

Language Model Alignment: Theory & Practice

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Alignment

Deep Reinforcement Learning from Human Preferences

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Direct Preference Optimization:

Your Language Model is Secretly a Reward Model

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SELF-INSTRUCT: Aligning Language Models with Self-Generated Instructions

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Training language models to follow instructions with human feedback

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OpenAI

A benign example of safety alignment

• We prefer the response to be safe.



No racism No stereotyping No profanity

....

Google

An adversarial example of safety alignment

• We would like the system to be robust to adversaries.



....

Outline

• Understand alignment through a simplified lens

• Introduce an inference-time alignment framework, called controlled decoding

• Shed light on the remarkable performance of best-of-n alignment

• Conclude with some practical issues of alignment

• A generative **language model p(.|x)** is a distribution over outcome y given x.

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- A reward model r(x,y) may be thought of as the log-likelihood of another generative alignment language model q(.|x)
 r(x,y) = log q(y|x)

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 $E_{y \sim p}[r(x, y)] = -H(p(.|x) || q(.|x))$

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• Alignment Goal: Sample from the **aligned distribution** π(.|x) that leads improve expected reward but remain "close to **p**."

Best-of-n: A simple baseline for alignment

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Let y_1, \ldots, y_n be *n* i.i.d. draws from $p(\cdot|x)$. The best-of-*n* strategy is denoted by $\pi^{(n)}$ and returns $y = y_{k^*}$ where $k^* := \arg \max_{k \in [n]} r(x, y)$.

Best-of-n: A simple baseline for alignment

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- Best-of-n is
 - \circ simple
 - effective
 - expensive in terms of throughput
 - incompatible with streaming

Markov Decision Process state: prompt x action: response y

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• Reward $r(\mathbf{x}, \mathbf{y})$

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Advantage

$$A(\mathbf{x};\pi) := E_{\mathbf{z} \sim \pi} \left\{ r(\mathbf{x}, \mathbf{z}) \right\} - E_{\mathbf{y} \sim p} \left\{ r(\mathbf{x}, \mathbf{y}) \right\}$$

Markov Decision Process state: prompt **x** action: response y

 $r(\mathbf{x}, \mathbf{y})$ Reward

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 $D(\mathbf{x}; \pi) := KL(\pi(\cdot|\mathbf{x}) \| p(\cdot|\mathbf{x}))$

Drift

- Markov Decision Process state: prompt x action: response y
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Drift

$$J_{\beta}(\mathbf{x};\pi) := A(\mathbf{x};\pi) - \beta D(\mathbf{x};\pi)$$

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RL objective

Drift

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Linear in π

Strongly convex in π

RL objective could be solved in closed form

Theorem 1. The optimal policy for the RL objective is unique and is given by $\pi^{\star}_{\beta}(\mathbf{y}|\mathbf{x}) \propto p(\mathbf{y}|\mathbf{x})e^{\frac{1}{\beta}r(\mathbf{x},\mathbf{y})}.$

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• At the sequence level, the solution is a tilted mismatched distribution²

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- Large $\beta \to \infty$ $\pi_{\beta}(y|x) \approx p(x)$
- Small β≈0

¹RL with KL penalties is better viewed as Bayesian inference (Korbak, Tomasz et al., EMNLP 2022).
 ²Mismatched Guesswork (Salamatian et al., Information Theory Workshop 2019).



Aligned family



Aligned family



Where does the alignment distribution come from?



No stereotyping No profanity

••••

Where does the alignment distribution come from?



- Reward may be trained on
 - (x, y, safe) tuples labeled for safety
 - (x, y⁺, y⁻, *preferred*) tuples depicting preference
- similar to a classifier
- using the Bradley Terry model

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- Reward doesn't have to be trained/differentiable, e.g., response length. Google

RLHF on side-by-side (s x s) preference data

• Bradley Terry model

$$p(y_1 \prec y_2 \mid x) = \sigma(r(x, y_2) - r(x, y_1))$$

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$$\mathcal{J}(r) = \mathbb{E}_{(x,y^+,y^-) \sim D} \left[\log p(y^- \prec y^+ \mid x) \right]$$

