

# Using chaos to characterize a programmable analog quantum simulator

**Adam Shaw**  
Endres Lab

**Mostly following:**

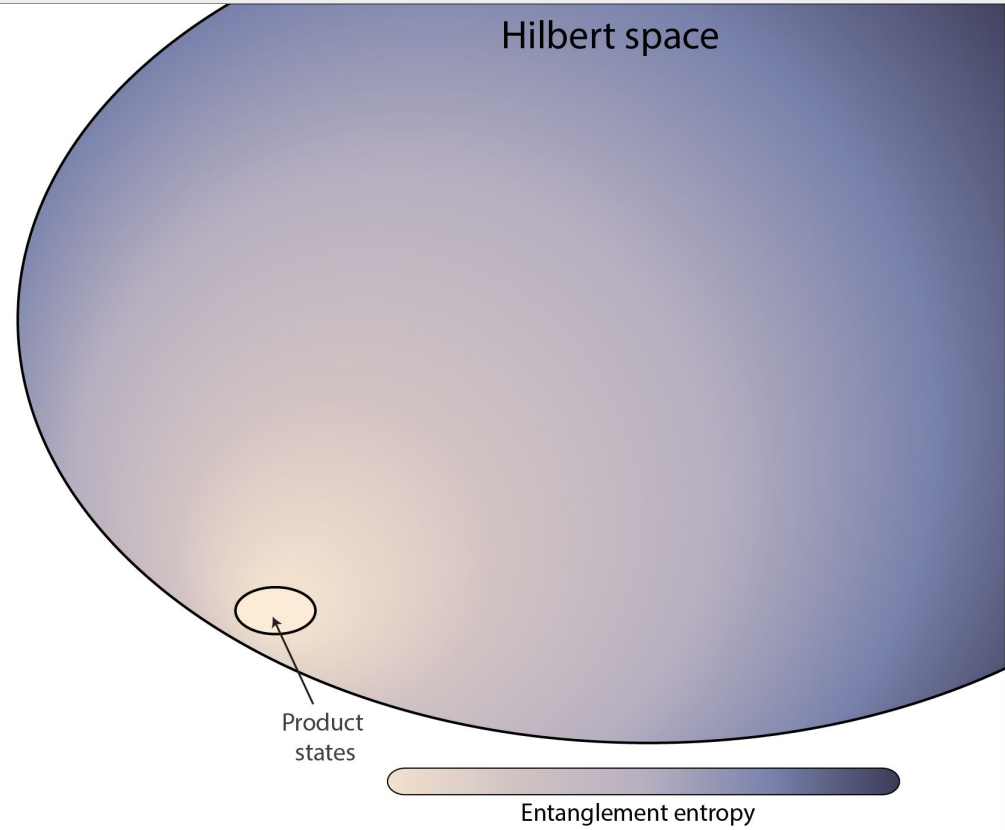
**Shaw\***, Chen\*, Choi\*, Mark\*, *et al*, Nature 628 (2024), arXiv:2308.07914

**Shaw\***, Mark\*, *et al*, arXiv:2403.11971

The Caltech logo is displayed in a large, bold, orange font. It consists of the word "Caltech" in a sans-serif typeface, with the "C" being significantly larger than the other letters.

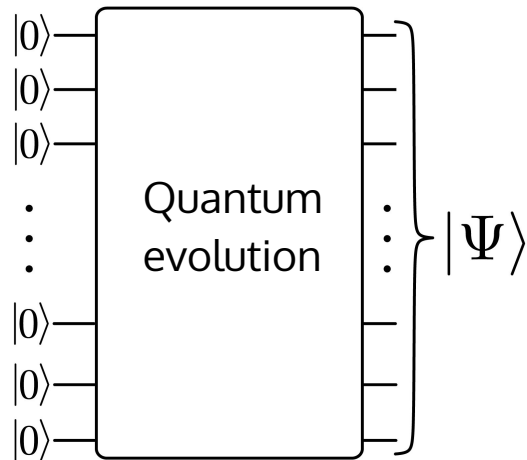
# The entanglement challenge

Start with some simple **initial** state...

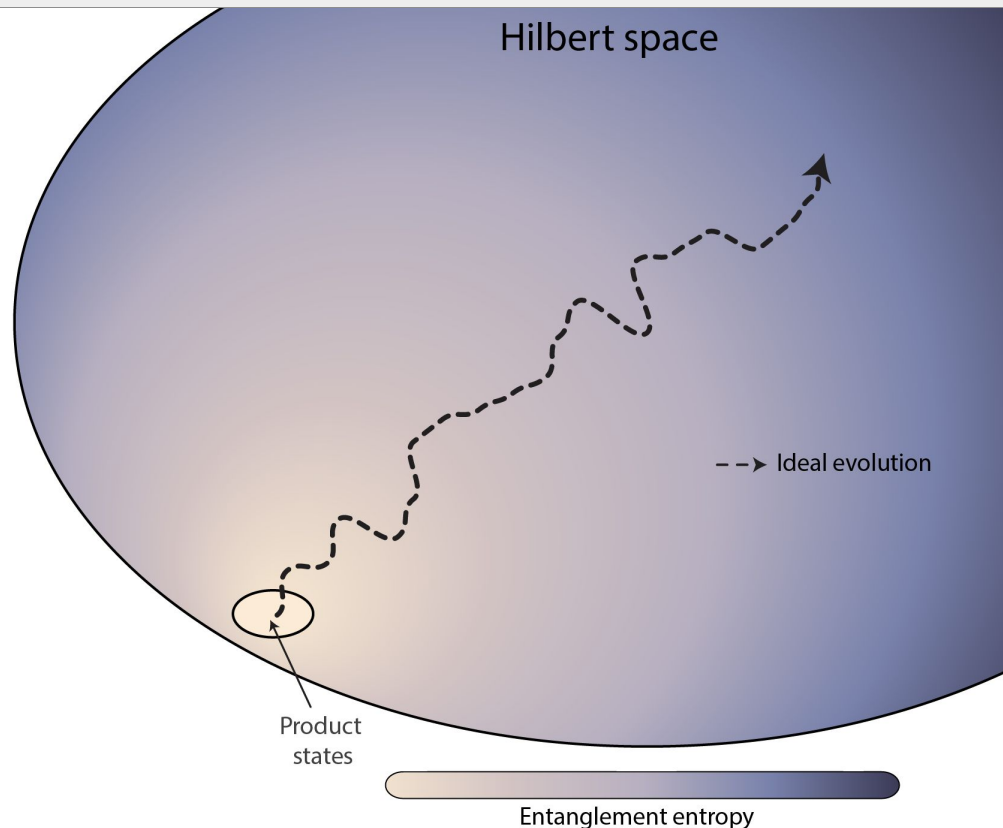


# The entanglement challenge

Start with some simple **initial** state...



... create a useful **entangled** state  
(as a resource for computation,  
simulation, metrology, etc...)



# The entanglement challenge

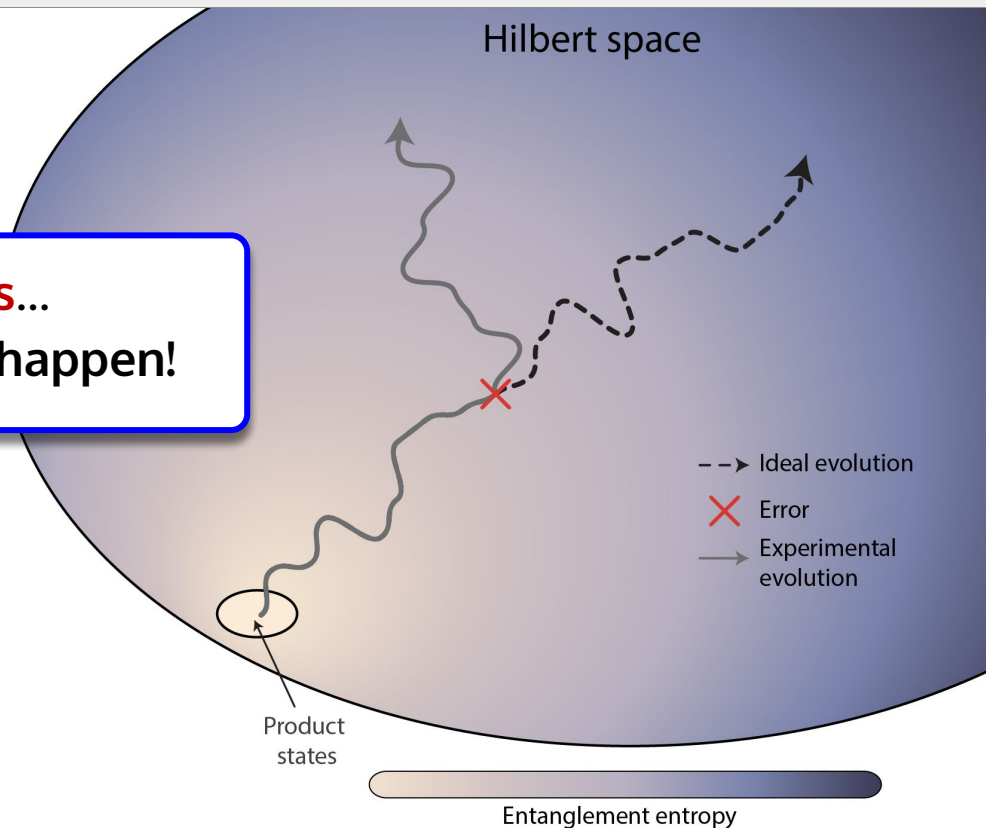
Start with some simple **initial** state...



But we are always prone to **errors**...  
And we can't even detect when they happen!



... create a useful **entangled** state  
(as a resource for computation,  
simulation, metrology, etc...)





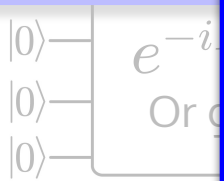
# The entanglement challenge

Start with some simple **initial** state...



But we are always prone to **errors**

And we can't even



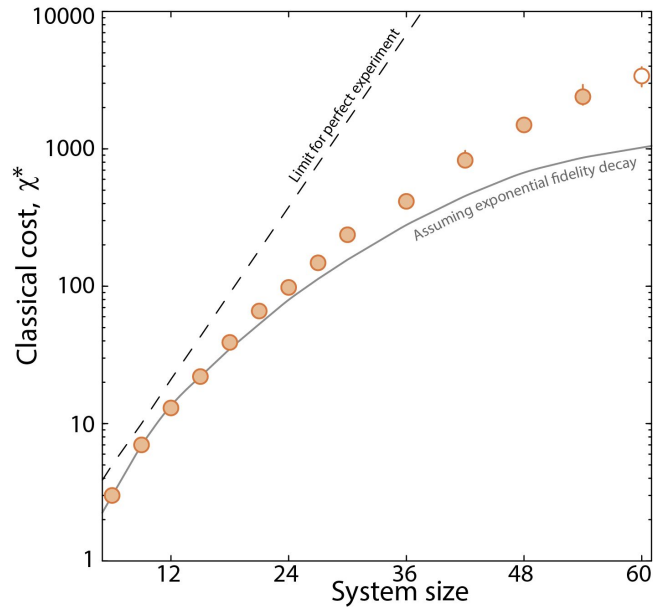
So how do we scale up quantum systems while **maintaining control**, **minimizing errors**, and **verifying** we've done the correct evolution?

... create a useful **entangled** state  
(as a resource for computation,  
simulation, metrology, etc...)



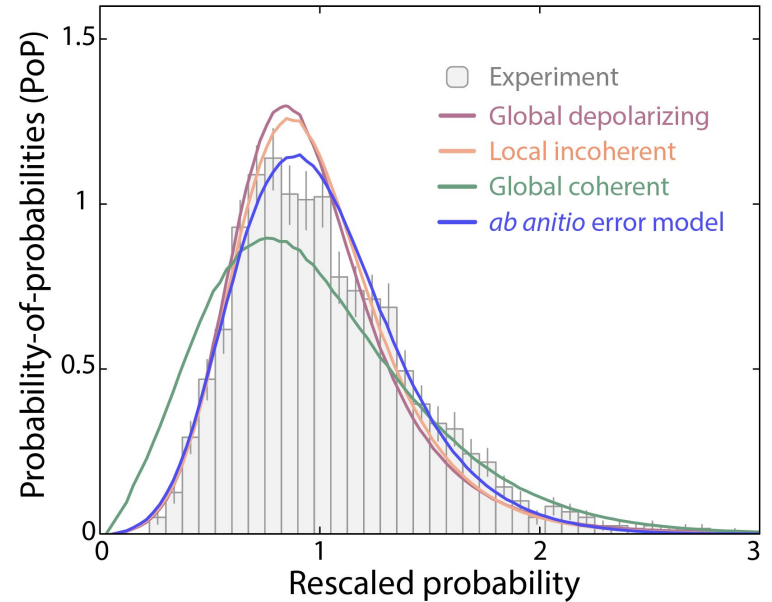
## Benchmarking

For a large scale analog quantum simulator



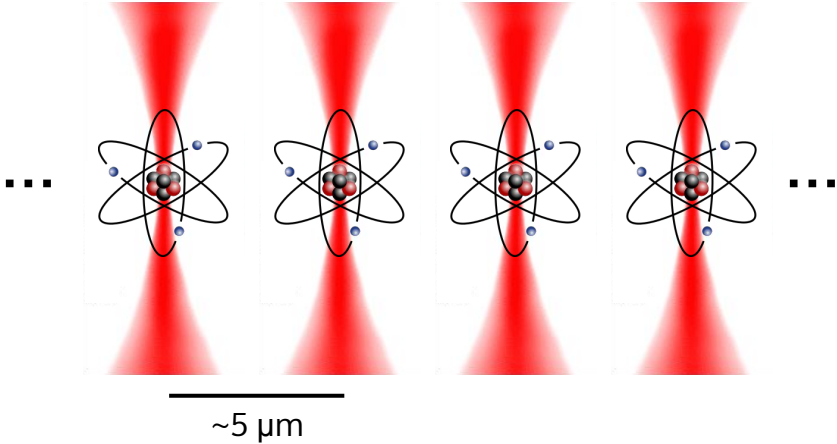
## Applications

Entanglement estimation, noise learning, etc

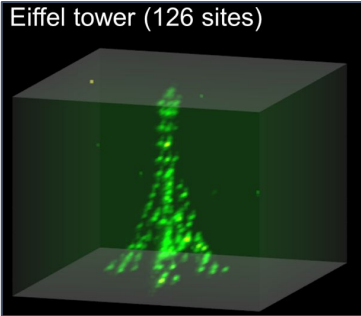
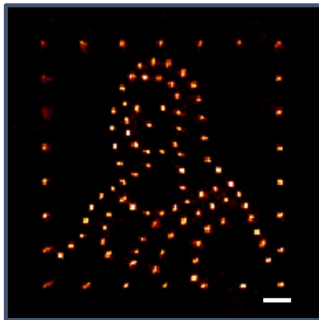
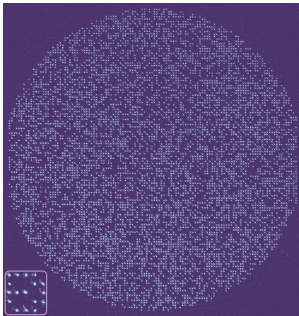


# Rydberg atom arrays

**Optical tweezers:**  
focused laser beams which can trap single atoms

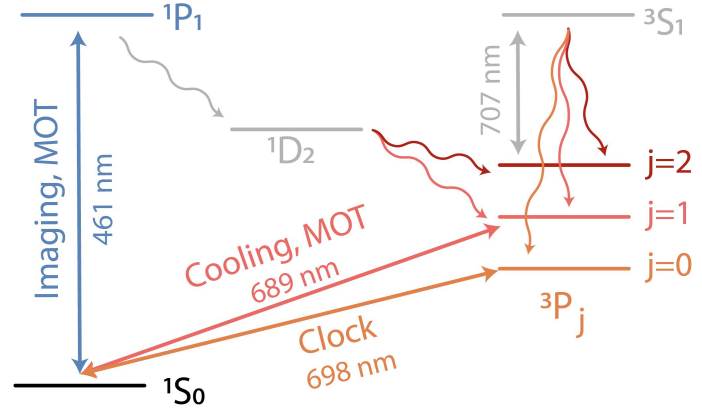


Can create large arrays in multiple dimensions with full positional control ("rearrangement")



# Rydberg atom arrays

1 <b>H</b> Hydrogen 1.008	
3 <b>Li</b> Lithium 6.941	4 <b>Be</b> Beryllium 9.012
11 <b>Na</b> Sodium 22.990	12 <b>Mg</b> Magnesium 24.305
19 <b>K</b> Potassium 39.098	20 <b>Ca</b> Calcium 40.078
37 <b>Rb</b> Rubidium 84.468	38 <b>Sr</b> Strontium 87.62

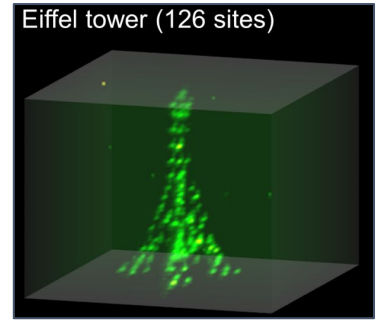
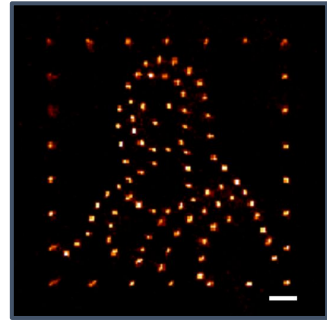
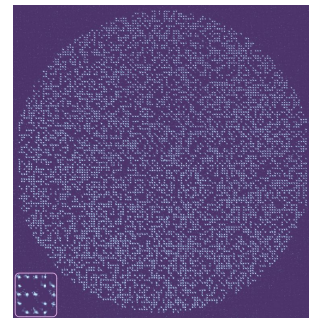


In our experiment we work with arrays of strontium atoms

Lots of interesting atomic physics to discuss...

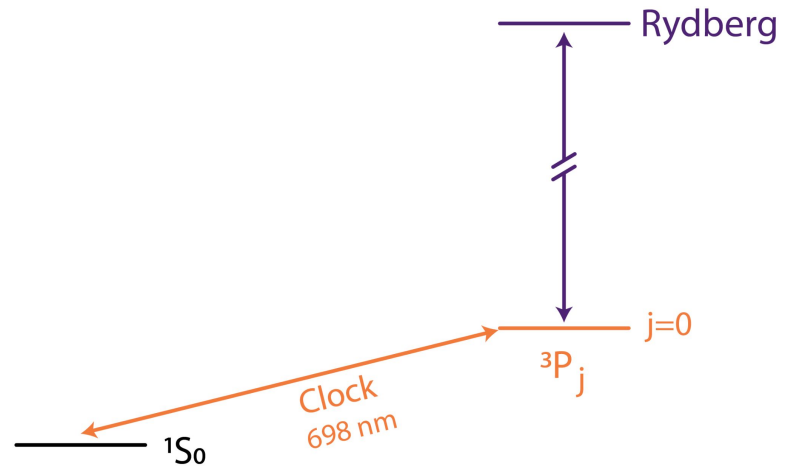


Can create large arrays in multiple dimensions with full positional control ("rearrangement")



# Rydberg atom arrays

1 <b>H</b> Hydrogen 1.008	
3 <b>Li</b> Lithium 6.941	4 <b>Be</b> Beryllium 9.012
11 <b>Na</b> Sodium 22.990	12 <b>Mg</b> Magnesium 24.305
19 <b>K</b> Potassium 39.098	20 <b>Ca</b> Calcium 40.078
37 <b>Rb</b> Rubidium 84.468	<b>38 Sr</b> Strontium 87.62

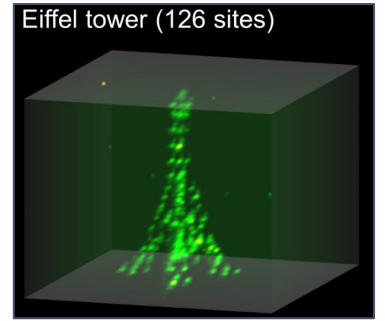
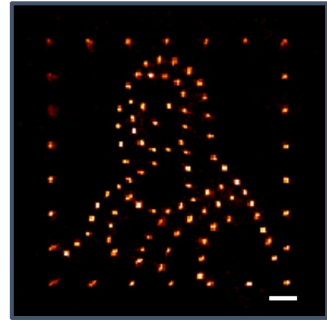
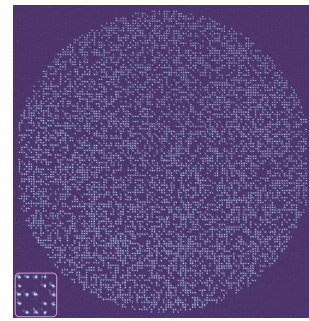


In our experiment we work with arrays of strontium atoms

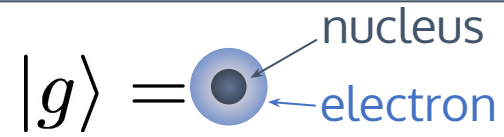
Lots of interesting atomic physics to discuss...

**But not for this talk!**

Can create large arrays in multiple dimensions with full positional control ("rearrangement")

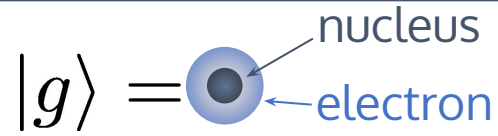


# Making atoms interact



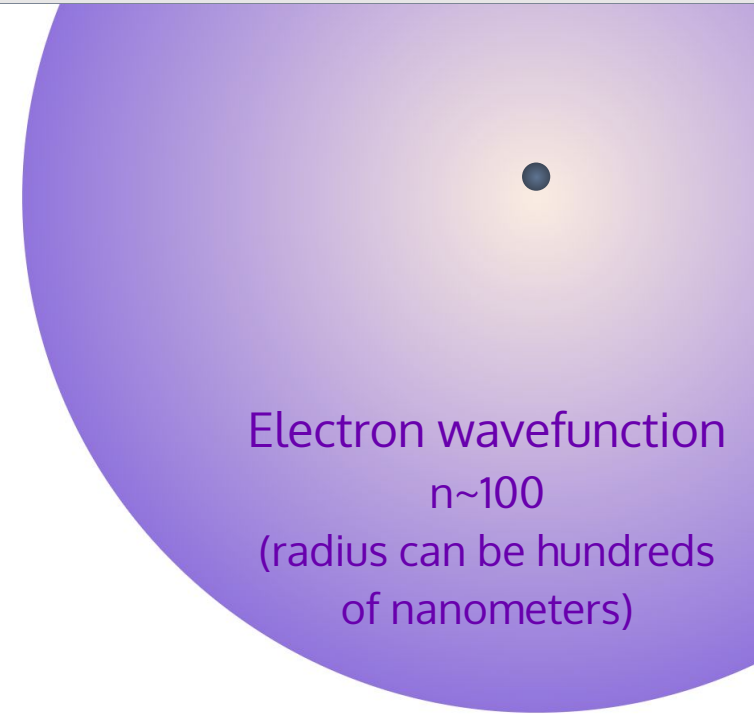
Atom in the ground state

# Making atoms interact



Atom in the ground state

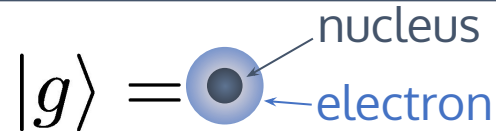
$$|r\rangle =$$



Electron wavefunction  
 $n \sim 100$   
(radius can be hundreds  
of nanometers)

Rydberg atom

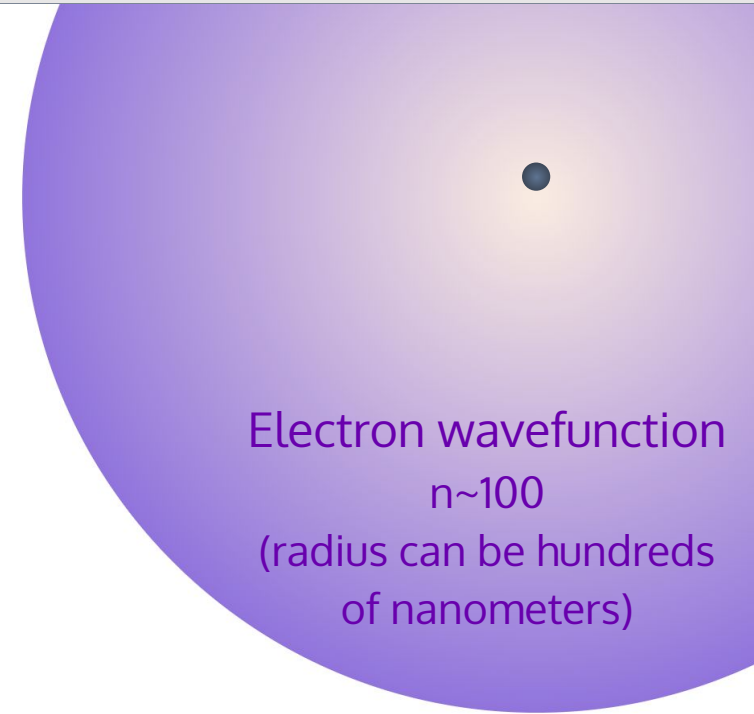
# Making atoms interact



Atom in the ground state

$$|r\rangle =$$

Try exciting two atoms...

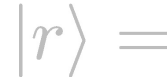
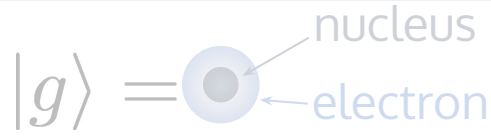


Electron wavefunction  
 $n \sim 100$   
(radius can be hundreds  
of nanometers)

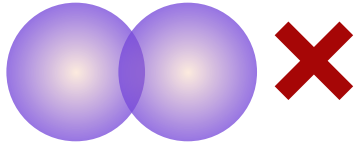
Rydberg atom



# Making atoms interact

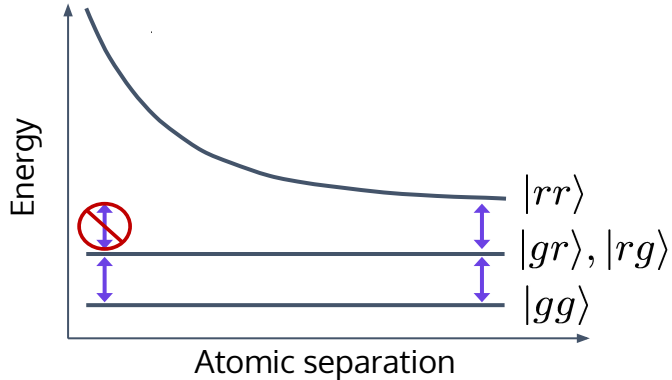


Small separations\*:  
Double excitation is **blocked**

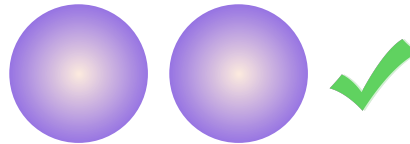


\*Visual is exaggerated

Try exciting two atoms...



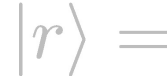
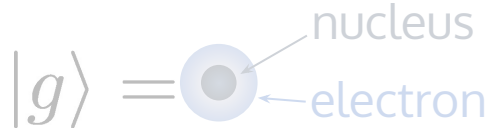
Large separations:  
Both atoms can be excited



Electron wavefunction  
 $n \sim 100$   
(radius can be hundreds  
of nanometers)

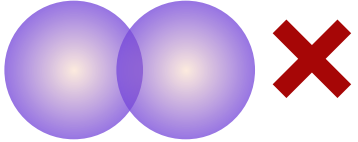
Rydberg atom

# Making atoms interact



state

Small separations\*:  
Double excitation is **blocked**

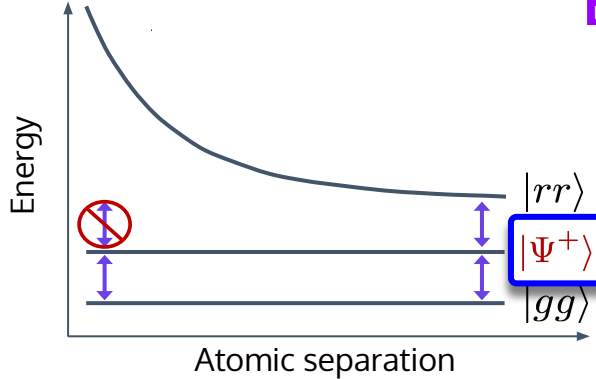


\*Visual is exaggerated

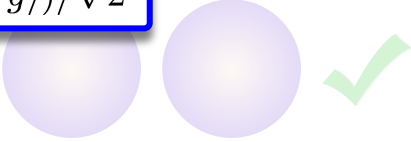
Try exciting two atoms...

One excitation becomes **shared** across both atoms...

