On Scrambling Times and Black Holes

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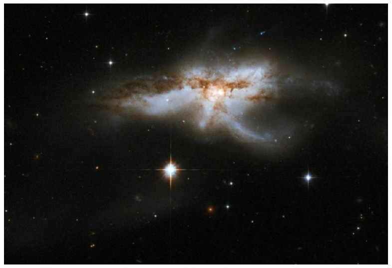
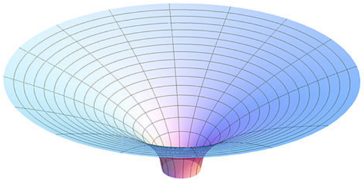
Outline

- History of black holes and information.
- Difference between weak scrambling time and strong scrambling time.
- An argument that fast strong scrambling implies non-standard physics.
- Some speculations.

Black Holes



Black Holes are a consequence of Einstein's theory of relativity.



A Schwatzschilld black hole is one which has no charge or spin. Its metric is given by

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2.$$

Black Hole Thermodynamics

We will mostly be using natural units where $c = \hbar = k_{\mathsf{B}} = G = 1$.

- The surface area of the horizon always increases. (Christodoulou)

  This is reminiscent of entropy.

- The entropy of a black hole is $S = A/4 = 4\pi M^2$ where A is the area of the horizon and M is the mass. (Bekenstein)

- Black holes radiate. Schwatzschilld black holes emit black-body radiation at a temperature of $T = 1/(8\pi M)$ where M is the mass. (Hawking)

Black hole information paradox

1. General relativity says that any information thrown into a black hole can never come out. So if the black hole evaporates, the information must be destroyed.

To support this, Hawking's derivation of Hawking radiation said it is uncorrelated with the stuff that fell into the black hole (but this derivation uses semi-classical gravity and is an approximation).

Black hole information paradox

2. Quantum mechanics says that physics is reversible, so information can never be destroyed.

This wasn't a problem before it was discovered that black holes evaporated.

Stephen Hawking, Kip Thorne, and John Preskill made a bet:
Hawking, Thorne: the information is destroyed.
Preskill: the information escapes.

Hawking conceded, but Thorne still has not.

Hawking/Unruh radiation

An observer hovering above the event horizon also sees Hawking radiation. To stay stationary above the event horizon, they need to be accelerating.

An accelerating observer sees Unruh radiation. For somebody hovering just above the event horizon, this is essentially equivalent to Hawking radiation.

The Hawking radiation seen at height h in Schwatzschilld coordinates (for small h) above the event horizon by a hovering observer is thermal radiation at a temperature

$$T = \frac{1}{4\pi\sqrt{2Mh}}$$

Susskind Complementarity (1993)

If somebody jumps into a black hole holding a qubit, for an outside observer, the qubit must emerge somehow, because information is never destroyed.

But the person falling in sees the qubit annihilated at the singularity.

Is the information destroyed?
Can we reconcile the point of view of the observer falling into a black hole and an observer staying outside the black hole?

Susskind's *complementarity principle* is the idea is that these observers can never get together and compare notes, so it doesn't matter if they have contradictory observations: They're both correct from their own point of view.

AMPS (Almheiri, Marolf, Polchinski, Sully, 2012)

Complementarity contradicts some fundamental theorems of quantum information theory.

One question: how much of complementarity can we save.

Does the outside observer's point of view make any sense if we completely ignore the point of view of somebody falling into the black hole.

“Black Holes as Mirrors” (Hayden and Preskill)

With quantum information theory, if you have a system that is maximally entangled with the black hole, and you throw something into the black hole, we can get the information on what was thrown in from just a few bits of the Hawking radiation.

This is true not just for a black hole, but for any quantum system maximally entangled with another system.

“Black Holes as Mirrors” (Hayden and Preskill)

Suppose you start with a black hole you know the state of. And suppose half of the information has escaped from the black hole (*the Page time*), and you have collected it. You now have Hawking radiation that is maximally entangled with the black hole.

Now, you throw a qubit into the black hole. You wait for the “scrambling time”, and collect order $\log M$ bits of additional Hawking information. You can then do a measurement that gives you information about what was thrown in.