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• KL-regularized Reinforcement learning

$$\max_{\pi} \mathbb{E}_{\substack{x \sim \rho \\ y \sim \pi}} [r(x, y)] - \beta \mathrm{KL}(\pi \| p)$$

Many alignment methods

• Direct preference optimization (DPO)¹

$$\max_{\pi} \mathbb{E}_{(x,y^+,y^-)\sim D} \left[\log \sigma \left(\beta \log \frac{\pi(y^+|x)}{p(y^+|x)} - \beta \log \frac{\pi(y^-|x)}{p(y^-|x)} \right) \right]$$

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• Identity preference optimization (IPO)²

$$\max_{\pi} \mathbb{E}_{\substack{y_1 \sim p \\ y_2 \sim \pi}} [\Phi(p(y_1 \prec y_2 \mid x))] - \lambda \mathrm{KL}(\pi \parallel p)$$

¹Direct Preference Optimization: Your Language Model is Secretly a Reward Model (Rafailov et al., NeurIPS 2023). ²A General Theoretical Paradigm to Understand Learning from Human Preferences (Azar et al., AISTATS 2024).

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Many more!!!

How do they differ?

- Deployment
 - Training-time solution, e.g., DPO/PPO
 - Inference-time solution, e.g., Best-of-n

- widely used, easy to deploy

- easy to adapt, modular

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All methods roughly solve a KL-regularized RL problem

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The role of reverse KL regularizer in alignment

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- Assume a logistic language model
- Supervised finetuning (SFT)

 $heta^*_{ ext{sft}} = rg\min_{ heta} \mathcal{L}_{ ext{sft}}(heta) \qquad ext{where} \qquad \mathcal{L}_{ ext{sft}}(heta) := E_{(x,y) \sim D_{ ext{sft}}}\{A(heta; x) - heta^ op g(x,y)\},$

• KL-regularized RL

 $heta^*_{ ext{bilevel},eta} = rg\min_{ heta} \mathcal{L}_{ ext{bilevel},eta}(heta) \qquad ext{where} \qquad \mathcal{L}_{ ext{bilevel}}(heta) := D_{ ext{KL}}(\pi_{ heta} \| \pi_{ ext{sft}}) + rac{1}{eta} \mathcal{L}_{ ext{ro}}(heta),$

• Multi-tasking SFT and reward optimization

 $\theta^*_{ ext{multi-task},\beta} = \operatorname*{arg\,min}_{ heta} \mathcal{L}_{ ext{multi-task},\beta}(heta) \qquad ext{where} \qquad \mathcal{L}_{ ext{multi-task}}(heta) := \mathcal{L}_{ ext{sft}}(heta) + rac{1}{eta} \mathcal{L}_{ ext{ro}}(heta).$

The role of reverse KL regularizer in alignment

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Proposition 1. For all $\beta \in \mathbb{R}$, we have $\theta^*_{bilevel,\beta} = \theta^*_{multi-task,\beta}$.



YAAM: Yet Another Alignment Method

Markov Decision Process state: prompt + decoded prefix x, y^t action: next token y_{t+1}

Markov Decision Process

state: prompt + decoded prefix **x**, y^t

action: next token y_{t+1}

• Reward

$$R([\mathbf{x}, y^t]) := \begin{cases} 0 & y_t \neq EOS \\ r([\mathbf{x}, y^t]) & y_t = EOS \end{cases}$$

Markov Decision Process

state: prompt + decoded prefix **x**, y^t

action: next token y_{t+1}

- Reward
- Value

$$R([\mathbf{x}, y^t]) := \begin{cases} 0 & y_t \neq EOS \\ r([\mathbf{x}, y^t]) & y_t = EOS \end{cases}$$
$$V^{\star}([\mathbf{x}, y^t]) := E_{z_1, z_2, \dots \sim p} \left\{ \sum_{\tau \ge 0} R([\mathbf{x}, y^t, z^{\tau}]) \right\}$$

