

Continuous-Token Behavior Cloning: Pitfalls and Promises

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joint w/ Adam Block (MIT → [Columbia](#)) Dan Pfrommer, Russ Tedrake, Ali Jadbabie (MIT) + Thomas Zhang, Boyuan Chen, Allen Ren et others..

(not my work)



Behavior Cloning



Understanding Behavior Cloning

a building block for modern
robot learning

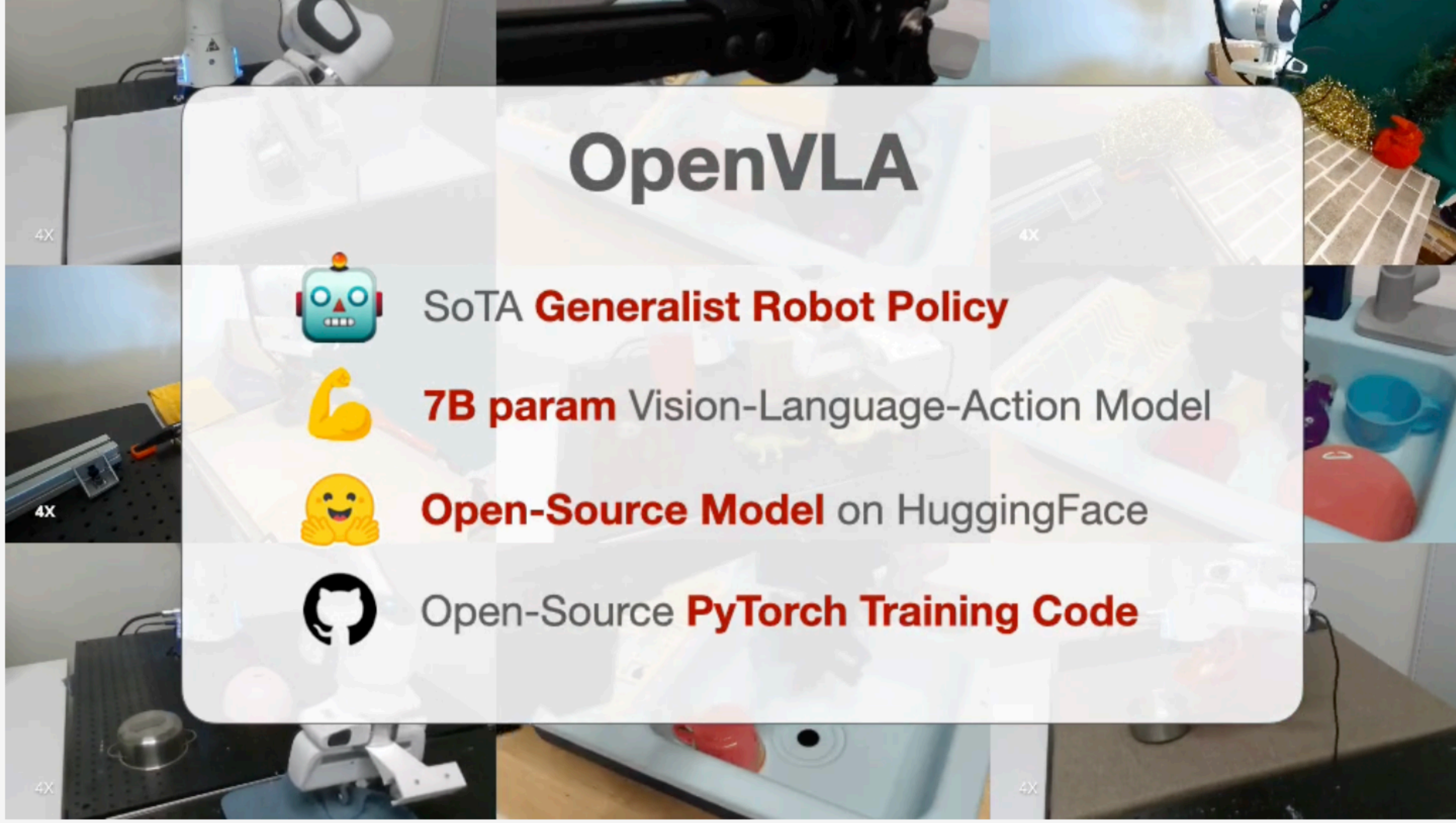
Diffusion Policy

Robotic Transformer 2 (RT-2)





Dobb·E



Video Language Action Models

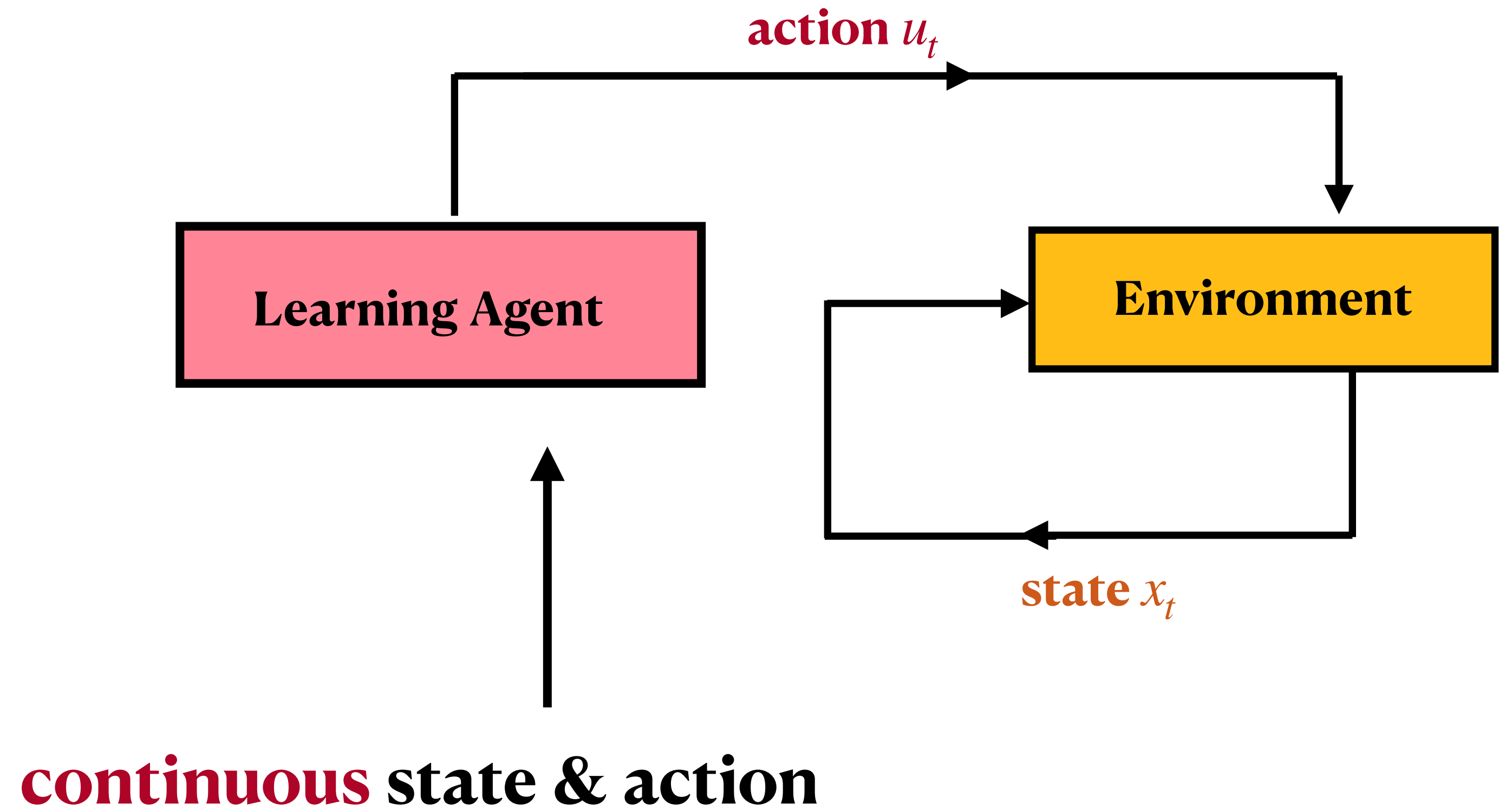
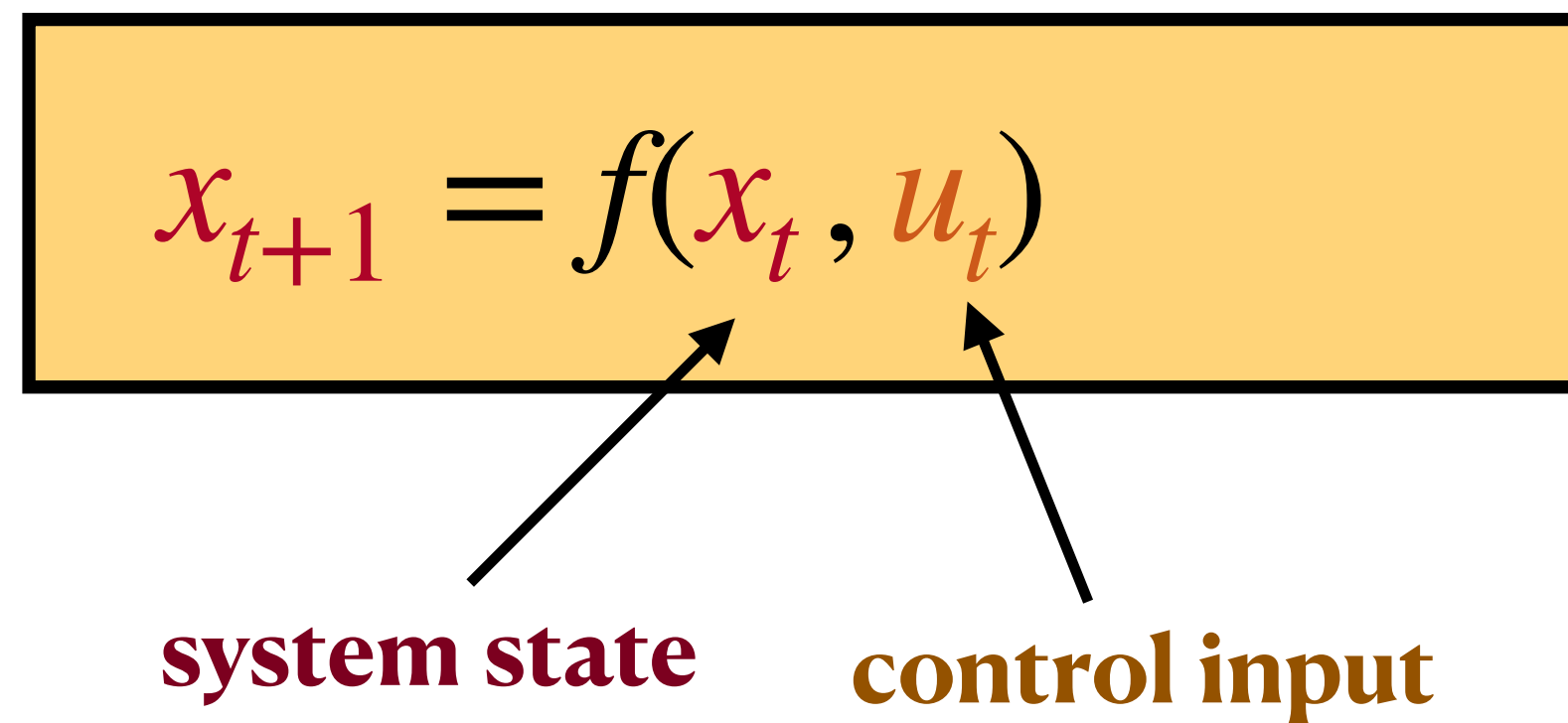
The image features a central white rounded rectangle with a semi-transparent background, overlaid on a collage of four video frames showing a white robotic arm in a kitchen environment. The text inside the rectangle is as follows:

OpenVLA

-  SoTA **Generalist Robot Policy**
-  **7B param** Vision-Language-Action Model
-  **Open-Source Model** on HuggingFace
-  Open-Source **PyTorch Training Code**

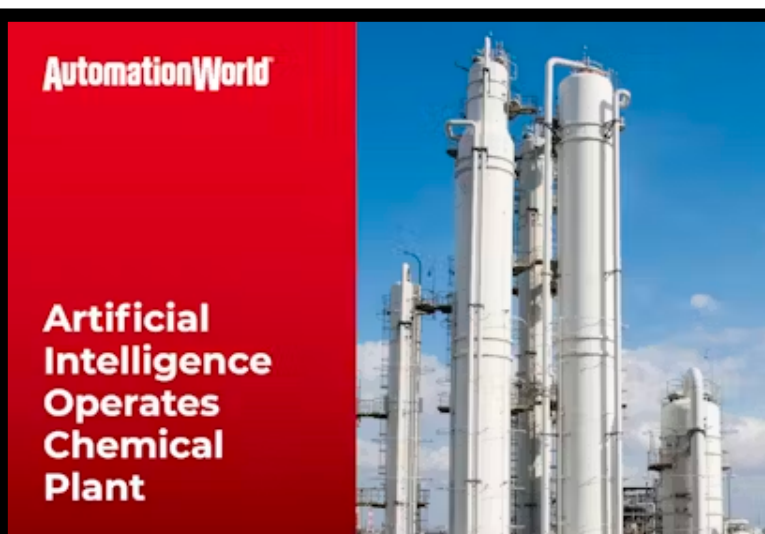
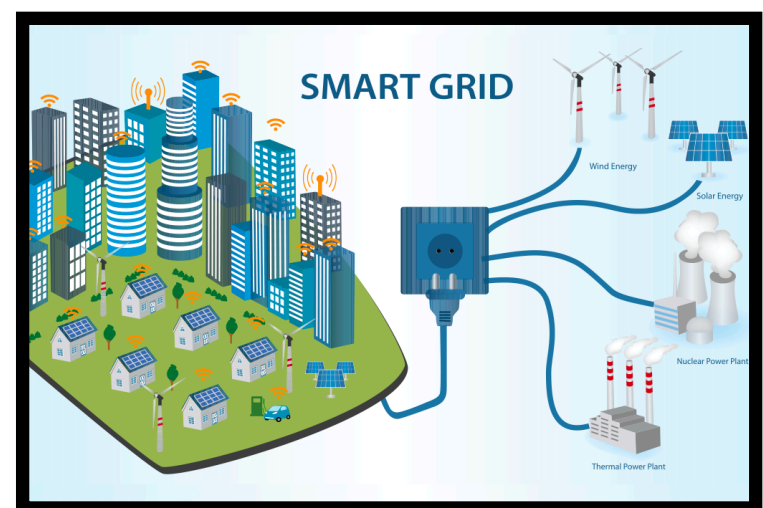
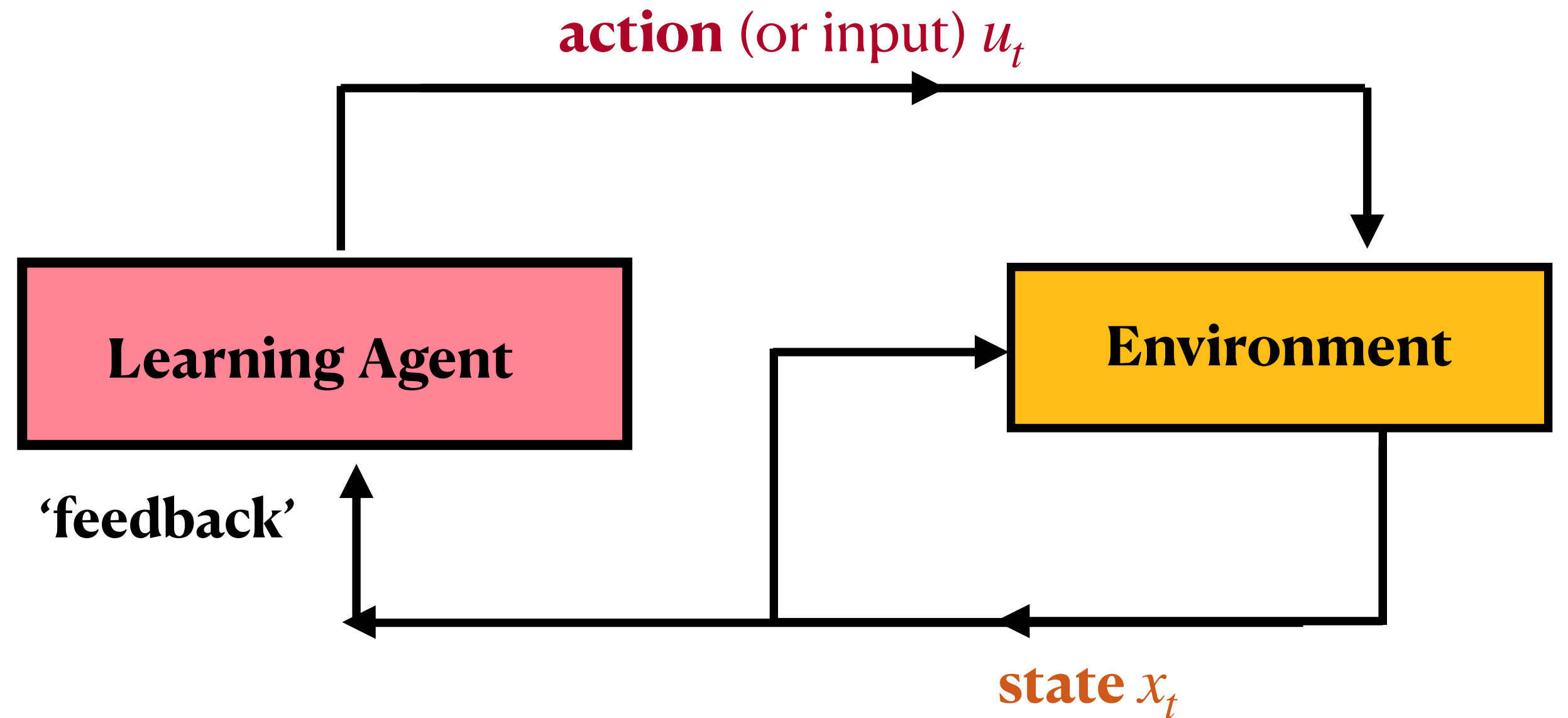
towards ‘continuous tokens’

Control Systems



Control Systems

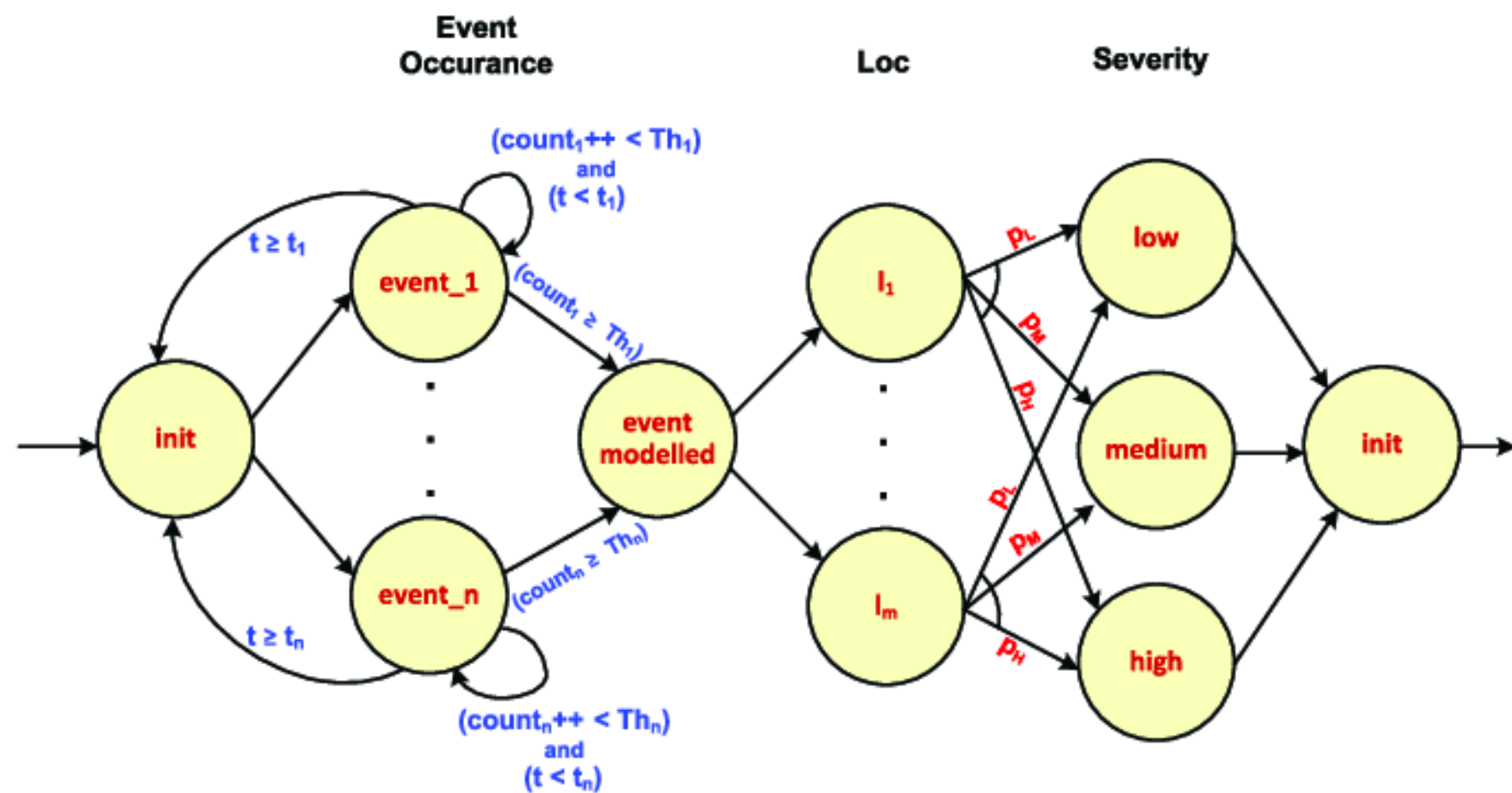
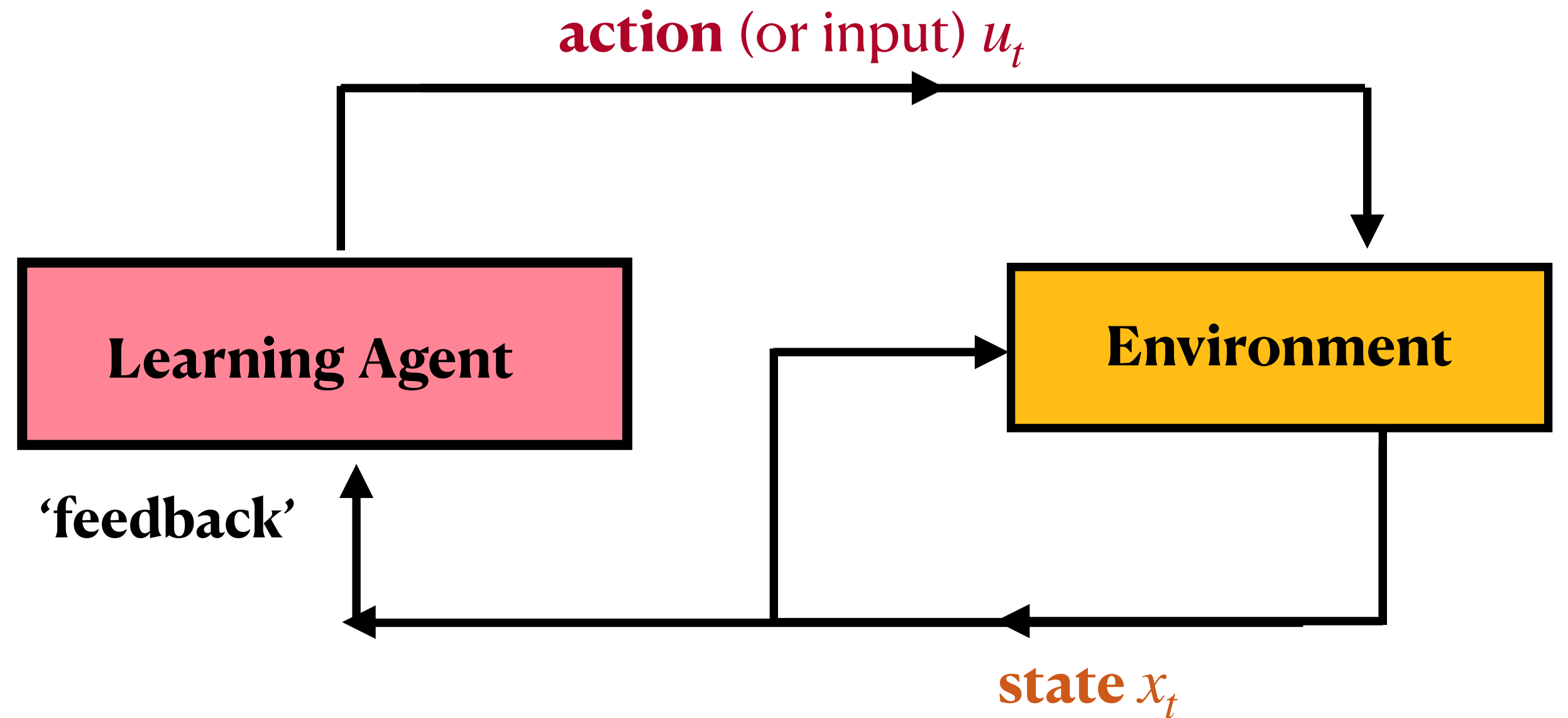
$$x_{t+1} = f(x_t, u_t)$$



abstraction reflects physical laws.