Because the Hawking radiation you collected is maximally entangled with the black hole, you can use it to make any measurement you could have made on Alice’s original qubit.

Test of complementarity principle

(Hayden and Preskill)

Idea: Bob throws in a qubit, waits until he recovers the information, and then jumps in, hoping to catch up the information he has thrown in.

If he does catch up, he has duplicated the information, and thus violated the no-cloning theorem.

Bob has to be quick
(Hayden and Preskill)

If he waits more than order $M \log M$ time, he can never catch up with the information he's thrown in.

So in order for the no-cloning theorem not to be violated, the scrambling time must be at least order $M \log M$.

Scrambling time:

What is “scrambling time”? There are several definitions that have been used.

1. The time it takes for the out-of-time-order correlator (OTOC) to equilibriate.

2. The time it takes for the system starting in an arbitrary product state $A \otimes B$ to become nearly maximally entangled.

Time (2) is at least as large as time (1).

Sekino and Susskind used a third definition.
A fourth definition is based on the tripartite mutual information.

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We will argue that for black holes, time (2) (strong scrambling time) must be large, although time (1) (weak scrambling time) may be small.

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Yoshida and Kitaev (arXiv:1710.033623) show that definition (1) is enough for the Hayden-Preskill protocol.

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With Aram Harrow, Linghang Kong, Zi-Wen Liu, and Saeed Mehraben, I have a toy model where we can prove these two scrambling times differ substantially.

Out of Time Order Correlators

We have two operators, W_1 (at time 0 and position x) and W_2 (at time t and position y). We look at their commutator. $[W_1(0,x), W_2(t,y)]$.

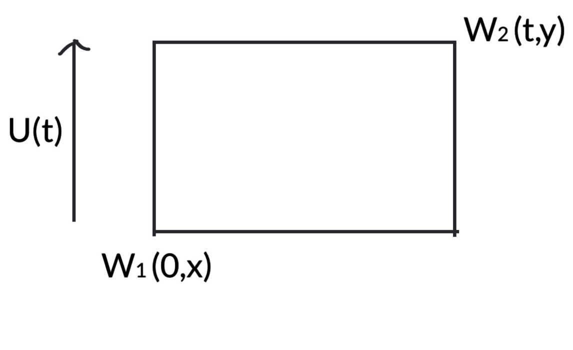
V and W have different spatial locations and times. This gives some indication of how long it takes the effects of V to reach the location of W — if t is too small for the effects of V to reach W, then the commutator is 0.

The OTOC is defined as

$$\frac{1}{2}\text{Tr } [W(t), V(0)]^{\dagger} \ [W(t), V(0)]$$

Out of Time Order Correlators

You can think of the OTOC as first applying $W_1(0, x)$, then applying unitary evolution going forward in time, then applying $W_2(t, y)$, then applying unitary evolution going backward in time. The OTOC measures how close this procedure comes to getting you back to the state you started with.



The scrambling time is defined as how long you need to make t before the OTOC equilibriates.

Lower bound on scrambling time.

A system with radius R clearly needs R/c time for information to pass from one side to the other. For black holes, this means the scrambling time (assuming relativity holds) has to be at least order M since $R = 2M$.

Hayden and Preskill showed a lower bound of order $M \log M$, assuming the no-cloning theorem holds.

Our Toy Example

We put qubits on the vertices of a binary tree.

Time evolution runs by choosing a random edge, and applying a random two-qubit Clifford group element to that edge.

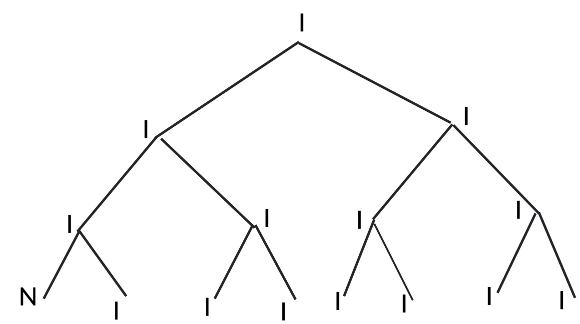
We choose edges according to a Poisson process with rate 1, so for every unit time, approximately one Clifford group element is applied to any edge.