**Entanglement!**



Large separations.  
Both atoms can be excited

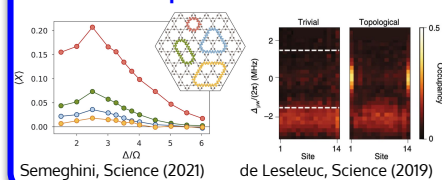


Electron wavefunction  
 $n \sim 100$   
(radius can be hundreds of nanometers)

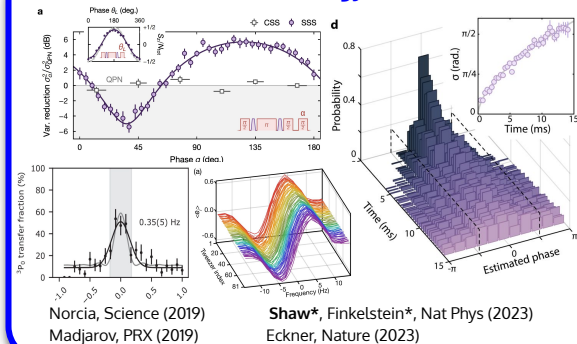
Rydberg atom

# Physics with atom arrays, a small selection (pre 2024)

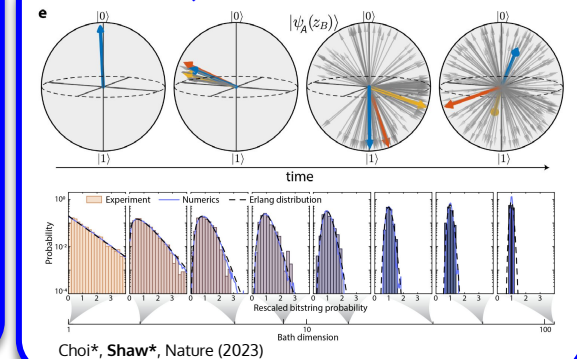
## New phases of matter



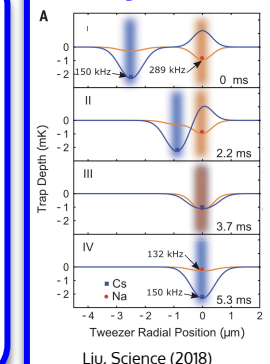
## Metrology



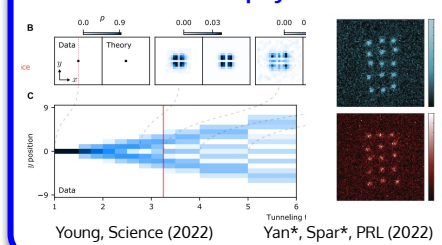
## Quantum randomness



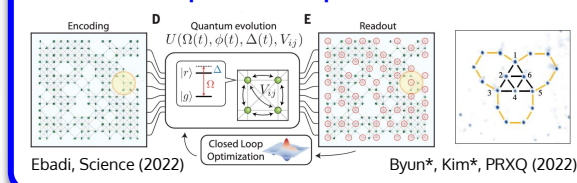
## Making molecules



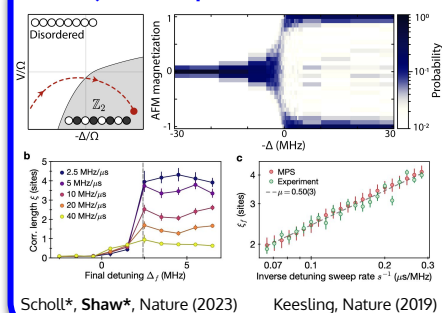
## Hubbard physics



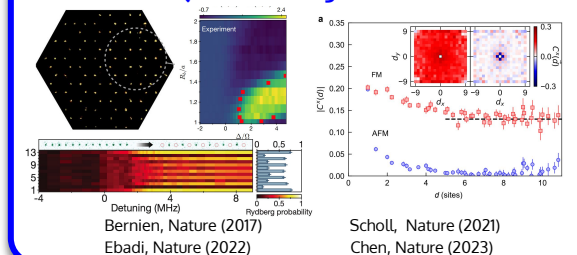
## Optimization problems



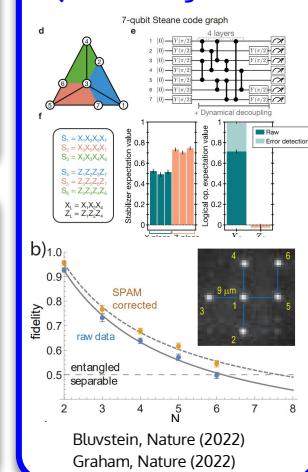
## Quantum phase transitions



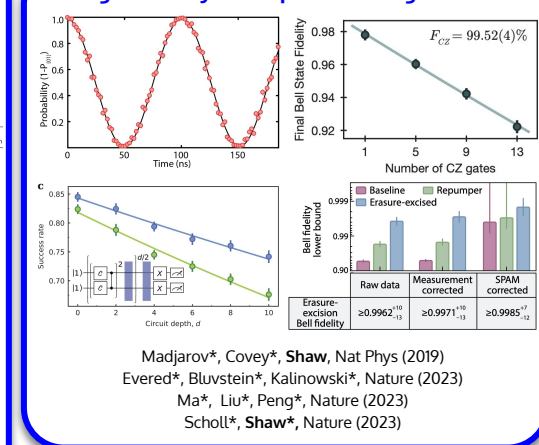
## Quantum magnetism



## Quantum algorithms

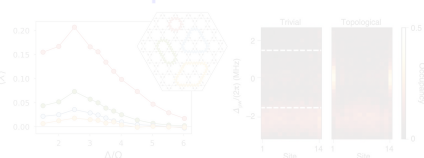


## High-fidelity two-qubit entanglement

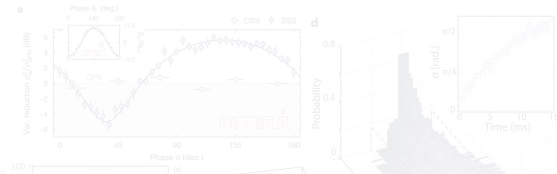


# Physics with atom arrays, a small selection (pre 2024)

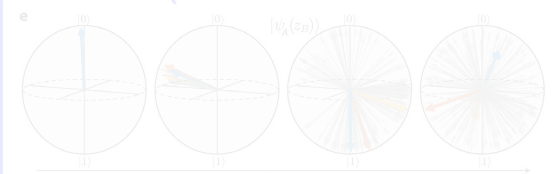
## New phases of matter



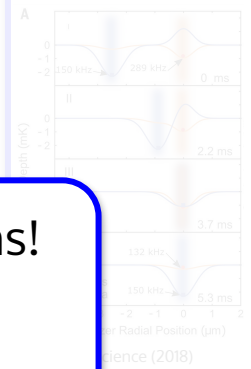
## Metrology



## Quantum randomness



## Making molecules



Both **quantum simulation** and **quantum computation** applications!

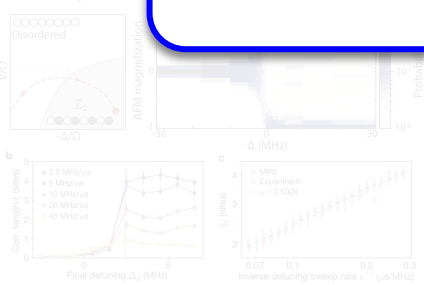
**Analog:**

Simulating complex quantum dynamics by mapping to the natural system evolution

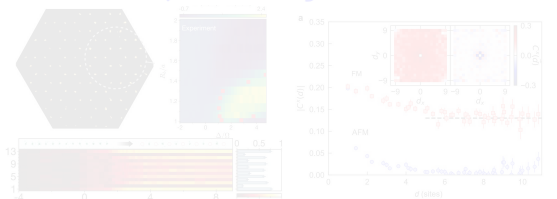
**Digital:**

Solving computational problems using a discrete and universal set of quantum operations

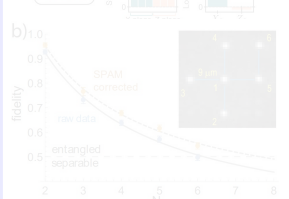
## Quantum magnetism



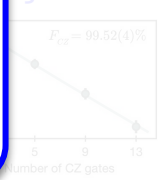
## Quantum magnetism



## Quantum computation



## Entanglement



Madjarov\*, Covey\*, Shaw, Nat Phys (2019)  
Evered\*, Bluvstein\*, Kalinowski\*, Nature (2023)  
Ma\*, Liu\*, Peng\*, Nature (2023)  
Scholl\*, Shaw\*, Nature (2023)

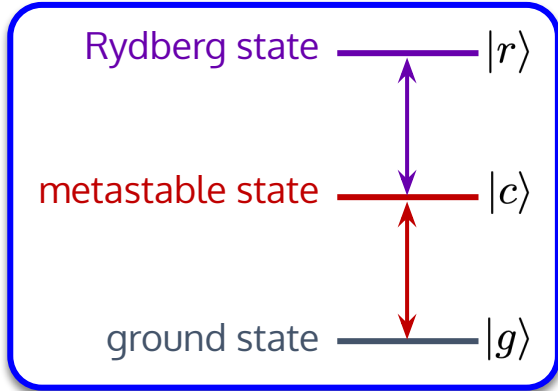
Scholl\*, Shaw\*, Nature (2023) Keesling, Nature (2019)

Bernien, Nature (2017) Ebadi, Nature (2022) Scholl, Nature (2021) Chen, Nature (2023)

Bluvstein, Nature (2022) Graham, Nature (2022)

# A tale of two qubits

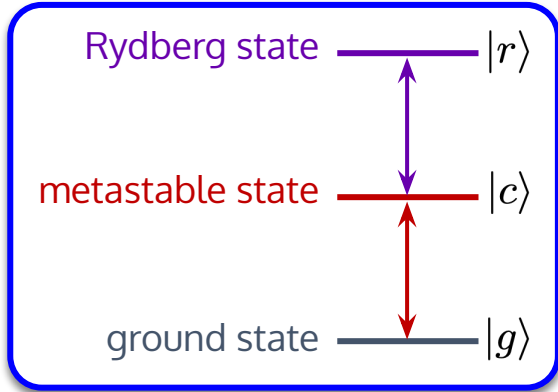
Two different  
choices of qubit...



Approximate energy levels of  
strontium, the atom we use

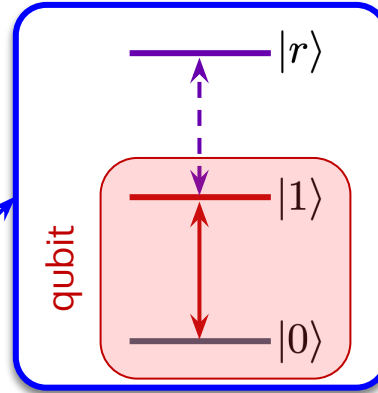
# A tale of two qubits

Two different choices of qubit...



Approximate energy levels of strontium, the atom we use

Digital

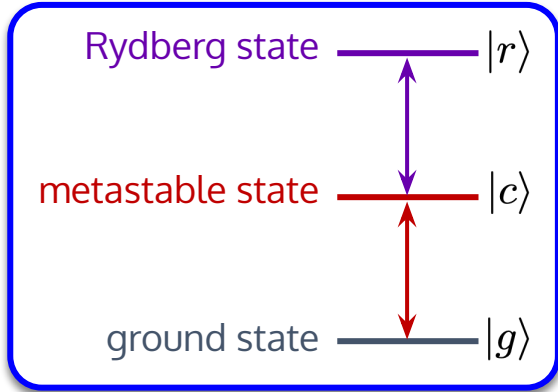


**Digital:**

Long-lived qubit, amenable to gate-based operation, Rydberg state is only excited transiently

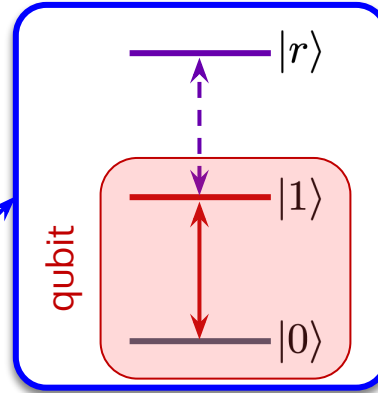
# A tale of two qubits

Two different choices of qubit...



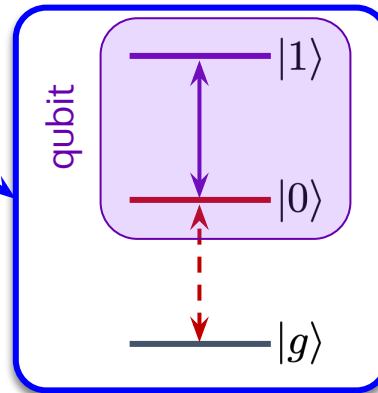
Approximate energy levels of strontium, the atom we use

Digital



**Digital:**  
Long-lived qubit, amenable to gate-based operation, Rydberg state is only excited transiently

Analog

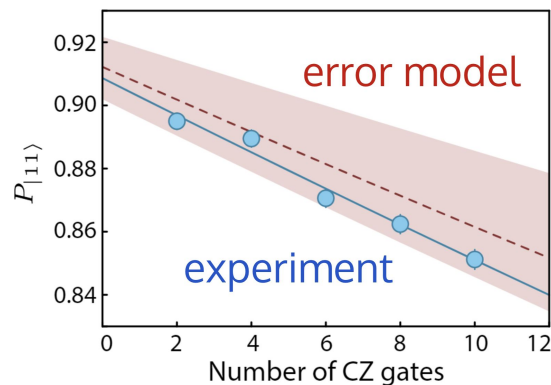


**Analog:**  
Strongly interacting spin system with an Ising-like Hamiltonian which exhibits critical and high-entanglement behavior

# A tale of two qubits

Digital qubit: CZ gate fidelity  
(measured with RB) of **0.9973(4)\***

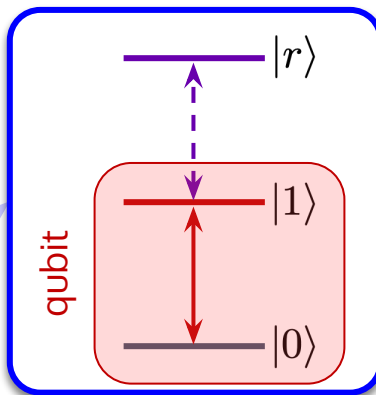
\*unpublished, published value is 0.9935



Finkelstein\*, Tsai\*,..., **Shaw**, Endres,  
arXiv:2402.16220 (2024)

Other gate results from: Saffman, Lukin, Thompson,  
Kaufman, Bernien, Zhan, and more

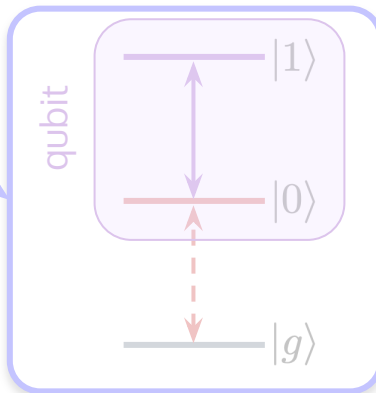
Digital



**Digital:**

Long-lived qubit, amenable to  
gate-based operation, Rydberg  
state is only excited transiently

Analog



**Analog:**

Strongly interacting spin system  
with an Ising-like Hamiltonian  
which exhibits critical and  
high-entanglement behavior



# A tale of two qubits

Analog qubit: Bell state fidelity of  $\sim 0.9992$  (with erasure conversion\*)

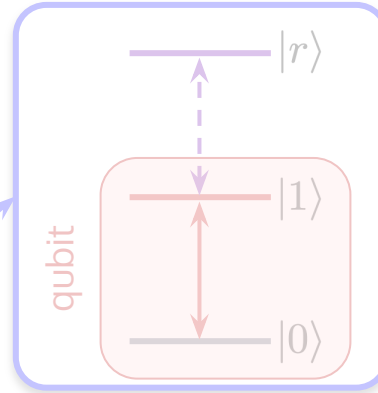


Detailed error modeling shows that  $\sim 0.9999$  is experimentally realistic

Scholl\*, Shaw\*, et al., Nature 622 (2023)

\*for erasure conversion, see also  
theory: Wu, ..., Thompson, Nat Comm (2022)  
experiment: Ma, ..., Thompson, Nature 622 (2023)

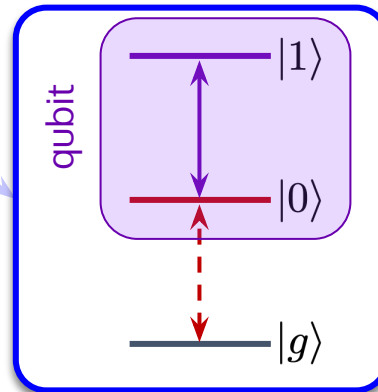
Digital



**Digital:**

Long-lived qubit, amenable to gate-based operation, Rydberg state is only excited transiently

Analog

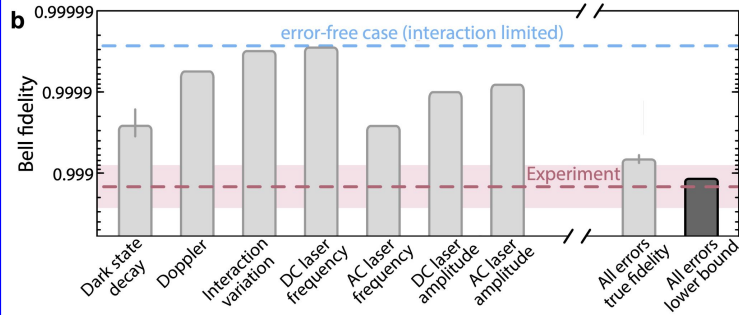


**Analog:**

Strongly interacting spin system with an Ising-like Hamiltonian which exhibits critical and high-entanglement behavior

# A tale of two qubits

Analog qubit: Bell state fidelity of  $\sim 0.9992$  (with erasure conversion\*)



Detailed error modeling shows that  $\sim 0.9999$  is experimentally realistic

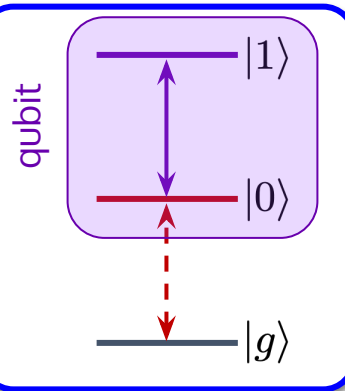
Scholl\*, Shaw\*, et al., Nature 622 (2023)

\*for erasure conversion, see also  
theory: Wu, ..., Thompson, Nat Comm (2022)  
experiment: Ma, ..., Thompson, Nature 622 (2023)

Digital

But what about larger systems?

Analog



**Analog:**

Strongly interacting spin system with an Ising-like Hamiltonian which exhibits critical and high-entanglement behavior

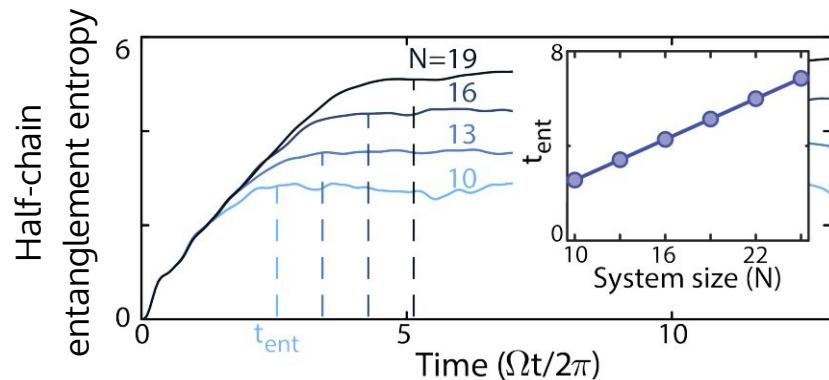
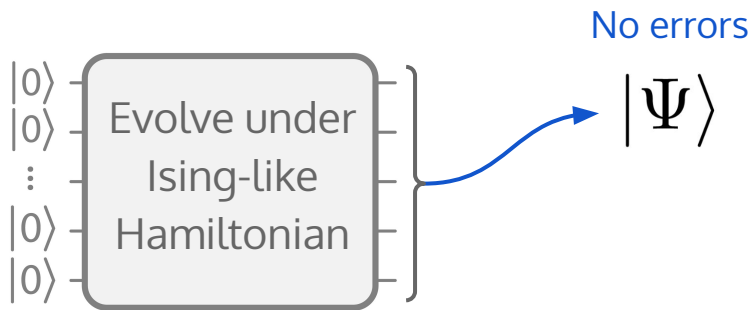
# Fidelity benchmarking

One-dimensional array of up to 60 atoms



# Fidelity benchmarking

One-dimensional array of up to 60 atoms

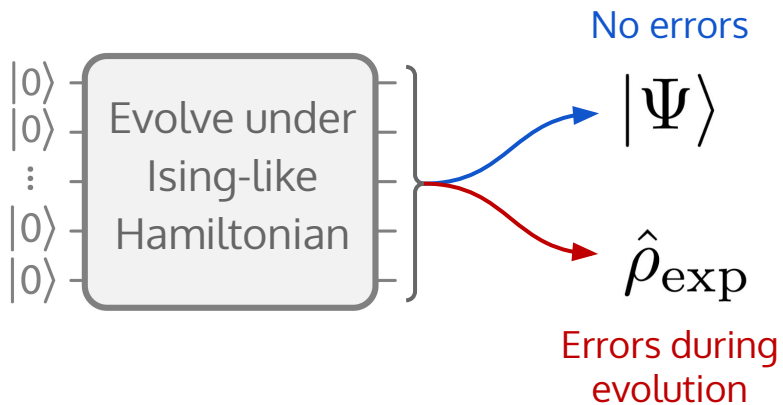


Entanglement entropy grows linearly with a system-size independent rate, but saturates at system size dependent time/level

$$\hat{H}/\hbar = \Omega \sum_i \hat{S}_i^x - \Delta \sum_i \hat{n}_i + \frac{C_6}{a^6} \sum_{i>j} \frac{\hat{n}_i \hat{n}_j}{|i-j|^6}$$

# Fidelity benchmarking

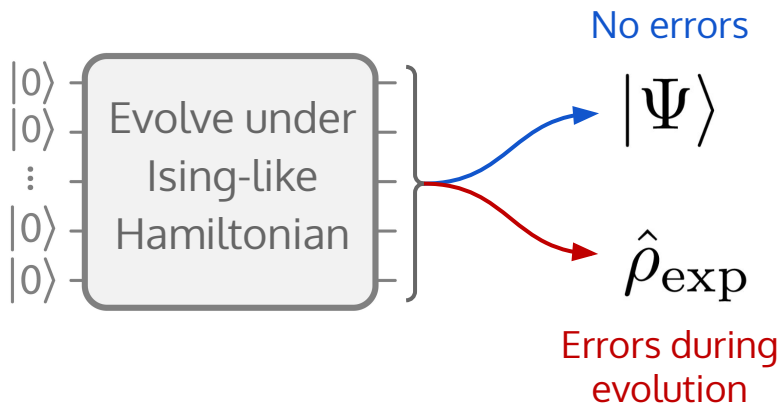
One-dimensional array of up to 60 atoms



$$\hat{H}/\hbar = \Omega \sum_i \hat{S}_i^x - \Delta \sum_i \hat{n}_i + \frac{C_6}{a^6} \sum_{i>j} \frac{\hat{n}_i \hat{n}_j}{|i-j|^6}$$

# Fidelity benchmarking

One-dimensional array of up to 60 atoms



$$\text{Fidelity: } F = \langle \Psi | \hat{\rho}_{\text{exp}} | \Psi \rangle$$

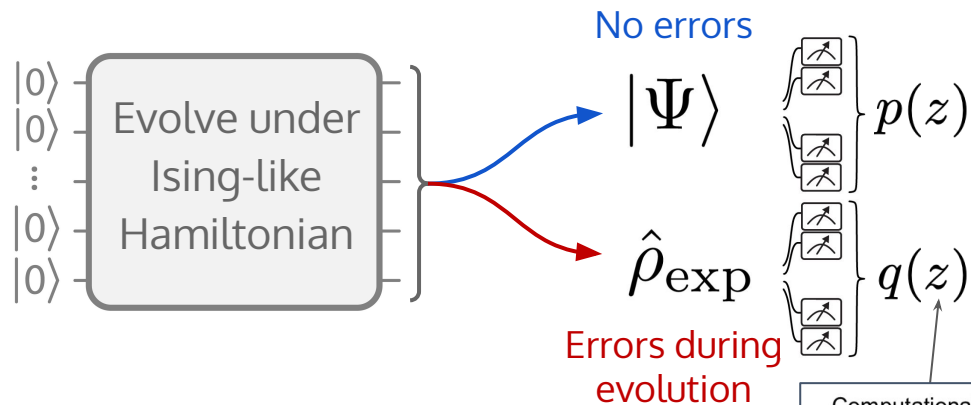
Fidelity is the probability  
that we don't make an error



Exponentially difficult to  
measure for large systems!

# Fidelity benchmarking

One-dimensional array of up to 60 atoms



$$\text{Fidelity: } F = \langle \Psi | \hat{\rho}_{\text{exp}} | \Psi \rangle$$

Fidelity is the probability that we don't make an error

Computational basis  
(Bitstring of length N)

$z_1 = |000 \cdots 000\rangle$

$z_2 = |000 \cdots 001\rangle$

$\vdots$

$z_D = |111 \cdots 111\rangle$

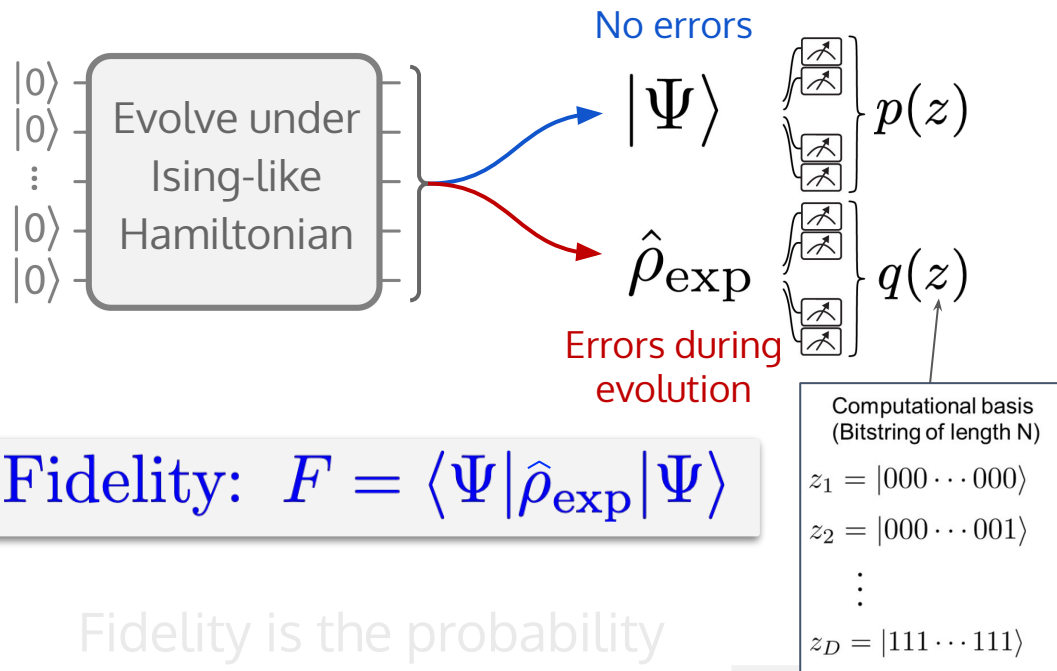
Exponentially difficult to measure for large systems!

