# Certifying almost all quantum states with few single-qubit measurements

Hsin-Yuan Huang (Robert)

with John Preskill and Mehdi Soleimanifar

# Motivation

- Quantum systems with **intricate entanglement** are pivotal in quantum information science.

# Motivation

- Quantum systems with intricate entanglement are pivotal in quantum information science.
- To understand if we have created the desired quantum system in the lab, we need to perform certification.

# What is Certification?

- We have a desired $n$-qubit state $|\psi\rangle$, which is our target state.
- We have an $n$-qubit state $\rho$ created in the experimental lab.
- **Task:** Test if $\rho$ is close to $|\psi\rangle\langle\psi|$ or not from data?

($\langle\psi|\rho|\psi\rangle$ is close to 1)

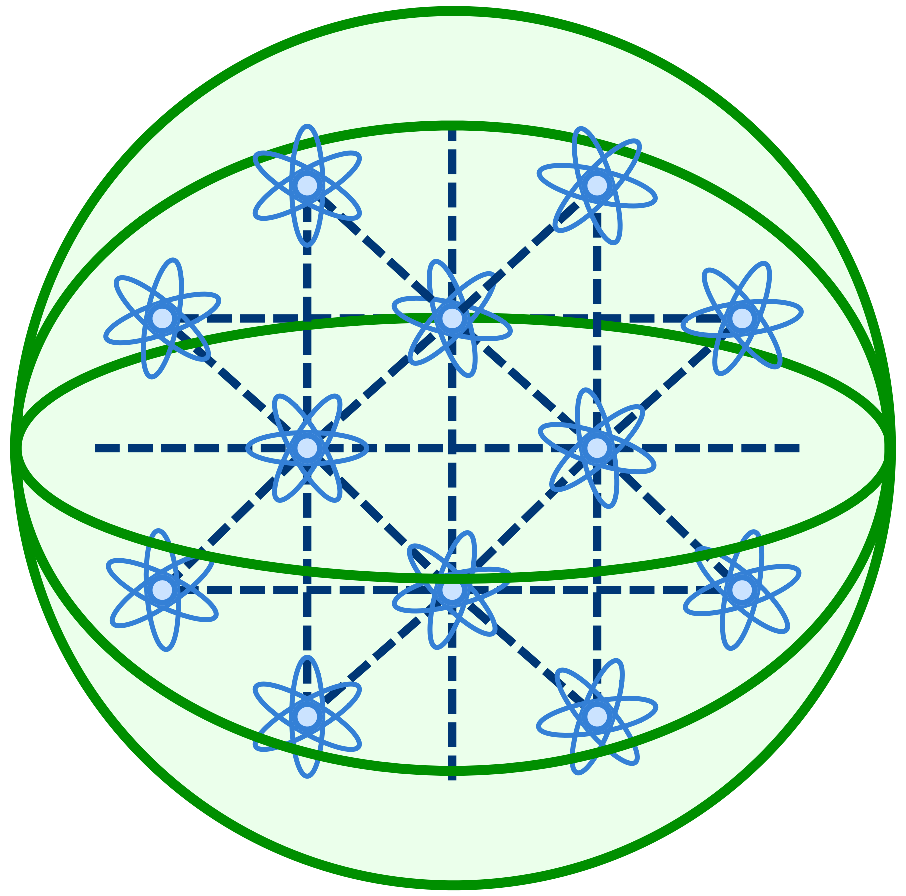
# Motivation

- Many techniques have been proposed for certification.
- However, it remains **experimentally challenging** to certify highly-entangled quantum many-body systems.

# How to Certify?

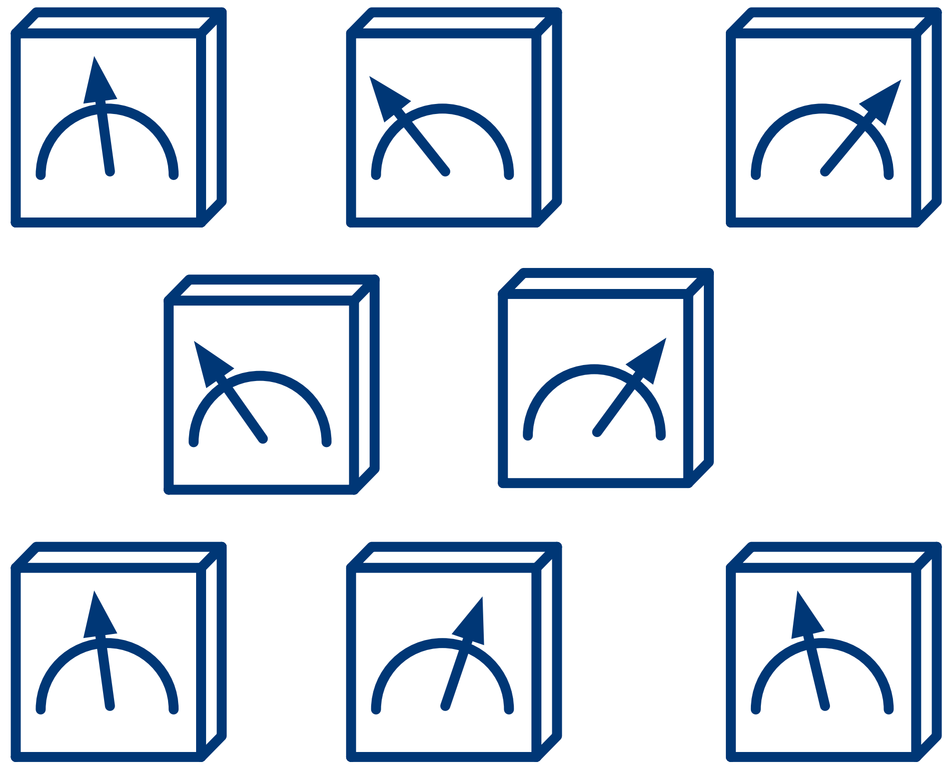
- **Approach 0:** Direct measurement

$|\psi\rangle = U|0^n\rangle$



Quantum state

$U^\dagger$



Single-qubit Measurement

# How to Certify?

- **Approach 0:** Direct measurement
- **Challenge:**

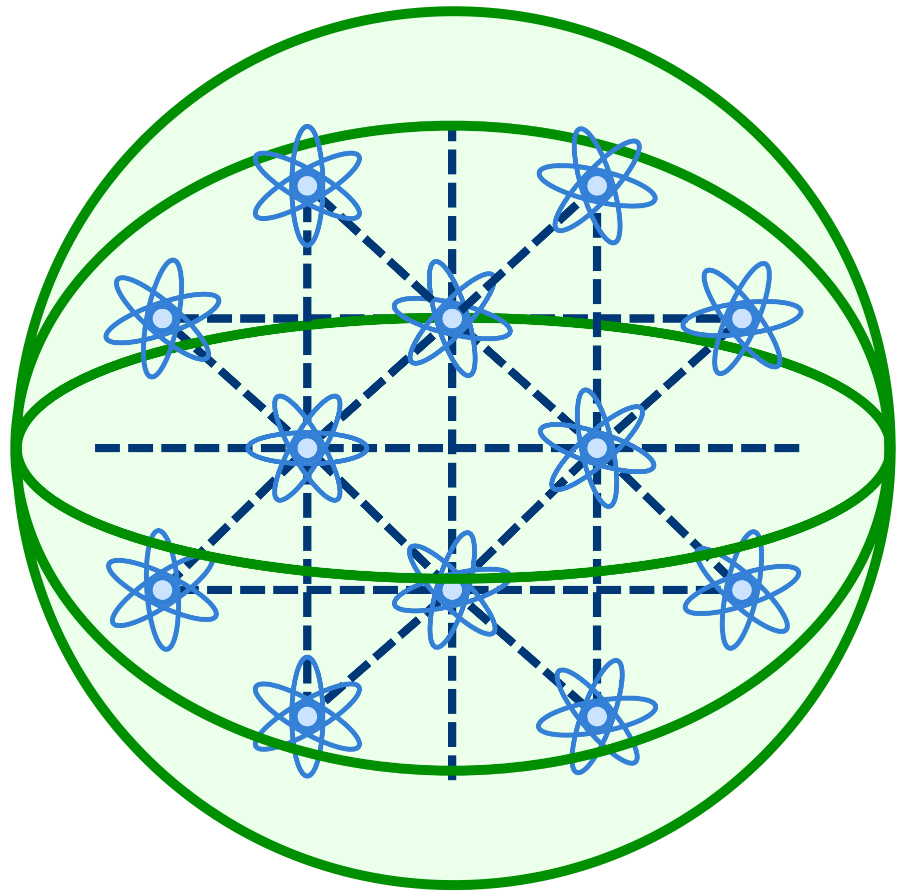
  If we can assume $U^\dagger$ is perfect, then $U$ should be perfect too.

  In this world, $\rho$ can be created to be $|\psi\rangle$ perfectly.

  So we don't need to do any certification.

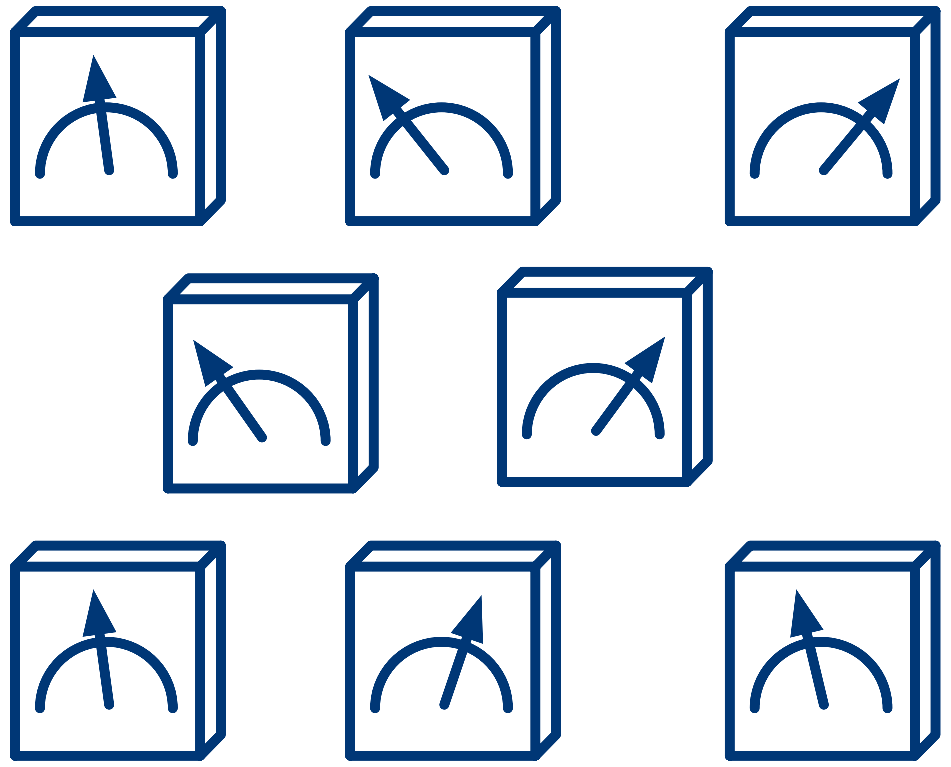
# How to Certify?

- **Approach 1:** Random Clifford measurements (classical shadow)



Quantum state

Random
Clifford Circuit



Single-qubit
Measurement

# How to Certify?

- **Approach 1:** Random Clifford measurements (classical shadow)
- **Advantage:**

  Only needs **depth-$n$** random Clifford circuits on $\rho$

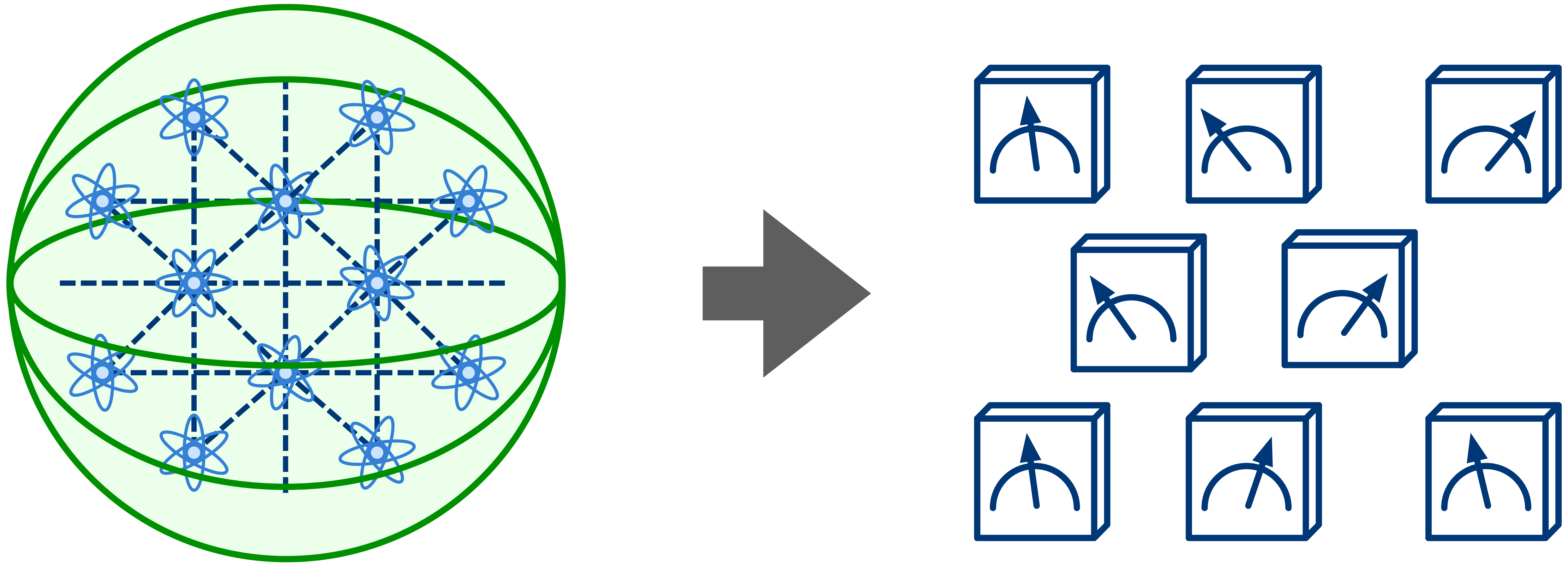
- **Challenge:**

  Implementing depth-$n$ random Clifford circuits

  is still experimentally challenging.

# How to Certify?

- **Approach 2:** Random Pauli measurements (classical shadow)



Quantum state

Single-qubit Measurement

# How to Certify?

- **Approach 2:** Random Pauli measurements (classical shadow)
- **Advantage:**

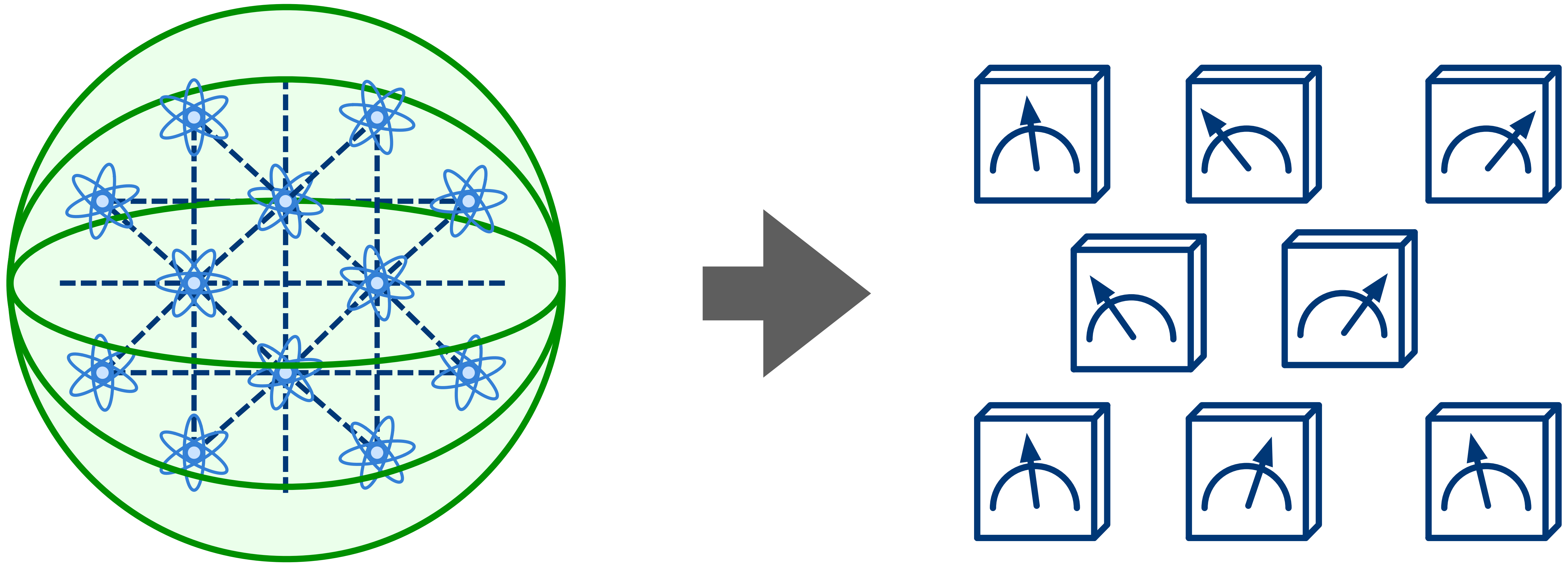
  Only needs **single-qubit** measurements on $\rho$
- **Challenge:**

  Requires **exp($n$)** measurements for most target $|\psi\rangle$

  especially when $|\psi\rangle$ is highly entangled.

# How to Certify?

- **Approach 3:** Cross-entropy benchmark (XEB)



Quantum state

Single-qubit
Measurement
(all Z bases)

# How to Certify?

- **Approach 3:** Cross-entropy benchmark (XEB)
- **Advantage:**

  Only needs single-qubit measurements (Z-basis) on $\rho$
- **Challenge:**

  Does not rigorously address the certification task.

  $\rho$ can be **far** from $|\psi\rangle\langle\psi|$ despite perfect XEB score.

# Existing Challenges

- All existing certification protocols either

# Existing Challenges

- All existing certification protocols either
    a. Require **deep** quantum circuits before measurements

# Existing Challenges

- All existing certification protocols either

  a. Require **deep** quantum circuits before measurements

  b. Use **exponentially** many measurements

# Existing Challenges

- All existing certification protocols either

    a. Require **deep** quantum circuits before measurements

    b. Use **exponentially** many measurements

    c. Apply only for **specialized** target state $|\psi\rangle$

# Existing Challenges

- All existing certification protocols either
  a. Require **deep** quantum circuits before measurements
  b. Use **exponentially** many measurements
  c. Apply only for **specialized** target state $|\psi\rangle$
  d. Lack rigorous guarantees

# Question

Can we rigorously certify **highly-entangled** quantum states from performing **few single-qubit** measurements?

# Question

Can we rigorously certify **almost all** quantum states from performing **few single-qubit** measurements?

# Outline

- Theorem
- Protocol
- Applications

# Outline

- Theorem
- Protocol
- Applications

# Certification

**Theorem 1**

For almost all $n$-qubit state $|\psi\rangle$, we can certify that $\rho$ is close to $|\psi\rangle\langle\psi|$ using only $\mathcal{O}(n^2)$ single-qubit measurements.

# Certification

**Theorem 1**

For almost all $n$-qubit state $|\psi\rangle$, we can certify that $\rho$ is close to $|\psi\rangle\langle\psi|$ using only $\mathcal{O}(n^2)$ single-qubit measurements.

- The certification procedure applies to any $\rho$.

# Certification

**Theorem 1**

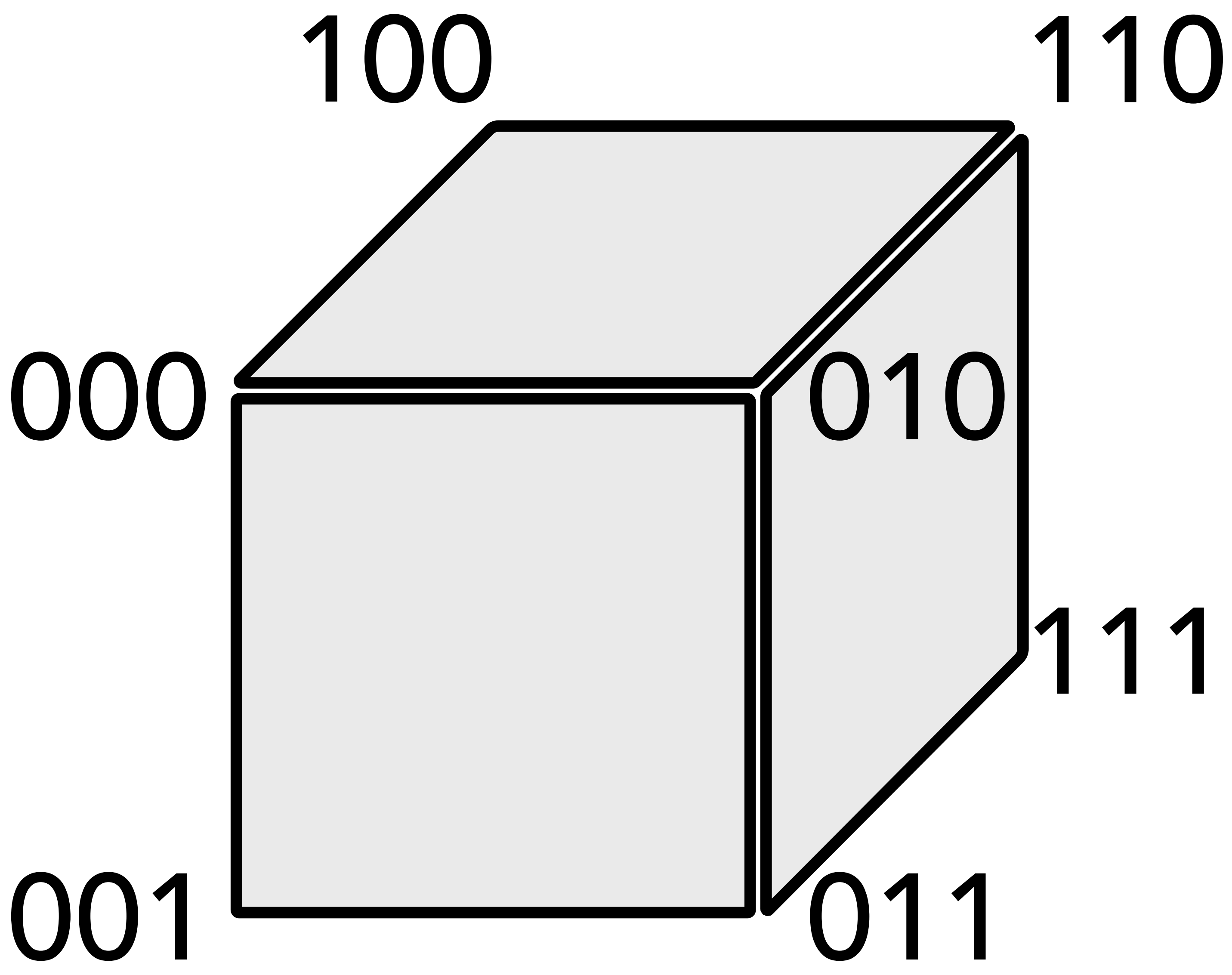
For almost all $n$-qubit state $|\psi\rangle$, we can certify that $\rho$ is close to $|\psi\rangle\langle\psi|$ using only $\mathcal{O}(n^2)$ single-qubit measurements.

- The certification procedure applies to any $\rho$.
- $\mathcal{O}(n^2)$ is enough even when $|\psi\rangle$ has $\exp(n)$ circuit complexity.

# Relaxation Time

- Consider an $n$-qubit target state $|\psi\rangle$.
- Choose a basis $|b\rangle$, where $b \in \{0,1\}^n$ is a bitstring.
- Let $\pi(b) = |\langle b|\psi\rangle|^2$ be the **measurement distribution**.