Our Toy Example

We let the operators $W_1(0, x)$ and $W_2(t, y)$ be Pauli operators, say σ_x.

When we apply a Clifford operator, it takes $id \otimes id$ to $id \otimes id$.
It takes any other $\sigma_a \otimes \sigma_b$ to all possible non-identity $\sigma_a \otimes \sigma_b$ with equal probability.
Let I stand for id,
Let N stand for σ_x, σ_y, or σ_x.

Then $N \otimes I$ has probability $\frac{1}{5}$
$I \otimes N$ has probability $\frac{1}{5}$
$N \otimes N$ has probability $\frac{3}{5}$



Starting Configuration

Our Toy Example

$N \otimes I$ has probability $\frac{1}{5}$
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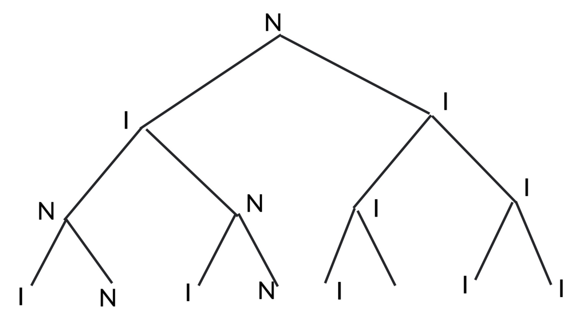


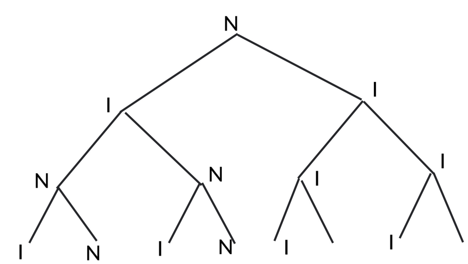
Consider the N closest to the point y. Let d be the distance from this N to y. Assume this N is unique. Then d goes up with probability $\frac{1}{5}$ if the Clifford gate is applied to an edge before this N, and goes down with probability $\frac{4}{5}$ if the Clifford gate is applied to the edge after this N.

So this is a random walk where the probability of going down is more than the probability of going up.

Our Toy Example

For the OTOC measure, the expected value of the OTOC reaches its final value when the random walk has hit point y with high probability. This is linear in the size of the tree, so linear in n.

For the entanglement to reach order n qubits between the two sides of the tree, we need to have Schmidt rank of order 2^n between the two halves of the binary tree. This can only happen if a qubit moves from the left to the right order 2^n times, which takes time 2^n.



So the entanglement scrambling time can be exponentially larger than the OTOC scrambling time.

Arbitrary Unitaries

Since the density matrix obtained by applying a random Clifford is equivalent to the one obtained by applying an arbitrary unitary, our lower bound also works for arbitrary unitaries.

Lower bound on scrambling time.

(Hayden and Preskill)

Hayden and Preskill claimed that the scrambling time has to be at least order $M \log M$ because in black holes, this is how long it takes mass and charge to distribute themselves equally on the surface of a black hole.

Fast Scrambling Conjecture

Sekino and Susskind: Black holes scramble in order $M \log M$ time (and are the fastest scramblers in existence).

Evidence:

If you drop some mass or charge onto a black hole, it takes order $M \log M$ time for the surface to equalize.

But ...

Suppose you put a drop of dye into a pitcher of water. The water level will equalize in a matter of seconds, while it takes much longer for the dye to diffuse evenly through the water.

This is because the change in water level is driven by an energy difference, while the diffusion of the dye is not.

Similarly, distributing mass or electric charge uniformly around the black hole decreases its energy, while spreading information does not.

An Argument for Fast Scrambling from AdS-CFT

You can show that out-of-time order correlation functions in CFT decay in time order $M \log M$ (Shenkar and Stanford). They argued that this means there should be fast decay in correlation functions and thus fast scrambling in the AdS side of the correspondence as well.