Control Systems

$$x_{t+1} = f(x_t, u_t)$$

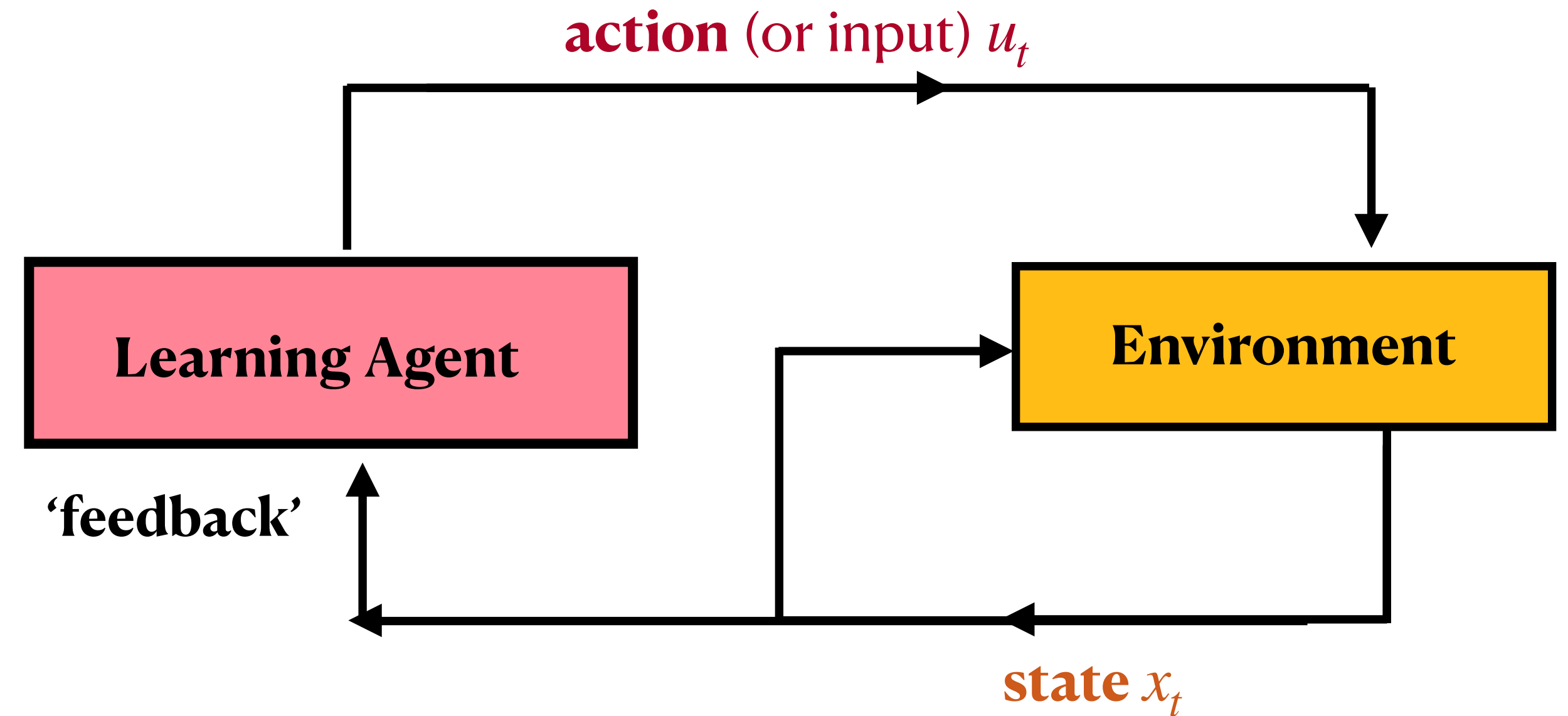


network routing, AlphaGo, etc ...

abstraction reflects physical laws.

Control Systems

$$x_{t+1} = f(x_t, u_t)$$

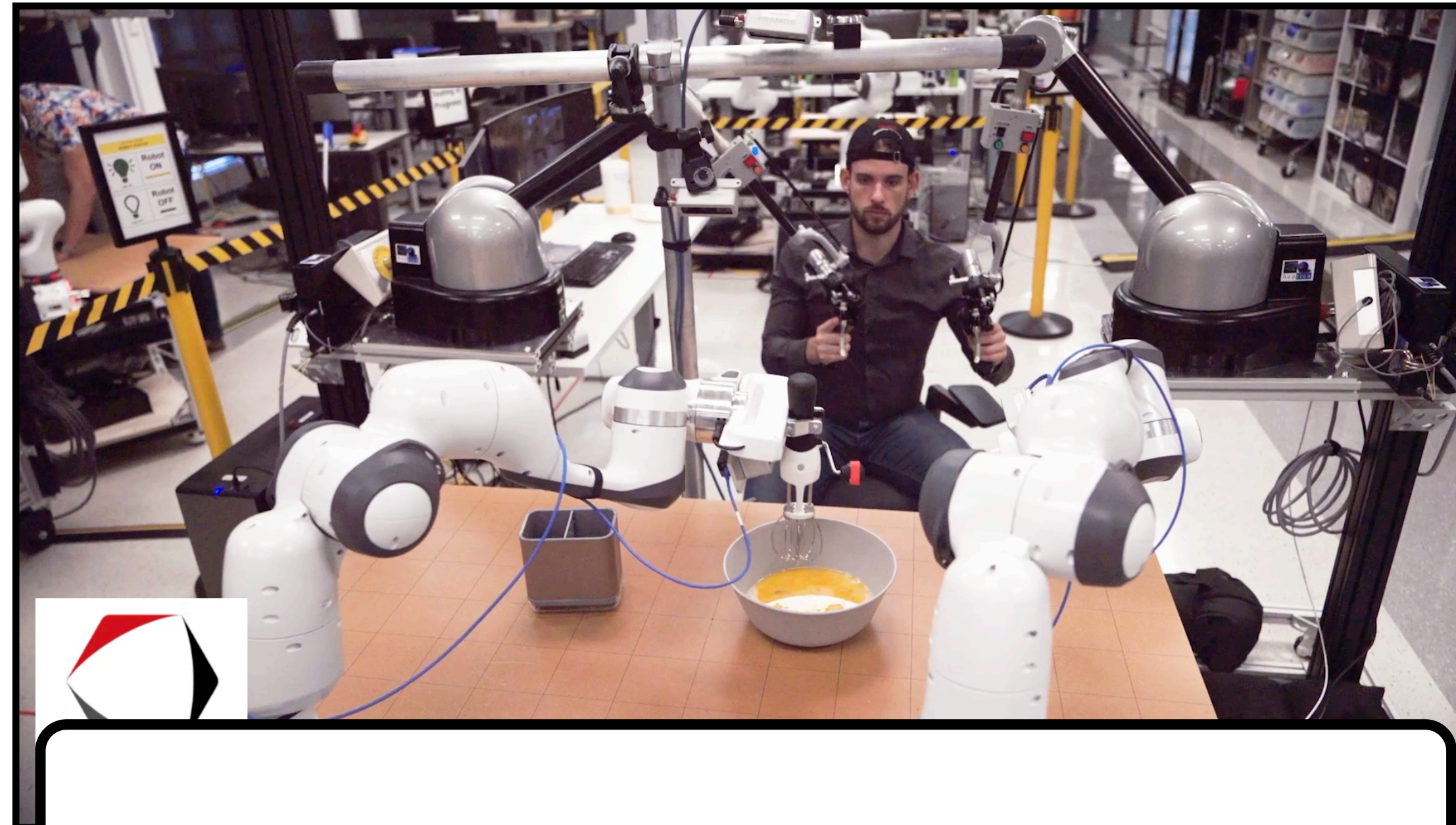


Defn. a **'policy'** maps {state x_t } \mapsto actions u_t

$$x_{t+1} = f(x_t, u_t)$$

nonlinear

Behavior Cloning



states x_t = state of robot + object

inputs u_t = robot action

'continuous tokens'

Themes



1. How things go **out-of-distribution**
2. How this differs from **discrete tokens**
3. Theoretical Guarantees
4. Some applications

$$x_{t+1} = f(x_t, u_t)$$

Behavior Cloning

- demonstrator $u^* \sim \pi^*(x^*)$
- robot imitation.

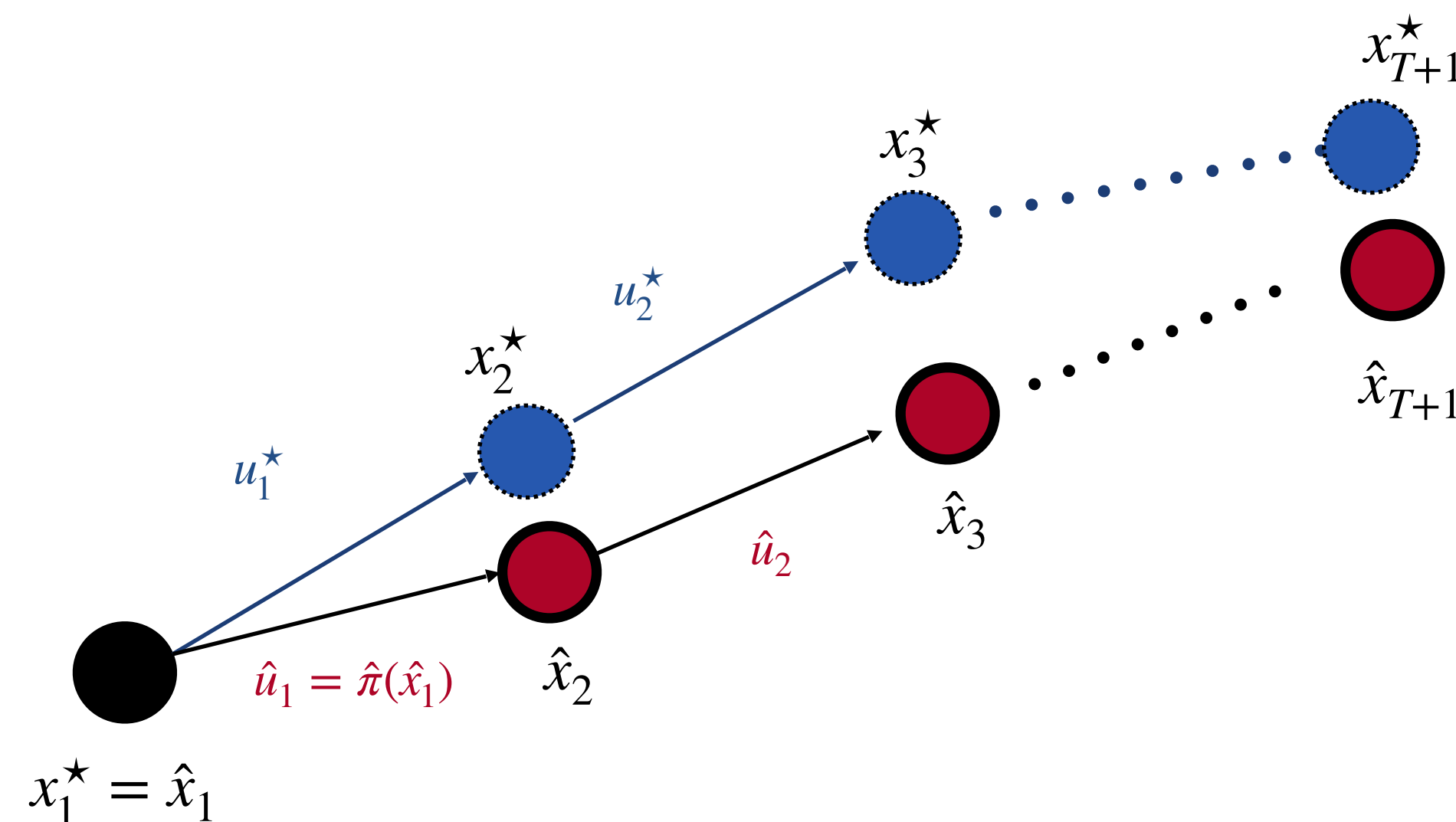
Algorithm Template:

(1) Collect N expert demonstrations

$$(x_{1:T}^*, u_{1:T}^*)$$

(2) train a **predictive model** to predict

$$\hat{\pi}(u | x) \approx \mathbb{P}[u_t^* = u | x_t^* = x]$$



1. supervised learning from **demonstration**

2. no **reward model** (given or inferred).

disclaimer: other approaches exist.

Behavior Cloning meets **Generative Models**

(as supervised learners)

Algorithm Template:

(1) Collect N expert demonstrations

$$(x_{1:T}^*, u_{1:T}^*)$$

(2) train a **generative model** to predict

$$\hat{\pi}(u | x) \approx \mathbb{P}[u_t^* = u | x_t^* = x]$$

$$\pi : x \mapsto u \sim P(x)$$

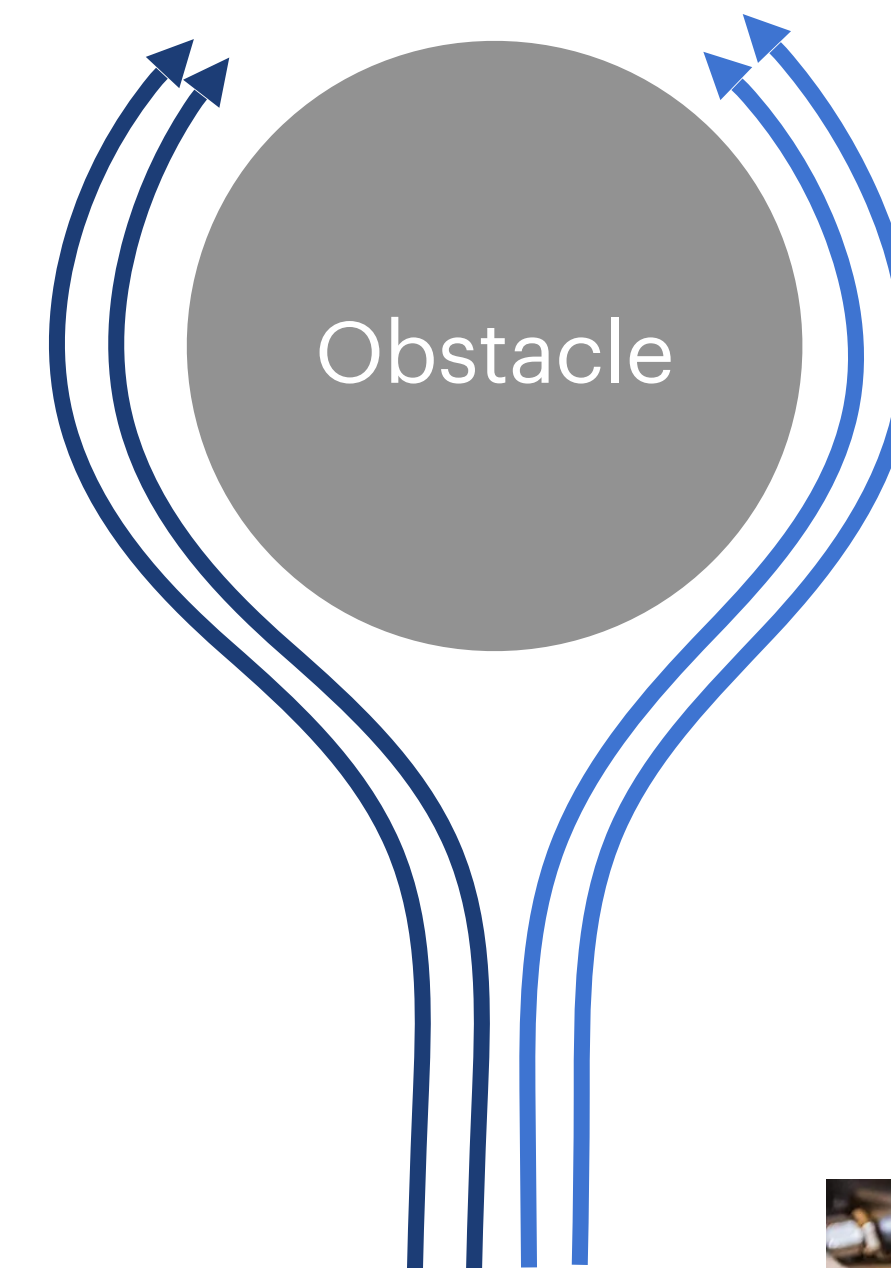
‘conditional sampling’

$$\pi : x \mapsto f(x) + \text{noise}$$

‘mean parametrization’



(e.g. Diffusion)



multi-modal expert data



Left Mode



Right Mode

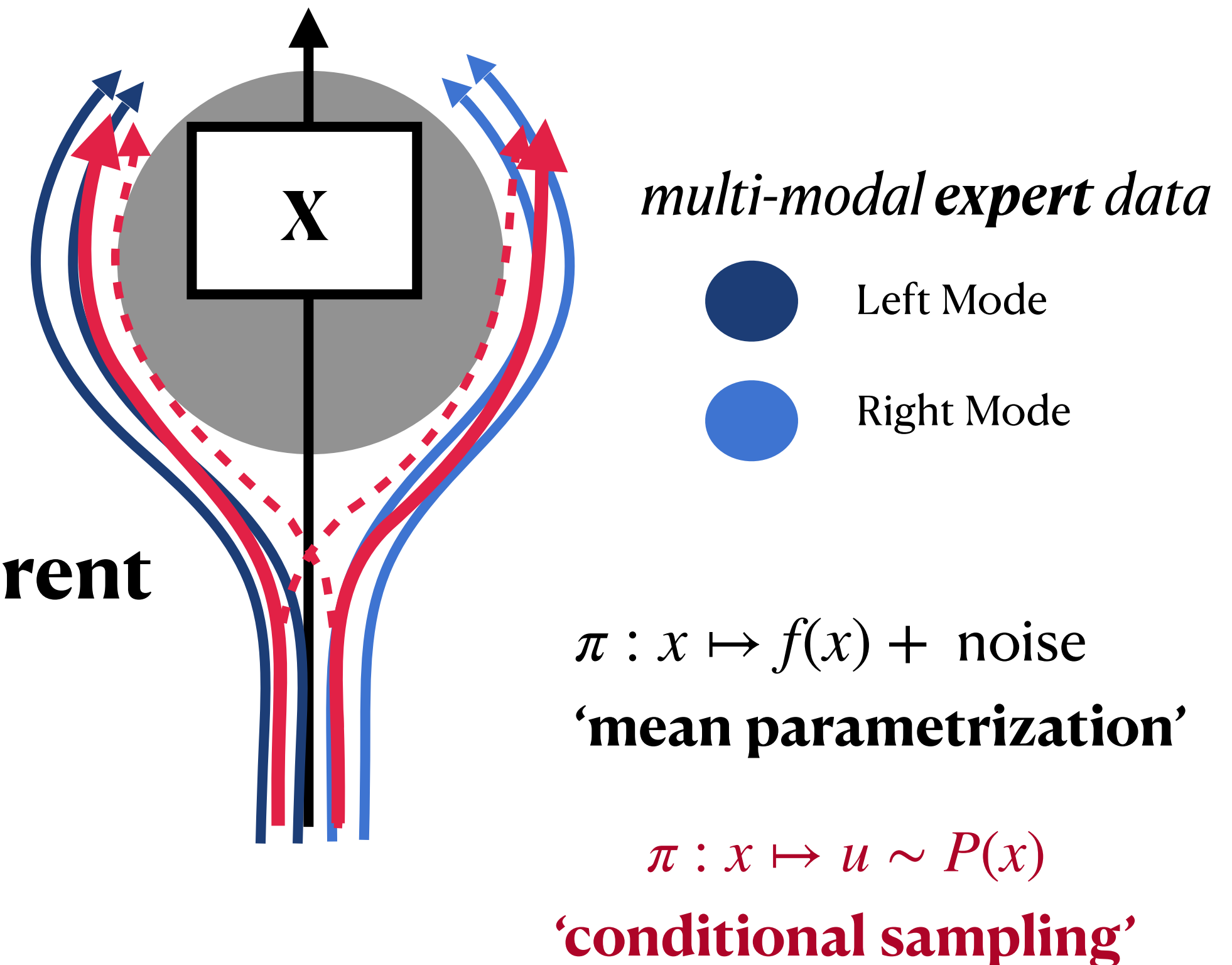


multiple strategies

Behavior Cloning meets **Generative Models**

Hypothesis 1: Condition sampling models can fit complex data distributions ('realizability')

Hypothesis 2: Condition sampling allow for different out-of-distribution inductive biases.

