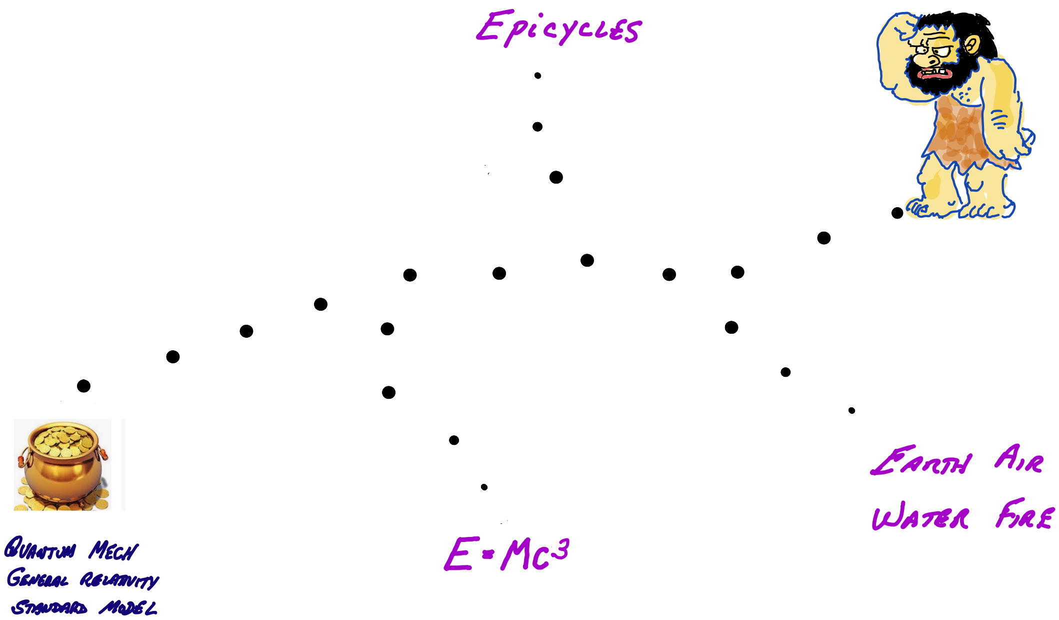
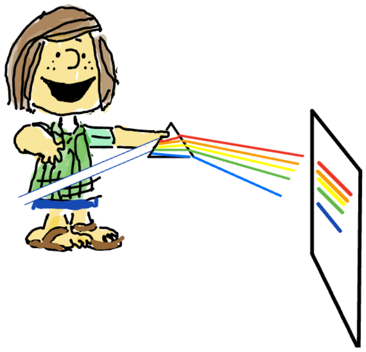
Welcome

THE BIGGEST PUZZLE OF PHYSICS IS THAT IT EXISTS AT ALL.



The Tools

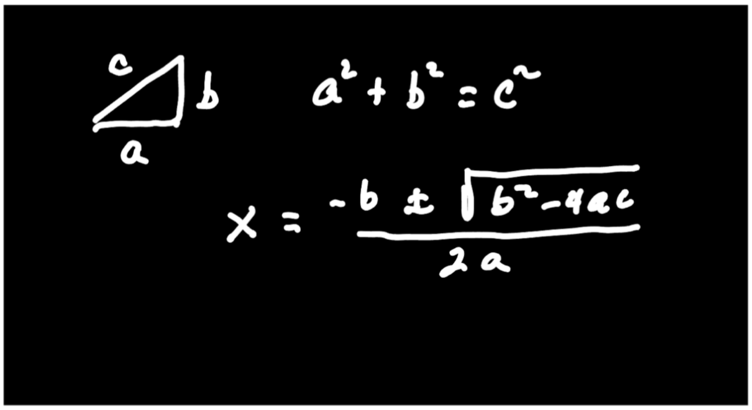
EXPERIMENT !



THOUGHT EXPERIMENTS

APPARENT CONFLICTS OF PRINCIPLES— PARADOXES

MATHEMATICS



Calculus, Geometry, Group Theory, Topology, Complex Functions, Differential equations, ...