If the dynamics are explicitly randomized, **you can estimate fidelity by measuring  $q(z)$** , the experimental bitstring probability distribution in a fixed basis

Arute et al, Nature (2019)

# Fidelity benchmarking

One-dimensional array of up to 60 atoms



$$\text{Fidelity: } F = \langle \Psi | \hat{\rho}_{\text{exp}} | \Psi \rangle$$

Fidelity is the probability that we don't make an error

If the dynamics are explicitly randomized, **you can estimate fidelity by measuring  $q(z)$** , the experimental bitstring probability distribution in a fixed basis

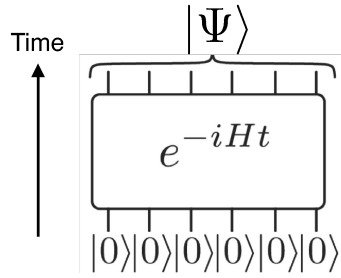
Arute *et al*, Nature (2019)

**We showed this also holds for time-independent Hamiltonian systems!**

Choi\*, Shaw\* *et al*, Nature (2023)  
Mark, Choi, Shaw *et al*, PRL (2023)  
Cotler, ...Shaw, *et al*, PRXQ (2023)



# Fidelities from bitstrings

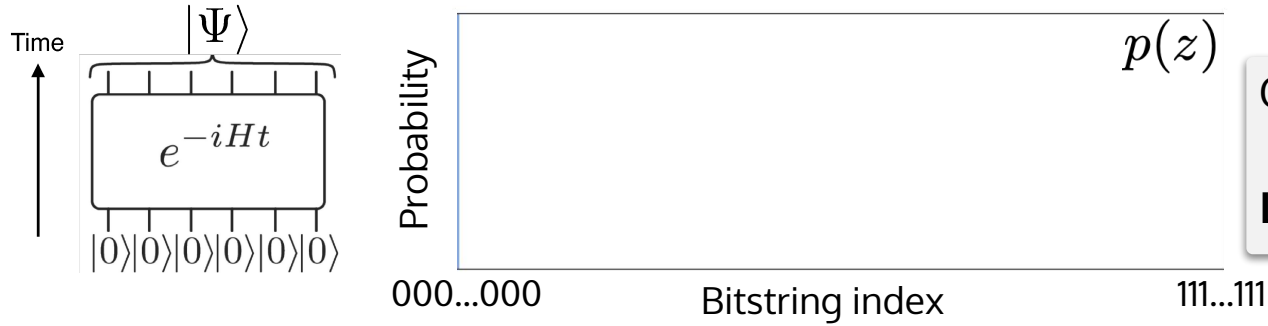


Choi\*, **Shaw\*** *et al*, Nature (2023)

Mark, Choi, **Shaw** *et al*, Phys Rev Lett (2023)

Also see: Arute *et al*, Nature (2019)

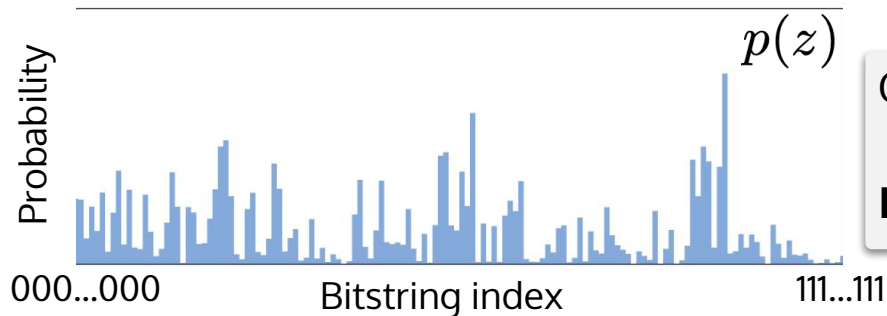
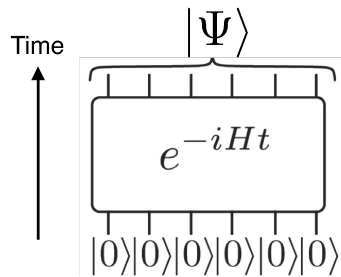
# Fidelities from bitstrings



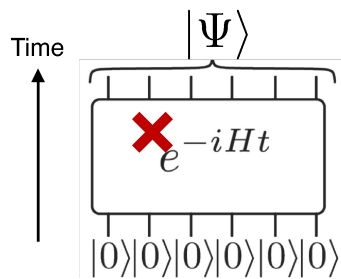
Quantum dynamics maps to near-random distribution...  
**Like a quantum fingerprint**

**Very sensitive to the exact initial state and dynamics!**

# Fidelities from bitstrings

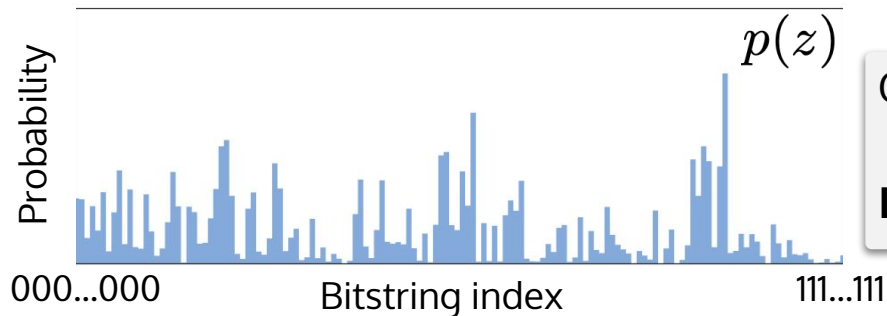
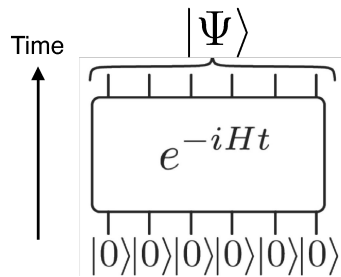


Quantum dynamics maps to near-random distribution...  
**Like a quantum fingerprint**

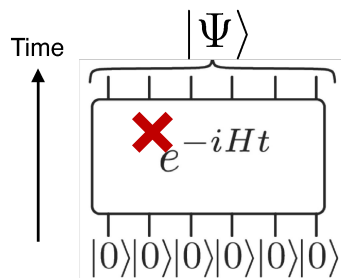


Impose a phase flip error on a single qubit: **initially not visible!**  
(because error is orthogonal to measurement basis)

# Fidelities from bitstrings

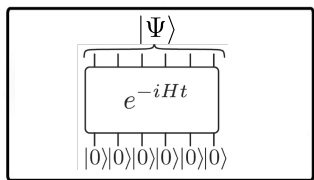


Quantum dynamics maps to near-random distribution...  
**Like a quantum fingerprint**

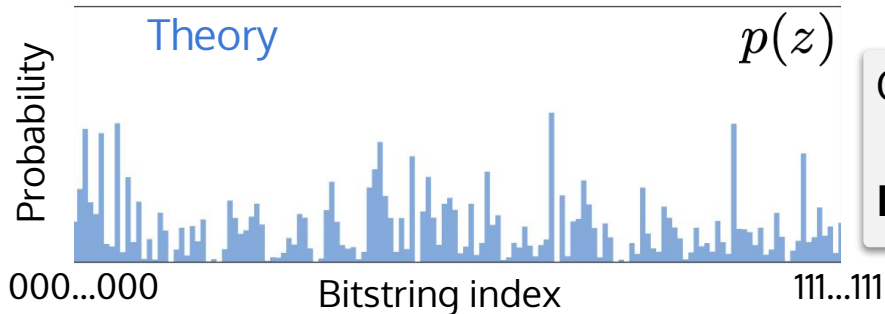


But error soon **scrambles**,  
changing the distribution!  
"Butterfly effect"

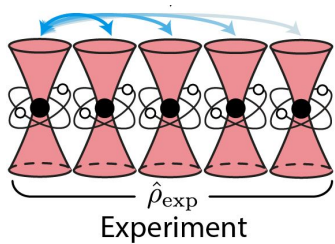
# Fidelities from bitstrings



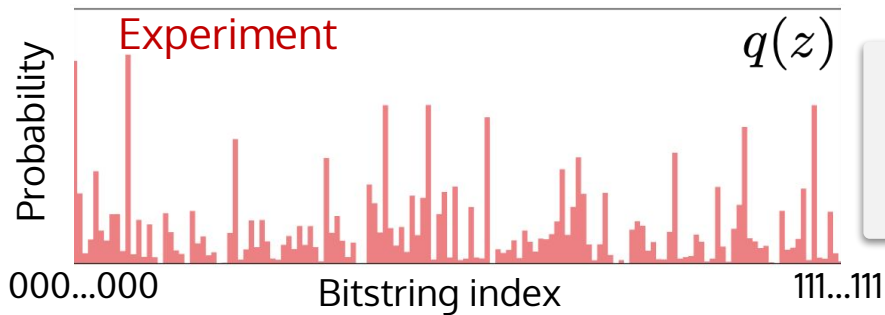
Classical simulation



Quantum dynamics maps to near-random distribution...  
**Like a quantum fingerprint**



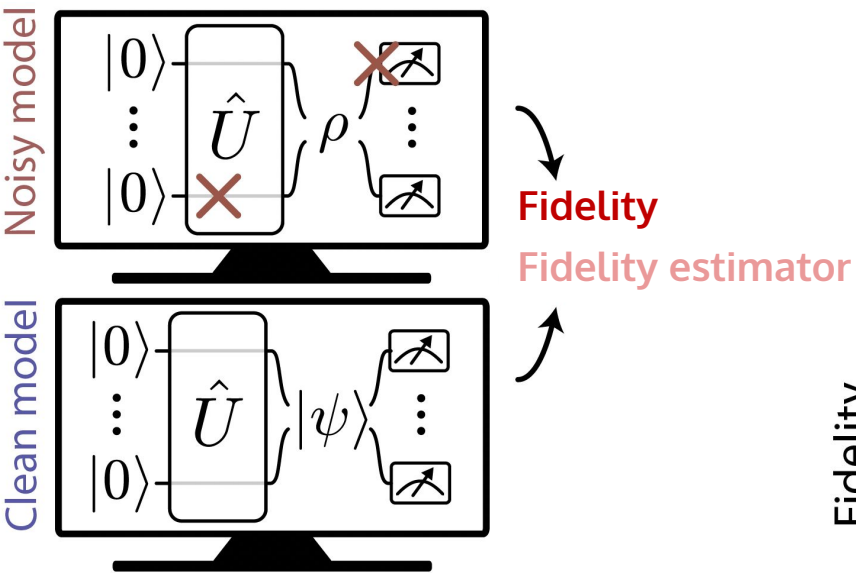
Experiment



But error soon **scrambles**,  
changing the distribution!  
"Butterfly effect"

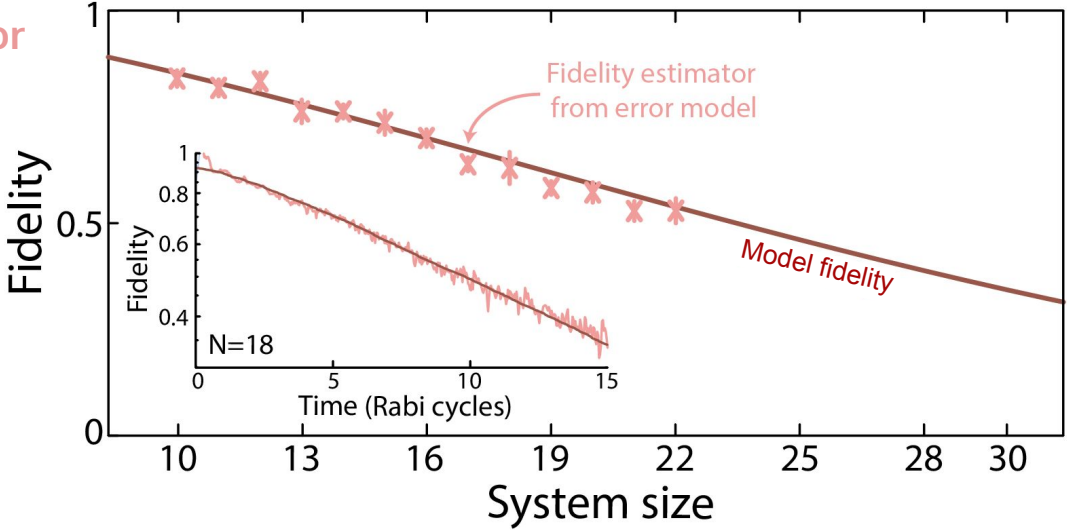
**Fidelity estimator** proportional to **Theory-Experiment** correlation!

# Demonstration of benchmarking with experiment



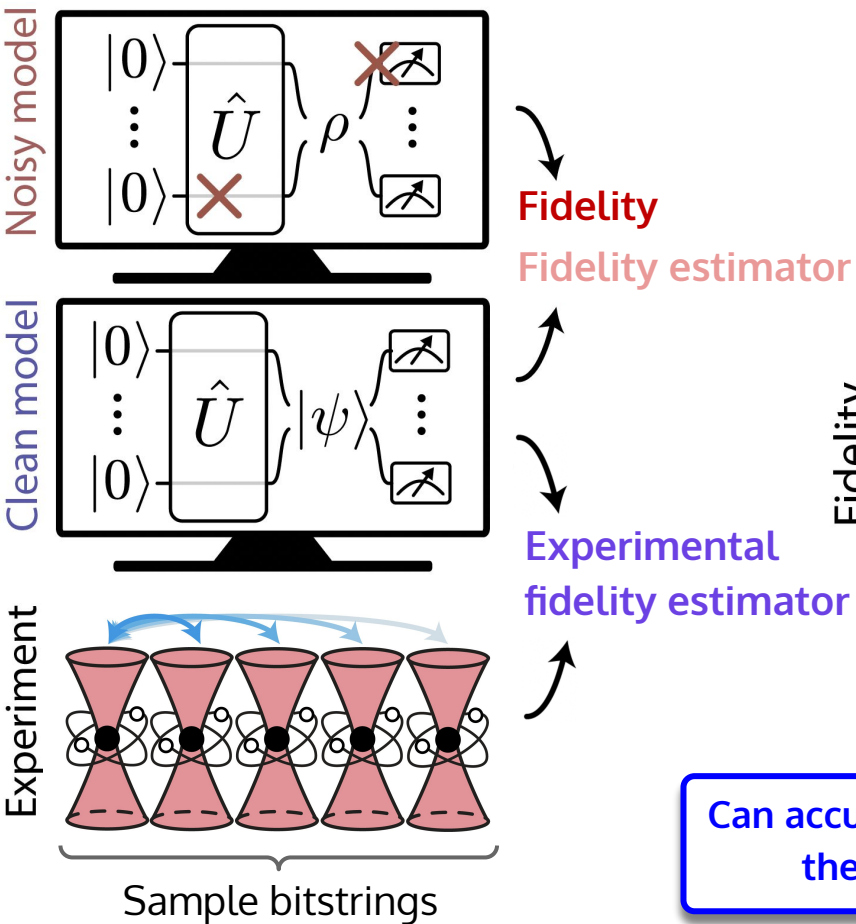
First, use noisy simulation to verify fidelity estimator is accurate to the model fidelity

Creating maximum entanglement entropy states

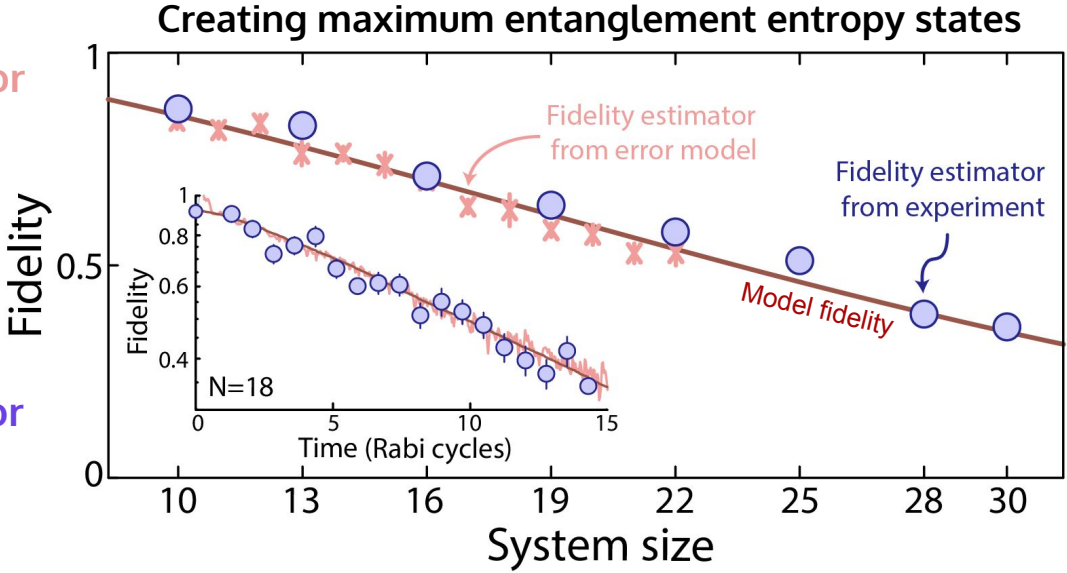


Choi\*, Shaw\* et al, Nature (2023)  
Mark, Choi, Shaw et al, PRL (2023)

# Demonstration of benchmarking with experiment



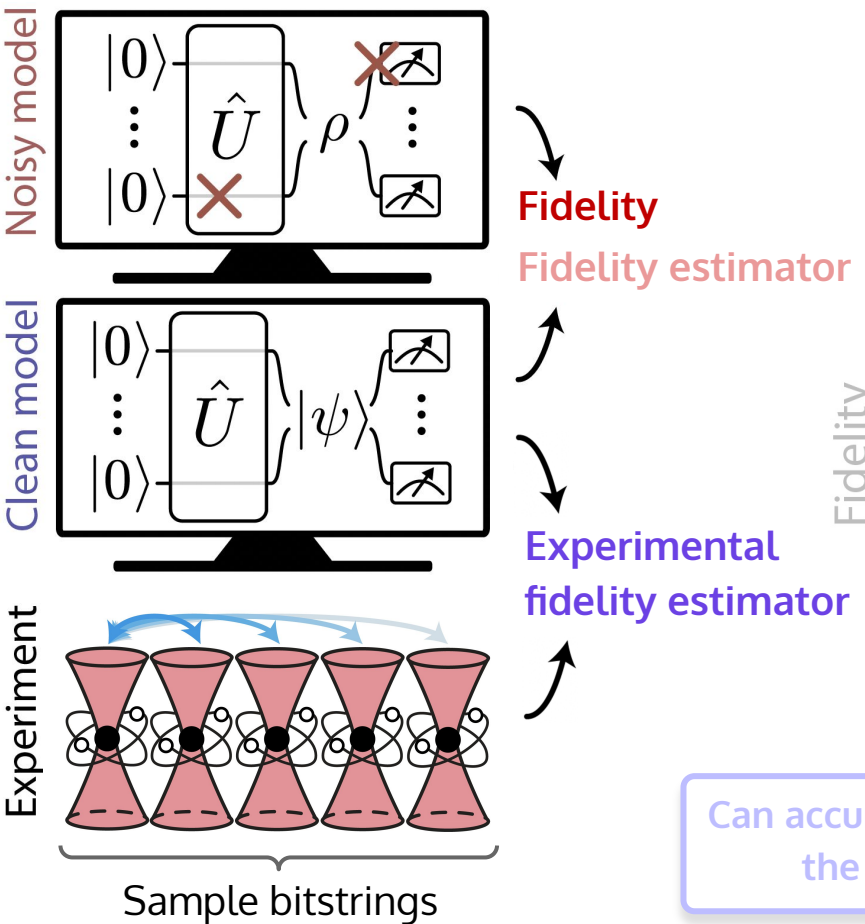
First, use noisy simulation to verify fidelity estimator is accurate to the model fidelity



**Can accurately benchmark the experiment!**

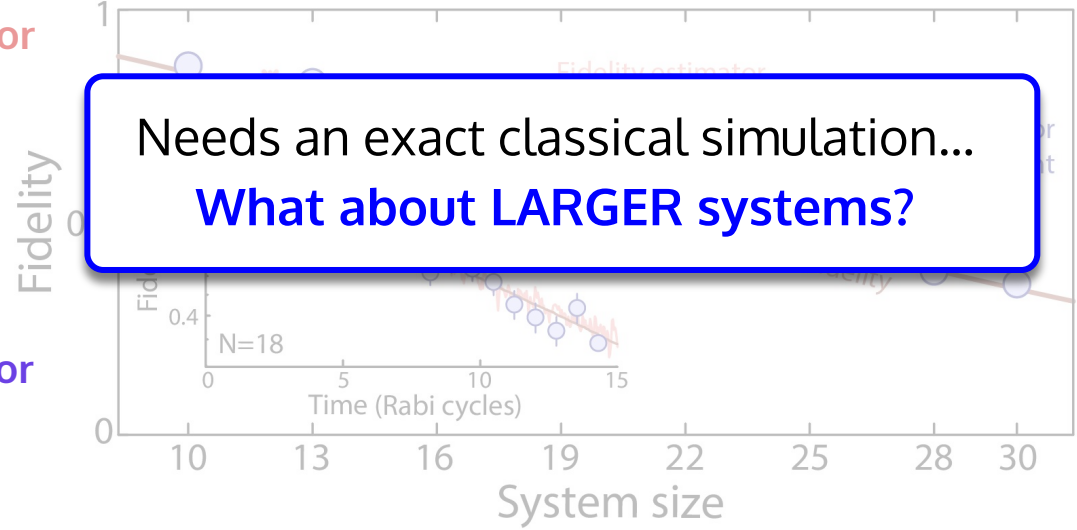
Choi\*, Shaw\* et al, Nature (2023)  
 Mark, Choi, Shaw et al, PRL (2023)

# Demonstration of benchmarking with experiment



First, use noisy simulation to verify fidelity estimator is accurate to the model fidelity

Creating maximum entanglement entropy states

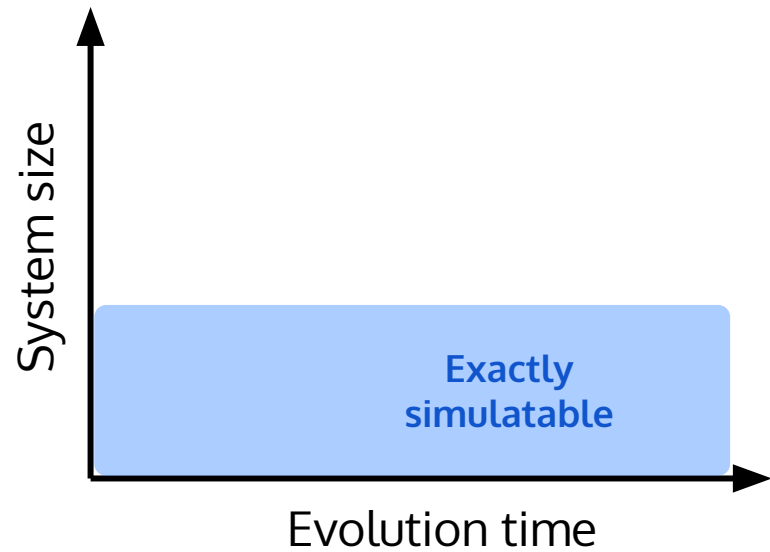


Can accurately benchmark the experiment!

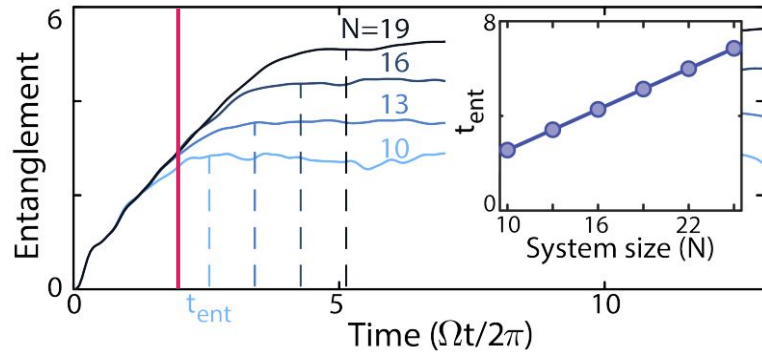
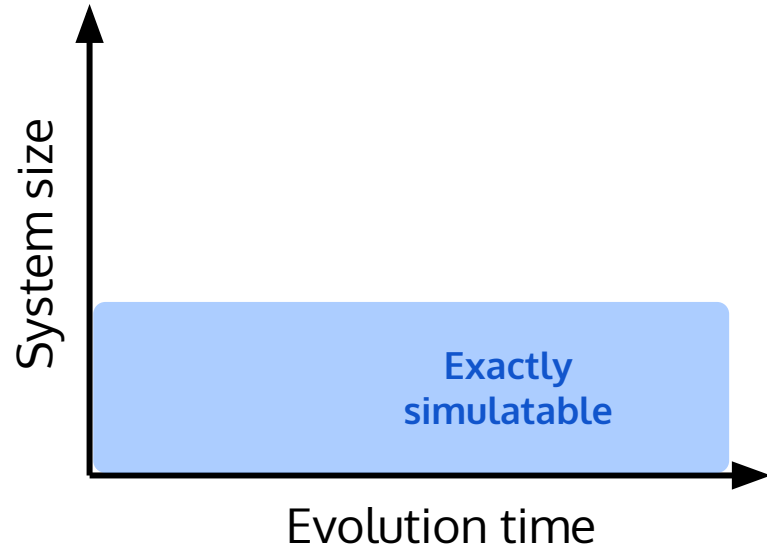
Choi\*, Shaw\* et al, Nature (2023)  
Mark, Choi, Shaw et al, PRL (2023)



# What makes classical simulation hard?

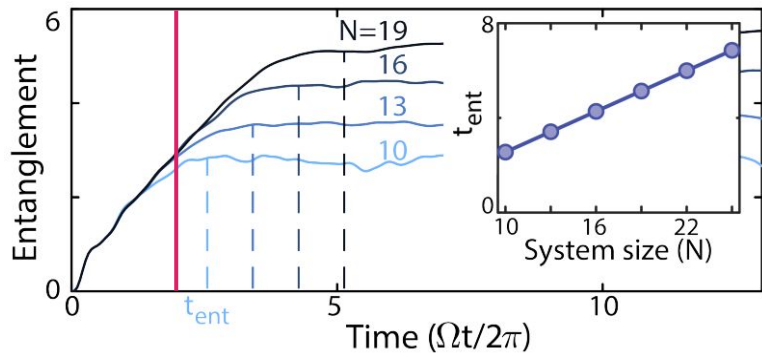
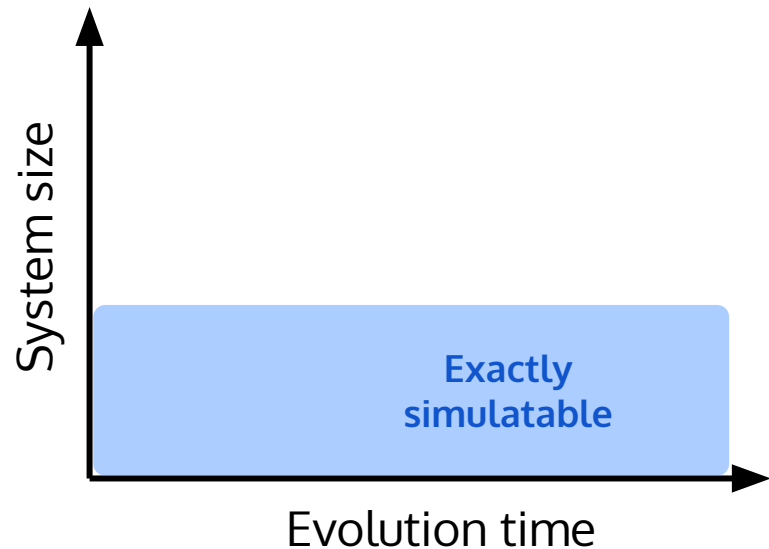


# What makes classical simulation hard?

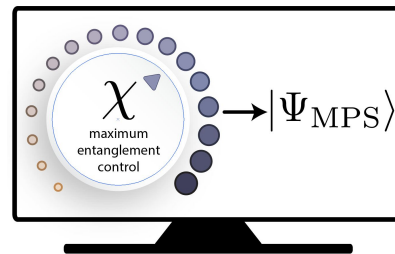


At **early times** the system hasn't built up much entanglement

# What makes classical simulation hard?

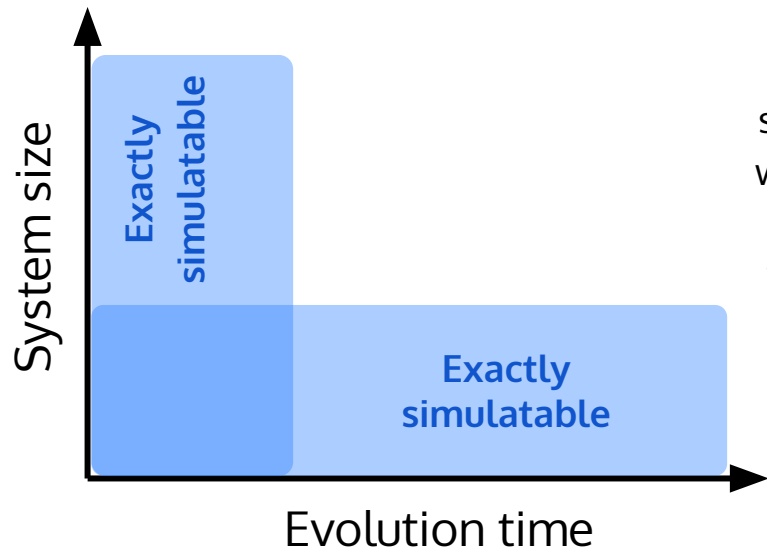


At **early times** the system hasn't built up much entanglement

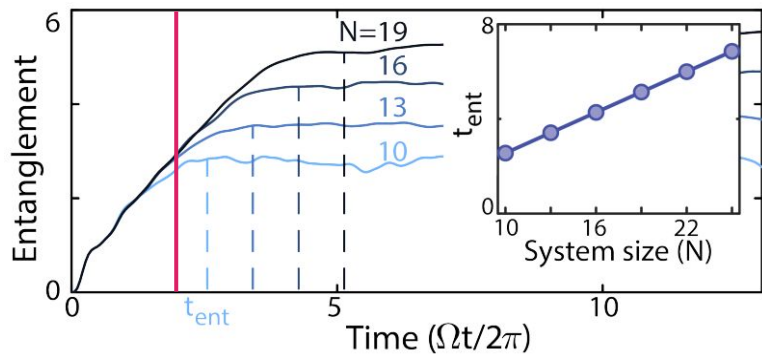
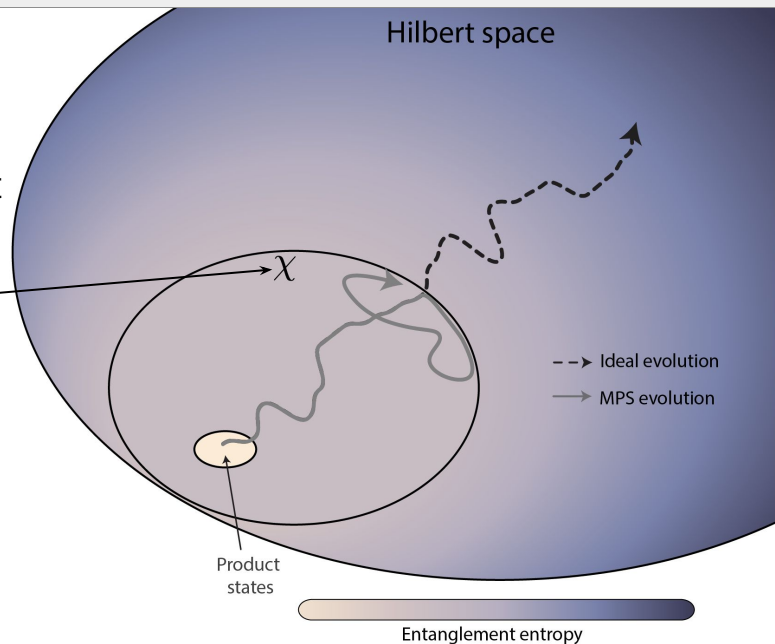


We can simulate by using **Matrix product states (MPS)**!

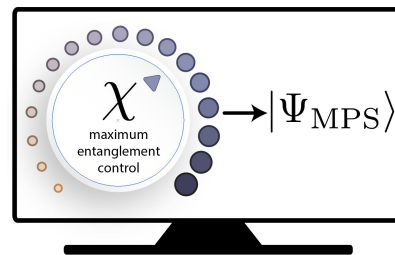
# What makes classical simulation hard?