Markov Decision Process

state: prompt + decoded prefix **x**, y^t

action: next token y_{t+1}

- Reward
- Value
- Advantage

$$\begin{aligned} R([\mathbf{x}, y^t]) &:= \left\{ \begin{array}{ll} 0 & y_t \neq EOS\\ r([\mathbf{x}, y^t]) & y_t = EOS \end{array} \right. \\ V^{\star}([\mathbf{x}, y^t]) &:= E_{z_1, z_2, \dots \sim p} \left\{ \sum_{\tau \geq 0} R([\mathbf{x}, y^t, z^{\tau}]) \right\} \\ A([\mathbf{x}, y^t]; \pi) &:= E_{z \sim \pi} \left\{ V^{\star}([\mathbf{x}, y^t, z]) - V^{\star}([\mathbf{x}, y^t]) \right\} \end{aligned}$$

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}

- Reward
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- Drift

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• Value

Advantage

Drift

RL objective

Reward

Google

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RL objective

J

Advantage

Linear in π

Strongly convex in π

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Controlled decoding (CD)

Theorem 2. The optimal policy for the RL objective is unique and is given by $\pi_{\beta}^{\star}(z|[\mathbf{x}, y^{t}]) \propto p(z|[\mathbf{x}, y^{t}])e^{\frac{1}{\beta}V^{\star}([\mathbf{x}, y^{t}, z])}.$

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• Learn the value function:

$$\mathcal{L}^{\star}(\boldsymbol{\theta}) = E_{\mathbf{x} \sim p_{\mathbf{x}}} E_{\mathbf{y} \sim p_{\mathbf{y}|\mathbf{x}}} \ell^{\star}(\mathbf{x}, \mathbf{y}; \boldsymbol{\theta}),$$

where $\ell^{\star}(\mathbf{x}, \mathbf{y}; \boldsymbol{\theta}) = \frac{1}{2} \sum_{t \in [|\mathbf{y}|]} (V_{\boldsymbol{\theta}}([\mathbf{x}, y^{t}]) - V^{\star}([\mathbf{x}, y^{t}]))^{2}$

CD-FUDGE

• Use an unbiased draw from the model as the target¹

$$\mathcal{L}_{F}(\boldsymbol{\theta}) = E_{\mathbf{x} \sim p_{\mathbf{x}}} \ell_{F}(\mathbf{x}, \mathbf{y}; \boldsymbol{\theta}), \quad \text{s.t. } \mathbf{y} \sim p,$$

where $\ell_{F}(\mathbf{x}, \mathbf{y}; \boldsymbol{\theta}) = \frac{1}{2} \sum_{t \in [|\mathbf{y}|]} \left(V_{\boldsymbol{\theta}}([\mathbf{x}, y^{t}]) - r([\mathbf{x}, \mathbf{y}]) \right)^{2}$

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Theorem 3.1 (informal). Under regularity assumptions, applying SGD on \mathcal{L}_F converges to a stationary point of $\mathcal{L}^*(\theta)$.

CD-Q

• Bellman identity

$$V^{\star}([\mathbf{x}, y^t]) = \begin{cases} E_{z \sim p(\cdot | [x, y^t])} V^{\star}([\mathbf{x}, y^t, z]), & y_t \neq EOS\\ r([\mathbf{x}, y^t]), & y_t = EOS \end{cases}$$

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• Train the value function similarly to DQN

$$\begin{split} \mathcal{L}_{Q}(\boldsymbol{\theta}) &= E_{\mathbf{x} \sim p_{\mathbf{x}}} \ell_{Q}(\mathbf{x}, \mathbf{y}; \boldsymbol{\theta}), \\ \text{where } \ell_{Q}(\mathbf{x}, y^{t}; \boldsymbol{\theta}) &= \frac{1}{2} \sum_{t \in [|\mathbf{y}|]} \left(V_{\boldsymbol{\theta}}([\mathbf{x}, y^{t}]) - \dot{v}_{t} \right)^{2}, \\ v_{t} &= \begin{cases} \sum_{z \in \mathcal{Y}} p(z | [x, y^{t}]) V_{\boldsymbol{\theta}}([\mathbf{x}, y^{t}, z]) & y_{t} \neq EOS \\ r([\mathbf{x}, y^{t}]) & y_{t} = EOS \end{cases} \end{split}$$

Token-wise control using CD

Theorem 2. The optimal policy for the RL objective is unique and is given by $\pi^{\star}_{\beta}(z|[\mathbf{x}, y^t]) \propto p(z|[\mathbf{x}, y^t])e^{\frac{1}{\beta}V^{\star}([\mathbf{x}, y^t, z])}.$

 Q Will this paper get accepted?