Entry into physics of a different kind of mathematics — mathematics that grew up together with computer science and the logic of computation

Turing
Church
Shannon
Godel
Von Neumann
Kolmogorov

Feynman
⋮

(Quantum) Computational complexity:

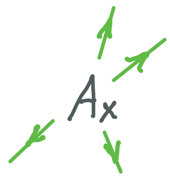
Why has it taken a prominent place in quantum theory of gravity and and black holes

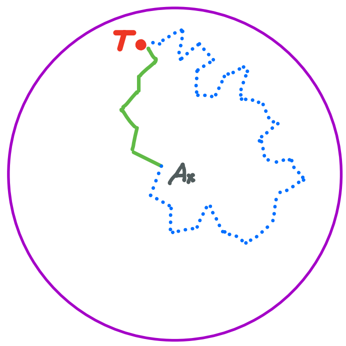
What is Computational complexity ?

How difficult is a theorem?

subjective
objective ?

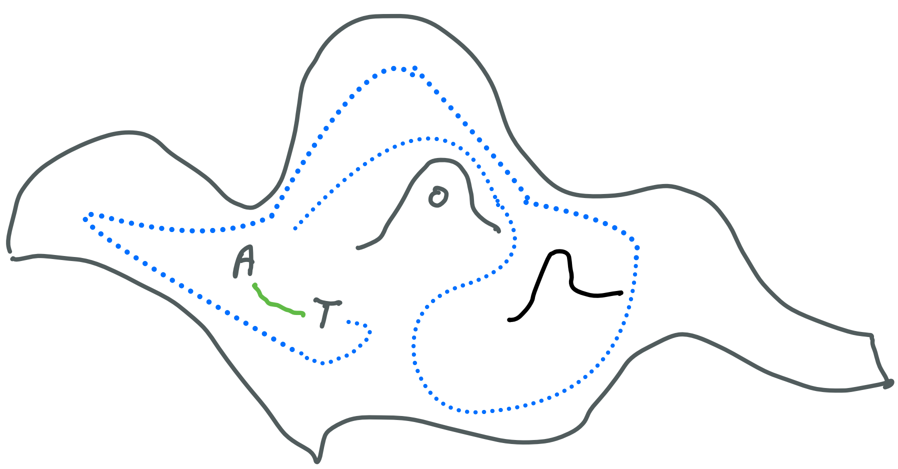
input — axioms and logical rules of inference.





Complexity = minimum number of steps $A_x \longrightarrow T$

Similar to what is the length of the absolute shortest path from A to T



That's the general Idea:

A simple starting point A.

A set of simple steps – gates.

A target T

The complexity of T is the absolute minimum number of steps to go from A to T.

Classical state complexity of N bits.

Simple state

↑↑↑↑↑↑↑↑↑↑↑↑↑↑↑↑ = A

Target

$\uparrow\downarrow\uparrow\uparrow\downarrow\downarrow\downarrow\uparrow\uparrow\downarrow\uparrow\downarrow\downarrow\uparrow\uparrow\uparrow \quad = T$

Simple (but universal) gates

flip $\uparrow \longleftrightarrow \downarrow$

Maximum Classical state complexity is N

Feynman: Quantum states can be vastly more complex than classical states.

Number of parameters for classical state = N binary digits.

Quantum state

$$|\psi\rangle = \sum_{\text{classical states}} \psi_i \, |\text{classical}\rangle_i$$

$\psi_i = 2^N$ complex amplitudes

Feynman: That's why quantum Mechanics is (computationally) hard.

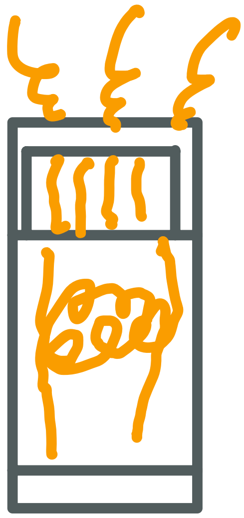
That's why we need quantum computers.

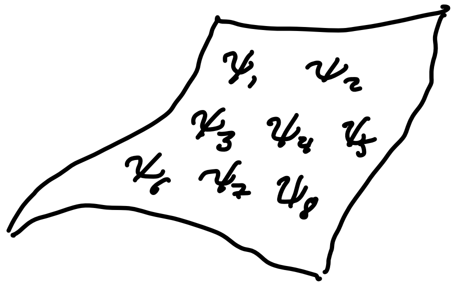
Complexity of a quantum state is the minimum number of gates needed to prepare $|T\rangle$ from some simple state $|A\rangle$.

$$\text{Maximum q-complexity} = e^N >>> N$$

Almost all states have maximum complexity.