MPS perfectly simulates dynamics while entanglement is below the "bond dimension"

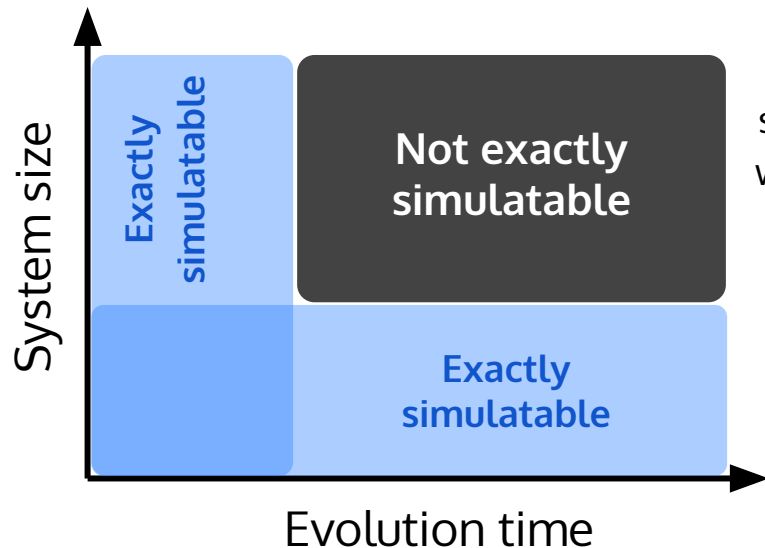


At **early times** the system hasn't built up much entanglement

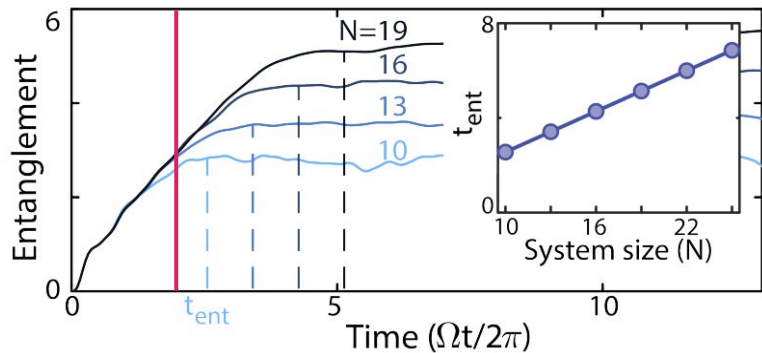
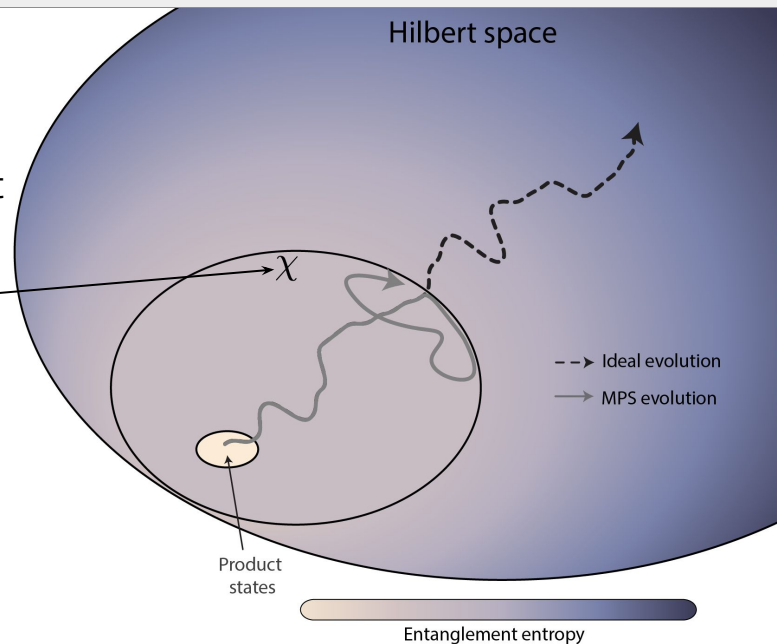


We can simulate by using **Matrix product states (MPS)**!

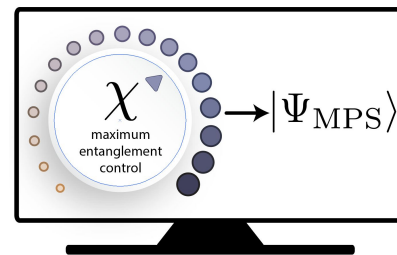
# What makes classical simulation hard?



MPS perfectly simulates dynamics while entanglement is below the "bond dimension"

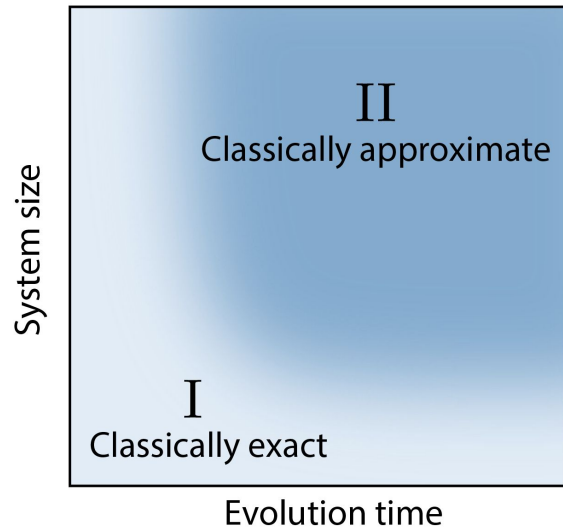


At early times the system hasn't built up much entanglement

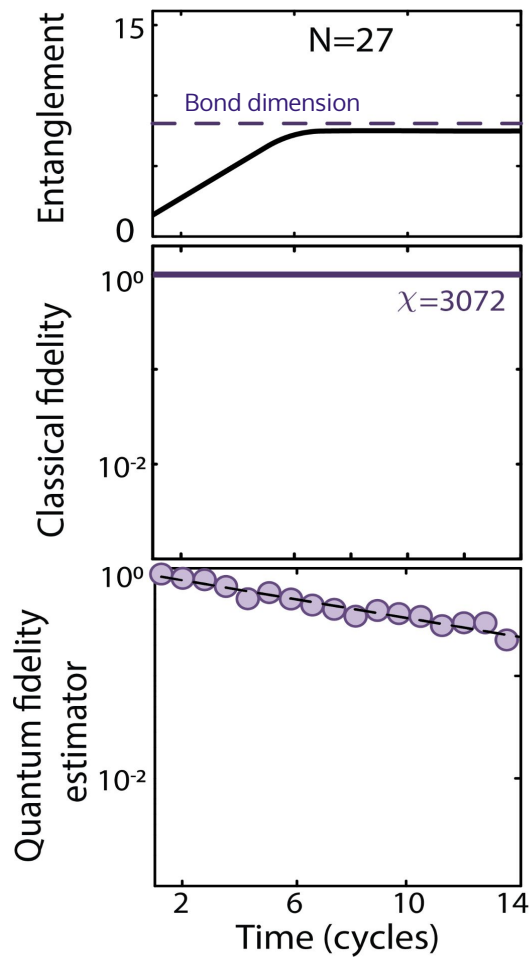
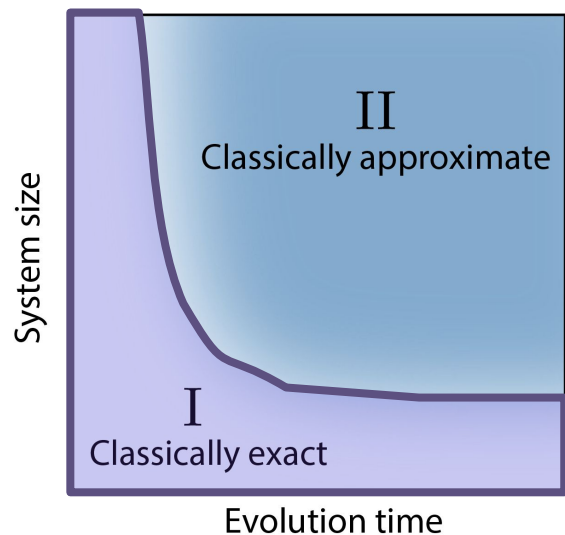


We can simulate by using **Matrix product states (MPS)**!

# Fidelity benchmarking breakdown

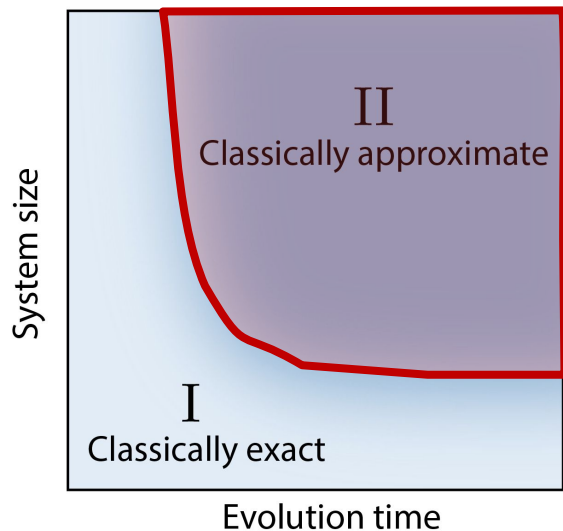


# Fidelity benchmarking breakdown



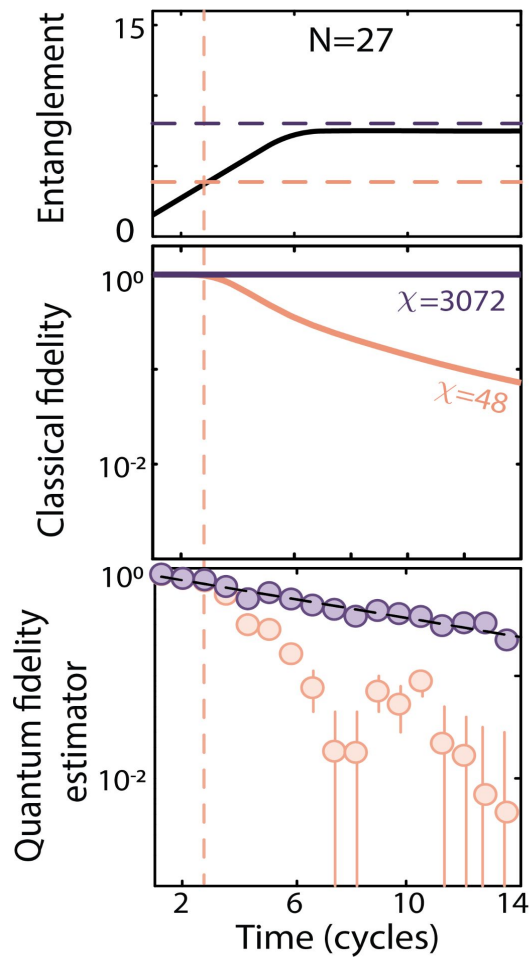
For large enough bond dimension, we exactly benchmark the system

# Fidelity benchmarking breakdown



$$\text{Fidelity estimate} \approx \text{Experimental true fidelity} \times \text{Simulation fidelity}$$

(Not exactly, but qualitatively)



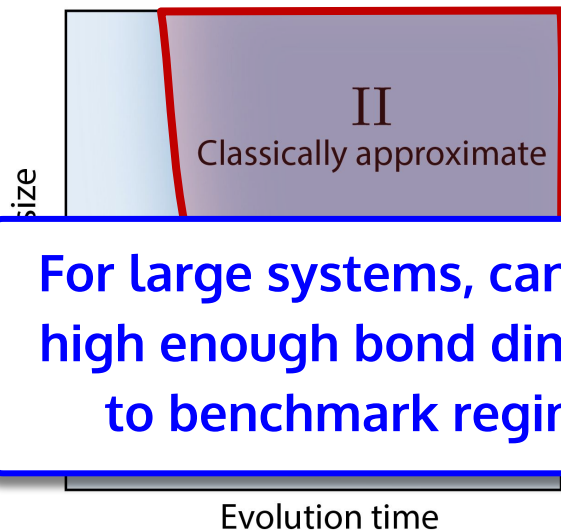
For large enough bond dimension, we exactly benchmark the system

When bond dimension is too small, classical accuracy drops

And so does the benchmarked quantum fidelity!



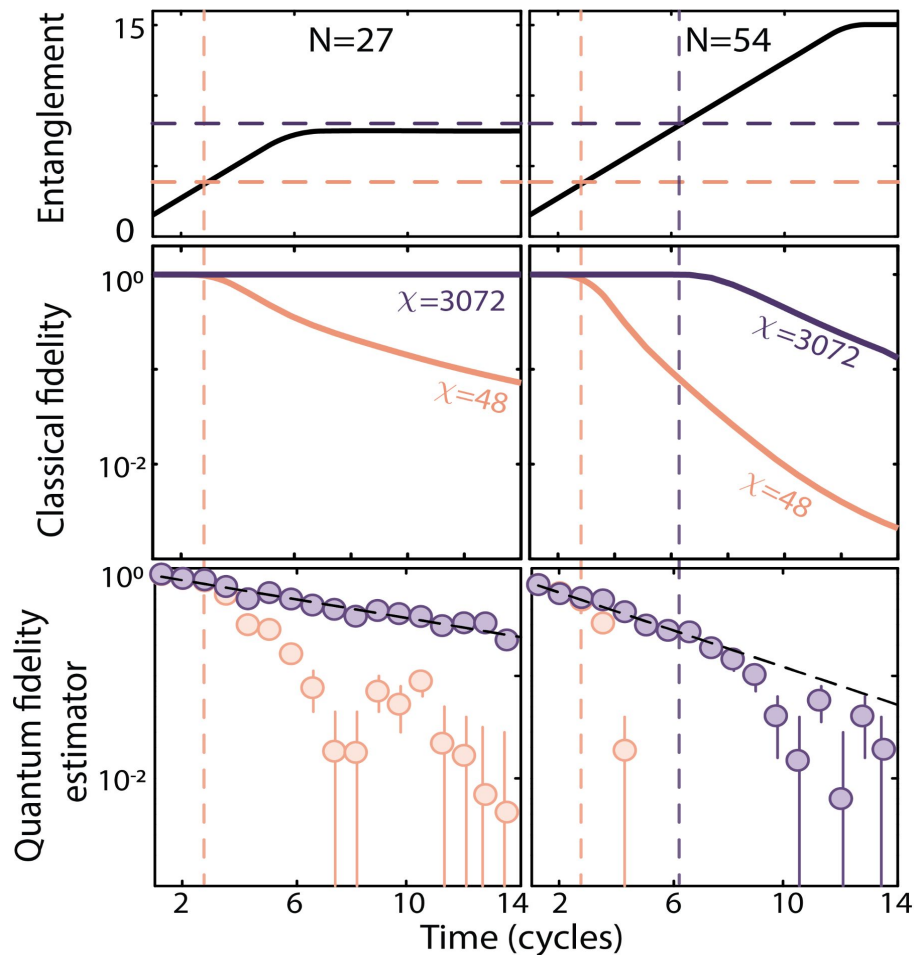
# Fidelity benchmarking breakdown



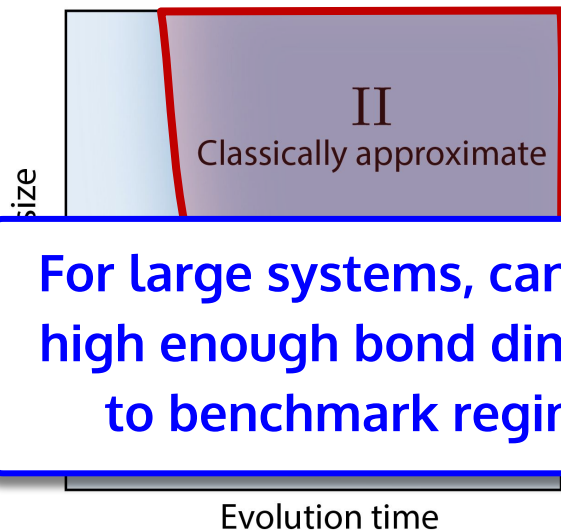
For large systems, can't go to high enough bond dimension to benchmark regime II!

$$\text{Fidelity estimate} \approx \text{Experimental true fidelity} \times \text{Simulation fidelity}$$

(Not exactly, but qualitatively)



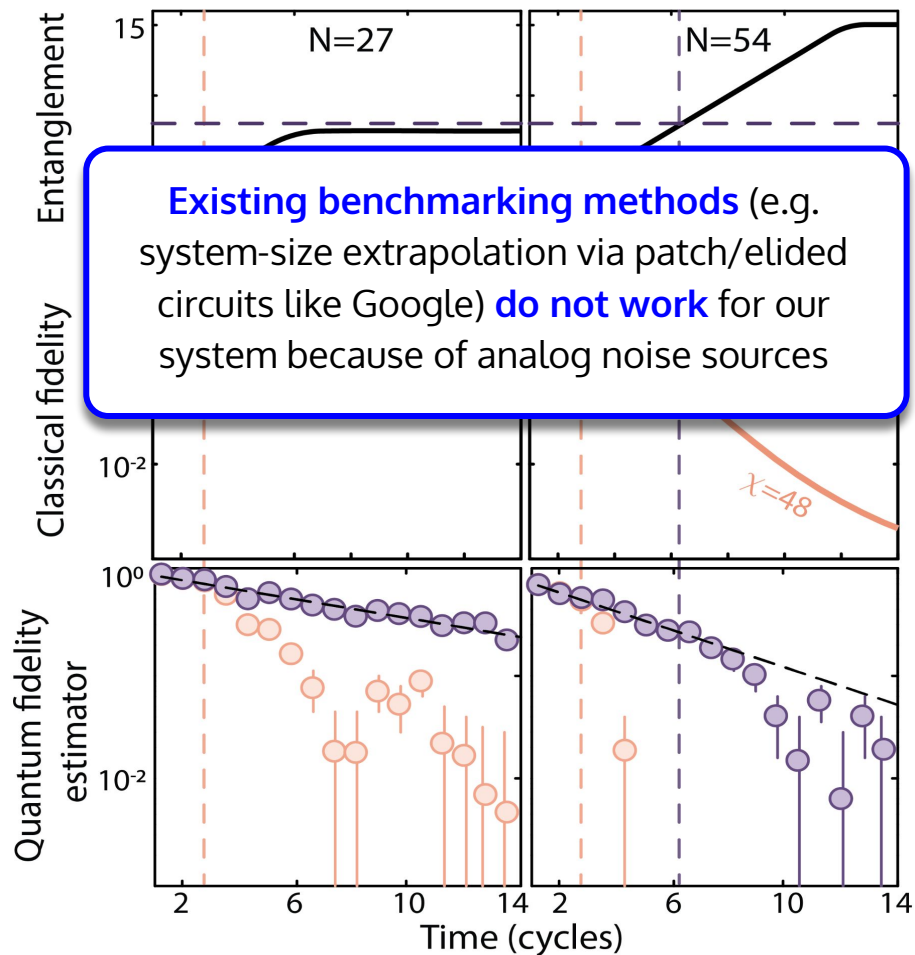
# Fidelity benchmarking breakdown



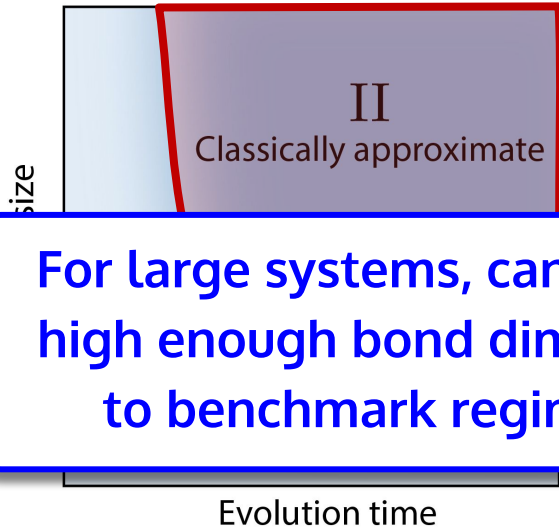
For large systems, can't go to high enough bond dimension to benchmark regime II!

$$\text{Fidelity estimate} \approx \text{Experimental true fidelity} \times \text{Simulation fidelity}$$

(Not exactly, but qualitatively)



# Fidelity benchmarking breakdown

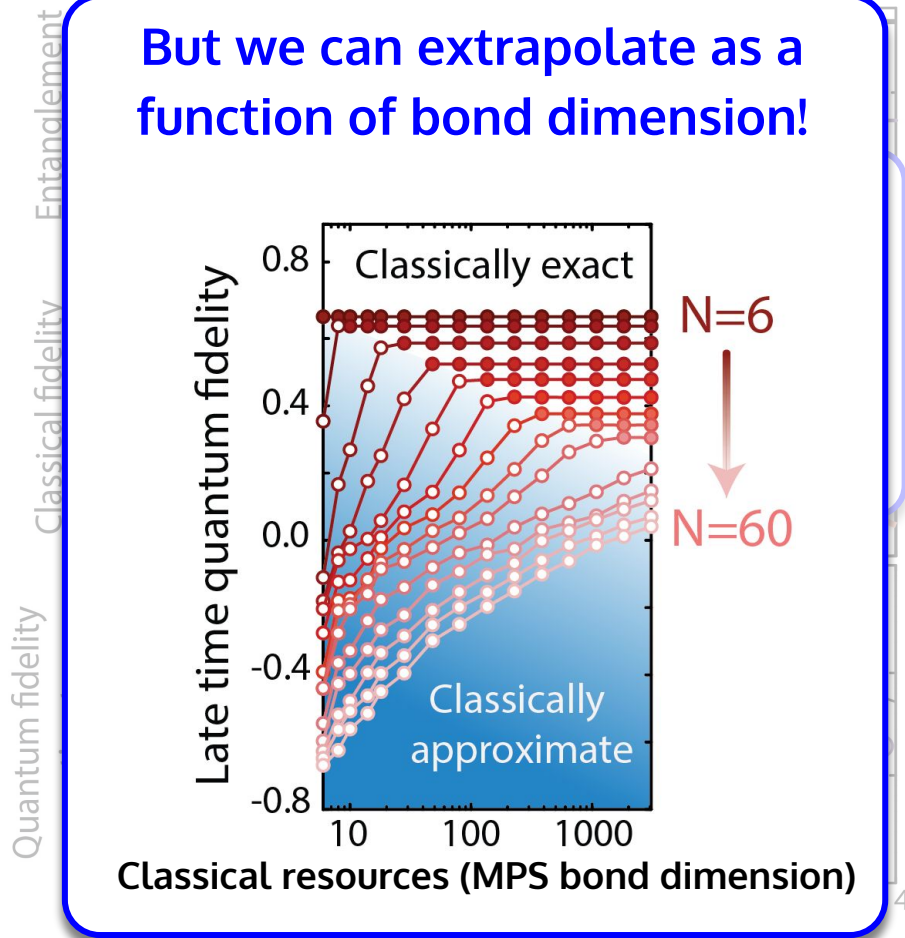


For large systems, can't go to high enough bond dimension to benchmark regime II!

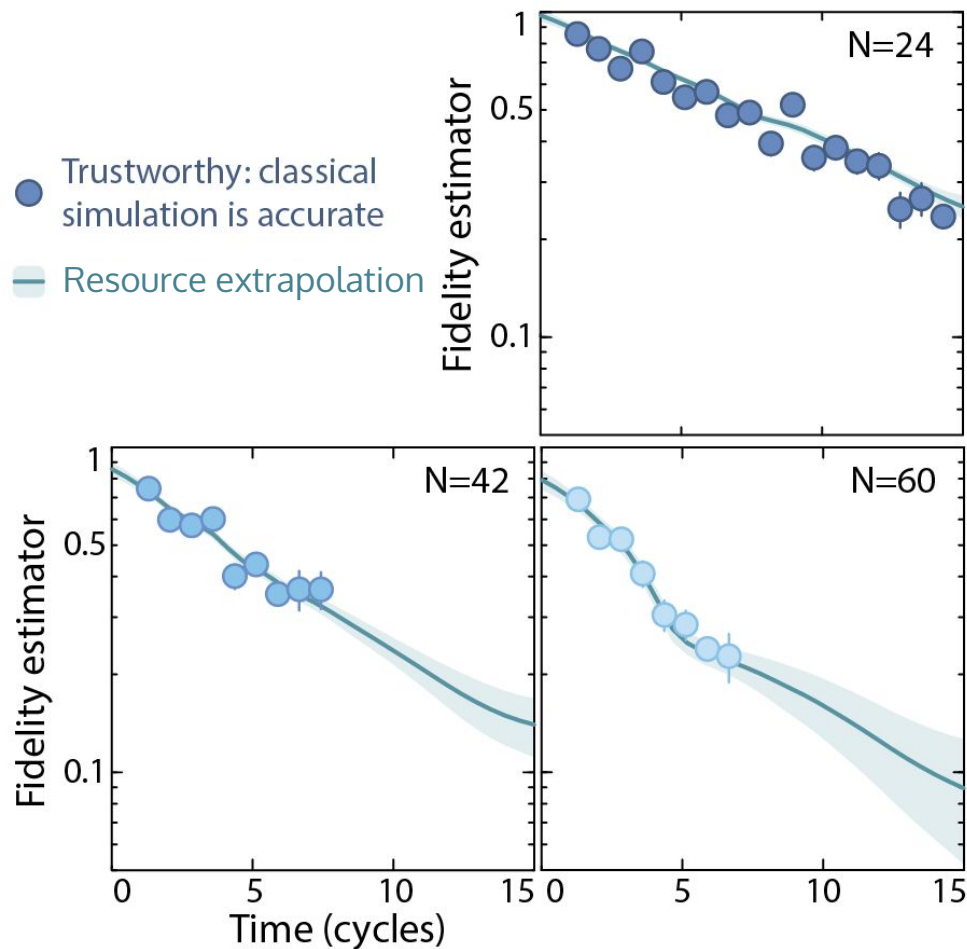
$$\text{Fidelity estimate} \approx \text{Experimental true fidelity} \times \text{Simulation fidelity}$$

(Not exactly, but qualitatively)

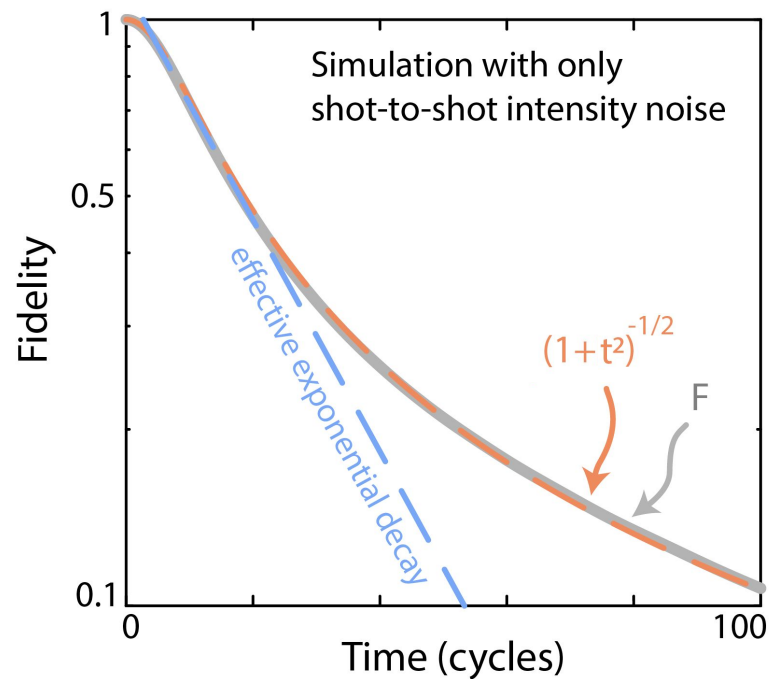
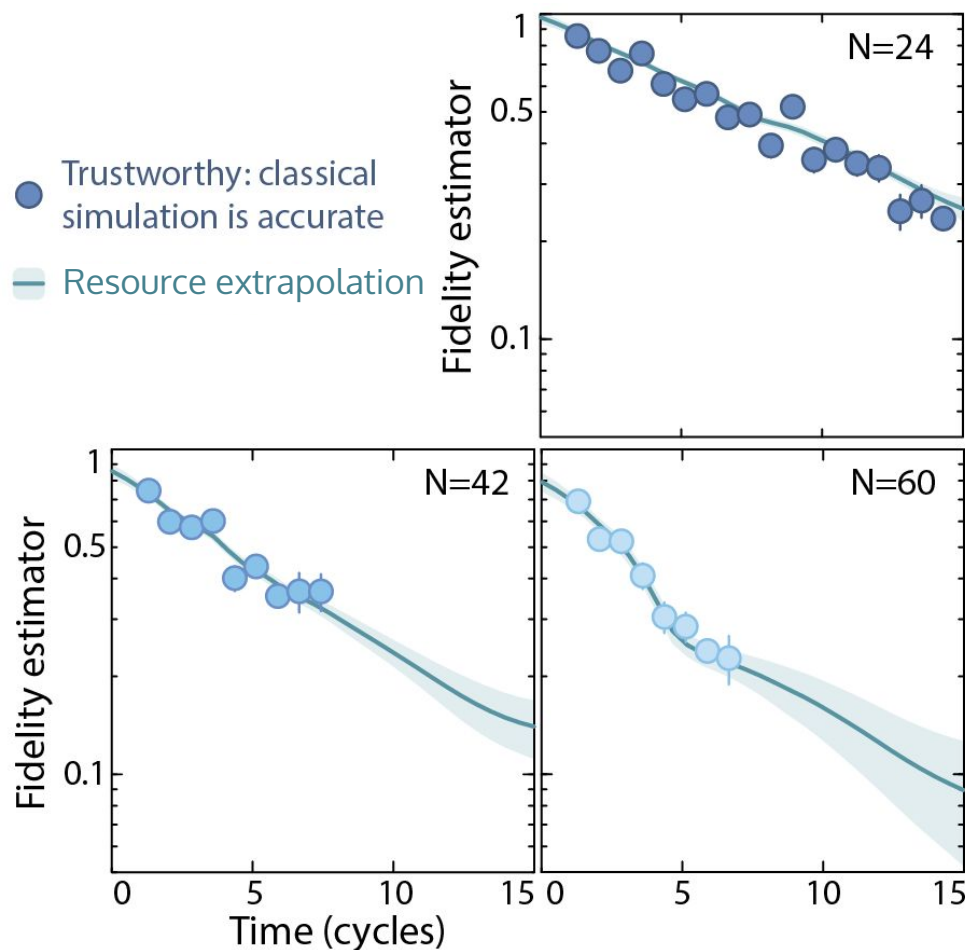
But we can extrapolate as a function of bond dimension!