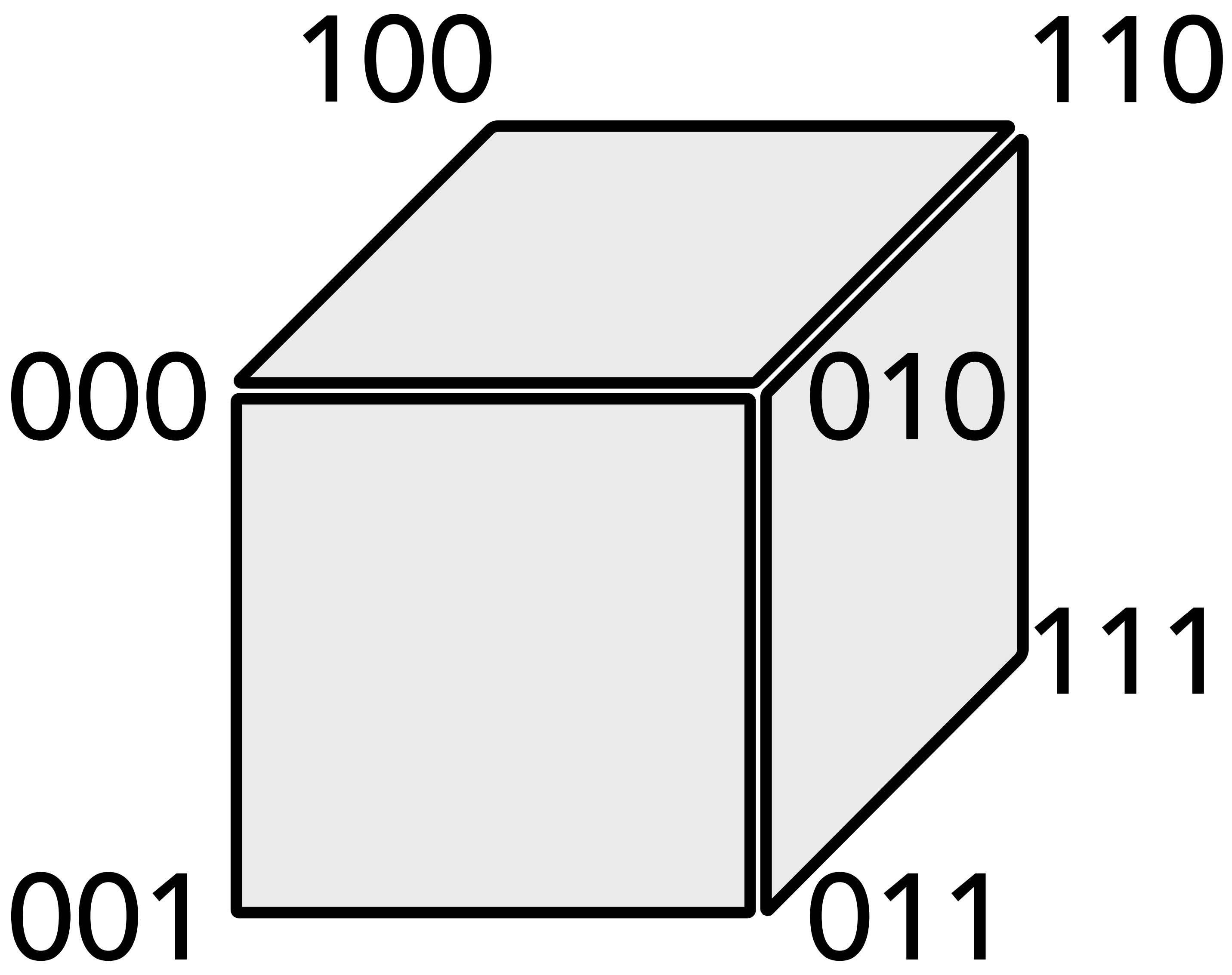
Boolean Hypercube



# Relaxation Time

- Let $\pi(b) = |\langle b|\psi\rangle|^2$ be the **measurement distribution**.
- Consider a random walk on $n$-bit Boolean hypercube.

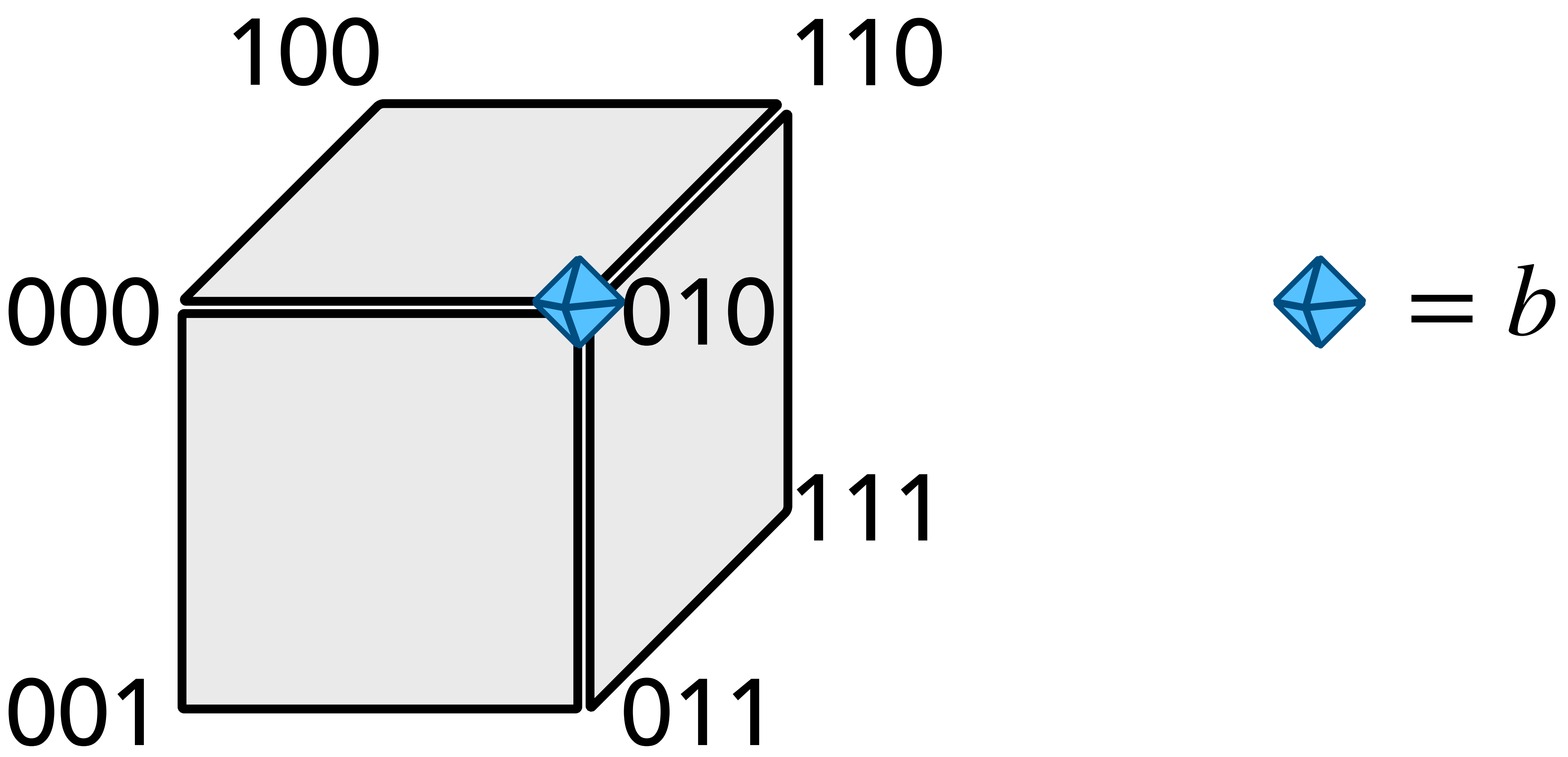
Boolean
Hypercube



# Relaxation Time

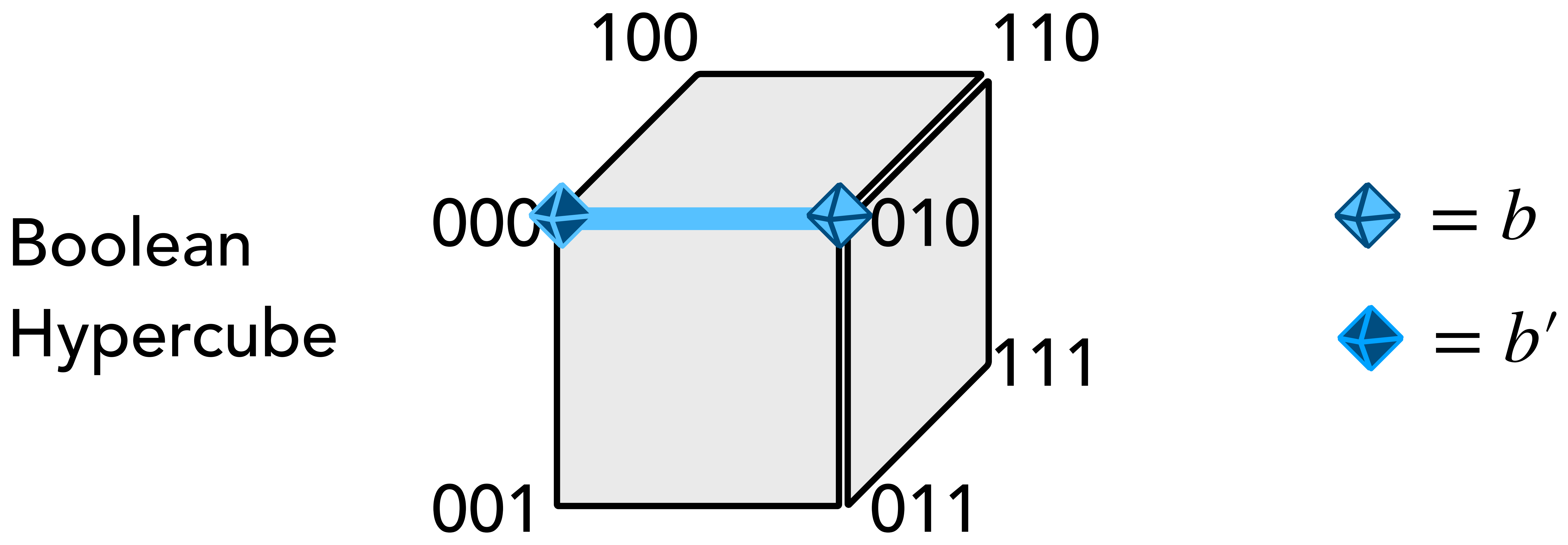
- Let $\pi(b) = |\langle b|\psi\rangle|^2$ be the **measurement distribution**.
- Consider a random walk on $n$-bit Boolean hypercube.

Boolean Hypercube



# Relaxation Time

- Let $\pi(b) = |\langle b|\psi\rangle|^2$ be the **measurement distribution**.
- Consider a random walk on $n$-bit Boolean hypercube.

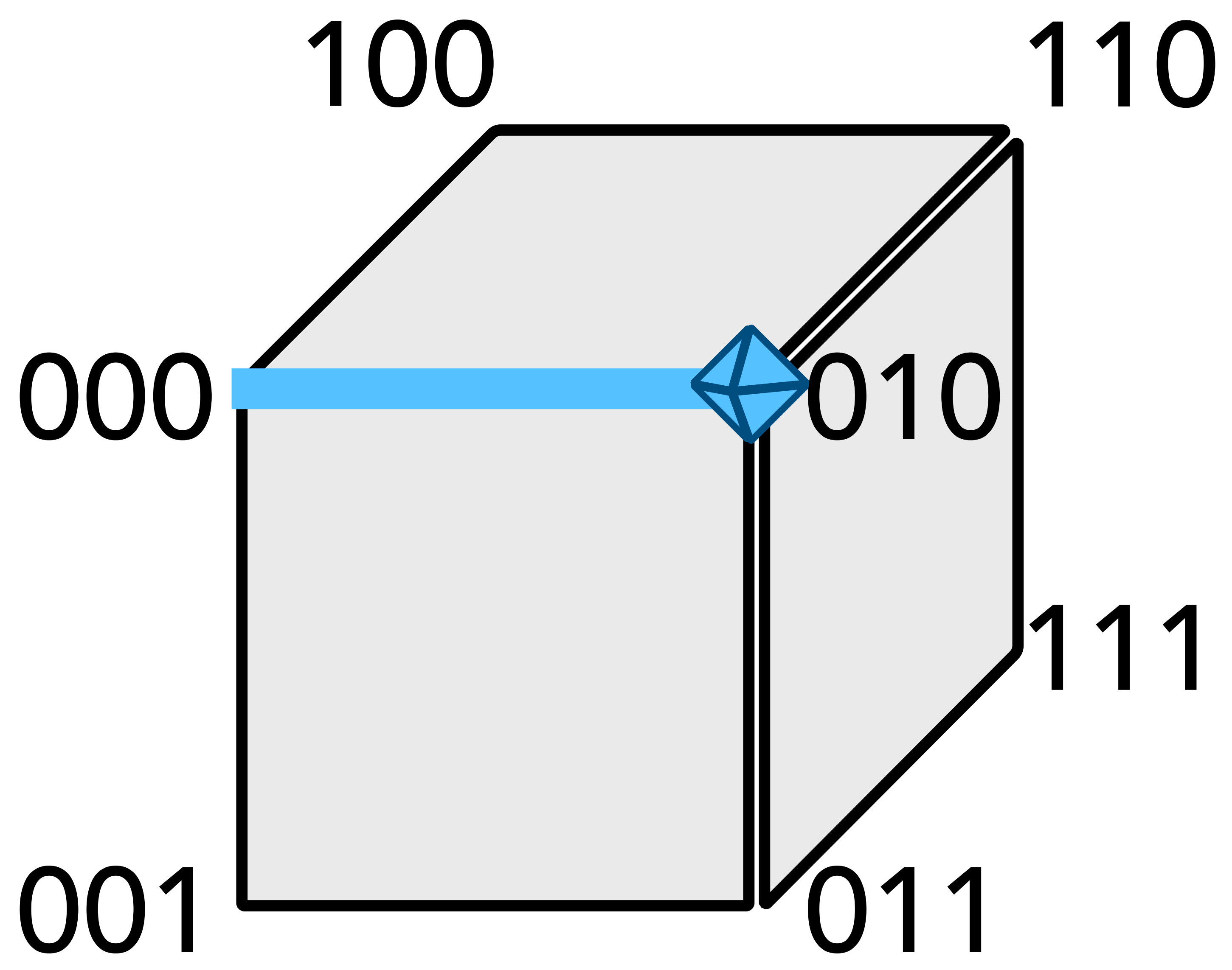


# Relaxation Time

- Let $\pi(b) = |\langle b|\psi\rangle|^2$ be the **measurement distribution**.
- Consider a random walk on $n$-bit Boolean hypercube.

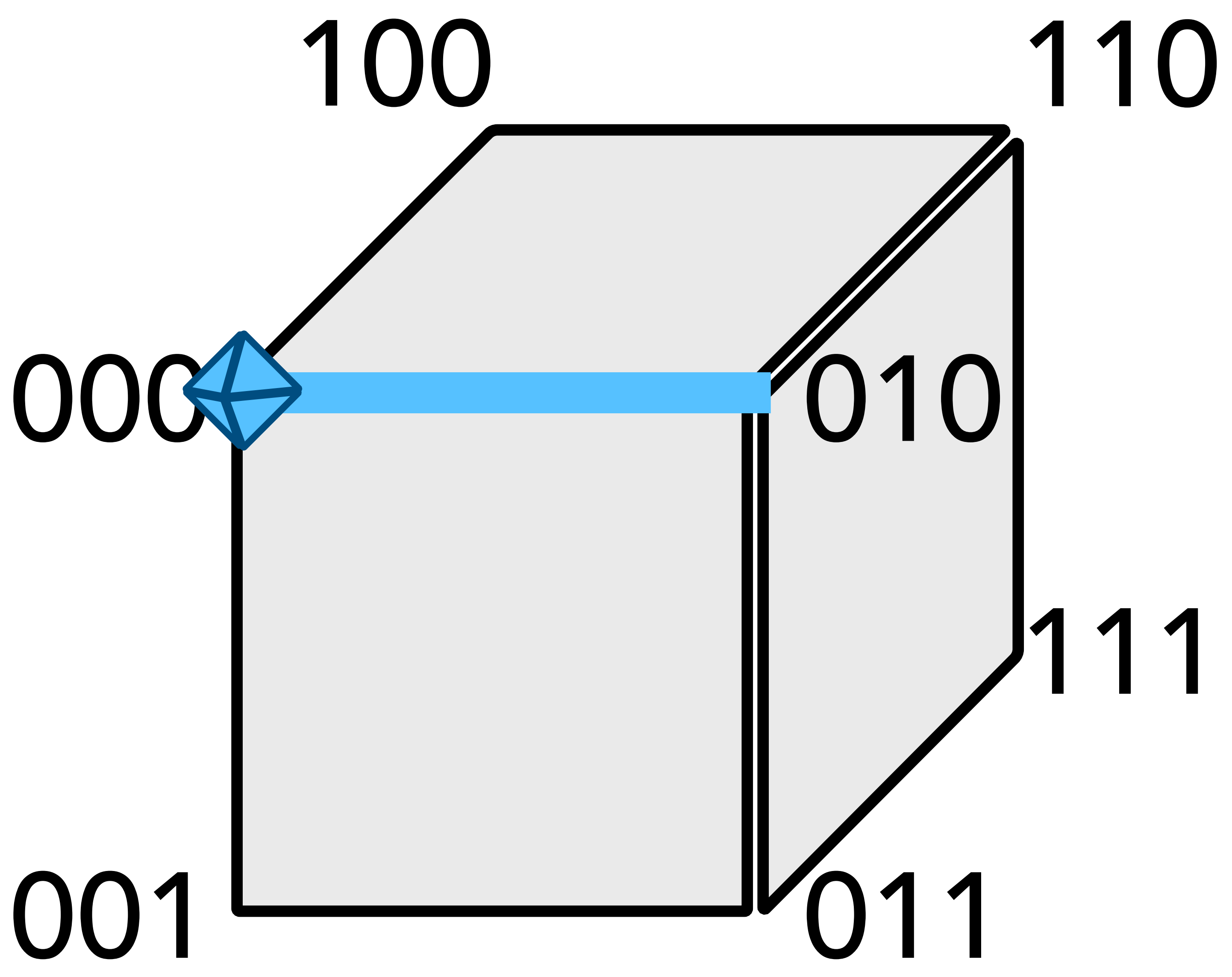
With prob.

$$\frac{\pi(b)}{\pi(b)+\pi(b')}$$




With prob.

$$\frac{\pi(b')}{\pi(b)+\pi(b')}$$

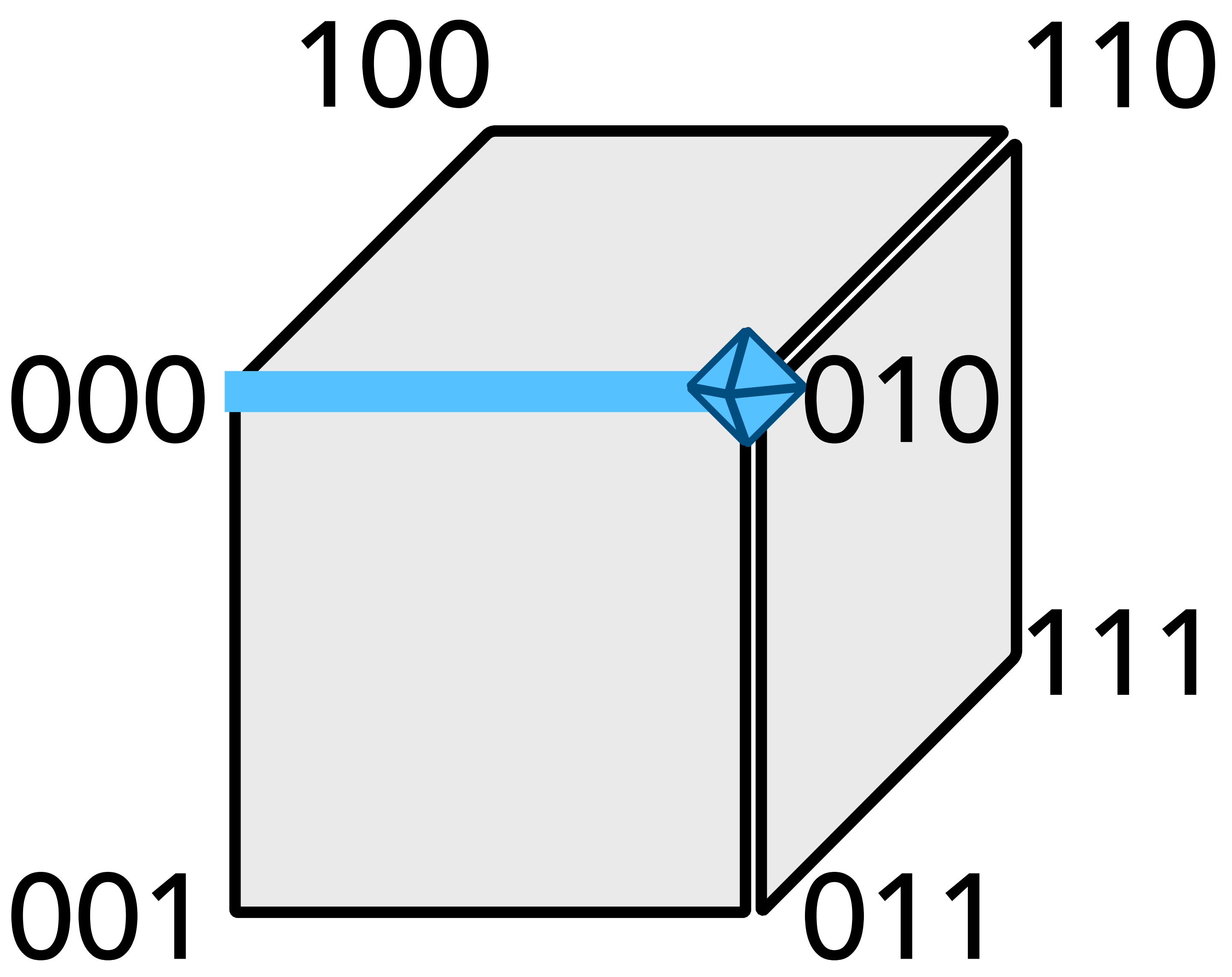


# Relaxation Time

- Let $\pi(b) = |\langle b|\psi\rangle|^2$ be the **measurement distribution**.
- Let $\tau$ be the time the random talk takes to relax to stationary $\pi$.

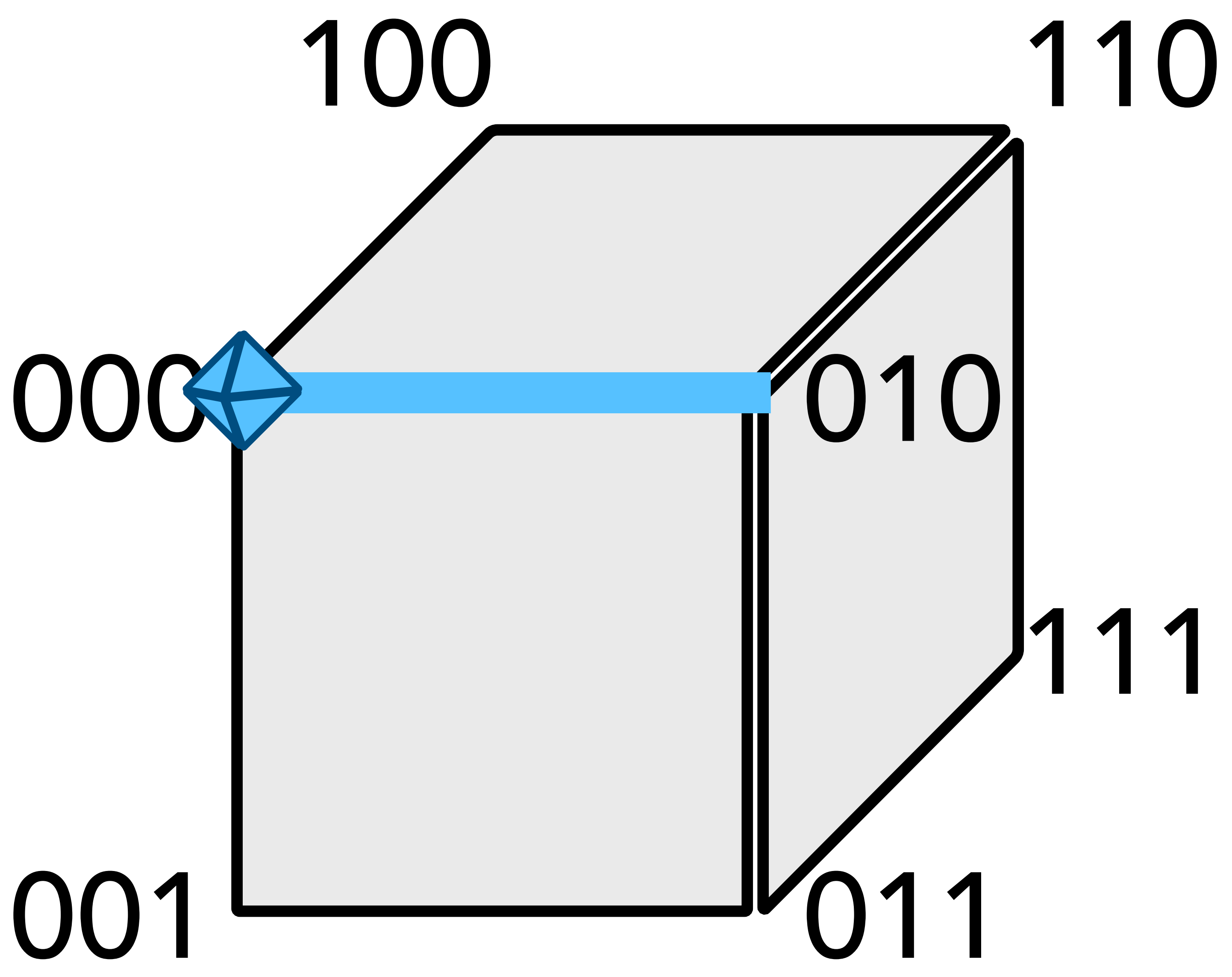
With prob.

$$\frac{\pi(b)}{\pi(b)+\pi(b')}$$




With prob.

$$\frac{\pi(b')}{\pi(b)+\pi(b')}$$



# Certification

**Theorem 2**

For an $n$-qubit state $|\psi\rangle$ with relax. time $\tau$, we can certify that $\rho$ is close to $|\psi\rangle\langle\psi|$ with $\mathcal{O}(\tau)$ single-qubit measurements.