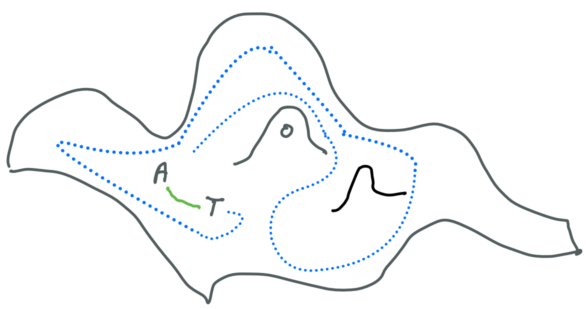
Suppose you are given a quantum state and you want to know how complex it is.

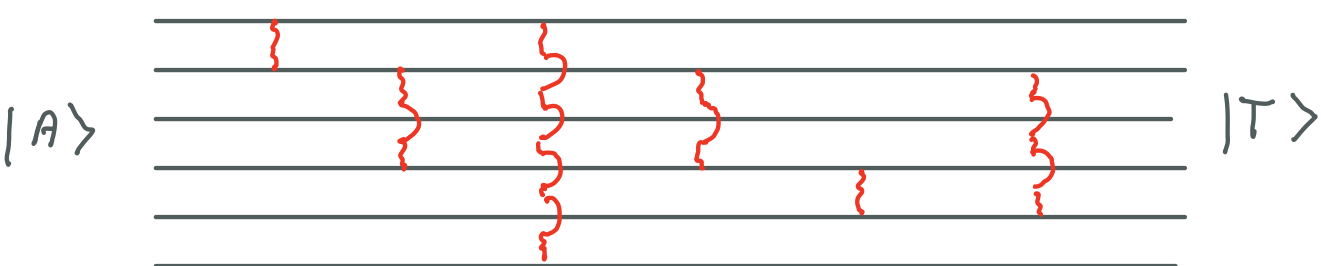




How do you compute complexity?

Try all possible circuits

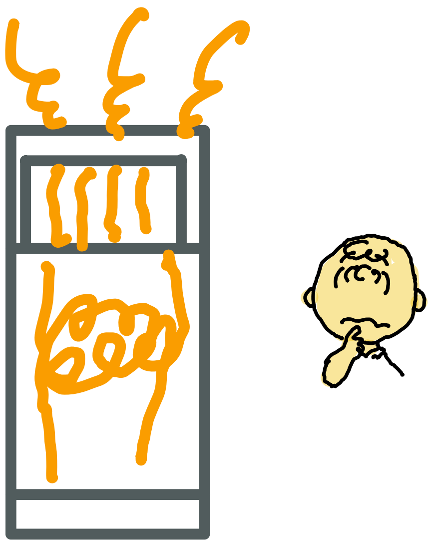




No shortcuts

How do you measure it?

Same way.



(A) polynomial N^{α} ?

(B) exponential e^{N} ?

Complexity of complexity CofC

How computationally complex is it to distinguish (A) from (B)? Surely not too hard to tell $C = N^{\alpha}$ from $C = e^{N}$.

$$\text{C of C} \sim e^{N}$$

Lesson:

Quantum Complexity is

subtle, illusive, Ghostly.
ethereal, tenuous,
fine-spun, hyper-refined.

Bouland Fefferman
Vazirani

unfeelable

so unfeelable that it can hardly be a interesting property of a physical system like energy, entropy, density.

Computer Science has been content to classify tasks into broad "complexity classes" e.g. P, NP, exp, BQP/poly

$$\frac{dC}{dt}$$

And yet...

