

# **Language Model Alignment: Theory & Practice**

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# Alignment

#### **Deep Reinforcement Learning** from Human Preferences

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**Direct Preference Optimization: Your Language Model is Secretly a Reward Model** 

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#### **SELF-INSTRUCT: Aligning Language Models** with Self-Generated Instructions

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#### Training language models to follow instructions with human feedback

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OpenAI

# A benign example of safety alignment

We prefer the response to be safe.



No stereotyping No profanity

….

# An adversarial example of safety alignment

We would like the system to be robust to adversaries.



….

### **Outline**

• Understand alignment through a simplified lens

● Introduce an inference-time alignment framework, called controlled decoding

• Shed light on the remarkable performance of best-of-n alignment

• Conclude with some practical issues of alignment

● A generative **language model p(.|x)** is a distribution over outcome y given x.

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- A **reward model r(x,y)** may be thought of as the log-likelihood of another generative **alignment language model q(.|x)**

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- Expected reward is the negative cross entropy

 $E_{y \sim p}[r(x, y)] = -H(p(.|x) || q(.|x))$ 

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● Alignment Goal: Sample from the **aligned distribution π(.|x)** that leads improve *expected reward* but *remain "close to p."*

#### Best-of-n: A simple baseline for alignment

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Let  $y_1, \ldots, y_n$  be *n* i.i.d. draws from  $p(\cdot|x)$ . The best-of-*n* strategy is denoted by  $\pi^{(n)}$  and returns  $y = y_{k^*}$  where  $k^* := \arg \max_{k \in [n]} r(x, y).$ 

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- Best-of-n is
	- simple
	- effective
	- expensive in terms of throughput
	- o incompatible with streaming

● Markov Decision Process state: prompt **x** action: response **y**

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 $A(\mathbf{x}; \pi) := E_{\mathbf{z} \sim \pi} \left\{ r(\mathbf{x}, \mathbf{z}) \right\} - E_{\mathbf{y} \sim p} \left\{ r(\mathbf{x}, \mathbf{y}) \right\}$ **Advantage** 

● Markov Decision Process state: prompt **x** action: response **y**

• **Reward** 
$$
r(\mathbf{x}, \mathbf{y})
$$

● Advantage

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$$

$$
D(\mathbf{x}; \pi) := KL(\pi(\cdot|\mathbf{x}) \| p(\cdot|\mathbf{x}))
$$

Drift

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	- $D(\mathbf{x}; \pi) := KL(\pi(\cdot|\mathbf{x}) || p(\cdot|\mathbf{x}))$

**RL objective**

Drift

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J_{\beta}(\mathbf{x}; \pi) := A(\mathbf{x}; \pi) - \beta D(\mathbf{x}; \pi)
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Linear in  $π$  Strongly convex in  $π$ 

#### RL objective could be solved in closed form

**Theorem 1.** The optimal policy for the RL objective is unique and is given by  $\pi_{\beta}^{\star}(\mathbf{y}|\mathbf{x}) \propto p(\mathbf{y}|\mathbf{x})e^{\frac{1}{\beta}r(\mathbf{x}, \mathbf{y})}.$ 

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At the sequence level, the solution is a tilted mismatched distribution<sup>2</sup>

 $\boldsymbol{\pi}_{\beta}(\mathsf{y}|\mathsf{x}) \boldsymbol{\propto} \mathsf{p}(\mathsf{y}|\mathsf{x})\,\mathsf{q}(\mathsf{y}|\mathsf{x})^{1/\beta}$ 

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 $\pi_{\beta}(y|x) \propto p(y|x) q(y|x)^{1/\beta}$ 

- Large  $\beta \rightarrow \infty$  $\pi_{\beta}(y|x) \approx p(y|x)$
- **Small**  $\beta \approx 0$  $\pi_{\beta}(y|x) \approx 1$  if y = argmax **q**(y|x)

[1RL with KL penalties is better viewed as Bayesian inference](https://arxiv.org/abs/2205.11275) (Korbak, Tomasz et al., EMNLP 2022). <sup>2</sup>Mismatched Guesswork (Salamatian et al., Information Theory Workshop 2019).



# Aligned family



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## Where does the alignment distribution come from?