- When restricted to independent Pauli-basis measurements, we need $\mathcal{O}(\tau^2)$ single-qubit measurements.

# Outline

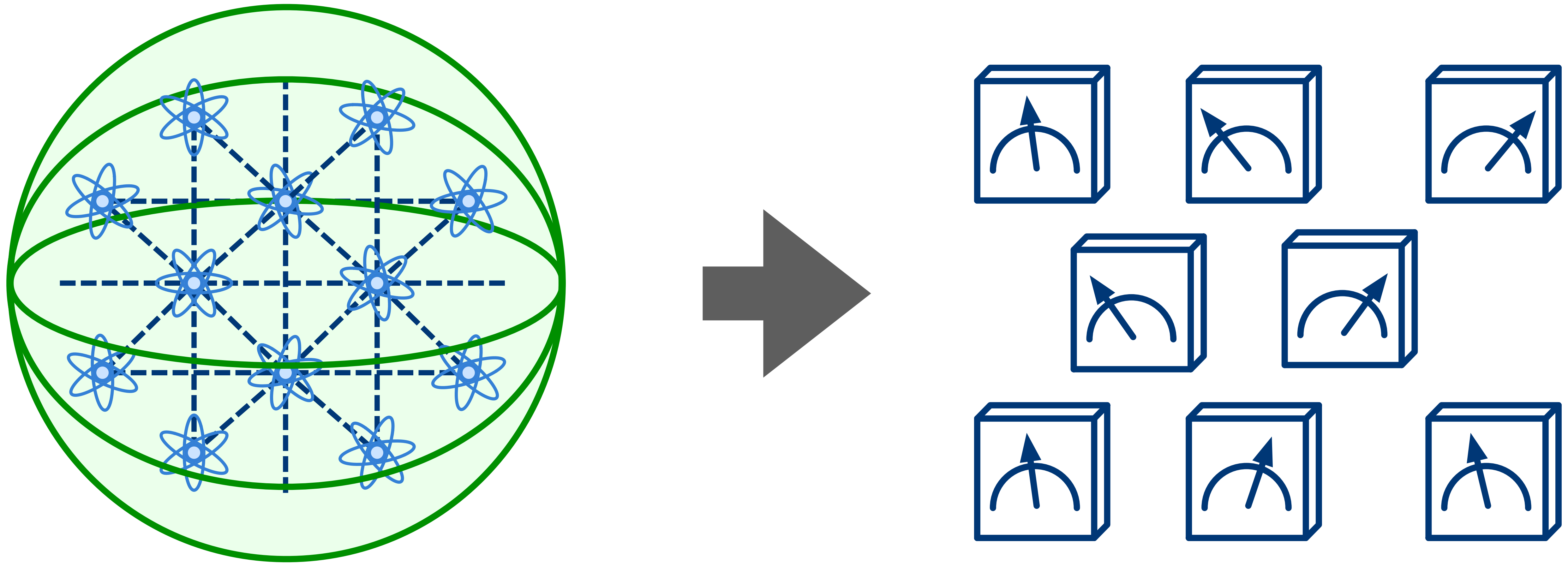
- Theorem
- Protocol
- Applications

# Outline

- Theorem
- Protocol
- Applications

# Measurement Protocol

- Repeat the following measurement a few times.

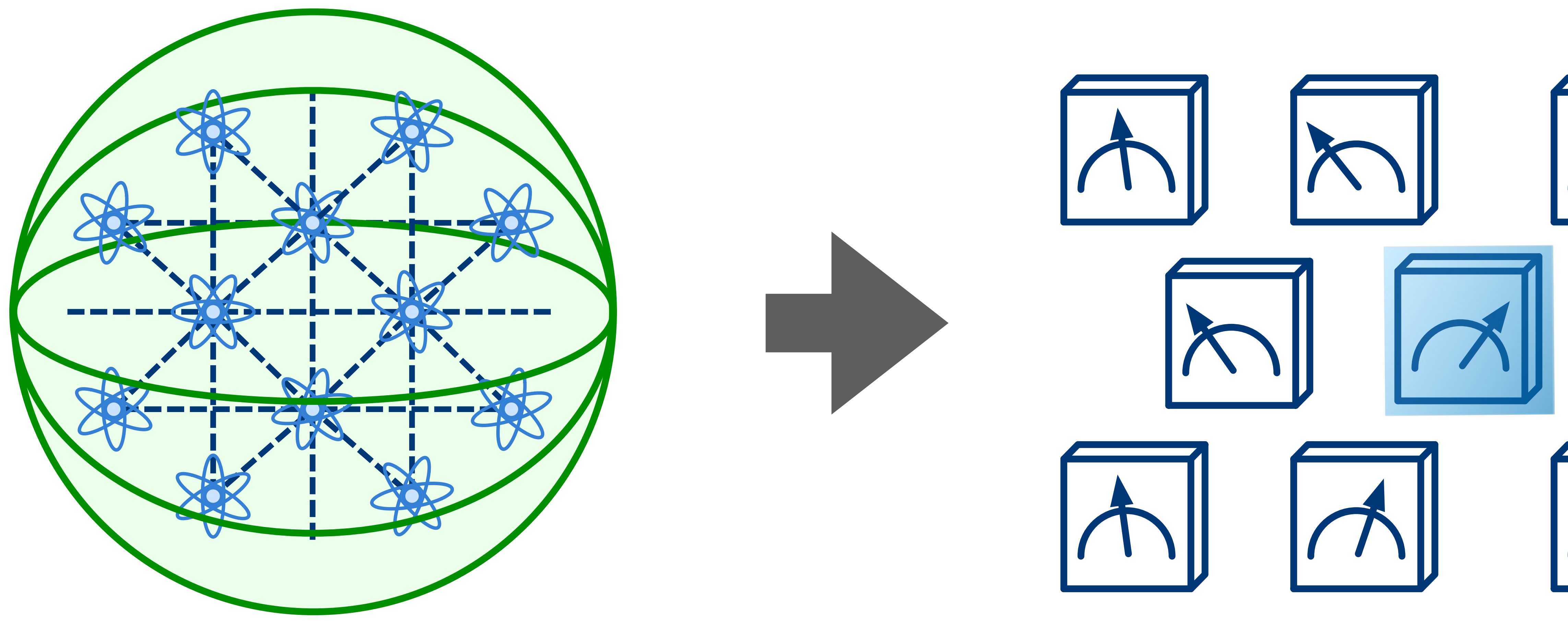


Quantum state

Single-qubit Measurement

# Measurement Protocol

- Pick a random qubit $x$.

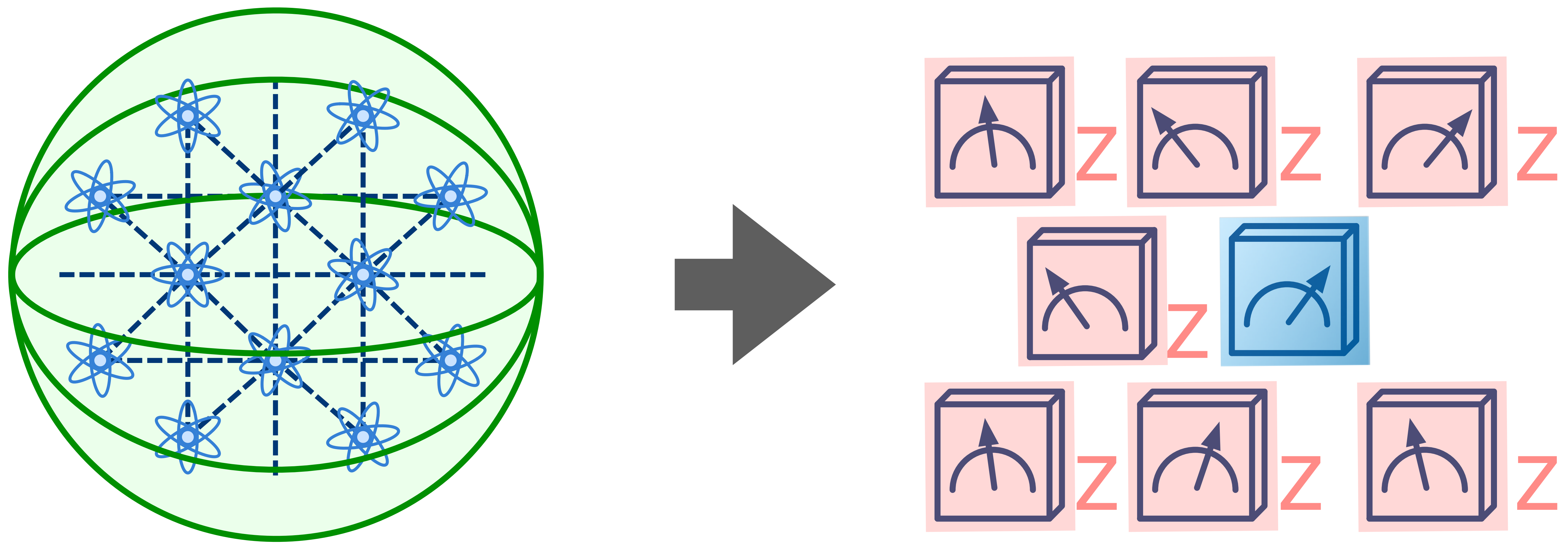


Quantum state

Single-qubit Measurement

# Measurement Protocol

- Pick a random qubit $x$. Measure all except qubit $x$ in Z basis.

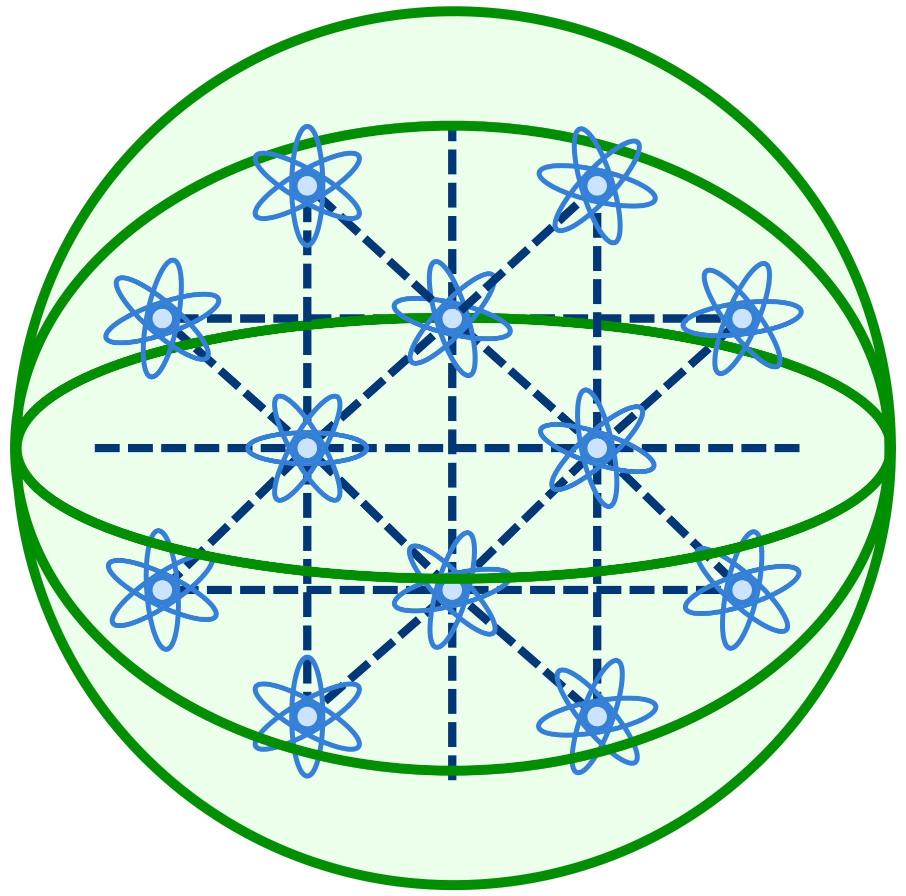



Quantum state

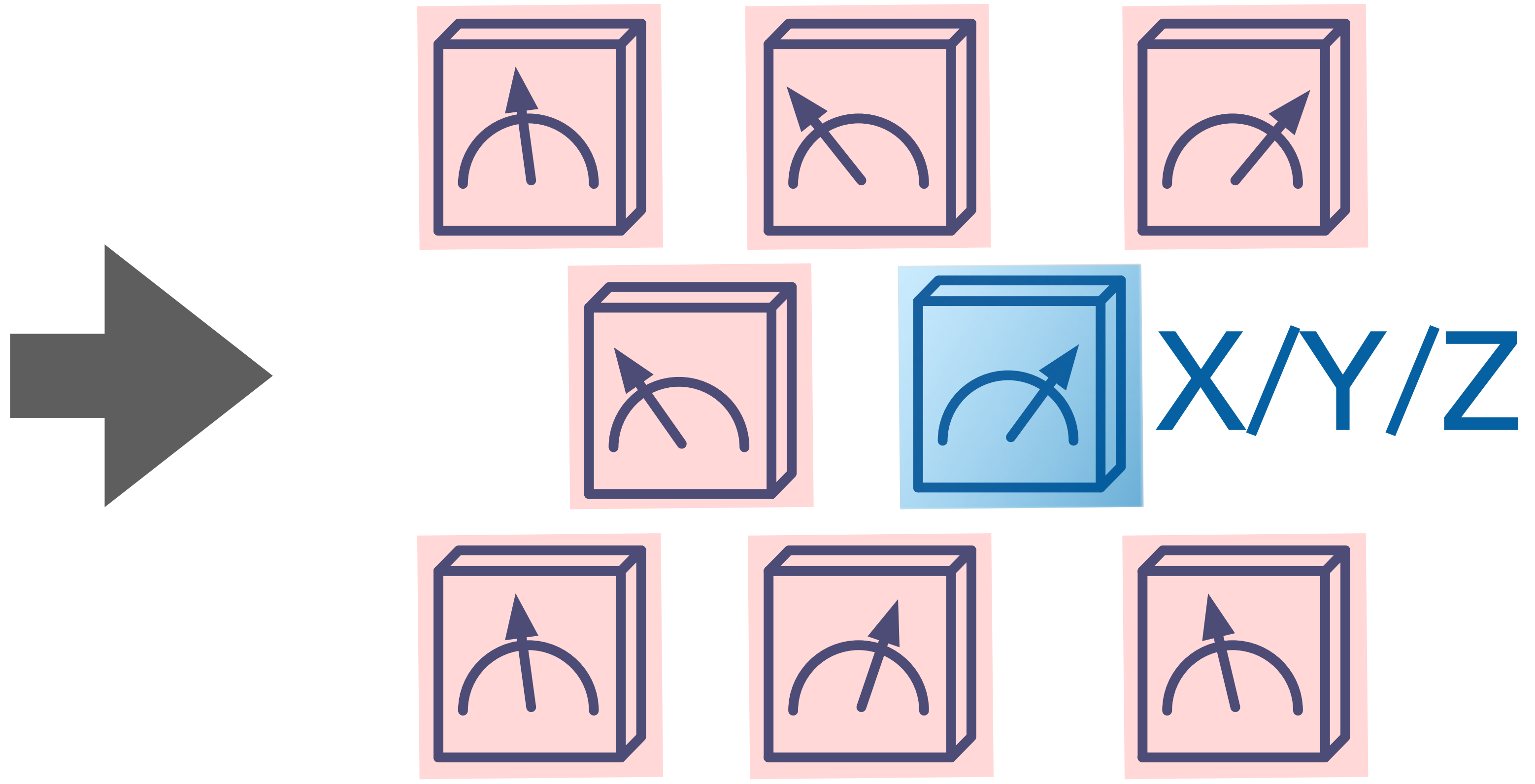
Single-qubit Measurement

# Measurement Protocol

- Pick a random qubit $x$. Measure $x$ in random X/Y/Z basis.



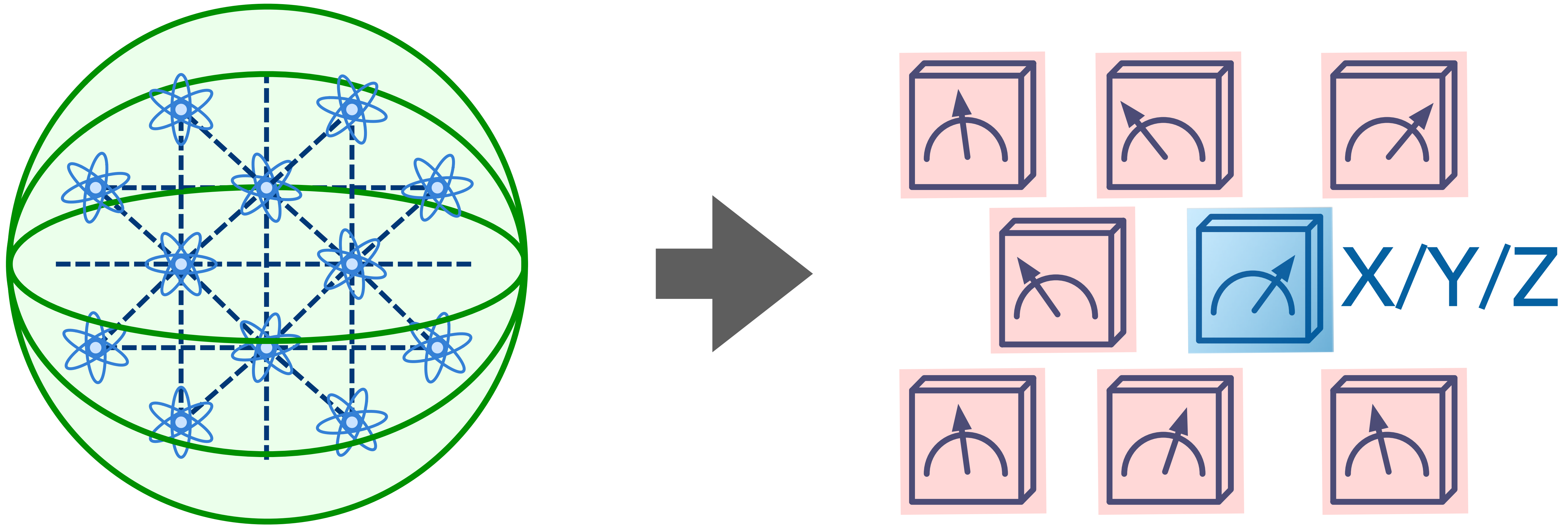
Quantum state




Single-qubit
Measurement

# Measurement Protocol

- That’s it.

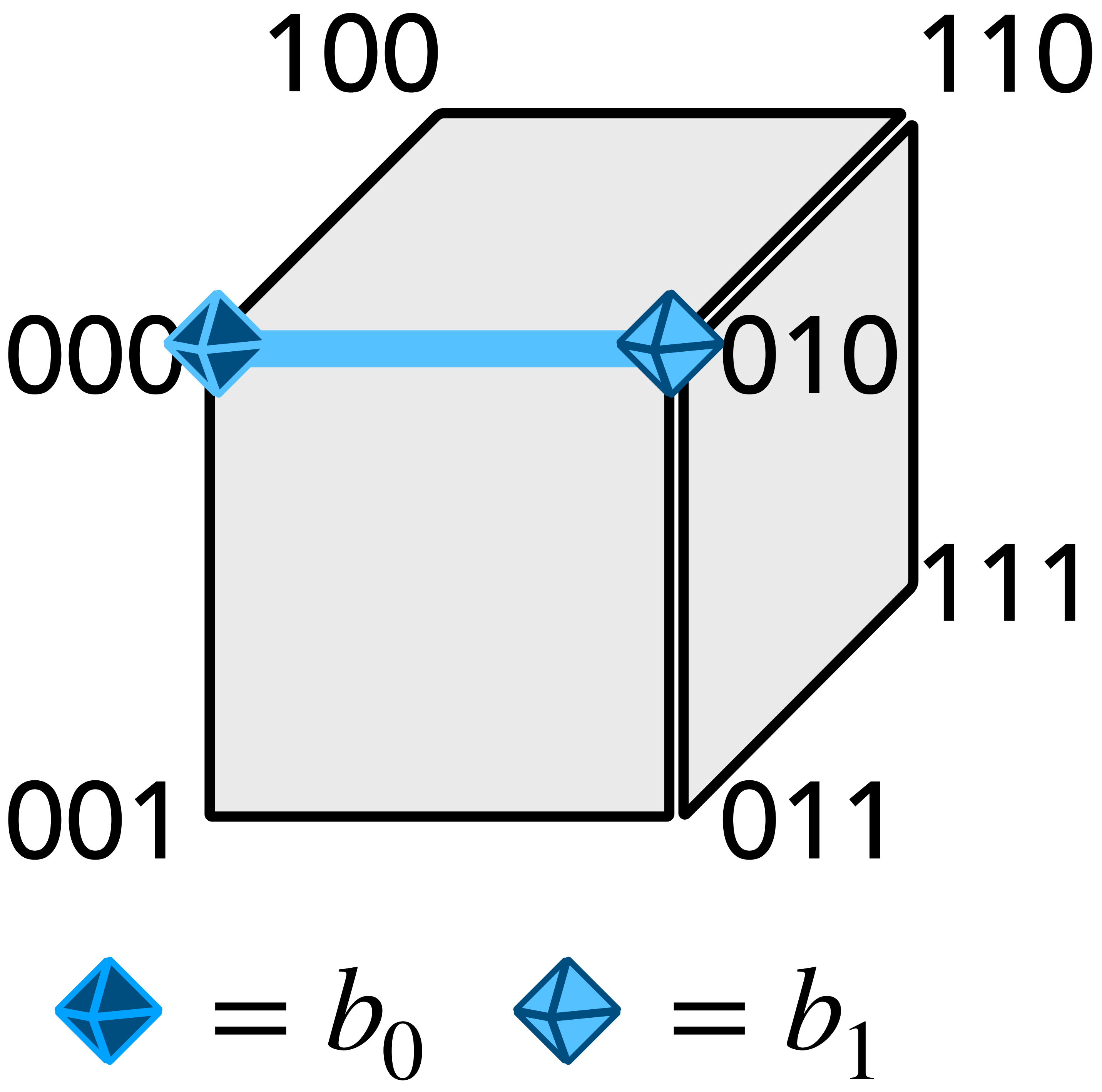


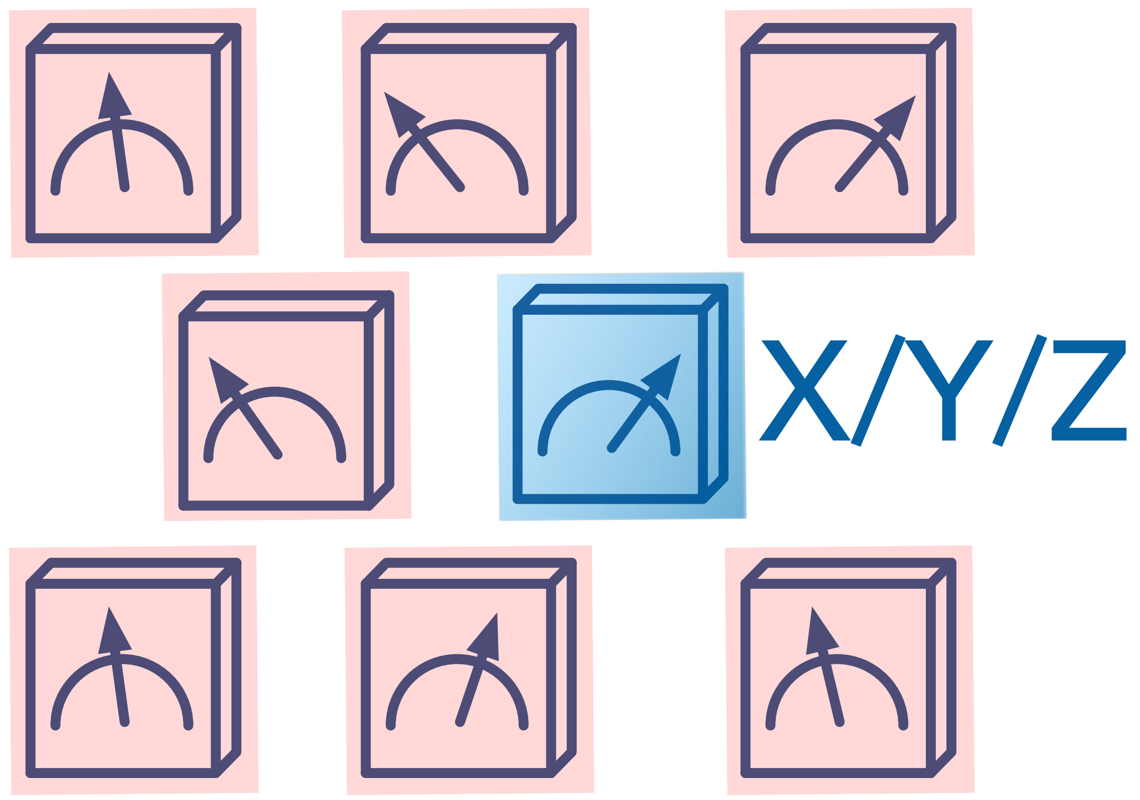

Quantum state

Single-qubit Measurement

# Postprocessing

- The measurement outcomes on  specifies an edge $(b_0, b_1)$ on the Boolean hypercube.