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Q Will this paper get accepted?

liked average very high high high disliked average 👗 This paper will be high average average reviewed very high very low average hated LM sentiment aligned likelihood prefix score score

Blockwise control using CD (best-of-n++)

• Draw K blocks of length M tokens

$$\left\{z_{(k)}^{M}\right\}_{k\in[n]} \stackrel{\text{i.i.d.}}{\sim} p(z^{M}|[\mathbf{x}, y^{t}]).$$

• Accept the continuation with the highest prefix score:

$$z^M := \arg \max_{\left\{z^M_{(k)}\right\}_{k \in [n]}} V_{\boldsymbol{\theta}}([\mathbf{x}, y^t, z^M_{(k)}])$$

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 $\boldsymbol{\zeta}$ Will this paper get accepted?



Advantages over best-of-n:

- Limits the user-facing latency to the decoding time of a single block.
- Makes a large effective n practically feasible.

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How do we evaluate alignment methods?

- Human evaluations
- Auto-evals that are correlated with human judgement

How do we evaluate alignment methods?



Google

How do we evaluate alignment methods?



Google

Estimating KL divergence

• We estimate KL divergence via aggregating the log-likelihood ratios between the aligned model and the base model

$$\begin{split} D([\mathbf{x}, y^t]; \pi) &:= \textit{KL}(\pi(\cdot | [\mathbf{x}, y^t]) \| p(\cdot | [\mathbf{x}, y^t])) \\ &= \sum_{z \in \mathcal{Y}} \pi(z | [\mathbf{x}, y^t]) \log \left(\frac{\pi(z | [\mathbf{x}, y^t])}{p(z | [\mathbf{x}, y^t])} \right) \quad \begin{array}{l} \text{Log-likelihood ratio} \\ \text{is an unbiased estimate of} \\ \text{KL divergence} \end{split}$$

Estimating KL divergence

• We estimate KL divergence via aggregating the log-likelihood ratios between the aligned model and the base model

$$\begin{split} D([\mathbf{x}, y^t]; \pi) &:= \mathit{KL}(\pi(\cdot | [\mathbf{x}, y^t]) \| p(\cdot | [\mathbf{x}, y^t])) \\ &= \sum_{z \in \mathcal{Y}} \pi(z | [\mathbf{x}, y^t]) \log \left(\frac{\pi(z | [\mathbf{x}, y^t])}{p(z | [\mathbf{x}, y^t])} \right) \quad \begin{array}{c} \text{Log-lik} \\ \text{is an unity} \\ \text{KL diverses} \end{split}$$

Log-likelihood ratio is an unbiased estimate of KL divergence

• We don't have the logits of best-of-n or blockwise CD. How can we estimate KL divergence?

Analytical formula for KL divergence of best-of-n

• An analytical formula that has appeared many times in the literature^{1,2}

$$\mathrm{KL}(\pi_{\boldsymbol{y}|\boldsymbol{x}}^{(n)} \| p_{\boldsymbol{y}|\boldsymbol{x}}) \stackrel{\mathrm{claim}}{=} \widetilde{\mathrm{KL}}_n := \log(n) - (n-1)/n.$$

- This formula remarkably
 - doesn't depend on the prompt **x** or its distribution $\mathbf{p}_{\mathbf{x}}$
 - \circ doesn't depend on the base policy **p**_{vlx}

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• Is this formula true?

Analytical formula is wrong!