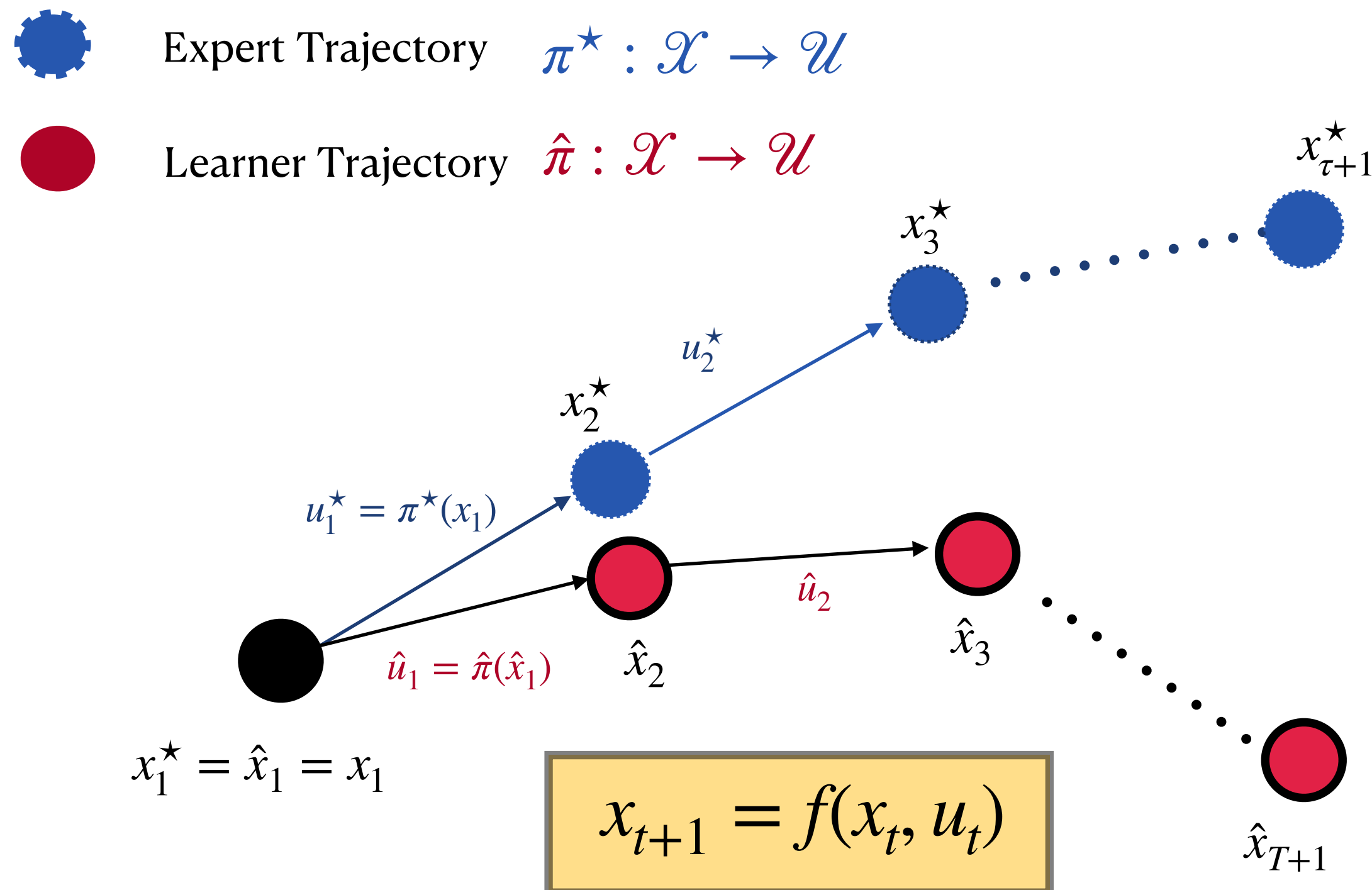


How distribution shift arises

Goal: Make trajectory distance small

$$\text{dist}(x_{1:T}^*, \hat{x}_{1:T}) = \max_t \|x_t^* - \hat{x}_t\|$$

(deterministic policies, in expectation over initial condition)



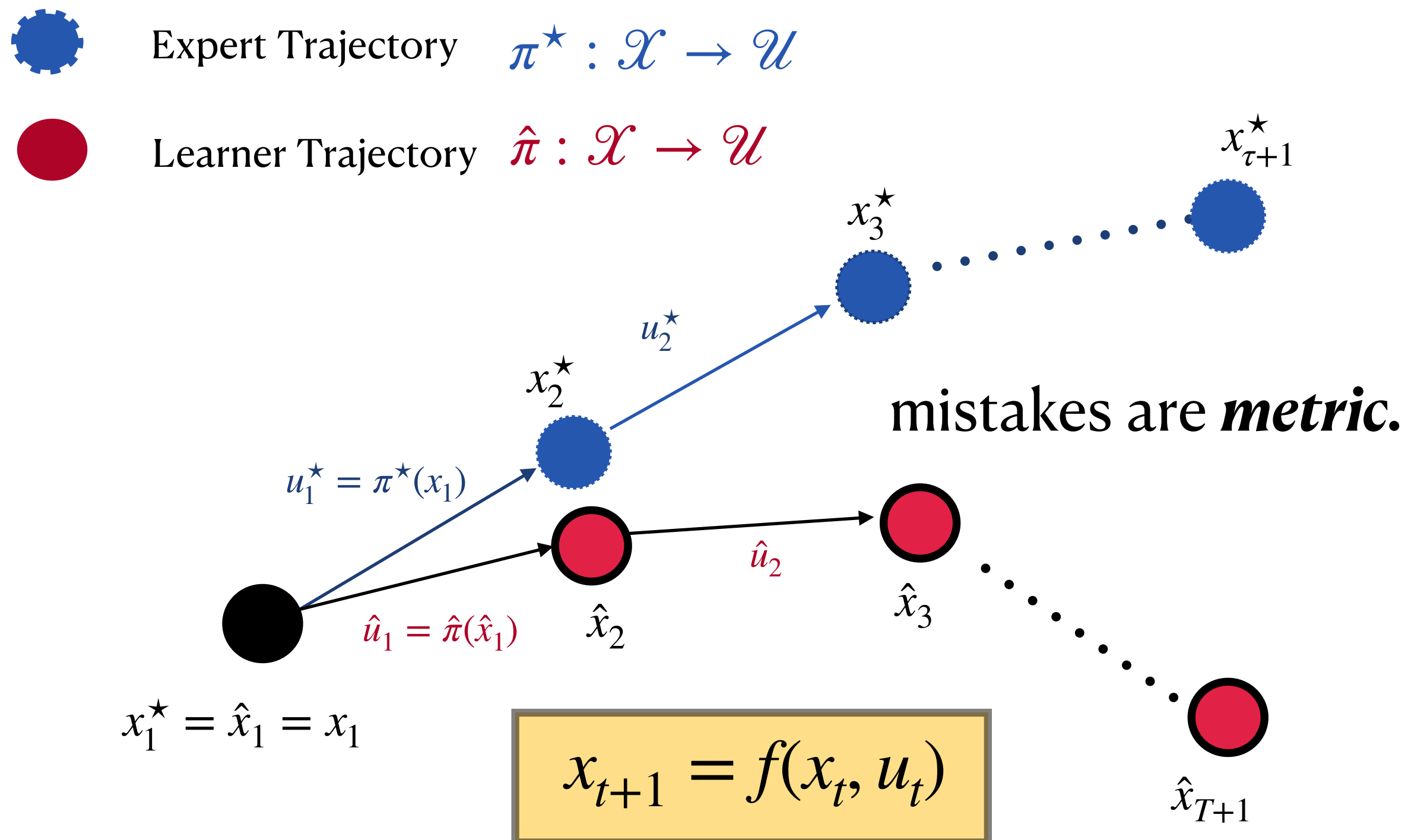
Challenge A: Error accumulates over time steps, possibly *exponentially in horizon!*

Challenge B: After error has accumulated, we are now **out of distribution.**

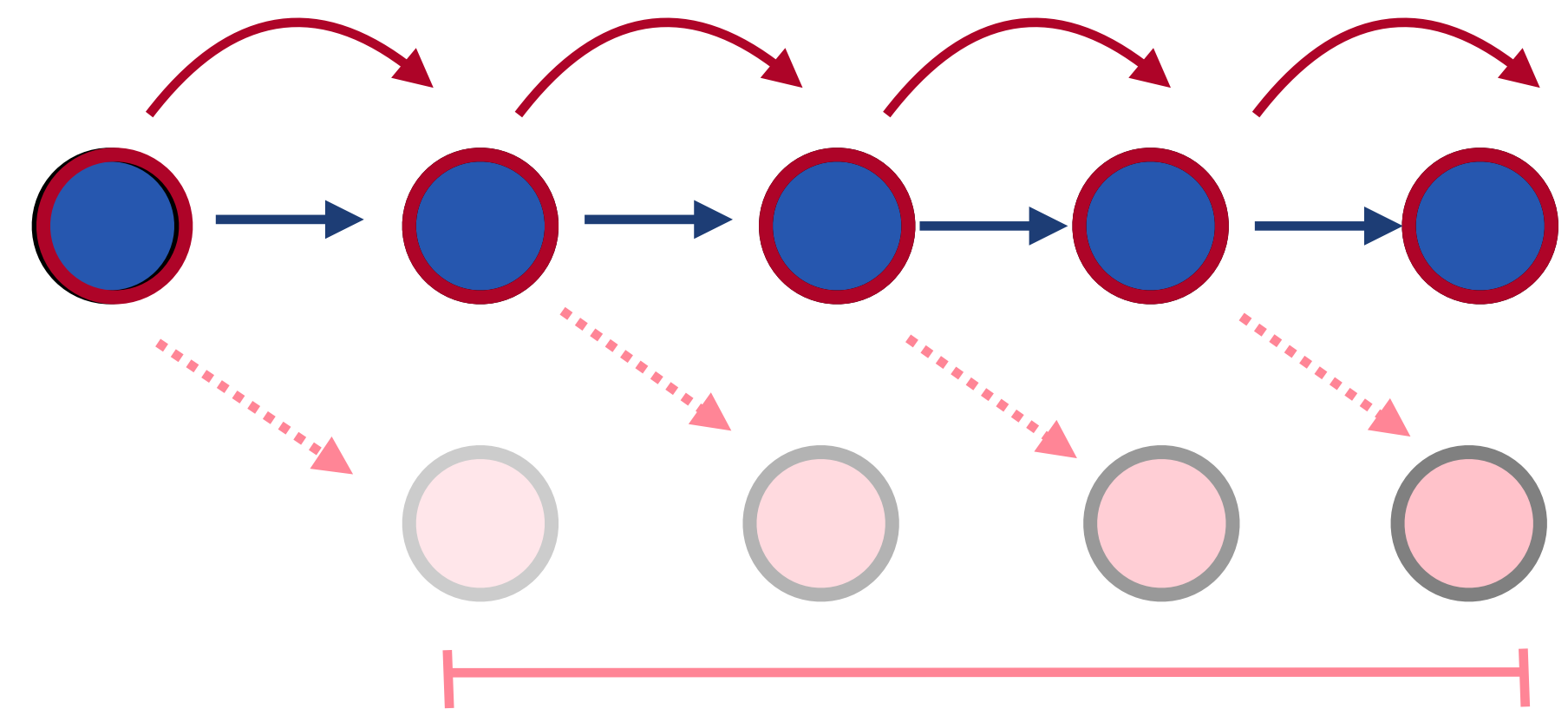
How this differs from **discrete tokens**

Goal: Make trajectory distance small

$$\text{dist}(x_{1:T}^*, \hat{x}_{1:T}) = \max_t \|x_t^* - \hat{x}_t\|$$



Contrast: Discrete Behavior Cloning



probabilistic mistakes accumulate at most linearly.

(e.g. DAGGER, see also Foster '24 et al.)

$$x_{t+1} = f(x_t, u_t)$$

Schematic of Results

Theorem 1 (informal): With **generative-model policies** (conditional sampling), we can imitate **without exponentially compounding error** in **contractive systems**.

Theorem 2 (informal): If **we know the dynamics**, there is a **reduction** to learning in contractive systems

Theorem 3 (informal): If **we don't know the dynamics**, learning is **hard**, even in “incrementally stable” but non-contractive systems.

$$x_{t+1} = f(x_t, u_t)$$

*all new results

Schematic of Results

Theorem 1 (informal): With **generative-model policies** (conditional sampling), we can imitate **without exponentially compounding error** in **contractive systems**.

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Theorem 3 (informal): If **we don't know the dynamics**, learning is **hard**, even in "incrementally stable systems."

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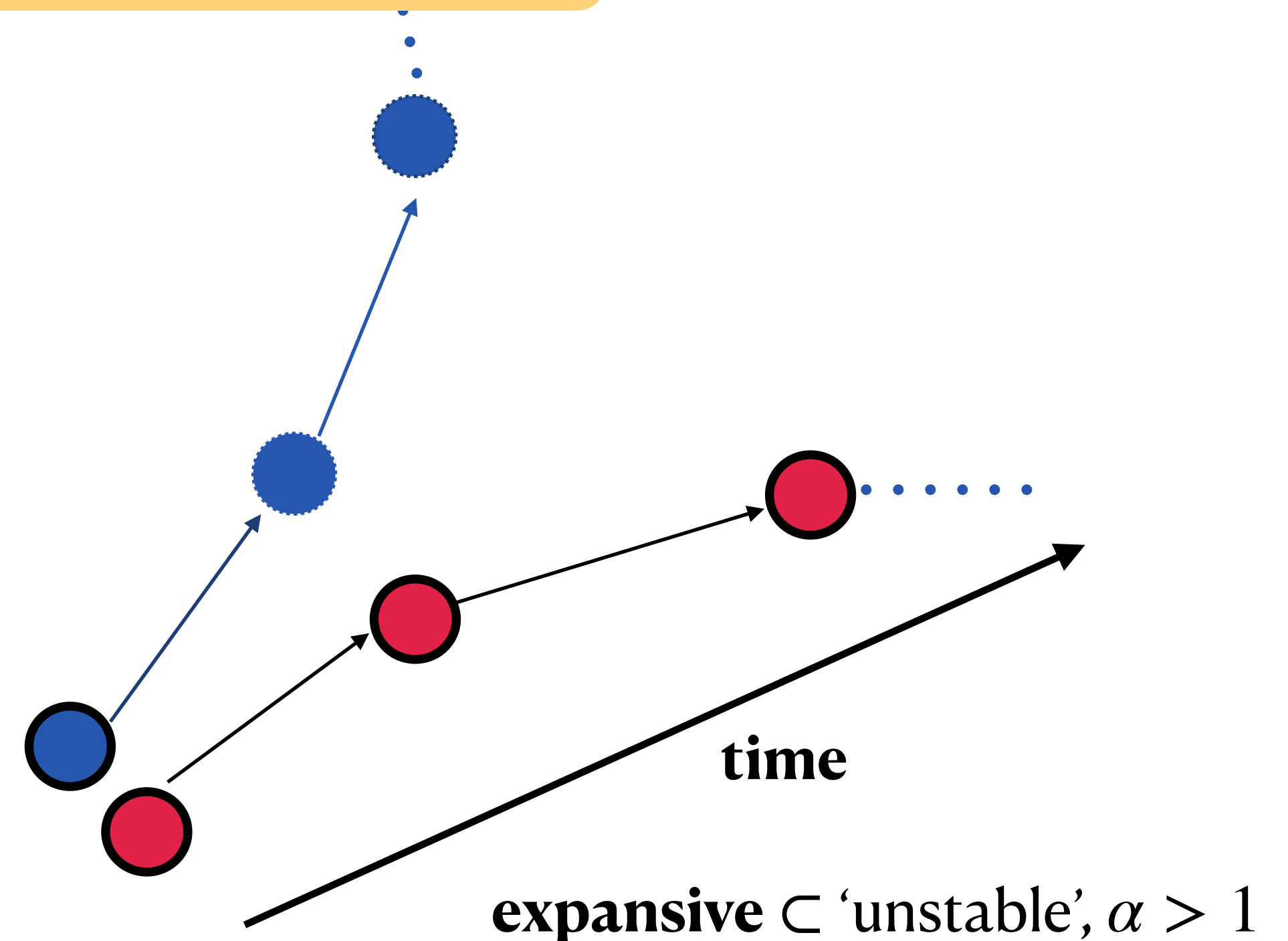
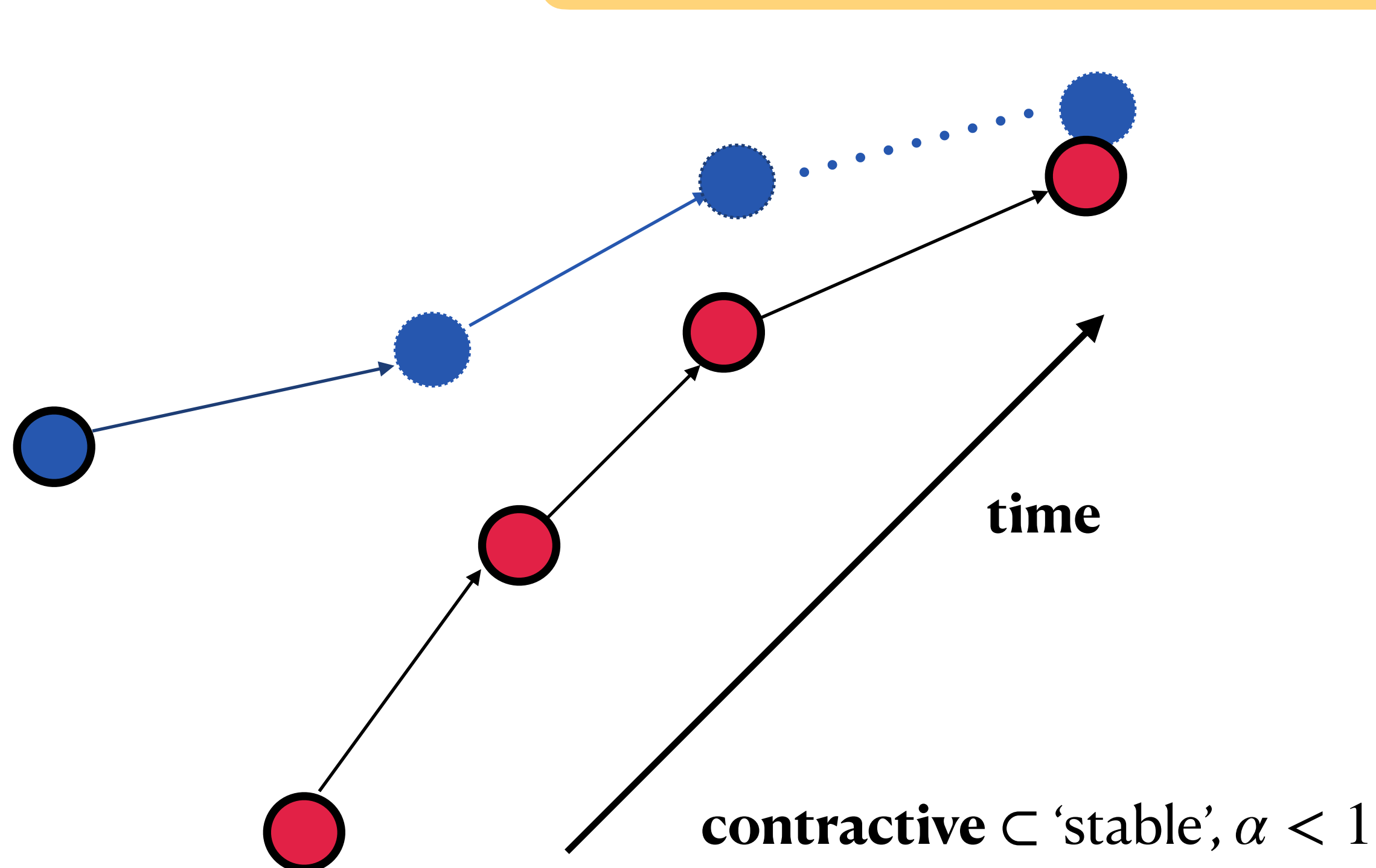
- A. Introduce contractive systems**
- B. Show just fitting the expert data **isn't enough**.**
- C. Introduce an inductive bias, **TVC**, guarantees imitation.**
- D. Given an algorithmic recommendation to ensure **TVC**.**

$$x_{t+1} = f(x_t, u_t)$$

Contractive Systems

Definition: We will say a system is (α, β) -contractive if

$$\|f(x', u') - f(x, u)\| \leq \alpha \|x - x'\| + \beta \|u - u'\|$$



$$x_{t+1} = f(x_t, u_t)$$

Contractive Systems

Definition: We will say a system is (α, β) -contractive if

$$\|f(x', u') - f(x, u)\| \leq \alpha \|x - x'\| + \beta \|u - u'\|$$

Lemma: If dynamics are (α, β) -contractive, given two sequences $(x_{1:T}^{\star}, u_{1:T}^{\star})$, $(\hat{x}_{1:T}, \hat{u}_{1:T})$ with $x_1^{\star} = \hat{x}_1$, and if $\alpha < 1$, we get

$$\max_{1 \leq t \leq T} \|x_t^{\star} - \hat{x}_t\| \leq \frac{\beta}{1 - \alpha} \max_{1 \leq t \leq T} \|u_t^{\star} - \hat{u}_t\|$$

special case of 'stability'

$$x_{t+1} = f(x_t, u_t)$$

Is Low Training Error Enough?

Example (*Contractive, Scalar Dynamics*):

(a) $f(x, u) = .9x + u$

(b) $\pi^*(x) = 0$

(c) *training data*: “0”-trajectory $x_1^* = x_2^* = \dots = 0$

Bad Learner Policy: $\hat{\pi}^{\text{Bad}}(x) = .15x + \epsilon$

(a) For all training x , $\pi^*(x) - \hat{\pi}^{\text{Bad}}(x) = \epsilon$

(b) On deployment, $\hat{x}_t \geq (1.05)^t \epsilon = e^{\Omega(t)} \cdot \epsilon$

‘feedback’: $f(x, \hat{\pi}(x))$



inductive bias creates ‘feedback’

$$x_{t+1} = f(x_t, u_t)$$

Is Low Training Error Enough?

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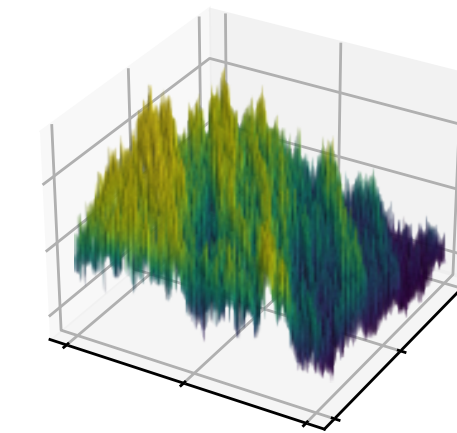
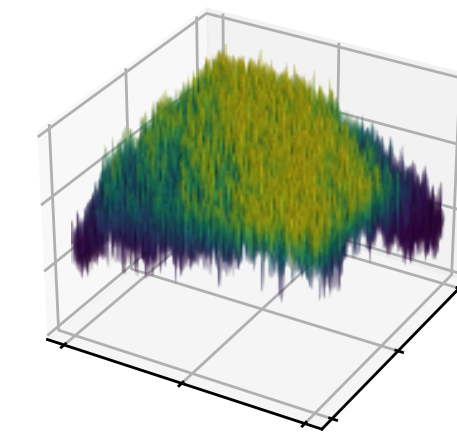
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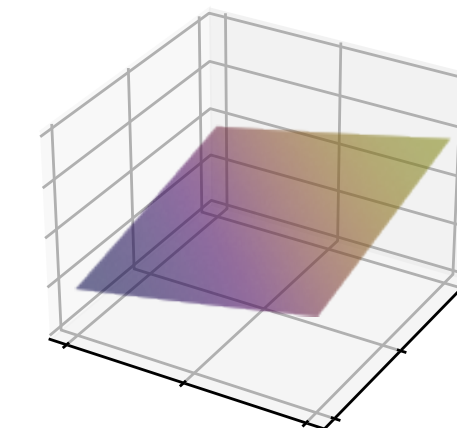
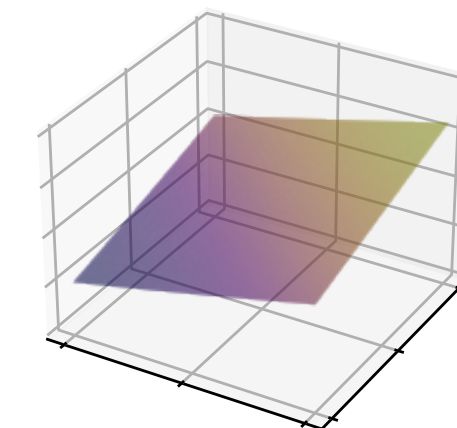
Butterfly Effects of SGD, Block ‘24

Step 115000

Step 120000



Reward



BC error



inductive bias creates ‘feedback’

can be improved by **better data coverage**

A different inductive bias.