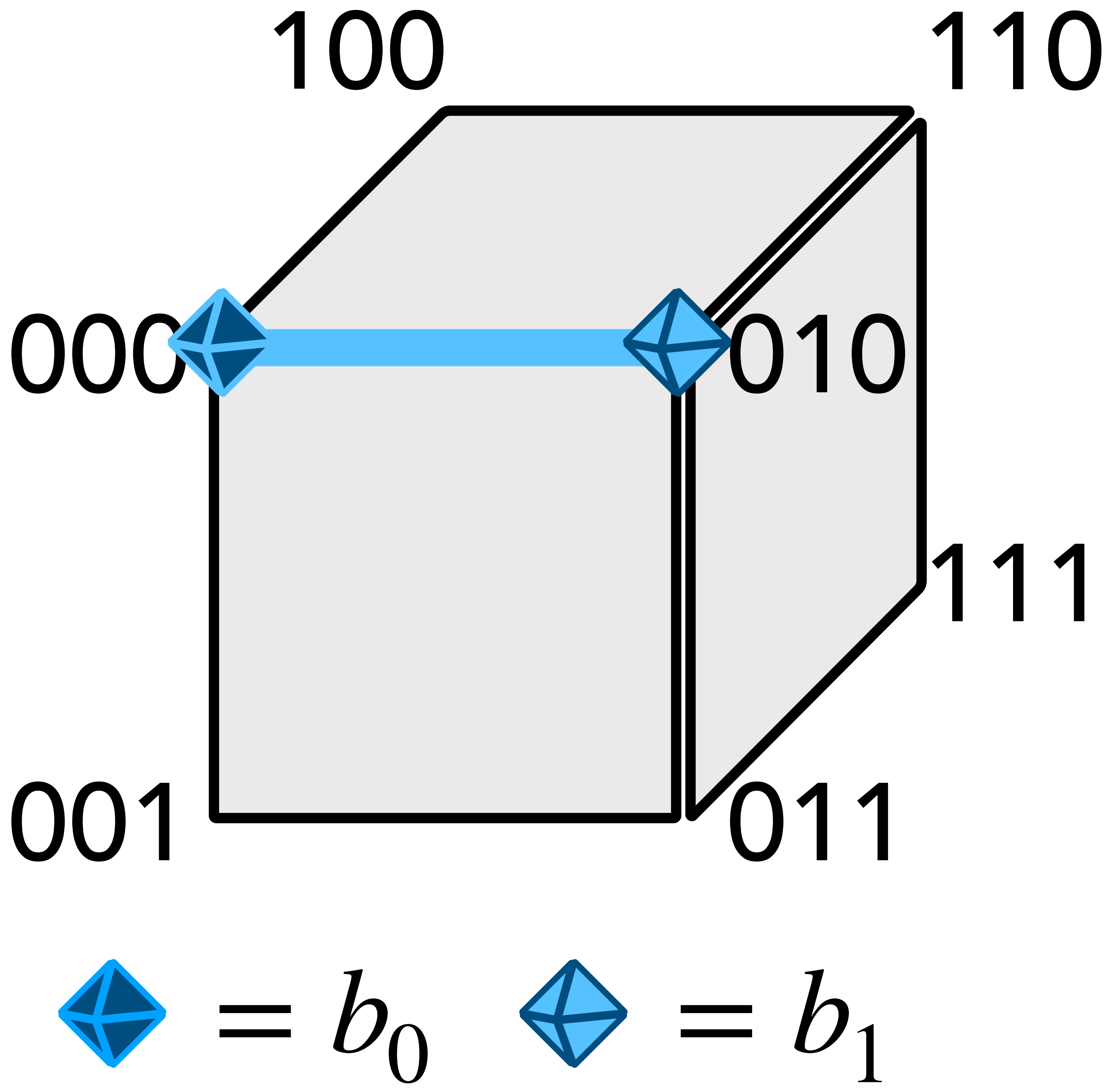


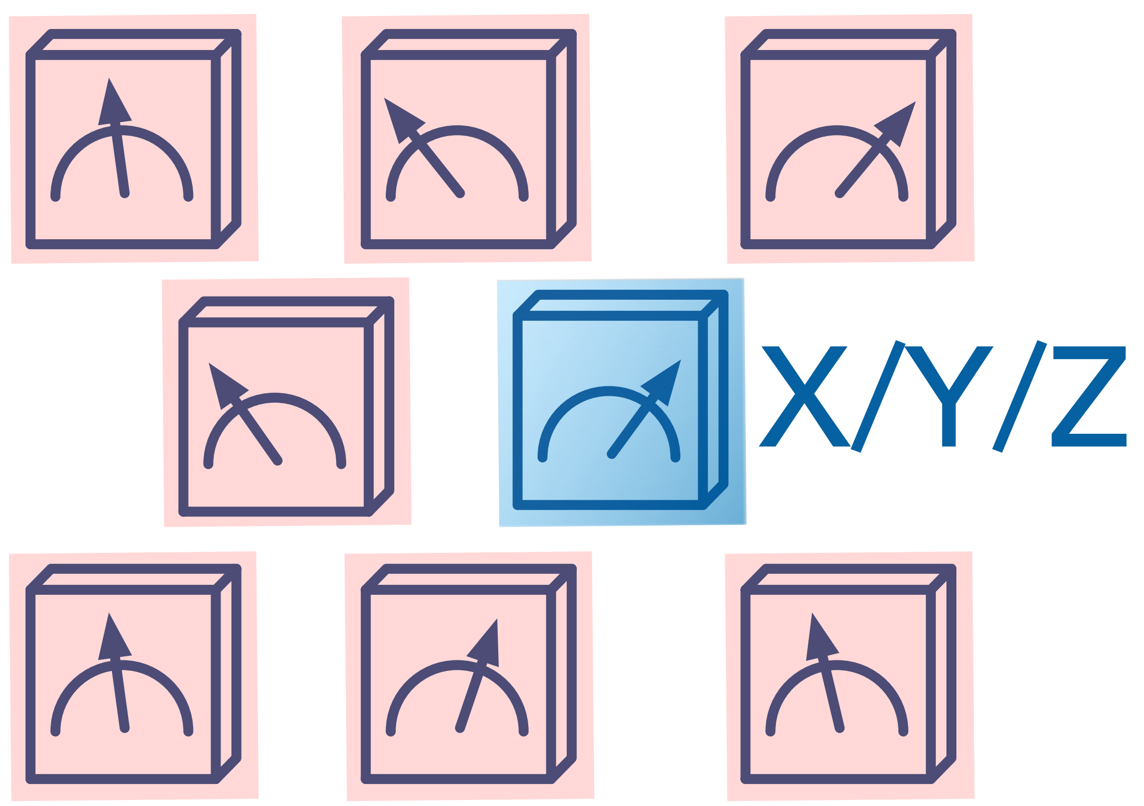




Single-qubit Measurement

# Postprocessing

- The **ideal** post-measurement 1-qubit state $|\psi_{b_0,b_1}\rangle$ on **qubit** $x$ is proportional to $\langle b_0|\psi\rangle|0\rangle + \langle b_1|\psi\rangle|1\rangle$.

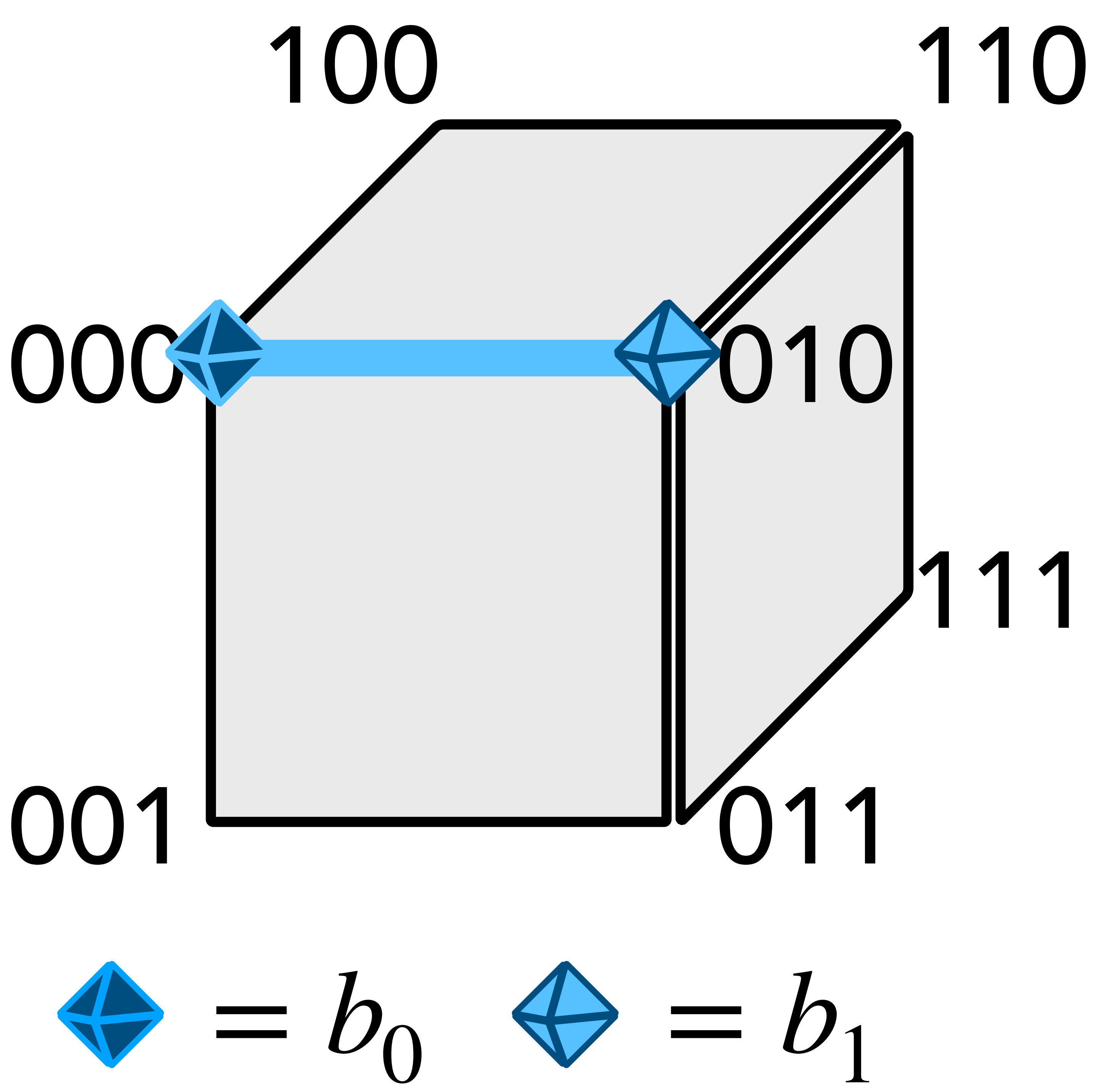


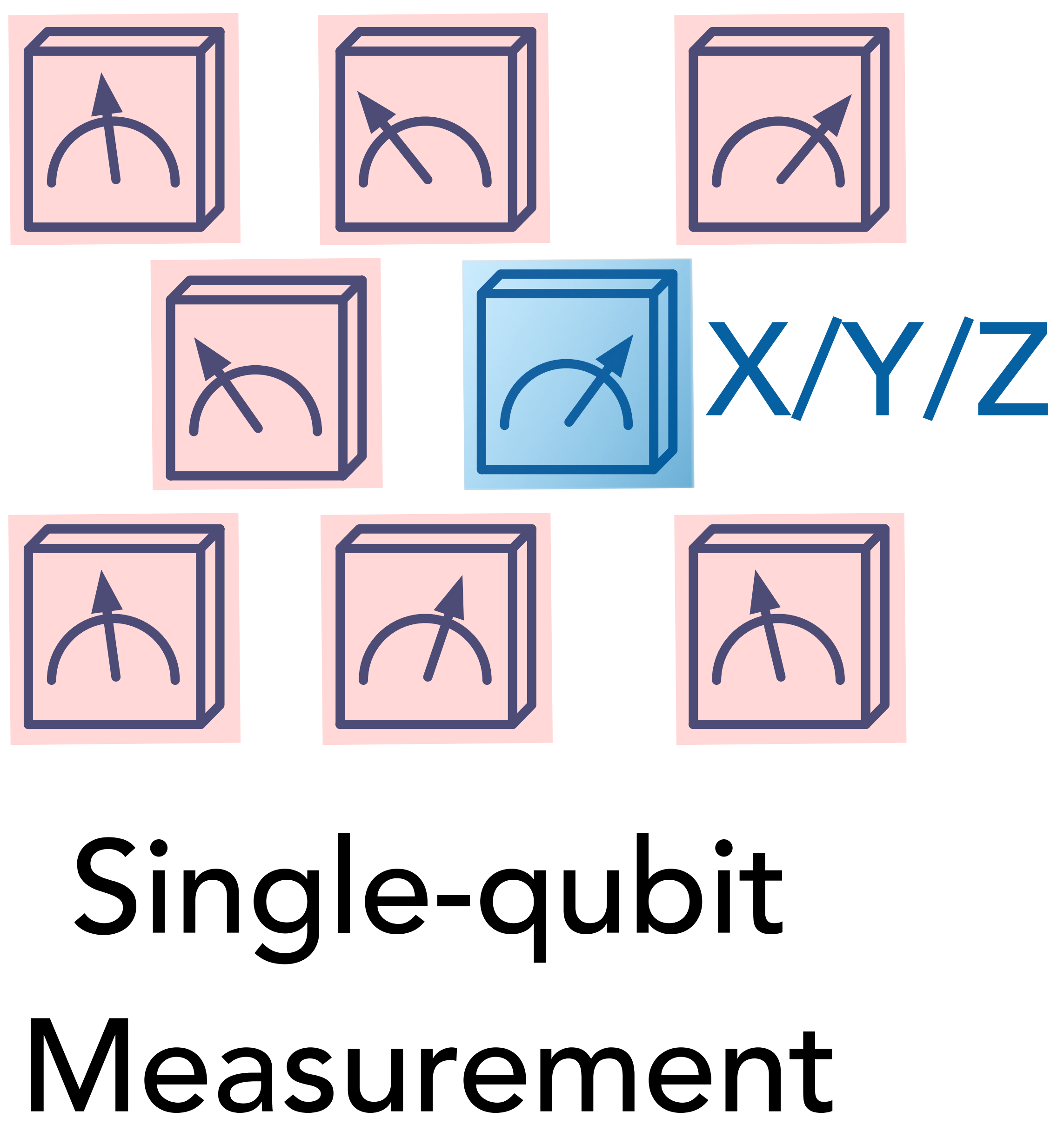




Single-qubit
Measurement

# Postprocessing

- Use randomized Pauli measurement (classical shadow) on **qubit** $x$ to predict the fidelity $\omega$ with the **ideal** 1-qubit state $|\psi_{b_0,b_1}\rangle$.

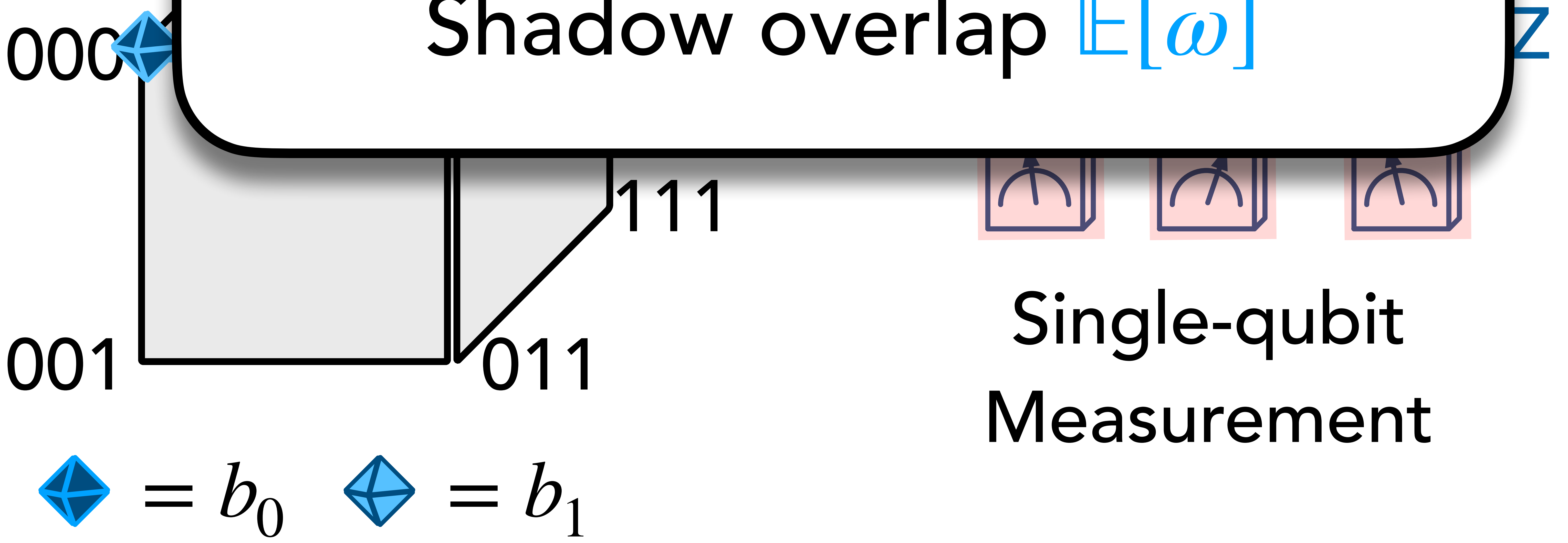


# Postprocessing

- Use randomized Pauli measurement (classical shadow) on **qubit $x$** to predict the fidelity $\omega$ with the **ideal** 1-qubit state $|\psi_{b_0,b_1}\rangle$.

Average over $\omega$ to get Shadow overlap $\mathbb{E}[\omega]$



# Key Feature

Shadow overlap $\mathbb{E}[\omega]$ accurately tracks the fidelity $\langle\psi|\rho|\psi\rangle$.

$\tau$ is the time the random talk takes to relax to stationary $\pi$

# Key Feature

Shadow overlap $\mathbb{E}[\omega]$ accurately tracks the fidelity $\langle\psi|\rho|\psi\rangle$.

$\mathbb{E}[\omega] \geq 1 - \epsilon$ **implies** $\langle\psi|\rho|\psi\rangle \geq 1 - \tau\epsilon$

$\tau$ is the time the random talk takes to relax to stationary $\pi$

# Key Feature

Shadow overlap $\mathbb{E}[\omega]$ accurately tracks the fidelity $\langle\psi|\rho|\psi\rangle$.

$\mathbb{E}[\omega] \geq 1 - \epsilon$ **implies** $\langle\psi|\rho|\psi\rangle \geq 1 - \tau\epsilon$

$\langle\psi|\rho|\psi\rangle \geq 1 - \epsilon$ **implies** $\mathbb{E}[\omega] \geq 1 - \epsilon$

$\tau$ is the time the random talk takes to relax to stationary $\pi$

# Physical Intuition

Shadow overlap $\mathbb{E}[\omega] = \frac{1}{n}\sum_{i=1}^{n}\sum_{b_{\neq i}\in\{0,1\}^{n-1}} \mathrm{Tr}\left(\langle b_{\neq i}|\rho|b_{\neq i}\rangle \frac{\langle b_{\neq i}|\psi\rangle\langle\psi|b_{\neq i}\rangle}{\mathrm{Tr}\langle b_{\neq i}|\psi\rangle\langle\psi|b_{\neq i}\rangle}\right)$

# Physical Intuition

Shadow overlap $\mathbb{E}[\omega] = \frac{1}{n}\sum_{i=1}^{n}\sum_{b_{\neq i}\in\{0,1\}^{n-1}} \mathrm{Tr}\left(\langle b_{\neq i}|\rho|b_{\neq i}\rangle \frac{\langle b_{\neq i}|\psi\rangle\langle\psi|b_{\neq i}\rangle}{\mathrm{Tr}\langle b_{\neq i}|\psi\rangle\langle\psi|b_{\neq i}\rangle}\right)$

- $|+\dots+\rangle\langle+\dots+|$ and $|-\dots-\rangle\langle-\dots-|$ has fidelity 0.
- $|+\dots+\rangle\langle+\dots+|$ and $|-\dots-\rangle\langle-\dots-|$ has $\mathbb{E}[\omega]$= 0.

# Physical Intuition

Shadow overlap $\mathbb{E}[\omega] = \frac{1}{n}\sum_{i=1}^{n}\sum_{b_{\neq i}\in\{0,1\}^{n-1}} \mathrm{Tr}\left(\langle b_{\neq i}|\rho|b_{\neq i}\rangle \frac{\langle b_{\neq i}|\psi\rangle\langle\psi|b_{\neq i}\rangle}{\mathrm{Tr}\langle b_{\neq i}|\psi\rangle\langle\psi|b_{\neq i}\rangle}\right)$

- $|+\ldots+\rangle\langle+\ldots+|$ and $|+\ldots+-\rangle\langle+\ldots+-|$ has fidelity 0.
- $|+\ldots+\rangle\langle+\ldots+|$ and $|+\ldots+-\rangle\langle+\ldots+-|$ has $\mathbb{E}[\omega] = \frac{n-1}{n}$.

# Physical Intuition

Shadow overlap $\mathbb{E}[\omega] = \frac{1}{n}\sum_{i=1}^{n}\sum_{b_{\neq i}\in\{0,1\}^{n-1}} \mathrm{Tr}\left(\langle b_{\neq i}|\rho|b_{\neq i}\rangle \frac{\langle b_{\neq i}|\psi\rangle\langle\psi|b_{\neq i}\rangle}{\mathrm{Tr}\langle b_{\neq i}|\psi\rangle\langle\psi|b_{\neq i}\rangle}\right)$

- $|+\ldots+\rangle\langle+\ldots+|$ and $|+\ldots+-\rangle\langle+\ldots+-|$ has fidelity 0.
- $|+\ldots+\rangle\langle+\ldots+|$ and $|+\ldots+-\rangle\langle+\ldots+-|$ has $\mathbb{E}[\omega] = \frac{n-1}{n}$.
- Shadow overlap has a Hamming distance nature.

# Outline

- Theorem
- Protocol
- Applications

# Outline

- Theorem
- Protocol
- Applications

# Applications

What can we use this new certification protocol for?