Black Holes

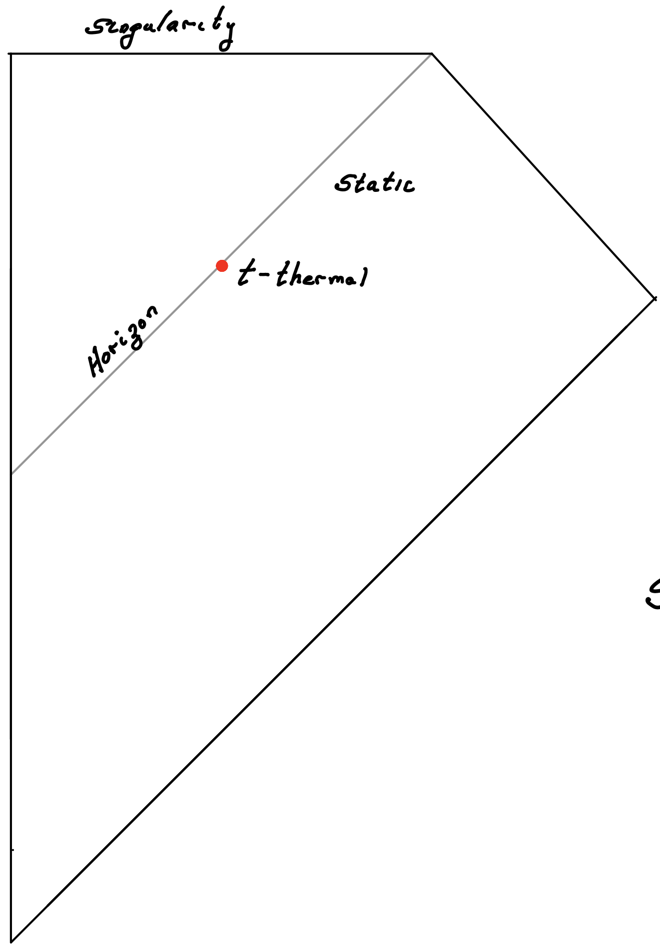
Conjecture 2014

The <u>volume</u> of the interior of a black hole is the complexity of its quantum state.

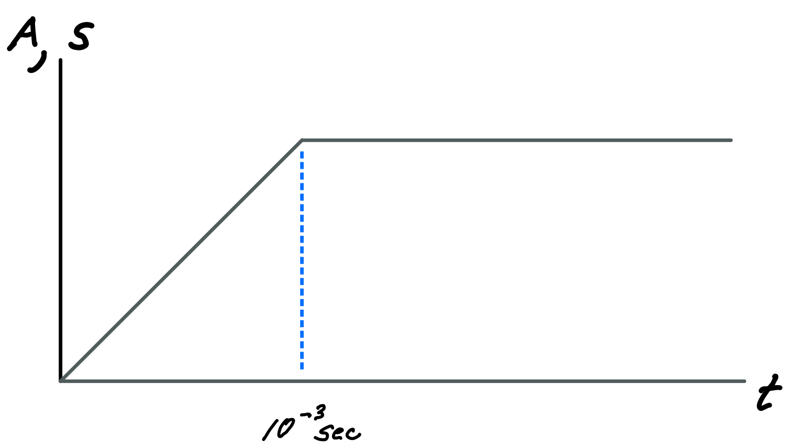
It grows with time as $V(t)$.

Ya Ya $V=C$

My response: Yes, complexity is inaccessible, but so is the interior of a black hole. I suspect the two inaccessibilities are dual to one another.