Our assumptions.

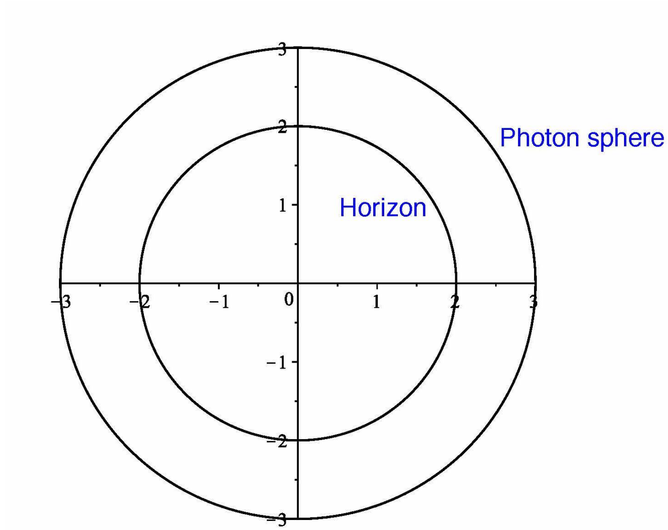
We will assume that from an outside observer's point of view, there is a consistent explanation of physics that makes sense where nothing falls past the horizon in finite time.

We further assume that low-energy physics looks more or less like the physics we know.

Finally, we assume that the causality structure of space–time behaves like general relativity says it does.

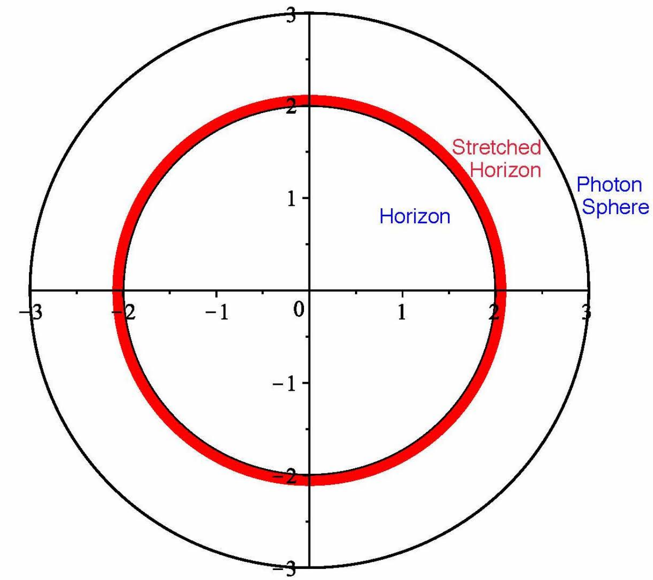
The photon sphere

Consider the space around a black hole. There is a region called the *photon sphere* which is the neighborhood of a black hole inside the smallest circular orbit. There are no orbits inside the photon sphere — either a photon will escape to infinity or it will fall into the black hole.



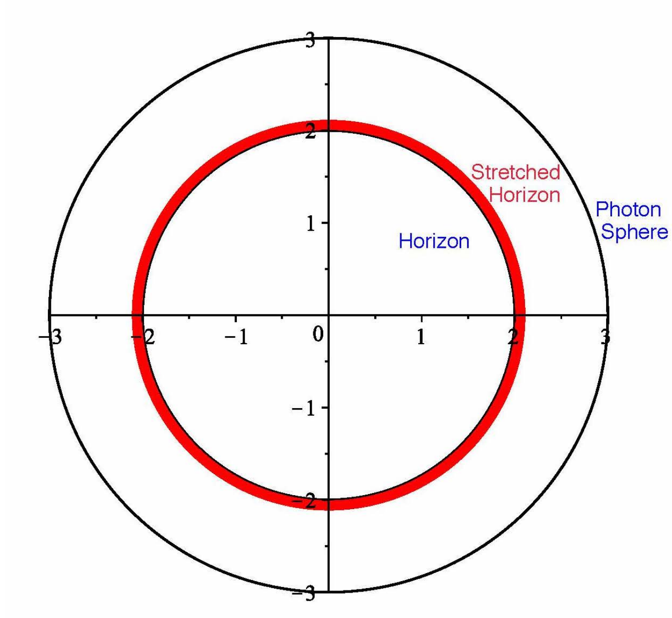
The stretched horizon

The Hawking radiation is only high-temperature very close to the horizon. Thus, except in the vicinity of the horizon (an area we will call the *stretched horizon*), physics should behave the way we're used to.