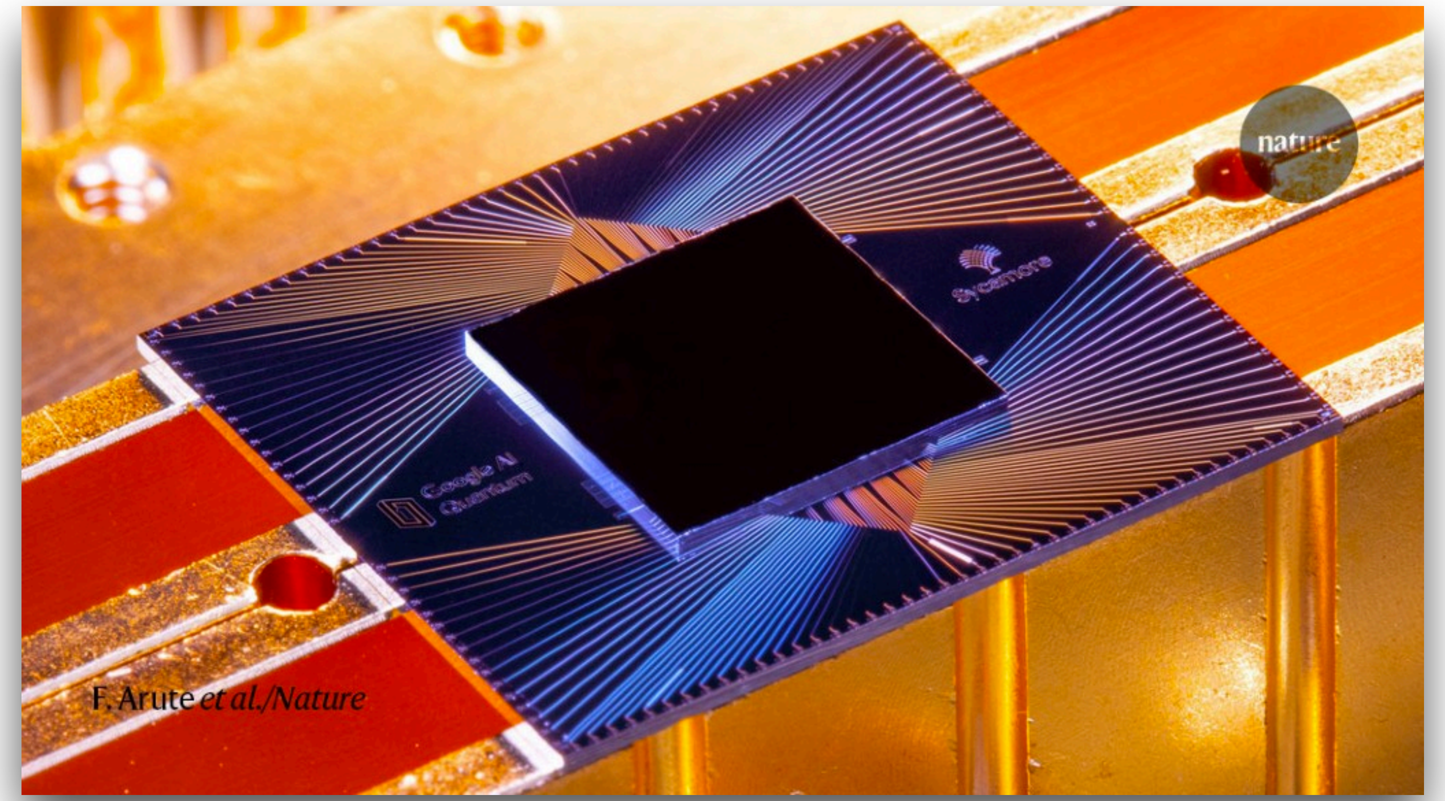
# Applications

What can we use this new certification protocol for?

**Example 1**

*Benchmarking*

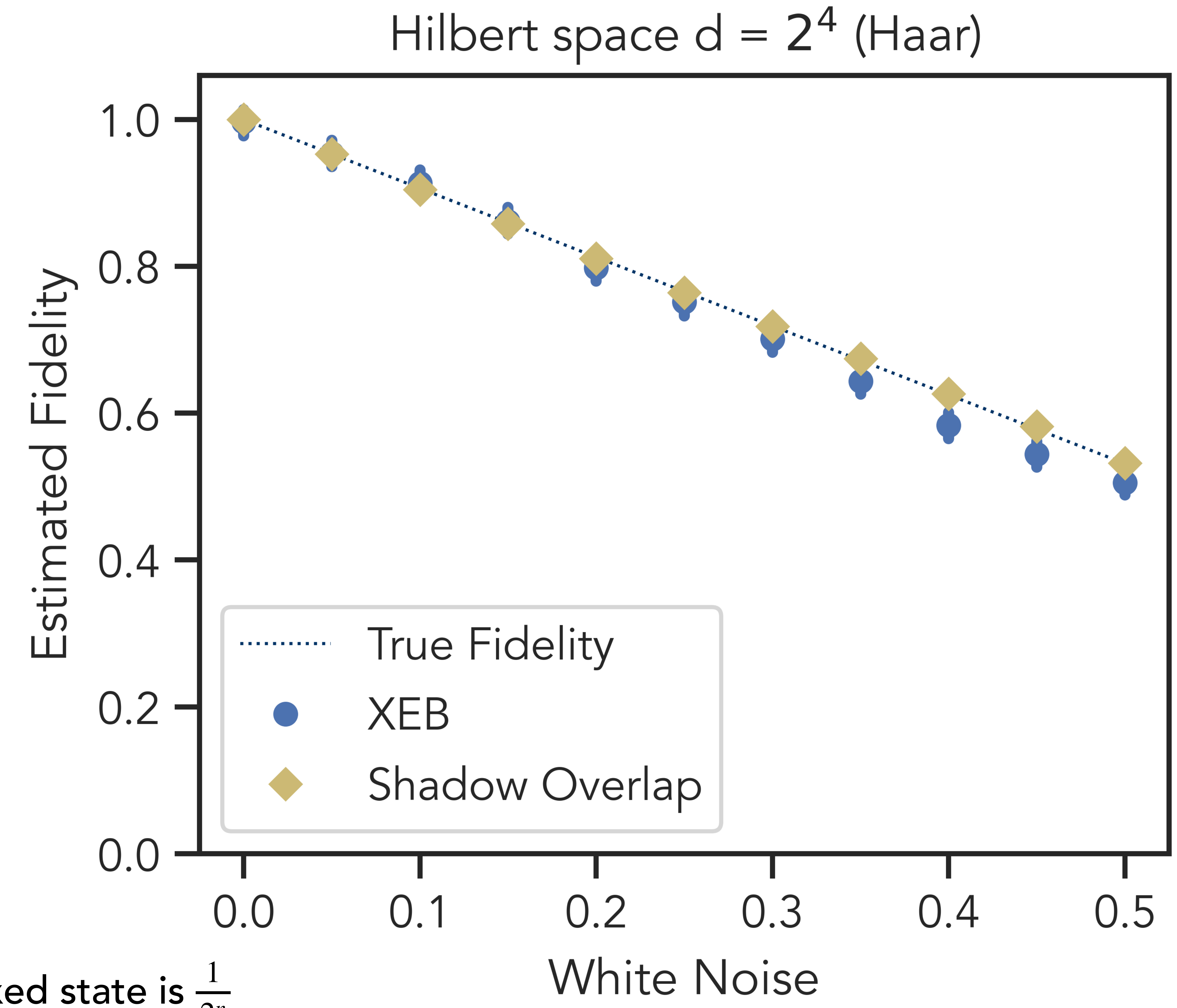
Shadow overlap $\mathbb{E}[\omega]$ certifies if the state has a high fidelity



# Benchmarking quantum devices

4-qubit Haar random state

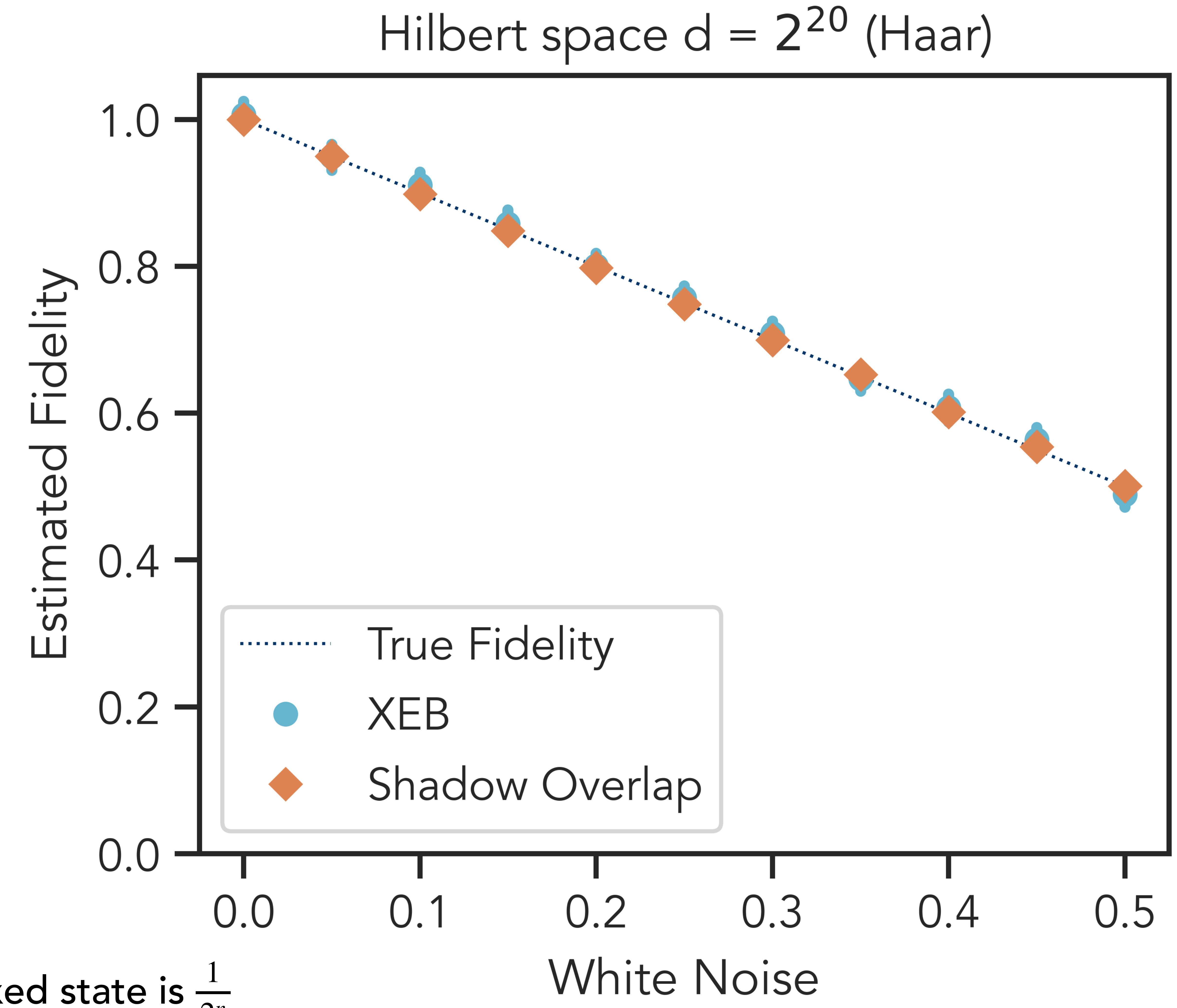
*White Noise*




*Shadow overlap normalized s.t., target state is 1, maximally mixed state is $\frac{1}{2^n}$

# Benchmarking quantum devices

20-qubit Haar random state
*White Noise*

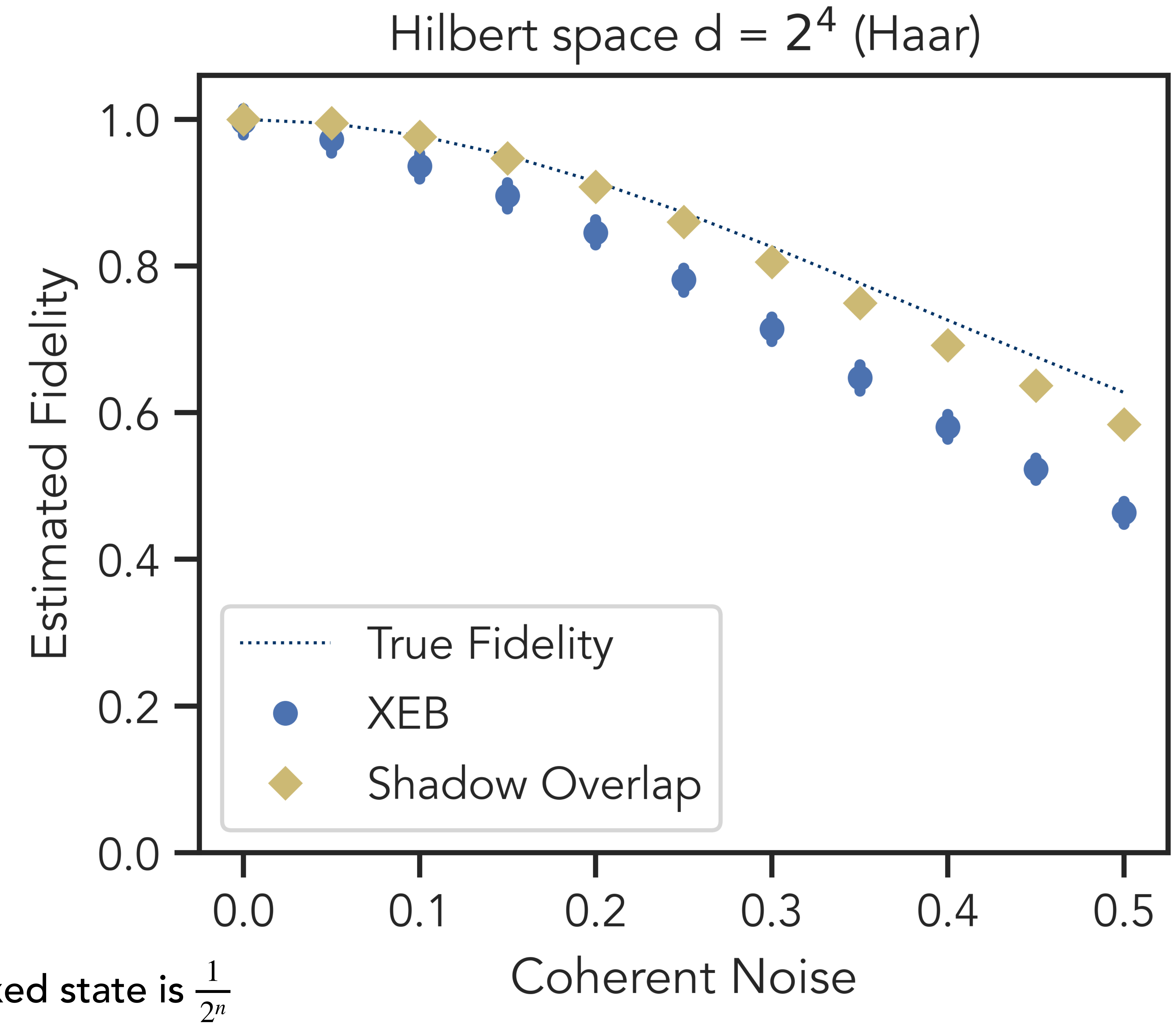



*Shadow overlap normalized s.t., target state is 1, maximally mixed state is $\frac{1}{2^n}$

# Benchmarking quantum devices

4-qubit Haar random state

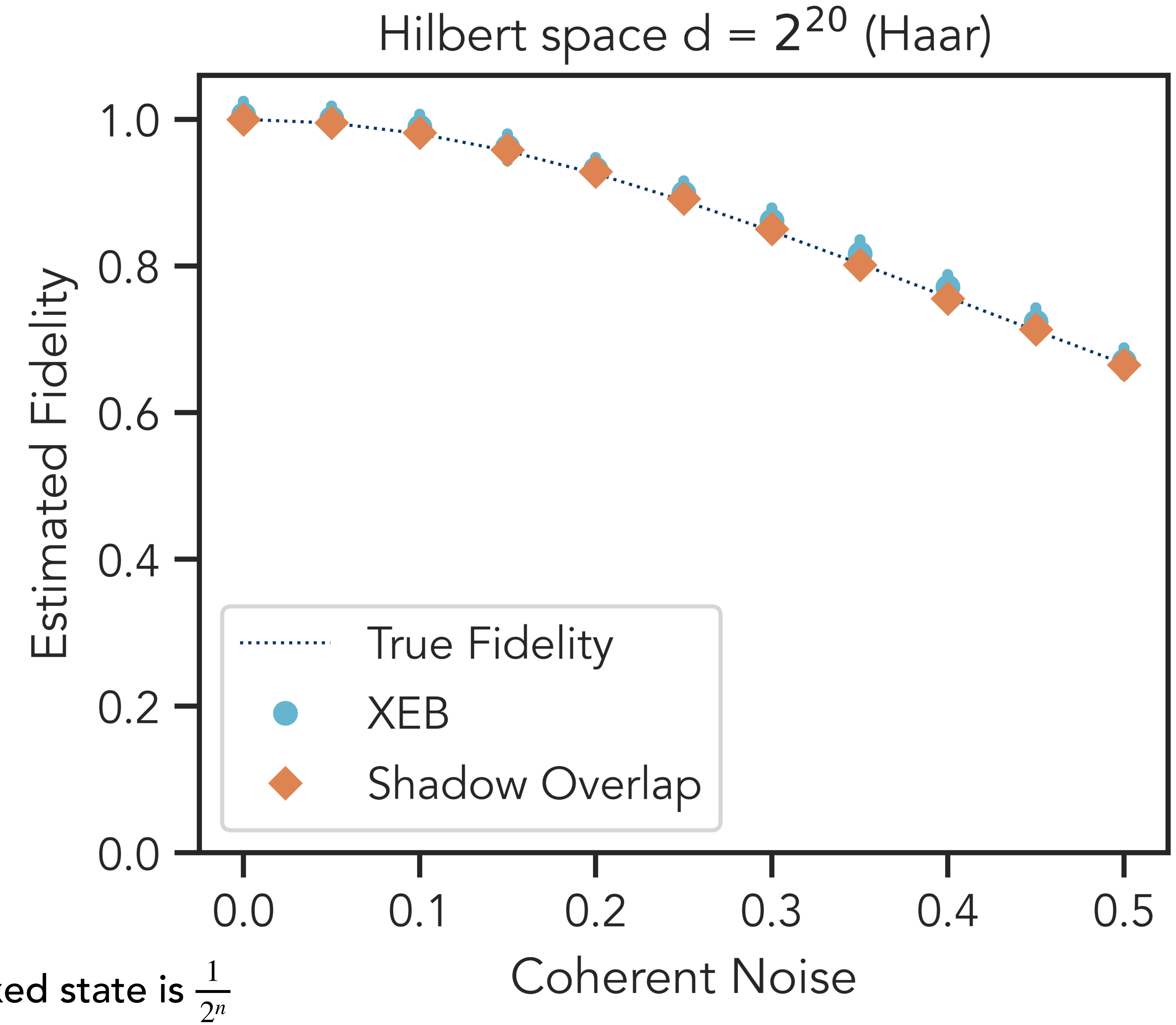
*Coherent Noise*




*Shadow overlap normalized s.t., target state is 1, maximally mixed state is $\frac{1}{2^n}$

# Benchmarking quantum devices

20-qubit Haar random state
*Coherent Noise*



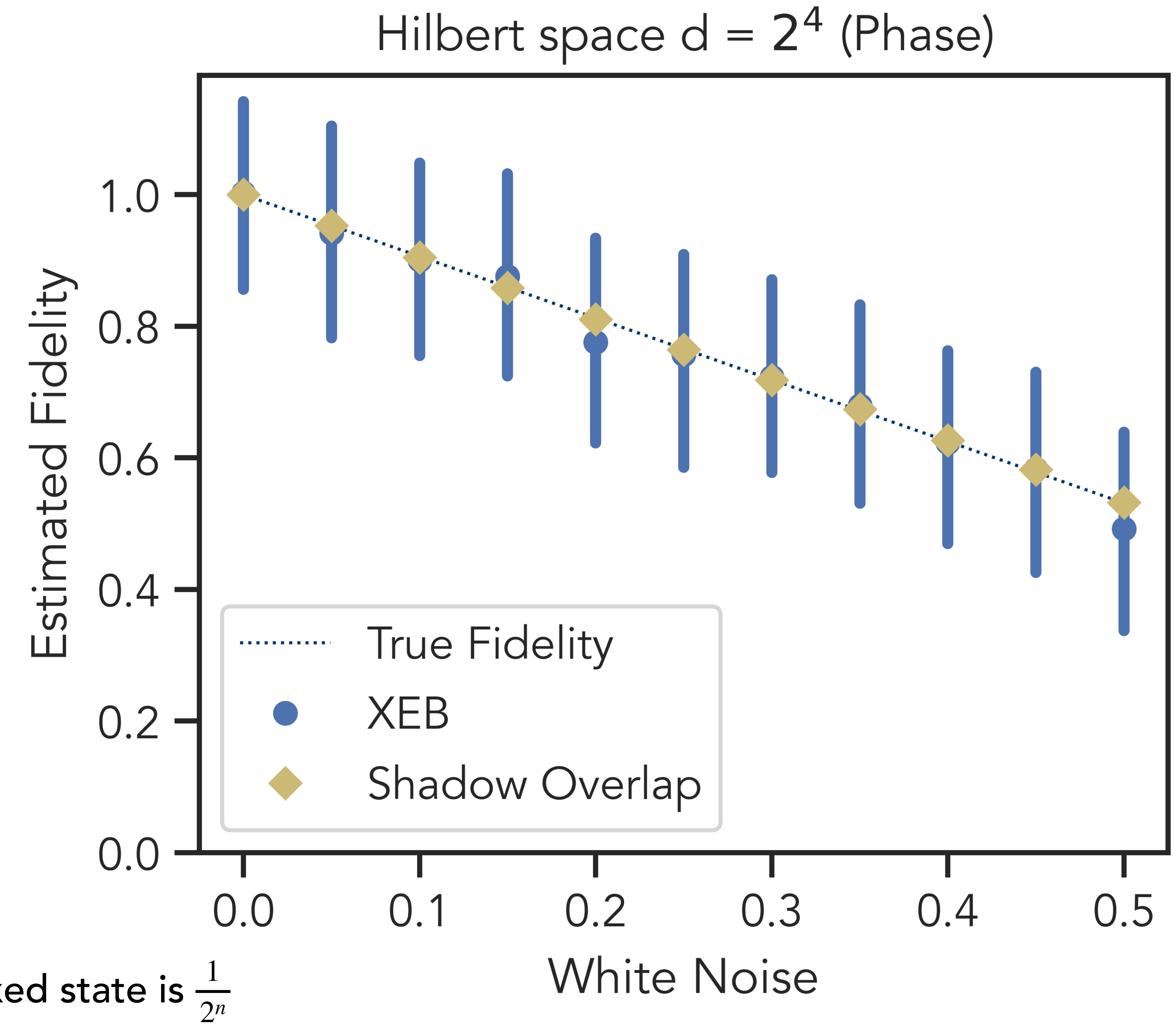

*Shadow overlap normalized s.t., target state is 1, maximally mixed state is $\frac{1}{2^n}$

# Benchmarking quantum devices

4-qubit random structured state

*White Noise*

$$|\psi\rangle = U_{\text{phase}} \bigotimes_{i=1}^{4} |\psi_i\rangle$$



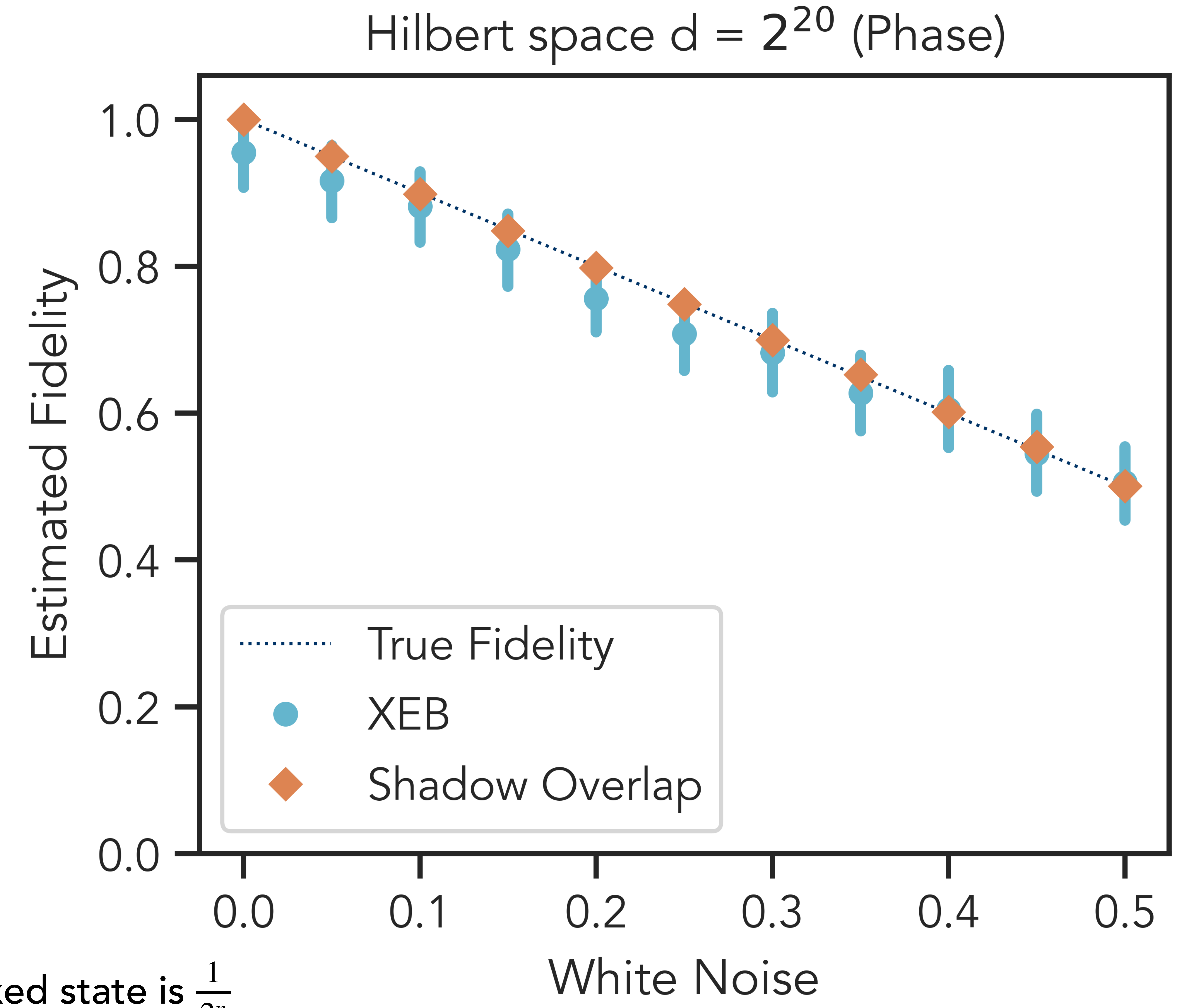

*Shadow overlap normalized s.t., target state is 1, maximally mixed state is $\frac{1}{2^n}$