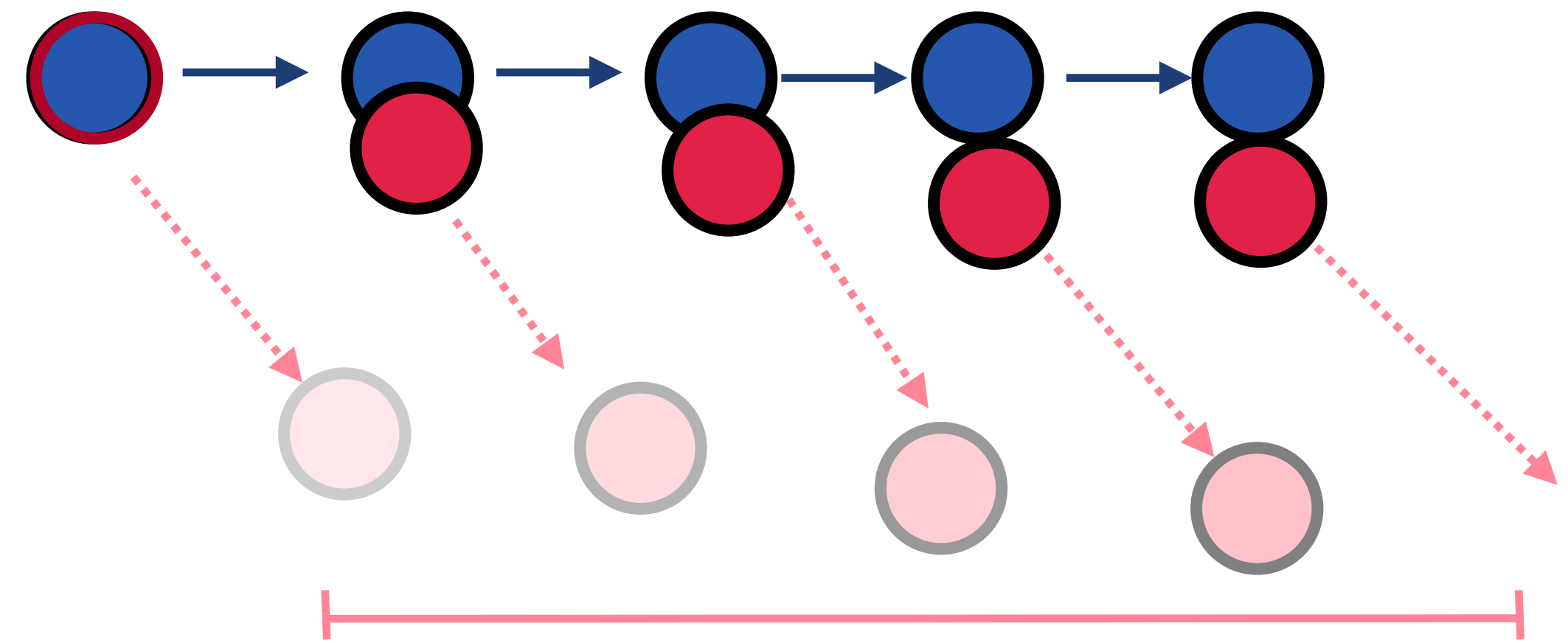
Example (*Contractive, Scalar Dynamics*):

$$f(x, u) = .9x + u, \pi^*(x) = 0$$

Not-So-Bad Learner Policy: $\hat{\pi}^{\text{NSB}}(x) = \text{Bernoulli}(\min\{1, .15x\}) + \epsilon$

(a) For all training x , $\pi^*(x) - \hat{\pi}(x) = \epsilon$

(b) On deployment, $\hat{x}_t \leq O(\epsilon)$ w.p. $1 - O(t\epsilon)$



probabilistic mistakes accumulate at most linearly.

'Discrete Token Error' ?

Example (*Contractive, Scalar Dynamics*):

$$f(x, u) = .9x + u, \pi^*(x) = 0$$

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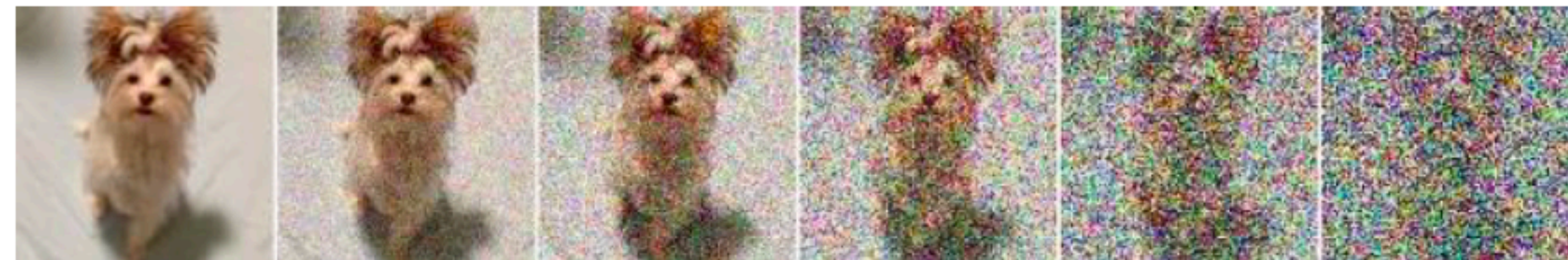
↑
convert 'metric mistakes' into 'probabilistic mistakes'

‘Discrete Token Error’ ?

Example (*Contractive, Scalar Dynamics*):

$$f(x, u) = .9x + u, \pi^*(x) = 0$$

Not-So-Bad Learner Policy: $\hat{\pi}^{\text{NSB}}(x) = \text{Bernoulli}(\min\{1, .15x\}) + \epsilon$



Generative models

For small enough \mathbf{x} , $\hat{\pi}^{\text{Bad}}(x) = \mathbb{E}[\hat{\pi}^{\text{NSB}}(x)] = .15x + \epsilon$ is the **OG bad policy**.

Total Variation Continuity

Definition: We say $\pi(x)$ is **L-TVC** if $\text{TV}(\pi(x), \pi(x')) \leq L\|x - x'\|$

$$\text{TV}(P, Q) := \inf_{(X_P, X_Q) \sim \mu} \Pr [X_P \neq X_Q]$$

Example 1: $\hat{\pi}^{\text{NSB}}(x) = \text{Bernoulli}(\min\{1, .15x\}) + \epsilon$ is $L = .15$ TVC

Example 2: $\hat{\pi}^{\text{Bad}}(x) = \mathbb{E}[\hat{\pi}^{\text{NSB}}(x)] = .15x + \epsilon$ is not TVC

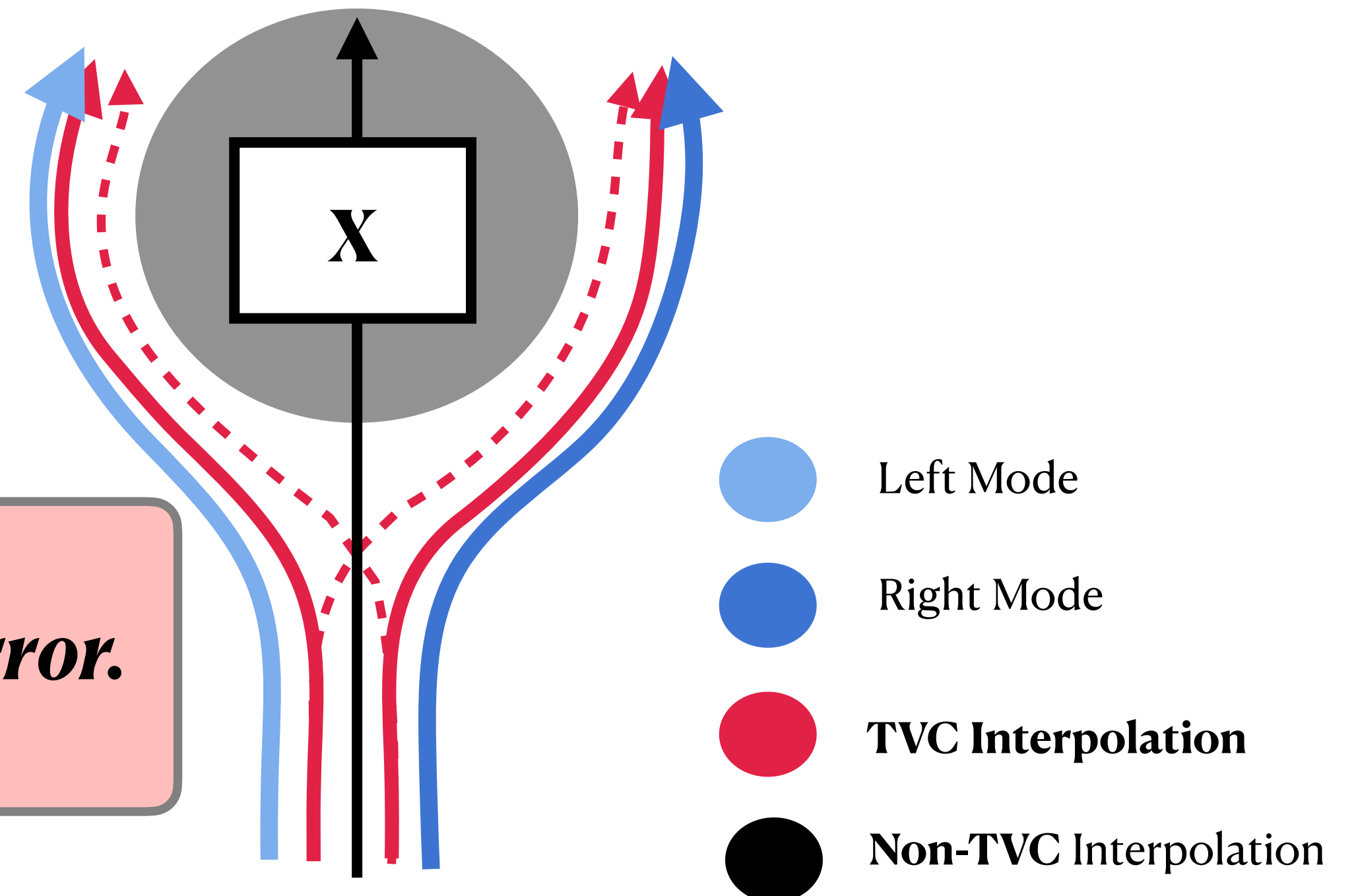
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TVC is the opposite of **mode-collapse**

We will show *TVC Policies have low execution error.*



$$x_{t+1} = f(x_t, u_t)$$

Problem Definition

Definition: Let \mathbf{P} , \mathbf{Q} be two distribution on the same normed space. We define

$$\text{TV}_\epsilon(P, Q) := \inf_{(X_P, X_Q) \sim \mu} \Pr \left[\|X_P - X_Q\| > \epsilon \right]$$

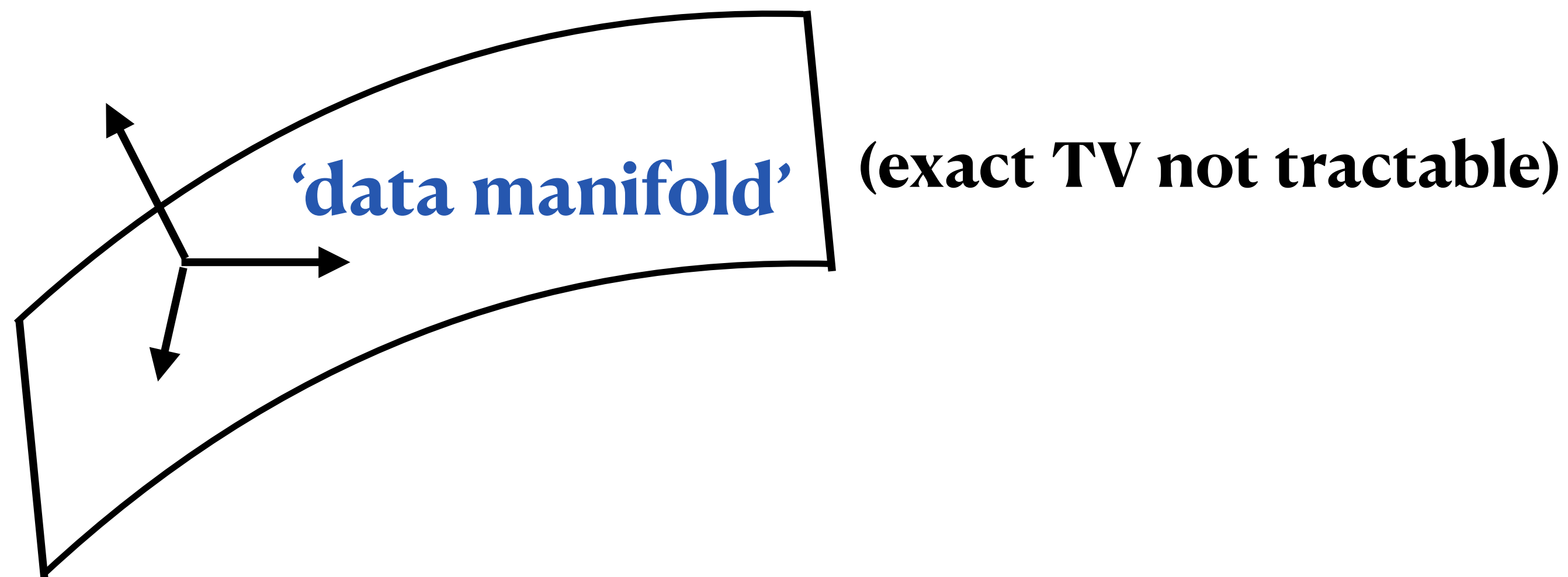
1. ‘Optimal Transport Distance’, reduces to regular **TV** for $\epsilon = 0$
2. A way of measuring distance between **continuous-valued R.V.s**
3. Like **Wasserstein**, but easier to work with for imitation learning

$$x_{t+1} = f(x_t, u_t)$$

Problem Definition

Training Error: Suppose we get trajectories $(x_1^*, u_1^*, x_2^*, u_2^*, \dots, x_H^*, u_H^*), u_t^* \sim \pi^*(x_t^*)$

$$D_{\text{train}, \epsilon}(\hat{\pi} \parallel \pi^*) := \max_t \mathbb{E}_{x_t^*} \text{TV}_\epsilon(\pi^*(x_t^*), \hat{\pi}(x_t^*)) \quad (\text{can be made small w/ DDPM})$$



$$x_{t+1} = f(x_t, u_t)$$

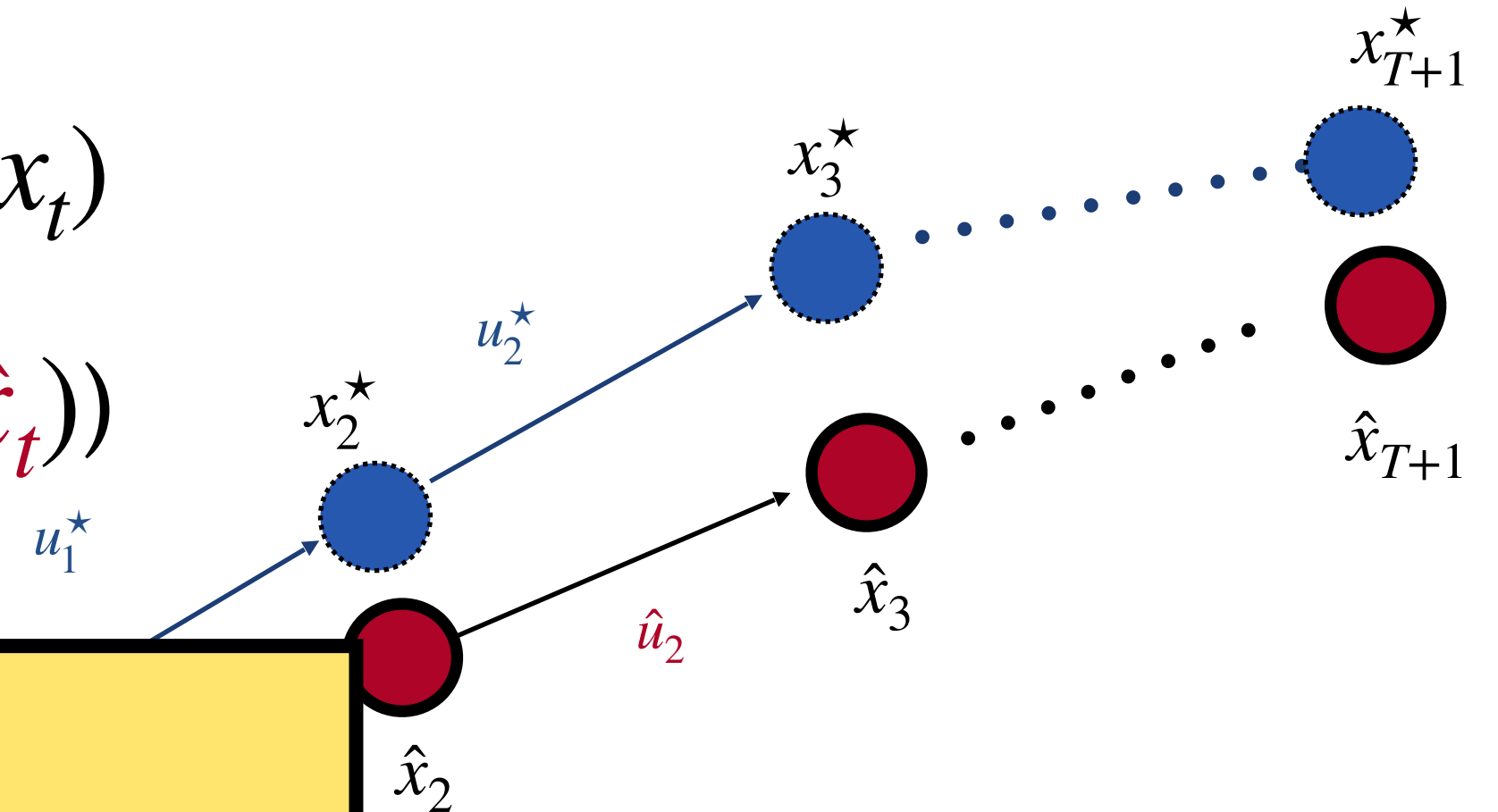
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Test Error: We roll out $(\hat{x}_1, \hat{u}_1, \hat{x}_2, \hat{u}_2, \dots, \hat{x}_H, \hat{u}_H), \hat{u}_t \sim \hat{\pi}(x_t)$

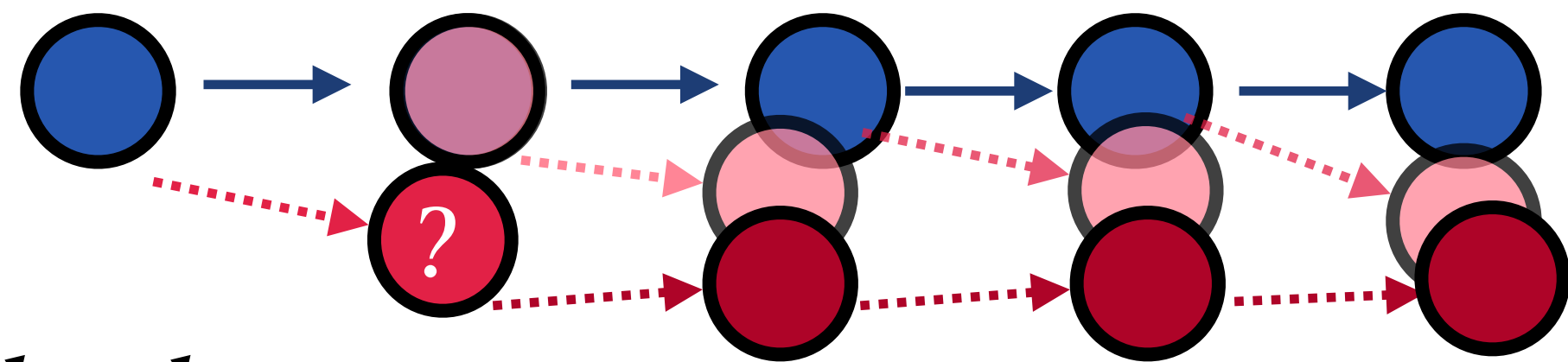
$$D_{\text{test}, \epsilon}(\hat{\pi} \parallel \pi^*) := \max_t \text{TV}_\epsilon(\text{Law}(x_t^*), \text{Law}(\hat{x}_t))$$



$$\text{Goal: } D_{\text{test}, \epsilon} \leq \text{poly}(H) \cdot D_{\text{train}, \epsilon'}$$

A First Guarantee

Theorem: If $\hat{\pi}$ is L-TVC,

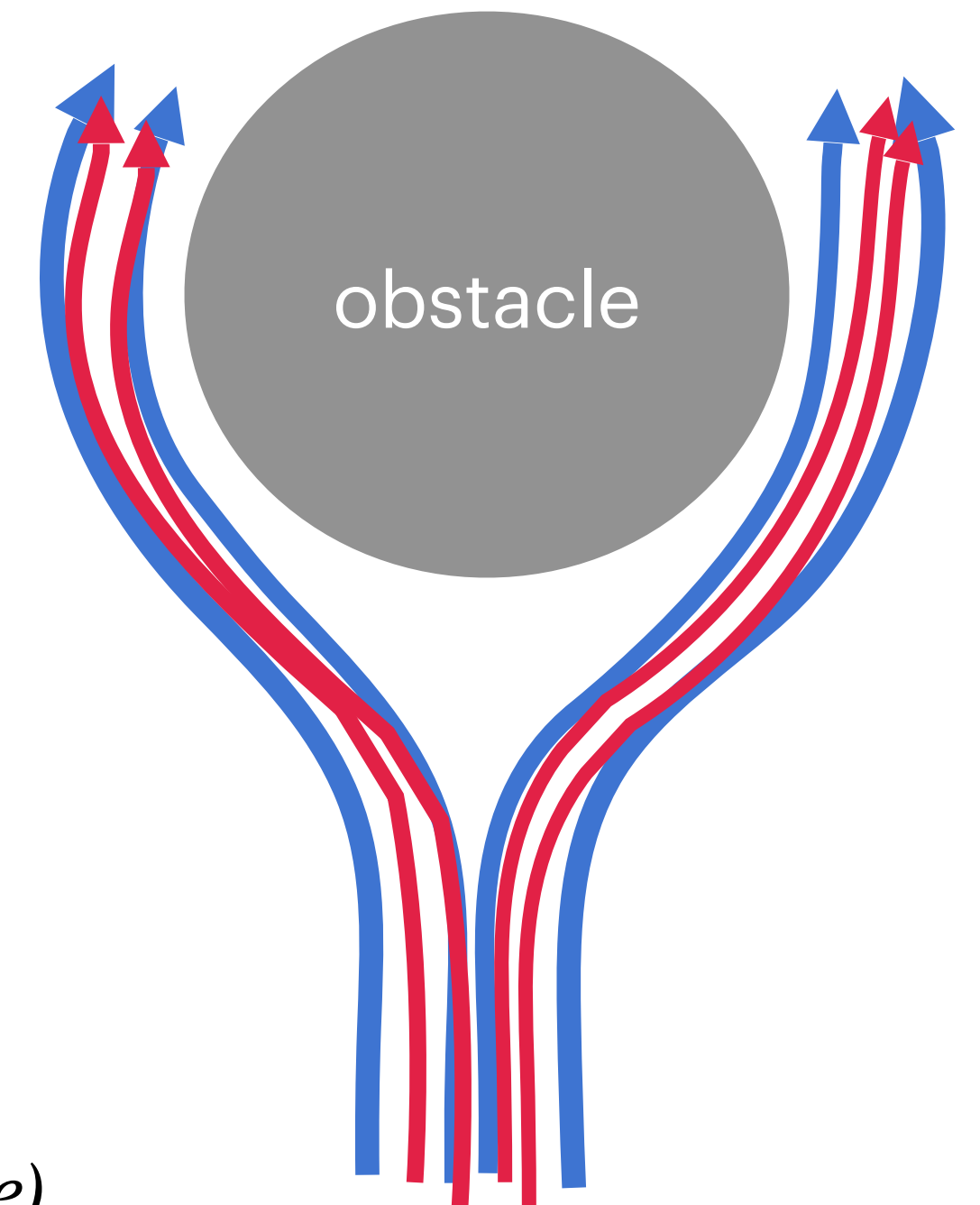


Proof Sketch:

(1) TVC implies **coupling** s.t. $\mathbb{P}[\hat{u}_t \sim \hat{\pi}(\hat{x}_t) \neq \hat{u}'_t \sim \hat{\pi}(x_t^*)] \leq L\epsilon$ (change of measure)

(2) Supervised Learning ensures that $\hat{u}'_t \sim \hat{\pi}(\hat{x}_t^*) \approx \pi^*(x_t^*)$

(3) Contractive of dynamics implies errors **compound** by at most **c-factor**



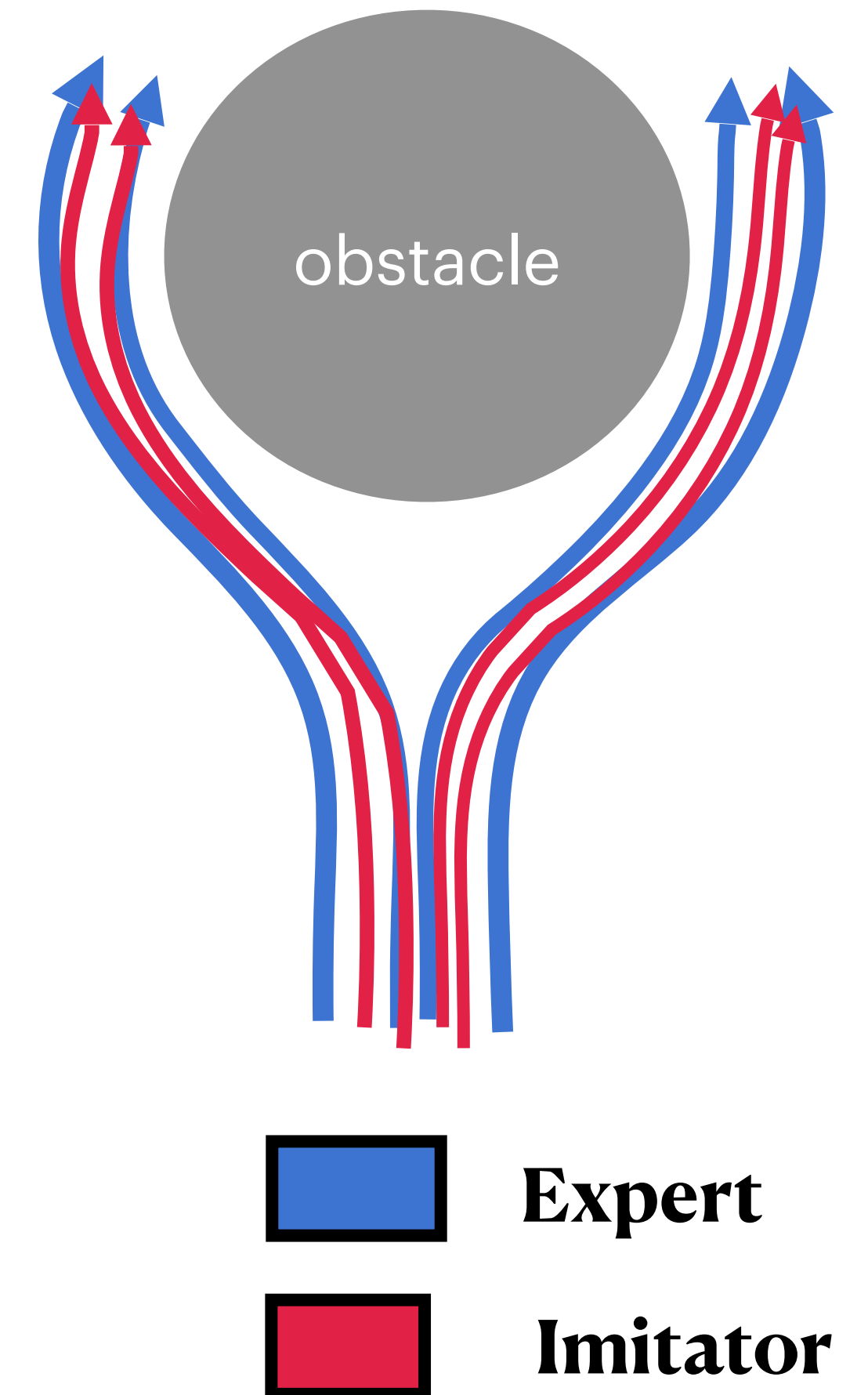
A Recap

Theorem: If $\hat{\pi}$ is L-TVC, and system is $(1 - c^{-1}, O(1))$ contractive

$$D_{\text{test},\epsilon}(\hat{\pi} \parallel \pi^{\star}) \leq O(cLH) \cdot D_{\text{train},\epsilon/c}(\hat{\pi} \parallel \pi^{\star})$$

- (1) **Distribution Shift can be bad in continuous-state BC**
- (2) **TVC + Contractive Dynamics*** gets us around the issue

TVC is a nice inductive bias. By how do we get it?