No stereotyping No profanity

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- Reward may be trained on
	- (x, y, *safe*) tuples labeled for safety similar to a classifier
	- (x, y+ , y- , *preferred*) tuples depicting preference using the Bradley Terry model
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- -
- Reward doesn't have to be trained/differentiable, e.g., response length. Google

#### RLHF on side-by-side (s x s) preference data

● Bradley Terry model

$$
p(y_1 \prec y_2 \mid x) = \sigma(r(x, y_2) - r(x, y_1))
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● KL-regularized Reinforcement learning

$$
\max_{\pi} \ \mathbb{E}_{\substack{x \sim \rho \\ y \sim \pi}}[r(x, y)] - \beta \text{KL}(\pi \| p)
$$

# Many alignment methods

• Direct preference optimization (DPO)<sup>1</sup>

$$
\max_{\pi} \ \mathbb{E}_{(x,y^+,y^-)\thicksim D} \left[ \log \sigma \left( \beta \log \frac{\pi (y^+|x) }{p(y^+|x)} - \beta \log \frac{\pi (y^-|x) }{p(y^-|x)} \right) \right]
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 $\bullet$  Identity preference optimization (IPO)<sup>2</sup>

$$
\max_{\pi} \mathbb{E}_{\substack{x \sim \rho \\ y_1 \sim p \\ y_2 \sim \pi}} [\Phi(p(y_1 \prec y_2 \mid x))] - \lambda \text{KL}(\pi \| p)
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[1Direct Preference Optimization: Your Language Model is Secretly a Reward Model](https://arxiv.org/abs/2305.18290) (Rafailov et al., NeurIPS 2023). <sup>2</sup>A General Theoretical Paradigm to Understand Learning from Human Preferences (Azar et al., AISTATS 2024).

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Many more!!!

# How do they differ?

- Deployment
	- Training-time solution, e.g., DPO/PPO widely used, easy to deploy
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All methods roughly solve a KL-regularized RL problem

## The role of reverse KL regularizer in alignment

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- Assume a logistic language model
- Supervised finetuning (SFT)

 $\theta_{\text{sft}}^* = \arg \min_{\theta} \mathcal{L}_{\text{sft}}(\theta) \quad \text{where} \quad \mathcal{L}_{\text{sft}}(\theta) := E_{(x,y) \sim D_{\text{sft}}} \{A(\theta; x) - \theta^{\top} g(x, y)\},$ 

KL-regularized RL

 $\theta^*_{\text{bilevel},\beta} = \arg \min_{\theta} \mathcal{L}_{\text{bilevel},\beta}(\theta) \qquad \text{where} \qquad \mathcal{L}_{\text{bilevel}}(\theta) := D_{\text{KL}}(\pi_{\theta} || \pi_{\text{sft}}) + \frac{1}{\beta} \mathcal{L}_{\text{ro}}(\theta),$ 

● Multi-tasking SFT and reward optimization

 $\theta^*_{\text{multi-task},\beta} = \argmin_{\theta} \mathcal{L}_{\text{multi-task},\beta}(\theta) \qquad \text{where} \qquad \mathcal{L}_{\text{multi-task}}(\theta) := \mathcal{L}_{\text{sft}}(\theta) + \frac{1}{\beta} \mathcal{L}_{\text{ro}}(\theta).$ 

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Multi-tasking SFT and reward optimization

 $\theta_{\text{multi-task},\beta}^* = \argmin_{\theta} \mathcal{L}_{\text{multi-task},\beta}(\theta) \quad \text{where} \quad \mathcal{L}_{\text{multi-task}}(\theta) := \mathcal{L}_{\text{sft}}(\theta) + \frac{1}{\beta} \mathcal{L}_{\text{ro}}(\theta).$ 

**Proposition 1.** For all  $\beta \in \mathbb{R}$ , we have  $\theta^*_{bilevel, \beta} = \theta^*_{multi\text{-}task, \beta}$ .