Example 1. Consider an unprompted model with $\mathbf{x} = \emptyset$ (no input) and binary output, $\mathbf{y} \in \{0, 1\}$. Let the two outcomes be equiprobable, i.e., $p_{\mathbf{y}|\mathbf{x}}(0) = p_{\mathbf{y}|\mathbf{x}}(1) = \frac{1}{2}$. Further, let r(0) = 0, and r(1) = 1, i.e., outcome 1 is more desirable than outcome 0. Here, we can compute $\pi_{\mathbf{y}|\mathbf{x}}^{(n)}$ in closed form. Specifically, we can see that $\pi_{\mathbf{y}|\mathbf{x}}^{(n)}(0) = \frac{1}{2^n}$ and $\pi_{\mathbf{y}|\mathbf{x}}^{(n)}(1) = 1 - \frac{1}{2^n}$. Thus,

$$KL(\pi_{\boldsymbol{y}|\boldsymbol{x}}^{(n)} \| p_{\boldsymbol{y}|\boldsymbol{x}}) = \log(2) - h\left(\frac{1}{2^n}\right)$$

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Guarantees on the analytical formula

Theorem 2. For any $n \in \mathbb{N}$ and any \boldsymbol{x} , $KL(\pi^{(n)} || p) \leq \widetilde{KL}_n = \log(n) - \frac{n-1}{n}.$

- Analytical formula is an upper bound
- Let $y \sim \pi^{(n)}$. Then, let $\varepsilon_n := p(y)$
 - Theorem: The gap is small if $\mathbf{n} \cdot \mathbf{\epsilon}_n \ll \mathbf{1}$
 - Theorem: The gap is large if $\mathbf{n} \cdot \mathbf{\epsilon}_n \gg \mathbf{1}$

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 - Theorem: The gap is large if $\mathbf{n} \cdot \mathbf{\epsilon}_n \gg 1$
- More recently, Mroueh showed that this result is an instance of strong data processing inequality²
New estimator for KL divergence of best-of-n

Approximation 1. Let $y \sim \pi^{(n)}$. Then, let $\varepsilon_n := p(y)$. We propose the following estimator for the KL divergence of the best-of-n policy and the base policy:

$$\widehat{KL}(\varepsilon_n) := (1 - \varepsilon_n)^n \left(\log n + (n - 1) \log(1 - \varepsilon_n) - \frac{n - 1}{n} \right) + (1 - (1 - \varepsilon_n)^n) \log \left(\frac{1 - (1 - \varepsilon_n)^n}{\varepsilon_n} \right).$$

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Win-rate of best-of-n

win-rate
$$(\pi \| p) := E_{y \sim \pi} E_{z \sim p} \{ \mathbf{1}(r(x, y) > r(x, z)) + \frac{1}{2} \mathbf{1}(r(x, y) = r(x, z)) \}$$

Lemma 1. The win-rate for best-of-n policy is given by

win-rate
$$(\pi^{(n)} || p) = 1 - \frac{1}{2} E_{y \sim p} \{ F(y|x)^n + F^-(y|x)^n \} \le \frac{n}{n+1}.$$

- Roughly the upper bound argument follows from
 - Draw (n+1) outcomes from the base model; and order them from the highest to lowest reward
 - Randomly assign 1 of the outcomes to base model
 - Choose the best of the remaining n to be a draw from the best-of-n model.
 - Best-of-n wins against the reference model with probability n/(n+1).
- The upper bound would be exact if w/p 1 no two outcomes were identical

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Win-rate vs KL tradeoffs (helpfulness & harmlessness)



Figure 4: Win rate vs. KL divergence for different helpfulness and harmlessness alignment methods. CD-Q (blockwise) vastly outperforms RL techniques such as IPO & PPO.

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Win-rate vs KL tradeoff for best-of-n



- KL values <10 are sufficient to reach a high win-rate against base policy
- This is the ideal setting ignoring noise in reward and generalization

Google

Win-rate vs KL tradeoffs



(b) RL

Google

Scaling Laws for Reward Model Overoptimization (Gao et al., ICML 2023).

Win-rate vs KL tradeoffs



(b) RL

Google

Scaling Laws for Reward Model Overoptimization (Gao et al., ICML 2023).

Win-rate vs KL tradeoffs (helpfulness & harmlessness)



- Best-of-n is better than state-of-the-art RL methods
- Blockwise CD bridges the gap between tokenwise control and best-of-n
- Token-wise CD is a good contender for token-wise control (on par with other methods)

Figure 4: Win rate vs. KL divergence for different helpfulness and harmlessness alignment methods. CD-Q (blockwise) vastly outperforms RL techniques such as IPO & PPO.

Modularity of CD for the win!



Multi-objective alignment

Modularity of CD for the win!



Modularity of CD for the win!