Replica Noising.

TVC via Noising

Elementary Lemma: Let $\hat{\pi} : x \in \mathbb{R}^d \mapsto \Delta(\mathcal{U})$

Define **smoothed policy** $\hat{\pi}_\sigma : x \mapsto \hat{\pi} \circ \mathcal{N}(x, \sigma^2 \mathbf{I})$

Then $\hat{\pi}_\sigma$ is $(1/2\sigma)$ - TVC

Proof: $\text{TV}(\hat{\pi}_\sigma(x), \hat{\pi}_\sigma(x')) \leq \text{TV}(\mathcal{N}(x, \sigma^2 \mathbf{I}), \mathcal{N}(x', \sigma^2 \mathbf{I}))$

$$\leq \left(\frac{1}{2} \text{KL}(\mathcal{N}(x, \sigma^2 \mathbf{I}), \mathcal{N}(x', \sigma^2 \mathbf{I})) \right)^{1/2}$$

$$= \frac{1}{2\sigma} \|x - x'\|$$

(Data Processing)

(Pinsker)

(Stat Class)

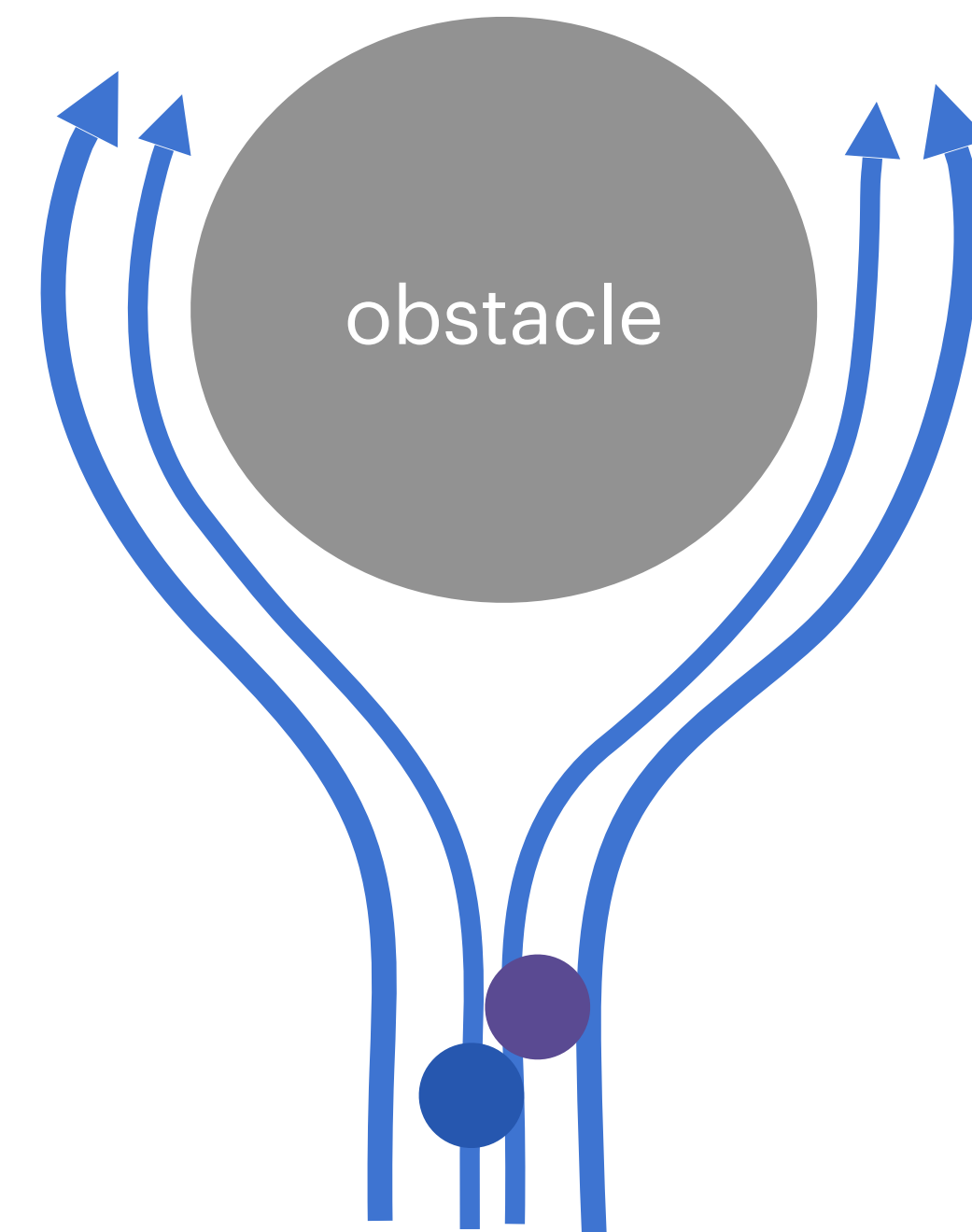
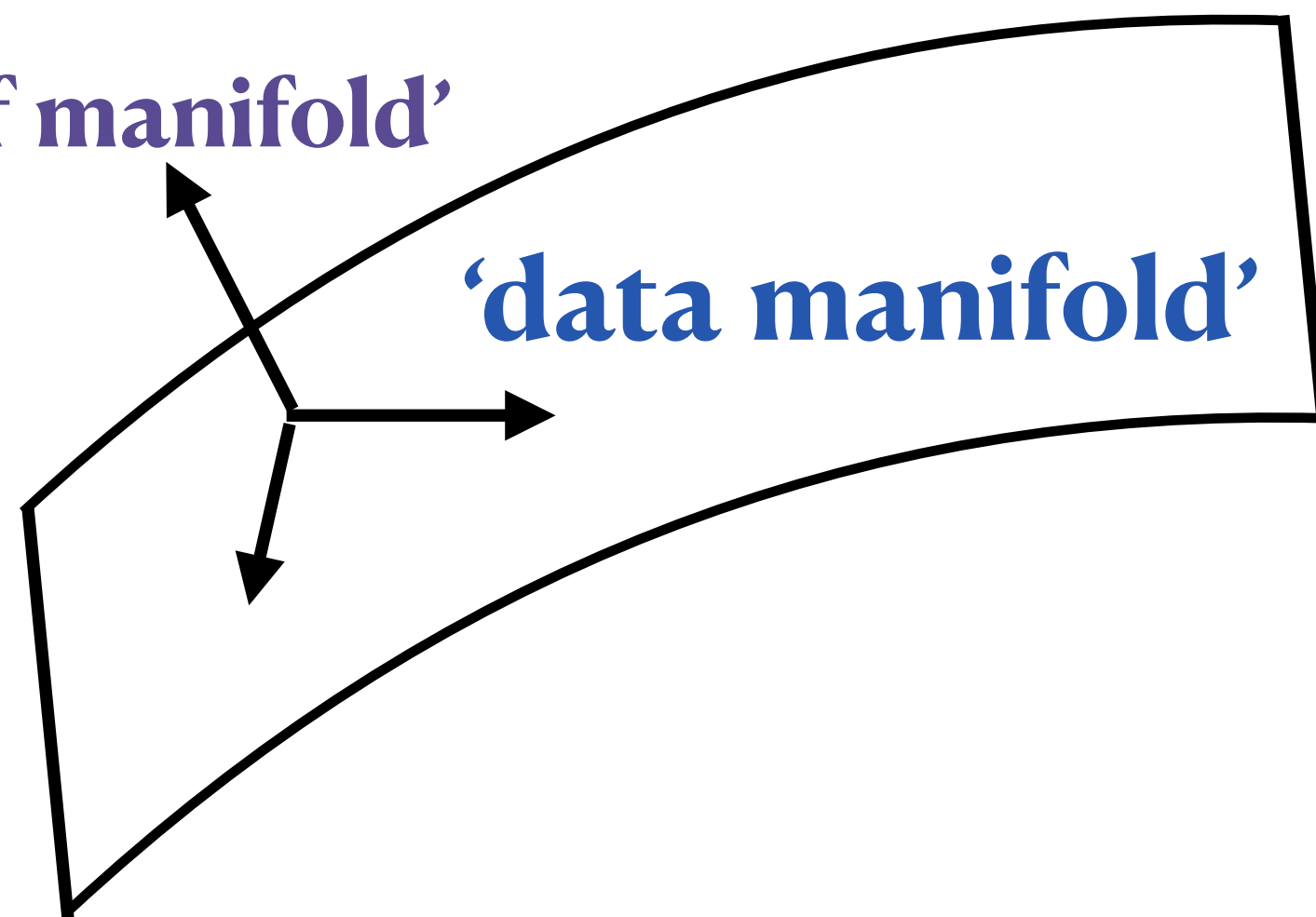
TVC via Noising

Smoothed policy $\hat{\pi}_\sigma : x \mapsto \hat{\pi}(x + \sigma w)$ is $(1/2\sigma)$ -TVC

1. **Nothing new here** - we know noising gives robustness
2. This might be a **terrible idea**:

'noise goes off manifold'

'data manifold'



Noise might knock me off modes

Replica Noising

Algorithm

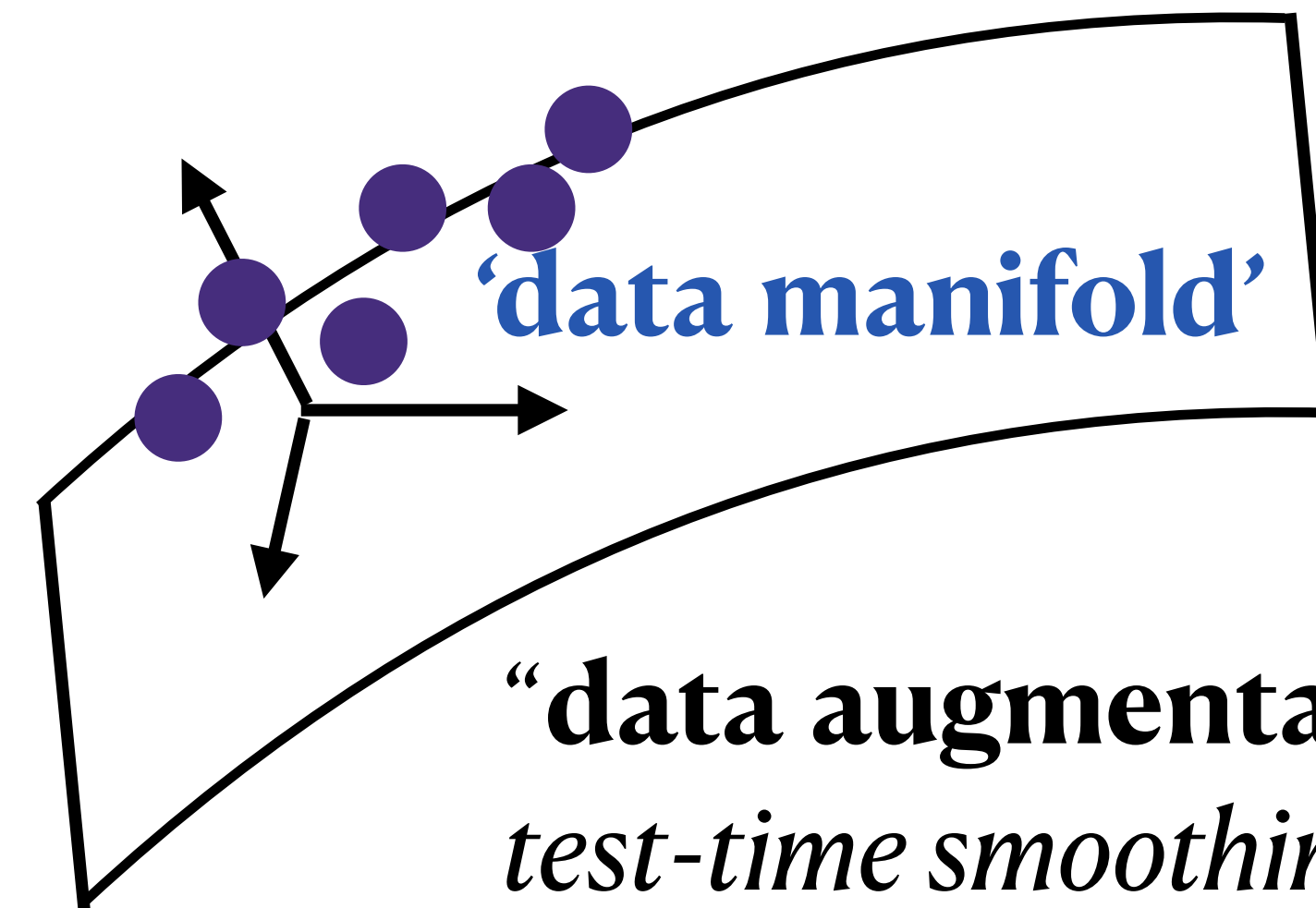
(1) Collect demonstrations $\{x^*, u^* \sim \pi^*(x^*)\}$

(2) Train **policy** (e.g. Diffusion)

$$\hat{\pi}(x^* + \sigma w) \approx \mathbb{P}[u^* \mid x^* + \sigma w]$$

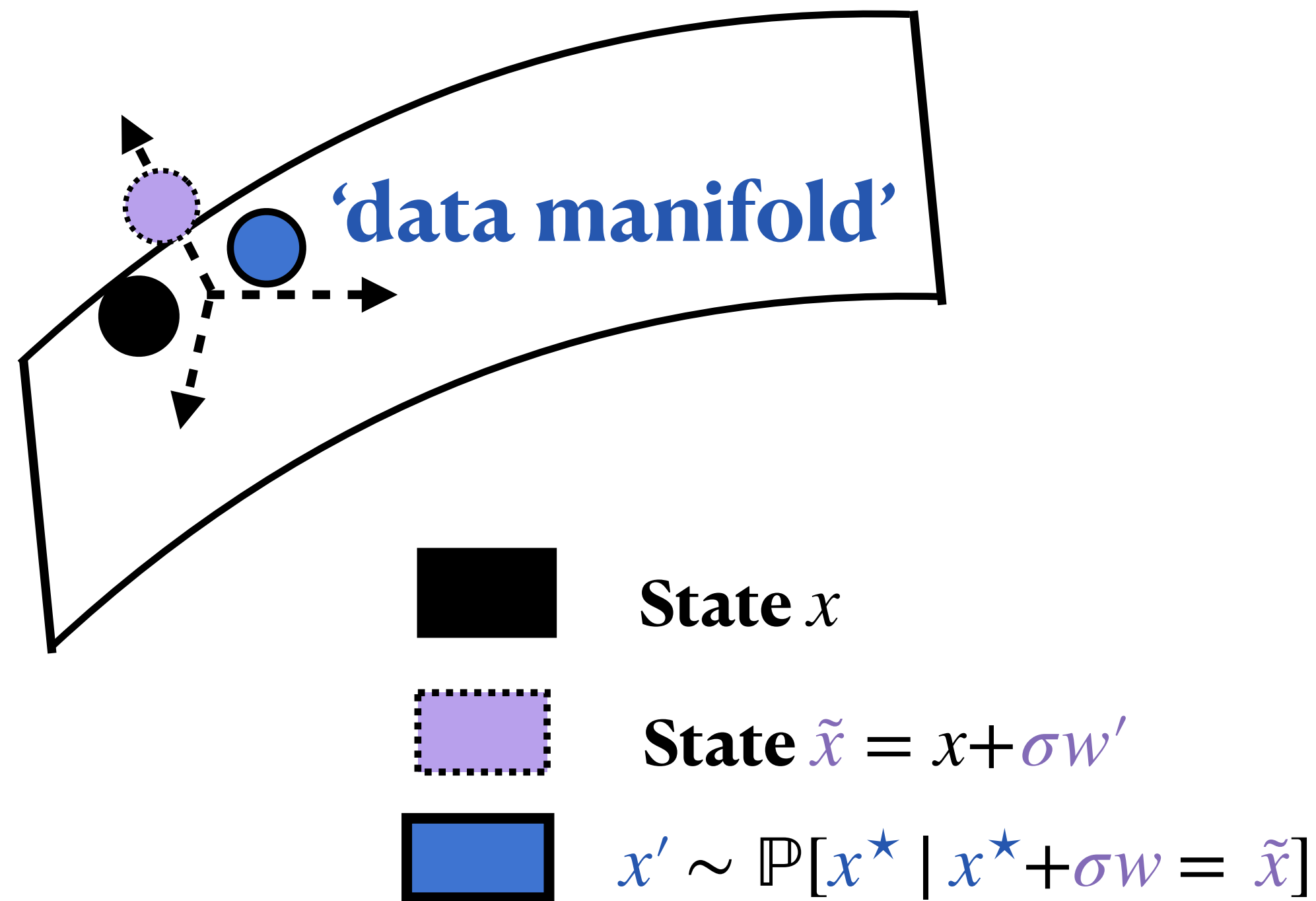
(3) Deploy $\hat{\pi}_\sigma(x) = \hat{\pi}(x + \sigma w')$

train with **same noise** as testing



'conditional sampling'

Replica Noising



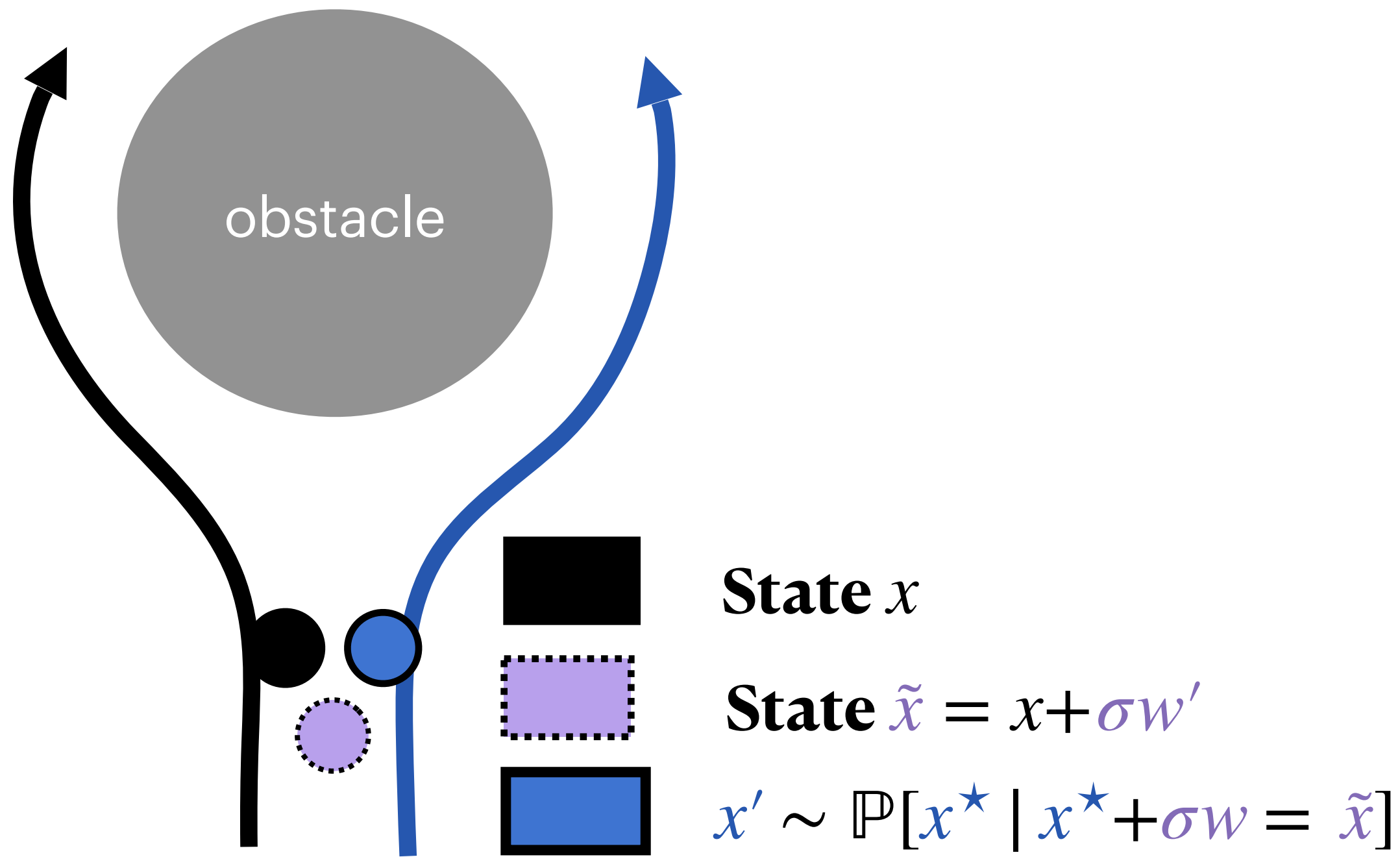
Observation: If $\hat{\pi}(x^* + \sigma w) = \mathbb{P}[u^* | x^* + \sigma w]$ is perfect, then,