# Large scale fidelity

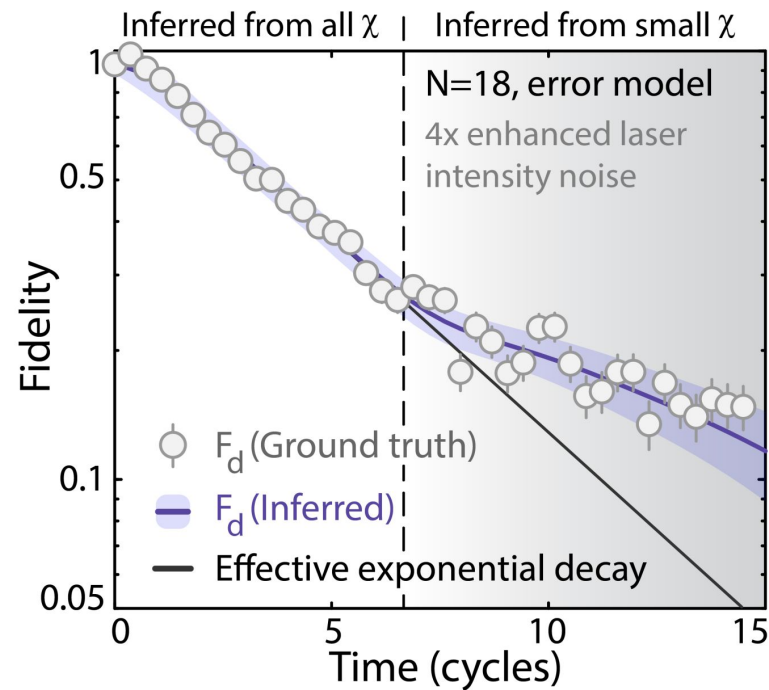
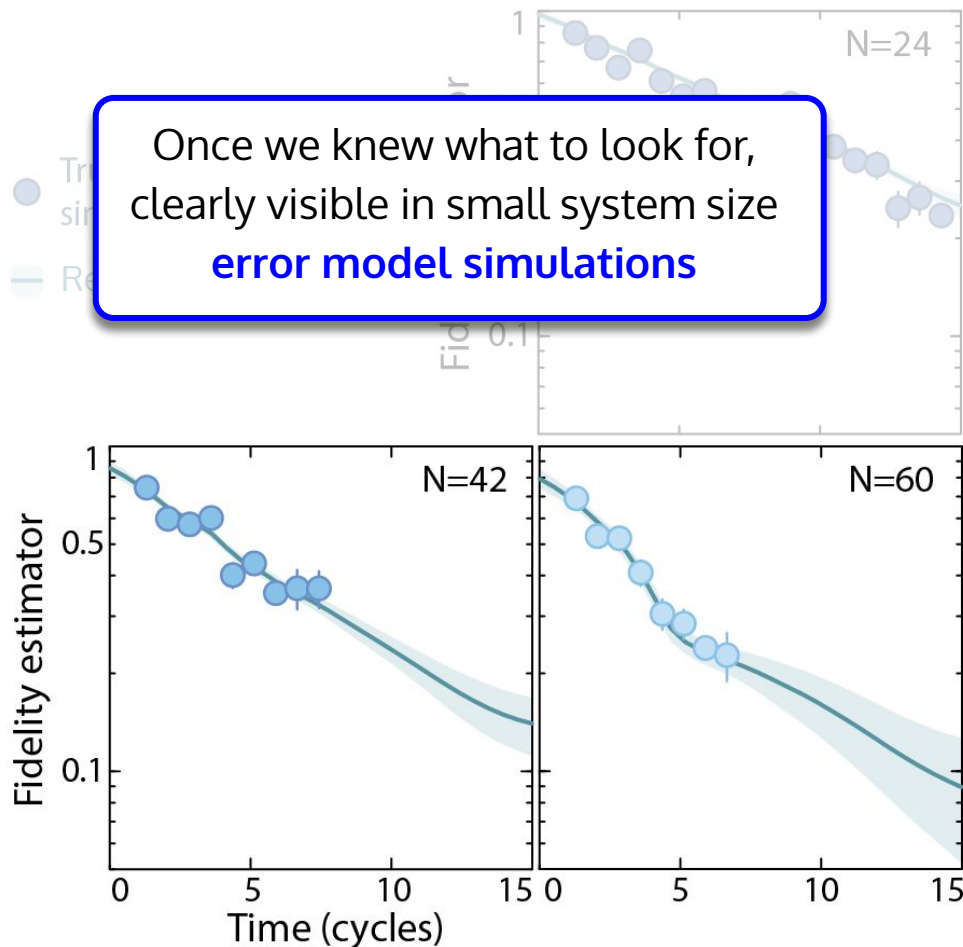


# Large scale fidelity



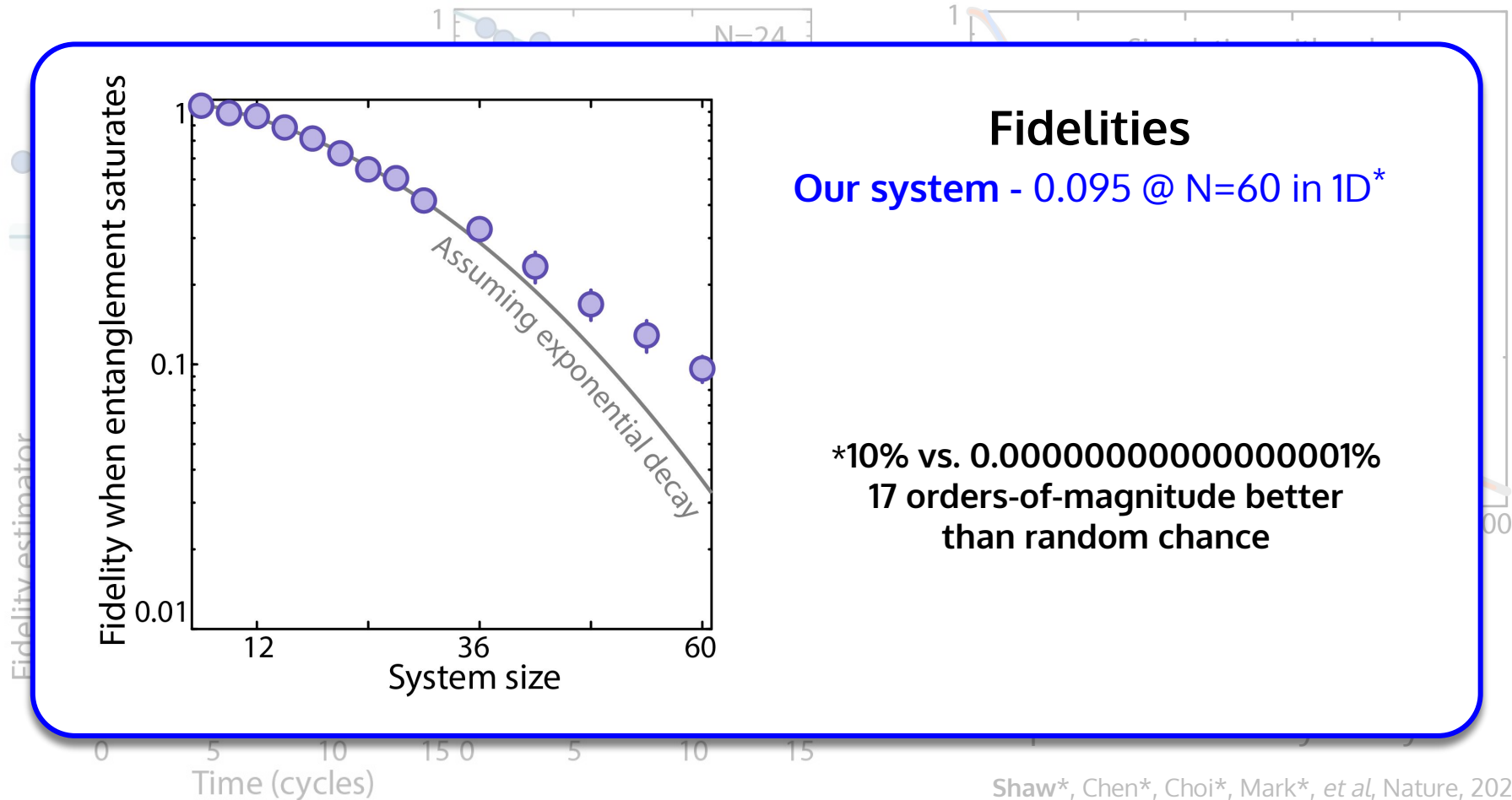
This led us to discover certain **non-Markovian** noise leads to power law fidelity decay!

# Large scale fidelity

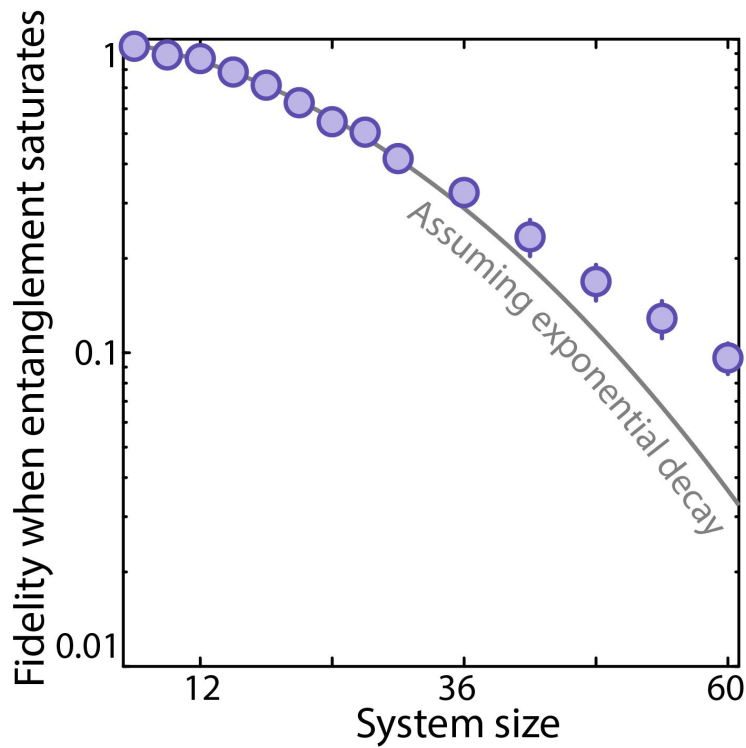


This led us to discover certain  
**non-Markovian** noise leads to  
power law fidelity decay!

# Large scale fidelity



# Large scale fidelity



## Fidelities

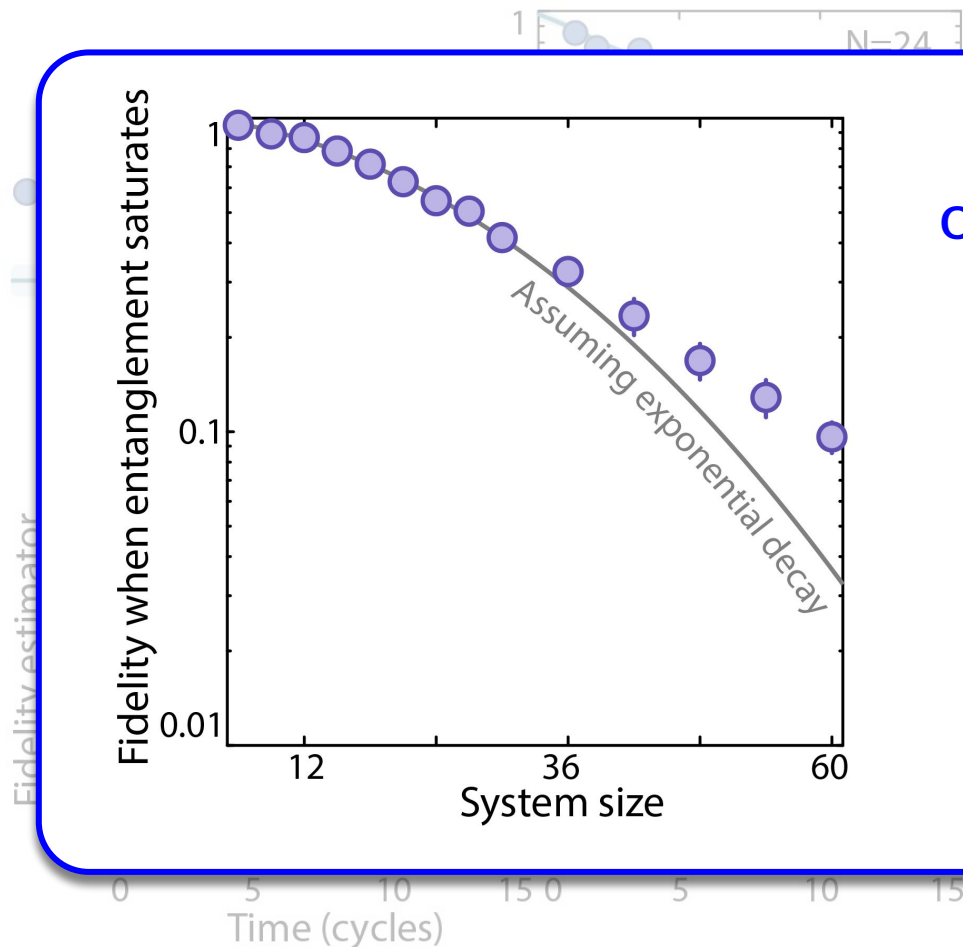
Our system - 0.095 @ N=60 in 1D\*

These results are consistent with a two-qubit fidelity of  $\sim 0.999$

**\*10% vs. 0.000000000000000001%  
17 orders-of-magnitude better  
than random chance**



# Large scale fidelity



## Fidelities

Our system - 0.095 @ N=60 in 1D\*

Google\*\* - 0.003 @ N=53 in 2D

These results are consistent with  
a two-qubit fidelity of  $\sim 0.999$

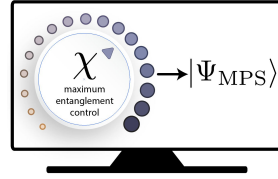
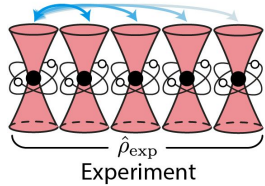
**\*10% vs. 0.0000000000000000001%**  
**17 orders-of-magnitude better  
than random chance**

\*\*Not a fair comparison because of  
different level of control, but gives a  
general sense of scale. Higher values in  
more recent papers (Morvan et al, 2023)

# What is the actual classical cost?

Which better represents the quantum world?

Quantum experiment or classical computer?

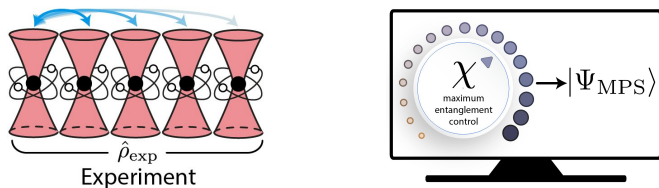


**Physical error vs approximation error**

# What is the actual classical cost?

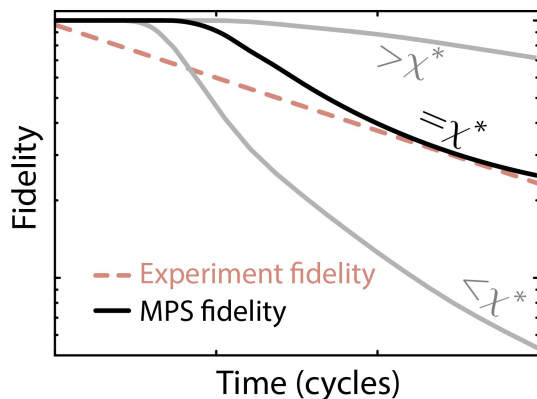
## Which better represents the quantum world?

Quantum experiment or classical computer?



**Physical error vs approximation error**

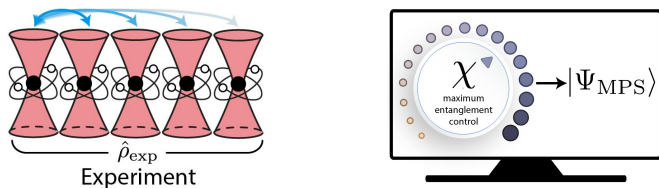
Find minimum classical resources for classical computer to have higher fidelity than experiment



# What is the actual classical cost?

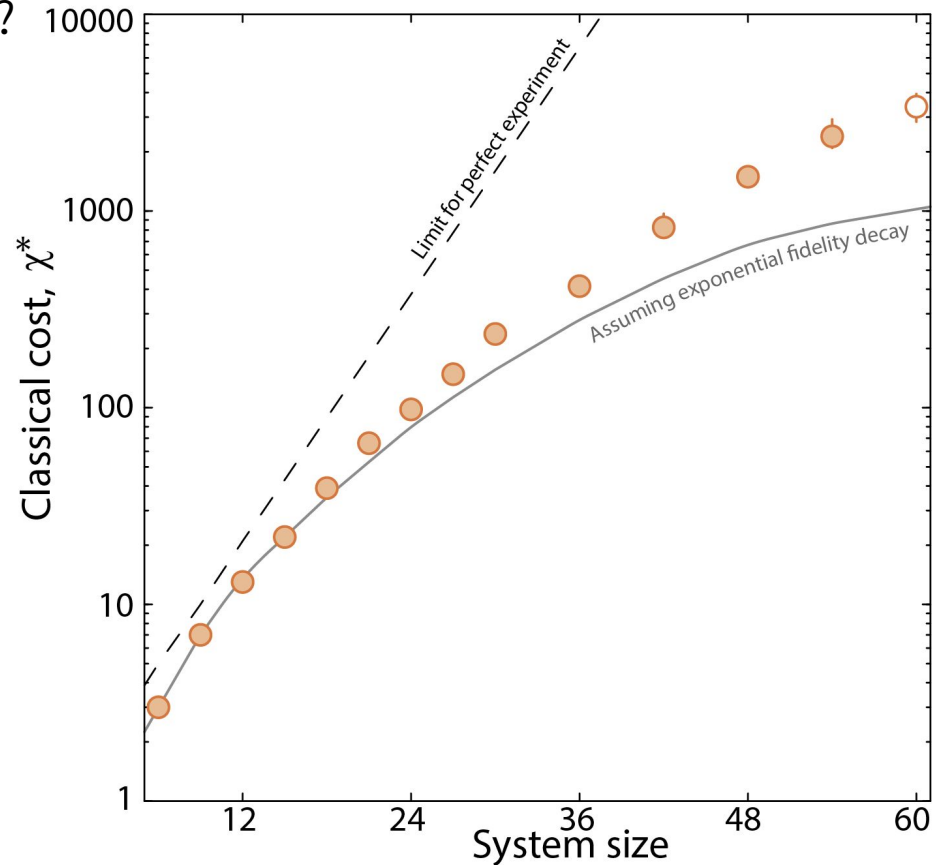
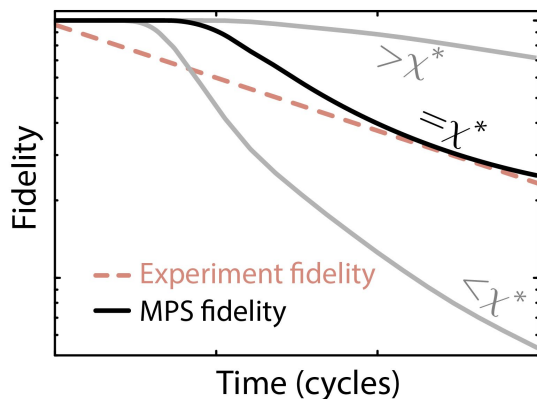
## Which better represents the quantum world?

Quantum experiment or classical computer?



**Physical error vs approximation error**

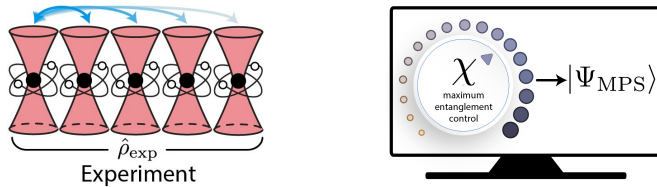
Find minimum classical resources for classical computer to have higher fidelity than experiment



# What is the actual classical cost?

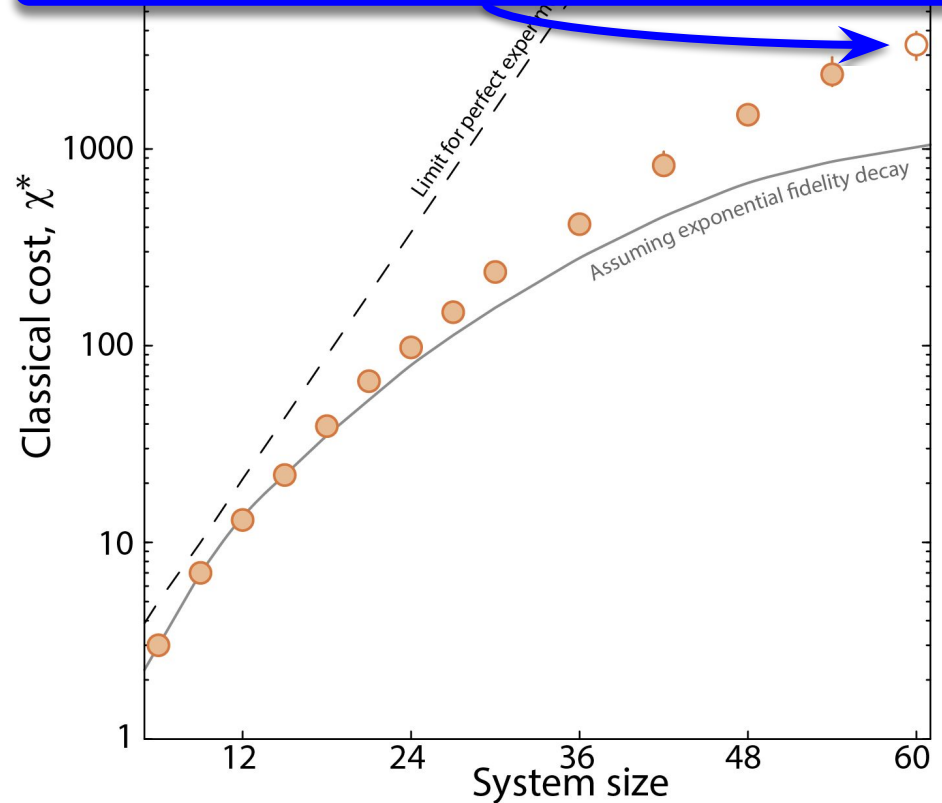
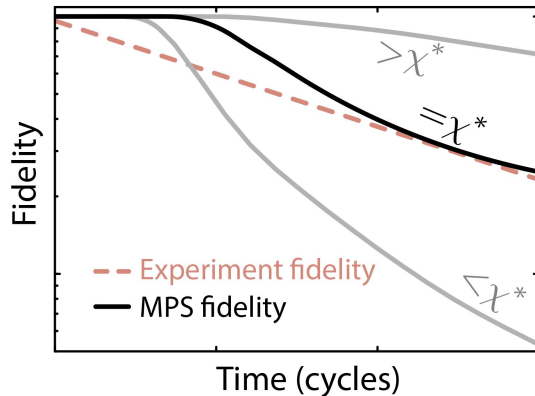
Which better represents the quantum world?  
Quantum experiment or classical computer?

**180 core-days on the Caltech supercomputer**  
(using a highly optimized algorithm)



**Physical error vs approximation error**

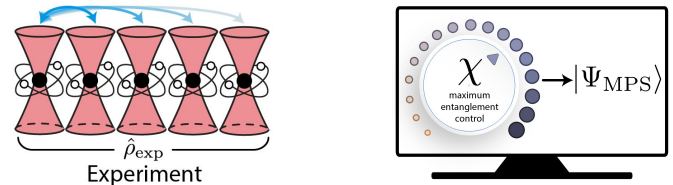
Find minimum classical resources for classical computer to have higher fidelity than experiment



# What is the actual classical cost?

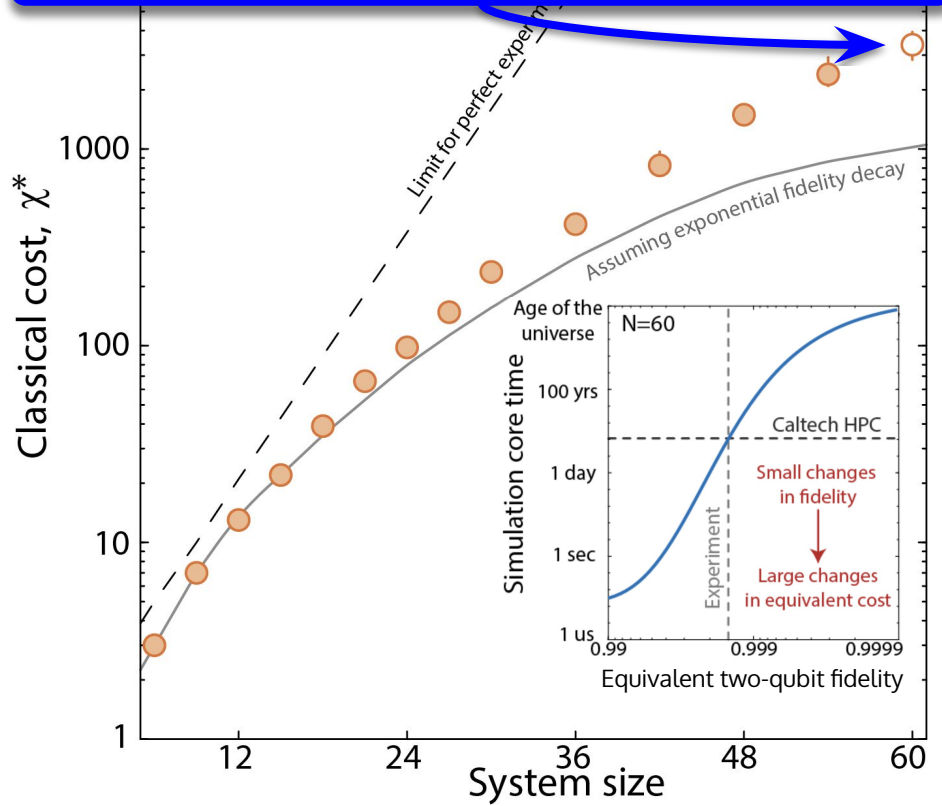
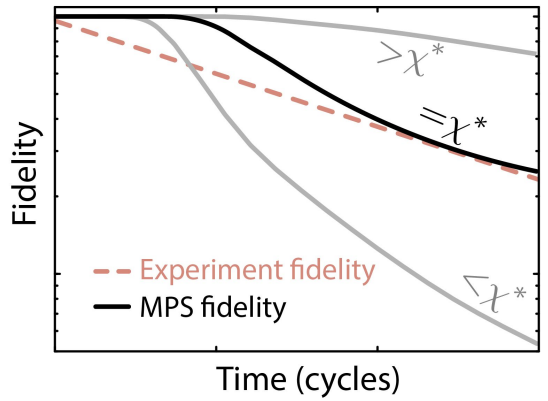
Which better represents the quantum world?  
 Quantum experiment or classical computer?