#### YAAM: Yet Another Alignment Method

• Markov Decision Process state: prompt + decoded prefix **x**,  $y^t$  action: next token  $y_{t+1}$ 

• Markov Decision Process state: prompt + decoded prefix **x**,  $y^t$  action: next token  $y_{t+1}$ 

● Reward

$$
R([\mathbf{x}, y^t]) := \begin{cases} 0 & y_t \neq EOS \\ r([\mathbf{x}, y^t]) & y_t = EOS \end{cases}
$$

• Markov Decision Process state: prompt + decoded prefix **x**,  $y^t$  action: next token  $y_{t+1}$ 

- Reward
- Value

$$
R([\mathbf{x}, y^t]) := \begin{cases} 0 & y_t \neq EOS \\ r([\mathbf{x}, y^t]) & y_t = EOS \end{cases}
$$

$$
V^*([\mathbf{x}, y^t]) := E_{z_1, z_2, \dots \sim p} \left\{ \sum_{\tau \ge 0} R([\mathbf{x}, y^t, z^{\tau}]) \right\}
$$

• Markov Decision Process state: prompt + decoded prefix **x**, y<sup>t</sup> action: next token y<sub>th</sub>

- Reward
- Value
- Advantage

$$
R([\mathbf{x}, y^t]) := \begin{cases} 0 & y_t \neq EOS \\ r([\mathbf{x}, y^t]) & y_t = EOS \end{cases}
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$$
V^{\star}([\mathbf{x}, y^t]) := E_{z_1, z_2, \dots \sim p} \left\{ \sum_{\tau \geq 0} R([\mathbf{x}, y^t, z^{\tau}]) \right\}
$$

$$
A([\mathbf{x}, y^t]; \pi) := E_{z \sim \pi} \left\{ V^{\star}([\mathbf{x}, y^t, z]) - V^{\star}([\mathbf{x}, y^t]) \right\}
$$

• Markov Decision Process state: prompt + decoded prefix **x**,  $y^t$  action: next token  $y_{t+1}$ 

- Reward
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R([\mathbf{x}, y^t]) := \begin{cases} 0 & y_t \neq EOS \\ r([\mathbf{x}, y^t]) & y_t = EOS \end{cases}
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D([\mathbf{x}, y^t]; \pi) := KL(\pi(\cdot | [\mathbf{x}, y^t]) || p(\cdot | [\mathbf{x}, y^t]))
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J_{\beta}([\mathbf{x}, y^t]; \pi) := A([\mathbf{x}, y^t]; \pi) - \beta D([\mathbf{x}, y^t]; \pi)
$$

● Value

● Reward

● Advantage

Drift

**RL objective**

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● Reward

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**RL objective**

 $J$ 

Linear in  $π$  Strongly convex in  $π$ 

 $;\pi)$ 

Google

## Controlled decoding (CD)

**Theorem 2.** The optimal policy for the RL objective is unique and is given by  $\pi_{\beta}^{\star}(z|[\mathbf{x},y^t]) \propto p(z|[\mathbf{x},y^t])e^{\frac{1}{\beta}V^{\star}([\mathbf{x},y^t,z])}.$ 

## Controlled decoding (CD)

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I earn the value function:

$$
\mathcal{L}^{\star}(\boldsymbol{\theta}) = E_{\mathbf{x} \sim p_{\mathbf{x}}} E_{\mathbf{y} \sim p_{\mathbf{y}|\mathbf{x}}} \ell^{\star}(\mathbf{x}, \mathbf{y}; \boldsymbol{\theta}),
$$
  
where 
$$
\ell^{\star}(\mathbf{x}, \mathbf{y}; \boldsymbol{\theta}) = \frac{1}{2} \sum_{t \in [|\mathbf{y}|]} (V_{\boldsymbol{\theta}}([\mathbf{x}, y^t]) - V^{\star}([\mathbf{x}, y^t]))^2
$$

#### CD-FUDGE

 $\bullet$  Use an unbiased draw from the model as the target<sup>1</sup>

$$
\mathcal{L}_{F}(\boldsymbol{\theta}) = E_{\mathbf{x} \sim p_{\mathbf{x}}} \ell_{F}(\mathbf{x}, \mathbf{y}; \boldsymbol{\theta}), \qquad \text{s.t. } \mathbf{y} \sim p,
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**Theorem 3.1** (informal). Under regularity assumptions, applying SGD on  $\mathcal{L}_F$  converges to a stationary point of  $\mathcal{L}^*(\theta)$ .