1. $\hat{\pi}(x) = \pi^* \circ \mathbb{P}[x^* | x^* + \sigma w = x]$

2. $\hat{\pi}_\sigma(x) = \pi^* \circ \mathbb{P}[x^* | x^* + \sigma w = x + \sigma w']$

$$\mathbb{K}^{\text{rep}} : \mathcal{X} \mapsto \Delta(\mathcal{X})$$

Replica Noising



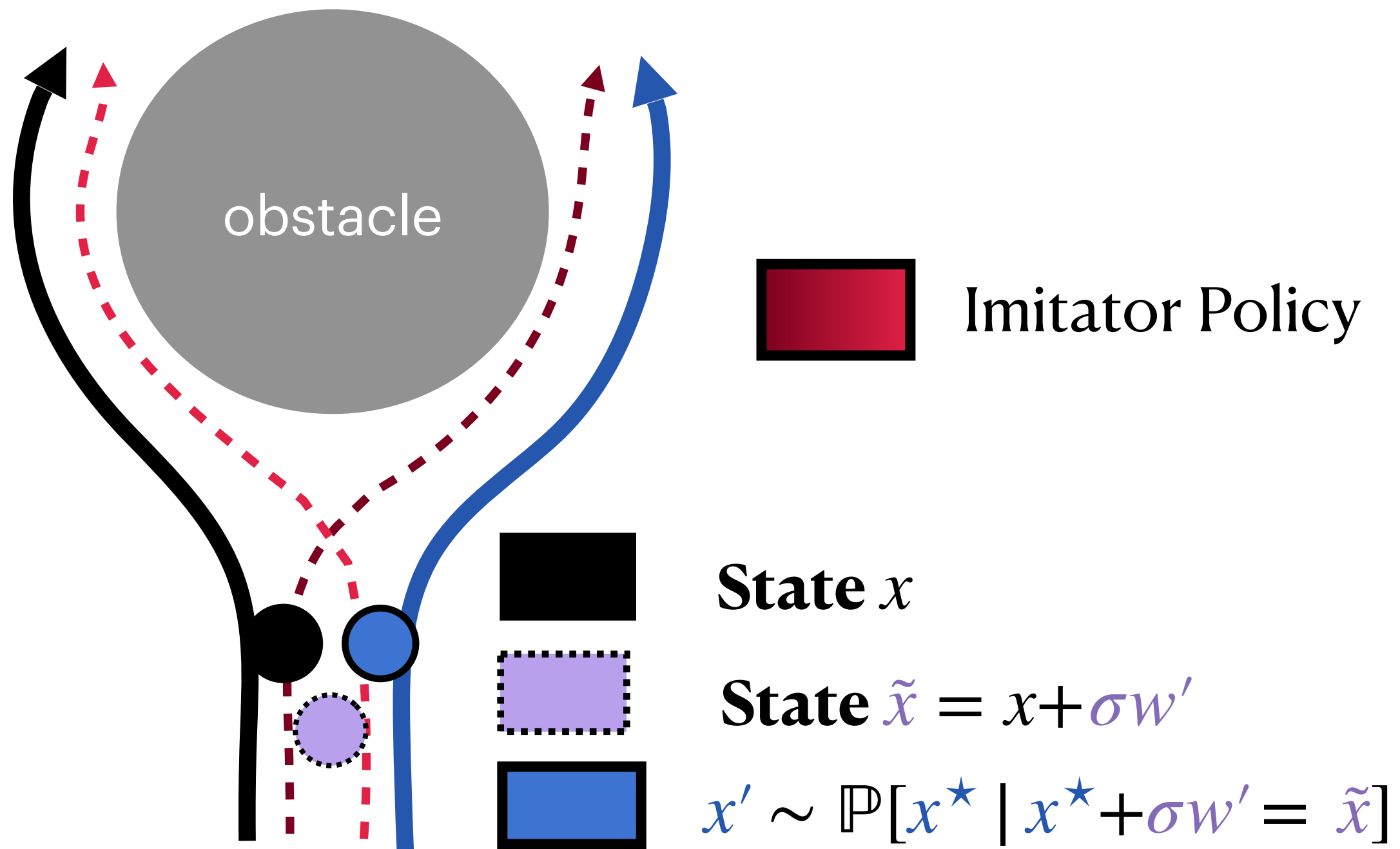
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2. $\hat{\pi}_\sigma(x) = \pi^* \circ \mathbb{P}[x^* \mid x^* + \sigma w = x + \sigma w']$

$$K^{\text{rep}} : \mathcal{X} \mapsto \Delta(\mathcal{X})$$

*proof via more complex
coupling argument using the
replica property*



Replica Noising

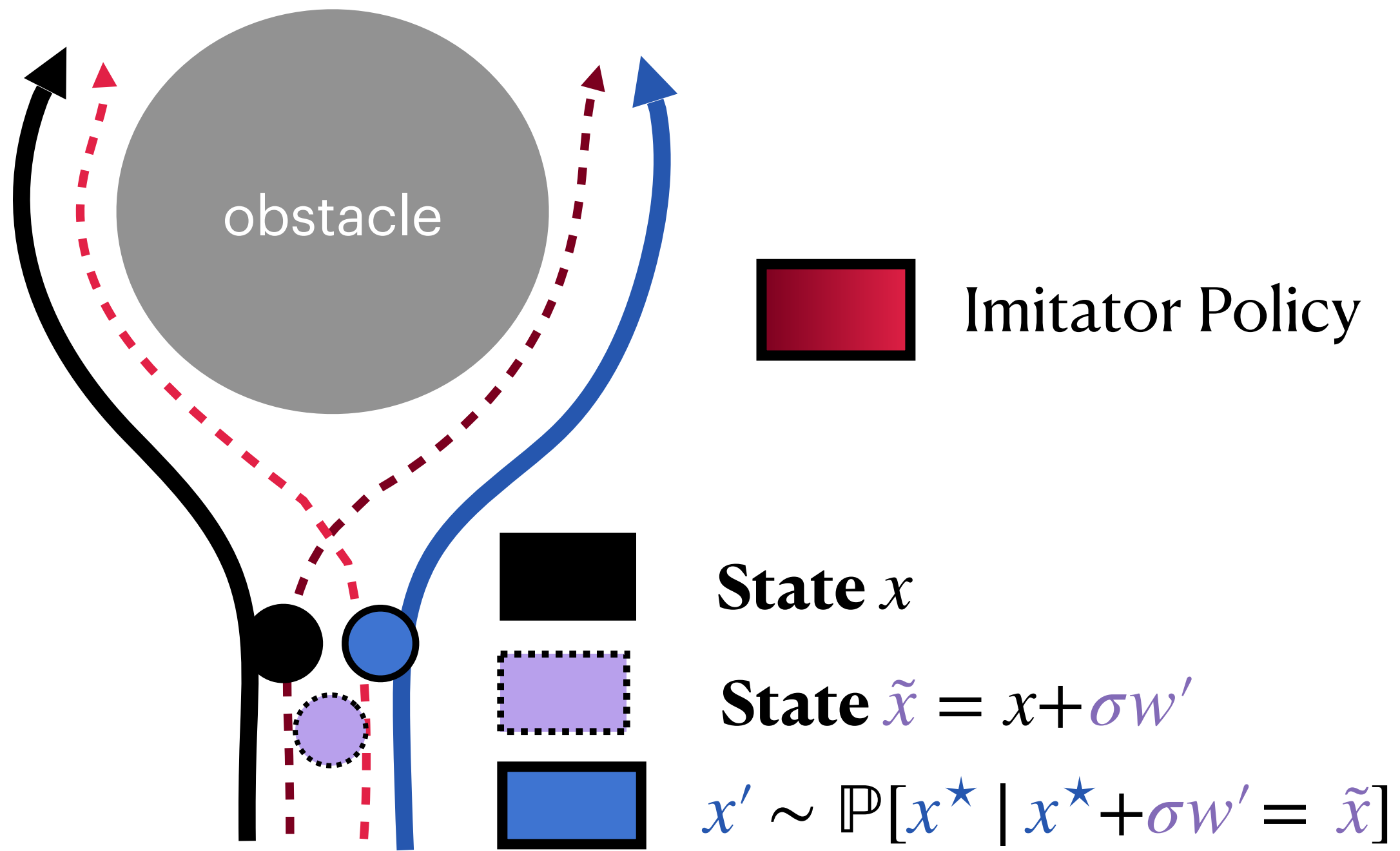
Lemma: Let $x \sim \text{Law}(x^\star)$, and let $x' \sim \mathbb{K}^{\text{rep}}(x)$
Then, (x, x') are

(1) **identically distributed** (and exchangeable)

(2) $\mathbb{P}[\|x - x'\| > 2\sigma\tau] \leq 2\mathbb{P}[\|w\| > \tau]$

With **perfect training**, $\hat{\pi}_\sigma(x) = \pi^\star \circ \mathbb{K}^{\text{rep}}(x)$ is **unbiased** at a distributional level (and TVC).

Replica Noising



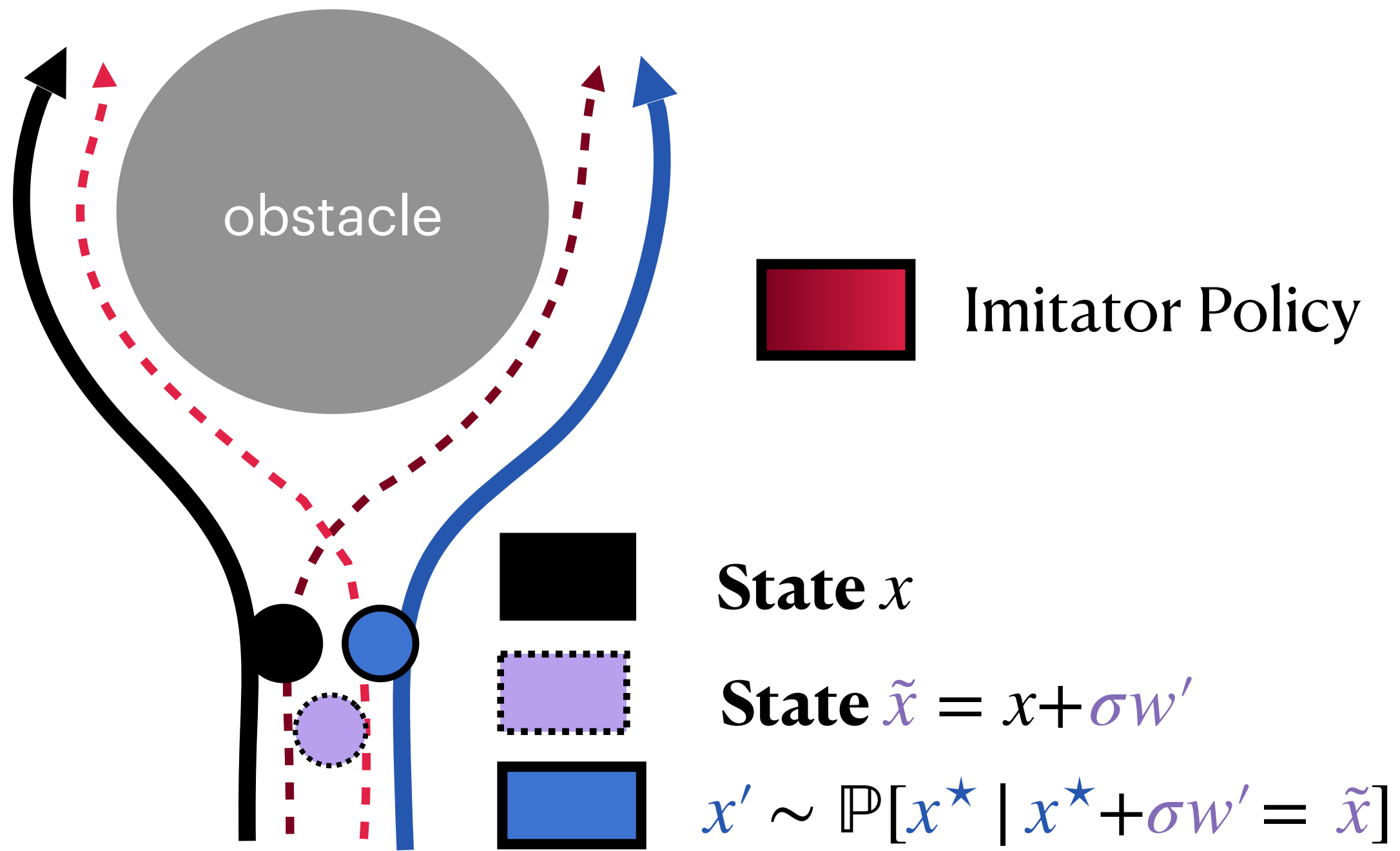
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This argument requires modeling
distributions, not simply ‘means’!

Replica Noising



Theorem: tuning $\sigma = \epsilon^{1/2}$, and with some caveats

$$D_{\text{test},\epsilon}(\hat{\pi} \parallel \pi^*) \leq O(H) \cdot D_{\text{train},\epsilon^2}(\hat{\pi} \parallel \pi^*)$$

1. **TVC** enforced, not **assumed!**
2. **Degradation** in rates due to noising parameter tradeoff
3. **Noising** introduces the possibility of ‘mode swapping’...

... which means we imitation **joint distributions**, not per-trajectory ones.

What did we do?

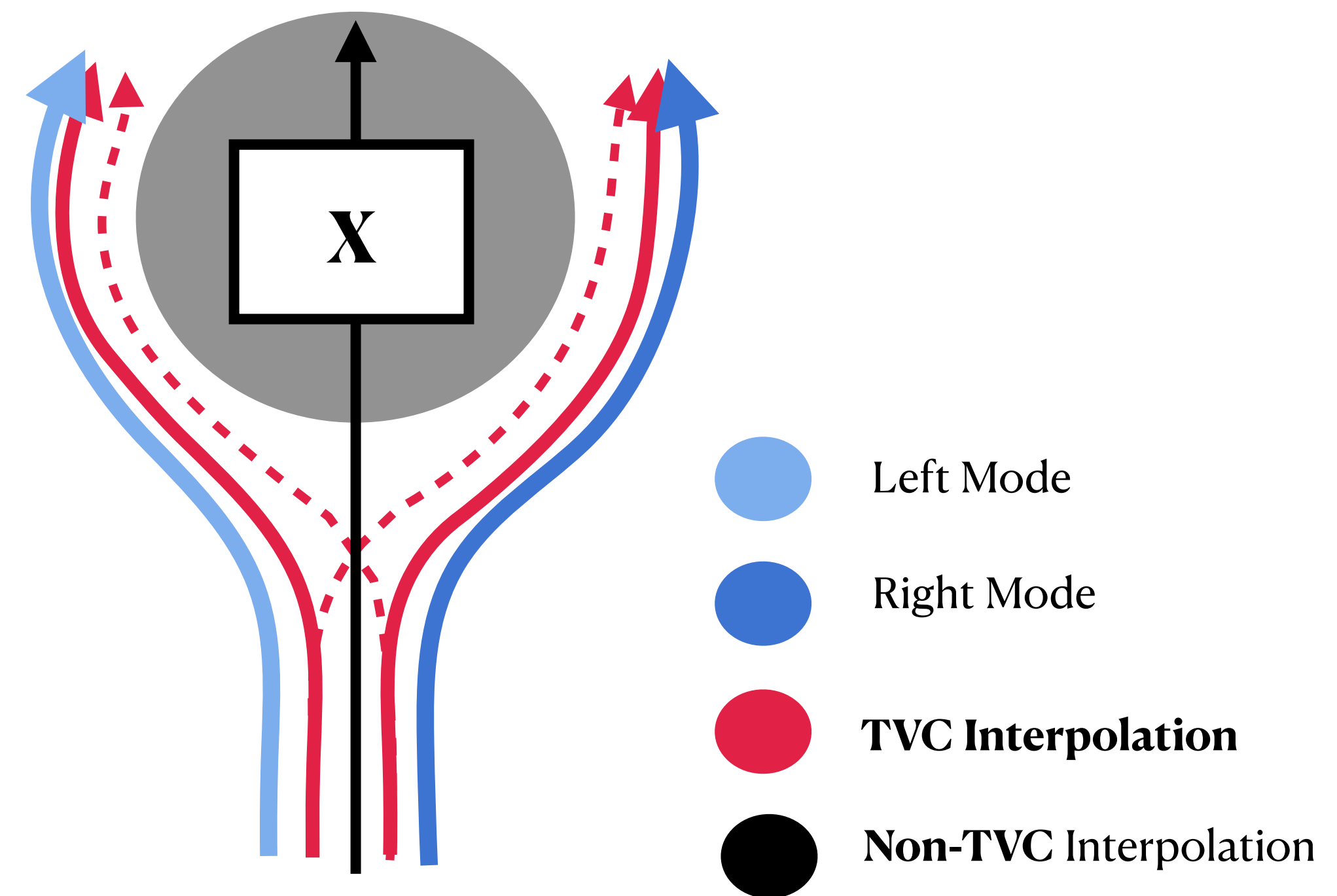
Theorem: tuning $\sigma = \epsilon^{1/2}$, and with some caveats

$$D_{\text{train},\epsilon}(\hat{\pi} \parallel \pi^{\star}) \leq O(H) \cdot D_{\text{train},\epsilon^2}(\hat{\pi} \parallel \pi^{\star})$$

Clever **smoothing** with noise induces **TVC**

TVC converts 'metric error' into '*discrete-token-error*'

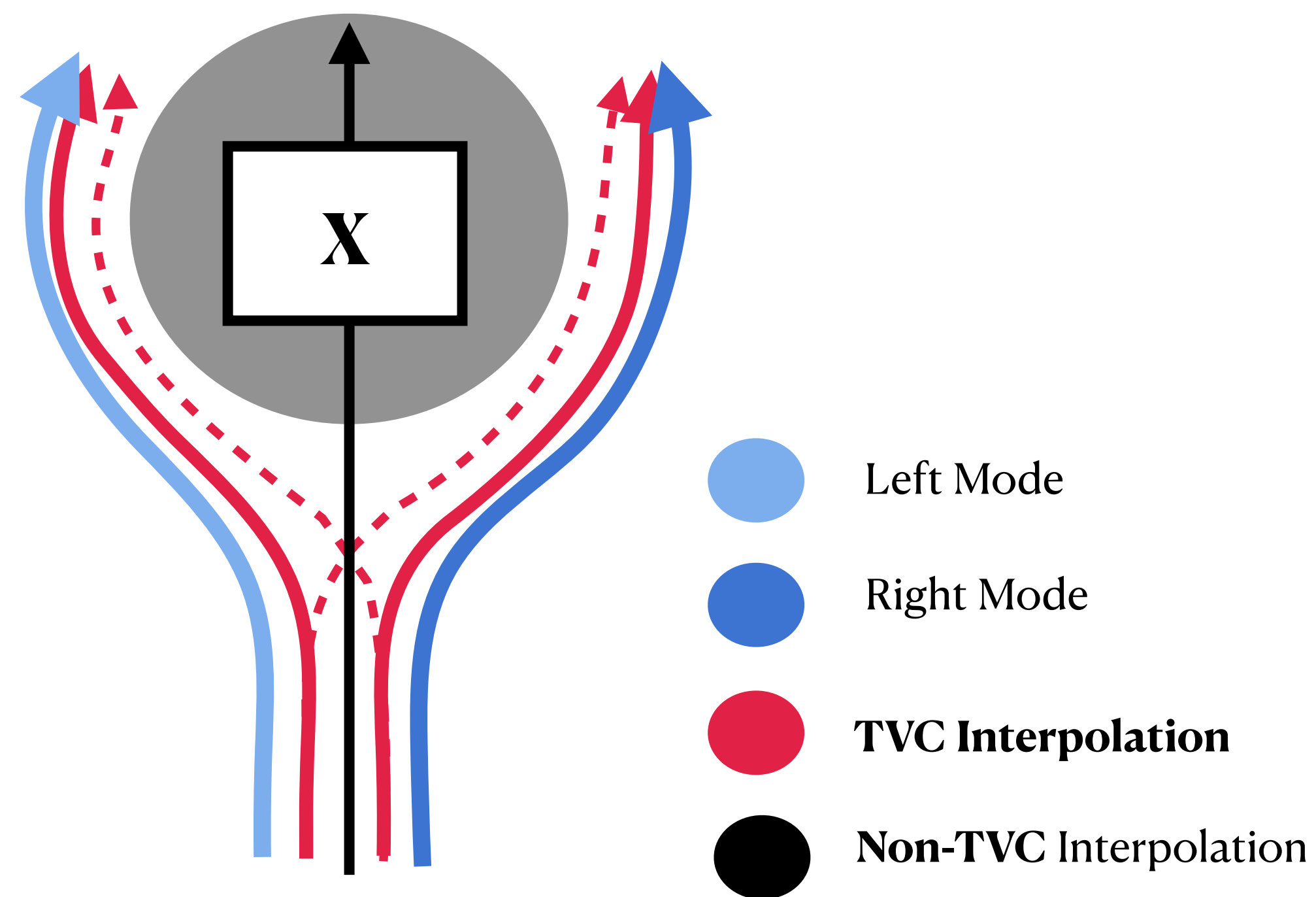
Imitation with '*discrete-token-error*' is easier



What did we do?

Theorem 1 (super informal): We can imitate without exponentially compounding error in contractive systems.

We algorithmically enforced the TVC inductive bias.



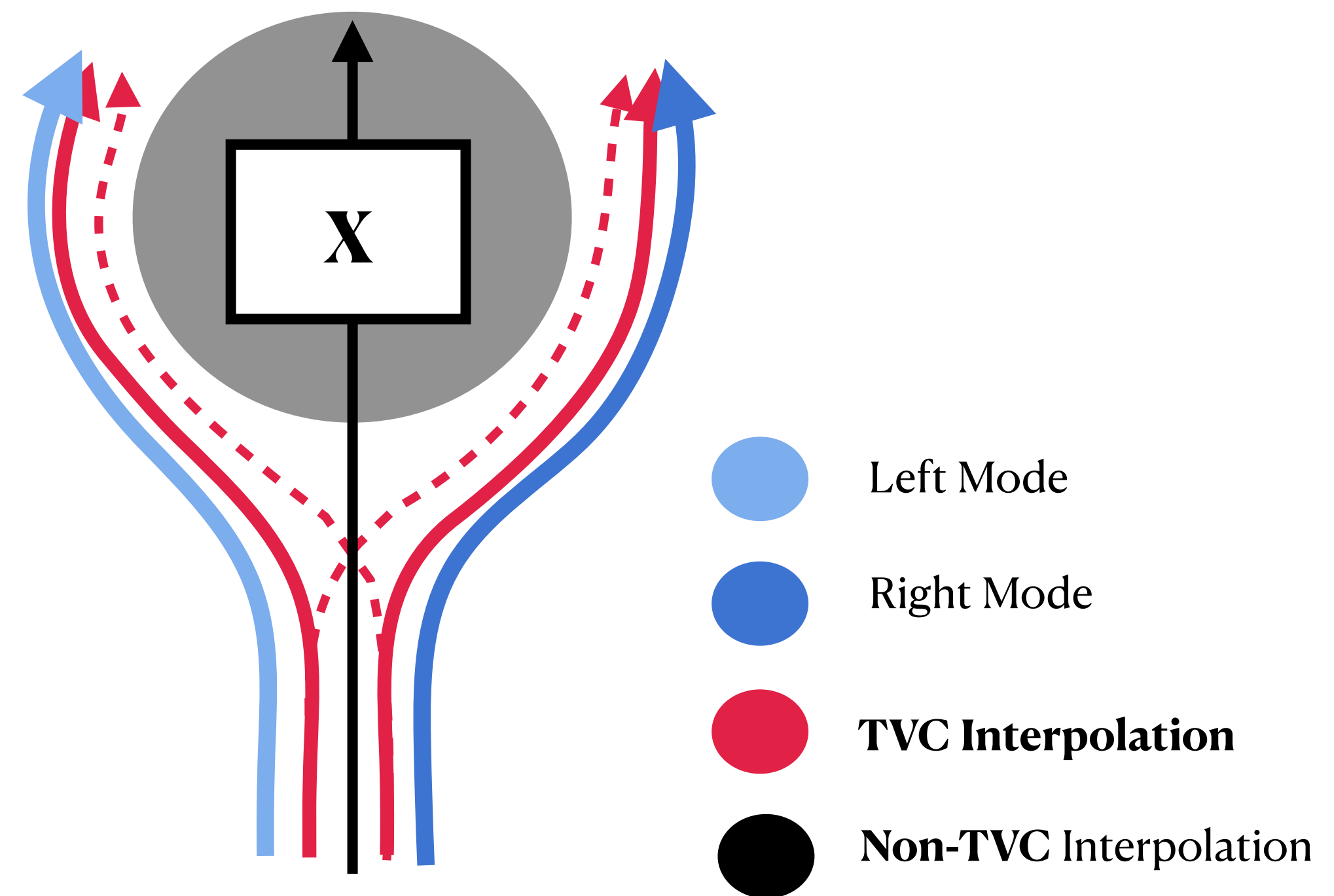
What did we do?