**180 core-days on the Caltech supercomputer**  
 (using a highly optimized algorithm)



**Physical error vs approximation error**

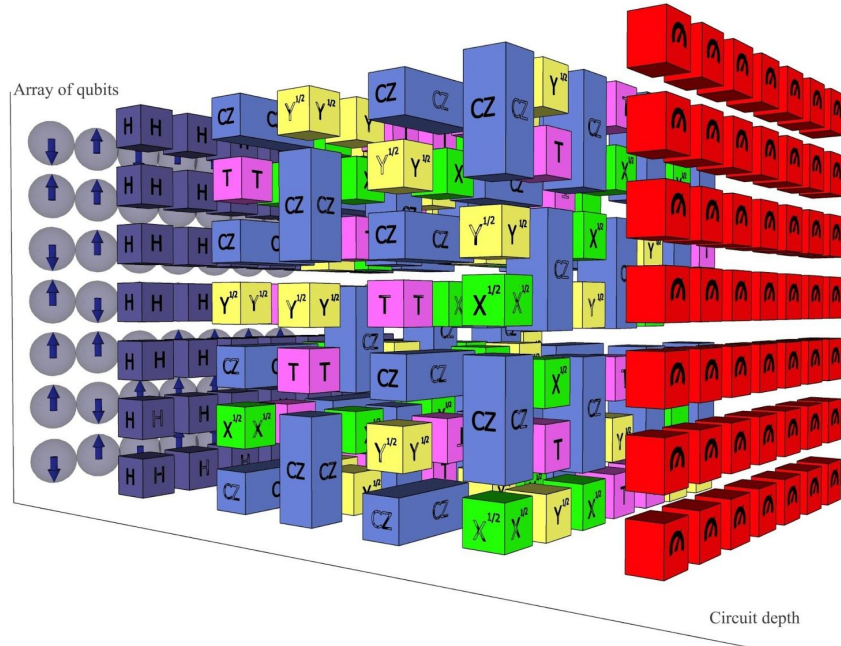
Find minimum classical resources for classical computer to have higher fidelity than experiment



# Quantum is hard

## Sycamore supremacy circuit

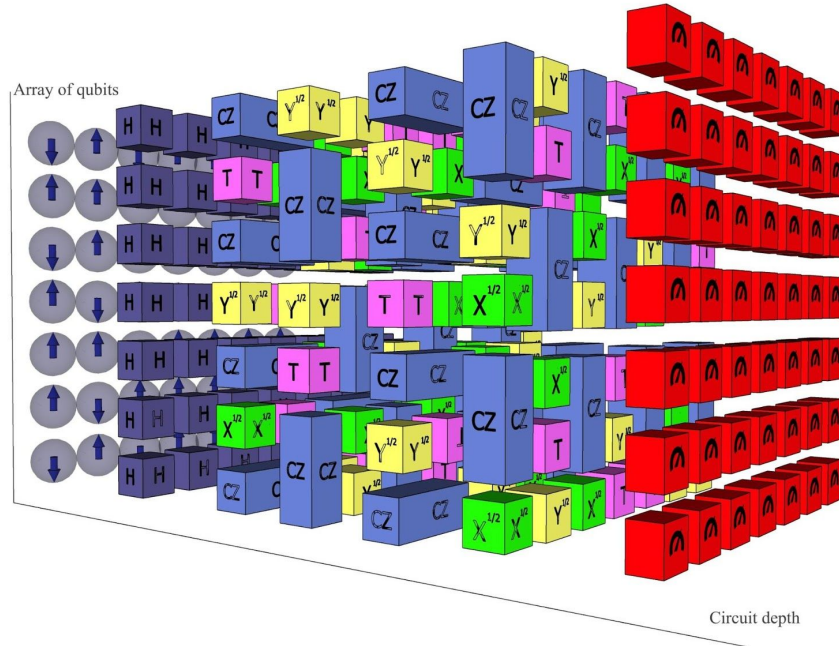
Complex 2D random unitary circuit



# Quantum is hard

## Sycamore supremacy circuit

Complex 2D random unitary circuit



## Our "circuit"

Time-independent, global, 1D evolution

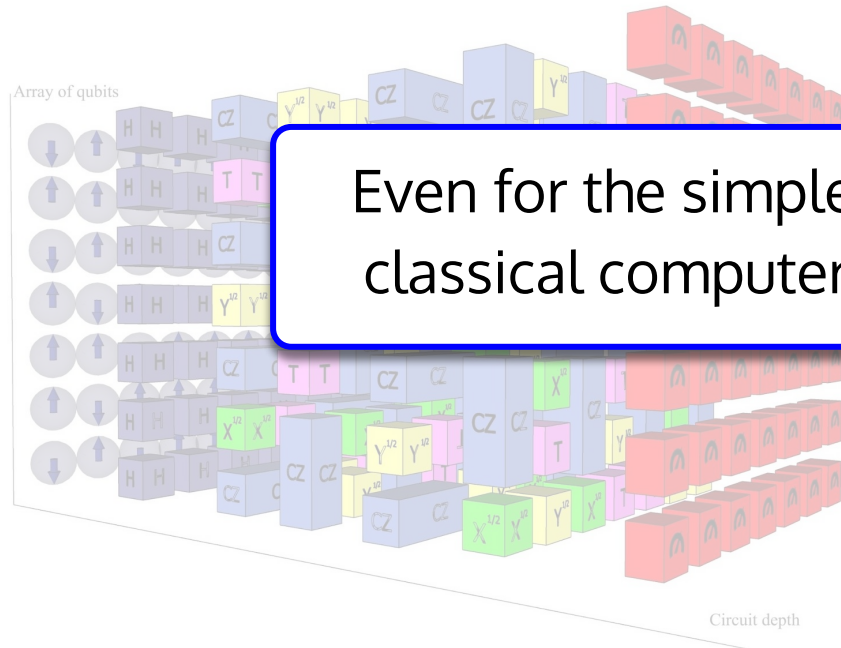
$$e^{-i\hat{H}t/\hbar}$$



# Quantum is hard

## Sycamore supremacy circuit

Complex 2D random unitary circuit



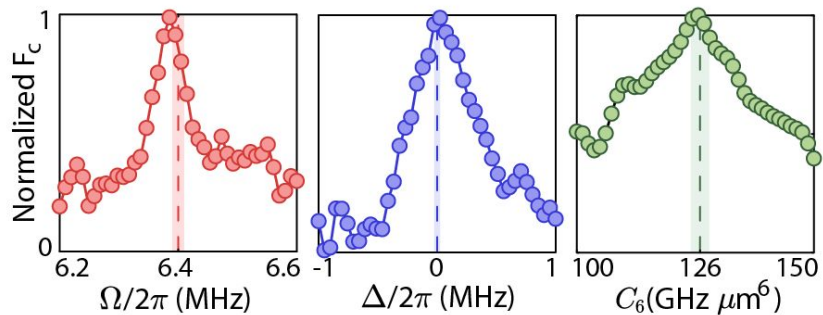
## Our "circuit"

Time-independent, global, 1D evolution

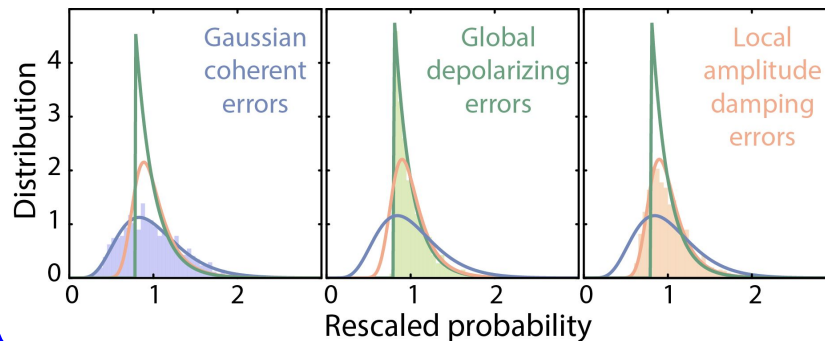
Even for the simplest quantum evolution,  
classical computers struggle to keep up!

# Applications!

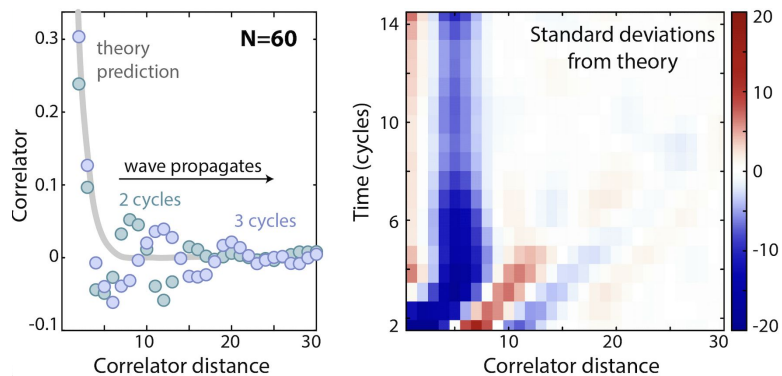
## Hamiltonian estimation



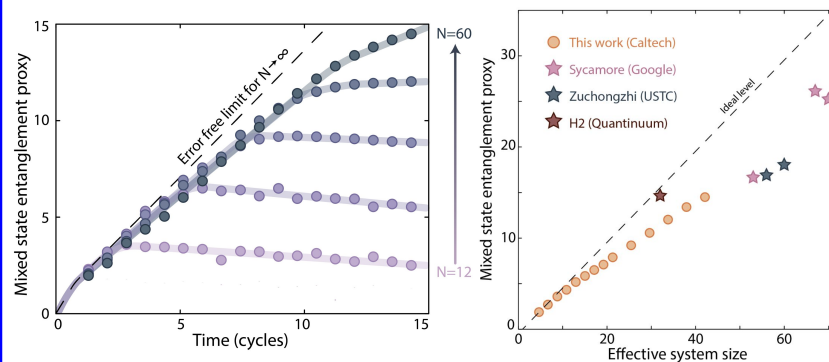
## Noise learning



## Unusual thermalization

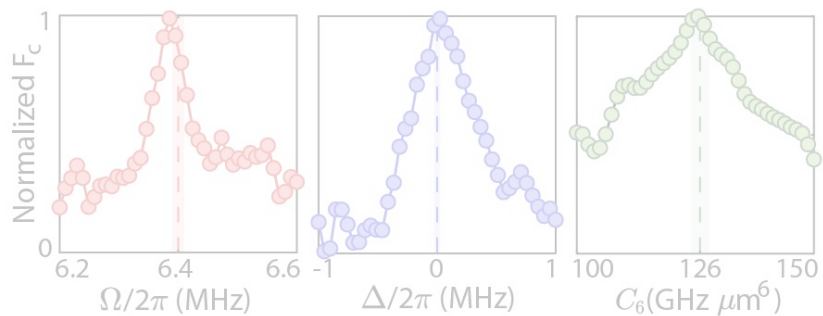


## Entanglement estimation

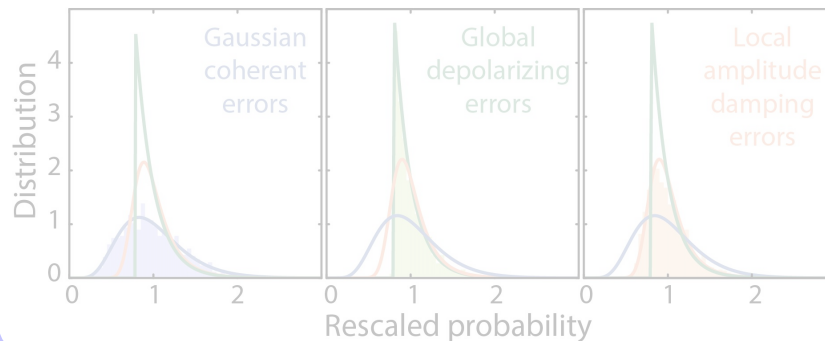


# Applications!

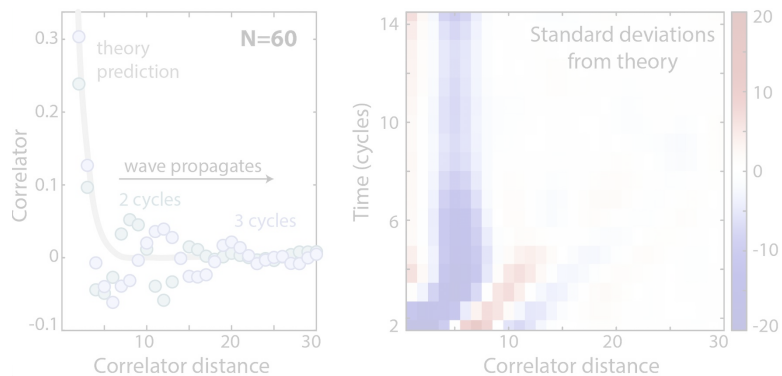
## Hamiltonian estimation



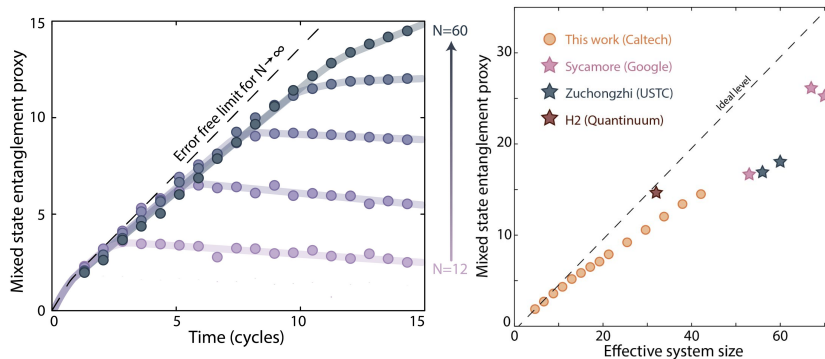
## Noise learning



## Unusual thermalization

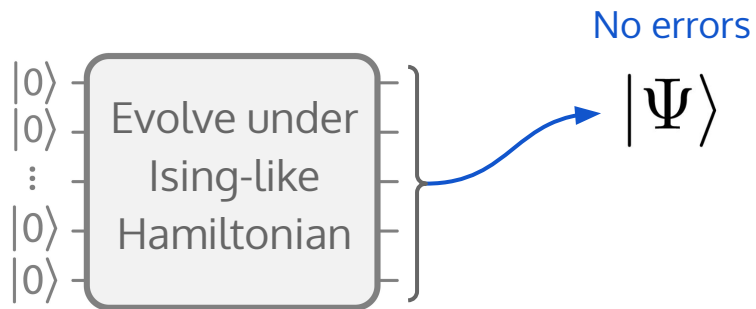


## Entanglement estimation



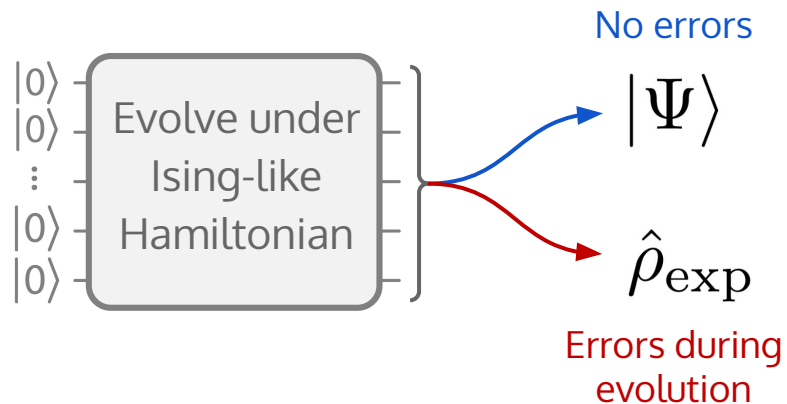
# How entangled are we?

Whenever we've talked about entanglement, we've meant **pure state entanglement**



# How entangled are we?

Whenever we've talked about entanglement, we've meant **pure state entanglement**

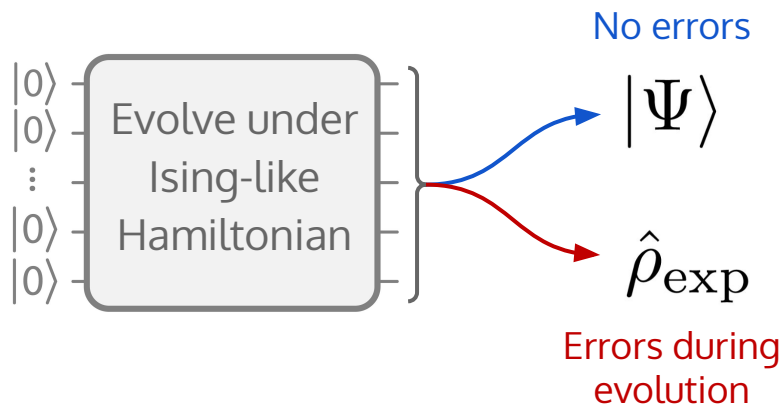


But what about the actual **experimental mixed state entanglement?**

Notoriously hard to measure,  
even theoretically!

# How entangled are we?

Whenever we've talked about entanglement, we've meant **pure state entanglement**



But what about the actual **experimental mixed state entanglement?**

Notoriously hard to measure, even theoretically!

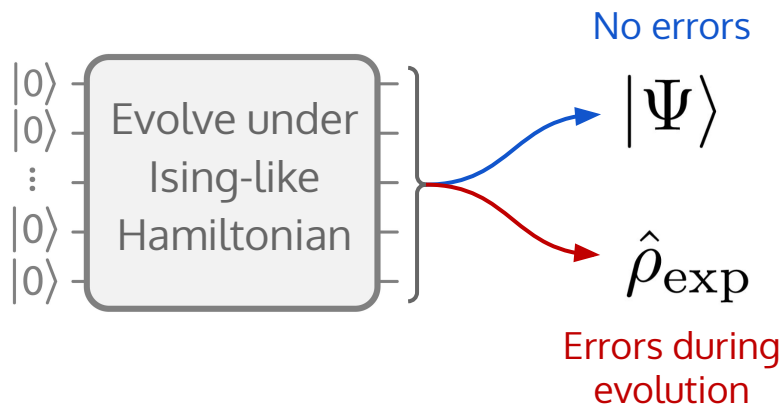
We developed a new **mixed state entanglement proxy\***

$$E_{\text{mixed}} = E_{\text{pure}} + \log(F)$$

\*For experts, this is a proxy for the **negativity**

# How entangled are we?

Whenever we've talked about entanglement, we've meant **pure state entanglement**

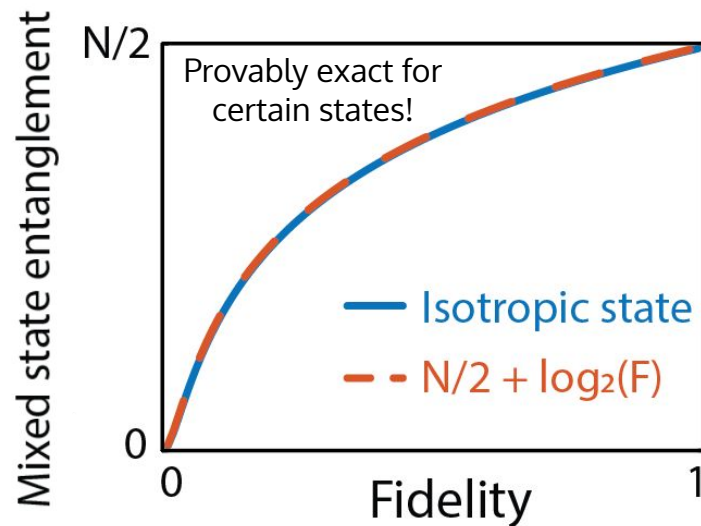


But what about the actual **experimental mixed state entanglement?**

Notoriously hard to measure, even theoretically!

We developed a new **mixed state entanglement proxy\***

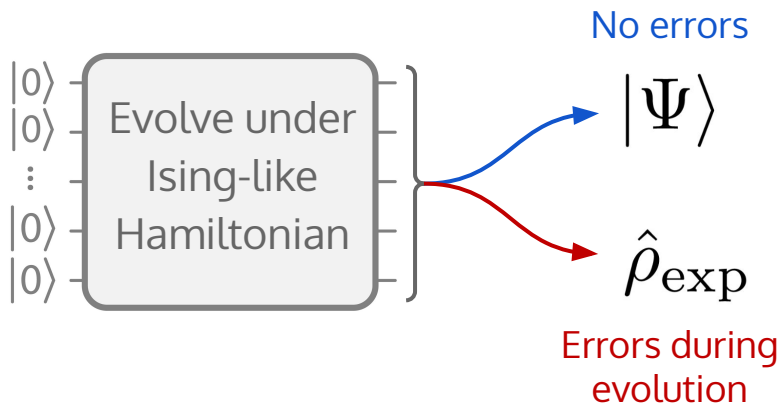
$$E_{\text{mixed}} = E_{\text{pure}} + \log(F)$$



\*For experts, this is a proxy for the **negativity**

# How entangled are we?

Whenever we've talked about entanglement, we've meant **pure state entanglement**

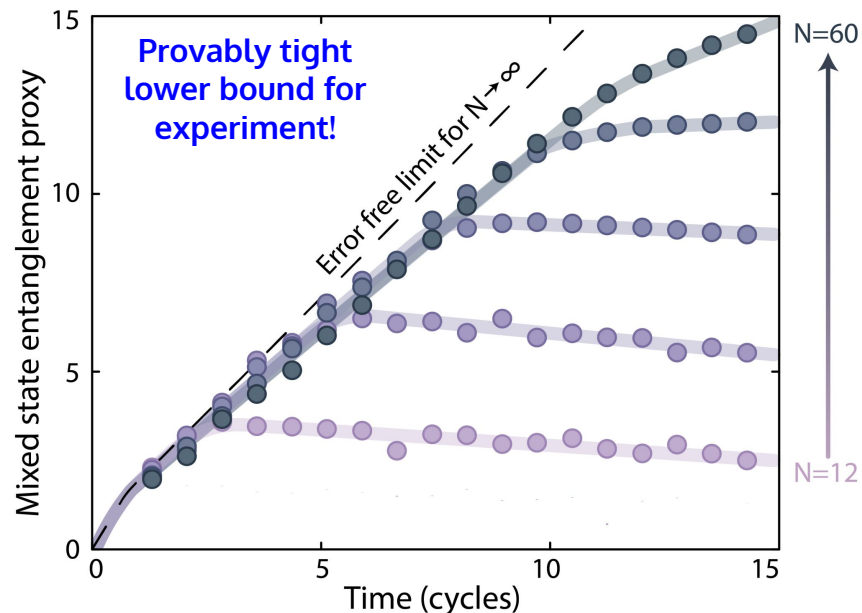


But what about the actual **experimental mixed state entanglement?**

Notoriously hard to measure, even theoretically!

We developed a new **mixed state entanglement proxy\***

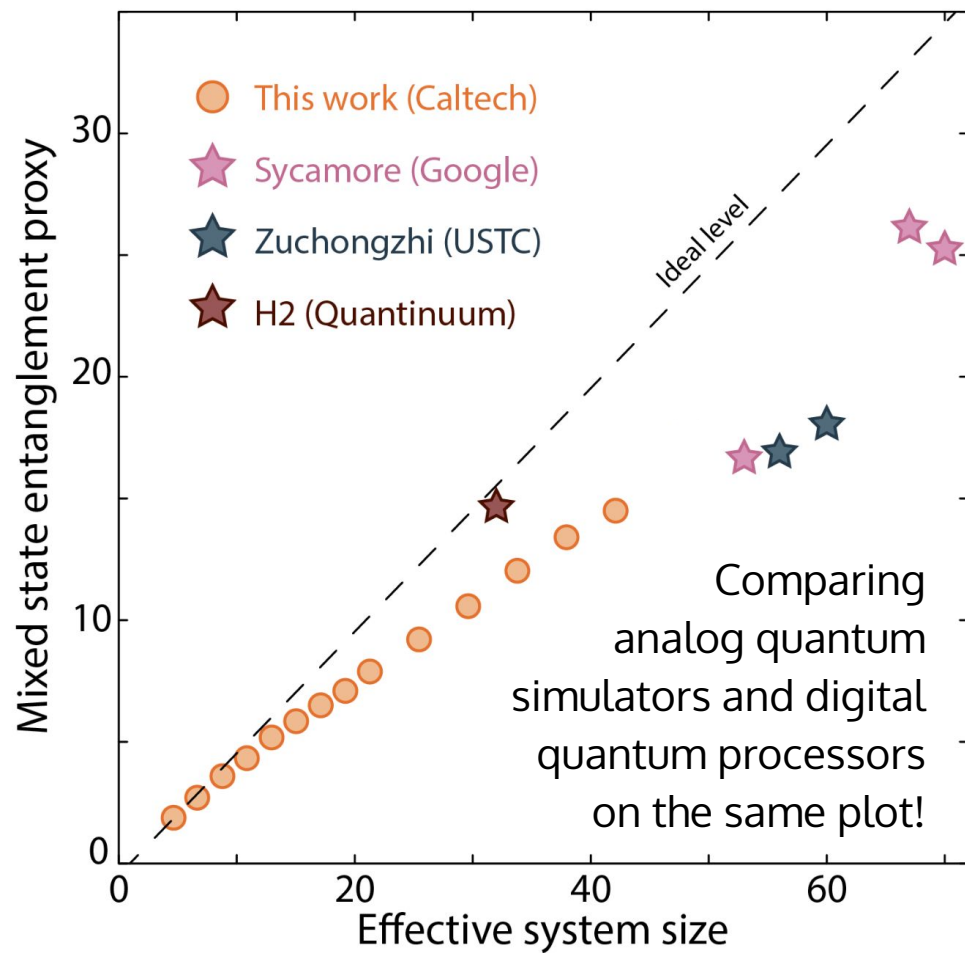
$$E_{\text{mixed}} = E_{\text{pure}} + \log(F)$$



\*For experts, this is a proxy for the **negativity**

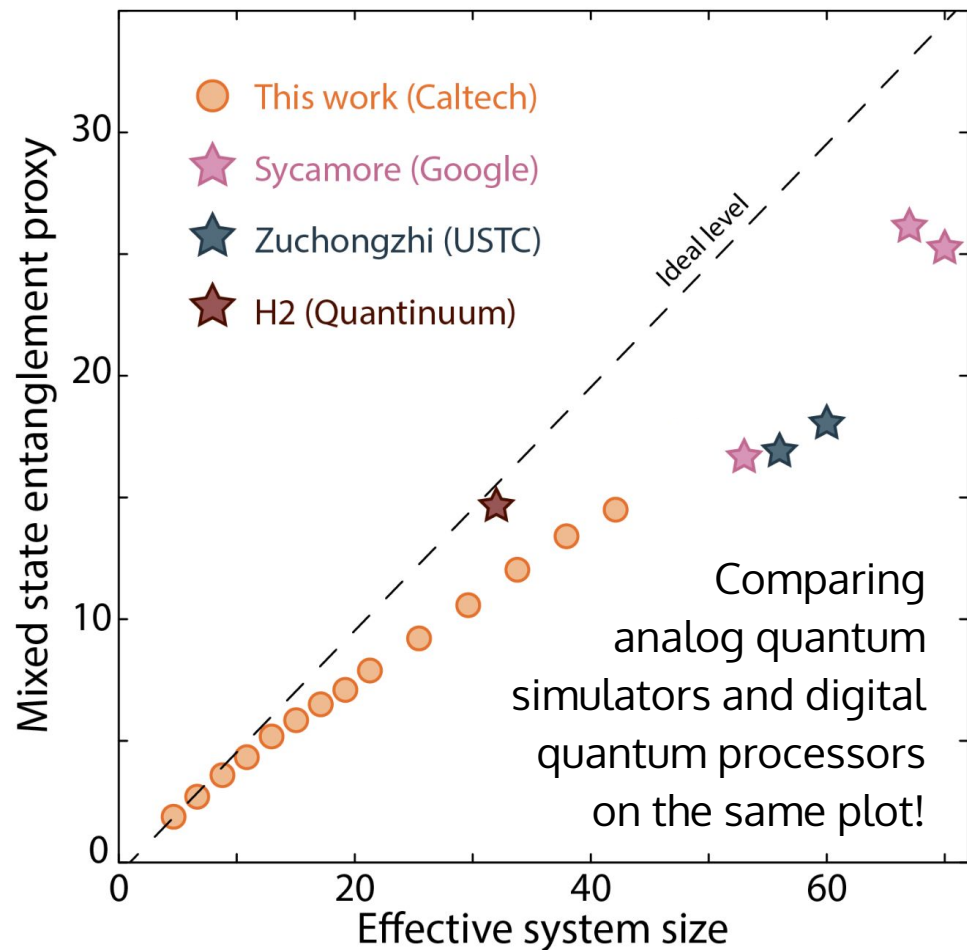


# How entangled are we?



General purpose,  
cross-platform, evaluation  
metric including both qubit  
quantity and quality!

# How entangled are we?



General purpose,  
cross-platform, evaluation  
metric including both qubit  
quantity and quality!

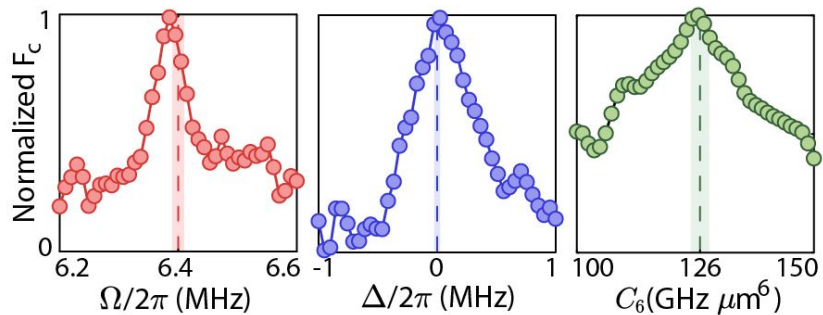
Closely related to questions of:

How many Bell pairs  
could we possibly extract?

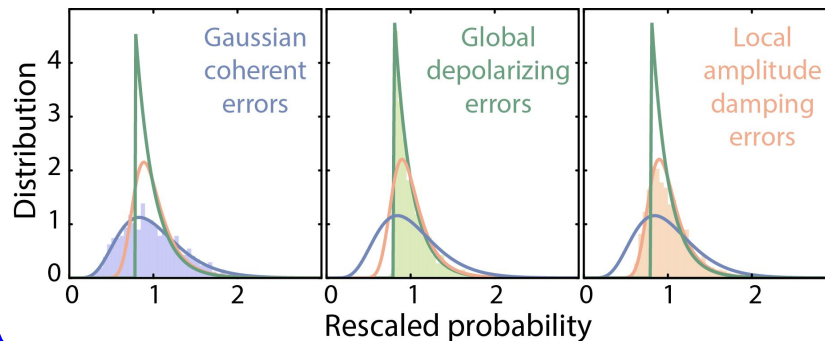
What is the classical  
simulation complexity?

# Applications!

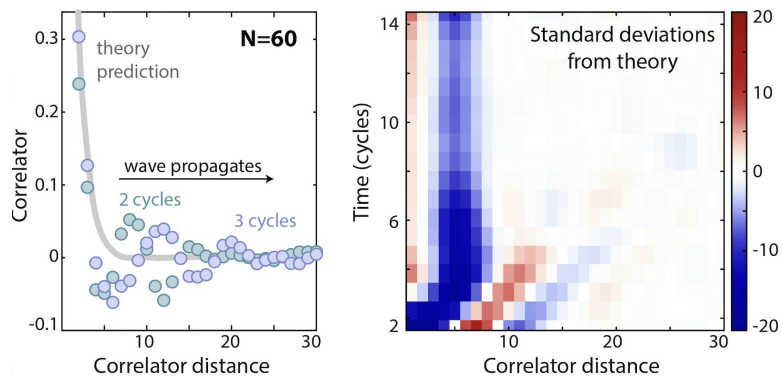
## Hamiltonian estimation



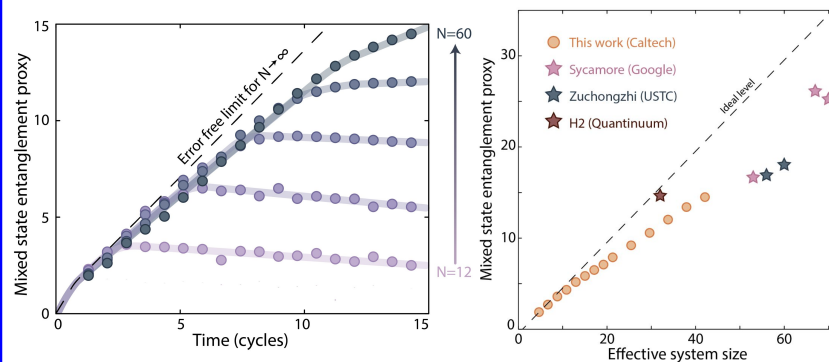
## Noise learning



## Unusual thermalization

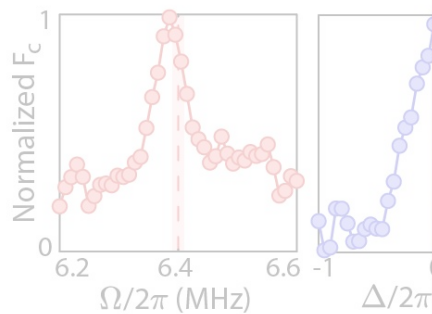


## Entanglement estimation



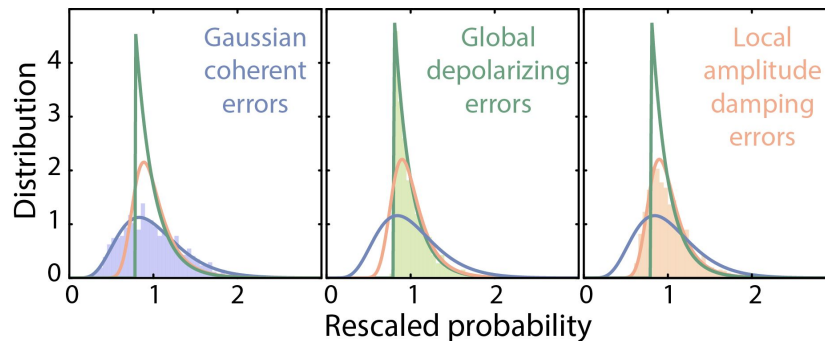
# Applications!