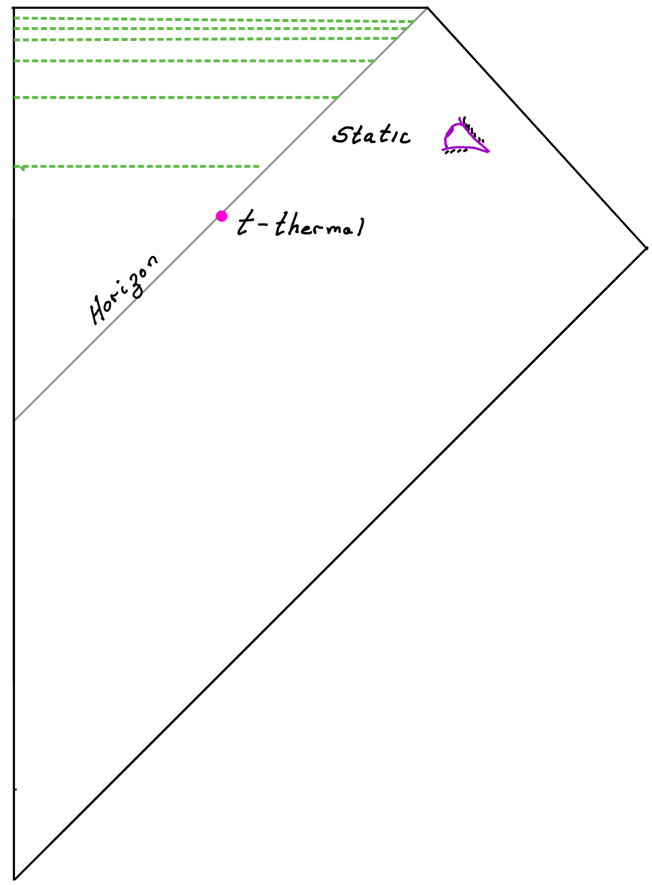
$$S = \frac{A}{4G\hbar}$$

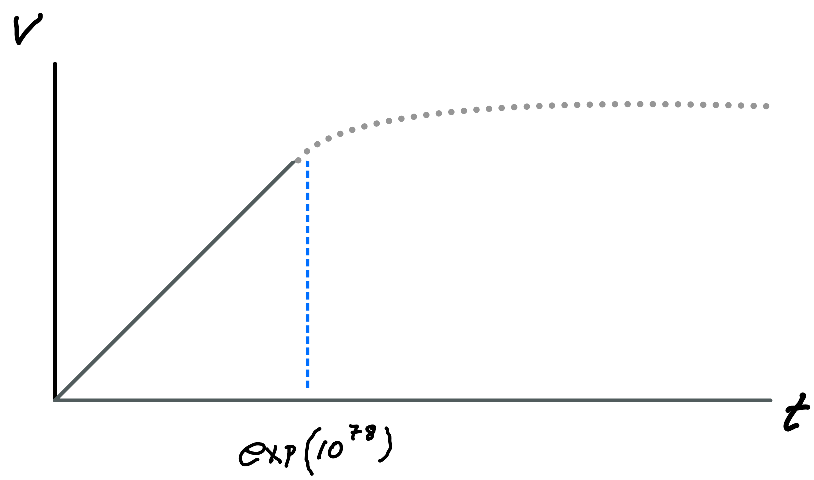


U

GR Thing: A ⧖ Quantum thing S

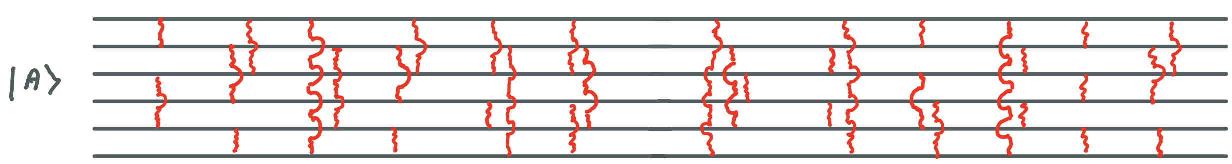
But something behind the horizon Continues to grow. The volume.

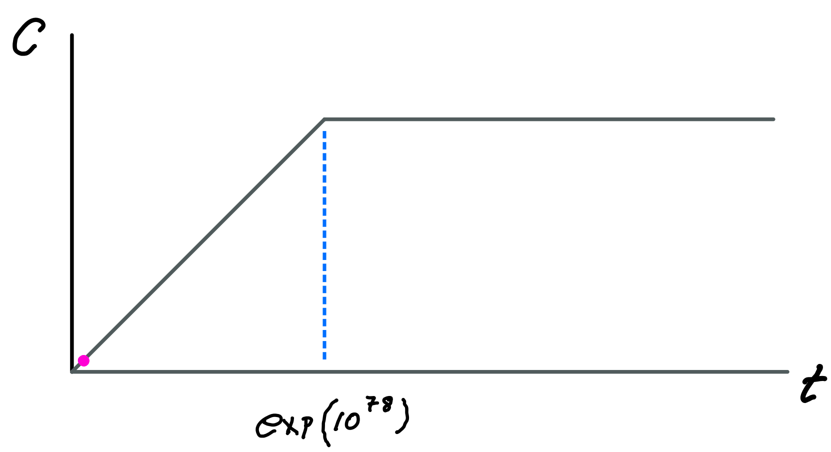




What is the quantum thing that grows linearly for an exponential time but which is invisible from the outside?

Complexity Growth





$C_{max} \approx e^S$

> The volume of the black hole interior
>
> $\approx$ Complexity of quantum state of BH.

What about unfeelability?

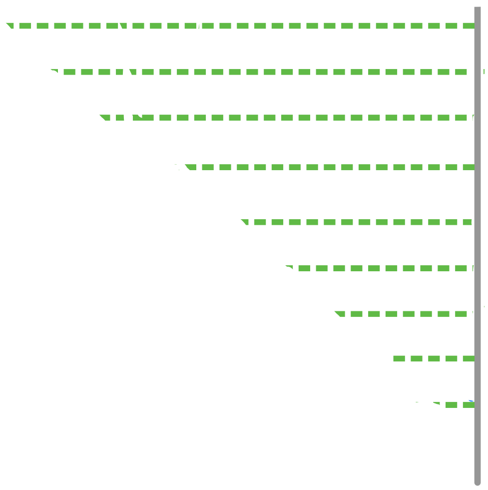
The Dictionary (Bouland, Fefferman, Vazirani)