#### CD-Q

• Bellman identity

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● Train the value function similarly to DQN

$$
\mathcal{L}_{Q}(\theta) = E_{\mathbf{x} \sim p_{\mathbf{x}}} \ell_{Q}(\mathbf{x}, \mathbf{y}; \theta),
$$
  
where  $\ell_{Q}(\mathbf{x}, y^{t}; \theta) = \frac{1}{2} \sum_{t \in [|\mathbf{y}|]} (V_{\theta}([\mathbf{x}, y^{t}]) - \dot{v}_{t})^{2},$   

$$
v_{t} = \begin{cases} \sum_{z \in \mathcal{Y}} p(z | [x, y^{t}]) V_{\theta}([\mathbf{x}, y^{t}, z]) & y_{t} \neq EOS \\ r([\mathbf{x}, y^{t}]) & y_{t} = EOS \end{cases}
$$

## Token-wise control using CD

**Theorem 2.** The optimal policy for the RL objective is unique and is given by  $\pi_{\beta}^{\star}(z|[\mathbf{x}, y^t]) \propto p(z|[\mathbf{x}, y^t]) e^{\frac{1}{\beta}V^{\star}([\mathbf{x}, y^t, z])}.$ 

Will this paper get accepted?



## Token-wise control using CD

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## Blockwise control using CD (best-of-n++)

• Draw K blocks of length M tokens

$$
\left\{z_{(k)}^M\right\}_{k\in[\mathsf{N}]} \overset{\text{i.i.d.}}{\sim} p(z^M|[\mathbf{x},y^t])
$$

● Accept the continuation with the highest prefix score:

$$
z^M := \arg\max_{\left\{z_{(k)}^M\right\}_{k \in [n]}} V_{\boldsymbol{\theta}}([\mathbf{x}, y^t, z_{(k)}^M])
$$

## Blockwise control using CD (best-of-n++)

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Accept the continuation with the highest prefix score:

$$
z^M := \arg\max_{\left\{z_{(k)}^M\right\}_{k \in [n]}} V_{\boldsymbol{\theta}}([\mathbf{x}, y^t, z_{(k)}^M])
$$





sentiment prefix score

## Blockwise control using CD (best-of-n++)

Draw K blocks of length M tokens

$$
\left\{z_{(k)}^M\right\}_{k\in[\mathsf{n}]} \stackrel{\text{i.i.d.}}{\sim} p(z^M|[\mathbf{x},y^t])
$$

Accept the continuation with the highest prefix score:

$$
z^M := \arg\max_{\left\{z_{(k)}^M\right\}_{k \in [1]}} V_{\boldsymbol{\theta}}([\mathbf{x}, y^t, z_{(k)}^M])
$$





#### Advantages over best-of-n:

- Limits the user-facing latency to the decoding time of a single block.
- Makes a lar[ge effective n pra](https://arxiv.org/abs/2310.17022)ctically feasible.

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## How do we evaluate alignment methods?

- Human evaluations
- Auto-evals that are correlated with human judgement

#### How do we evaluate alignment methods?



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#### How do we evaluate alignment methods?



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## Estimating KL divergence

● We estimate KL divergence via aggregating the log-likelihood ratios between the aligned model and the base model

$$
D([\mathbf{x}, y^t]; \pi) := KL(\pi(\cdot | [\mathbf{x}, y^t]) || p(\cdot | [\mathbf{x}, y^t]))
$$
  
= 
$$
\sum_{z \in \mathcal{Y}} \pi(z | [\mathbf{x}, y^t]) \log \left( \frac{\pi(z | [\mathbf{x}, y^t])}{p(z | [\mathbf{x}, y^t])} \right)
$$
 Log-likelihood ratio  
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Log-likelihood ratio unbiased estimate of livergence

● We don't have the logits of best-of-n or blockwise CD. How can we estimate KL divergence?

## Analytical formula for KL divergence of best-of-n

An analytical formula that has appeared many times in the literature $1,2$ 

$$
\text{KL}(\pi^{(n)}_{\bm{y}|\bm{x}}\|p_{\bm{y}|\bm{x}}) \stackrel{\text{claim}}{=\!\!=} \widetilde{\text{KL}}_n := \log(n)-(n-1)/n.
$$

- This formula remarkably
	- **b** doesn't depend on the prompt **x** or its distribution **ρ**<sub>x</sub>
	- o doesn't depend on the base policy  $p_{v|x}$

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Is this formula true?