Optimal reward-KL tradeoff

• Theorem: KL-regularized RL solution is optimal for reward-KL tradeoff

$$J_{\beta}(\mathbf{x}; \pi) := A(\mathbf{x}; \pi) - \beta D(\mathbf{x}; \pi)$$

$$E_{\mathbf{z} \sim \pi} \{r(\mathbf{x}, \mathbf{z})\} - E_{\mathbf{y} \sim p} \{r(\mathbf{x}, \mathbf{y})\}$$

$$KL(\pi(\cdot |\mathbf{x})||p(\cdot |\mathbf{x}))$$

Optimal reward-KL tradeoff

to the optimal trade-off

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$$Linear in \pi$$
Empirically, best-of-n is strikingly close to the optimal trade-off
$$\int_{0}^{0} \int_{0}^{0} \int_{0}^{0} \int_{0}^{0} \int_{0}^{1} \int_$$

Why does best-of-n work so well?

- Let's revisit the example
- The probability of type **t** is given by e^{-mKL(**t**||p)}



Why does best-of-n work so well?

- Let's revisit the example
- The probability of type **t** is given by e^{-mKL(**t**||p)}
- Let $n = e^{m\Delta}$, then
 - \circ Lemma: Any type t in the KL ball of radius Δ is sampled almost surely
 - \circ Lemma: No type t outside the KL ball of radius Δ is sampled almost surely



Why does best-of-n work so well?

- Let's revisit the example
- The probability of type **t** is given by e^{-mKL(**t**||p)}
- Let $n = e^{m\Delta}$, then
 - Lemma: Any type t in the KL ball of radius Δ is sampled almost surely
 - \circ Lemma: No type t outside the KL ball of radius Δ is sampled almost surely

Theorem 2. Let ϕ_{Δ} be the optimal solution to Definition 2, and π_N^m be the distribution of the best-of-N, with $N = \exp(m\Delta)$. Under Assumption 1, we have that for all x,

$$\lim_{m \to \infty} \frac{1}{m} D_{\mathrm{KL}}(\pi_N^m(\cdot | \boldsymbol{x}) \| \boldsymbol{\phi}_{\Delta}^m(\cdot | \boldsymbol{x})) = 0.$$
 (5)





Can we distill best-of-n into a new model?

Can we distill best-of-n into a new model?

[Submitted on 8 Jul 2024] Variational Best-of-N Alignment

Afra Amini, Tim Vieira, Ryan Cotterell

[Submitted on 19 Jul 2024]

BOND: Aligning LLMs with Best-of-N Distillation

Pier Giuseppe Sessa, Robert Dadashi, Léonard Hussenot, Johan Ferret, Nino Vieillard, Alexandre Ramé, Bobak Shariari, Sarah Perrin, Abe Friesen Geoffrey Cideron, Sertan Girgin, Piotr Stanczyk, Andrea Michi, Danila Sinopalnikov, <u>Sabela Ramos</u>, Amélie Héliou, Aliaksei Severyn, Matt Hoffman, Nikola Momchev, Olivier Bachem • The PMF of best-of-n suggests a way



Does alignment work in practice?



Why is (safety) alignment hard?

$$\max_{\pi} \mathbb{E}_{\substack{x \sim \rho \\ y \sim \pi}} [r(x, y)] - \beta \mathrm{KL}(\pi \| p)$$

- Reward modeling
 - Reward models are noisy. Does reward ensembling help?^{1,2}
 - Train rewards from a handful of loss patterns.³
- Choosing the prompt set
 - Does automated red teaming help uncover prompts that trigger the model?⁴
 - Safety alignment is shallow, need to think about diverse training prompts.⁵
- Online vs offline
 - Offline methods (e.g., DPO) are not robust.²
- How to think about multi-lingual alignment?⁶

¹Helping or Herding? Reward Model Ensembles Mitigate but do not Eliminate Reward Hacking (Eisenstein et al., 2024).
²Robust Preference Optimization through Reward Model Distillation (Fisch et al., 2024).
³Improving Few-shot Generalization of Safety Classifiers via Data Augmented Parameter-Efficient Fine-Tuning (Balashankar et al., 2024)
⁴Gradient-Based Language Model Red Teaming (Wichers et al., 2024).
⁵Safety Alignment Should Be Made More Than Just a Few Tokens Deep (Qi et al., 2024).
⁶Reuse Your Rewards: Reward Model Transfer for Zero-Shot Cross-Lingual Alignment (Wu et al., 2024).