Quantum State $\Longleftrightarrow$ Exterior: Low Complexity

Quantum State $\Longleftrightarrow$ interior: exp complexity

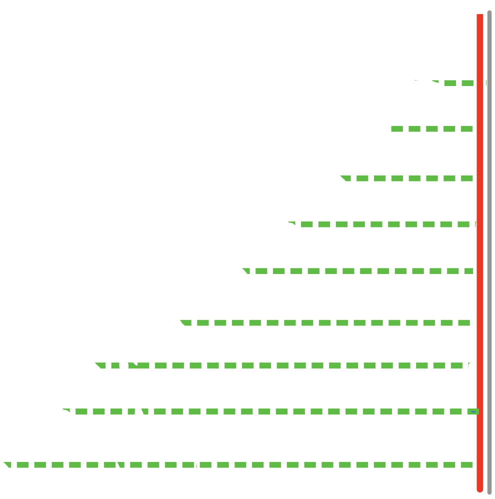
Firewalls

Almheri, Marolf,
Polchinski, Sully



complexity/volume increasing

Horizon normal



complexity/volume decreasing

Horizon firewall

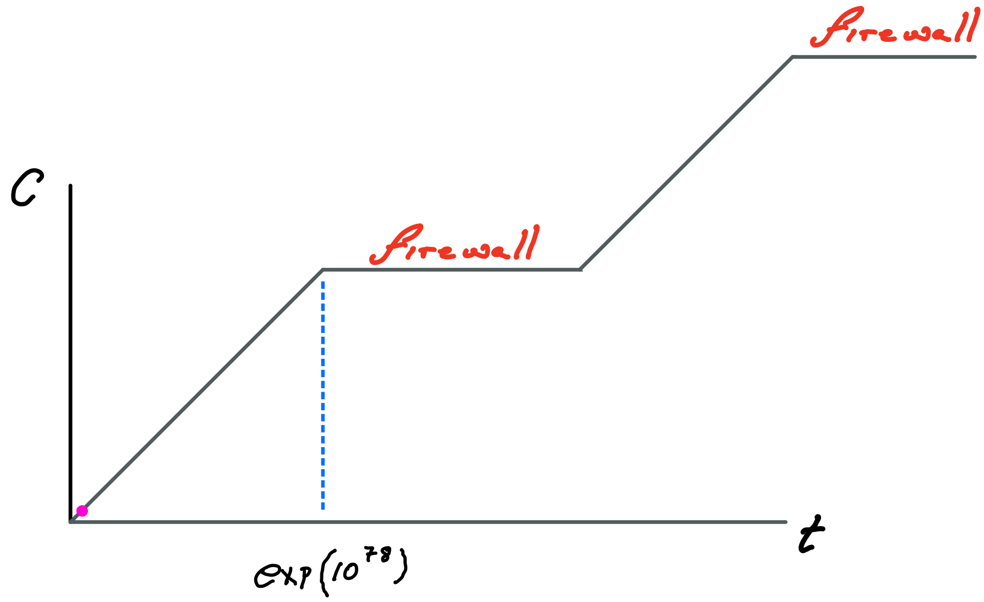
Suppose that at time $t > e^S$ there is a firewall that Alice wants to get rid of before she pushes Bob into the black hole. All she has to do is drop a single very low energy photon in and wait a scrambling time.

Complexity Analysis

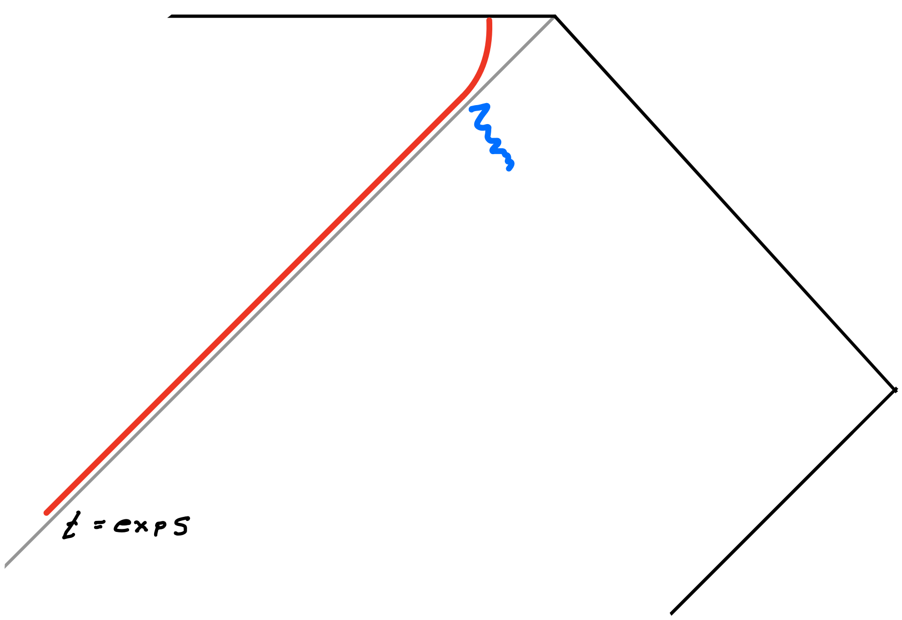
$$S \rightarrow S + 1 \text{ bit} = S + \log 2$$