# Benchmarking quantum devices

20-qubit random structured state

*White Noise*

$$|\psi\rangle = U_{\text{phase}} \bigotimes_{i=1}^{20} |\psi_i\rangle$$



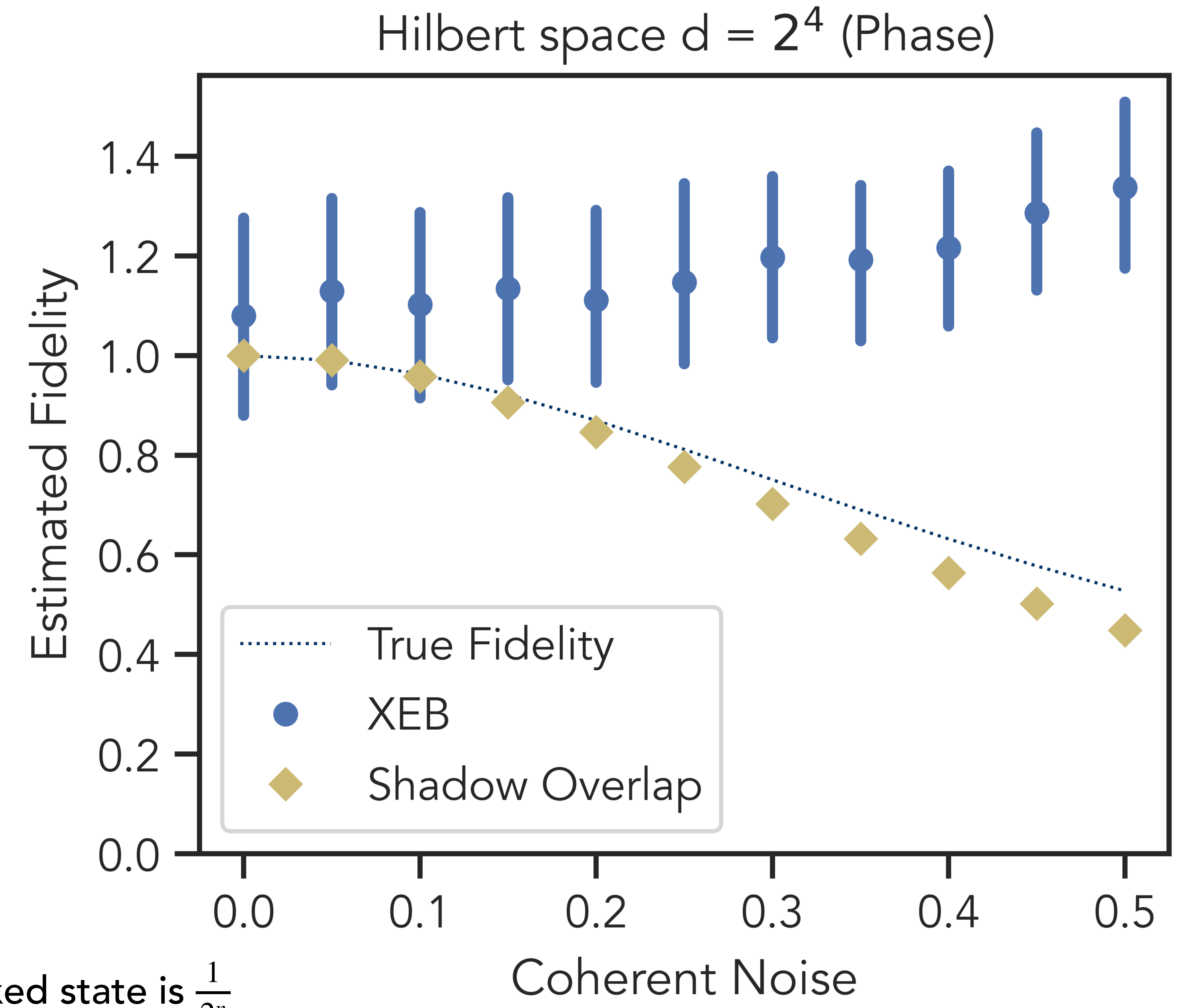

*Shadow overlap normalized s.t., target state is 1, maximally mixed state is $\frac{1}{2^n}$

# Benchmarking quantum devices

4-qubit random structured state

*Coherent Noise*

$$|\psi\rangle = U_{\text{phase}} \bigotimes_{i=1}^{4} |\psi_i\rangle$$



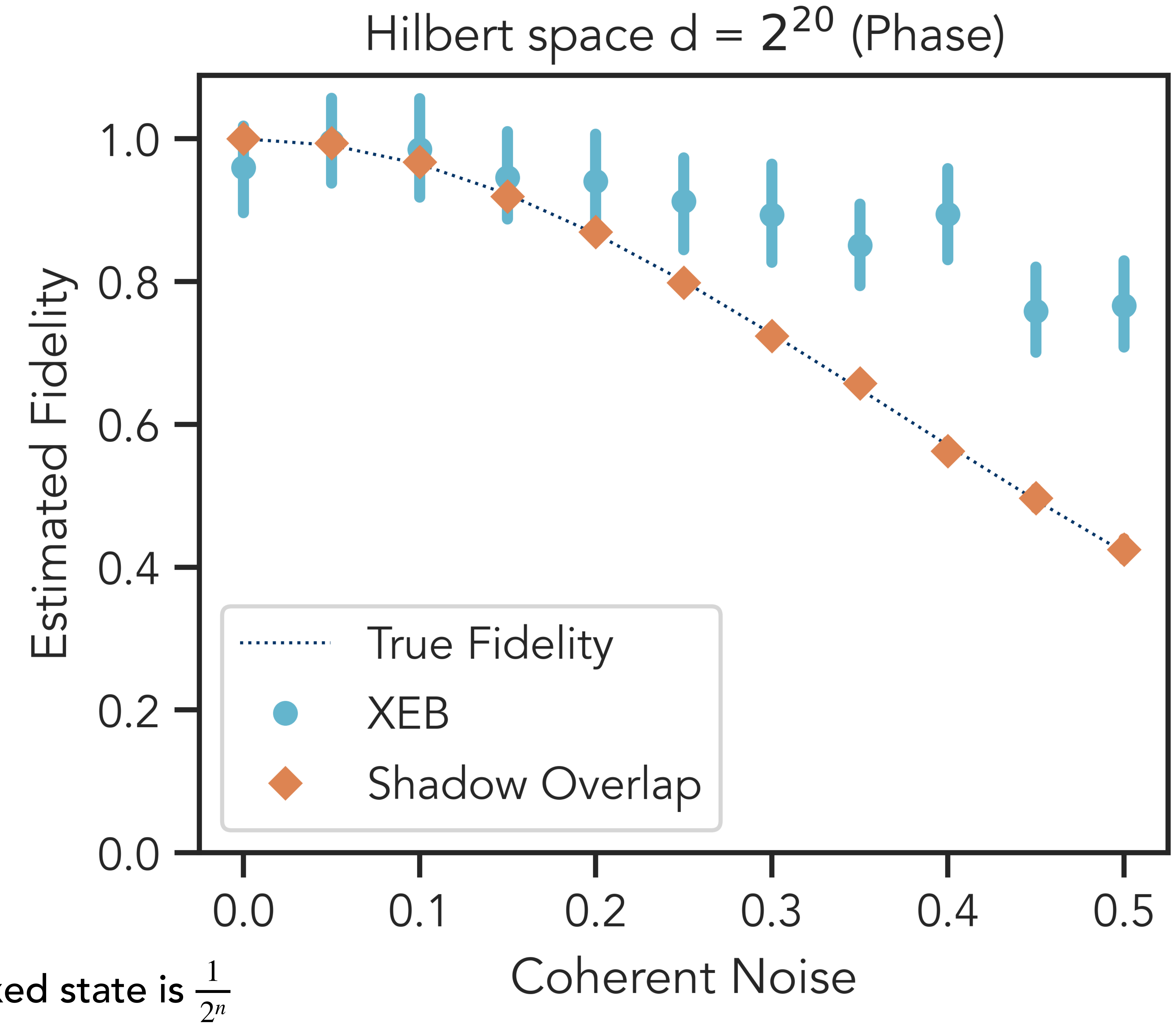

*Shadow overlap normalized s.t., target state is 1, maximally mixed state is $\frac{1}{2^n}$

# Benchmarking quantum devices

20-qubit random structured state

*Coherent Noise*

$$|\psi\rangle = U_{\text{phase}} \bigotimes_{i=1}^{20} |\psi_i\rangle$$




*Shadow overlap normalized s.t., target state is 1, maximally mixed state is $\frac{1}{2^n}$

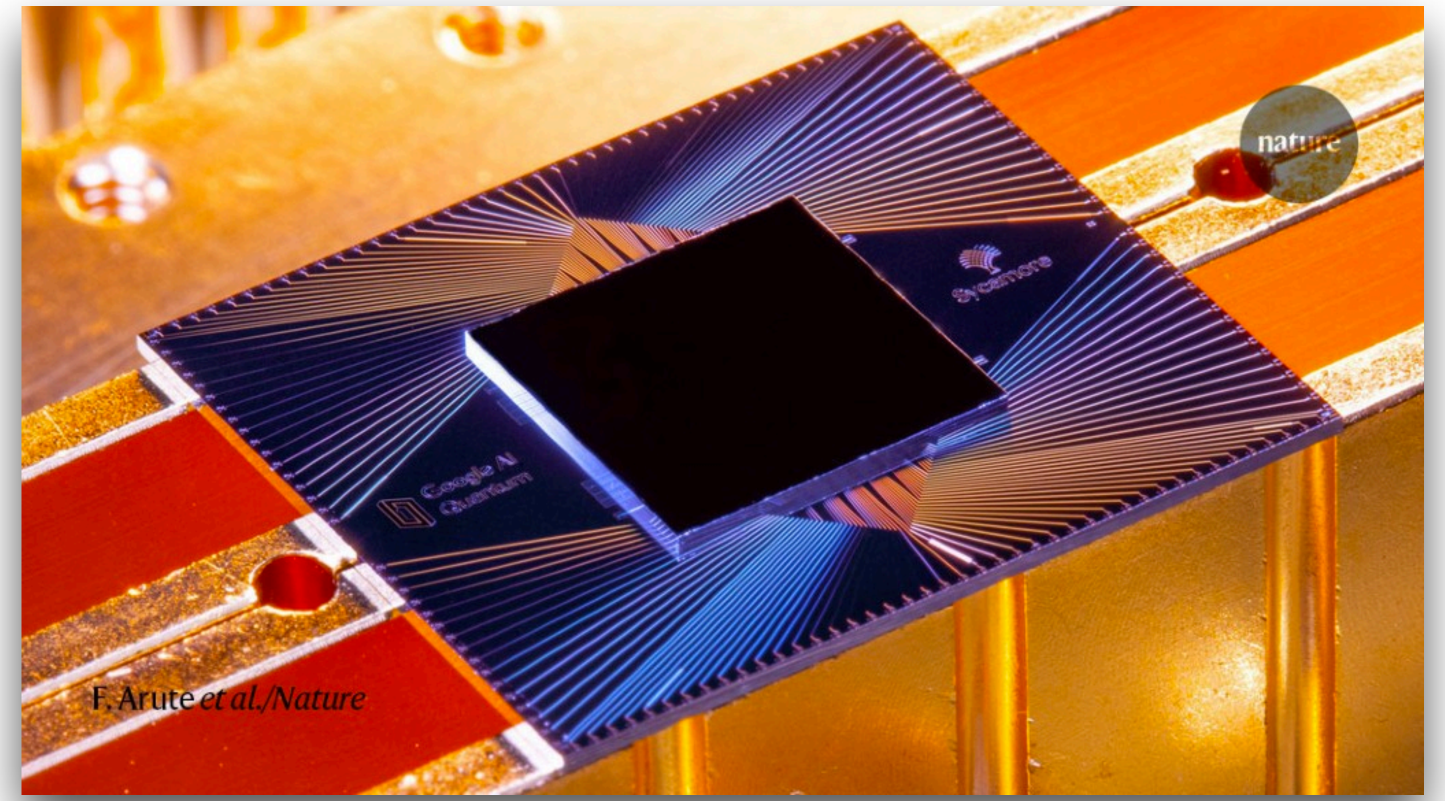
# Applications

What can we use this new certification protocol for?

**Example 1**

*Benchmarking*

Shadow overlap $\mathbb{E}[\omega]$ certifies if the state has a high fidelity



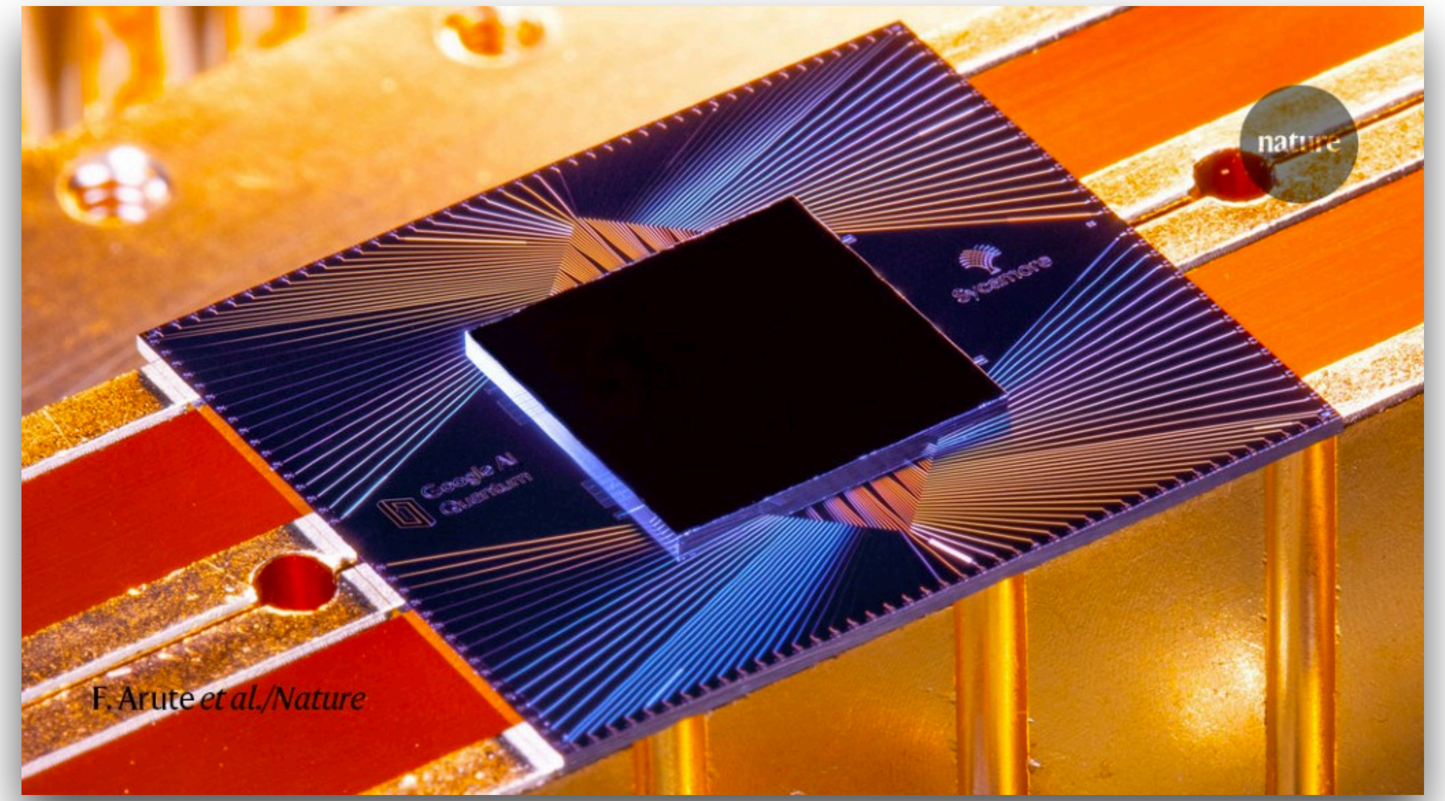
# Applications

What can we use this new certification protocol for?

**Example 1**

*Benchmarking*

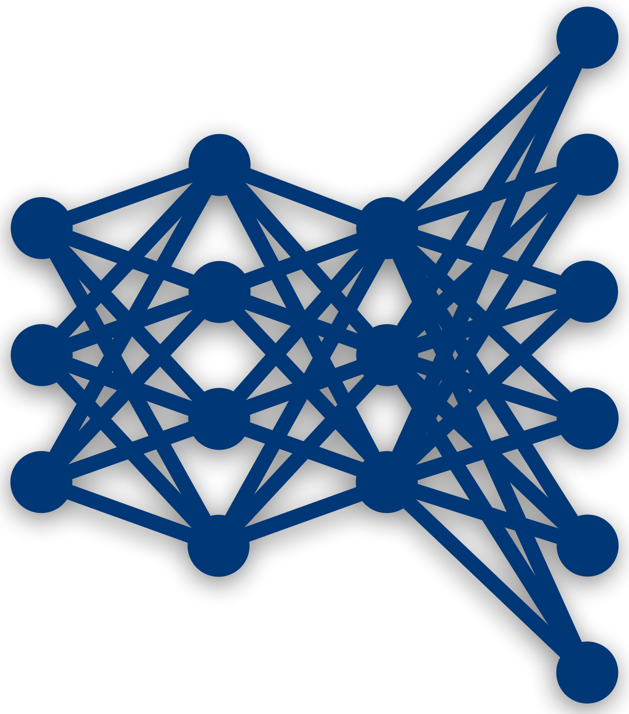
Shadow overlap $\mathbb{E}[\omega]$ certifies if the state has a high fidelity




**Example 2**

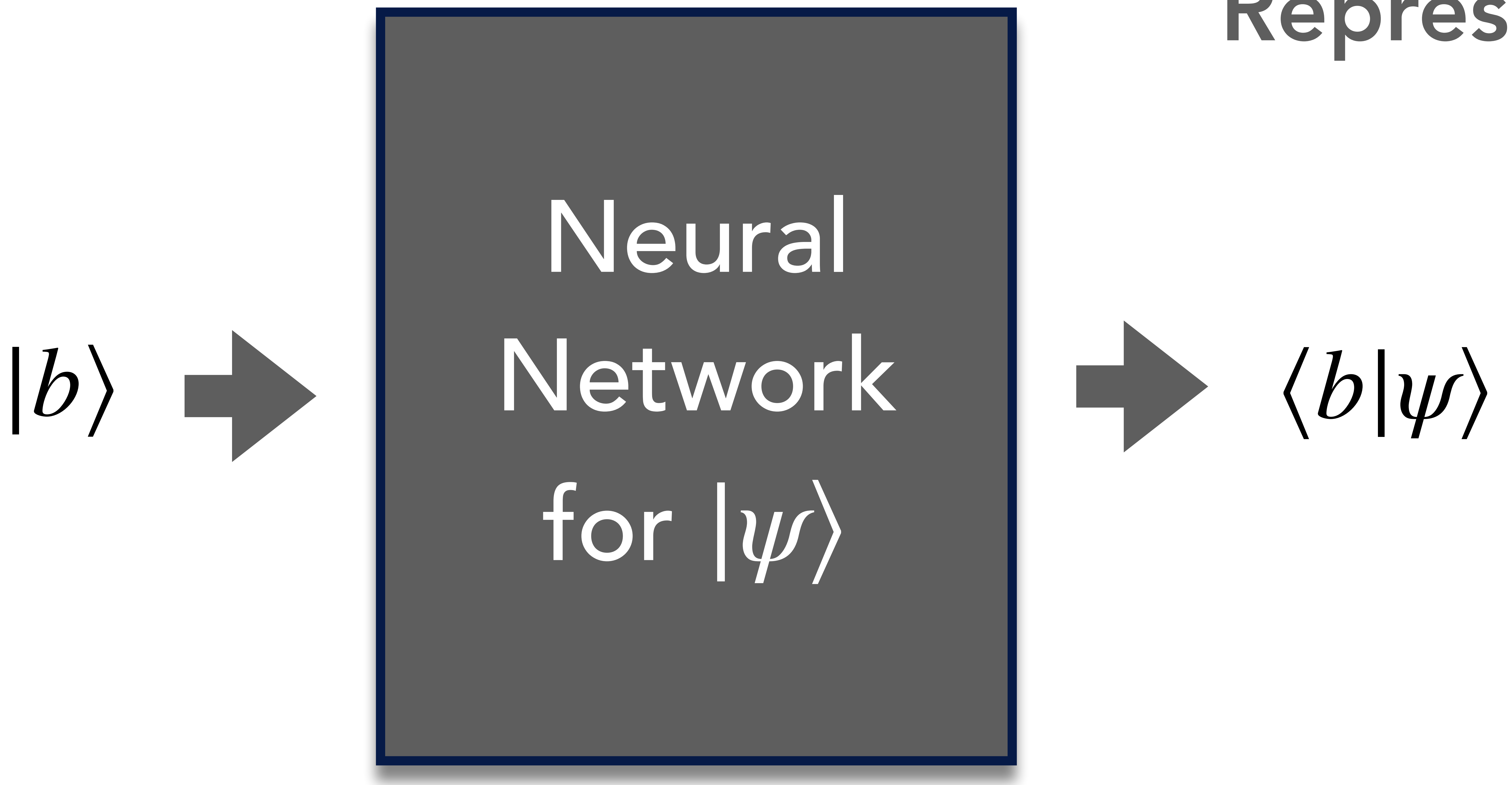
*ML tomography*

Train/certify ML models, such as neural quantum states, using shadow overlap $\mathbb{E}[\omega]$