#### Analytical formula is wrong!

 $\sim$   $\sim$   $\sim$ 

**Example 1.** Consider an unprompted model with  $x = \emptyset$  (no input) and binary output,  $y \in \{0, 1\}$ . Let the two outcomes be equiprobable, i.e.,  $p_{y|x}(0) = p_{y|x}(1) = \frac{1}{2}$ . Further, let  $r(0) = 0$ , and  $r(1) = 1$ , i.e., outcome 1 is more desirable than outcome 0. Here, we can compute  $\pi_{y|x}^{(n)}$  in closed form. Specifically, we can see that  $\pi_{\mathbf{y}|\mathbf{x}}^{(n)}(0) = \frac{1}{2^n}$  and  $\pi_{\mathbf{y}|\mathbf{x}}^{(n)}(1) = 1 - \frac{1}{2^n}$ . Thus,

$$
KL(\pi_{\boldsymbol{y}|\boldsymbol{x}}^{(n)} \| p_{\boldsymbol{y}|\boldsymbol{x}}) = \log(2) - h\left(\frac{1}{2^n}\right)
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$$

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$$



#### Guarantees on the analytical formula

**Theorem 2.** For any  $n \in \mathbb{N}$  and any  $x$ ,  $KL(\pi^{(n)} \| p) \leq \widetilde{KL}_n = \log(n) - \frac{n-1}{n}.$ 

- Analytical formula is an upper bound
- Let  $y \sim \pi^{(n)}$ . Then, let  $\varepsilon_n := p(y)$ ●
	- Theorem: The gap is small if **n.ε <sup>n</sup>** ≪ **1**
	- Theorem: The gap is large if **n.ε <sup>n</sup>** ≫ **1**

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- More recently, Mroueh showed that this result is an instance of strong data processing inequality<sup>2</sup>
### New estimator for KL divergence of best-of-n

**Approximation 1.** Let  $y \sim \pi^{(n)}$ . Then, let  $\varepsilon_n := p(y)$ . We propose the following estimator for the KL divergence of the best-of-n policy and the base policy:

$$
\widehat{KL}(\varepsilon_n) := (1-\varepsilon_n)^n \left( \log n + (n-1) \log(1-\varepsilon_n) - \frac{n-1}{n} \right) + (1-(1-\varepsilon_n)^n) \log \left( \frac{1-(1-\varepsilon_n)^n}{\varepsilon_n} \right).
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$$



### Win-rate of best-of-n

$$
\text{win-rate}(\pi \| p) := E_{y \sim \pi} E_{z \sim p} \{ \mathbf{1}(r(x, y) > r(x, z)) + \frac{1}{2} \mathbf{1}(r(x, y) = r(x, z)) \}
$$

**Lemma 1.** The win-rate for best-of-n policy is given by

$$
win\text{-}rate(\pi^{(n)}\|p) = 1 - \frac{1}{2}E_{y \sim p}\{F(y|x)^n + F^-(y|x)^n\} \le \frac{n}{n+1}.
$$

- Roughly the upper bound argument follows from
	- Draw (n+1) outcomes from the base model; and order them from the highest to lowest reward
	- Randomly assign 1 of the outcomes to base model
	- Choose the best of the remaining n to be a draw from the best-of-n model.
	- Best-of-n wins against the reference model with probability n/(n+1).
- The upper bound would be exact if w/p 1 no two outcomes were identical

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### Win-rate vs KL tradeoffs (helpfulness & harmlessness)



Figure 4: Win rate vs. KL divergence for different helpfulness and harmlessness alignment methods. CD-Q (blockwise) vastly outperforms RL techniques such as IPO & PPO.

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### Win-rate vs KL tradeoff for best-of-n



- KL values <10 are sufficient to reach a high win-rate against base policy
- This is the ideal setting ignoring noise in reward and generalization

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### Win-rate vs KL tradeoffs



 $(b)$  RL

Google

[Scaling Laws for Reward Model Overoptimization](https://arxiv.org/abs/2210.10760) (Gao et al., ICML 2023).