$$e^S \longrightarrow 2e^S$$

After a scrambling time t_* the Maximum complexity doubles. There is lots of room for complexity to increase for another $t = \exp S$



Gravity Analysis



Precise match!

The QECTT

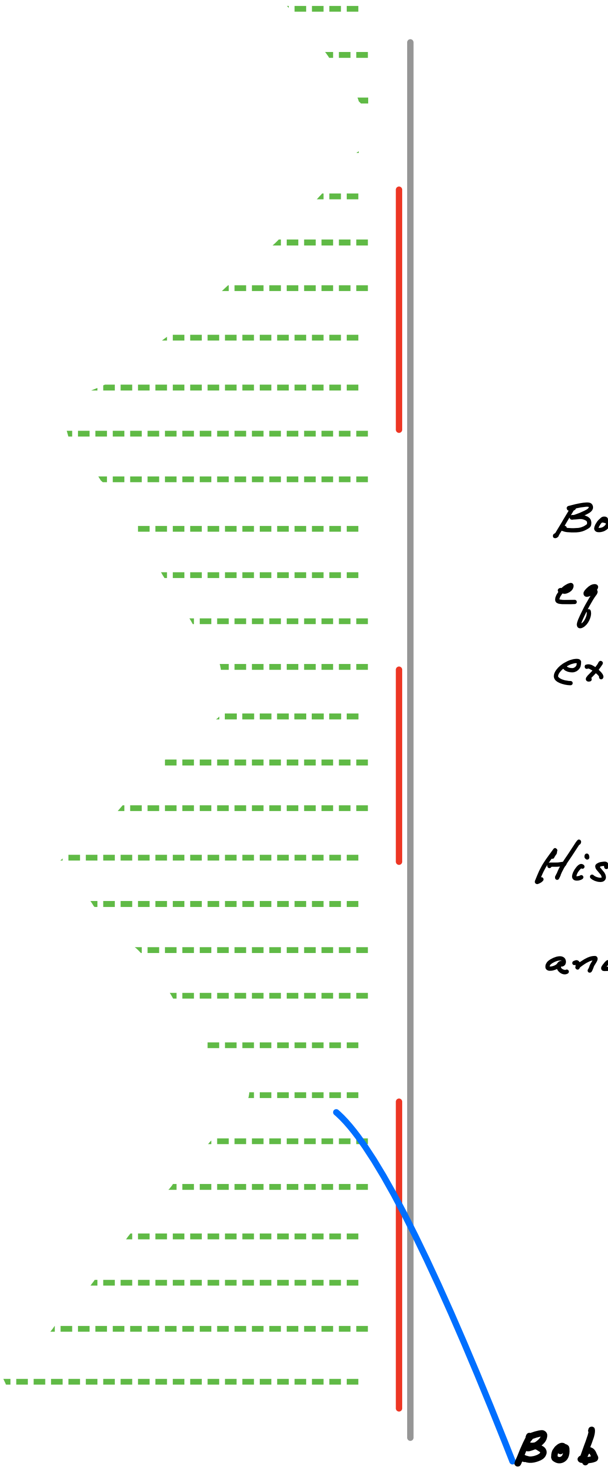
CTT No physical system can do a calculation that's impossible for a Turing Machine.