# Training/Certifying NN tomography

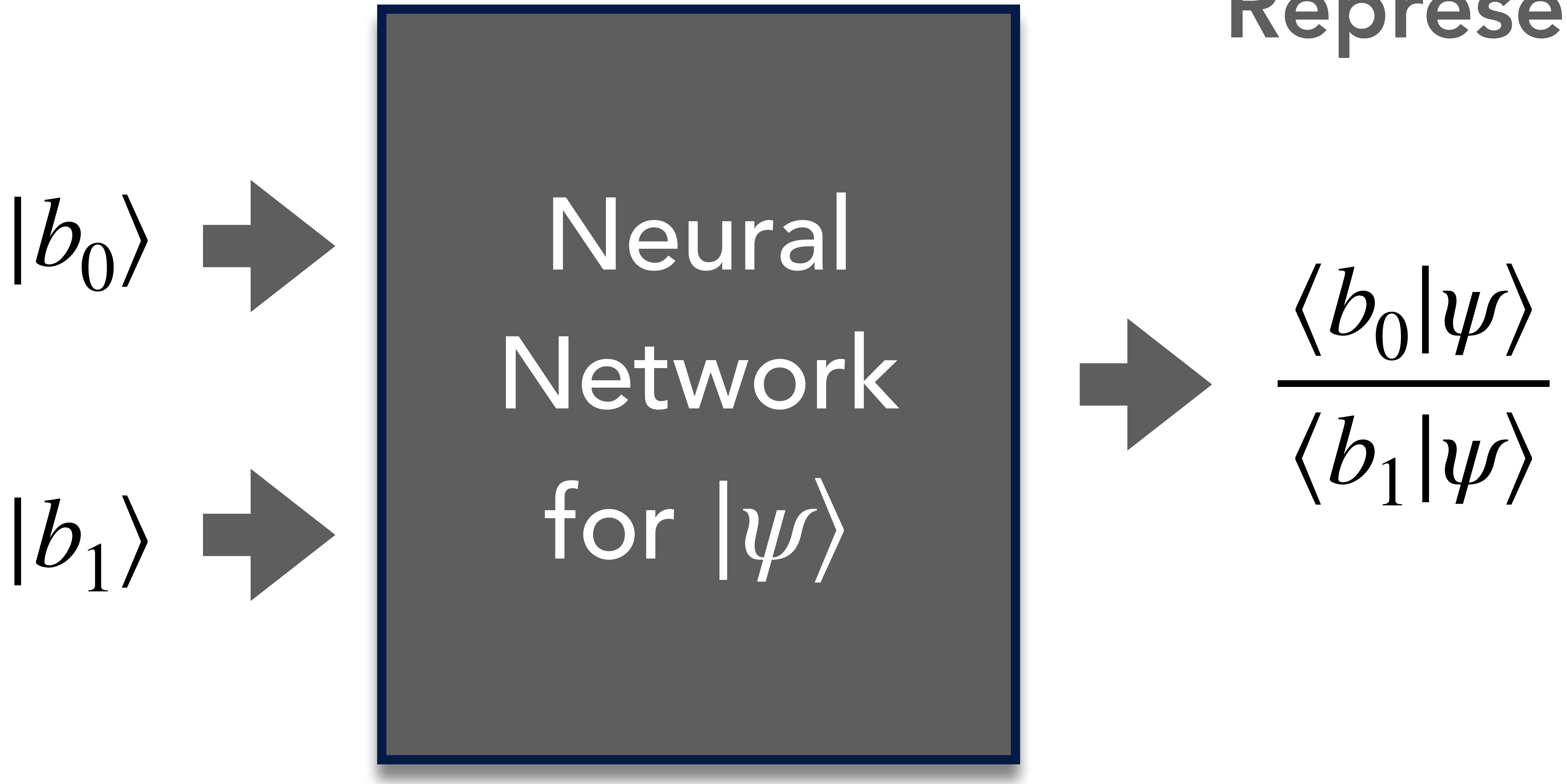
Represent $|\psi\rangle$




Standard Neural Quantum State

# Training/Certifying NN tomography

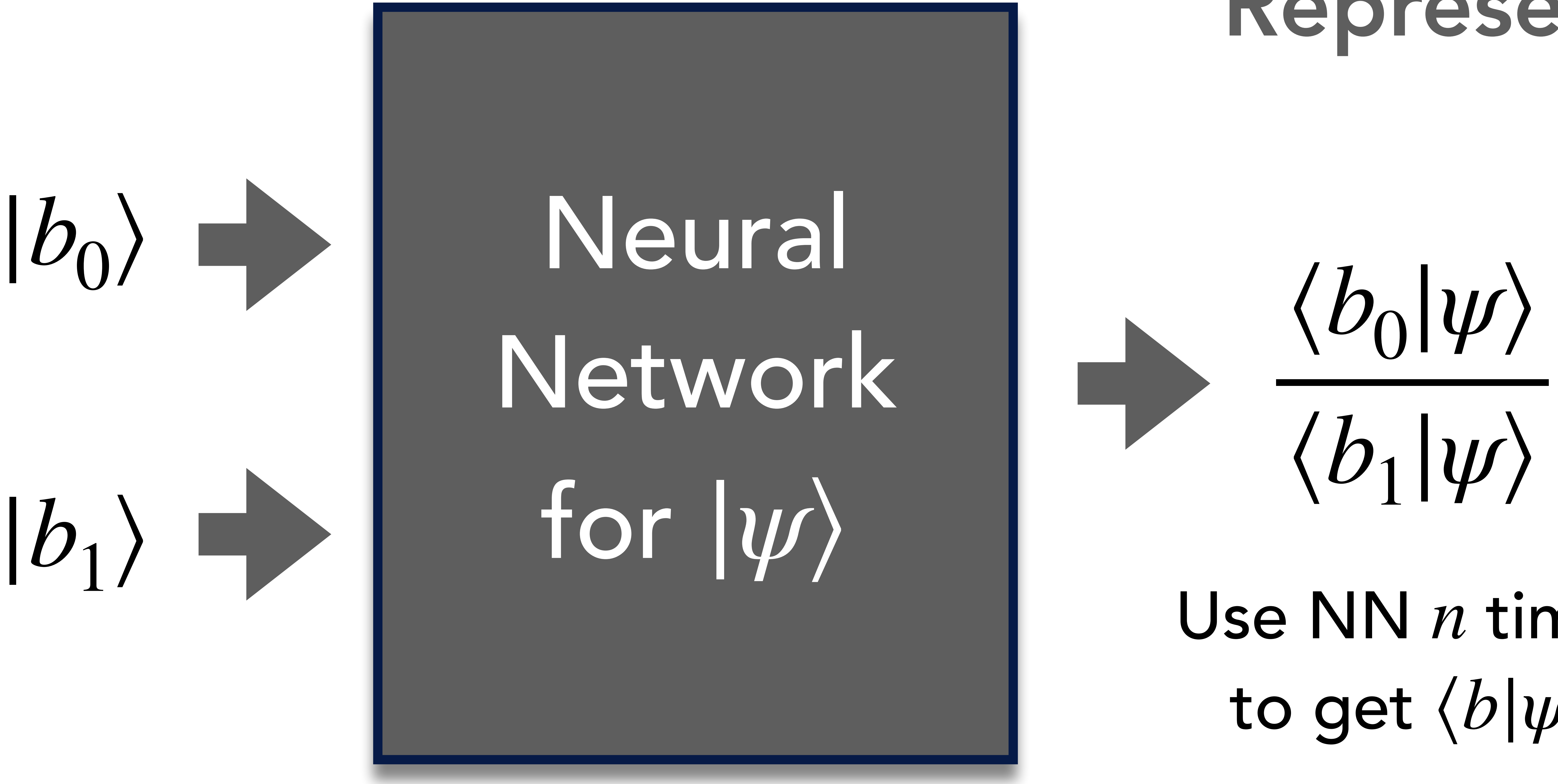
Represent $|\psi\rangle$




Relative Neural Quantum State

# Training/Certifying NN tomography

Represent $|\psi\rangle$

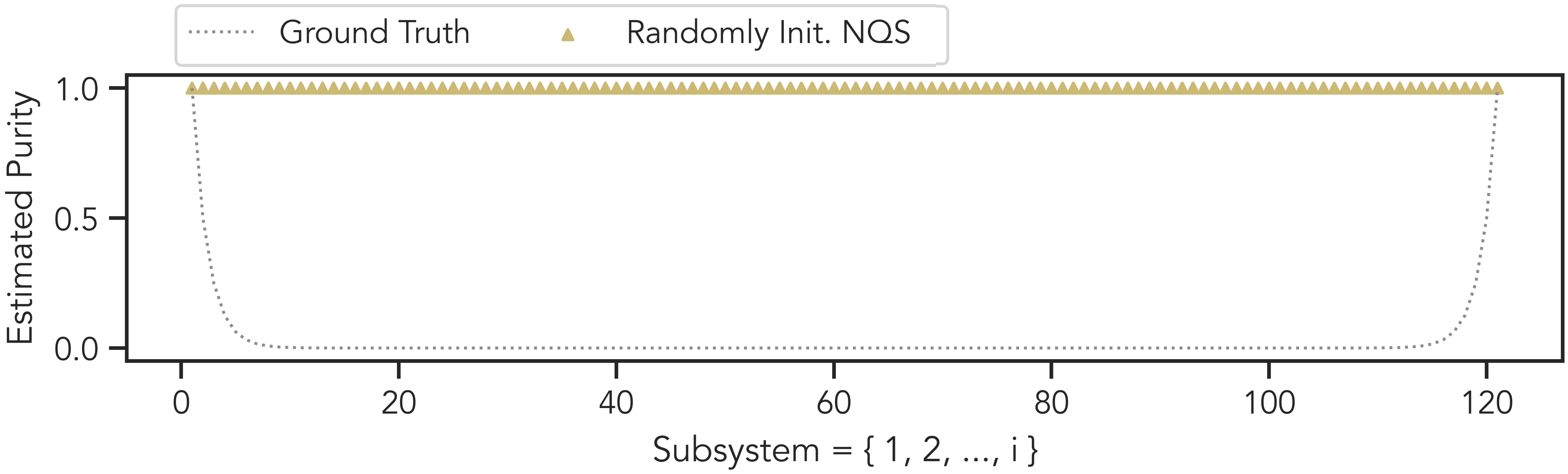



Use NN $n$ times to get $\langle b|\psi\rangle$

Relative Neural Quantum State

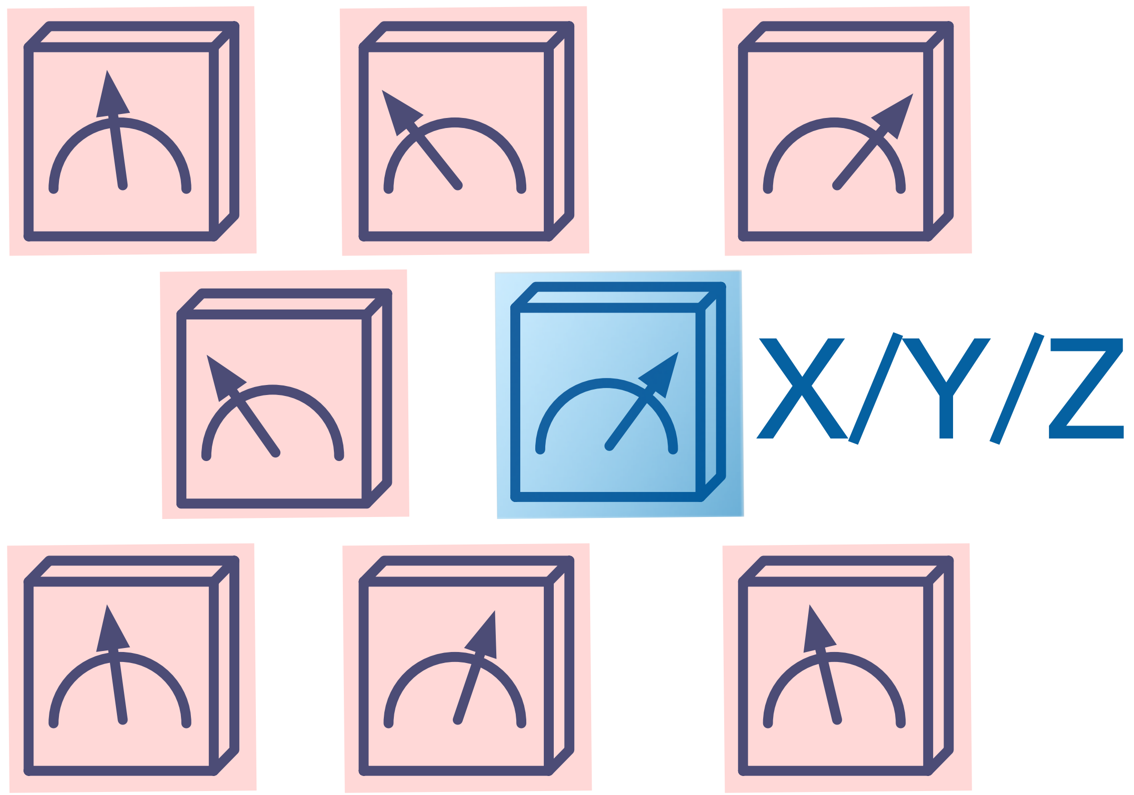
# Training/Certifying NN tomography

We consider learning a class of 120-qubit states with exponentially high circuit complexity.

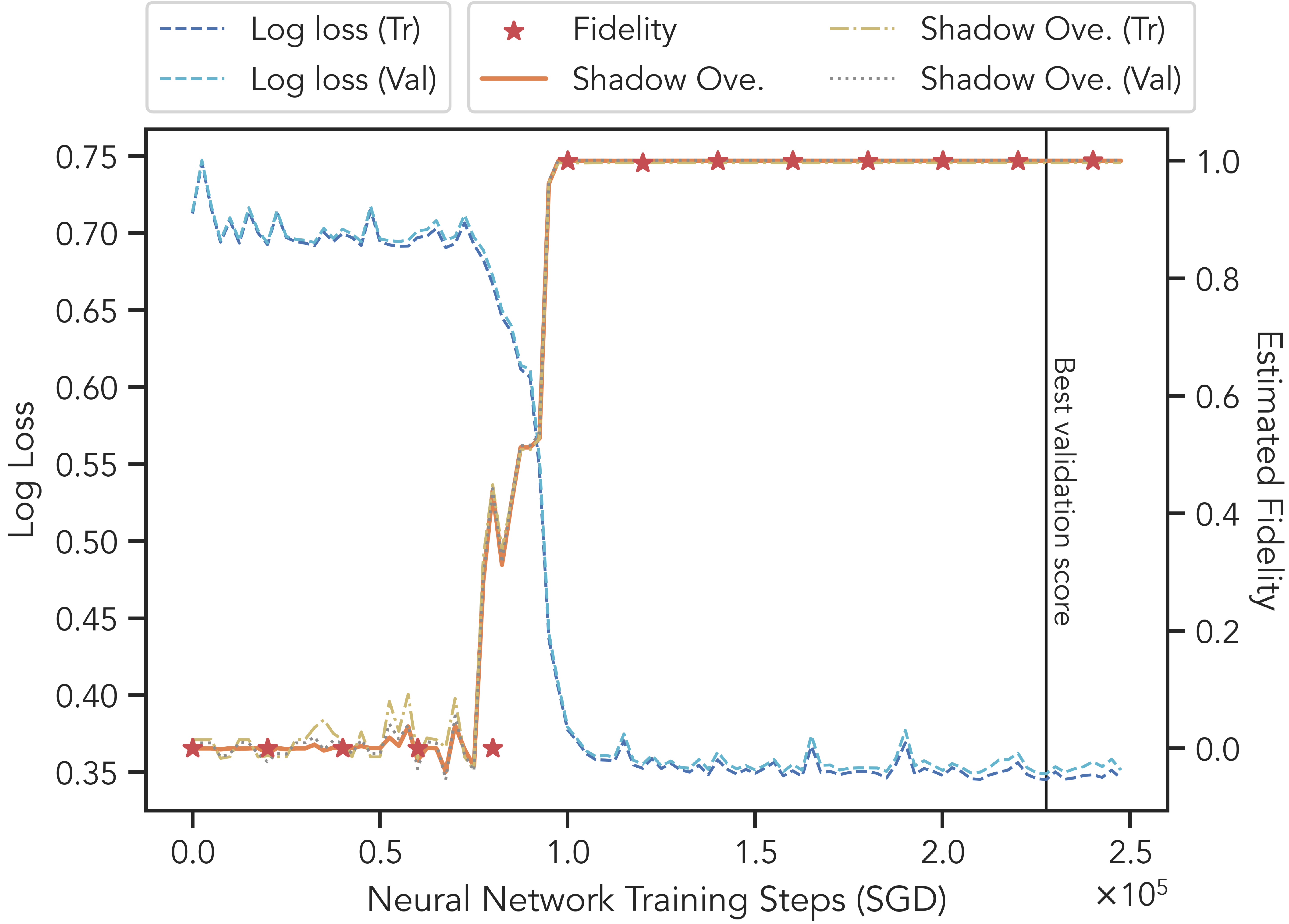


# Training/Certifying NN tomography

Trained using
shadow-overlap-based loss

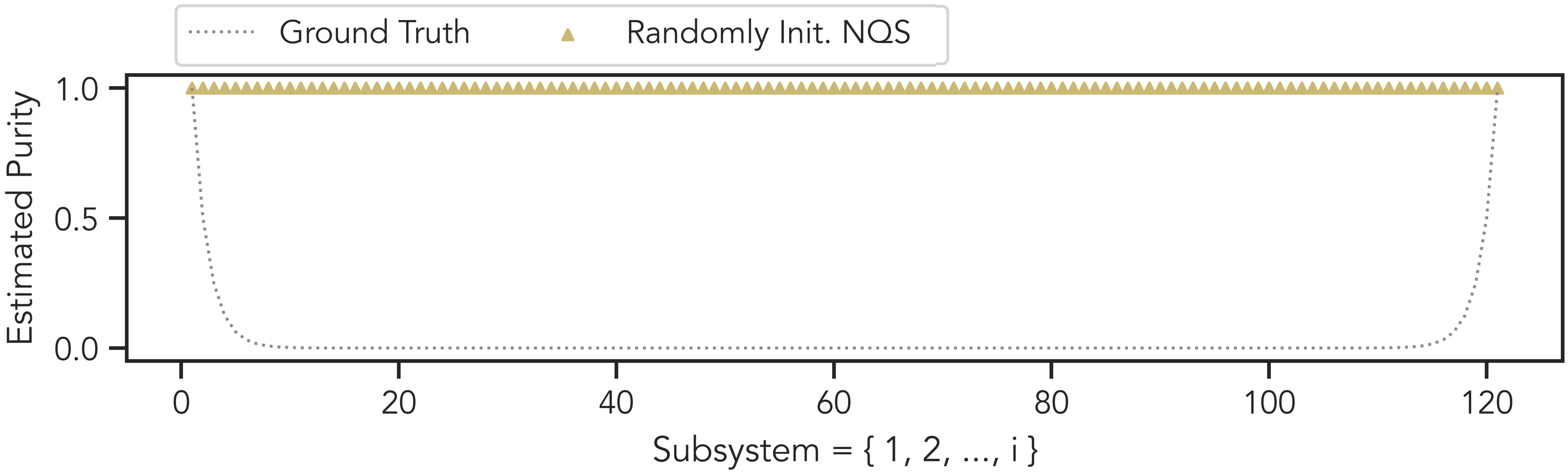



Certified using
shadow overlap



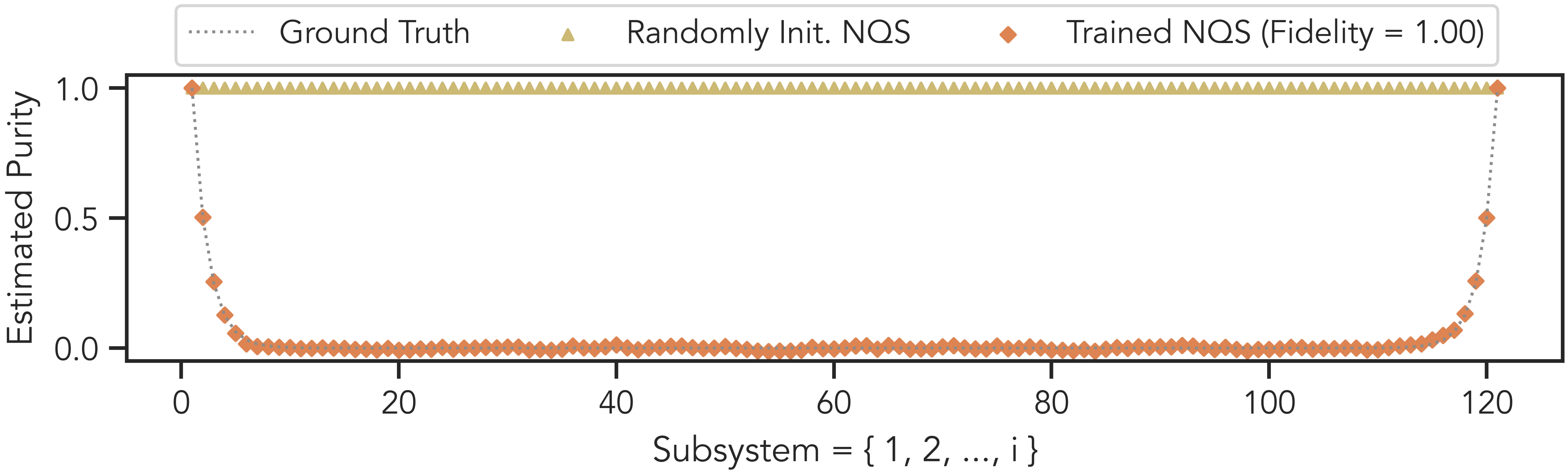
# Training/Certifying NN tomography

We consider learning a class of 120-qubit states with exponentially high circuit complexity.



# Training/Certifying NN tomography

We consider learning a class of 120-qubit states with exponentially high circuit complexity.



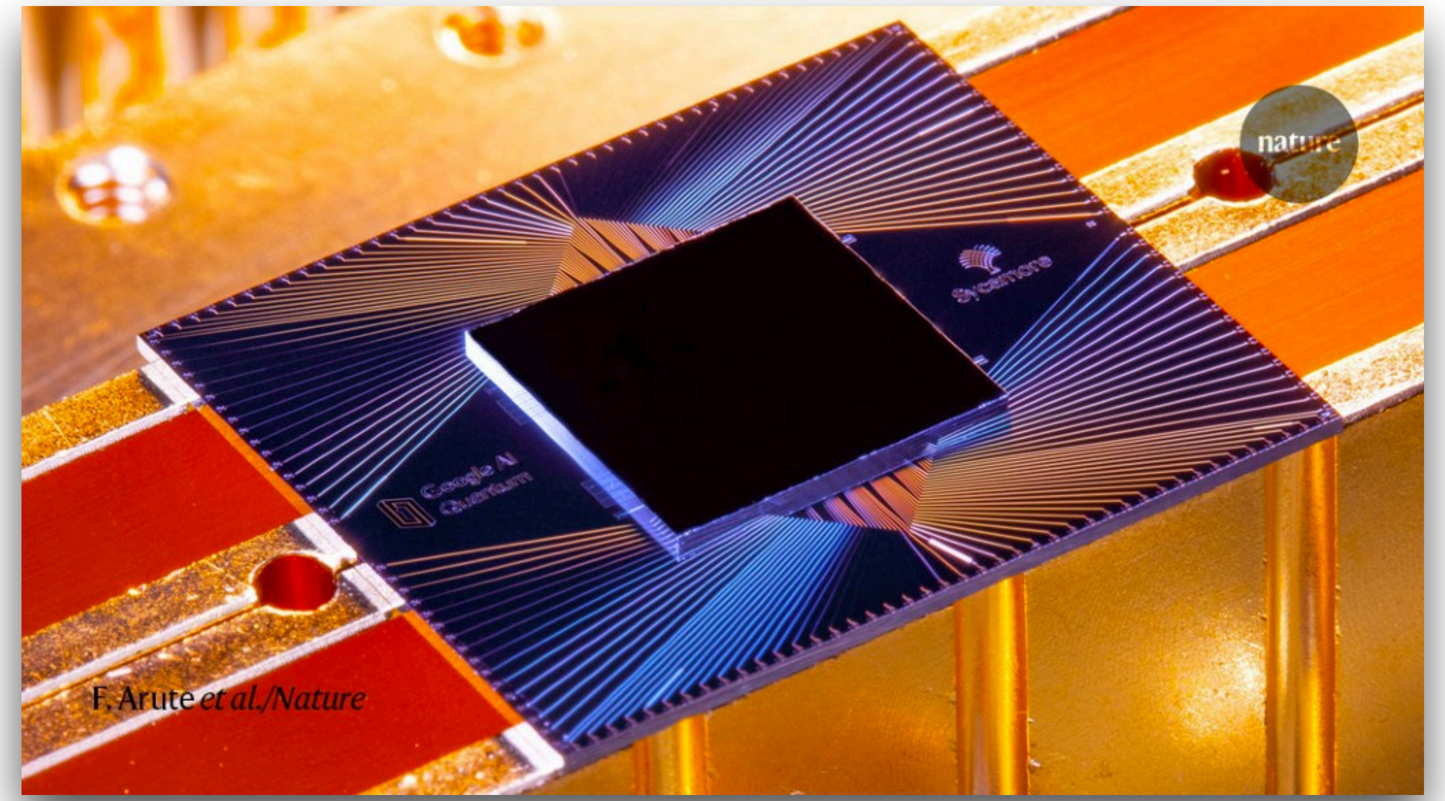
# Applications

What can we use this new certification protocol for?