### Win-rate vs KL tradeoffs



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Google

[Scaling Laws for Reward Model Overoptimization](https://arxiv.org/abs/2210.10760) (Gao et al., ICML 2023).

### Win-rate vs KL tradeoffs (helpfulness & harmlessness)



- Best-of-n is better than state-of-the-art RL methods
- Blockwise CD bridges the gap between tokenwise control and best-of-n
- Token-wise CD is a good contender for token-wise control (on par with other methods)

Figure 4: Win rate vs. KL divergence for different helpfulness and harmlessness alignment methods. CD-Q (blockwise) vastly outperforms RL techniques such as IPO & PPO.

# Modularity of CD for the win!



Multi-objective alignment

# Modularity of CD for the win!



# Modularity of CD for the win!



# Optimal reward-KL tradeoff

● Theorem: KL-regularized RL solution is optimal for reward-KL tradeoff

Linear in π Strongly convex in π

# Optimal reward-KL tradeoff

• Theorem: KL-regularized RL solution is optimal for reward-KL tradeoff

$$
J_{\beta}(\mathbf{x};\pi) := A(\mathbf{x};\pi) - \beta D(\mathbf{x};\pi)
$$
  
\n
$$
E_{\mathbf{z} \sim \pi} \{r(\mathbf{x}, \mathbf{z})\} - E_{\mathbf{y} \sim p} \{r(\mathbf{x}, \mathbf{y})\}
$$
  
\n
$$
E_{\text{Linear in }\pi}
$$
  
\n
$$
M(\pi(\cdot|\mathbf{x})||p(\cdot|\mathbf{x}))
$$
  
\n
$$
S_{\text{trongly convex in }\pi}
$$

• Empirically, best-of-n is striking to the optimal trade-off

# Why does best-of-n work so well?

- Let's revisit the example
- The probability of type **t** is given by e<sup>-mKL(t||p)</sup>



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- Let's revisit the example
- The probability of type **t** is given by e<sup>-mKL(t||p)</sup>
- Let  $n = e^{m\Delta}$ , then
	- Lemma: Any type **t** in the KL ball of radius Δ is sampled almost surely
	- Lemma: No type **t** outside the KL ball of radius Δ is sampled almost surely



# Why does best-of-n work so well?

- Let's revisit the example
- The probability of type **t** is given by e<sup>-mKL(t||p)</sup>
- Let  $n = e^{m\Delta}$ , then
	- Lemma: Any type **t** in the KL ball of radius Δ is sampled almost surely
	- Lemma: No type **t** outside the KL ball of radius Δ is sampled almost surely

**Theorem 2.** Let  $\phi_{\Delta}$  be the optimal solution to Definition 2, and  $\pi_N^m$  be the distribution of the best-of-N, with  $N =$  $\exp(m\Delta)$ . Under Assumption 1, we have that for all x,

$$
\lim_{m\to\infty}\frac{1}{m}D_{\mathrm{KL}}(\pi_N^m(\cdot|\boldsymbol{x})\|\boldsymbol{\phi}_{\Delta}^m(\cdot|\boldsymbol{x}))=0. \hspace{1cm} (5)
$$



### Can we distill best-of-n into a new model?

# Can we distill best-of-n into a new model?

#### [Submitted on 8 Jul 2024] Variational Best-of-N Alignment

#### Afra Amini, Tim Vieira, Ryan Cotterell

[Submitted on 19 Jul 2024]

#### **BOND: Aligning LLMs with Best-of-N Distillation**

Pier Giuseppe Sessa, Robert Dadashi, Léonard Hussenot, Johan Ferret, Nino Vieillard, Alexandre Ramé, Bobak Shariari, Sarah Perrin, Abe Friesen Geoffrey Cideron, Sertan Girgin, Piotr Stanczyk, Andrea Michi, Danila Sinopalnikov, Sabela Ramos, Amélie Héliou, Aliaksei Severyn, Matt Hoffman, Nikola Momchev, Olivier Bachem

• The PMF of best-of-n suggests a way



# Does alignment work in practice?



# Why is (safety) alignment hard?

$$
\max_{\pi} \ \mathbb{E}_{\substack{x \sim \rho \\ y \sim \pi}}[r(x, y)] - \beta \text{KL}(\pi \| p)
$$