Safety alignment should be made deeper



Figure 2: ASR vs. Number of Prefilled Harmful Tokens, with $\hat{y} \sim \pi_{\theta}(\cdot | \boldsymbol{x}, \boldsymbol{y}_{\leq k})$ on Harmful HEx-PHI.



Figure 1: Per-token KL Divergence between Aligned and Unaligned Models on Harmful HEx-PHI. Prefilling attacks and finetuning do away safety alignment

Alignment only touches the first few tokens of the model distribution

Offline policy optimization beyond DPO



- Explicit reward modeling through BT model is crucial
- Reward ensembling and pessimistic rewards help a lot!

Further understanding DPO



Further understanding DPO

Further understanding DPO

Takeaways (alignment recipe)

- Step 1: Perform Best-of-n and make sure it works as desired.
 - Inspect a few responses and verify that the ranking induced by reward makes sense.
 - Best-of-n essentially gives the best tradeoffs you can hope for so if best-of-n doesn't work for your problem, no other fancy method will!
 - You'd also be able to debug best-of-n much faster.

Takeaways (alignment recipe)

- Step 1: Perform Best-of-n and make sure it works as desired.
 - Inspect a few responses and verify that the ranking induced by reward makes sense.
 - Best-of-n essentially gives the best tradeoffs you can hope for so if best-of-n doesn't work for your problem, no other fancy method will!
 - You'd also be able to debug best-of-n much faster.
- Step 2: Only then train your favorite alignment method.
 - Track KL($\pi \parallel p$) throughout training
 - KL > 100 The results are unlikely to be any useful!
 - KL > 15 Inspect the outcomes for reward hacking!
 - KL < 8 You are probably OK!</p>

References & Acknowledgments

Controlled Decoding from Language Models	Helping or Herding? 🗟 Reward Model Ensembles Mitigate but do n Eliminate Reward Hacking			
Sidharth Mudgal *1 Jong Lee *1 Harish Ganapathy ¹ YaGuang Li ¹ Tao Wang ² Yanping Huang ¹ Zhifeng Chen ¹ Heng-Tze Cheng ¹ Michael Collins ¹ Trevor Strohman ¹ Jilin Chen ¹ Alex Beutel ² Ahmad Beirami ¹	Jacob Eisenstein ^{1,*} Ahmad Beirami ²	Chirag Nagpal ^{2,*} Alekh Aga Alex D'Amour ¹ DJ Dvijotham ¹ Adam		Alekh Agarwa Adam Fis
Theoretical guarantees on the best-of-n alignment policy	Katherine Heller ² Jonathan Berant ^{1,*}	Stephen Pfohl ²	Deepak Ramachandran ²	Peter Sh
Ahmad Beirami [†] Alekh Agarwal [†] Jonathan Berant [§] Alexander D'Amour [§] Jacob Eisenstein [§] Chirag Nagpal [†] Ananda Theertha Suresh [†]				
Asymptotics of Language Model Alignment	Improving Few-shot Generalization of Safety Class via Data Augmented Parameter-Efficient Fine-Tuning			
Joy Qiping Yang Salman Salamatian Ziteng Sun Ananda Theertha Suresh Ahmad Beirami	Yao Qin ¹ , Jilin Chen ¹ ,	and Alex Beutel* ²	nina , Annaŭ Den ann ,	
Reuse Your Rewards: Reward Model Transfer for Zero-Shot Cross-Lingual Alignment	Grae Warning: th	dient-Based La	nguage Model Rec	I Teaming
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or Herding? 🐂 D MODEL ENSEMBLES MITIGATE BUT DO NOT ATE REWARD HACKING

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Robust Preference Optimization through Reward Model Distillation

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Safety Alignment Should Be Made More Than Just a Few Tokens Deep

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Slides for this talk & more in a language model inference tutorial at ISIT : http://theertha.info/papers/isit 2024 tutorial.pdf (w/ Ananda Theertha Suresh)