What carries the information to scramble it?

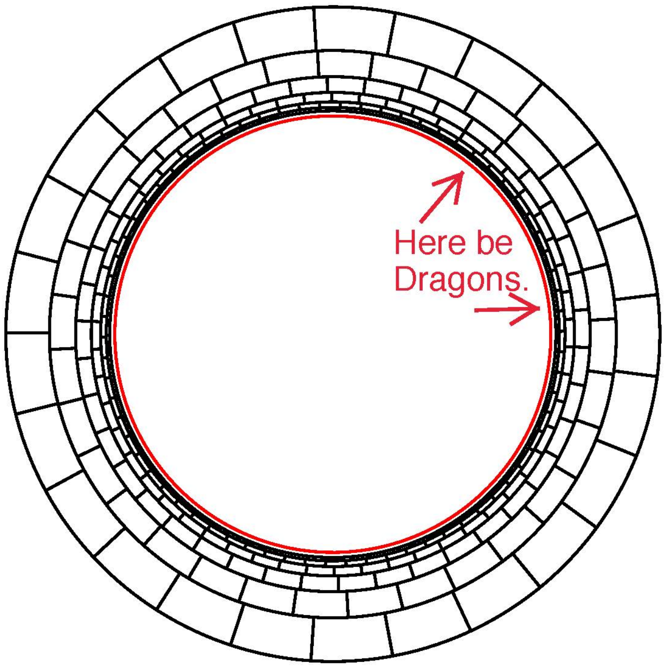
If the causality structure of general relativity holds, information cannot be transmitted quickly if it stays within the stretched horizon.



Outside the stretched horizon, the only thing available to carry information is Hawking radiation.

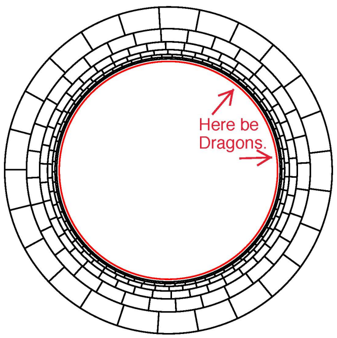
We will assume that the Hawking radiation carries the black hole information to scramble it (it has enough qubits).

Subdividing the photon sphere



We can divide the photon sphere into cells, so that the round trip from one side of the cell to the other and back takes time order M when viewed by a distant observer.

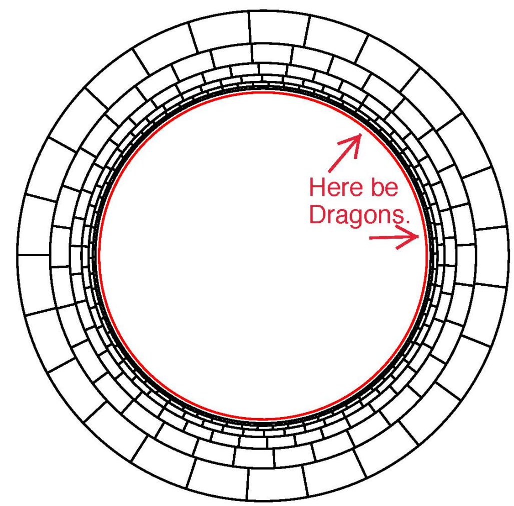
Information structure of the photon sphere



Each of these cells is filled with Hawking radiation, as seen from an outside observer. The number of qubits contained in the Hawking radiation inside each cell is $O(1)$. There are αA cells altogether, the vast majority very near the black hole. Within a constant factor, this is the amount of information contained by the black hole.