- Reward modeling
	- $\circ$  Reward models are noisy. Does reward ensembling help?<sup>1,2</sup>
	- $\circ$  Train rewards from a handful of loss patterns.<sup>3</sup>
- Choosing the prompt set
	- Does automated red teaming help uncover prompts that trigger the model?<sup>4</sup>
	- $\circ$  Safety alignment is shallow, need to think about diverse training prompts.<sup>5</sup>
- Online vs offline
	- Offline methods (e.g., DPO) are not robust.<sup>2</sup>
- How to think about multi-lingual alignment? $6$

[1Helping or Herding? Reward Model Ensembles Mitigate but do not Eliminate Reward Hacking](https://arxiv.org/abs/2312.09244) (Eisenstein et al., 2024). [2Robust Preference Optimization through Reward Model Distillation](https://arxiv.org/abs/2405.19316) (Fisch et al., 2024). <sup>3</sup>[Improving Few-shot Generalization of Safety Classifiers via Data Augmented Parameter-Efficient Fine-Tuning](https://arxiv.org/abs/2310.16959) **(Balashankar et al., 2024)** [4Gradient-Based Language Model Red Teaming](https://arxiv.org/abs/2401.16656) (Wichers et al., 2024). <sup>5</sup>[Safety Alignment Should Be Made More Than Just a Few Tokens Deep](https://arxiv.org/abs/2406.05946) (Qi et al., 2024). [6Reuse Your Rewards: Reward Model Transfer for Zero-Shot Cross-Lingual Alignment](https://arxiv.org/abs/2310.17022) (Wu et al.,2024).

### Safety alignment should be made **deeper**



Figure 2: ASR vs. Number of Prefilled Harmful Tokens, with  $\hat{y} \sim$  $\pi_{\theta}(\cdot|\boldsymbol{x},\boldsymbol{y}_{< k})$  on Harmful HEx-PHI.



Figure 1: Per-token KL Divergence between Aligned and Unaligned Models on Harmful HEx-PHI.

**Prefilling attacks** and finetuning do away safety alignment

Alignment only touches the first few tokens of the model distribution

# Offline policy optimization beyond DPO



- Explicit reward modeling through BT model is crucial
- Reward ensembling and pessimistic rewards help a lot!

# Further understanding DPO



# Further understanding DPO

# Further understanding DPO

# Takeaways (alignment recipe)

- Step 1: Perform Best-of-n and make sure it works as desired.
	- Inspect a few responses and verify that the ranking induced by reward makes sense.
	- Best-of-n essentially gives the best tradeoffs you can hope for so if best-of-n doesn't work for your problem, no other fancy method will!
	- You'd also be able to debug best-of-n much faster.

# Takeaways (alignment recipe)

- Step 1: Perform Best-of-n and make sure it works as desired.
	- Inspect a few responses and verify that the ranking induced by reward makes sense.
	- Best-of-n essentially gives the best tradeoffs you can hope for so if best-of-n doesn't work for your problem, no other fancy method will!
	- You'd also be able to debug best-of-n much faster.
- Step 2: Only then train your favorite alignment method.
	- Track KL(**π** || **p**) throughout training
		- KL > 100 The results are unlikely to be any useful!
		- KL > 15 Inspect the outcomes for reward hacking!
		- KL < 8 You are probably OK!

# References & Acknowledgments



or Herding? MODEL ENSEMBLES MITIGATE BUT DO NOT TE REWARD HACKING



**Carson Denison** 

#### **Robust Preference Optimization** through Reward Model Distillation

Adam Fisch\* Jacob Eisenstein\* Vicky Zayats\* Alekh Agarwal Ahmad Beirami Chirag Nagpal Pete Shaw Jonathan Berant\*

ankar<sup>1</sup>, Xiao Ma<sup>1</sup>, Aradhana Sinha<sup>1</sup>, Ahmad Beirami<sup>1</sup>,

#### **Safety Alignment Should Be Made** More Than Just a Few Tokens Deep

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**Ahmad Beirami** 

Slides for this talk & more in a **language model inference tutorial** at ISIT : [http://theertha.info/papers/isit\\_2024\\_tutorial.pdf](http://theertha.info/papers/isit_2024_tutorial.pdf) (w/ Ananda Theertha Suresh)