## Hamiltonian

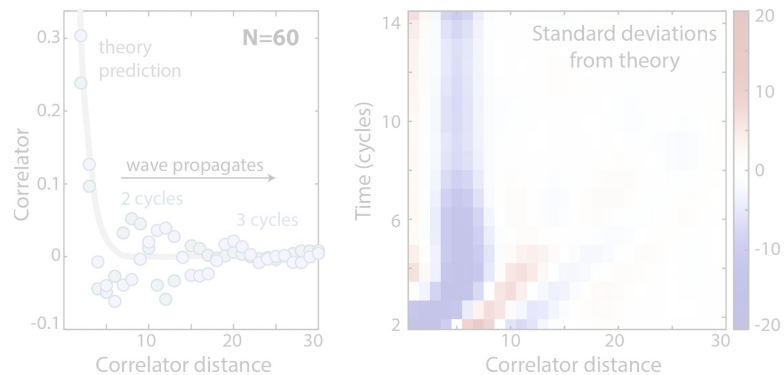


theory by Daniel Mark

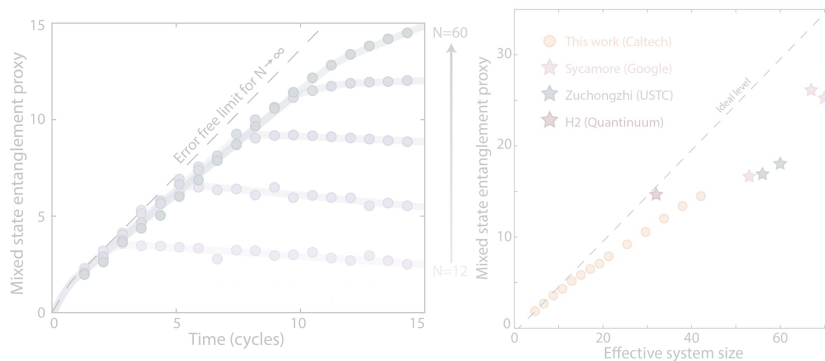
## Noise learning



## Unusual thermalization

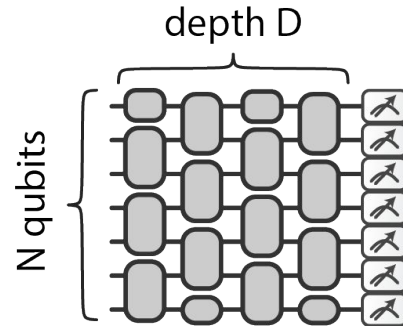


## Entanglement estimation



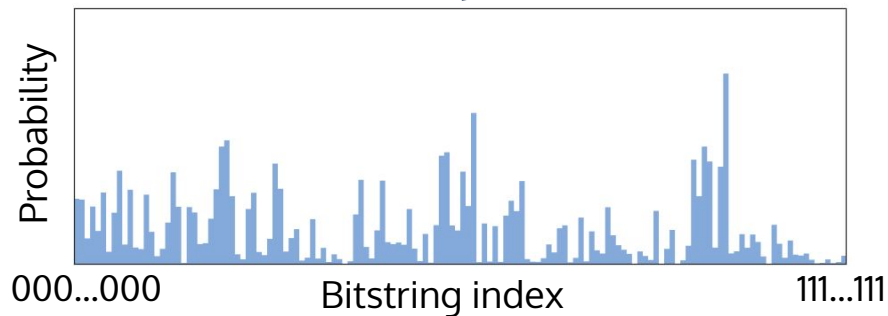
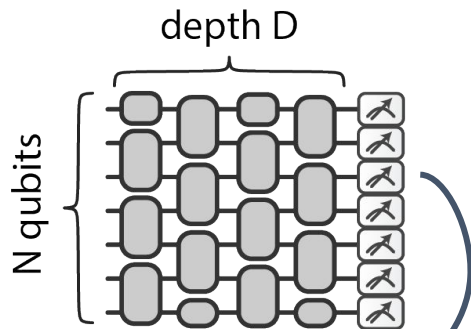
# Learning noise from bitstring measurements

Consider random  
unitary circuit  
(RUC) evolution



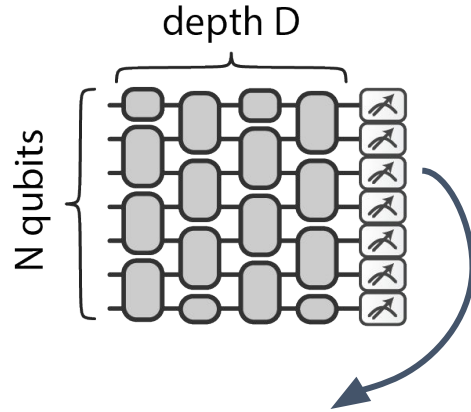
# Learning noise from bitstring measurements

Consider random  
unitary circuit  
(RUC) evolution

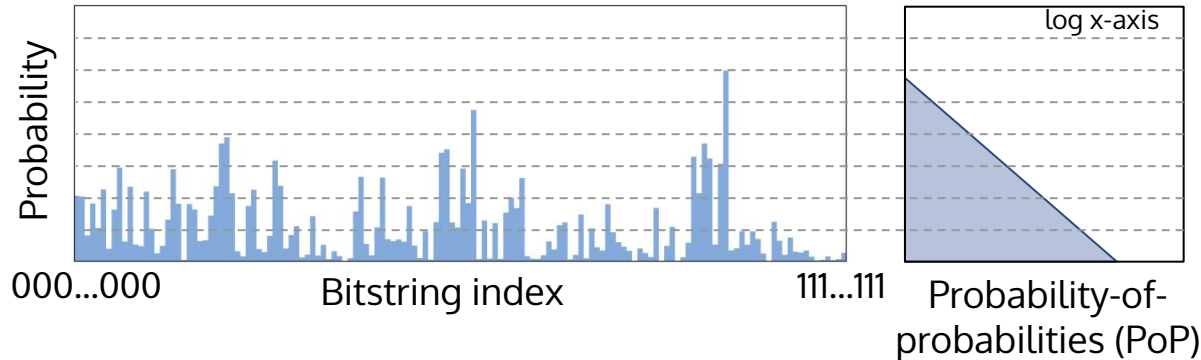


# Learning noise from bitstring measurements

Consider random  
unitary circuit  
(RUC) evolution



We will be studying the  
**probability-of-probabilities (PoP)**  
distribution... For RUCs, this is well known  
to be an **exponential distribution**\*

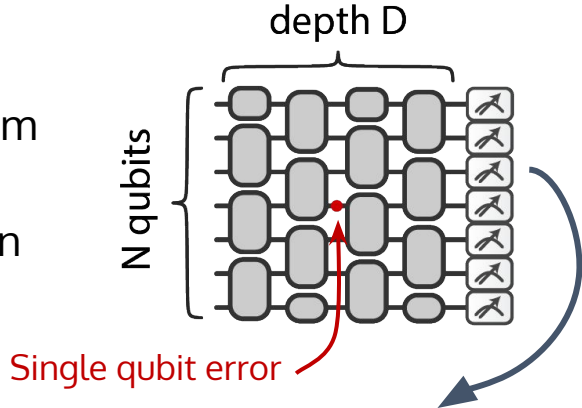


The PoP counts how  
many probabilities  
fall into each bin  
(gray lines)

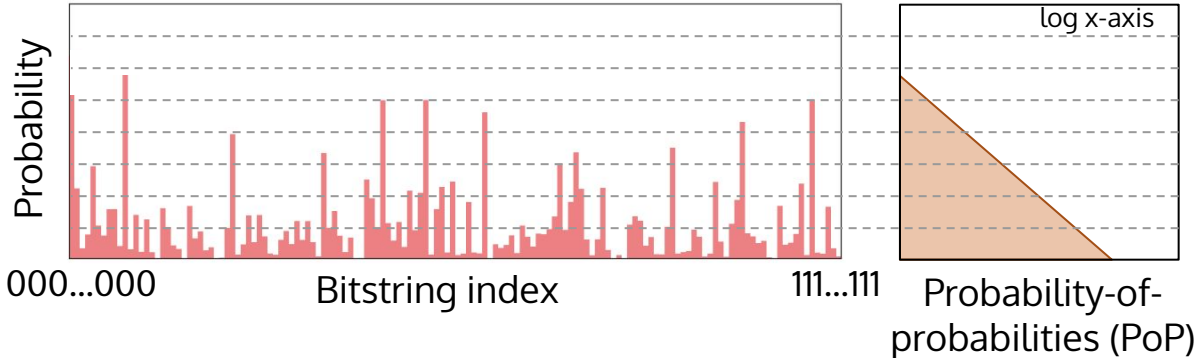
\*a Porter-Thomas distribution

# Learning noise from bitstring measurements

Consider random unitary circuit (RUC) evolution



If **one** error occurs, the PoP distribution will still be an **independent**\*\* exponential distribution\*!



The PoP counts how many probabilities fall into each bin (gray lines)

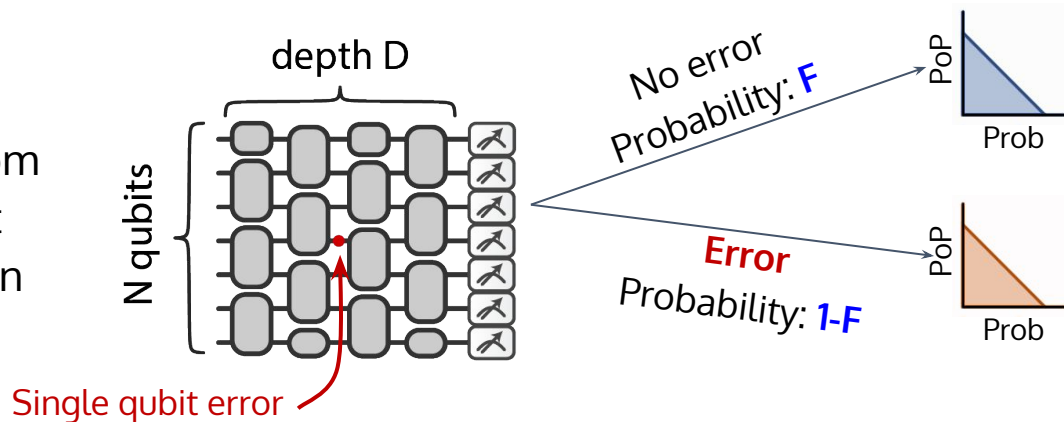
Shaw\*, Mark\*, et al, arXiv:2403.11971

\*a Porter-Thomas distribution  
\*\*Assuming dynamics is sufficiently scrambling



# Learning noise from bitstring measurements

Consider random  
unitary circuit  
(RUC) evolution

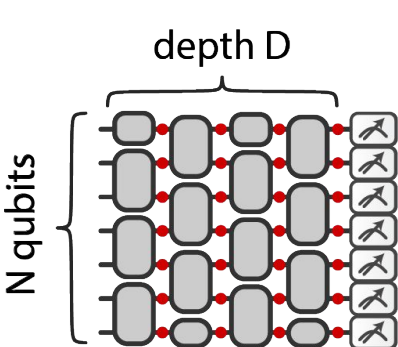


The **probability** of observing the different distributions is given by **the fidelity,  $F$**

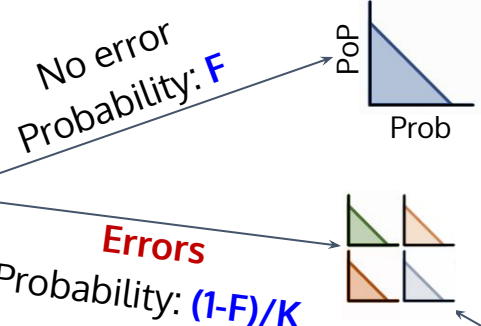
If **one** error occurs, the probability-of-probabilities (PoP) distribution will be an **independent**\*\* exponential distribution\*

# Learning noise from bitstring measurements

Consider random unitary circuit (RUC) evolution



$K = N \times D$  possible error locations  
(spacetime volume of circuit)



The probability of observing the different distributions is given by the fidelity,  $F$

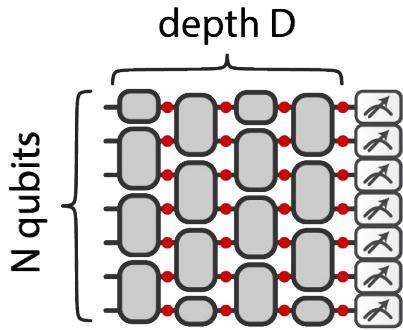
$K$  possibilities

If **many independent** errors occur, the probability-of-probabilities (PoP) distributions will be **many independent**<sup>\*\*</sup> exponential distributions<sup>\*</sup>

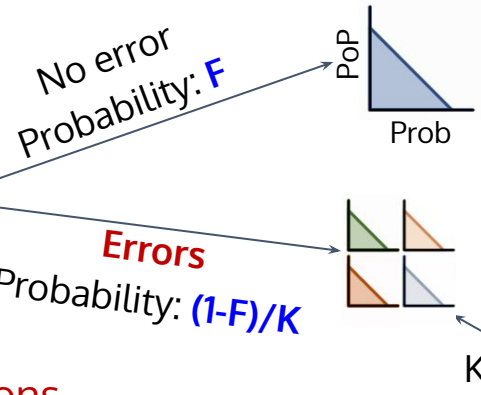
\*a Porter-Thomas distribution  
\*\*Assuming dynamics is sufficiently scrambling

# Learning noise from bitstring measurements

Consider random unitary circuit (RUC) evolution



$K = N \times D$  possible error locations (spacetime volume of circuit)



The probability of observing the different distributions is given by the fidelity,  $F$

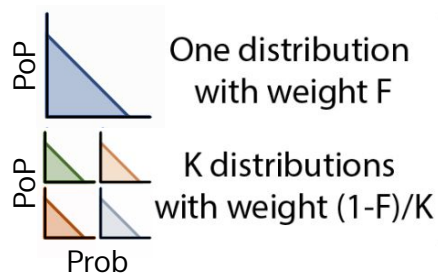
We can't keep track of all microscopic errors, so the aggregate PoP is an incoherent sum over all of them!

If many independent probability-of-prob distributions will be ma exponential distributions

\*a Porter-Thomas distribution  
\*\*Assuming dynamics is sufficiently scrambling

# Learning noise from bitstring measurements

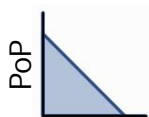
Incoherent sum over



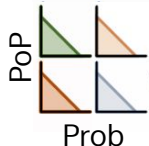
distributions will be **many independent**<sup>\*\*</sup>  
exponential distributions<sup>\*</sup>

# Learning noise from bitstring measurements

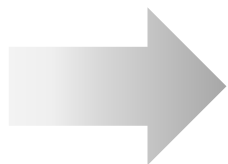
Incoherent sum over



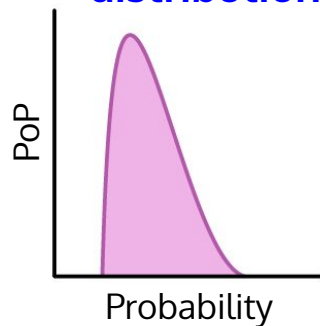
One distribution  
with weight  $F$



$K$  distributions  
with weight  $(1-F)/K$



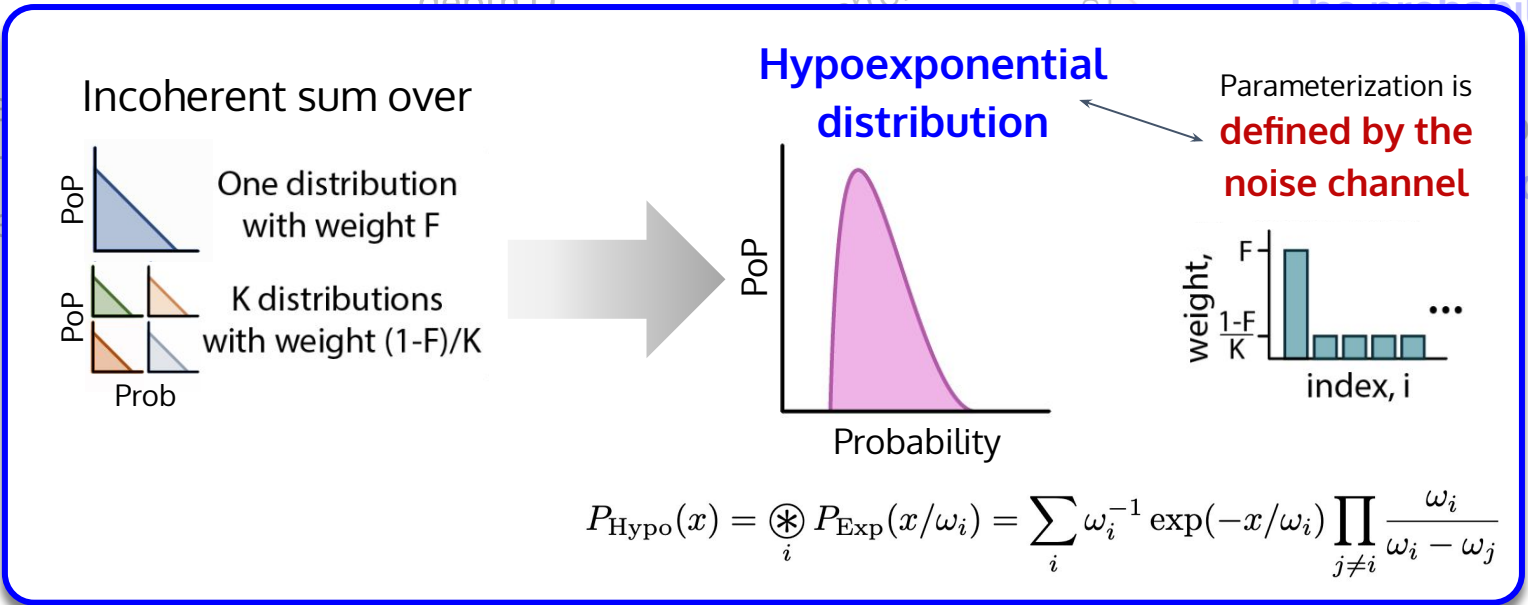
**Hypoexponential  
distribution**



$$P_{\text{Hypo}}(x) = \bigotimes_i P_{\text{Exp}}(x/\omega_i) = \sum_i \omega_i^{-1} \exp(-x/\omega_i) \prod_{j \neq i} \frac{\omega_i}{\omega_i - \omega_j}$$

distributions will be **many independent**<sup>\*\*</sup>  
exponential distributions<sup>\*</sup>

# Learning noise from bitstring measurements

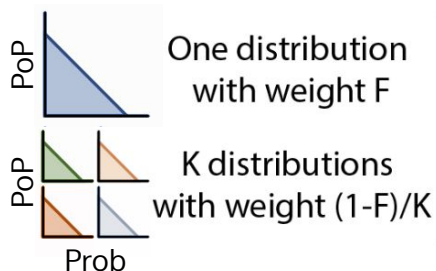


$$P_{\text{Hypo}}(x) = \bigotimes_i P_{\text{Exp}}(x/\omega_i) = \sum_i \omega_i^{-1} \exp(-x/\omega_i) \prod_{j \neq i} \frac{\omega_j}{\omega_i - \omega_j}$$

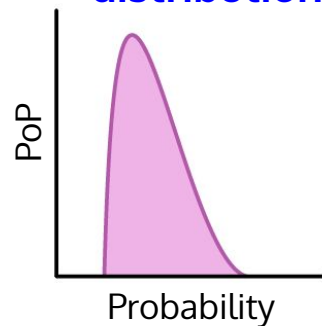
distributions will be **many independent**<sup>\*\*</sup>  
exponential distributions<sup>\*</sup>

# Learning noise from bitstring measurements

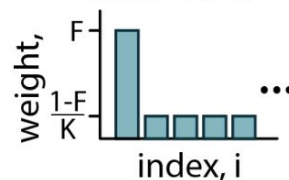
Incoherent sum over



Hypoexponential distribution



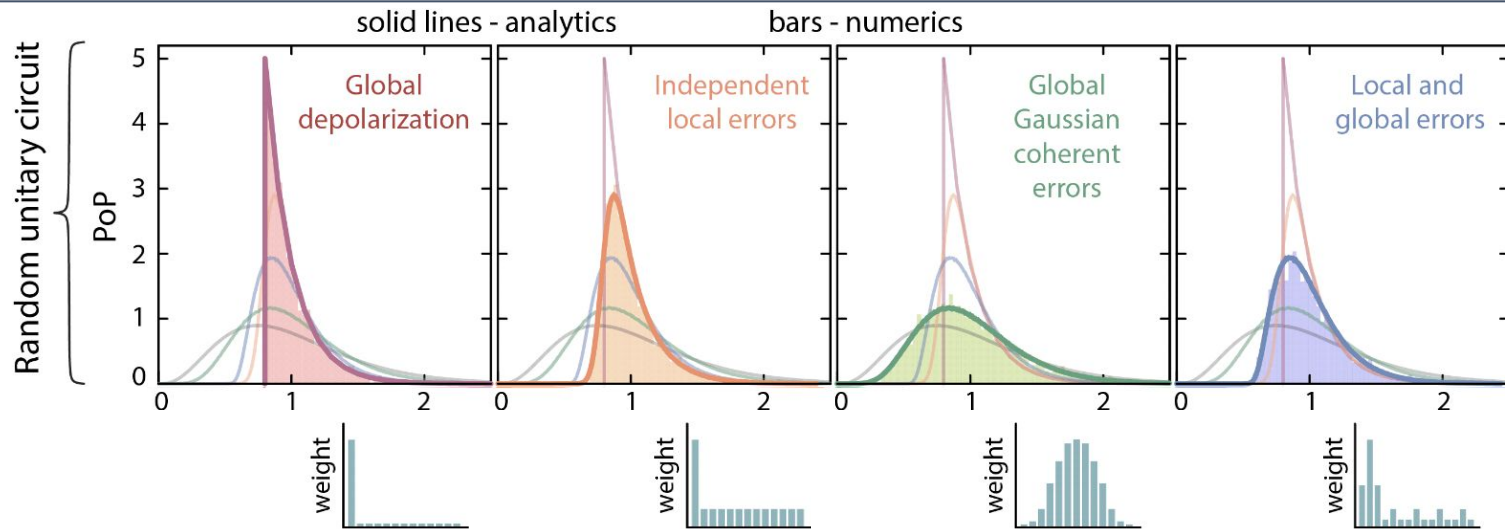
Parameterization is defined by the noise channel



**Technical details aside, the take-home message is:**

Given a noise channel (and a measured fidelity) we can always write the corresponding hypoexponential weights and **analytically predict the PoP distribution**

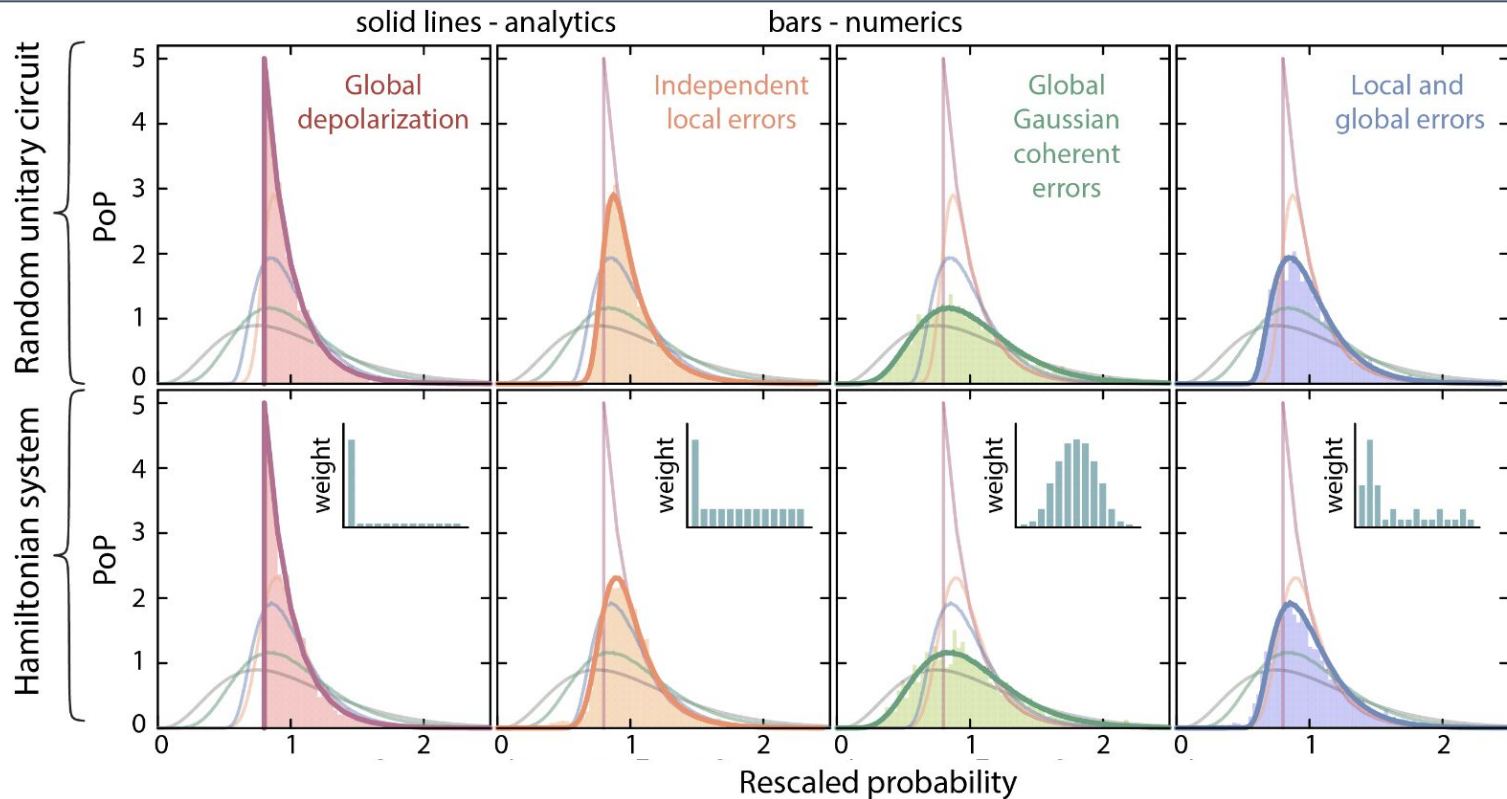
# Learning noise from bitstring measurements



**What to notice:** results from **numerical simulations (bars)** agree very well with corresponding analytical predictions (**same color lines**), while being clearly distinct from analytical predictions for other noise channels (faint lines)

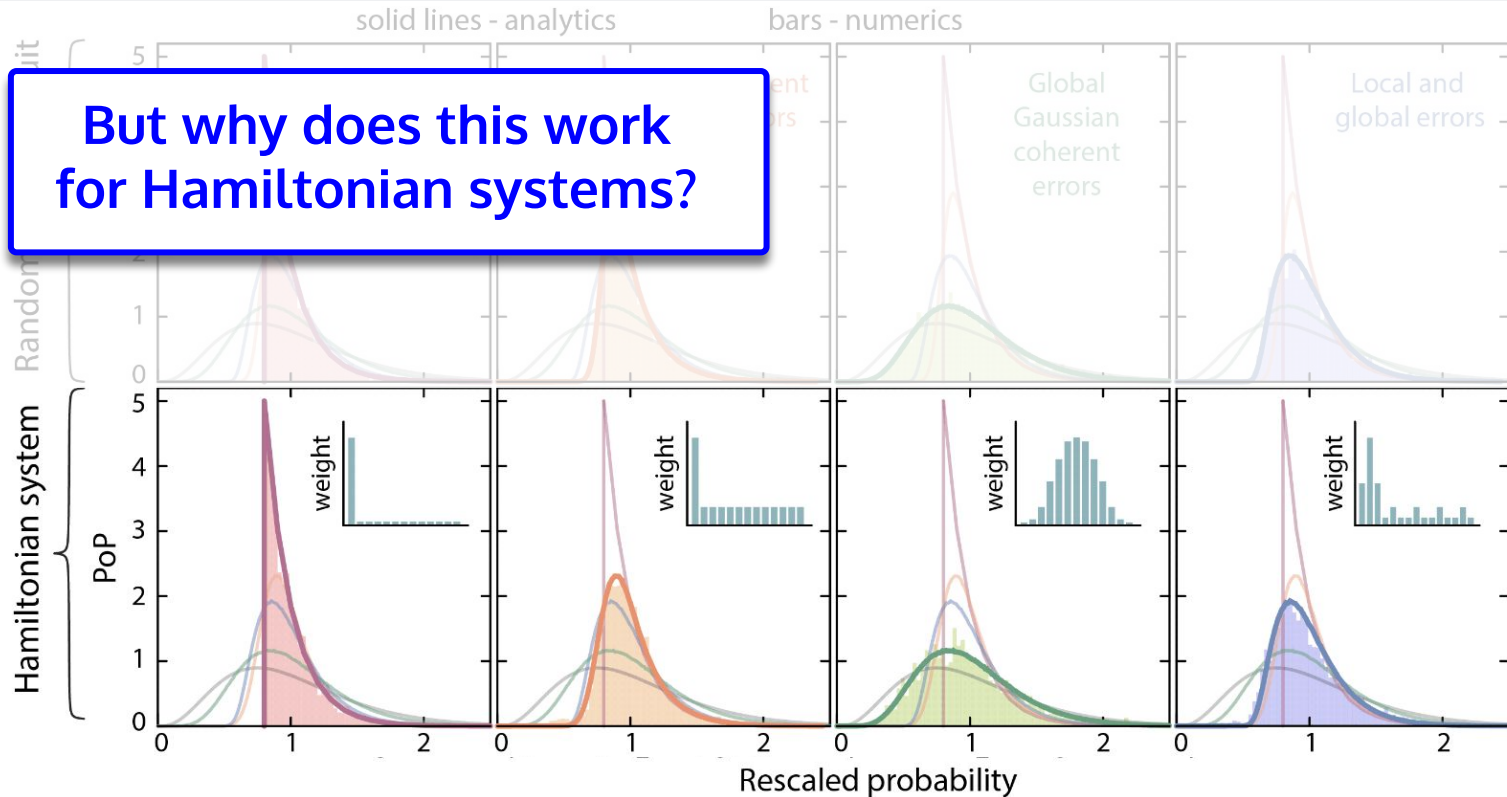


# Learning noise from bitstring measurements



**What to notice:** results from **numerical simulations (bars)** agree very well with corresponding analytical predictions (**same color lines**), while being clearly distinct from analytical predictions for other noise channels (faint lines)