ECTT (False)

No physical system can beat a Turing Machine (parametrically)

QECTT

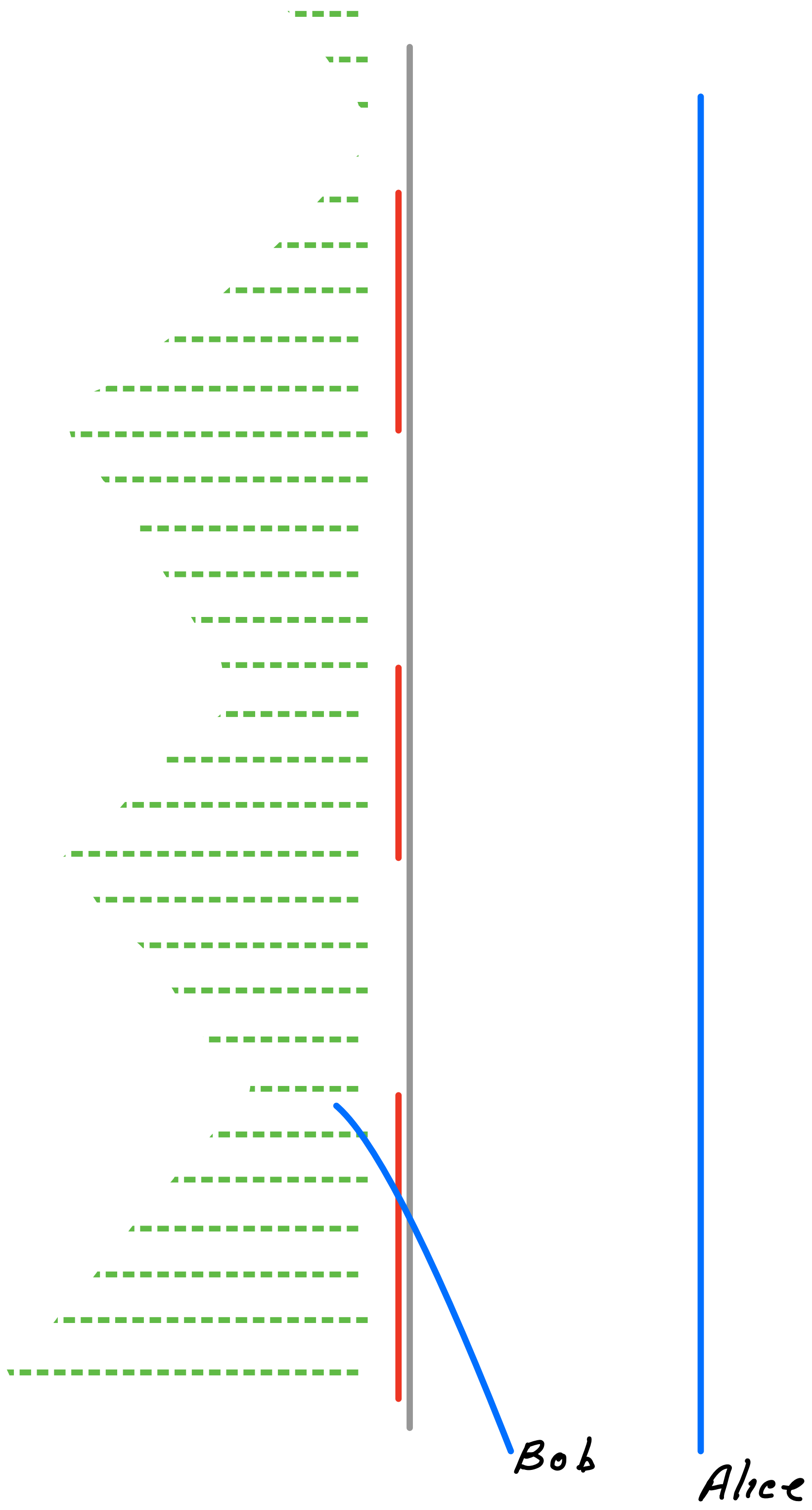
No physical system can beat a Quantum computer (parametrically)



Bob wants to solve the B.H. schroedinger eq and determine if $\dot{C}(t) > 0$. But that's exponentially hard. even for a Q.C.

His curiosity gets the better of him and he jumps in.

Bob learns the answer immediately! thus violating QECTT.


Bob
Alice

Prediction: Extracting Bob from behind the horizon* is as complex* as calculating $\frac{dC}{dt}$.

What happens behind the horizon stays behind the horizon (at least for exp-time).

* Let the black hole evaporate and study the evaporation products

* Harlow Hayden: $C \sim e^S$

The horizon is the "censor" that prevents the violation of QECTT.


?
QUANTUM MECHANICS
GRAVITY
THEN
QM = GR
NOW