Theorem 1 (super informal): We can imitate without exponentially compounding error in contractive systems.

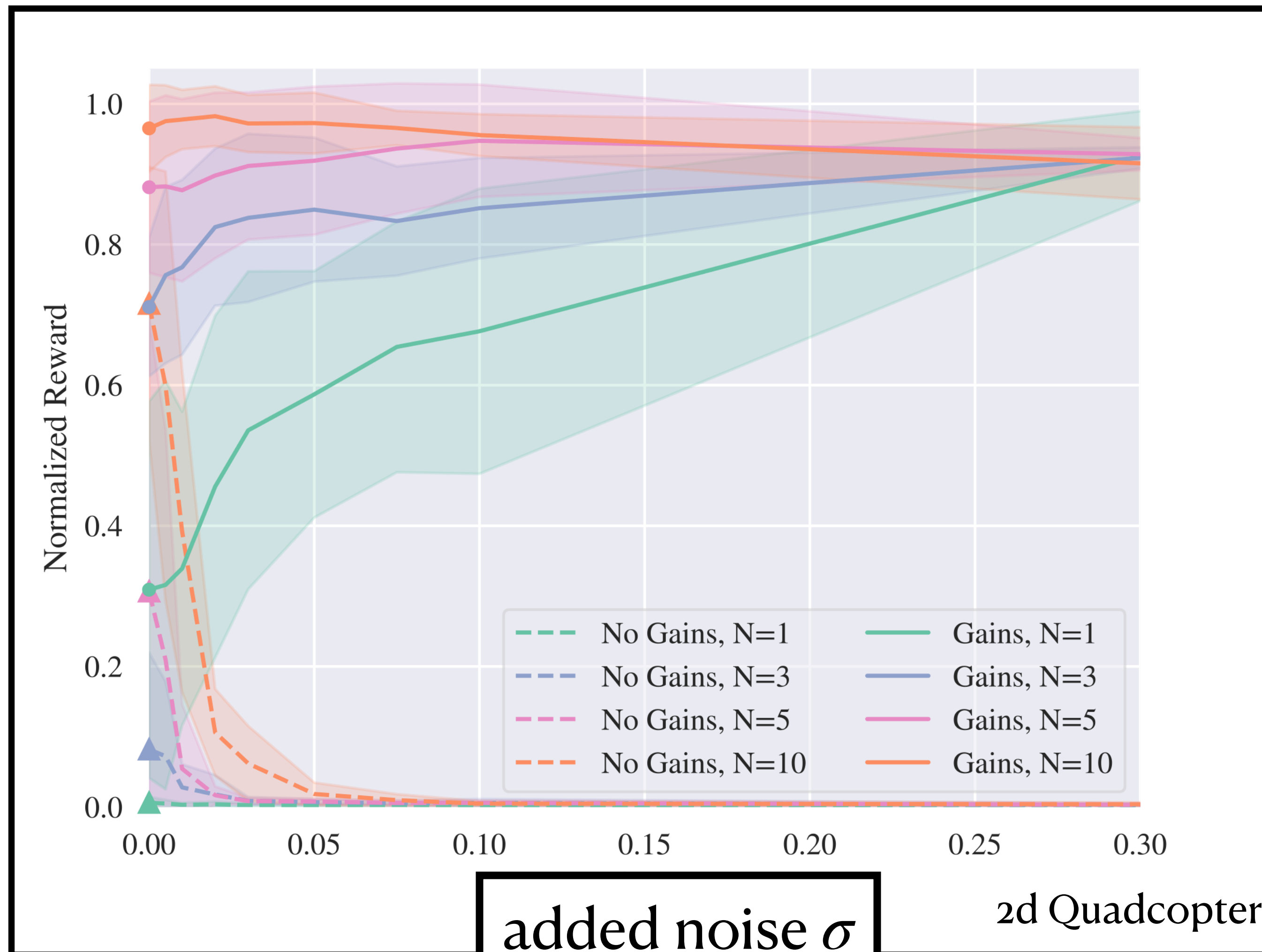
Open Question: What are the **intrinsic inductive biases** of diffusion models?

$$x \mapsto u \sim P(x)$$

Forthcoming work: Validates that diffusion models are not just ‘more expressive’, but have different inductive biases OOD.



Simulation Study.



low-level control helps!
data noising helps!

data noising hurts without stabilization

Applications?



ChatGPT

**discrete-token
sequence model.**



**continuous-token
sequence model.**

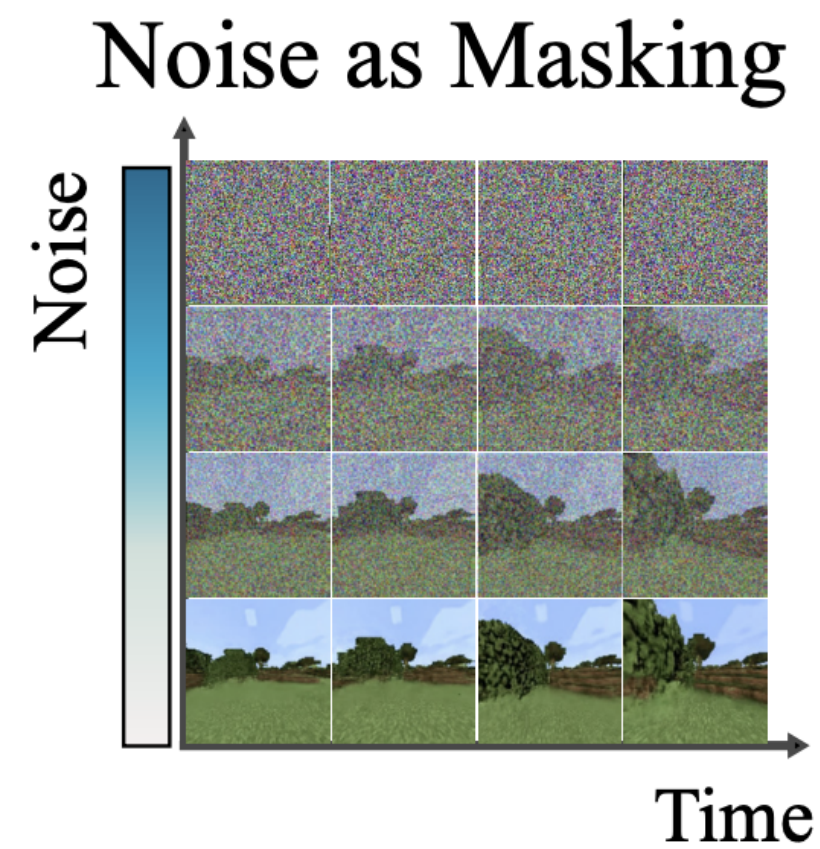
Recap: Diffusion

Training



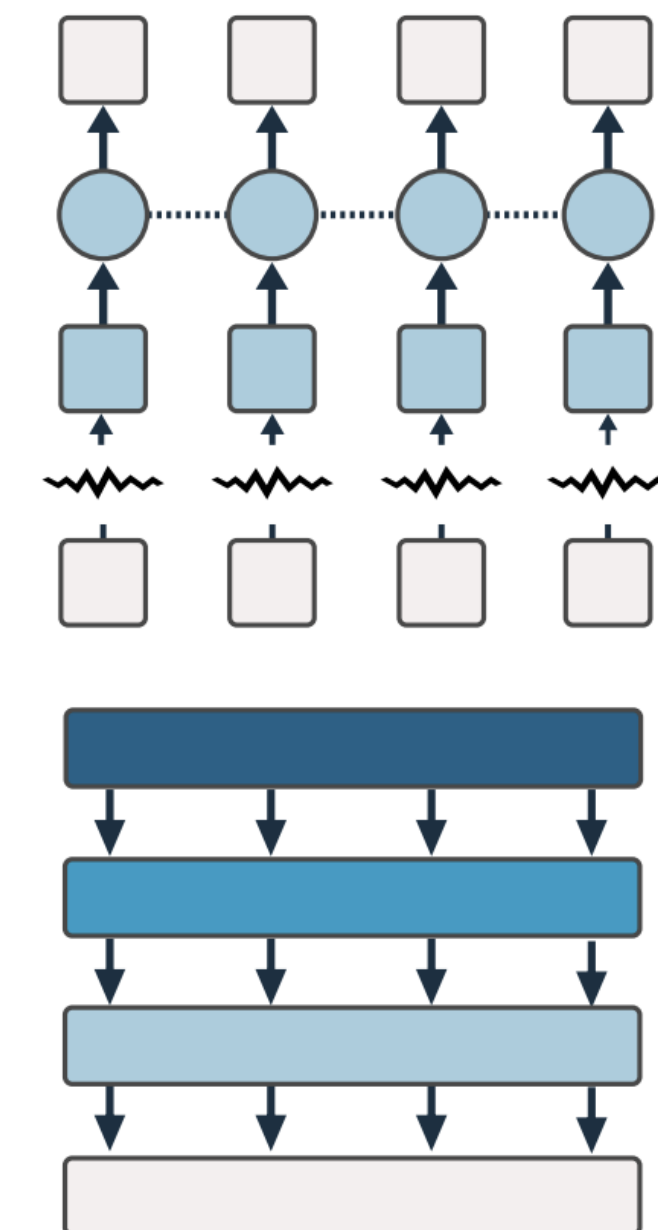
Inference

Diffusion for Sequences



- Observation
- Latent State
- Generation
- ⚡ Add Noise

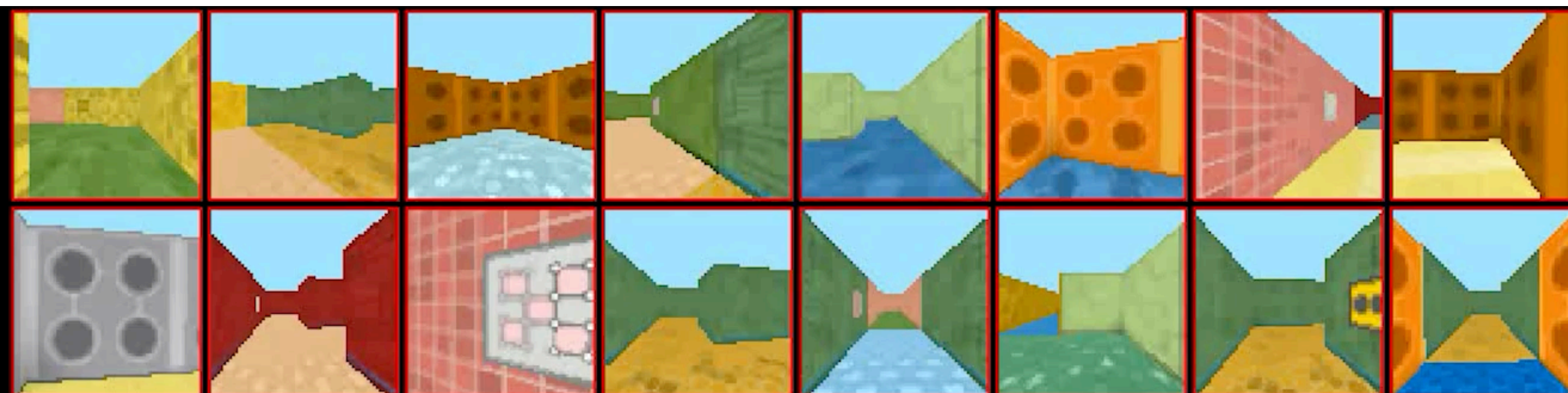
Full-Seq. Diffusion



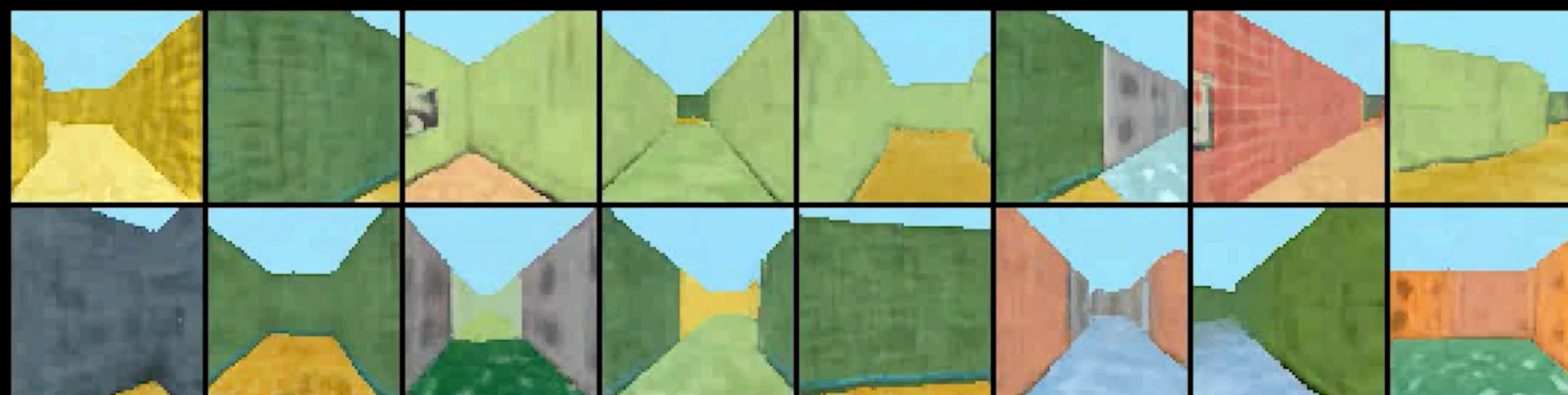
(*Boyuan Chen ... S ... et al. '24*)

*we **tell** the model the noise level

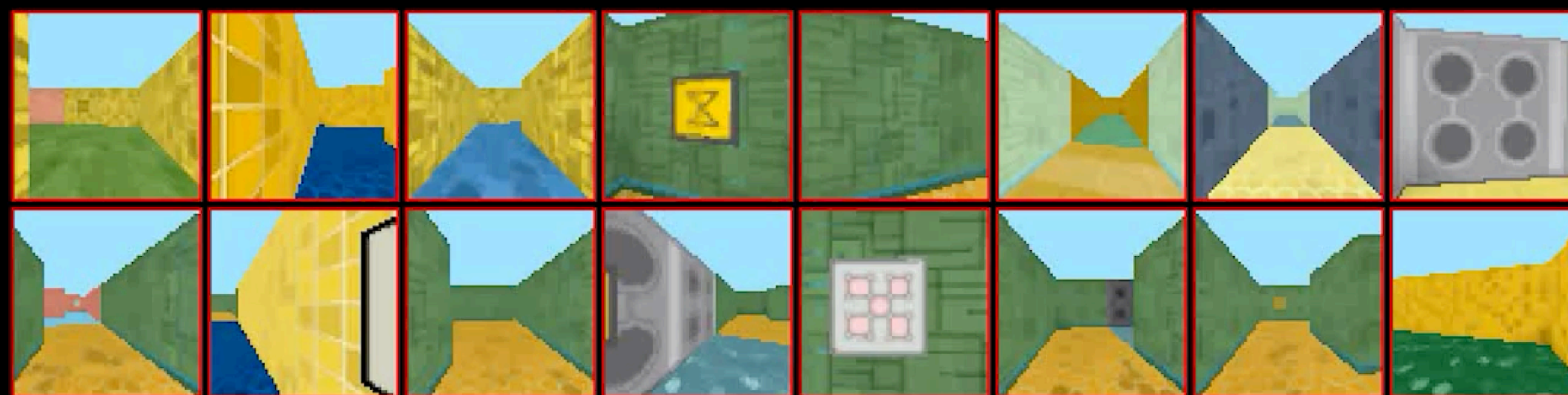
Diffusion Forcing
(Ours)



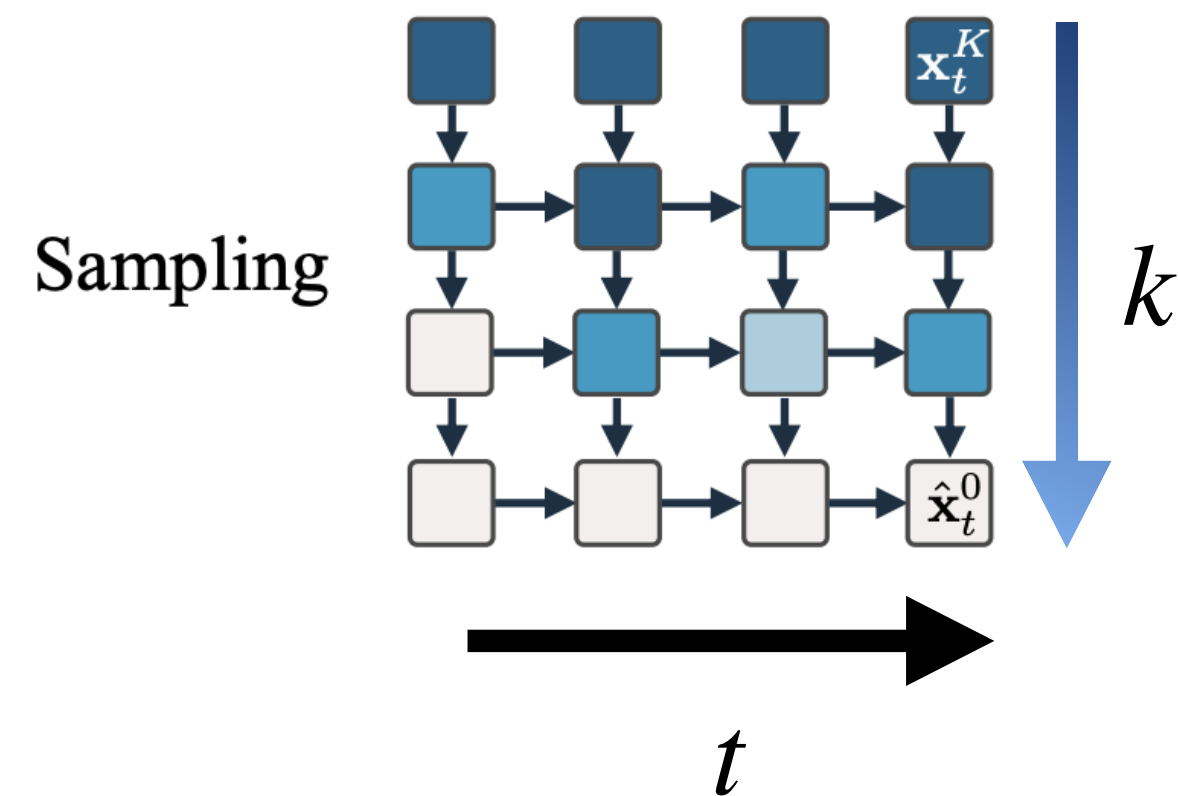
Full-Seq Diffusion



Teacher Forcing



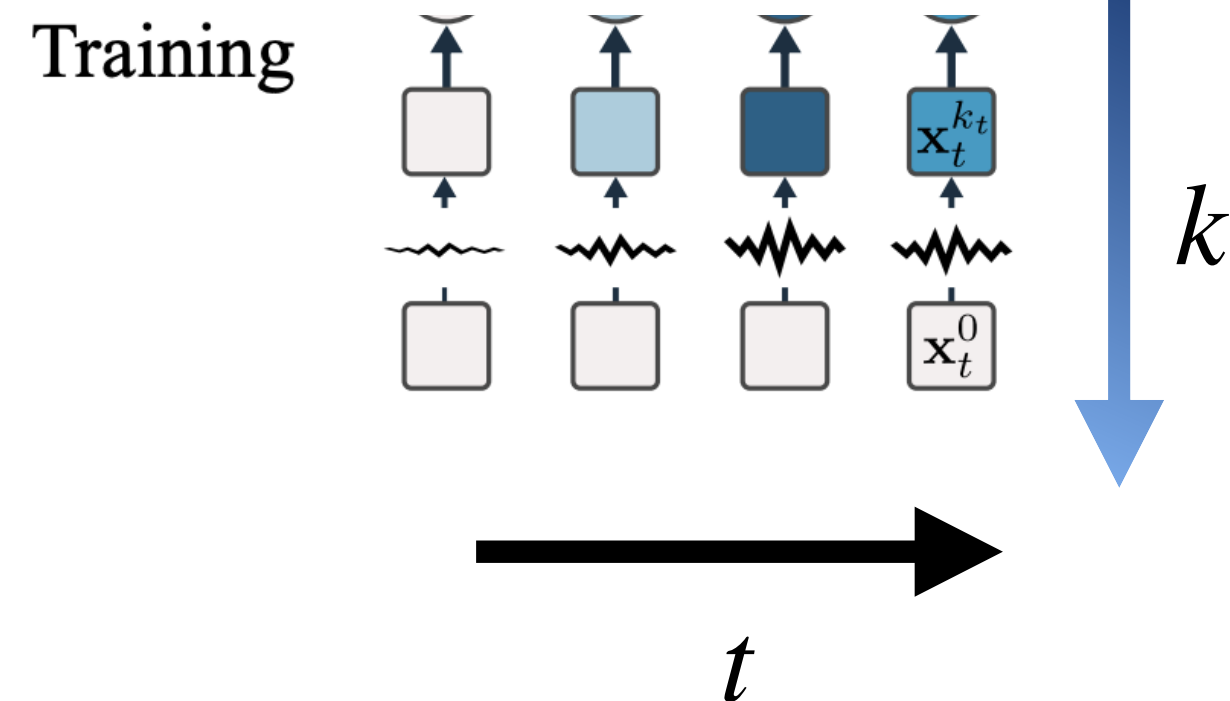
Example: **Explicit Algorithmic Modification** enabled by Generative Model



Inference

Predict token $x_t^0 \mid x_{t-1}^{k_0}, x_{t-2}^{k_0}, \dots$

$$\approx x_t^0 \mid x_{t-1}^0 + \sigma w'_{t-1}, x_{t-2}^0 + \sigma w'_{t-2}, \dots$$



Training

Predict token $x_t^0 \mid x_{t-1}^{k_0}, x_{t-2}^{k_0}, \dots$

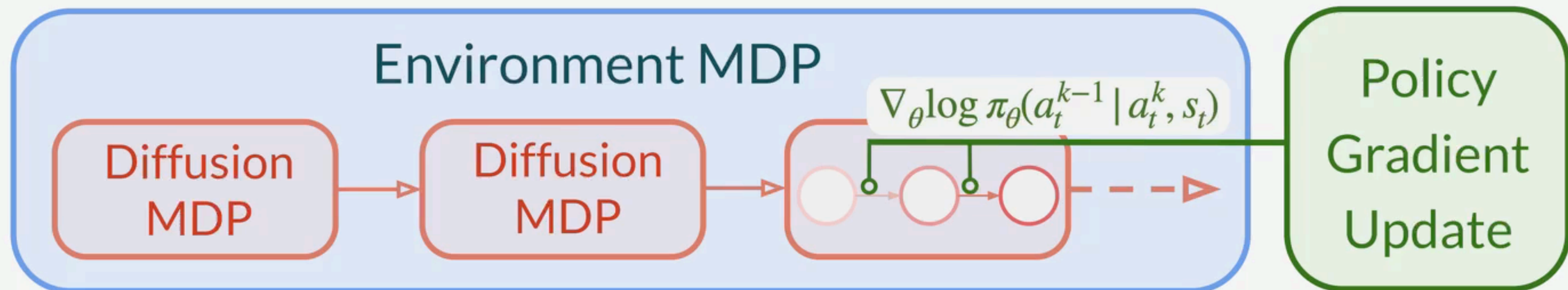
$$= x_t^0 \mid x_{t-1}^0 + \sigma w_{t-1}, x_{t-2}^0 + \sigma w_{t-2}, \dots$$

Replica Noising!