# Learning noise from bitstring measurements



**What to notice:** results from **numerical simulations (bars)** agree very well with corresponding analytical predictions (**same color lines**), while being clearly distinct from analytical predictions for other noise channels (faint lines)

# Learning noise from bitstring measurements

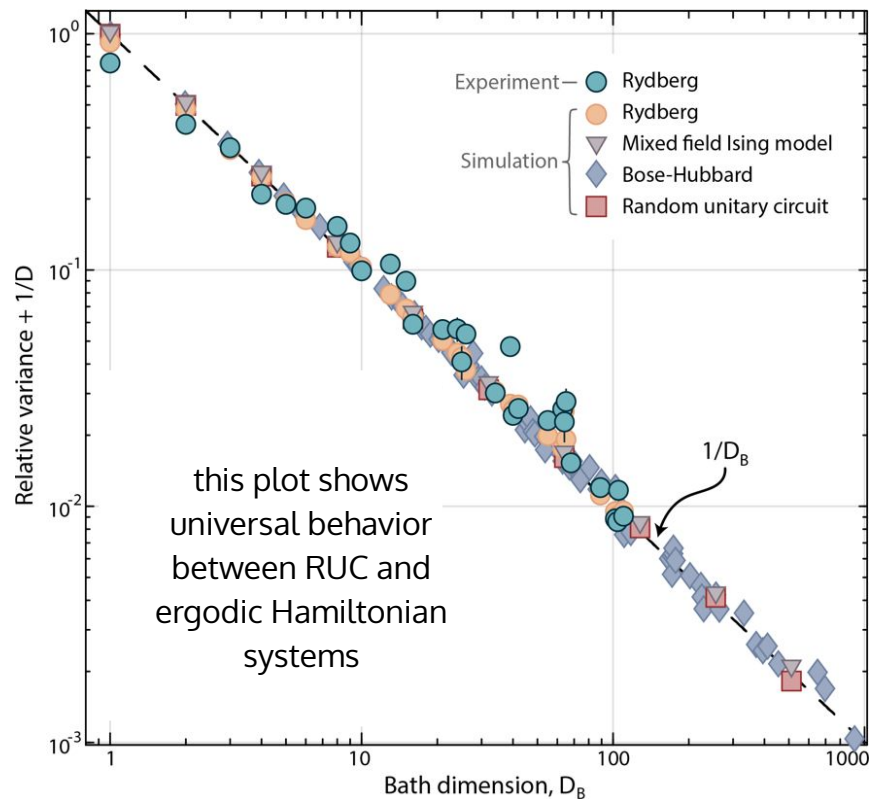
solid lines - analytics

bars - numerics

See Soonwon's talk just before mine about new theoretical discoveries (and experimental confirmations) that **ergodic Hamiltonian systems universally behave like random unitary circuits...**

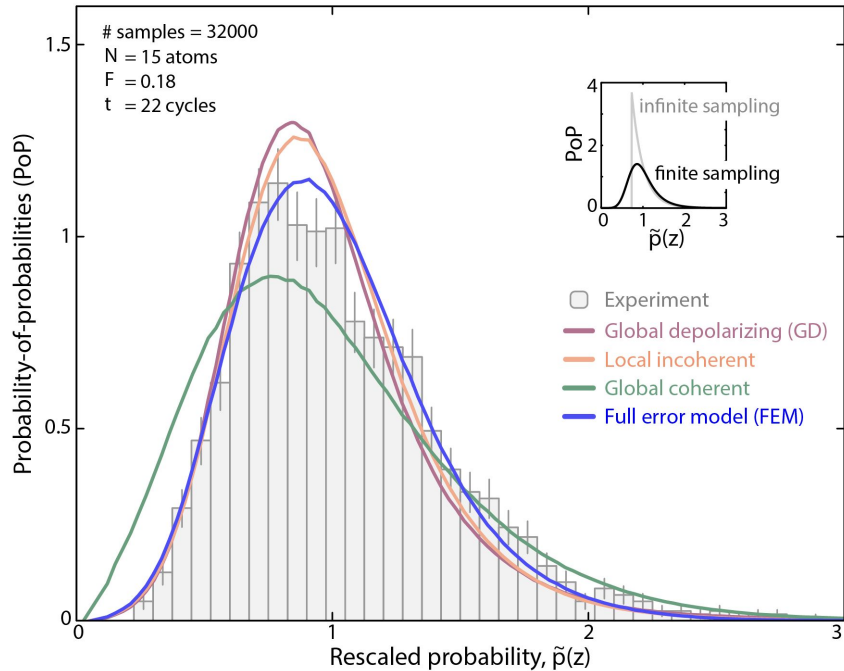


**Shaw\***, Mark\*, *et al*, arXiv:2403.11971  
Mark, Elben, Surace, **Shaw** et al, arXiv:2403.11970



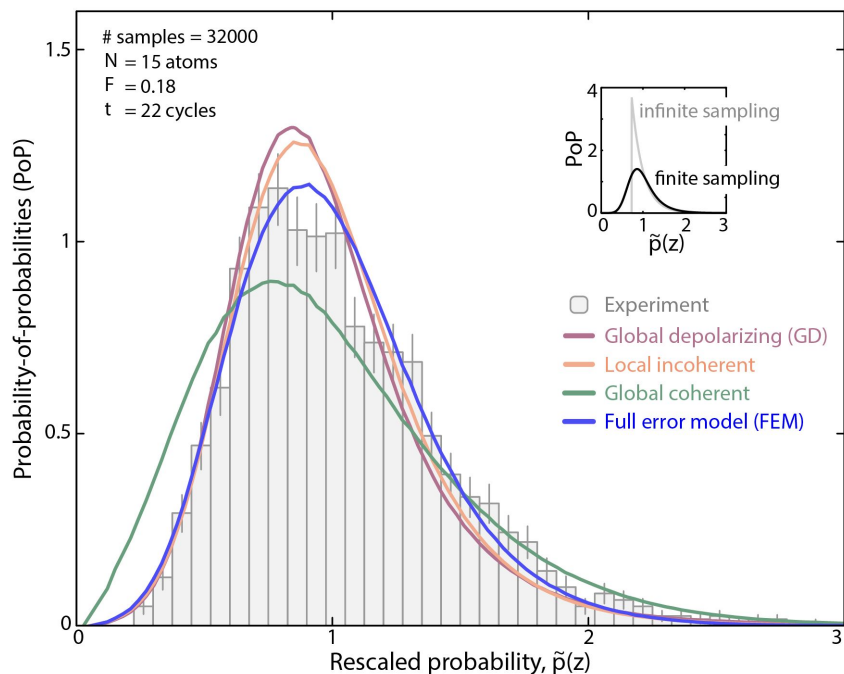
# Learning noise from bitstring measurements

Can apply to experiment\* using measured fidelity!

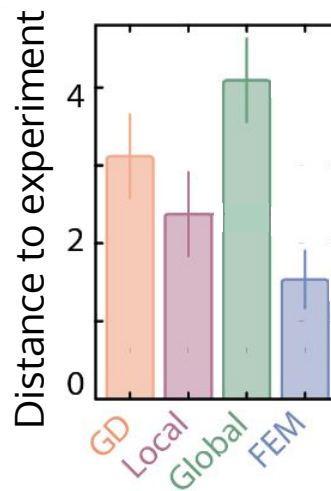


# Learning noise from bitstring measurements

Can apply to experiment\* using measured fidelity!

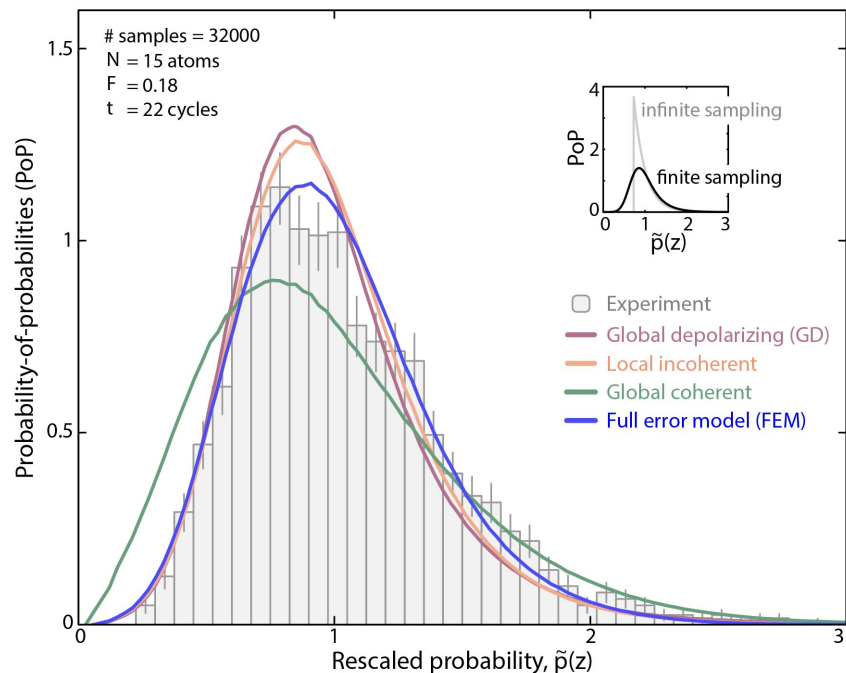


Can define a distance from experimental to predicted PoP



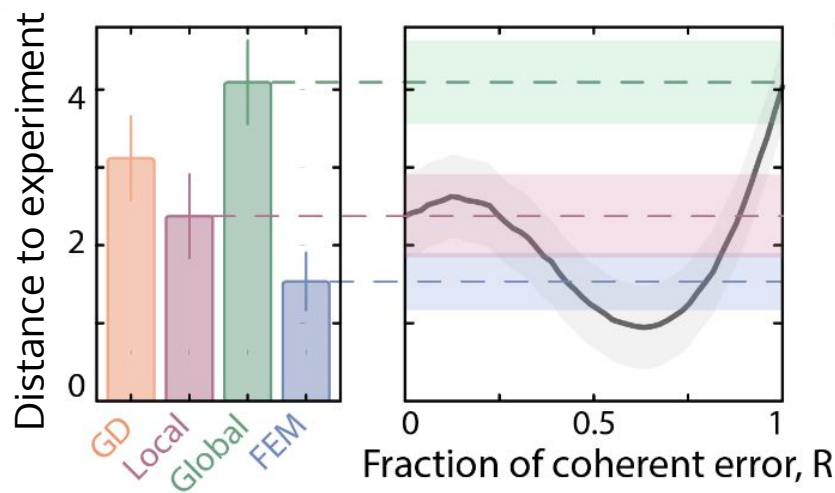
# Learning noise from bitstring measurements

Can apply to experiment\* using measured fidelity!



Can define a distance from experimental to predicted PoP

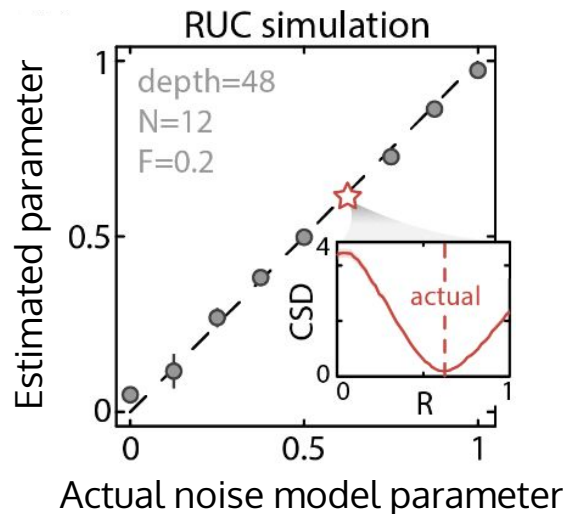
And learn experimentally consistent error models!



\*Because of finite-sampling costs, should actually compare low-order moments of the PoP, which are sample-efficient and still predictive

# Learning noise from bitstring measurements

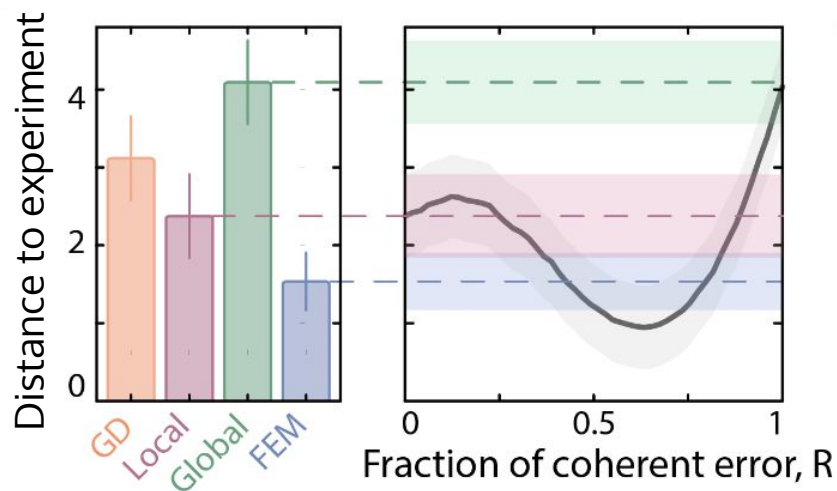
Can apply to experiment\* using measured fidelity!



RUC simulation shows learning noise models in this way is accurate!

Can define a distance from experimental to predicted PoP

And learn experimentally consistent error models!

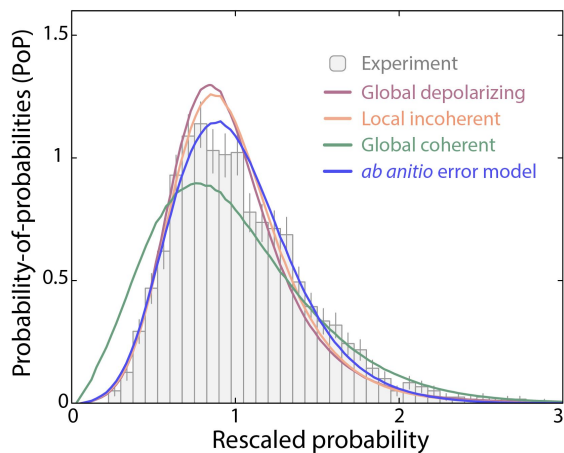
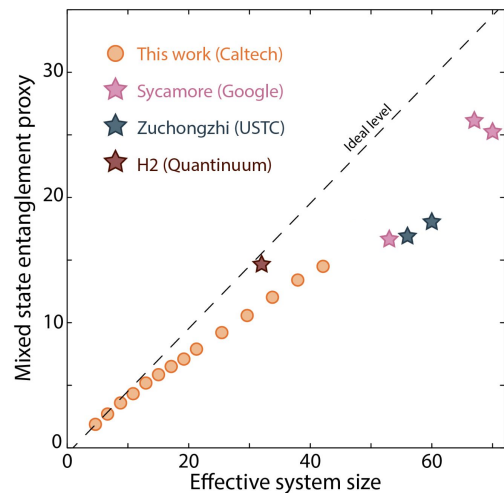


\*Because of finite-sampling costs, should actually compare low-order moments of the PoP, which are sample-efficient and still predictive

# Summary

Quantitative benchmarking enables both **improving quantum science**, and realizing **new science applications**

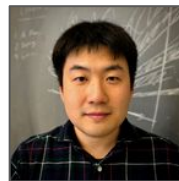
Shaw\*, Chen\*, Choi\*, Mark\*, *et al*, Nature 628 (2024), arXiv:2308.07914  
Shaw\*, Mark\*, *et al*, arXiv:2403.11971



## Thank you!



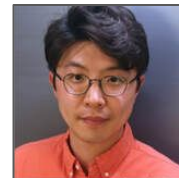
Manuel Endres



Soonwon Choi



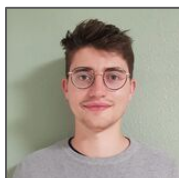
Daniel Mark



Joonhee Choi



Zhuo Chen



Pascal Scholl



Ran Finkelstein



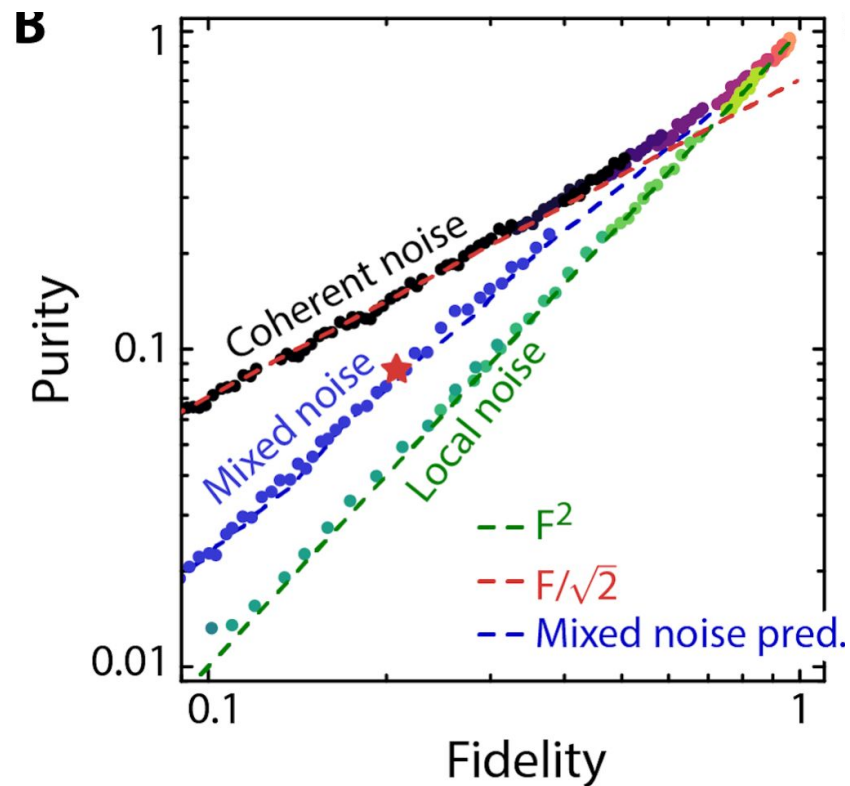
Andreas Elben



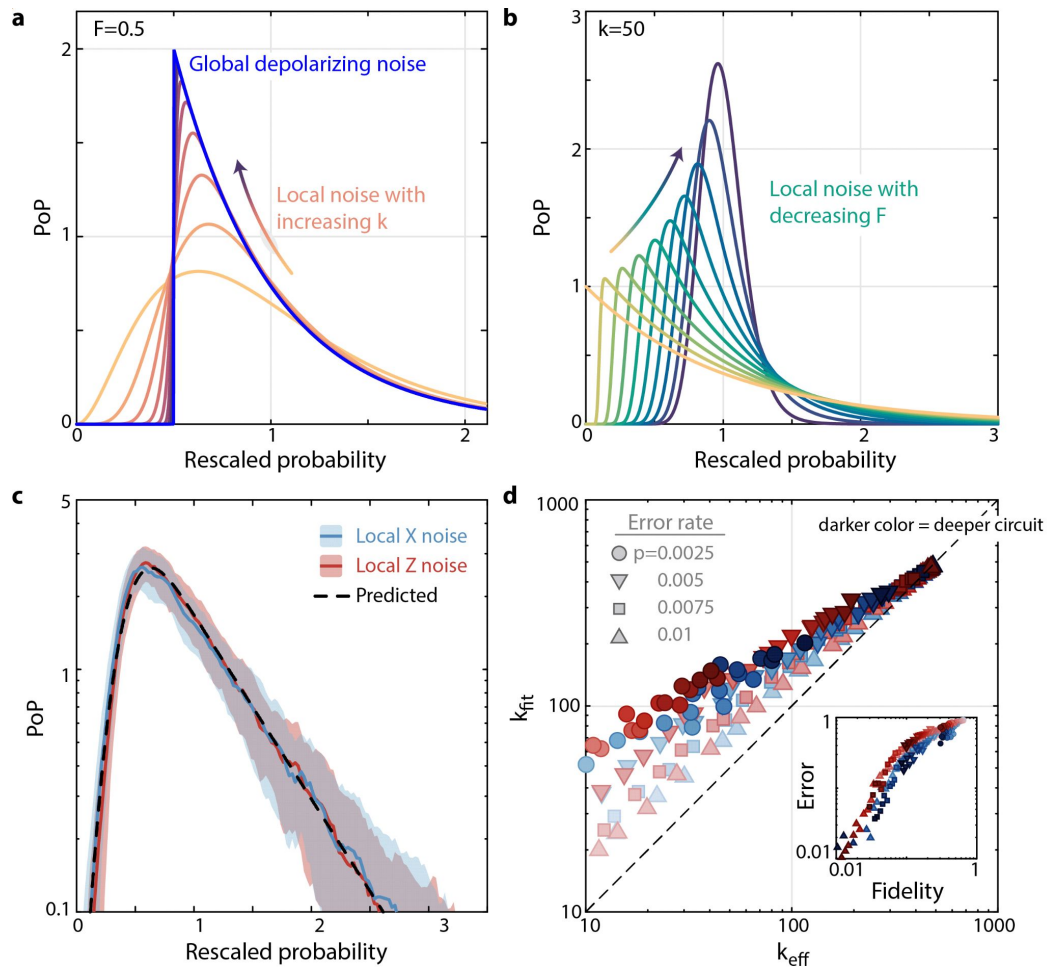
# Moments analysis

Can more efficiently learn noises  
just from their effects on the  
**moments** of the PoP

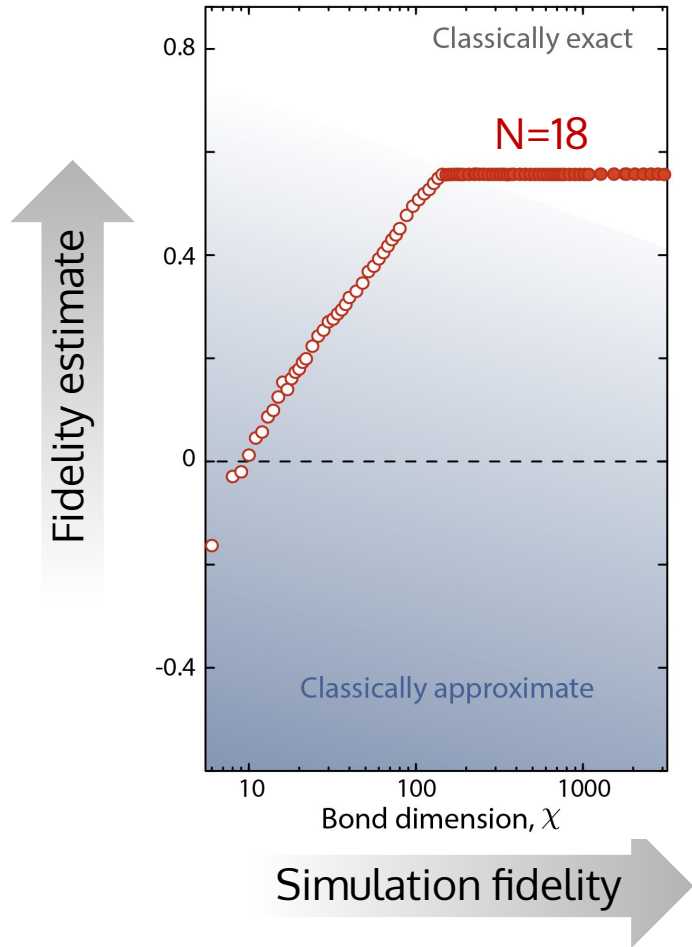
Noise channel	$\kappa_2$	$\kappa_3$
Global depolarization	$F^2$	$2F^3$
Local incoherent	$F^2 + (1 - F)^2/k$	$2(F^3 + (1 - F)^3/k^2)$
Global Gaussian coherent	$F/\sqrt{2}$	$2F^2/\sqrt{3}$



# Local noise -> Global depolarizing

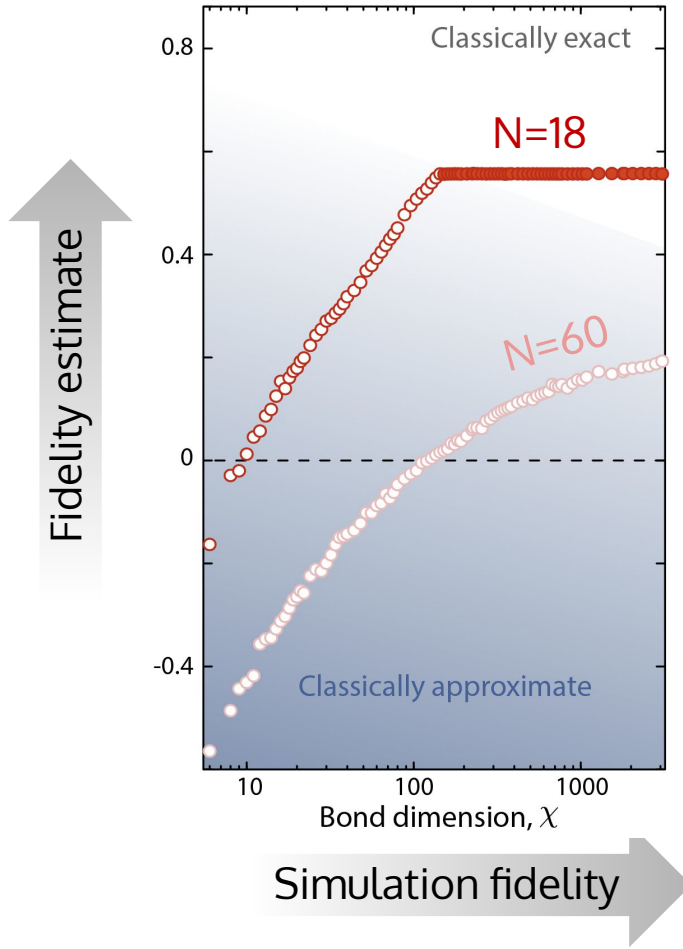


# Identifying scaling behavior



As the bond dimension is increased, fidelity estimate rises before saturating...

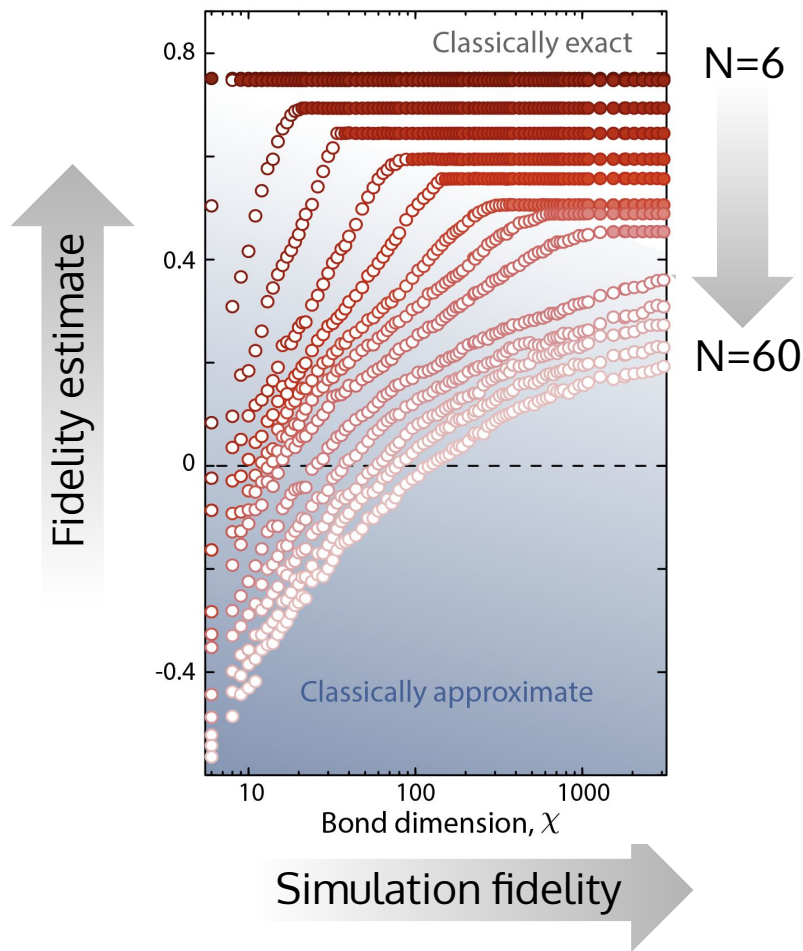
# Identifying scaling behavior



As the bond dimension is increased, fidelity estimate rises before saturating...

For large systems, we can't reach the "saturation bond dimension"

# Identifying scaling behavior



As the bond dimension is increased, fidelity estimate rises before saturating...

For large systems, we can't reach the "saturation bond dimension"

But we can extrapolate as a function of the bond dimension!