Details of the subdivision

At height h (in Schwatzschilld coordinates) above the event horizon, a cell has height $c_1 h$ and radial diameter $c_2\sqrt{Mh}$, where c_1, c_2 are constants. This ensures that the Hawking radiation in each cell contains a constant number of qubits.



The cells get smaller as you approach the horion, but from the viewpoint of an observer hovering inside one of these cells, its aspect ratio is constant.

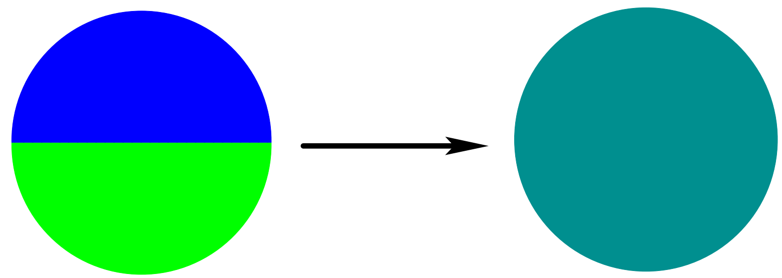
More details of the subdivision

As seen from an outside observer's viewpoint, the Hawking radiation has the same frequency everywhere. The redshift makes it much greater frequency to an observer hovering over the horizon.

Since we designed these cells so that the round trip from one side of the cell to the other takes time M when viewed by a distant observer, they all are of the same size as the dominant wavelength in the Hawking radiation seen by an observer hovering inside the cell.

This means that each of them contains a constant number of bits of information.

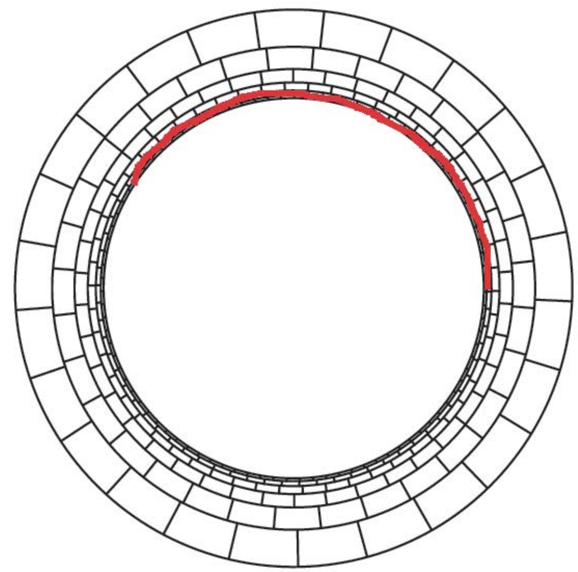
Requirements for Strong Scrambling



To scramble the information, if we start out with a separable state (nearly no entanglement between the top and bottom halves) and end with a generic pure state (nearly maximal entanglement between the top half and the bottom half) we need to carry $O(A)$ information from the top half to the bottom half.

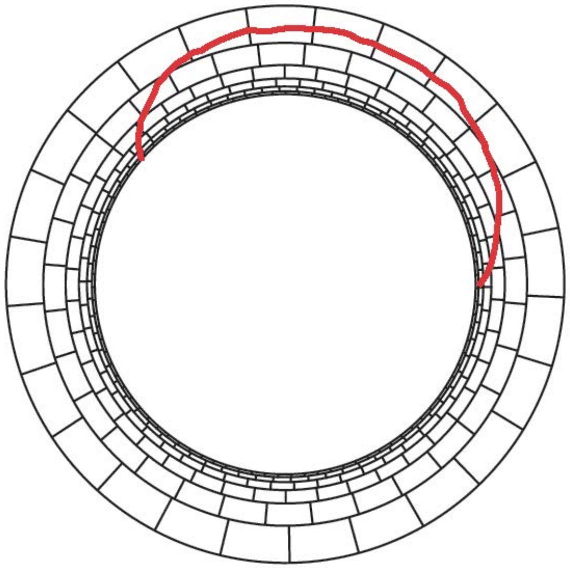
Intuition: Two paths I

Suppose we try to move the information along paths near the horizon of the black hole. The number of cells each of these paths crosses is order M, meaning we need order M^2 time to traverse this path.