**Example 1**

*Benchmarking*

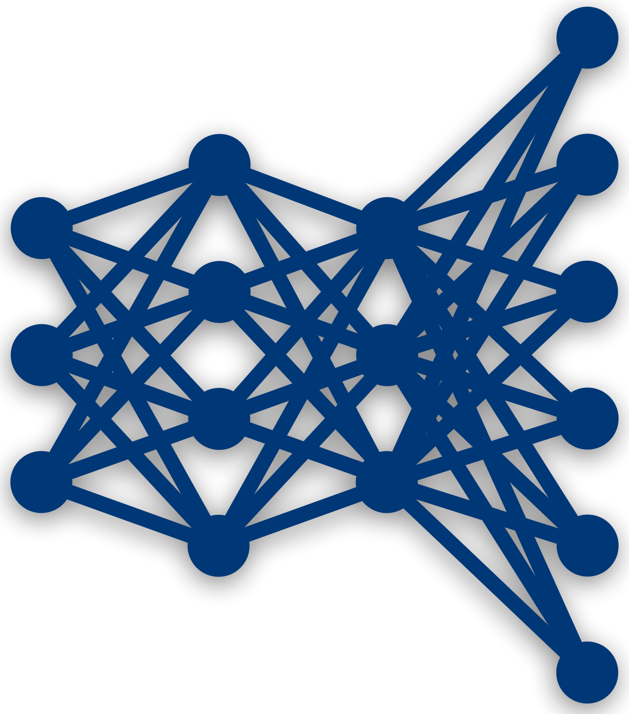
Shadow overlap $\mathbb{E}[\omega]$ certifies if the state has a high fidelity




**Example 2**

*ML tomography*

Train/certify ML models, such as neural quantum states, using shadow overlap $\mathbb{E}[\omega]$



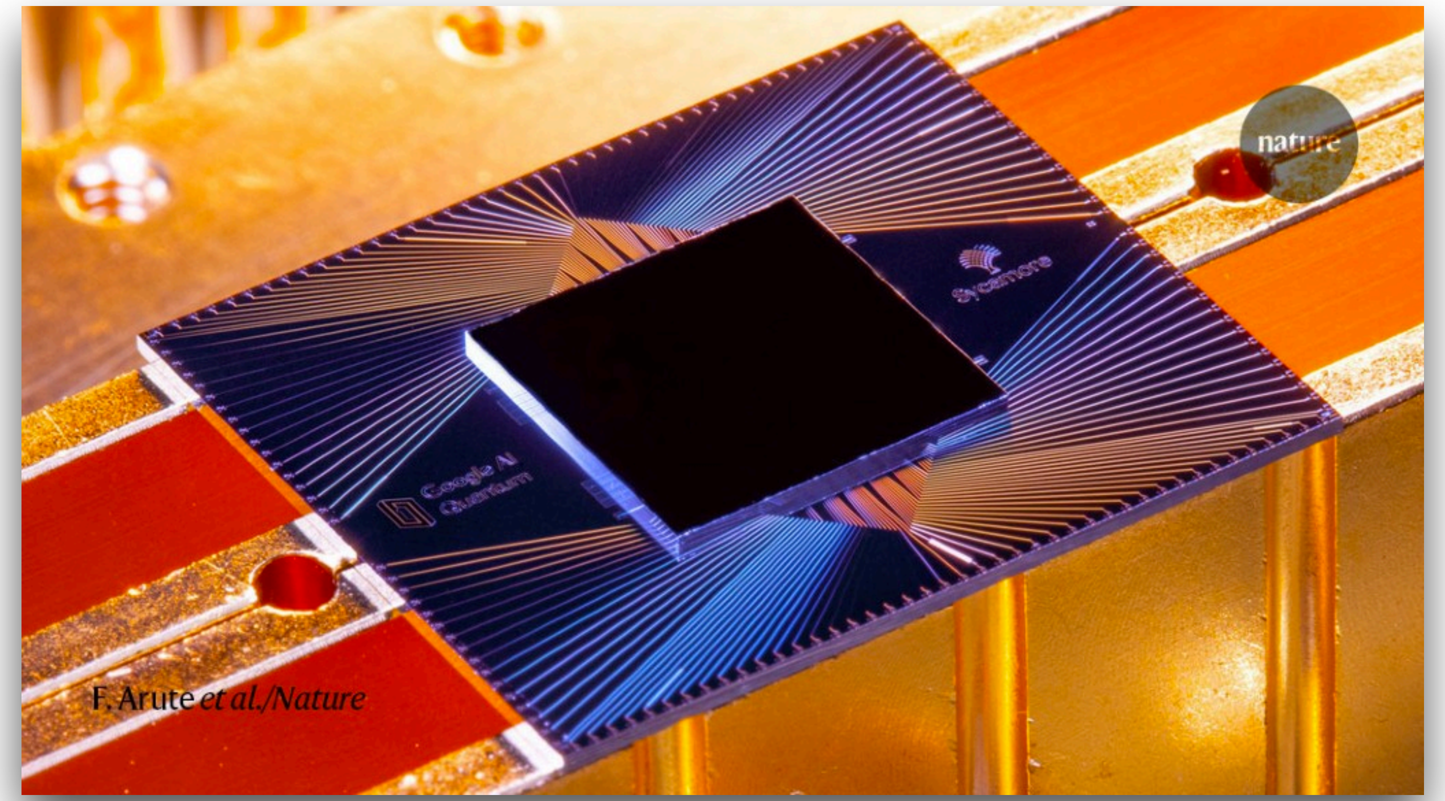
# Applications

What can we use this new certification protocol for?

## Example 1

*Benchmarking*

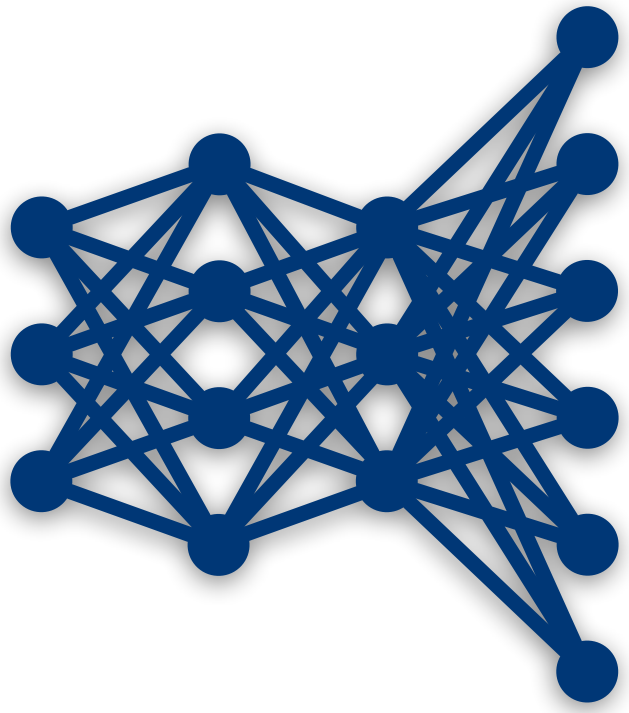
Shadow overlap $\mathbb{E}[\omega]$ certifies if the state has a high fidelity




## Example 2

*ML tomography*

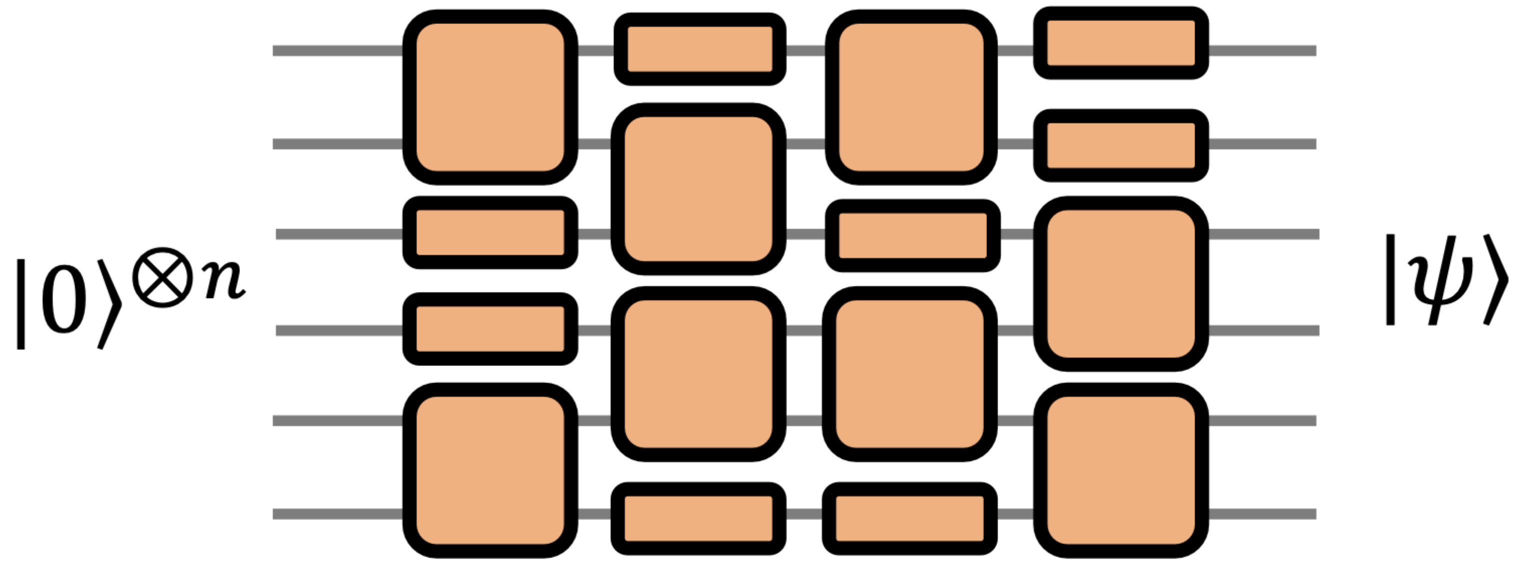
Train/certify ML models, such as neural quantum states, using shadow overlap $\mathbb{E}[\omega]$



## Example 3

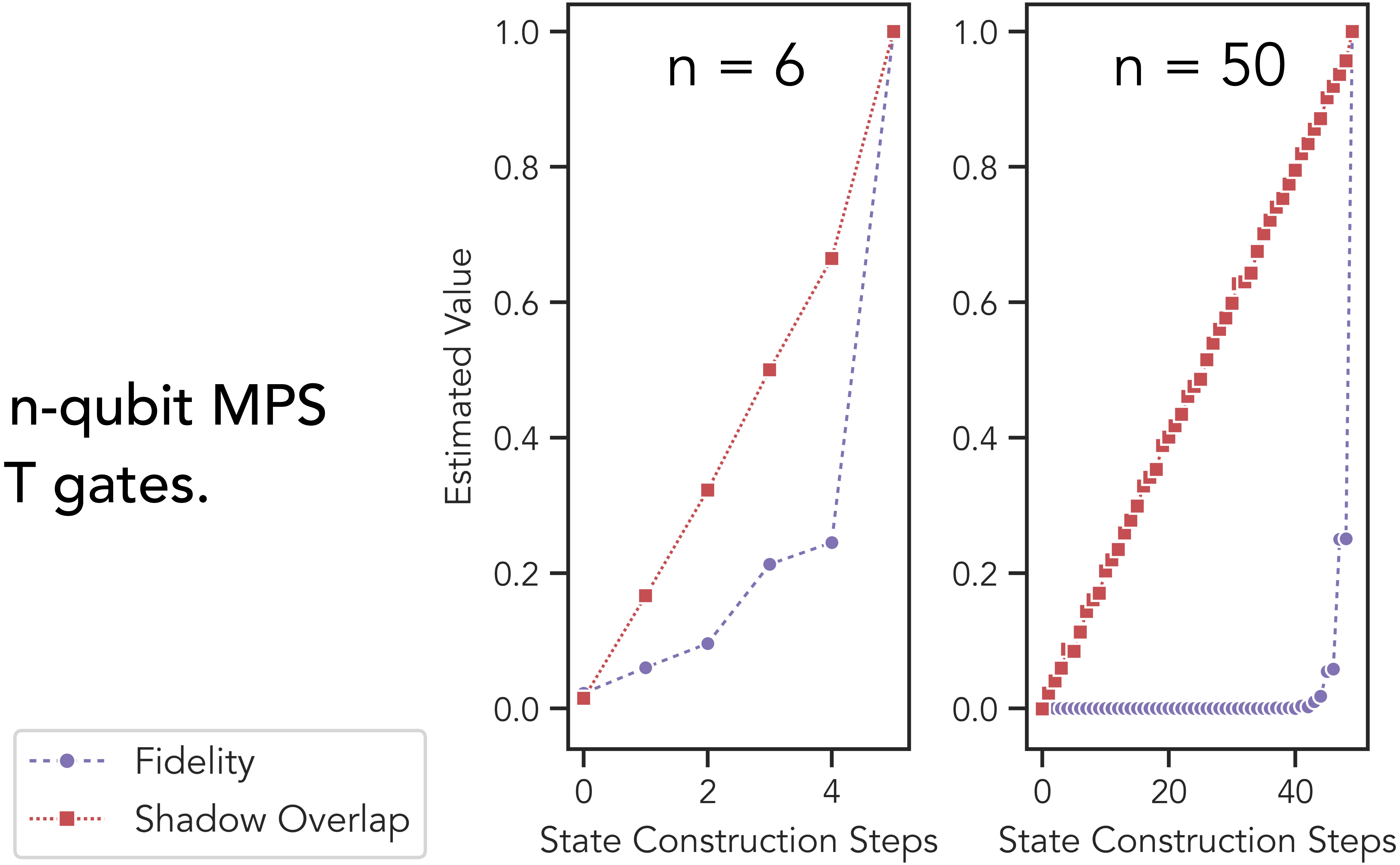
*Optimizing circuits*

To prepare a target state $|\psi\rangle$, we can optimize the circuit to max shadow overlap $\mathbb{E}[\omega]$



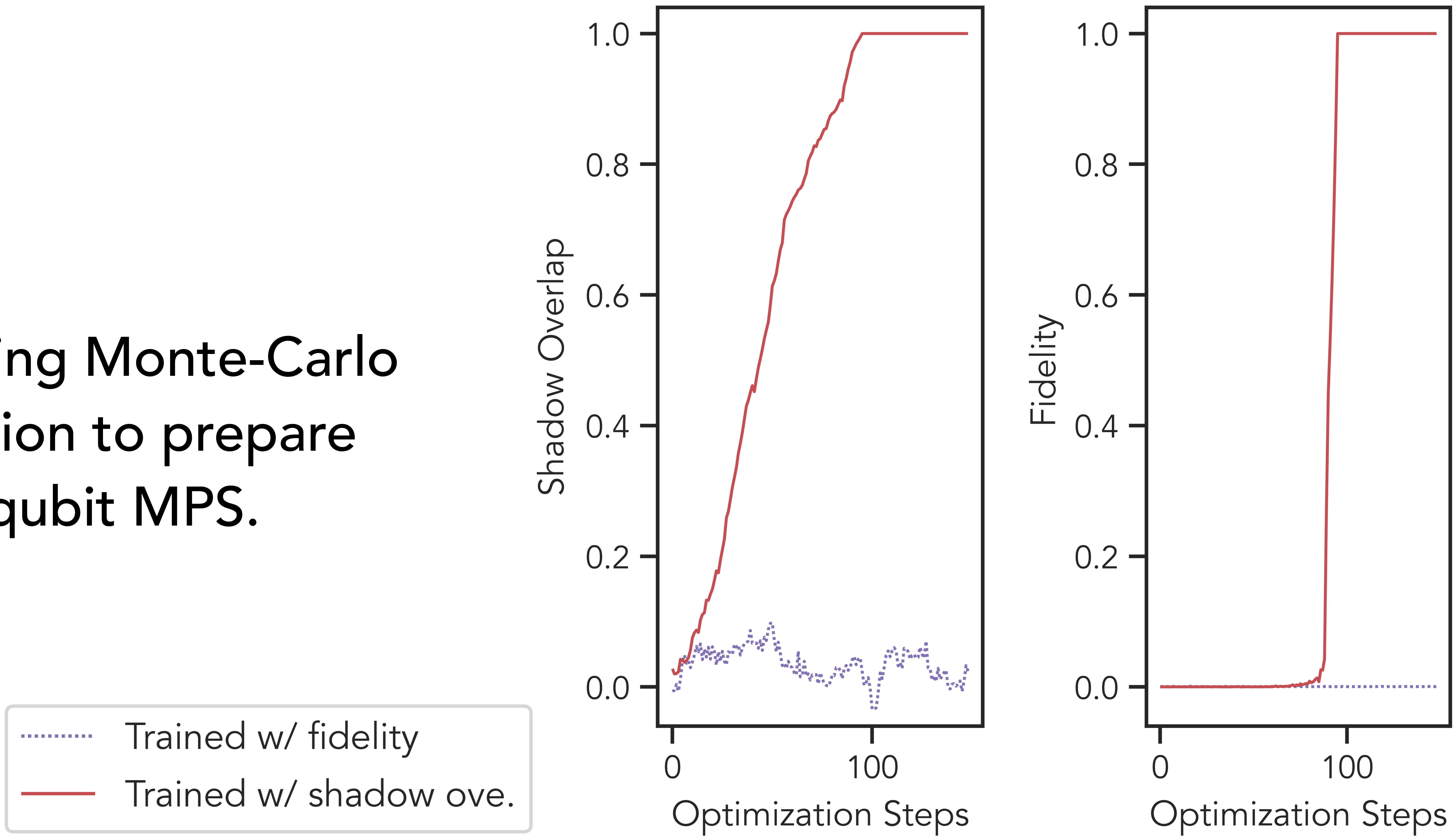
# Optimizing state-preparation circuit

Constructing an n-qubit MPS with H, CZ, T gates.



# Optimizing state-preparation circuit

Training using Monte-Carlo optimization to prepare a 50-qubit MPS.



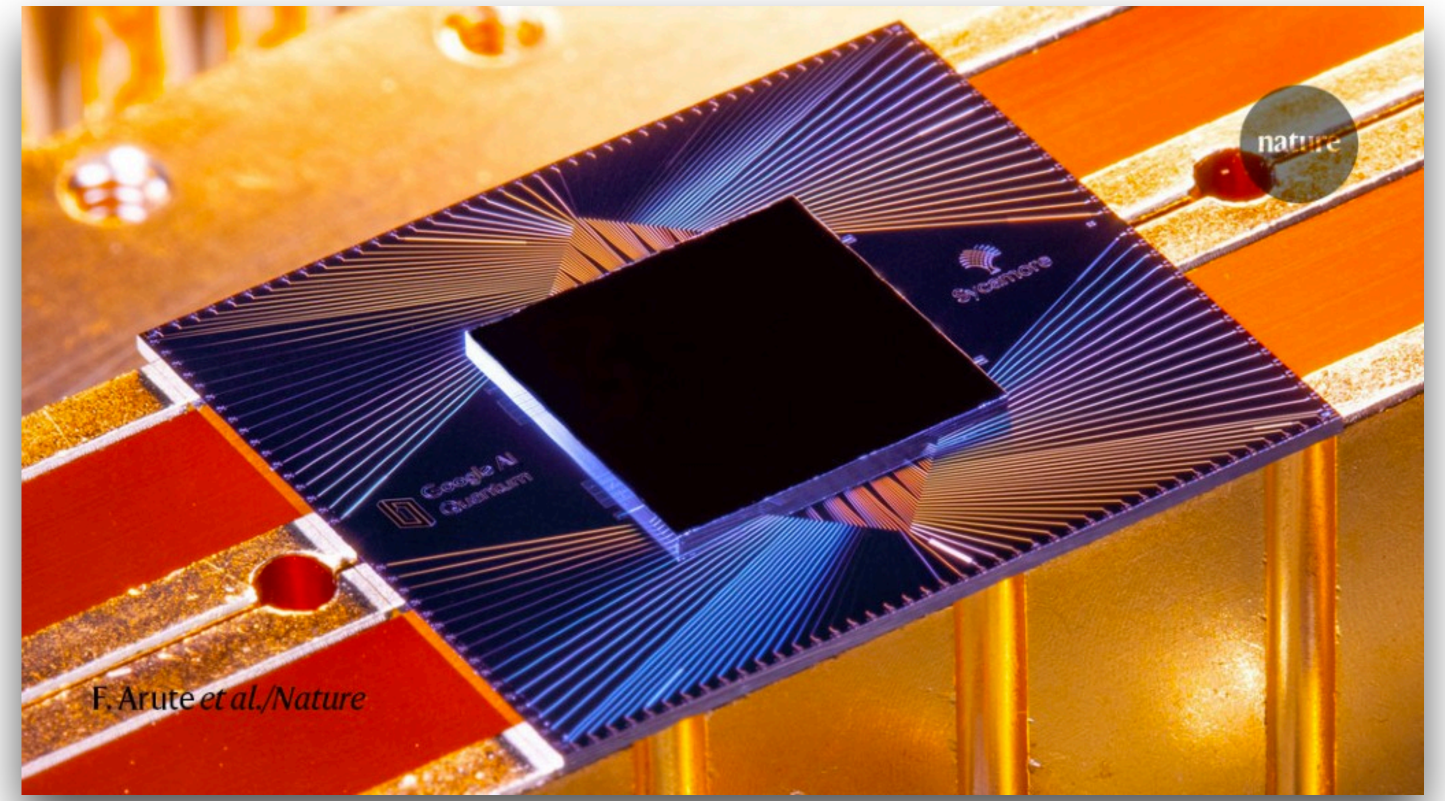
# Applications

What can we use this new certification protocol for?

## Example 1

*Benchmarking*

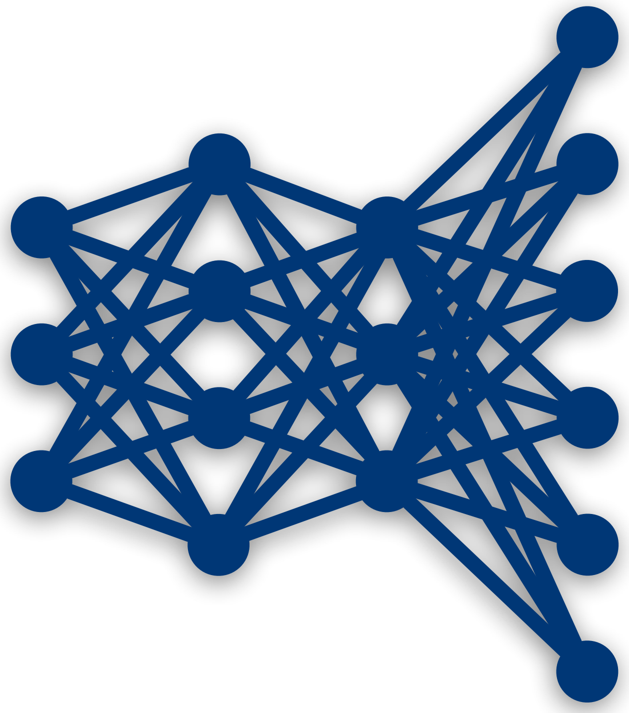
Shadow overlap $\mathbb{E}[\omega]$ certifies if the state has a high fidelity




## Example 2

*ML tomography*

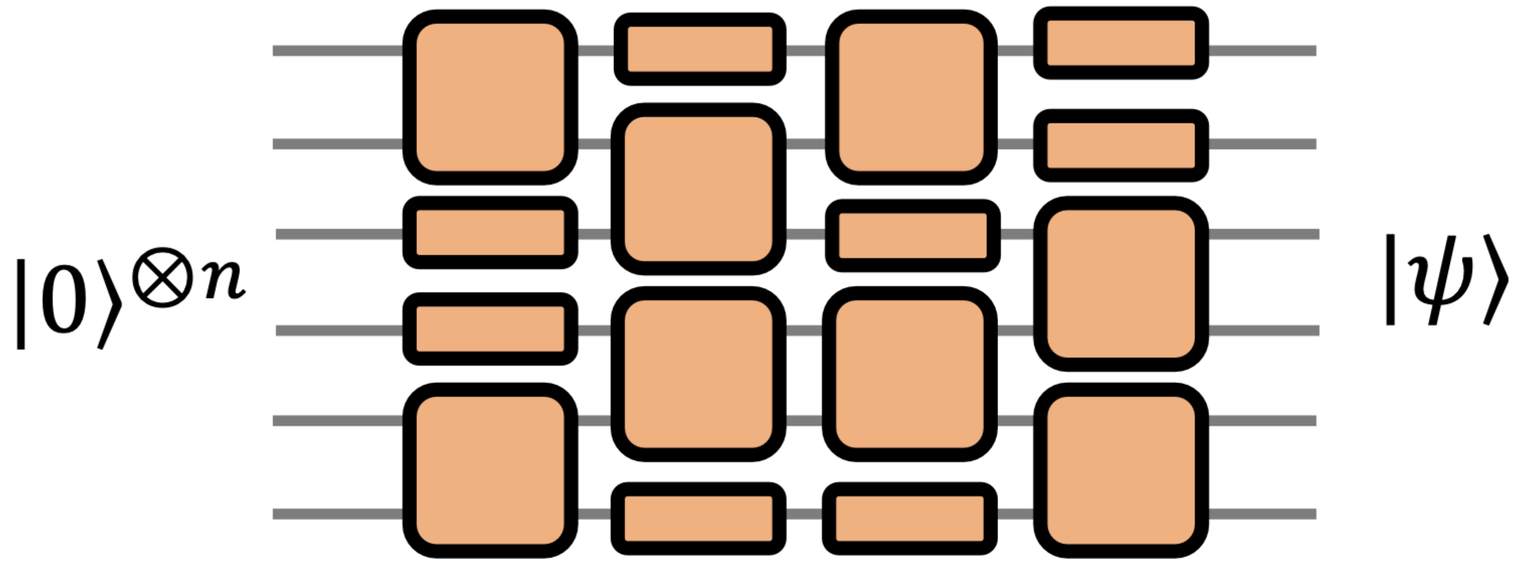
Train/certify ML models, such as neural quantum states, using shadow overlap $\mathbb{E}[\omega]$



## Example 3

*Optimizing circuits*

To prepare a target state $|\psi\rangle$, we can optimize the circuit to max shadow overlap $\mathbb{E}[\omega]$



# Conclusion

- We prove that almost all quantum states can be efficiently certified from few single-qubit measurements.
- Are there states not certifiable with few single-qubit measurements?