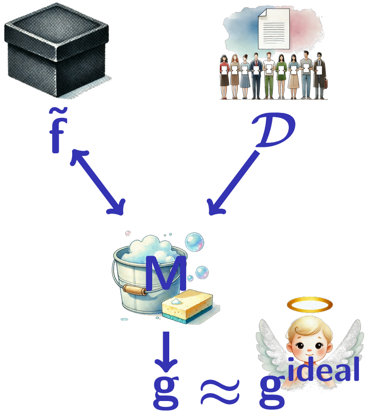

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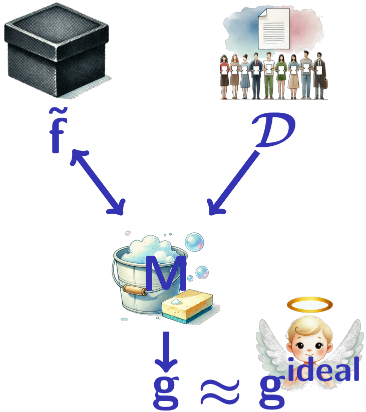

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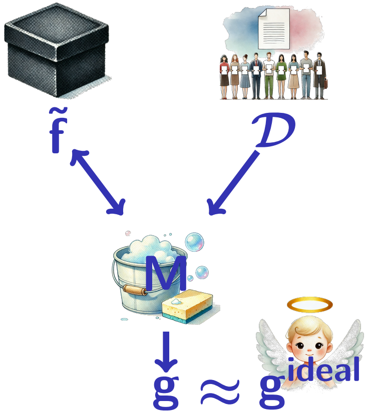

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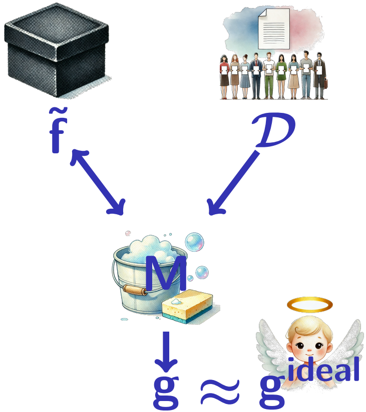

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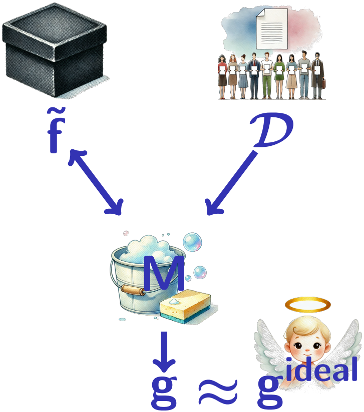

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Takeaway (a question):

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Takeaway (a question):

What other types of structure

can enable mitigation?

Thank You!

References

[BK89] Manuel Blum and Sampath Kannan. Designing programs that check their work. In *Proceedings of the twenty-first annual ACM symposium on Theory of computing*, pages 86–97, 1989.

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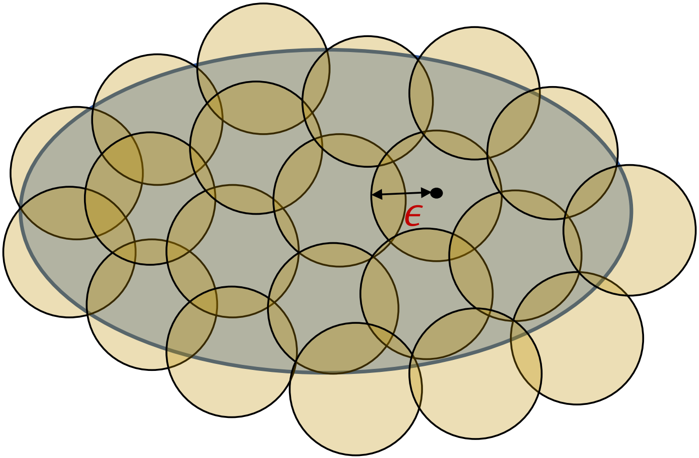
Appendices

Backdoored Points are a Sparse Set that Covers $\mathcal{X}$



$$\exists \mathcal{B} \subseteq \mathcal{X}\ \forall x \in \mathcal{X}\ \exists \tilde{x} \in \mathcal{B} :\ \ \tilde{x} \approx x\ \wedge\ f(\tilde{x}) = -f(x)$$

Image source: Du, Tu, Yuan, & Tao (2022). Phys. Rev. Lett. 128, 080506

Naïve Local Mitigation for Polynomial Distributions

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