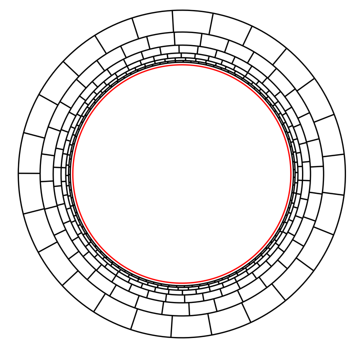
Intuition: Two paths II

Suppose we try to move the information on paths that go near the outside of the photon sphere. There are only a constant number of cells on the outside boundary of the photon sphere, meaning that it takes order M^2 time steps to move all the information through these cells, leading to order M^3 time total.



The argument

Now, suppose we divide the photon sphere of the black hole into two sections by a plane. We can consider the Hawking radiation as a channel carrying information from one side of the plane to the other. There are order M cells separating the top half and the bottom half. Quantum information theory says this can carry at most order M bits in a timestep of length M. Thus, to get order $A \approx M^2$ information from one side to the other, we need time order M^2



Question

Could the Hawking/Unruh radiation (which has $O(M)$ entropy according to our calculation) be the source of the microstates of black holes?

Objection: But the Hawking/Unruh radiation is virtual.

Counter-objection: Why does this matter?

How can information come out of black holes?

One possibility is that quantum fluctuations of the horizon let information that fell into black holes long ago come out.

In order for causality to be preserved, this information would have to come out essentially at the point where it entered.

Speculation

Maybe the strong scrambling time is really order M^2.

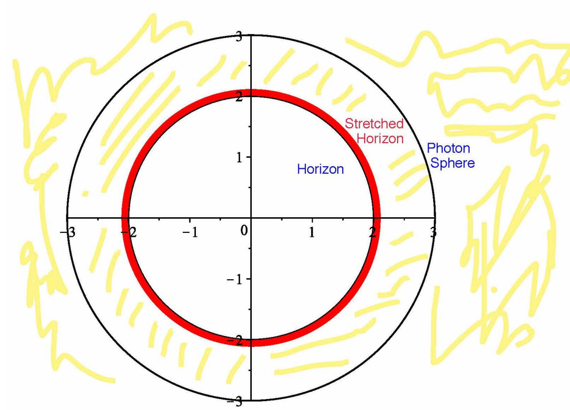
Maybe unitary breaks down for black holes, and the information doesn't come out.

Maybe there is new physics operating near the horizon.

Thank You

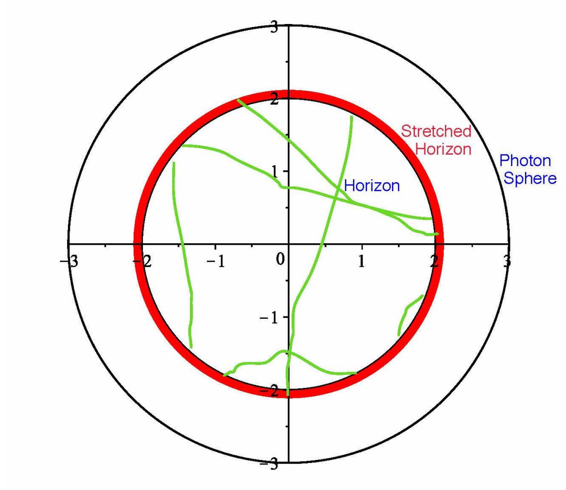
One Possibility

There is unknown physics going on everywhere; the universe is massively non-local (despite what it looks like). Possibly the only way of understanding it is to figure out what the CFT is in the AdS-CFT correspondence and use that to deduce the physics of our universe.



Another Possibility

The unknown physics is confined to the stretched horizon, and somehow information is being communicated from one part of it to another.



This seems to violate relativity

But maybe, from the viewpoint of an outside observer, space is connected strangely near the black hole horizon.

Thank You