Replica Noising 'Mathematical Foundation' for why this works....



DPPPO: Diffusion Policy Policy Optimization

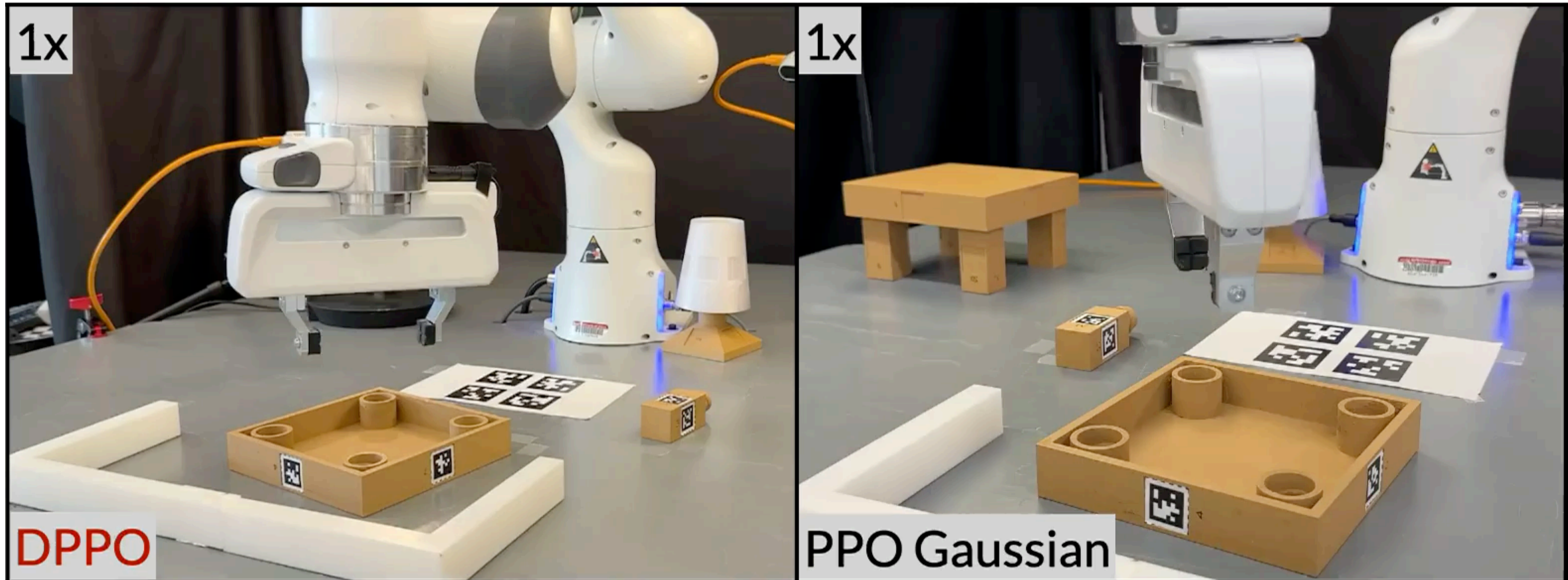


Structured exploration ✓ Training stability ✓ Policy robustness ✓

(Allen Ren ... S et al. '24)

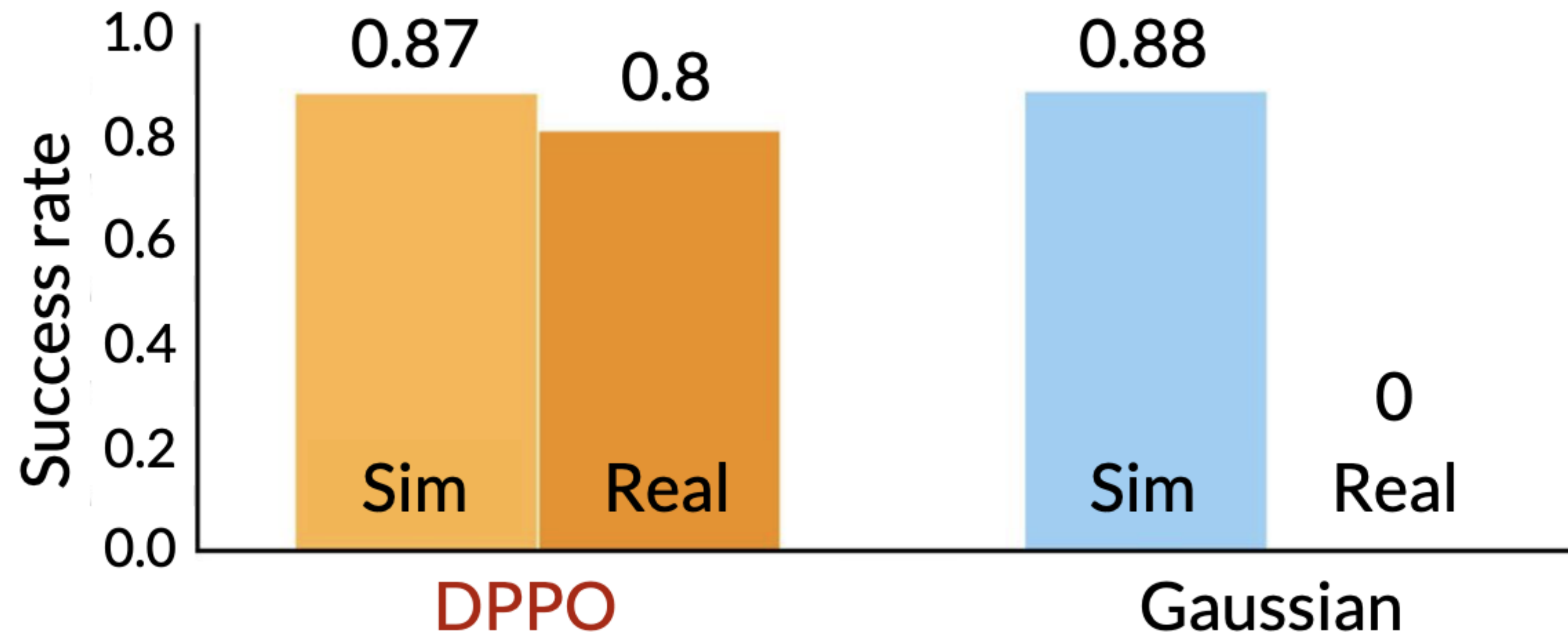
conditional sampling

$$\pi : x \mapsto f(x) + \text{noise}$$



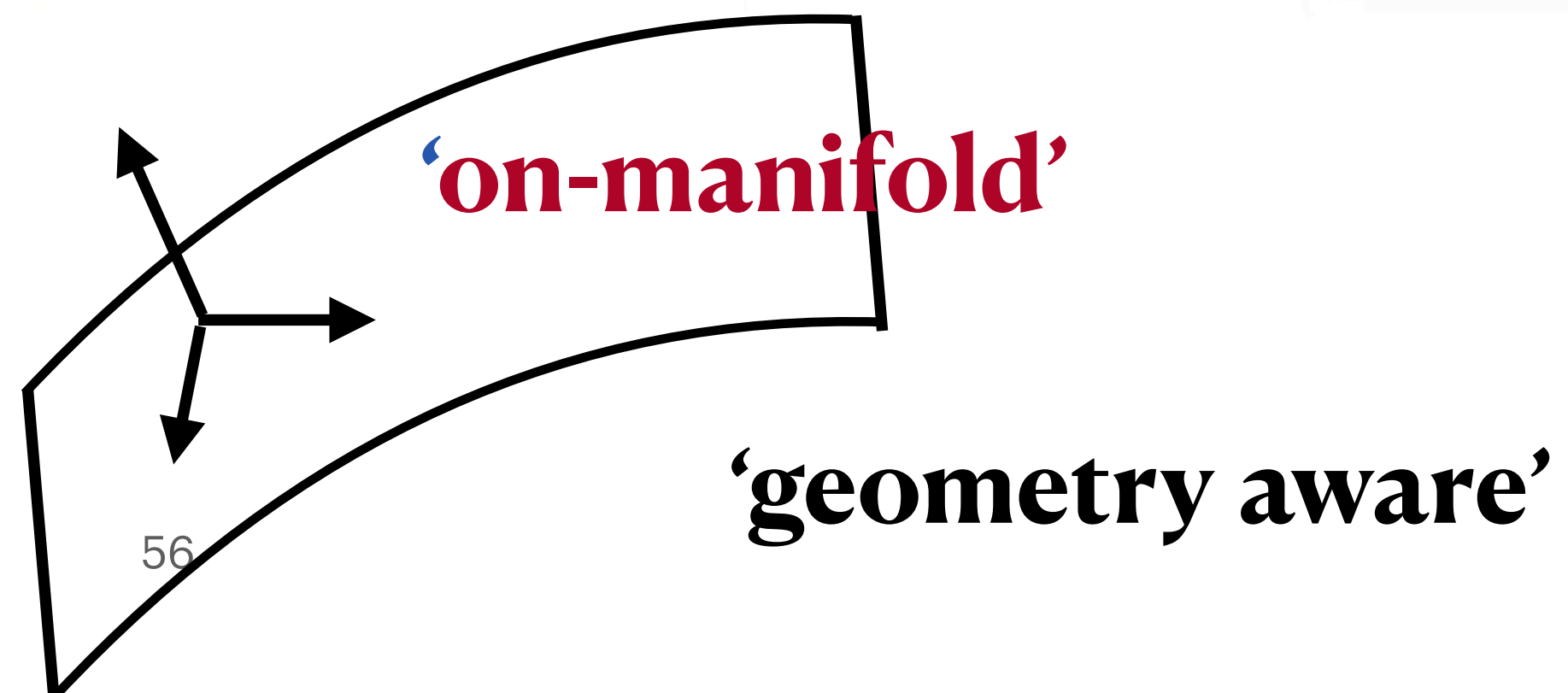
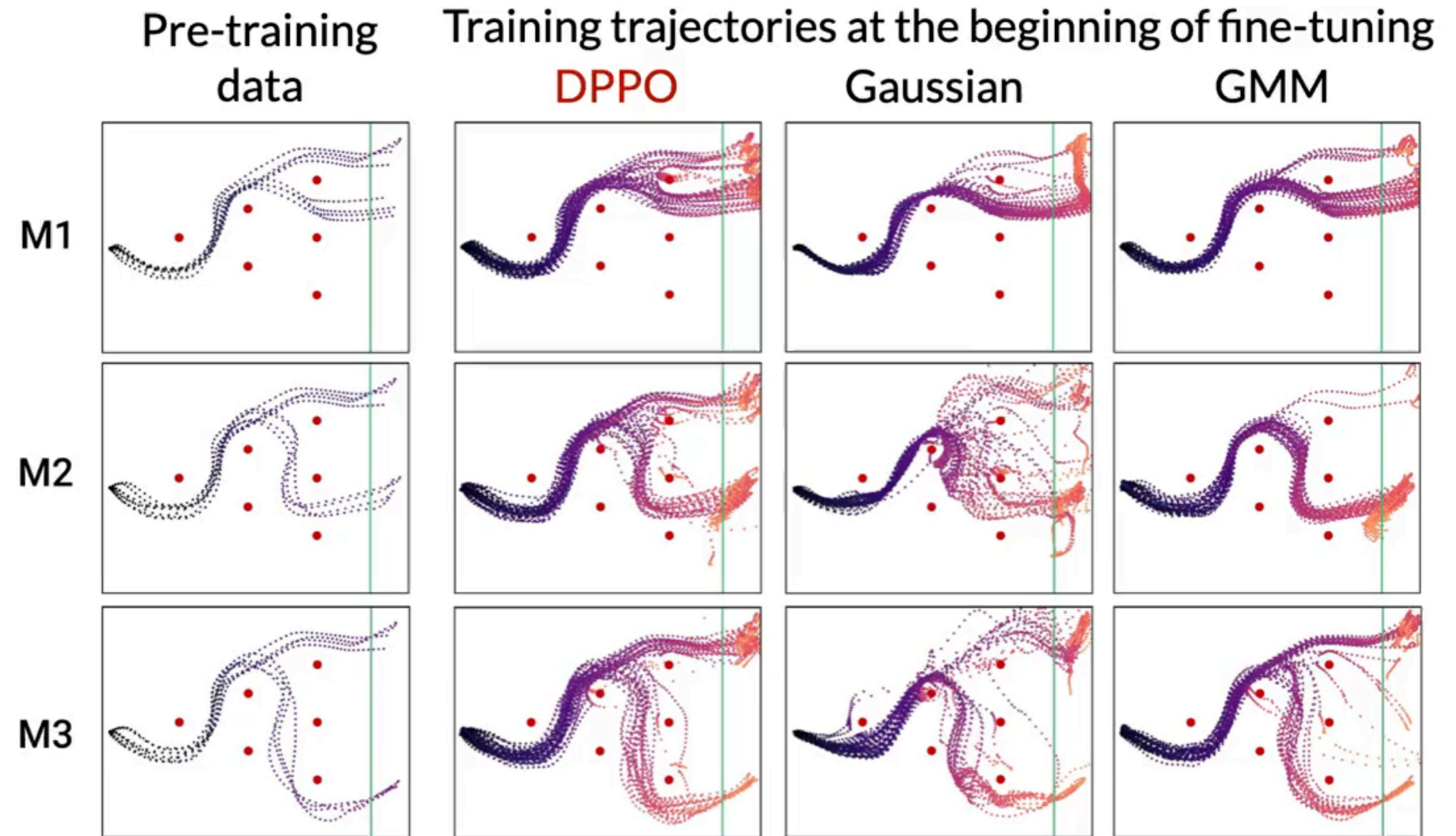
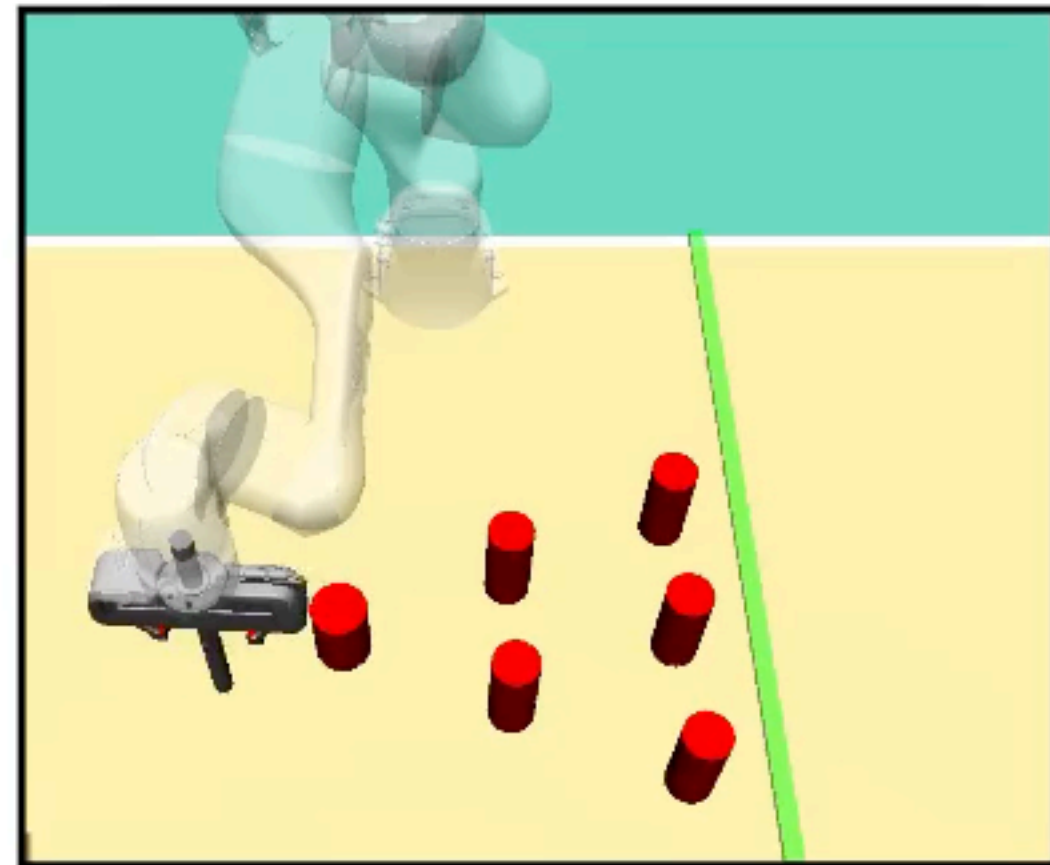
Real Hardware!

Simply that Diffusion Policies can ‘represent’ better performing policies?

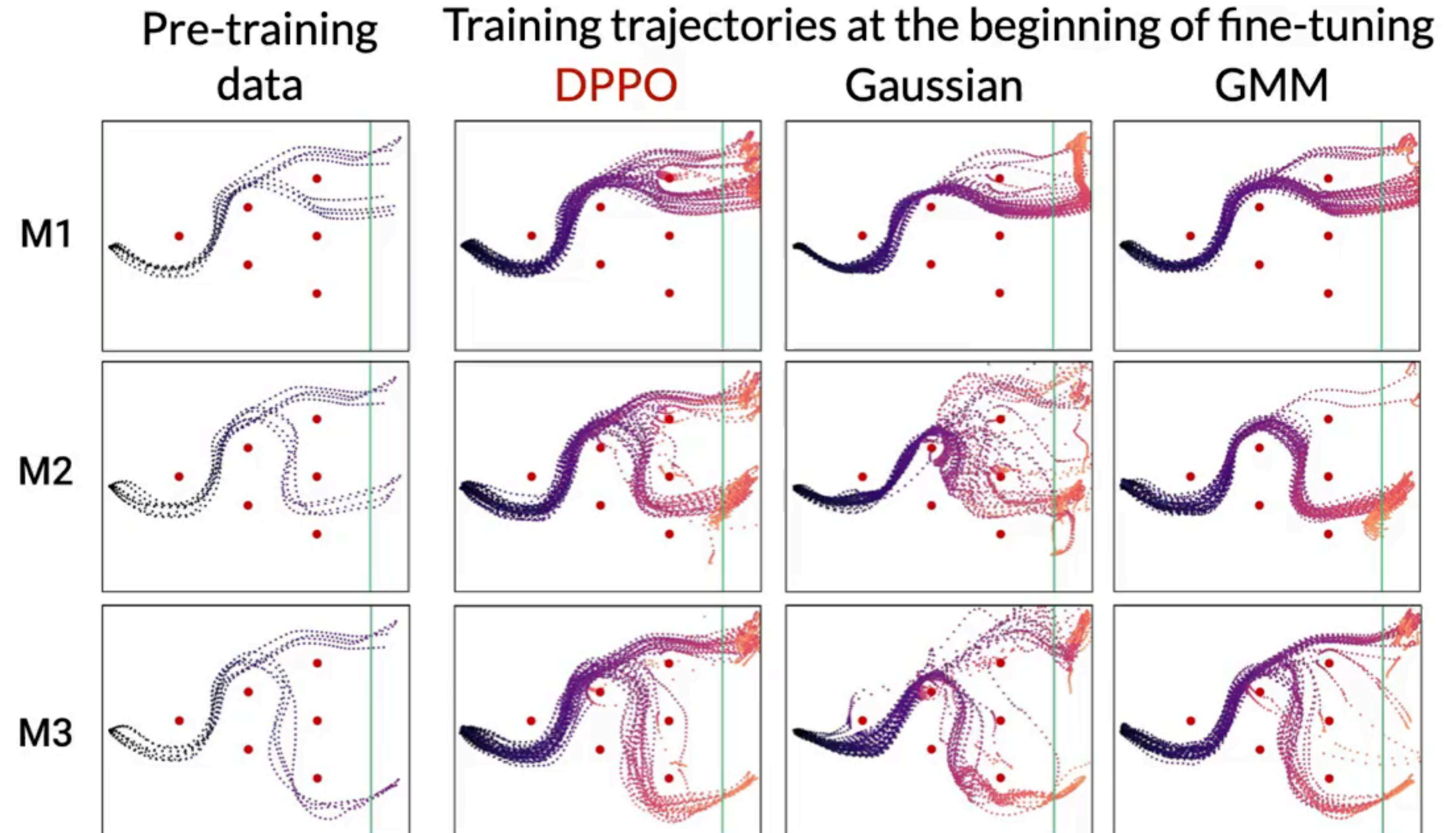
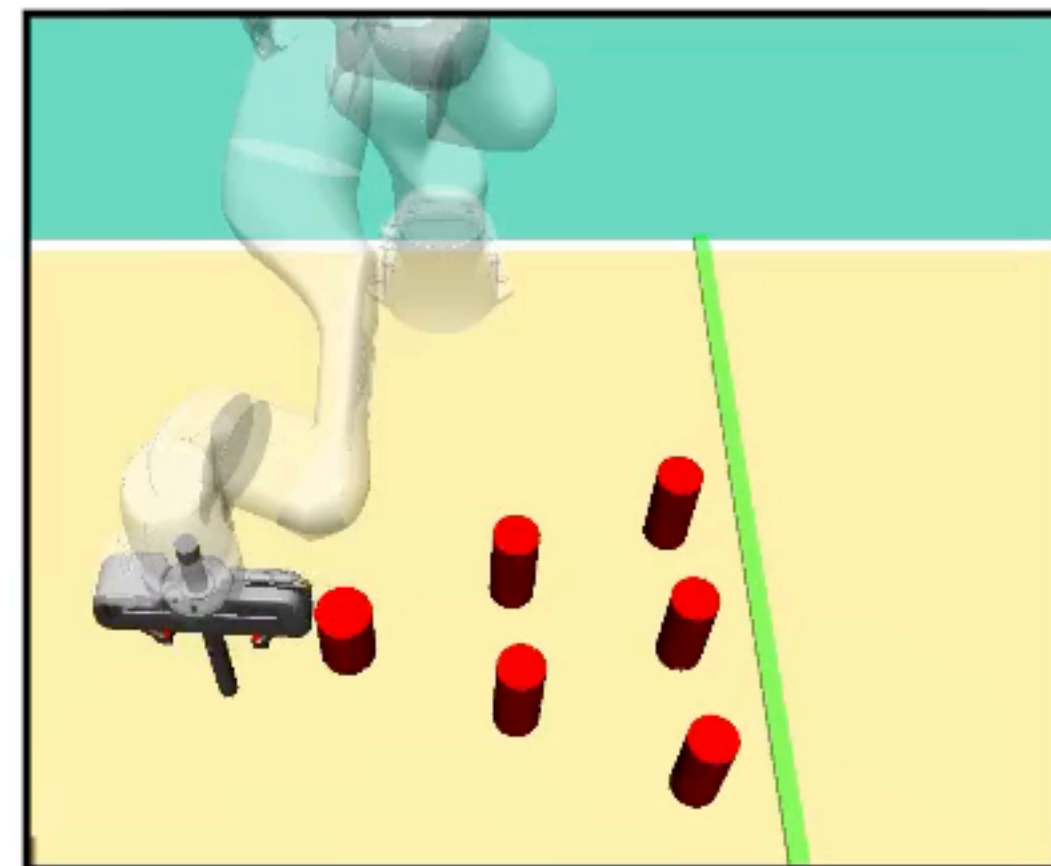


Example of richer models having better ‘intrinsic’ O.O.D. inductive bias

Avoid Environment
from D3IL



Avoid Environment
from D3IL



Open Question: Richer Models = More 'Reasonable' Exploration!

Pontification...

1. Lot's of exciting questions in **continuous-token prediction!**

(robots, video, climate, AI4Science, conditional diffusion...)

2. More expressive models + alg. choices = **richer O.O.D. inductive biases!**

3. How can we take full advantage of large/rich models **for exploration?**

(this should be true in LLMs!)

Provable Guarantees for Generative Behavior Cloning: Bridging Low-Level Stability and High-Level Behavior

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Diffusion Policy Policy Optimization

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Pulkit Agrawal², Anirudha Majumdar¹, Benjamin Burchfiel³, Hongkai Dai³, Max Simchowitz^{2,4}

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Diffusion Forcing: Next-token Prediction Meets Full-Sequence Diffusion

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Butterfly Effects of SGD Noise: Error Amplification in Behavior Cloning and Autoregression

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Enjoy the weekend!